Market Design, The Kidney Exchange Problem and the Role of Software in Analytical Business Modelling

April 2017

Shane McCarthy (14200512) & Sean Tully (15205062)

Table of Contents

1 Introduction

In 2012, Alvin Roth and Lloyd Shapley were jointly awarded the Nobel Prize in Economics for their work on market design and matching theory. Unlike traditional economics, which has a naïve view of markets as the intersection of supply and demand, this new field recognises that well-functioning markets are complex and depend on a detailed set of rules. For example, the housing market and job market are both driven by supply and demand; both a house buyer and a job seeker go through a very different process compared to someone trying to buy a house. The fields of market design and matching theory evolved out of both game theory and the advancements in experimental economics enabled by maturity of computing and software.

In (Roth, 2008) the requirements for markets to work well are outlined:

1. Marketplaces must provide sufficient thickness – essentially, they need to attract enough buyers and sellers.
2. They need to prevail over the congestion that thickness can bring to allow individuals to evaluate enough alternative options before landing on the good ones.
3. The marketplace must be a safe environment where individuals are free to share or act on confidential information they may hold. The market must incentivise the sharing of information by participants.

Although abstract, these are the hidden economic rules that govern several diverse areas including online dating sites like Tinder, job marketplaces like LinkedIn, electricity markets, university application processes and the topic of this paper – Kidney Exchanges.

2 Background

2.1 Kidney Exchange

Globally, more than 200,000 people are on waiting lists for kidneys (Garwood, 2016); many more, particularly in third world countries, don't have access to dialysis or transplantation facilities. When the kidneys stop working, whether from diabetes, high blood pressure or genetic disorders, the body begins to accumulate toxic waste products. This can result in end-stage renal disease. Although dialysis can be used to treat end-stage renal disease, transplantation is the only cure. Kidneys are one of the few organs where donation from both deceased and living donors is possible however the majority of kidneys come from deceased donors. There is a large shortage of supply compared to demand and many patients die waiting for a kidney or become too sick for a transplant every year.

Often, a patient will have a loved one who is willing to donate a kidney to save the patient's life however their loved one is not compatible with patient due to blood type or a high percentage of reactive antibodies. Figure 1(a) below outlines the compatibility between blood types. Note that type "O" donors are compatible with all other blood types however type "O" patients can only receive a kidney from type "O" donors – therefore type "O" patients are at a disadvantage in finding compatible kidneys. The patient's percentage of reactive antibodies or sensitivity also needs to be considered, typically patients who have been ill for a long period of time or those where their body had previously rejected a kidney tend to have a higher sensitivity. An unfortunate correlation between the time spend on a kidney exchange list and the probability of a successful transplant exists, some patients may be prioritised over others due to their overall health condition. A common two-pair kidney exchange is shown in Figure 1(b) where recipient 1 can't receive a kidney from her husband due to sensitivity, however another recipient-donor pair is found who can't exchange directly due to blood type compatibility - a cross-exchange is performed resulting in both patients getting a kidney. Scaling this simple example up to thousands of pairs it can be expressed

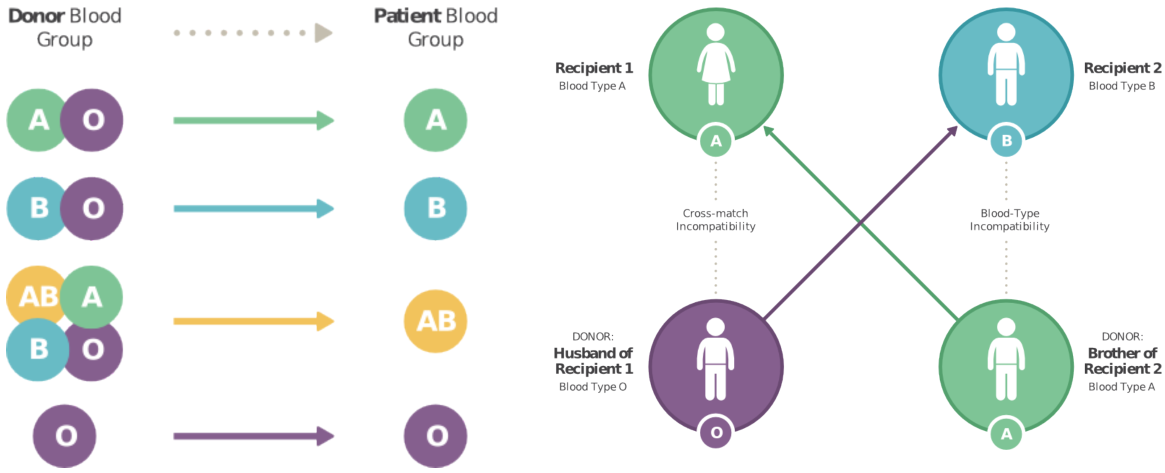

Figure 1 (a) Blood type compatibility (b) Simple two-pair kidney exchange[1]

as a graph $G(V, E)$, where each node is a recipient-donor pair and their match compatibility is expressed as a weighted directed edge. As tempting as it is to jump into theoretical solution mode we first most consider the practicalities of how a large kidney-exchange "market" would work.

2.2 Kidney Exchange in the Market Design Context

The emergence of Market Design theory allows us to apply insights and tools from economic theory to solve difficult real-life allocation problems like Kidney Exchange. In the seminal paper (Roth, et al., 2003) the authors drew parallels between the two main types of kidney exchanges (conducted in the US) and the most basic forms of exchanges in the classic housing allocation problem. The housing allocation problem is used to illustrate the trading of indivisible items, where *n* individuals living in *n* houses each assign a strict preference over a house – the goal is to realise all mutually-beneficial exchanges. The authors demonstrated that the Kidney Exchange problem could be solved using the same Top Trading Cycles (TTC) algorithm used to solve the housing allocation problem. Over the following years, the authors continued to build on theory analysing how an efficient and incentivised system could work and what its health outcome implications might be. Much of this theory forms the backbone of several Kidney Exchange programmes (KEPs) today however anumber of key challenges still prevent these programmes from working well.

Market thickness (number of pairs in a programme database) and the incentive for local hospitals to push their pairs to a national database are two significant challenges. As the market thickens the probability of find a match increases and the efficiency of matches also improves which particularly benefits hard to match (highly sensitised) patients. For example, as the thickness increases the cycle lengths also have the potential to increase, short cycles tend to follow a path of easy to match (low sensitised) pairs of all blood types as these are more frequent, as the cycle length increases the probability of included a highly sensitised pair increase. Unfortunately today (again taking the US as an example), a number of Kidney Exchange silos exists at a national, state and hospital level – this means instead of having one large national database of all possible pairs they have multiple databases containing

[1] Image sourced from http://www.transplantinterface.com/journey/living-donor

subsets of the overall number of pairs. This is contradictory to vision set out by the pioneers of this research. One reason for this current fragmented state is due to hospitals being individually rational[2] (IR). Hospitals lack incentive to put forward their easy-to-match pairs and typically only contribute their hard-to-match pairs to exchange programmes.

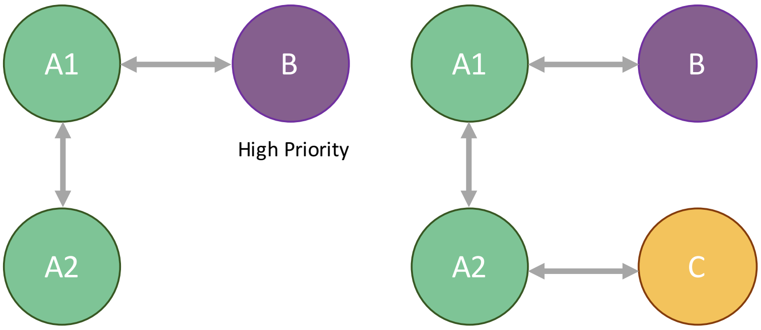

Figure 2 Incentives for hospitals not to fully participate even when there are only 2-way exchanges (a) 2-way exchange with priority patient (b) 4-way exchange

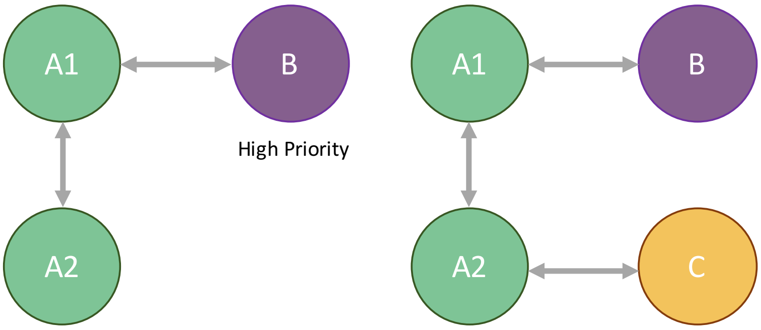

Figure 2 above illustrates the incentive challenge with hospitals. In (a) pairs A1 and A2 are at the same hospital therefore the hospital may be hesitant to submit them to a regional or national programme as the exchange A1-B may potentially get prioritised, meaning A2 (the hospital's patient) might have to wait longer for a match. However, in (b) the exchange A2-C now means there are a total of 4 exchanges which couldn't have been performed if the initial A1-A2 2-way exchange was not submitted. A number of solutions have been proposed to address this challenge including a "frequent flier" type accounting solution for hospitals and developing policy to mandate hospitals to submit all pairs. The challenge is compounded by the fact that hospitals don't have a set procedure; fees, nephrectomy and kidney transplant procedures vary which essentially equates to some matches costing more than others, a discouraging ethical dilemma. In the US attempts have been made to standardise procedure costs and the introduction of the Norwood Act[3] in 2007 provided clarity regarding the legal considerations of KEPs.

The ethical considerations of KEPs are considerable, it is illegal to pay someone for a kidney in almost every country in the world except for Iran, even drafting a contract to ensure a donor doesn't renege on a commitment to cross-exchange after their loved one receives a kidney is against the law. So, ensuring the market is *safe* is another challenge. Initially, paired exchanges were performed simultaneously to avoid the worst-case scenario of a pair donating a kidney on day one but not receiving a kidney in return on day two due the donor reneging or becoming ill. However, simultaneous exchanges don't scale well and are very impractical for hospitals; even for a two-way exchange a hospital would require four

[2] In game theory "individually rational" refers to not accepting distributions where a player (a hospital in our case) receives less than it could obtain on its own, without cooperating with anyone else.

[3] An Act to amend the US National Organ Transplant Act to provide that criminal penalties do not apply to human organ paired donation, and for other purposes.

operating theatres and four surgical teams. KEPs quickly transitioned to non-simultaneous exchanges using cycles whereby the first pair doesn't receive a Kidney until the cycle completes or an altruistic[4] kidney donor could be used to start a chain of donation. Chains started by altruistic kidney donors proved to be very successful because, unlike cycles, they could theoretically continue on indefinitely. In 2012, one altruistic kidney donor started a chain of 30 transplants. The characteristics and behavior of these long chains is worthy of investigation with several questions still unanswered such as when is the optimal time to end a chain.

2.3 Role of Software in Analytical Business Modelling

"In OR practice and research, software is fundamental. The dependence of OR on software implies that the ways in which software is developed, managed, and distributed can have a significant impact on the field."

It's hard to disagree with the above extract from (Lougee-Heimer, 2003), the author goes on to assert that the current practice (in 2003) of not publishing code along with theory and not making commercial source code available was stunting Operations Research progression. In the context of Analytic Business Modelling (ABM), the above extract is also true; advances in software and hardware have revolutionised the field particularly within supervised and unsupervised learning which is more dependent on big data sets for training models. If we consider "Deep Learning" (which has received a lot of coverage in mainstream media) as an example, much of the theory behind artificial neural networks has been around since the 1980s. However it took some time for the software and hardware to catch up with the theory. When Google[5] made their machine learning library Tensorflow open-source, it allowed the masses (academics, business, students) to start exploring and building neural networks ultimately driving the field forward. In contrast to our neural networks example, many would argue that we currently lack new theory within ABM, the software and hardware that has been developed in recent years (including many open-source libraries) allows us to store and process big data, however there is a disconnect between "big data" and big insight. Despite storing increasingly enormous amounts of data, organisations are struggling to extract meaningful insight and, ultimately, business value. Is this because we're essentially using the models previously used on "small data" and now we're just pointing them towards our "big data"? Is a new wave of theory required to enable big data to live up to its inflated expectations or like neural networks, will the existing models and theory evolve to become more powerful and insightful? The role of open-source will be instrumental, after all it is equivalent to *the rising tide that lifts all boats*.

To solve complex problems, we very often draw on theory from multiple fields, sometimes the models and software only make a small but critical part of the overall solution. For example, if we consider the Kidney Exchange solution problem outlined in sections 1 and 2.1 we use concepts from game theory, economics, graph theory and mixed integer programming to develop the overall solution to the problem. In the business world, a common mistake is for organisations to invest heavily in an expensive suite of software that uses a very specific technique; the users get comfortable with the software and familiar with the technique, leading to the golden hammer effect i.e. *if the only tool you have is a hammer, you treat everything as if it were a nail.* Good business analytics practitioners develop a deep understanding of the business problem they are trying to solve before determining what modelling approaches could be used or on what software to implement the model. Surprisingly, in the business world (with the exceptions of tech giants Google, Facebook,

[4] An altruistic kidney donor is a person who donates one of his or her own kidneys to a stranger.
[5] Google were not the only organisation to publish open-source deep learning libraries. A number of others exist including Deeplearning4j, Torch, Theano, Caffe, Paddle, MxNet, Keras & CNTK

Twitter et al.), more often than not, the proposed software is typically not open source. This rationale is counter-intuitive; why spend millions on software licenses when you could download R or Python and a few open-source libraries for free? The truth is that the directors of organisations that make the buying decisions like to talk to the sales engineers, they get comfort from having a support team contactable by phone 24/7 as part of their licence fee and, most of all, it's the software vendors fault if it all goes wrong (it reduces their own accountability). In addition, across industries subject to regularity audits such banking risk and pharmaceuticals, the regulators often insist that a particular suite of software is used, as if a p-value calculated using proprietary software was better measure of significance that one calculated using open-source software.

In summary, advancements in software and hardware will continue to shape the field of Analytic Business Modelling, open-source transparent software will push forward the art of the possible within academia. In the business world, until open-source is fully embraced and owned by organisations (apart from the top echelon) they will continue to struggle on with issues arising from propriety software such as poor integration, restricted functionality and vendor lock in[6].

3 Technical Report

3.1 Overview

Our goal in this section is to demonstrate that optimal cycles can found in a network consisting strictly of recipient-donor pairs using a model similar to that of the Prize Collecting Travelling Salesman Problem (PC-TSP). As the name suggests the PC-TSP is a variation of the TSP where the objective is to find optimal cycles visiting each city at most once but now the salesman has the option of skipping some cities entirely and paying a penalty for doing so.

3.2 Approach

We express the Kidney Exchange problem as a graph $G(V, E)$, where each node is a incompatible recipient-donor pair and an weighted directed edge exists between two nodes (i, j) if the donor from *node* i is compatible with the recipient from *node* j. The directed edge weight is some score that quantifies the quality of match between the donor from node i and recipient from *node* j, this score is a function of the blood type and percentage of reactive antibodies of the recipient (as discussed in section 2.1). The higher the score the better the match.

3.3 Data Preparation

No Kidney Exchange data is publically available, therefore we generated a number of syntactic edge-lists of varying sizes using python, specifying a common match probability of 0.02. This is clearly a simplification. In reality, a recipient's match probability depends on a large number of factors including blood type, percentage of reactive antibodies, tissue and time on the waiting list – it is beyond the scope of this assignment to derive individual probabilities per node. Table 1 below shows a sample of one such an edge-list, to test and validate the model small example edge-lists with known optimal solutions were used.

[6] "Vendor lock-in" is a term from economics and refers when an organisation becomes overly dependent on a particular vendor (or a software solution in this context), the organisation is unable to use another vendor without substantial switching costs.

Table 1 Sample of one synthetic Kidney Exchange edge-list generated

From Pair	To Pair	Weight (quality of the match)
0	23	0.672130977
2	61	0.68194823
4	93	0.125439685
5	19	0.296388544
5	98	0.888646935
6	90	0.272647967
8	34	0.679963311
9	30	0.199119962
[illegible]	40	0.399337109
		0.75267[illegible]

3.4 Assumptions

The following are the primary assumption made;

- No altruistic kidney donors are present in the graph, if they were present we would have to alter our model to find chains not cycles.
- There is no priority order of recipients, we assume they all have been on the waiting list for the same duration and all have the same level of health.
- No matches are declined, in reality a doctor may decline a Kidney despite it being a match because of the health condition or age of the donor.
- The cycles do not get interrupted i.e. once a recipient-donor pair receive a kidney, they don't renege on donating one.

3.5 Algebraic formulation

The following formulation upon which our model is based has been adapted from (Galati, 2015)

Let M be an index set of matches (candidate disjoint unions of short cycles) such that $M = \{1, 2, 3, \dots, |N|/2\}$

Let x_{ijm} be a binary decision variable with the value 0 or 1, if set to 1 then $edge(i, j)$ is a in a matching m

Let y_{im} be a binary decision variable with the value 0 or 1, if set to 1 then $node\ i$ is covered by match m.

Let S_i be a binary slack variable with the value 0 or 1, if set to 1 it indicates that $node\ i$ is not covered by a match.

The objective function is to find a maximum weight node-disjoint union of directed cycles, we require the union to be node-disjoint to prevent any kidney from being donated more than once.

$$Maximize \sum_{(i,j)\in A} \sum_{m \in M} w_{ij} x_{ij}$$

$$subject\ to: \quad \sum_{m \in M} y_{im} + s_i = 1 \qquad i \in N \qquad (1)$$

$$\sum_{(i,j) \in A} x_{ijm} = y_{im} \qquad i \in N, m \in M \qquad (2)$$

$$\sum_{(i,j) \in A} x_{ijm} = y_{jm} \qquad j \in N, m \in M \qquad (3)$$

$$\sum_{(i,j) \in A} x_{ijm} \leq L \qquad m \in M \qquad (4)$$

$$where: \quad x_{ijm} \in \{0,1\} \qquad (i,j) \in A, m \in M$$
$$y_{im} \in \{0,1\} \qquad i \in N, m \in M$$
$$s_i \in \{0,1\} \qquad i \in N$$

Constraint (1) is the packing constraint[7] that prevents each node receiving more than one match. Note that the slack variable equates to one when there is no match.
Constraint (2) specifies the condition that if a pair receive a kidney they need to then donate one, that is if $node\ i$ is covered by a matching m, then matching m must use exactly one edge that leaves $node\ i$ and one edge entrees $node\ i$. Conversely if a pair does not receive a match then no edges that enter or leave $node\ i$ can be used in matching. The final constraint (4) ensures the number of edges in matching m does not exceed L, that is it ensures the cycle does not get exceed the specified threshold.

3.6 Model Parameters

The maximum acceptable length of a cycle L is the only specific MIP model parameter that be modified. Typically, L is kept low (≤ 5) for simultaneous transplants for practicality reason, for non-simultaneous transplants L can be increased but is constrained by the wait time and health of the recipient from the first pair that commenced the cycle. In appendix 6.1 graph plots are show for $L = 10, 20, 100$.

3.7 Implementation

Figure 3 below provides an overview of how the end-end solution was implemented including how data is passed seamlessly between Python, Model and Gephi[8]. One of the challenges was dealing with the sparse edge-lists, because the probability of a match was set to 0.02 for this set of simulations the graph adjacency matrix and edge-list were very sparse. We were unable to define a data structure in Mosel that could loop over values indexed by i, j where $i\ and\ j$ were from a list of possible values, it was only possible to loop over all edges i, j where $i\ and\ j$ were from a range $\{0,1,2,3 \ldots, n\}$. The implications of this are significant as it increases each set of constraints by order of n. It also requires the setting of an additional set of constraints to set the $x\ and\ y$ of invalid edges to zero.

[7] The packing constraint is a series of constraints

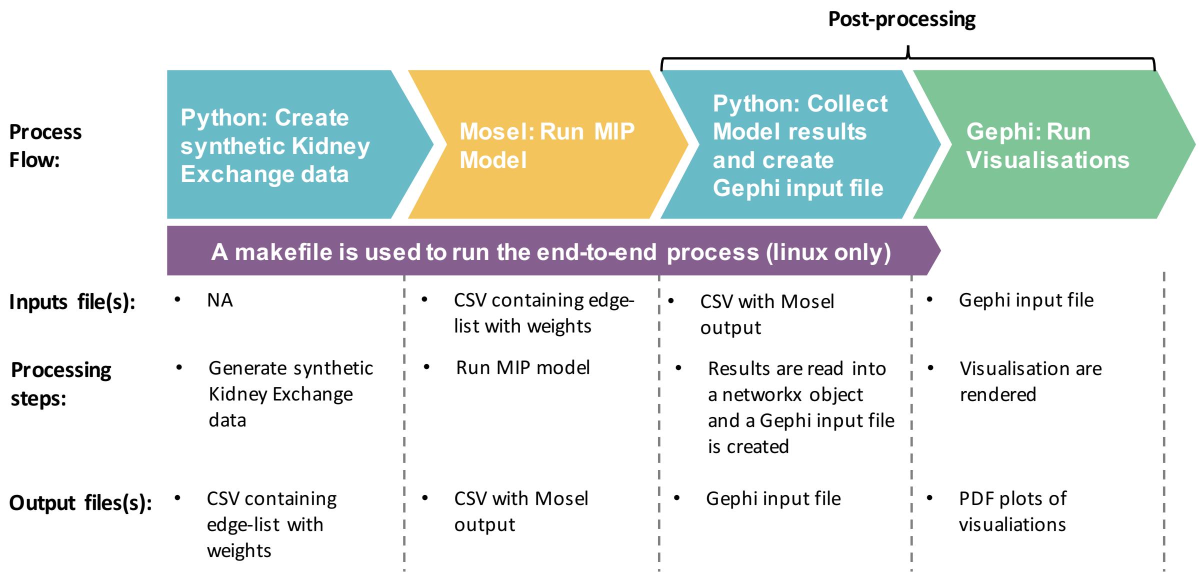

Figure 3 Solution implementation using Python, Mosel & Gephi

3.8 Results

The output from the MIP model visualised using Gephi is show in X below for a network of 100 recipient-donor pairs with a common match probability of .02 and maximum acceptable length of a cycle $L = 5$. In total 6 optimal cycles are found consisting of 24 pairs, if this were a real KEP database this insight could result in 24 lives being saved.

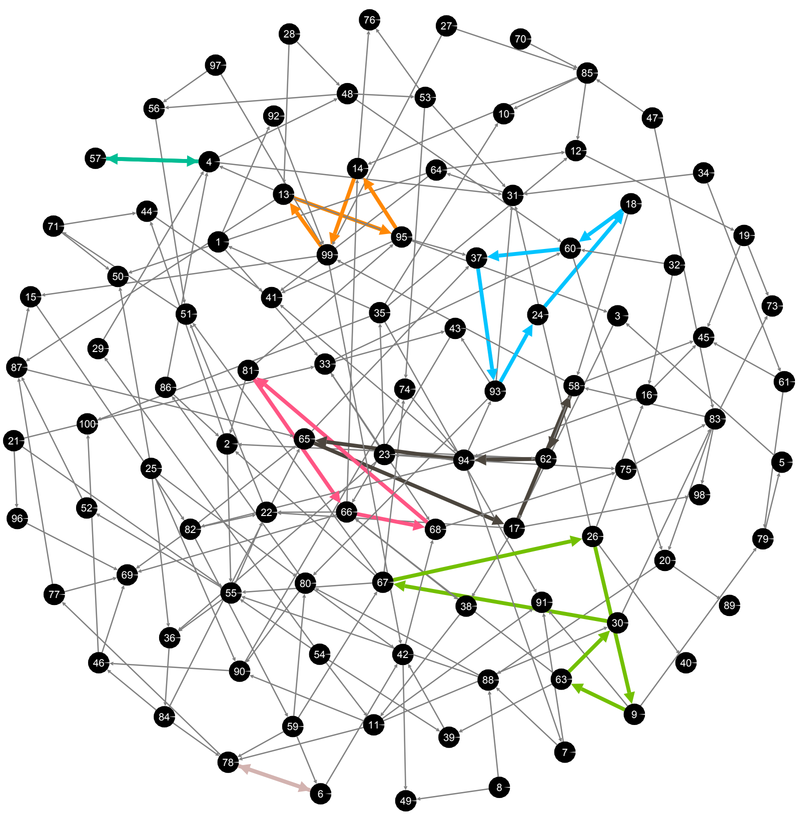

Figure 4 Output from the MIP model visualised using Gephi

3.9 Evaluating the Optimal Solution

In (Abraham, et al., 2007) the authors proved that this problem was NP-hard, meaning that it is unlikely there will be an algorithm that always finds for the optimal solution quickly for every possible permutation of the problem. Though our testing of this model a high variation of runtime was observed, although the model did always provide a solution. If we were to evaluate this model across the infinite potential datasets we're confident it would fail to find a solution in some cases.

To deepen our understanding of the effects of increasing the maximum acceptable length of a cycle L, we ran several simulations varying L. It can be observed in Figure 5(a) that as L increases the objective function Z increases before flattening off – this could be interrupted as the quality of matches increasing as L increases.

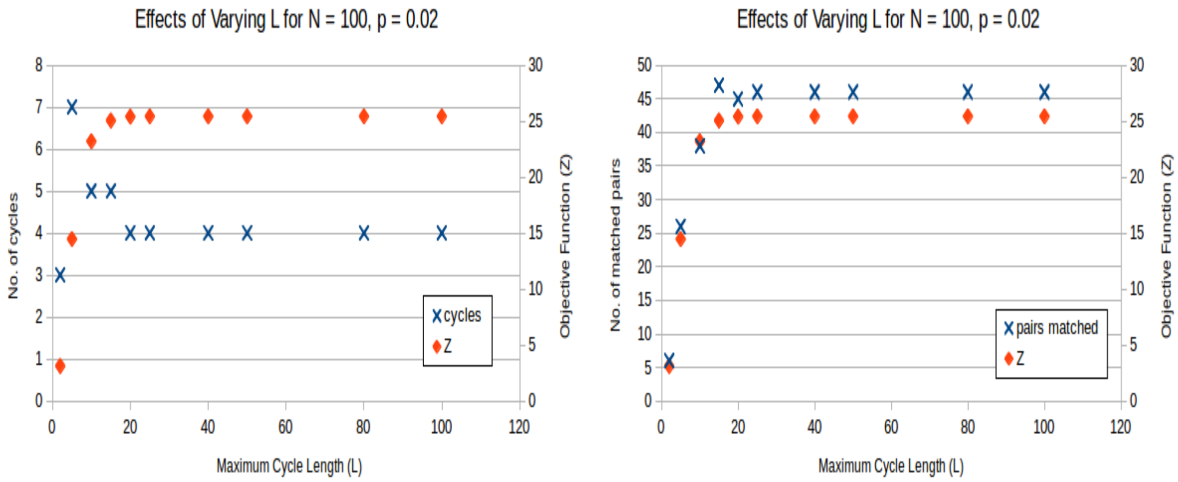

Figure 5 Effects of varying L (a) No. of cycles (b) No. of matched pairs

3.10 Extensions

Time permitting it would be interesting to see if this model could be adapted to address networks with altruistic kidney donors, in this case we would no longer require complete cycles, instead chains would suffice. To do this we would start by building out logic to identify altruistic kidney donors the network potentially using a bipartite graph structure. Once identified we could relax constrains (2) and (3) discussed in section 3.5 as these ensure that all nodes in a cycle have one input edge and one output edge i.e. if a pair receive a kidney they must donate a kidney. We then would need to identify isolated recipients in the graph who don't have a paired donor to end the chain.

4 Conclusion

In summary, we've introduced the key concepts of market design and dived deep into the theory behind Kidney Exchange Programmes before demonstrating that optimal cycles can found in a network consisting strictly of recipient-donor pairs using a model similar to that of the Prize Collecting Travelling Salesman Problem (PC-TSP). In section 2.3 we argue that software and hardware will continue to shape the field of Analytic Business Modelling and outline a number of challenges particularly for organisations.

4.1 Lessons learned

Overall, this proved to be very interesting and beneficial assignment. Getting the opportunity to tackle a topic that has already saved thousands of lives is extra special. Exploring the problem and solution outside the strict technical sense was eye-opening, it is clear that sometimes the perfect technical solution can fail due to complex "business issues" as seen in this case with hospitals being individually rational. On reflection, as business analytics practitioners we need to bare this in mind, the best technical solution in the world won't be a success without business buy-in and the right incentives.

5 Works Cited

Abraham, D., Blum, A. & Sandholm, T., 2007. Clearing algorithms for barter exchange markets: Enabling nationwide kidney exchanges. *In Proceedings of the 8th ACM conference on Electronic commerce,* June, .(.), pp. pp. 295-304.

Galati, M., 2015. *http://blogs.sas.com.* [Online]
Available at: http://blogs.sas.com/content/operations/2015/02/06/the-kidney-exchange-problem/#prettyPhoto
[Accessed 20 4 2016].

Garwood, P., 2016. *World Health Organization.* [Online]
Available at: http://www.who.int/bulletin/volumes/85/1/07-020107/en/

Lougee-Heimer, R., 2003. The Common Optimization INterface for Operations Research: Promoting open-source software in the operations research community. *BM Journal of Research and Development,* pp. 57-66.

Roth, A., 2008. What have we learned from market design?. *The Economic Journal,* pp. pp.285-310..

Roth, A., Sonmez, T. & Unver, M., 2003. Kidney exchange. *National Bureau of Economic Research,* p. w10002.

6 Appendix

6.1 Increasing maximum acceptable length of a cycle

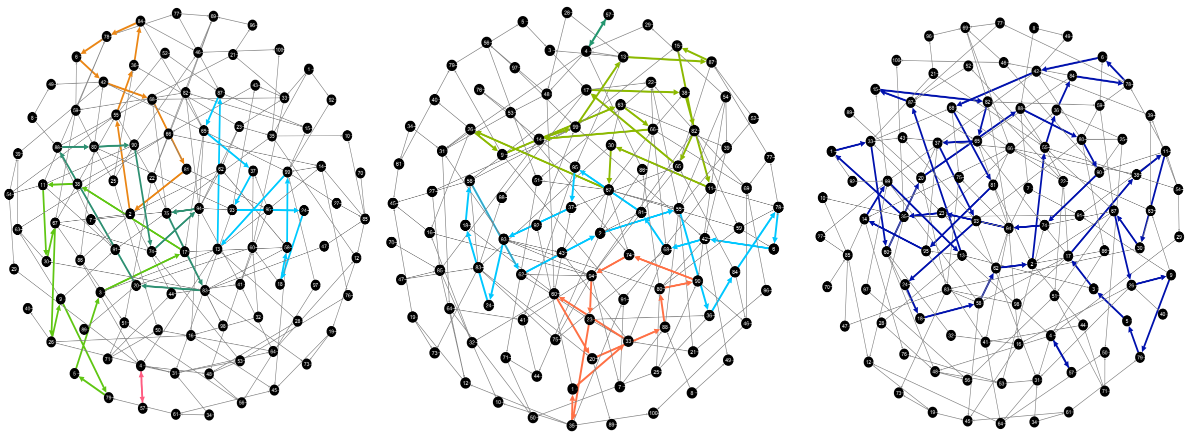

Figure 6(a) L =5 (b) L=10 (c) L=50