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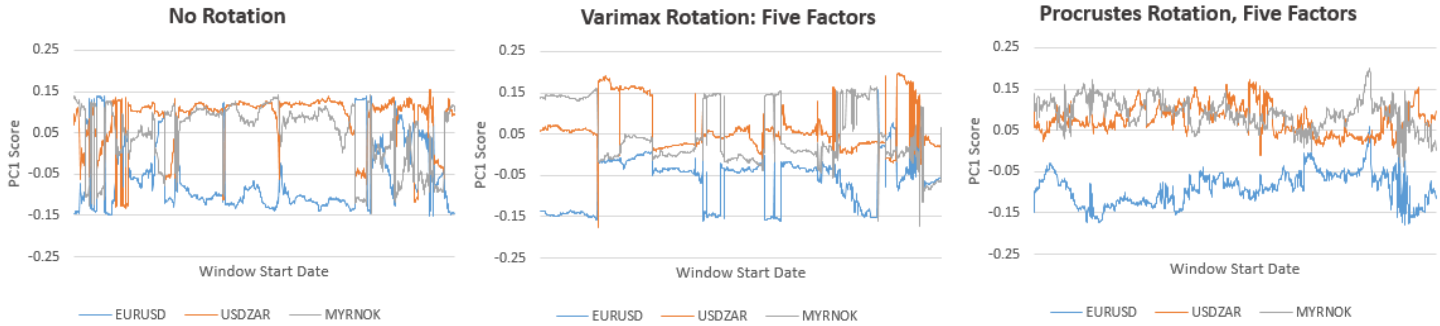
Overview

Principal component analysis is a widely used tool for analyzing risk common to groups of securities, but the factors it generates are too unstable to allow for rolling window analysis. This instability stems from random rotation of factors throughout time, although the sub-space defined by the dominant risk factors remains relatively stable.

Through collaborative research with Upgrade Capital we have derived a Procrustes rotation algorithm that allows us to look at factors from smaller sample sizes, compare factors from windows of different length, and observe meaningful changes in factor scores throughout time.

Practictioners typically use Varimax rotation to improve the interpretability of factors loadings, but as can be seen below the problem of factor switching *worsens* with rolling window. Procrustes rotation eliminates this issue.



Data Set

To showcase our work we looked at log returns of 171 daily spot cross currency log returns derived from 18 major currency fixings to the Euro from April 2005 to present. Plotted above is the leading component factor score for three crosses which historically carry the heaviest weight.

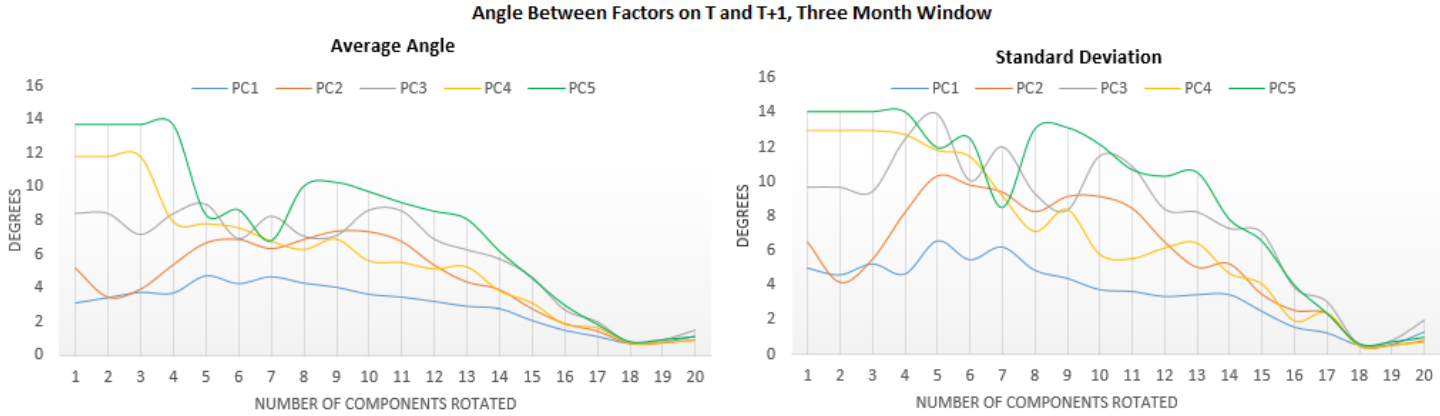
Rolling Window Approach

We run PCA for each date twice – once with a one year window and one more time with a three month window. For the first date, we rotate the three month window factors towards one year window factors to make the short-term factors more consistent with the long term ones.

Rolling into the next window, we rotate the new one year factors towards the one year factors from the previous window. We then rotate the new three month window factors to the new rotated one year factors.

Results

Looking at angles between each of the five dominant Principal Components:



Following rotation we are left with stable and directly comparable global risk factors without a numeraire currency. We see that the ideal number of factors to use in the Procrustes rotation is 18- which makes sense because the data set began with 18 time series; note that changing the number of components rotated does not alter the amount of variance explained by the subspace being rotated.

Conclusion

With the ability to map different window lengths to each other we can measure the stability of volatility relationships among securities in order to better grasp outlook on investment time-horizons.

Follow-on work should aim to understand changes in the distribution of variance explained by individual factors, and to further stabilize risk measures through time by varying window lengths and changing the sequence in which factors are rotated. Doing so would allow the evolution of factors through time to be understood much more clearly, making possible the creation of new kinds of risk and regime models.