APPM 4650 Homework #1 Fall 2016

Due Friday, September 2, 2016

Please read the comments at the bottom of the assignment. The problems listed below should be edition independent for the Burden and Faires textbook.

1. Use a tolerance of 10^{-4} on the relative change in x_n (specifically, $\dfrac{x_n - x_{n-1}}{x_{n-1}} \leq 10^{-4}$) to find the root of $f(x) = x + \exp(x)$ using the Bisection, Fixed Point Iteration, Newton-Raphson, Secant, and False Position methods. How many iterations are required for convergence using each method assuming you start each with the same initial guess(es)?

2. Section 2.1 material

   (a) Use the Bisection method to find an approximation to within 10^{-5} to the first positive value of x with $x = \tan(x)$.

   (b) Find an approximation to $\sqrt[3]{25}$ correct to within 10^{-4} using the Bisection Algorithm. Hint: Don't simply try to find the root of $f(x) = x - \sqrt[3]{25} = 0$. Use another function that has the same root.

3. Section 2.2 material

   (a) Use the fixed–point iteration method to determine the root(s) of $2\sin(\pi x) + x = 0$. Your answer(s) should be accurate to within 10^{-2}.

4. Section 2.3 material

   (a) The iteration equation for the Secant method can be written in the simpler form

   $$x_n = \frac{f(x_{n-1})\, x_{n-2} - f(x_{n-2})\, x_{n-1}}{f(x_{n-1}) - f(x_{n-2})}\,.$$

   Explain why, in general, this iteration equation is likely to be less accurate than the one given in Algorithm 2.4.

   (b) Use Newton's method for this problem. Problems involving the amount of money required to pay off a mortgage over a fixed period of time involve the formula

   $$A = \frac{P}{i}\left[1 - (1+i)^{-n}\right]\,,$$

   known as an *ordinary annuity equation*. In this equation, A is the amount of the mortgage, P is the amount of each payment, and i is the interest rate per period for the n payment periods. Suppose that a 30-year home mortgage in the amount of \$135,000 is needed and that the borrower can afford house payments of at most \$1000 per month. What is the maximal interest rate the borrower can afford to pay?

5. In class we used Fixed Point Iteration to find the roots of $f(x) = (x+1)(x-3)$. One of the more obvious choices lead us to $x = g(x) = \dfrac{x^2-3}{2}$. Explore the convergence characteristics of $x = g(x) = \dfrac{x^2-3}{2}$ relative to the magnitude of $|g'|$ near each of the roots of $f(x)$. Also please include "spider web" graphs for several initial guesses to help support your conclusions.

Here are a few comments about homework assignments.

- First, note that I am asking you to write your programs such that only one convergence condition is satisfied instead of two or three of the possible conditions, as mentioned in class. If you already wrote your code to use two convergence tests, please state this in your hard copy of the homework.

- If necessary, you can use the code supplied by the text for most of the root finding problems. I would prefer that you write your own code from scratch. Which route you pick will probably depend on your programming background and experience. I strongly encourage you to write as much of your own code as possible. What I don't want though, is for you to use built–in functions of a language. For example, please don't simply use Matlab's built–in functionality for root finding, integration, etc.

- When you write up your results, you do not need to include code. For results, I would like you to report the sequence of root estimates. For example, when using bisection, a table of n, a, b and the resulting c, for each iteration would suffice. Or for Newton's method, a table of n and x_n would also be fine. Use common sense when reporting the results. For example, if it took 3000 iterations, please include the first and last 5 or 6 iteration results. You may omit everything in between. On the other hand, if it only took 10 iterations, then please show the results of all 15 iterations. Finally, be sure to clearly label the tables.