CKY Parsing & CNF Conversion

LING 571 — Deep Processing Techniques for NLP

October 5, 2022

Shane Steinert-Threlkeld

Announcements

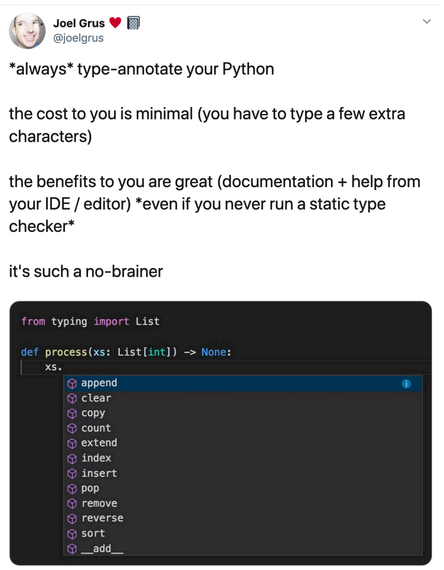
- **HW #1** due tonight at **11:59pm**.
- Python on Patas: installed versions `ls /opt | grep python`. E.g., invoke by:
  - **`/opt/python-3.6/bin/python3`**
  - `nltk` is installed.
- [For personal projects, but not 571 HW, you can use the latest of everything via Anaconda (download with `wget`).]
- When in doubt, use *full paths* for everything (python binary, file names, etc)
- check_hwX.sh: invoke from your local directory (for permission reasons)

Type Hinting in Python

- Supported in ≥3.6 [tutorial]

```
from typing import List
from nltk.grammar import Production

def fix_hybrid_production(hybrid_prod: Production) -> List[Production]:
    …
```



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    …
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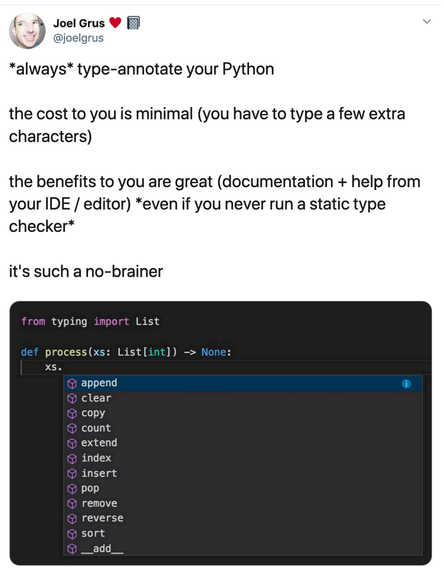
- Also available in PyCharm through docstrings and/or comments:

```
def fix_hybrid_productions(hybrid_prod):
    """
    This function takes a hybrid production and
    returns a list of new CNF productions
    :type hybrid_prod: Production
    :rtype: list[Production]
    """
```



Joke of the Week (PP Attachment Ambiguity)

tott @crazytott · Oct 5

A cop just knocked on my door and told me that my dogs were chasing people on bikes???? Wtf??? My dogs don't even own bikes tf

Roadmap

- **Parsing-as-Search**
- Parsing Challenges
- Strategy: Dynamic Programming
- Grammar Equivalence
- CKY parsing algorithm

Computational Parsing

- Given a body of (annotated) text, how can we derive the grammar rules of a language, and employ them in automatic parsing?
  - Treebanks & PCFGs
- Given a grammar, how can we derive the analysis of an input sentence?
  - Parsing as search
  - CKY parsing
    - Conversion to CNF

What is Parsing?

- CFG parsing is the task of assigning trees to input strings
  - For any input A and grammar G
    - …assign ≥0 parse trees T that represent its syntactic structure, and…
    - Cover all and only the elements of A
    - Have, as root, the start symbol S of G
    - …do not necessarily pick one single (or correct) analysis
- Subtask: Recognition
  - Given input A, G – is A in language defined by G or not?

Motivation

- Is this sentence in the language — i.e. is it “grammatical?”
  - ** I prefer United has the earliest flight.*
  - FSAs accept regular languages defined by finite-state automata.
  - Our parsers accept languages defined by CFG (equiv. pushdown automata).

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- Is this sentence in the language — i.e. is it “grammatical?”
  - ** I prefer United has the earliest flight.*
  - FSAs accept regular languages defined by finite-state automata.
  - Our parsers accept languages defined by CFG (equiv. pushdown automata).
- What is the syntactic structure of this sentence?
  - *What airline has the cheapest flight?*
  - *What airport does Southwest fly from near Boston?*
  - Syntactic parse provides framework for semantic analysis
    - What is the subject? Direct object?

Parsing as Search

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Parsing as Search

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- Formally, search problems are defined by:
  - Start state S
  - Goal state G (with a test)
  - Set of actions that transition from one state to another
    - "Successor function"
  - A path cost function

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- Path cost:
  - …ignored for now.

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  - Start symbol of CFG
- Goal node:
  - Full parse tree: covering all of, and only the input, rooted at S

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- Depth First
  - Keep expanding nonterminals until they reach words
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- Other alternatives, if have associated path costs.

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- Two constraints on parsing:
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- Correspond to main parsing search strategies
  - Top-down search (Goal-directed)
  - Bottom-up search (Data-driven search)

A Grammar

Grammar	Lexicon
$S \rightarrow NP\ VP$	$Det \rightarrow that \mid this \mid a$
$S \rightarrow Aux\ NP\ VP$	$Noun \rightarrow book \mid flight \mid meal \mid money$
$S \rightarrow VP$	$Verb \rightarrow book \mid include \mid prefer$

Jurafsky & Martin, Speech and Language Processing, p.390

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$NP \rightarrow Pronoun$	$Pronoun \rightarrow I \mid she \mid me$
$NP \rightarrow Proper\text{-}Noun$	$Proper\text{-}Noun \rightarrow Houston \mid NWA$
$NP \rightarrow Det\ Nominal$	$Aux \rightarrow does$
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$Nominal \rightarrow Nominal\ PP$	
$VP \rightarrow Verb$	

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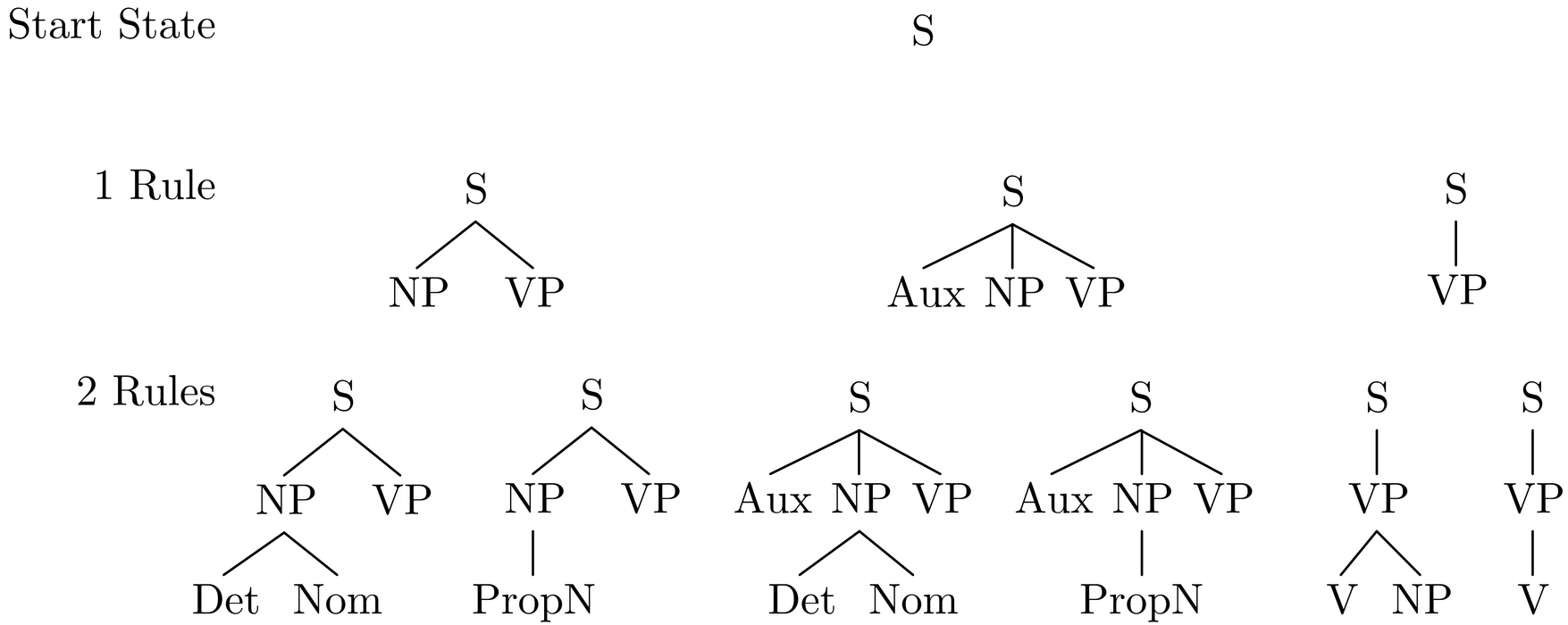
- Successively expand nonterminals
  - e.g. $NP \rightarrow Det\ Nominal;\ VP \rightarrow V\ NP$
- Terminate when all leaves are terminals

Depth-First Search



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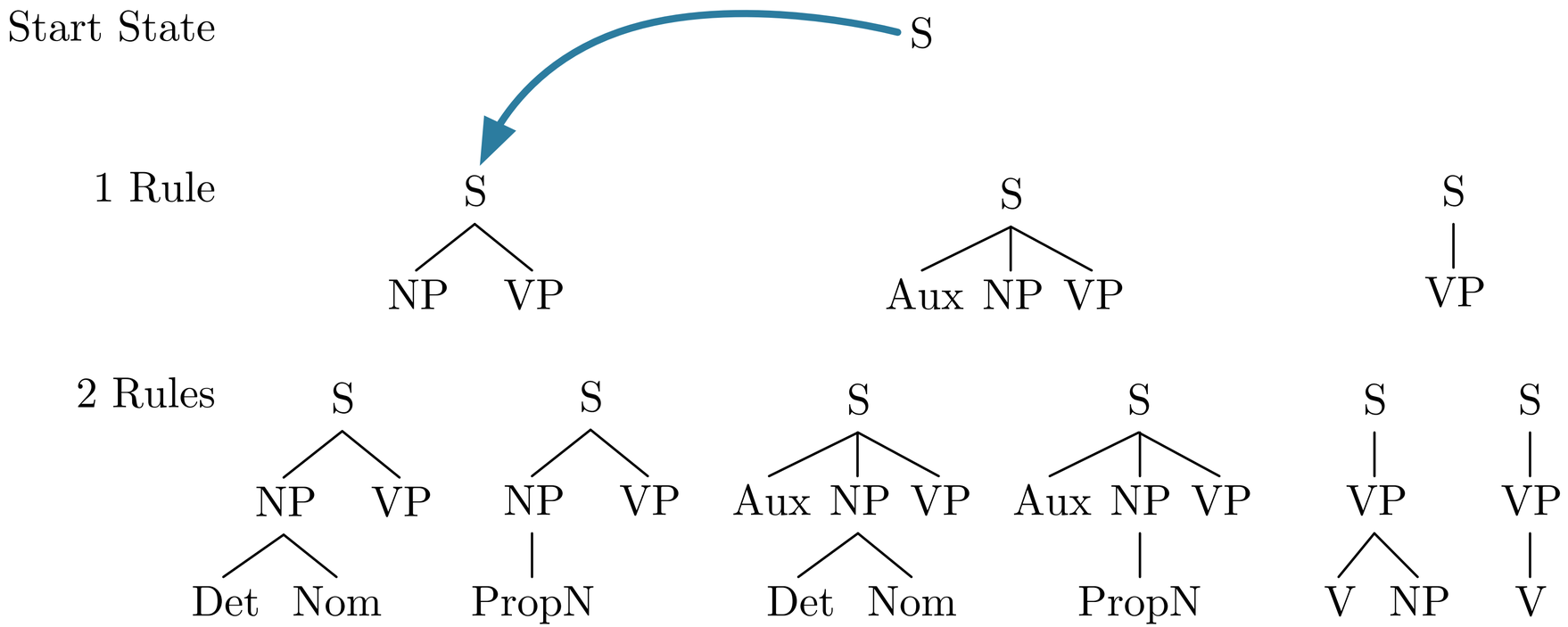

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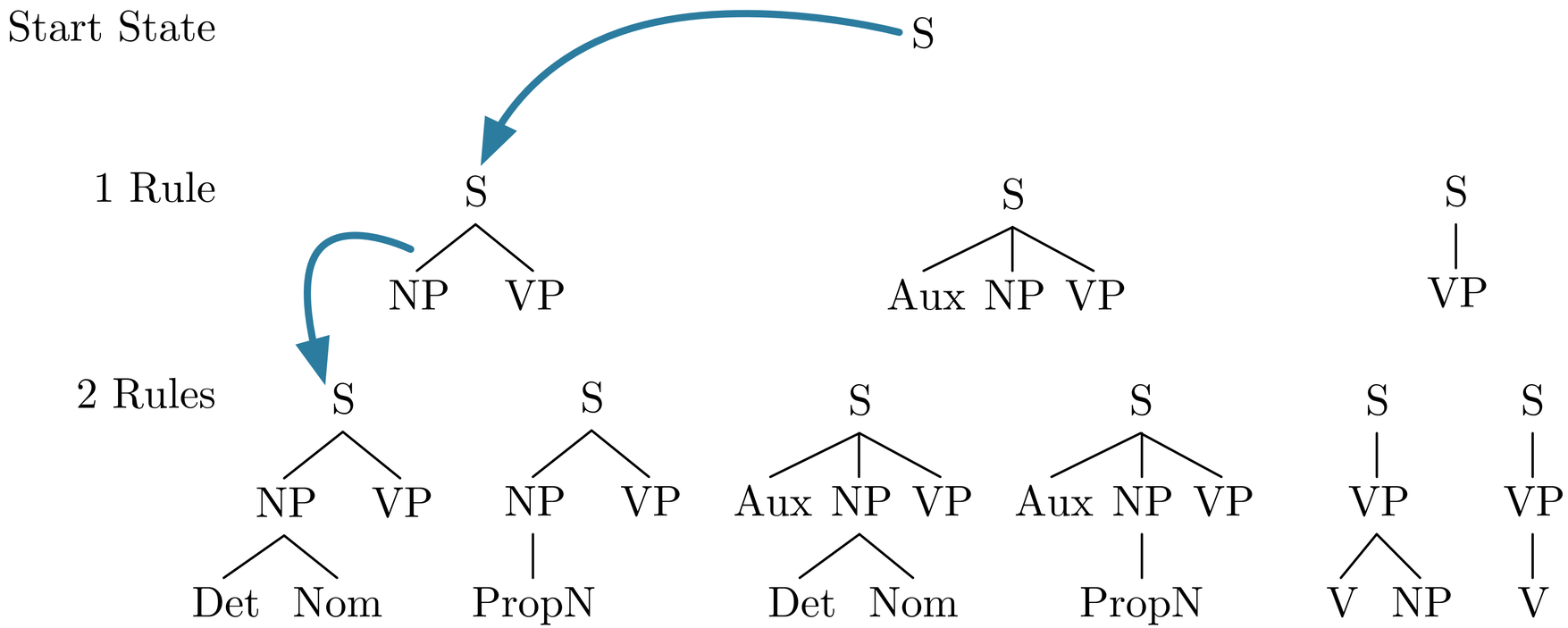

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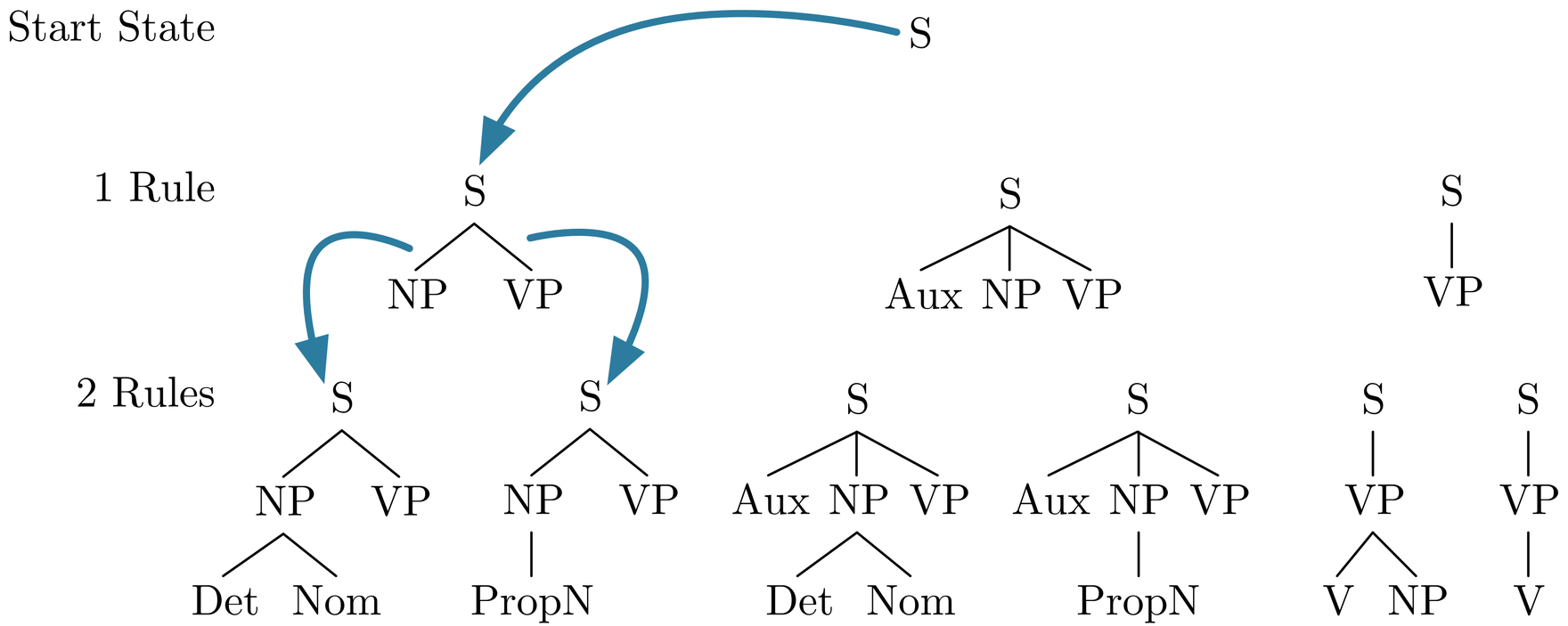

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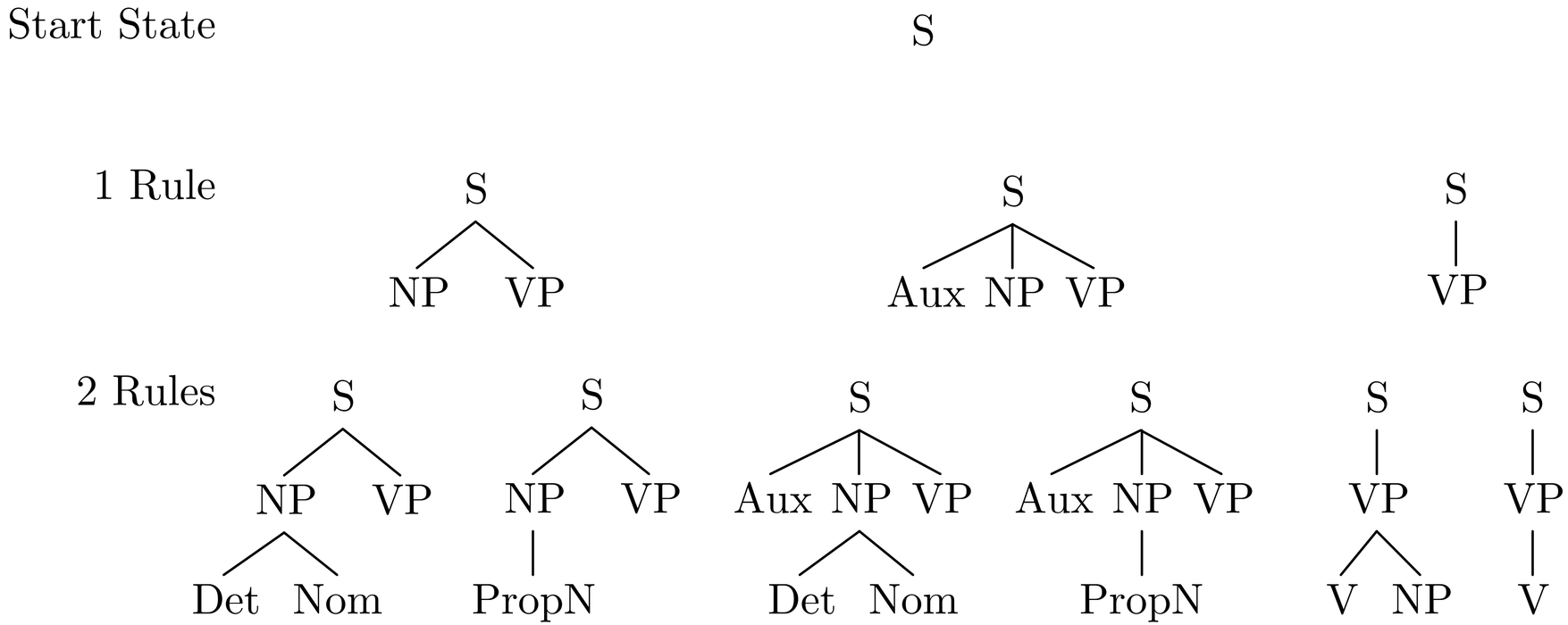

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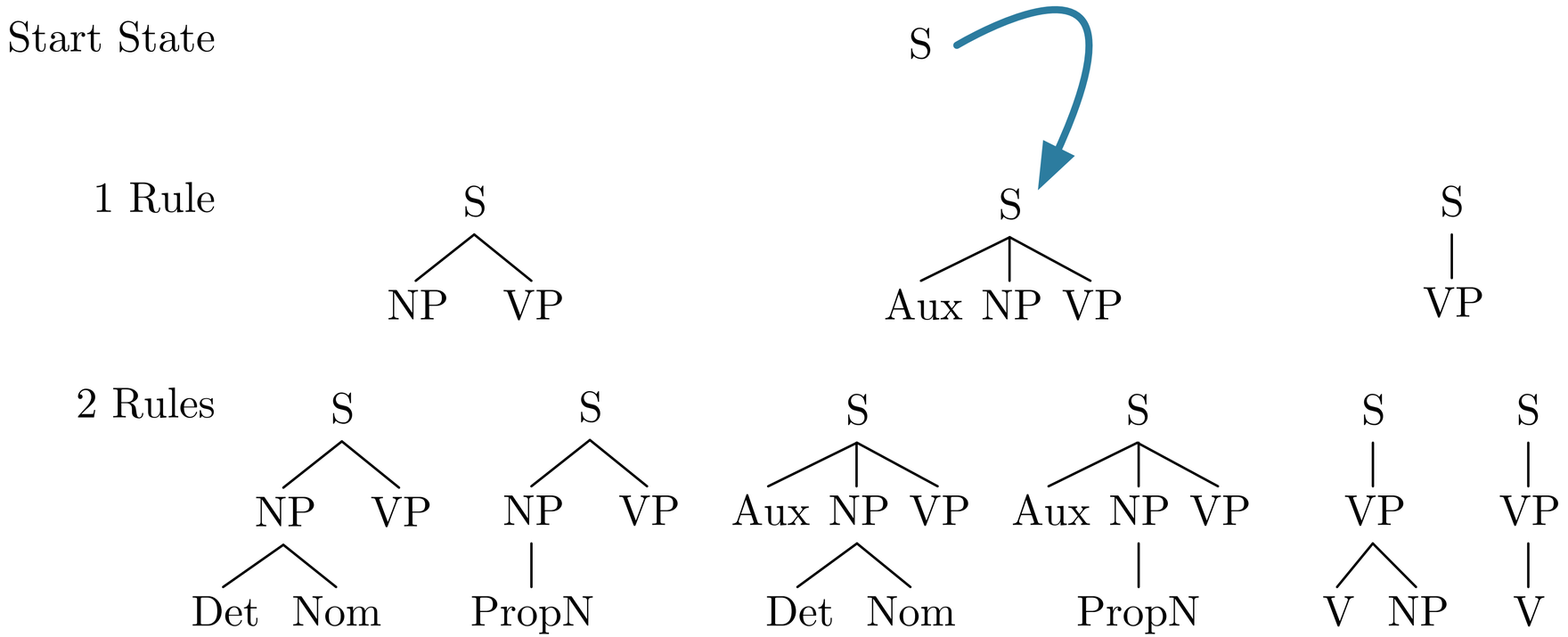

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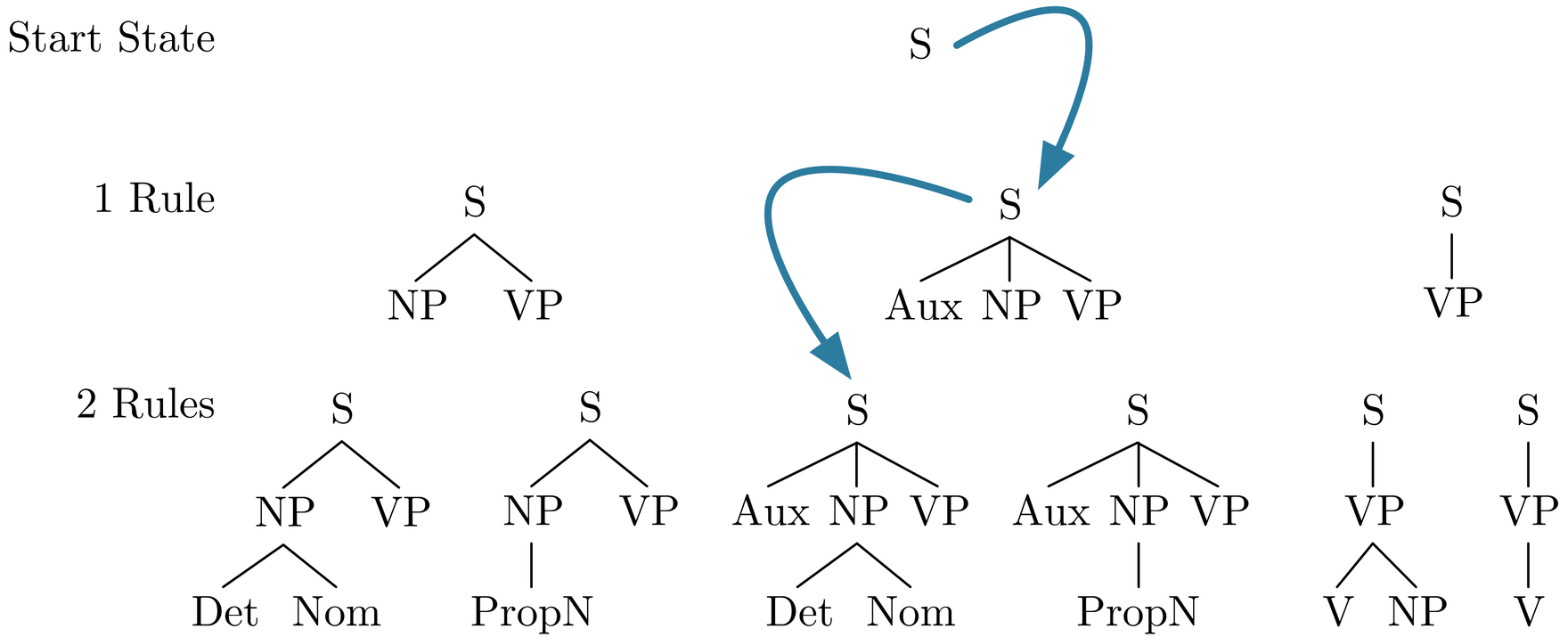

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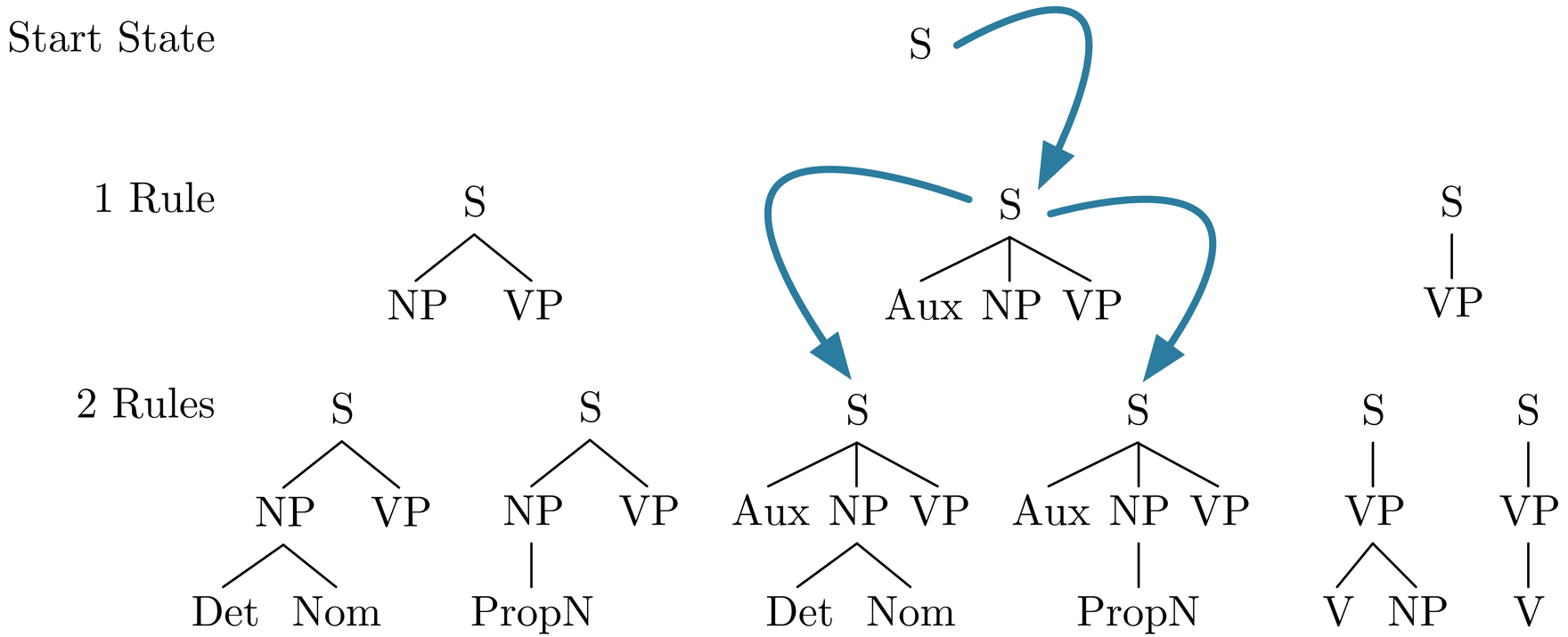

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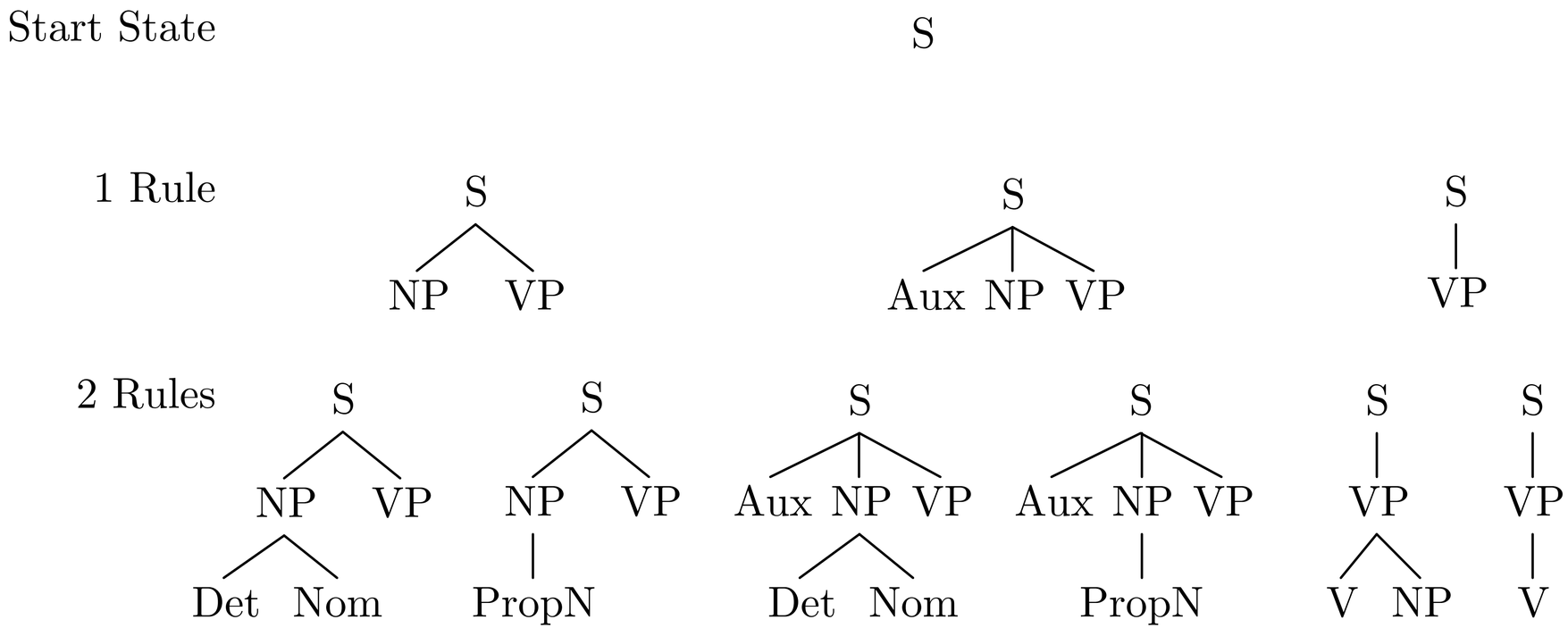

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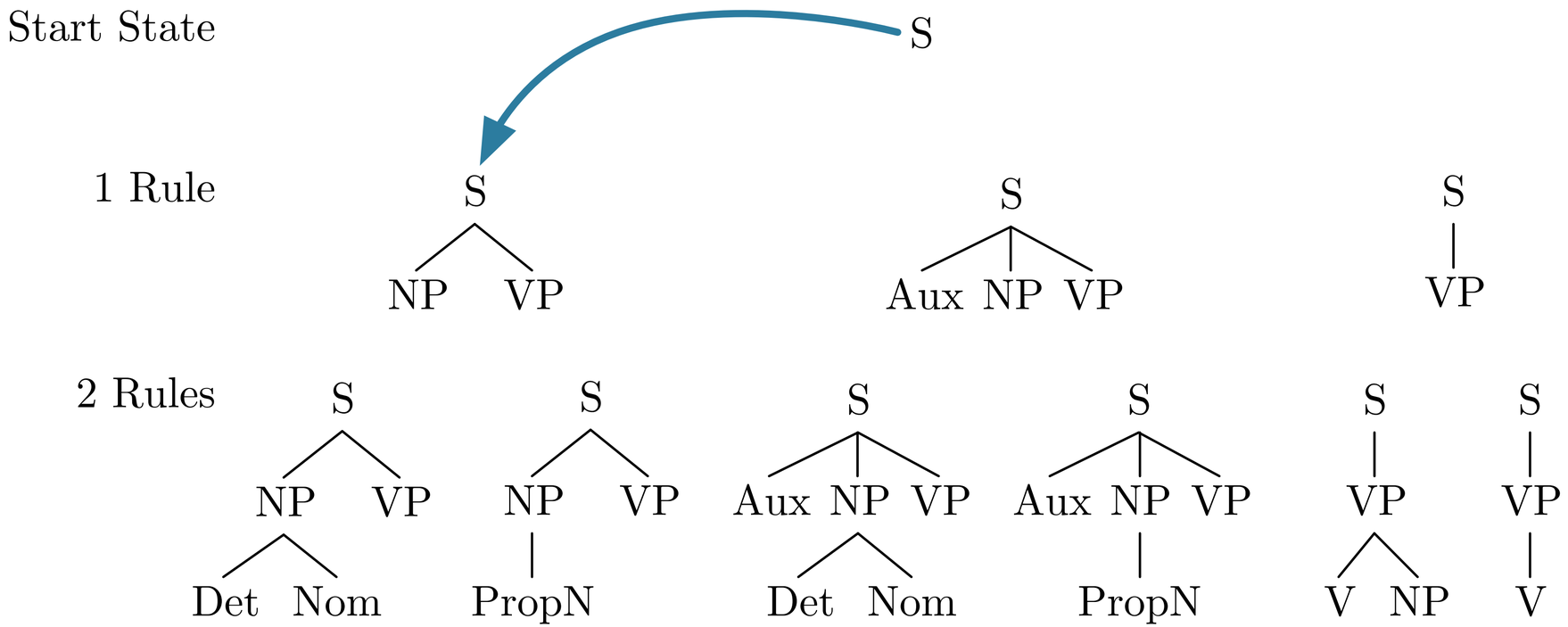

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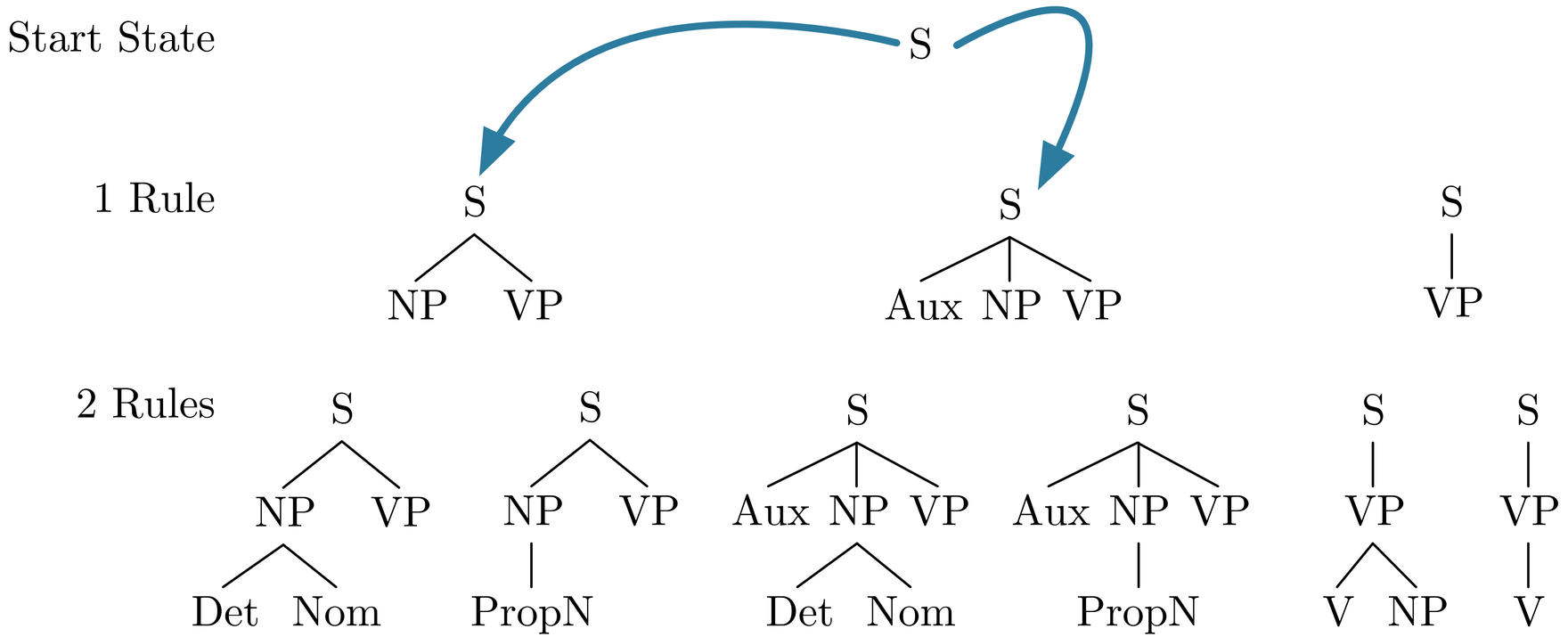

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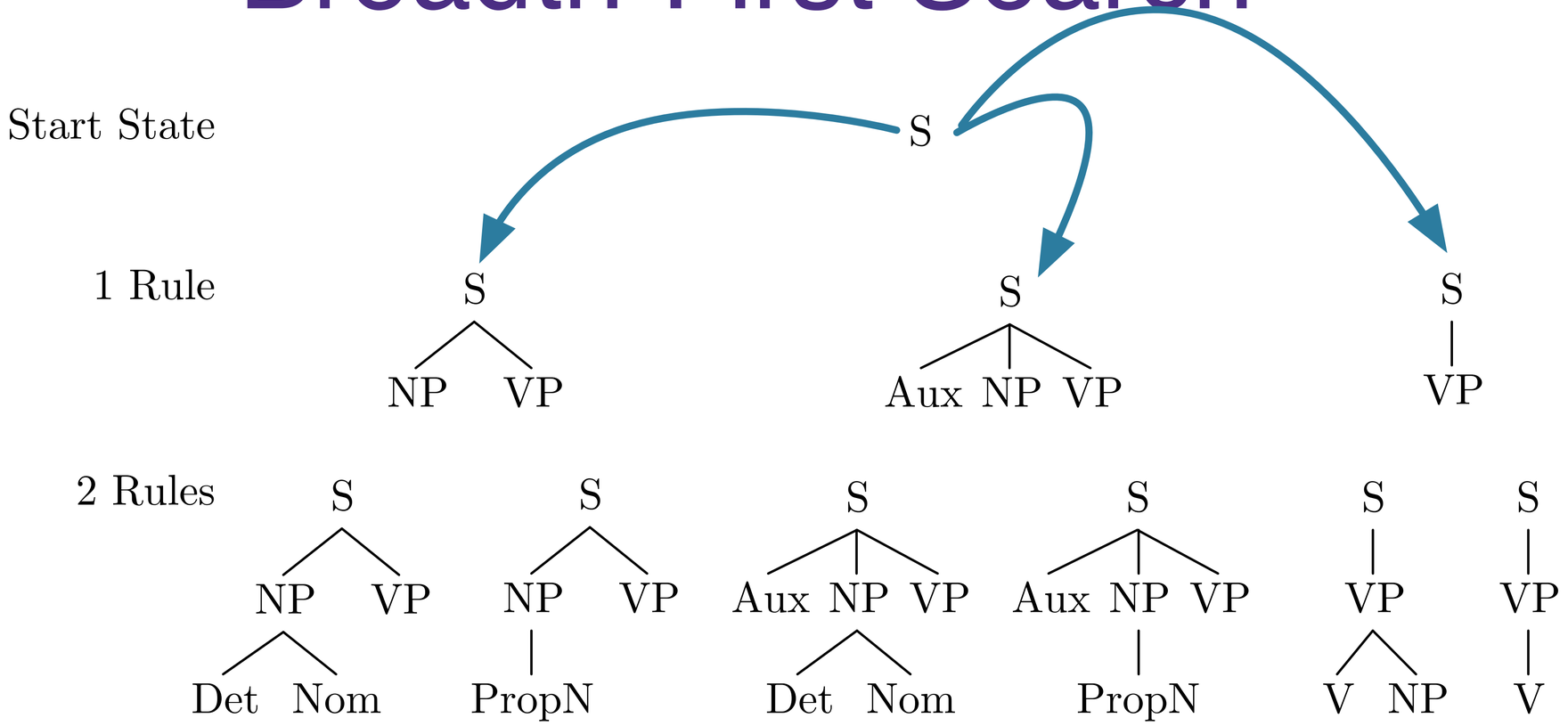

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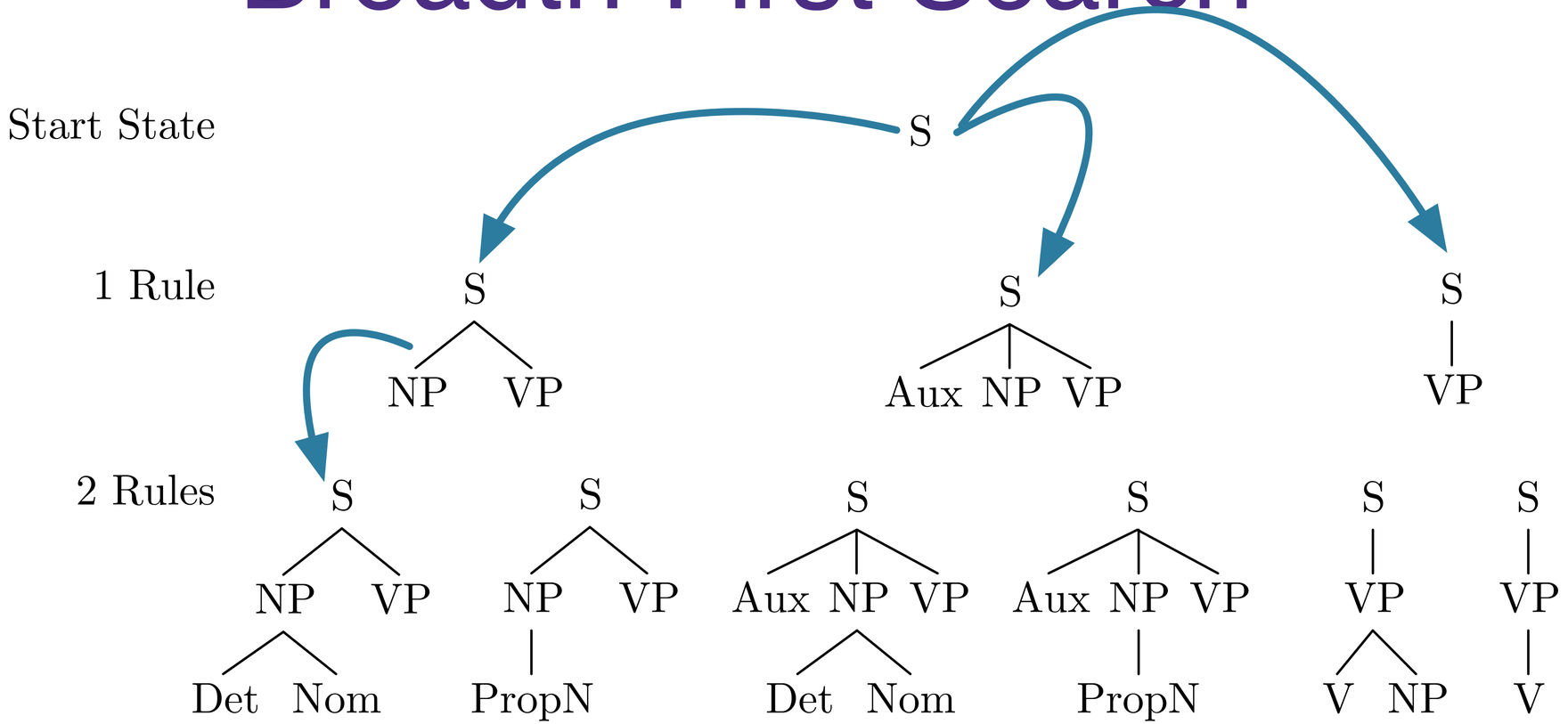

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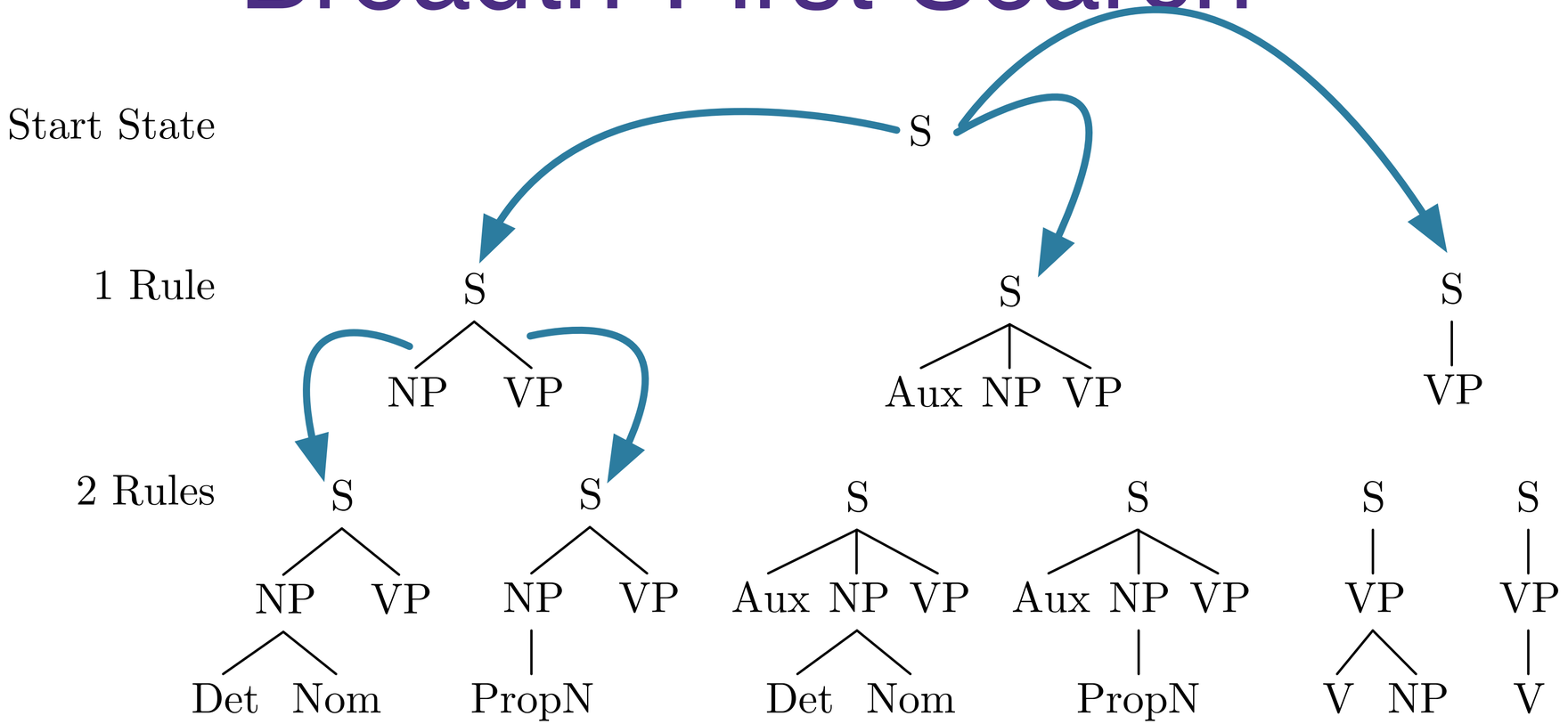

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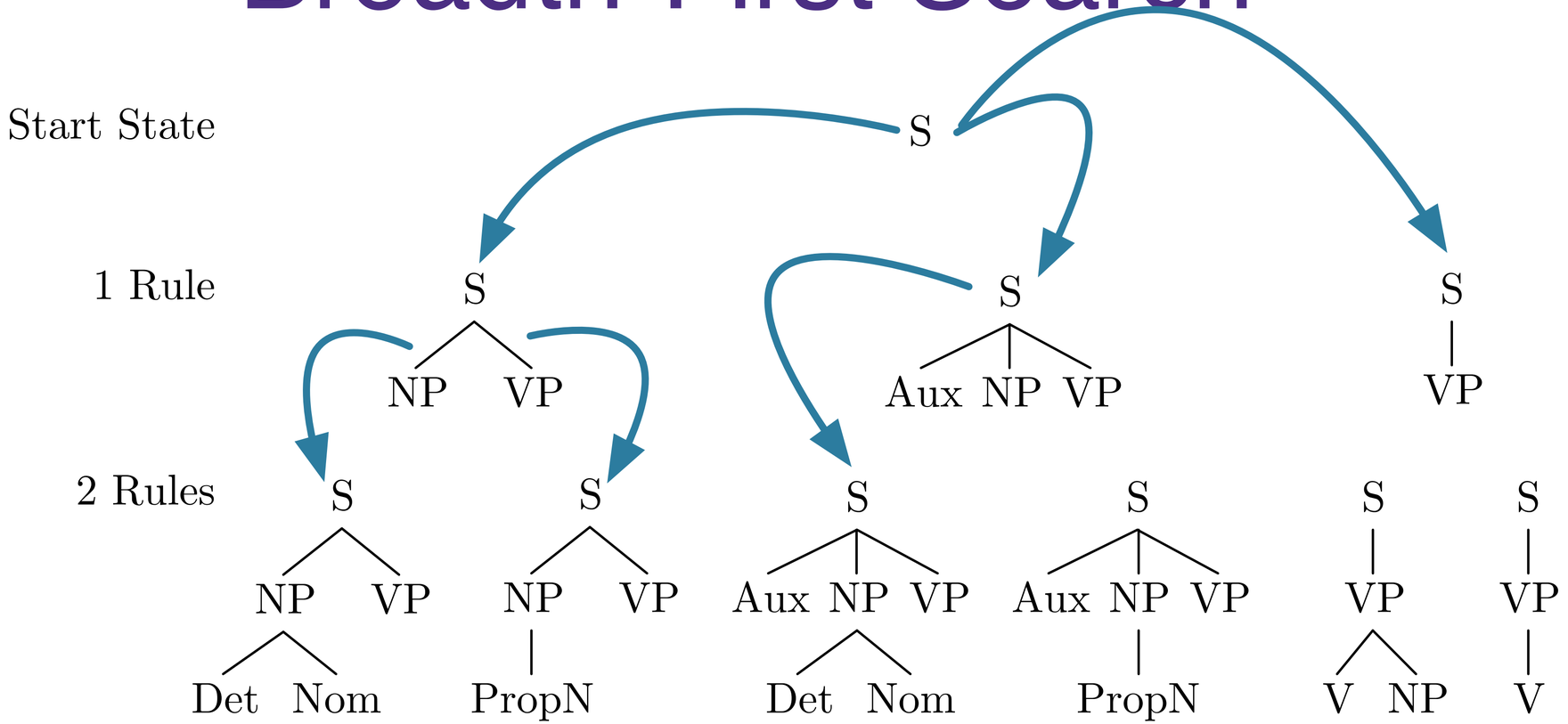

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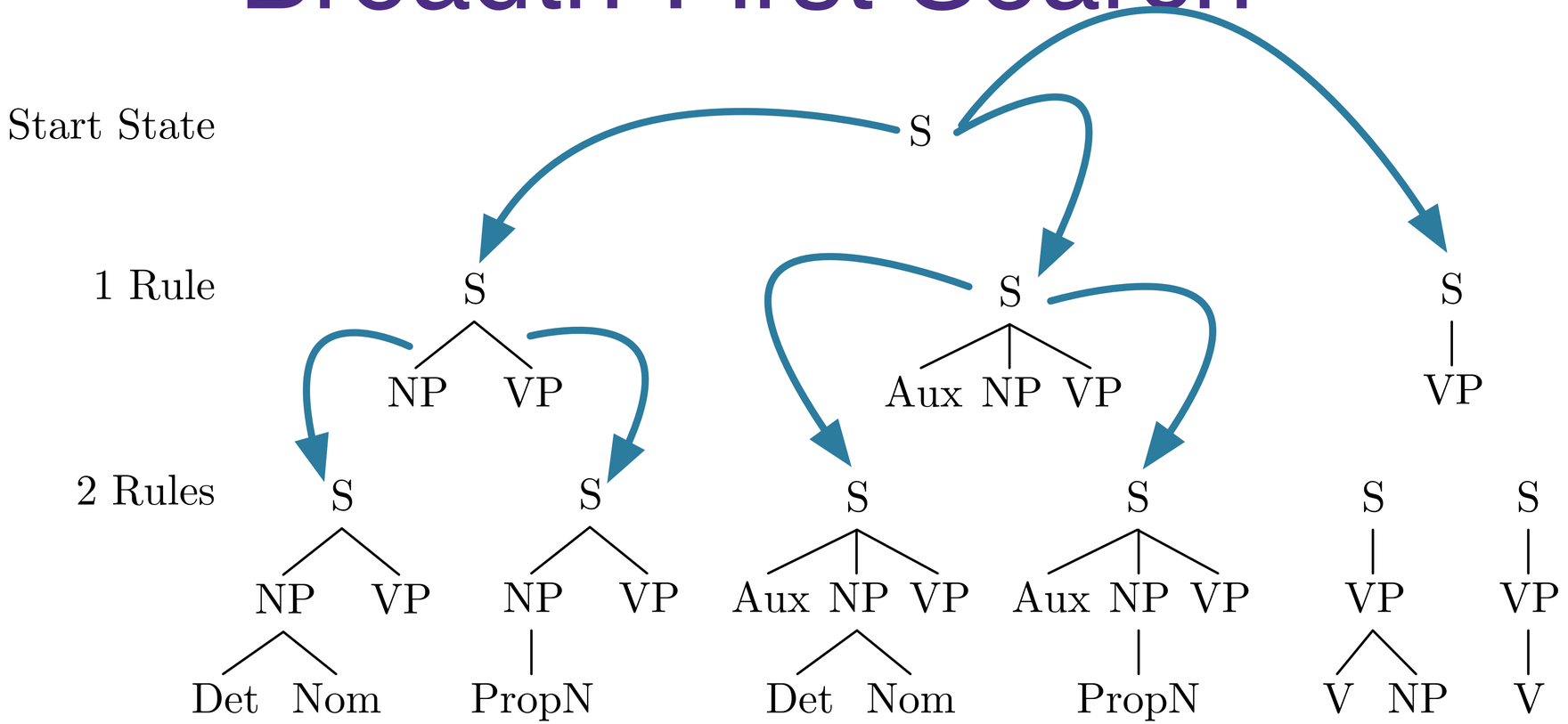

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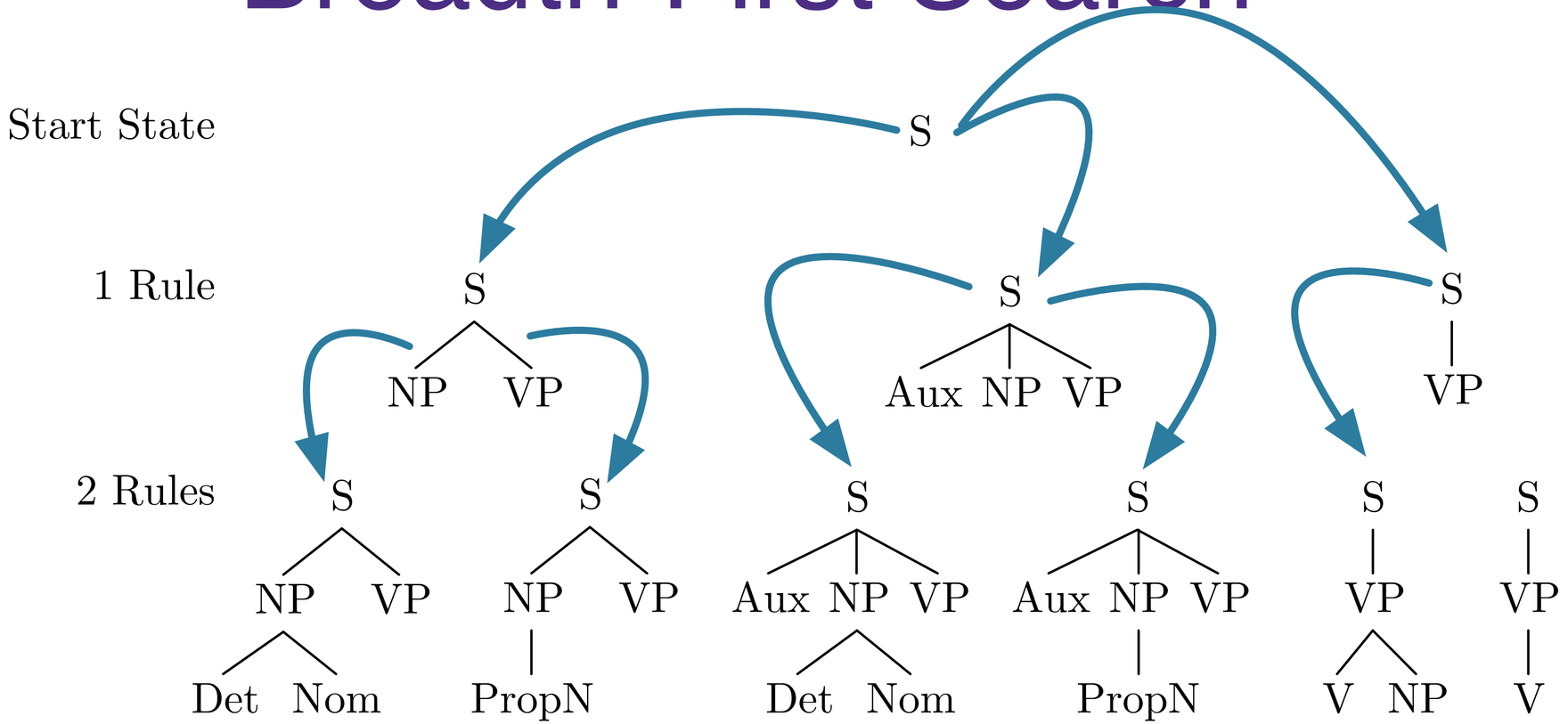

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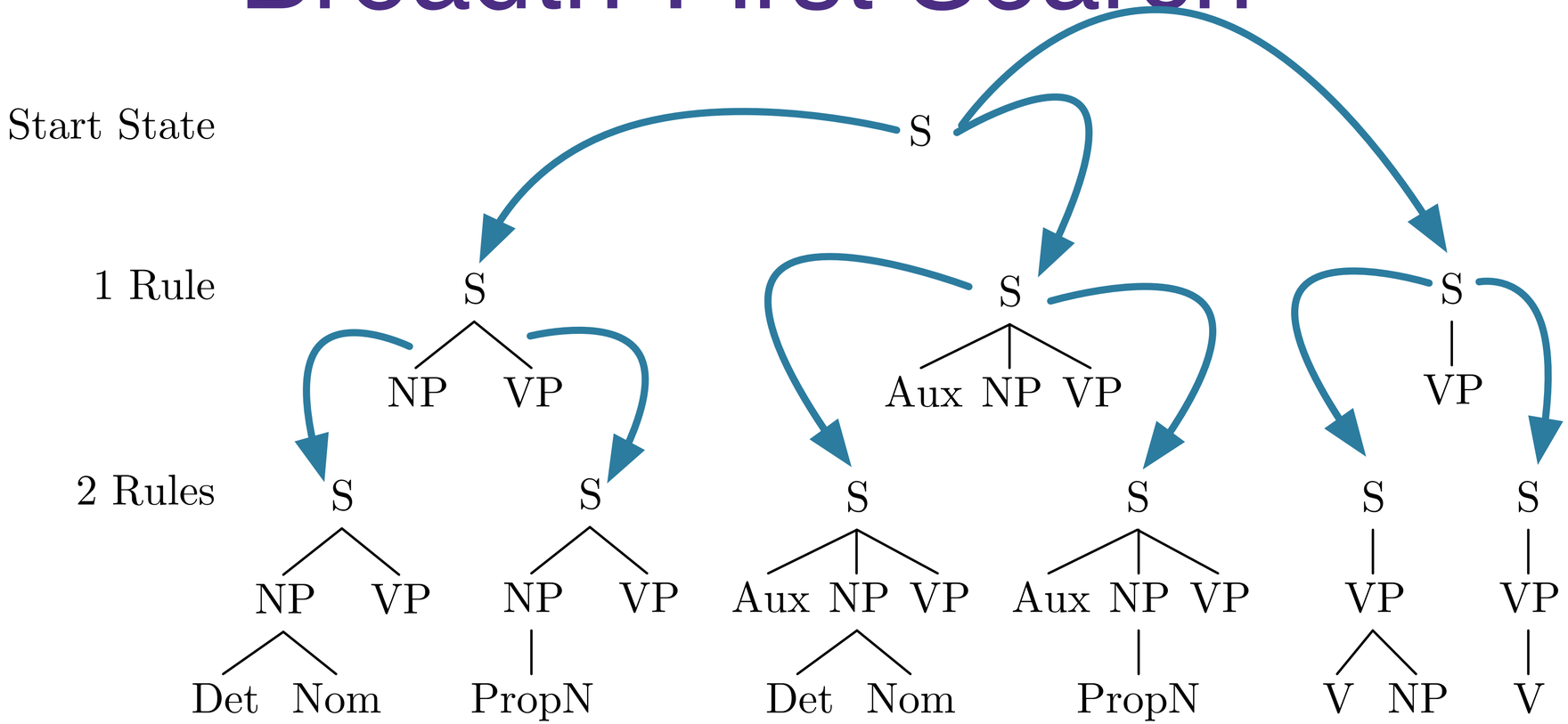

Pros and Cons of Top-down Parsing

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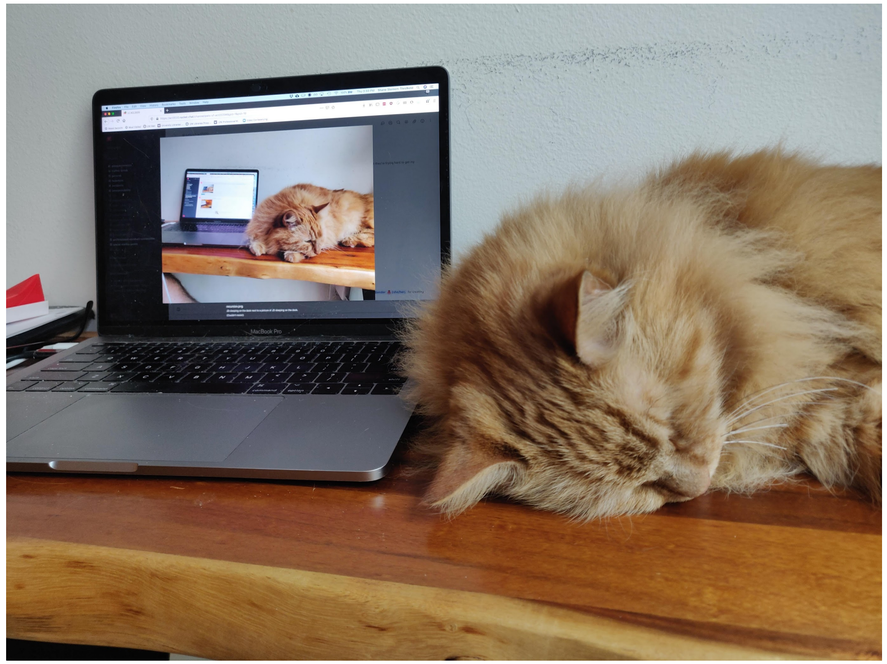

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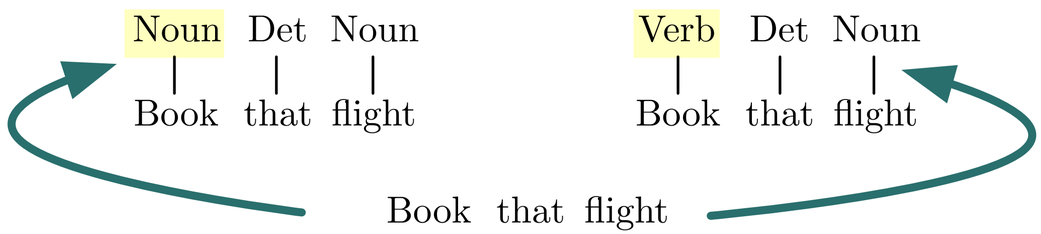
- Stop when spanned by S, or no more rules apply

Book that flight


Noun Det Noun
Book that flight
Verb Det Noun
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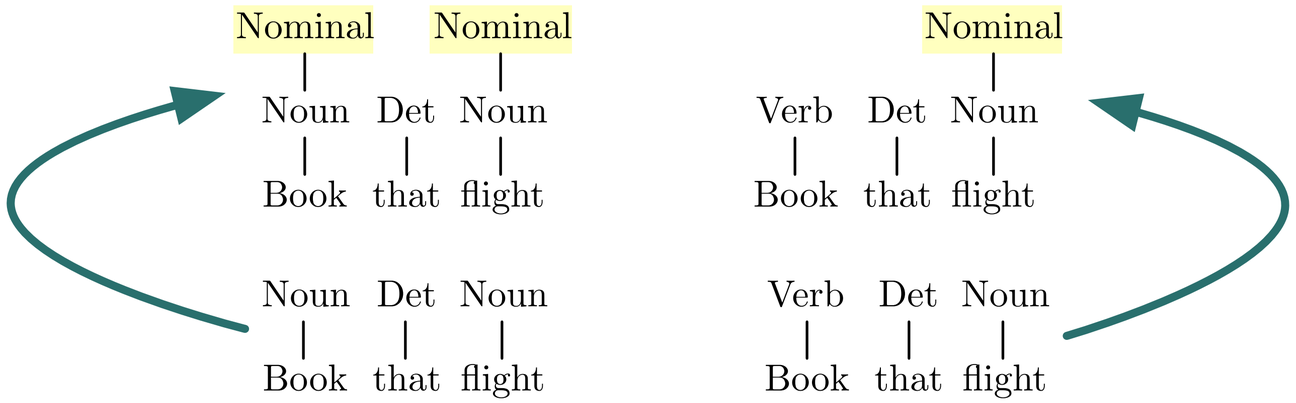

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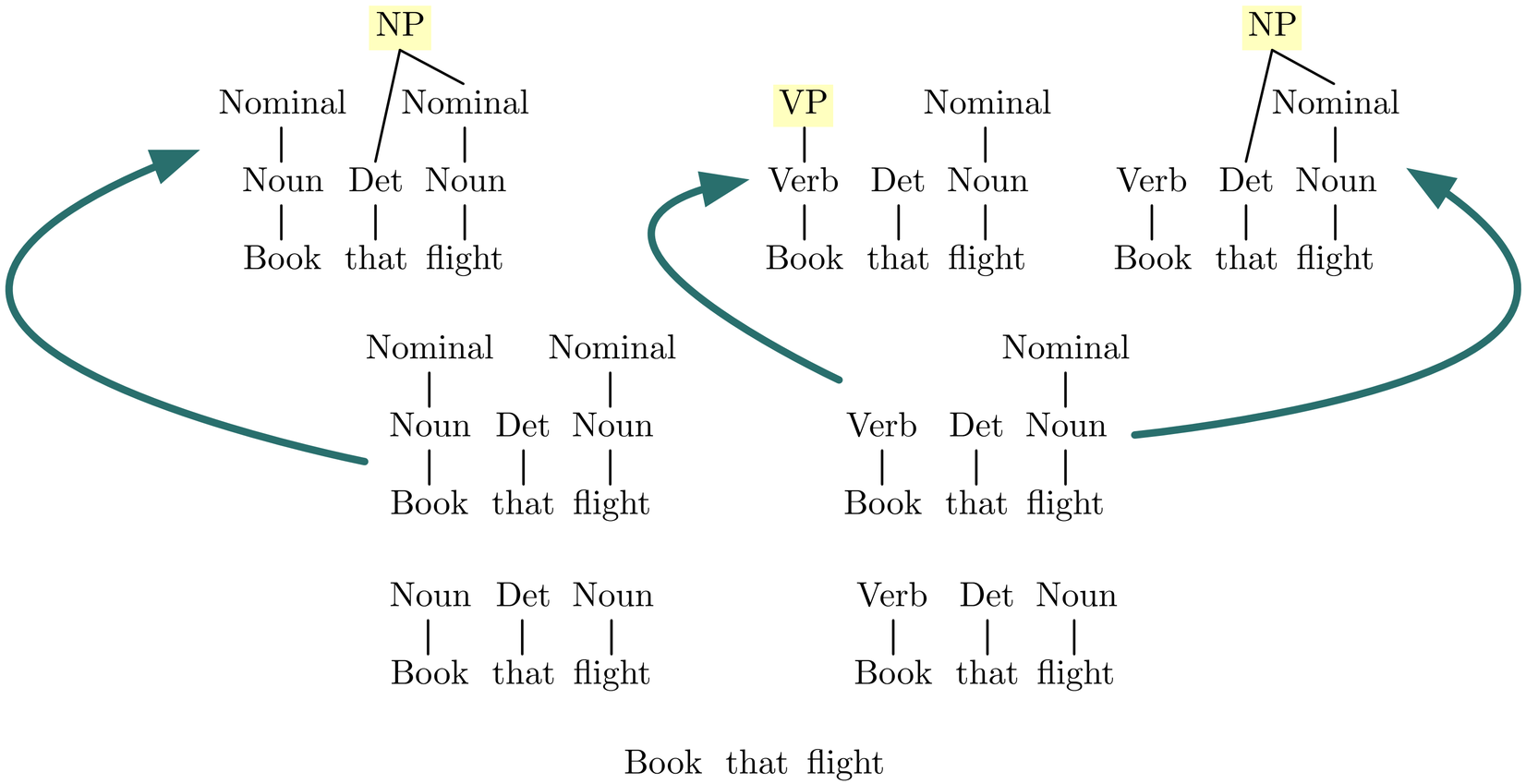
NP
Nominal
Nominal
Noun
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Noun
Book
that
flight
VP
Nominal
Verb
Det
Noun
Book
that
flight
NP
Nominal
Verb
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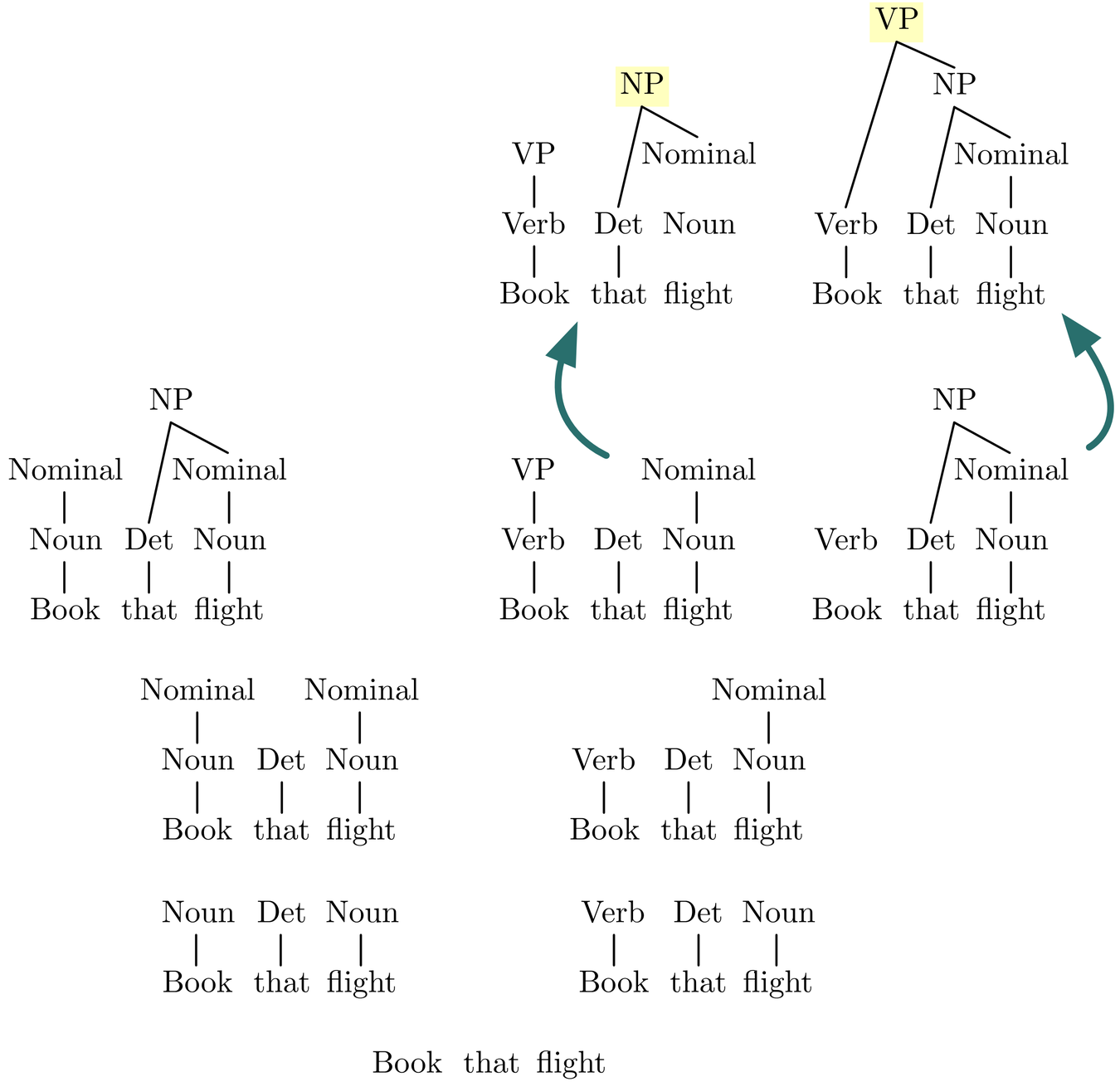
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- Cons:
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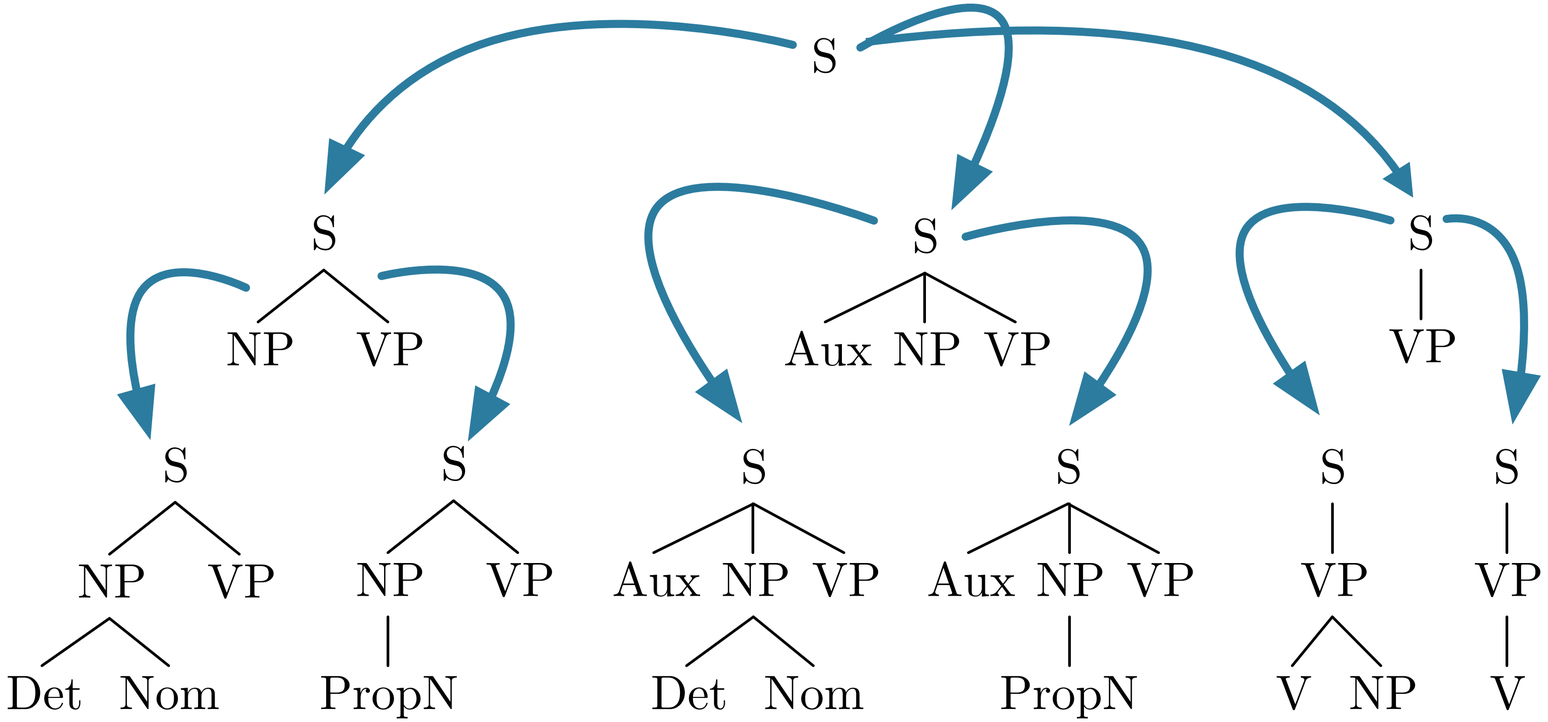
Recap: Parsing as Search



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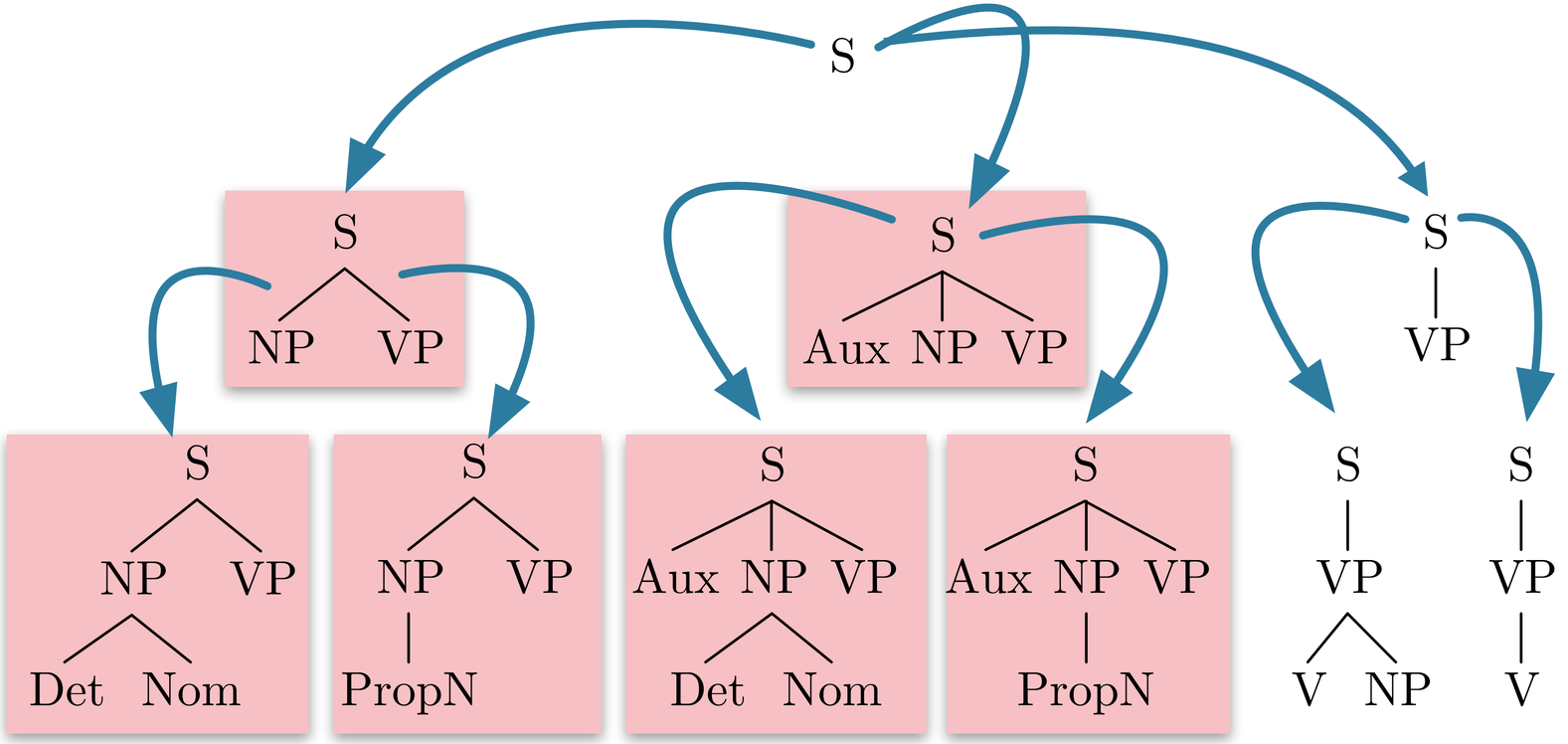
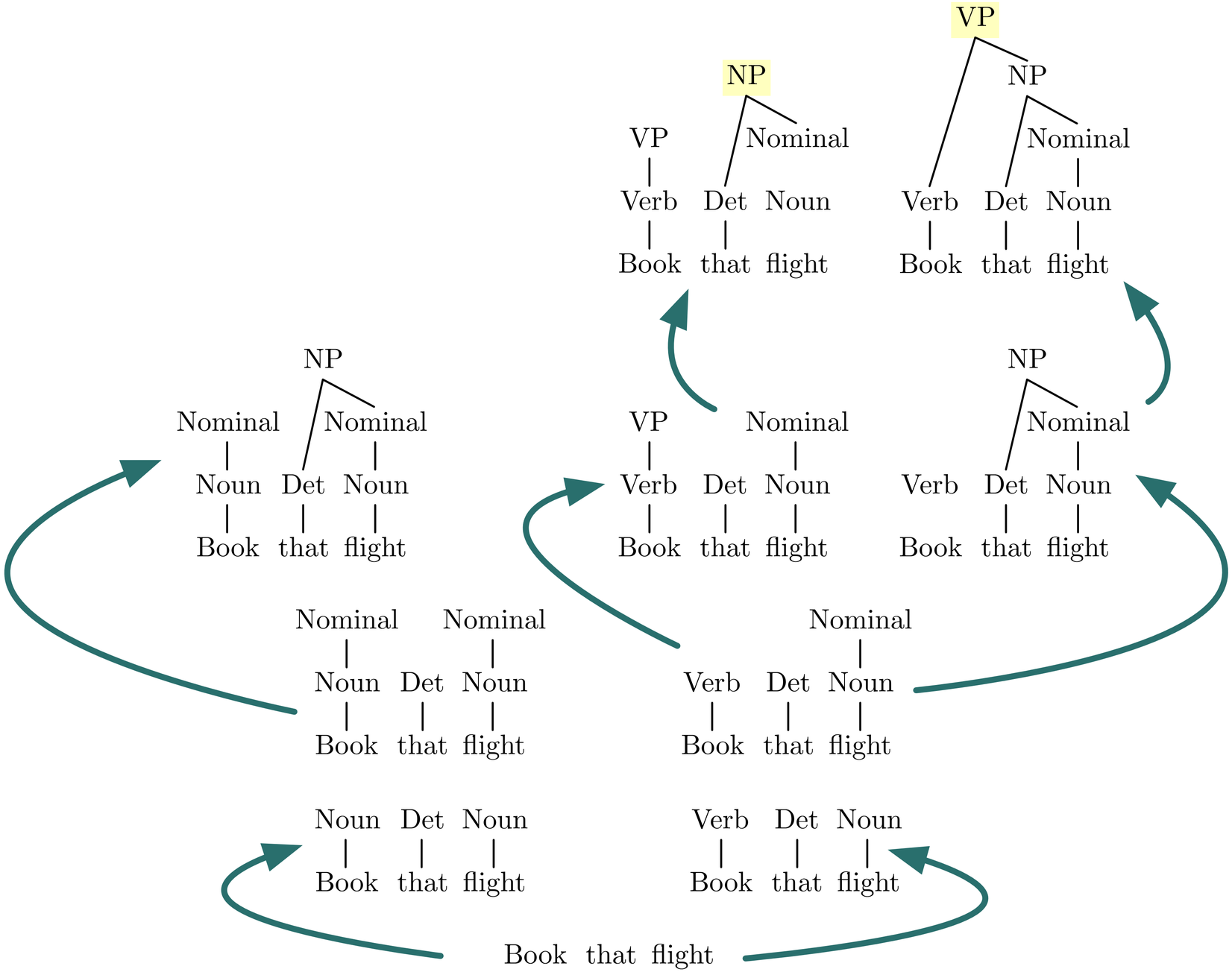


None of these nodes can produce *book* as first terminal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Nominal
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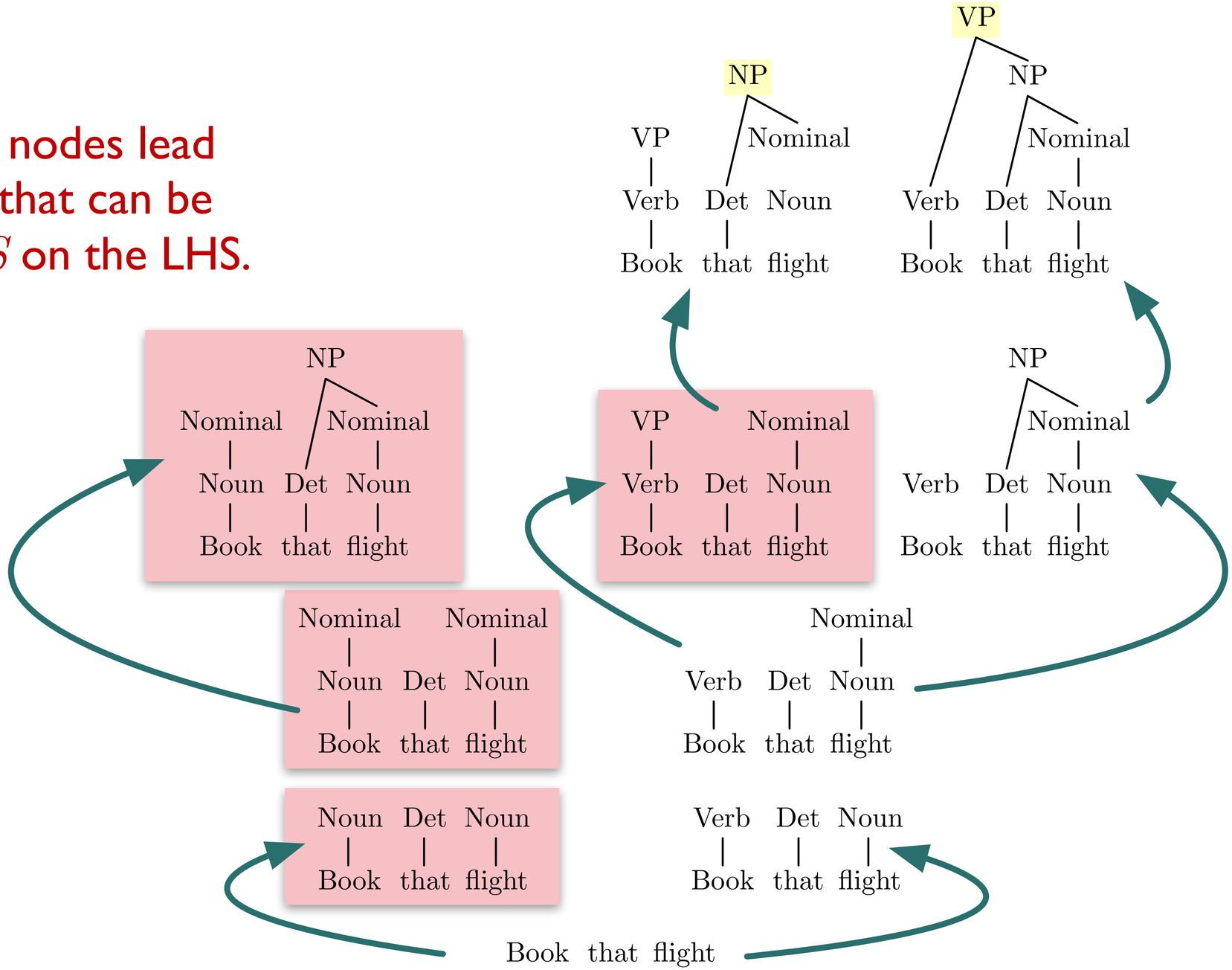
Book that flight

None of these nodes lead lead to a RHS that can be combined with S on the LHS.



Parsing Challenges

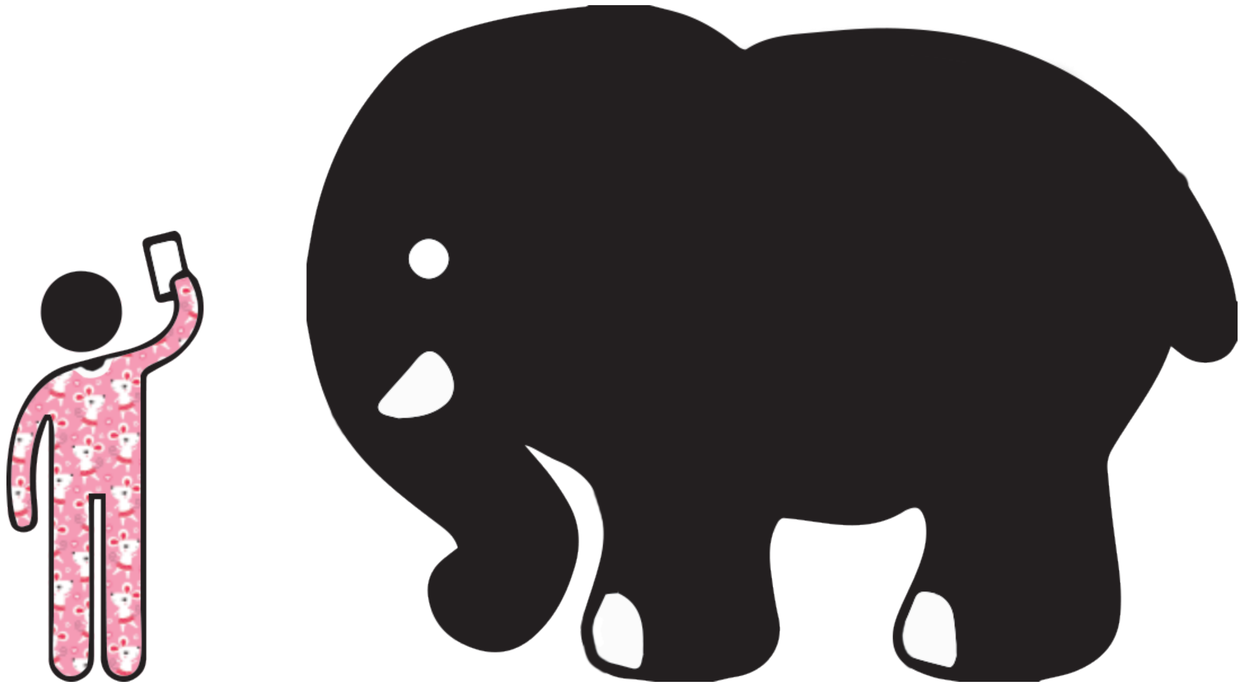
- Parsing-as-Search
- **Parsing Challenges**
  - **Ambiguity**
  - Repeated Substructure
  - Recursion
- Strategy: Dynamic Programming
- Grammar Equivalence
- CKY parsing algorithm

Parsing Ambiguity

- **Lexical Ambiguity**:
  - Book/NN → *I left a **book** on the table.*
  - Book/VB → ***Book** that flight.*
- Structural Ambiguity

Attachment Ambiguity

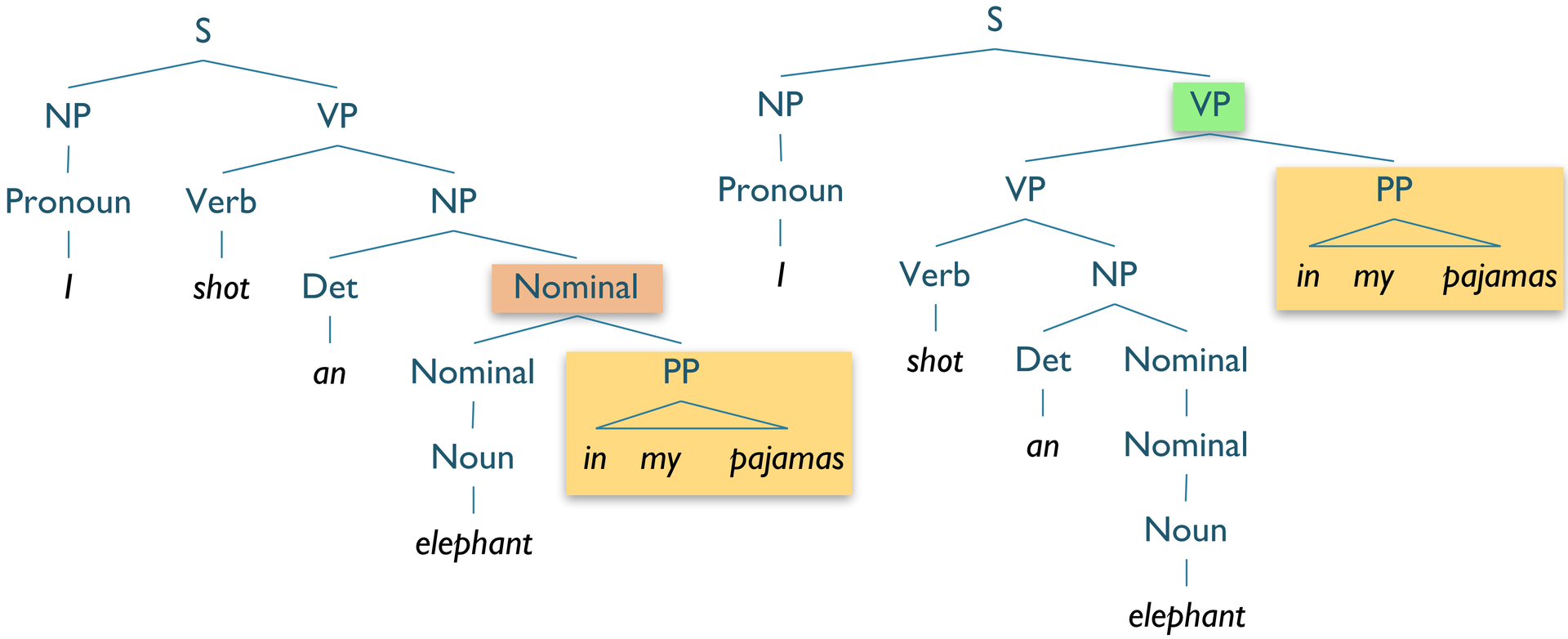
“One morning, I shot an elephant in my pajamas.



Attachment Ambiguity

“One morning, I shot an elephant in my pajamas.
How he got into my pajamas, I’ll never know.” — *Groucho Marx*

Attachment Ambiguity



“We saw the Eiffel Tower flying to Paris”

“We saw the Eiffel Tower flying to Paris”

Coordination Ambiguity:

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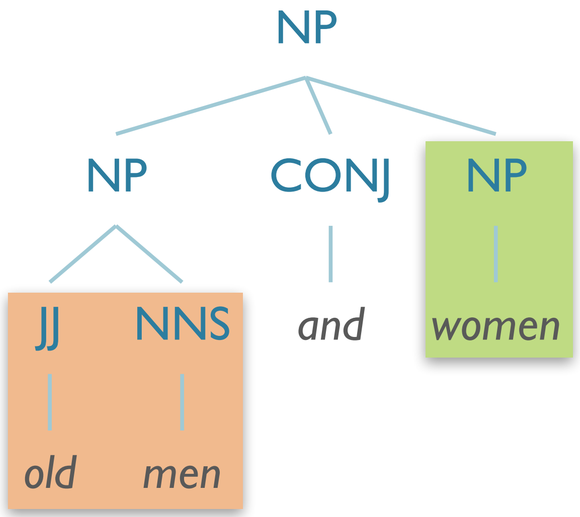
[old men] *and* ***[women]***



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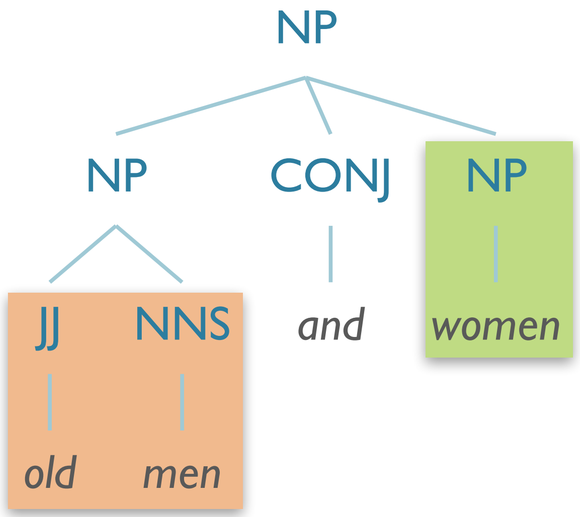
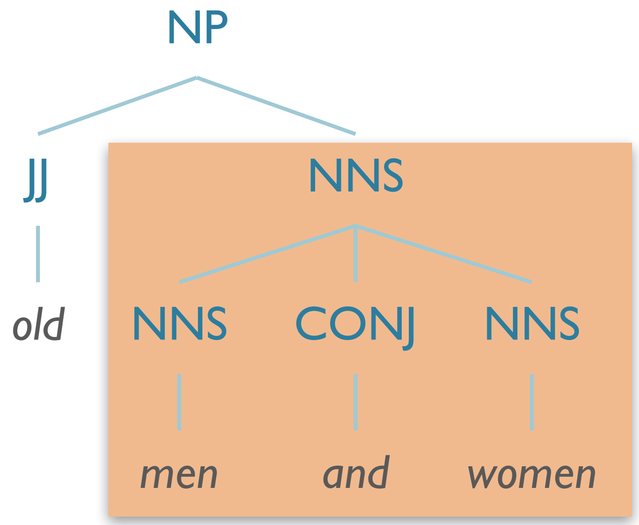


[old ***[men and women]****]*



Local vs. Global Ambiguity

- ***Local*** ambiguity:
  - Ambiguity that cannot contribute to a full, valid parse
  - e.g. *Book/NN* in *“Book that flight”*

Local vs. Global Ambiguity

- ***Local*** ambiguity:
  - Ambiguity that cannot contribute to a full, valid parse
  - e.g. *Book/NN* in *“Book that flight”*
- ***Global*** ambiguity
  - Multiple valid parses

Why is Ambiguity a Problem?

- *Local* ambiguity:
  - increased processing time

- *Global* ambiguity:
  - Would like to yield only “reasonable” parses
  - Ideally, the one that was intended*

Solution to Ambiguity?

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- ***Dis***ambiguation!

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- ***Dis***ambiguation!
- Different possible strategies to select correct interpretation:

Disambiguation Strategy: Statistical

- Some prepositional structs more likely to attach high/low

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  - *John was thought to have been seen by Mary*
    - Mary could be doing the seeing or thinking — seeing more likely

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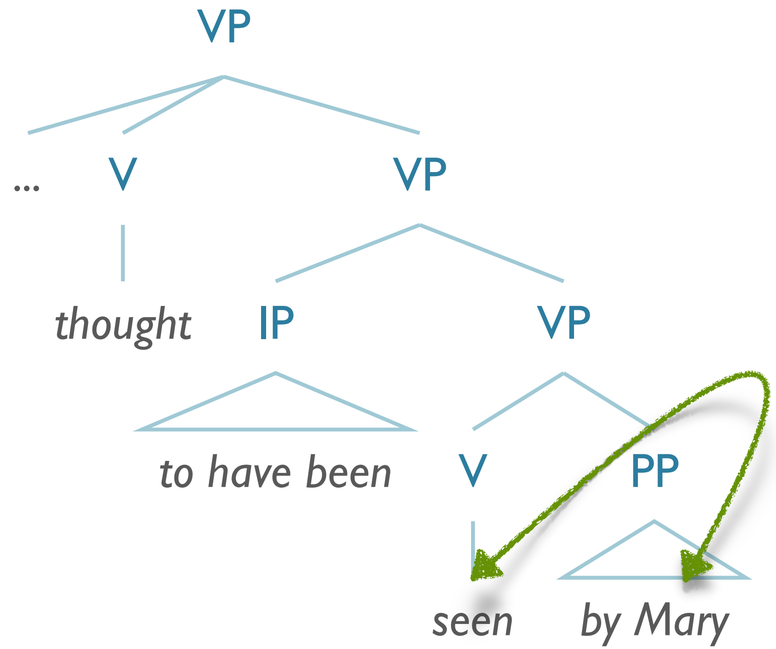

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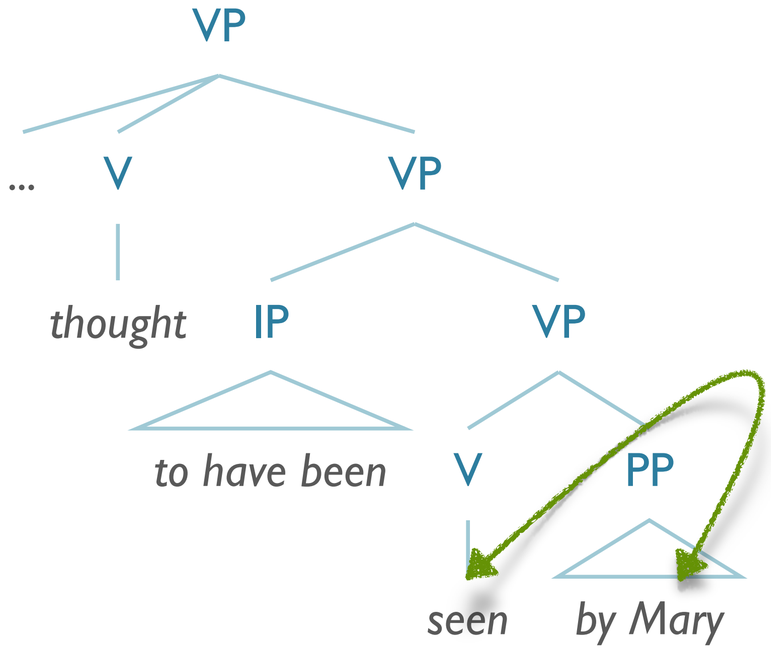
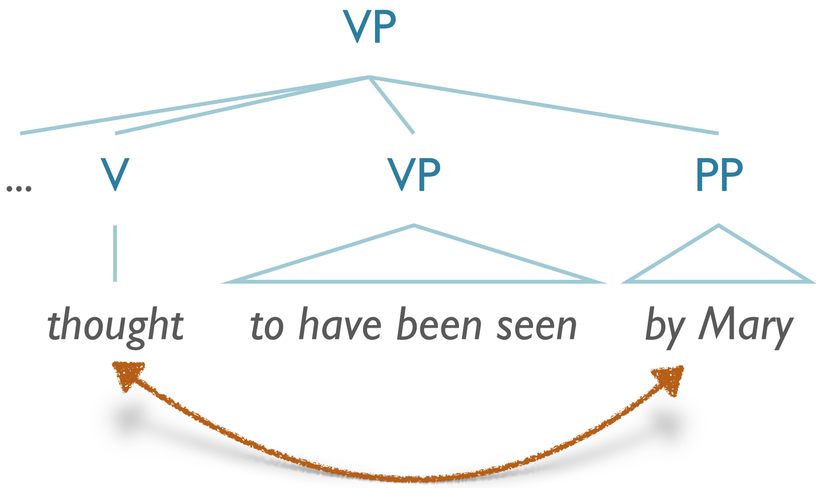


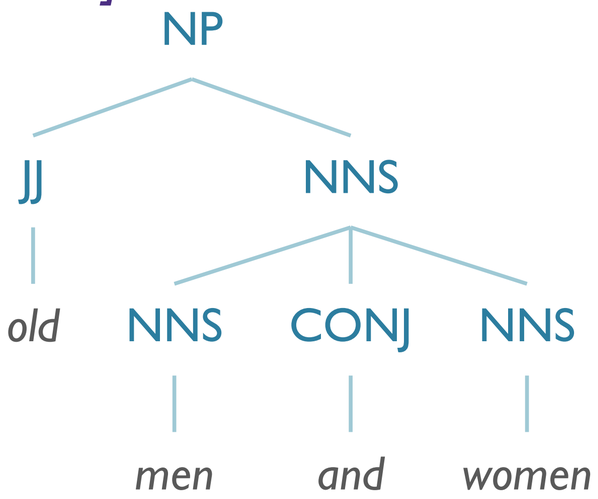


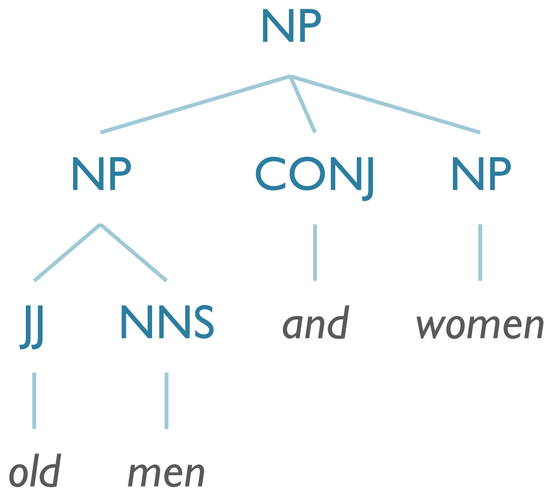
Disambiguation Strategy: Statistical

- Some phrases more likely overall

Disambiguation Strategy: Statistical

- Some phrases more likely overall
  - *[old [men and women]] is a more common construction than [old men] and [women]*





Disambiguation Strategy: Semantic

- Some interpretations we know to be semantically impossible

Disambiguation Strategy: Semantic

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  - *Eiffel tower* as subject of *fly*

Disambiguation Strategy:
Pragmatic

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Pragmatic

- Some interpretations are possible, unlikely given world knowledge
  - e.g. elephants and pajamas

Incremental Parsing and Garden Paths

- Idea: model *left-to-right* nature of (English) text
- Problem: “garden path” sentences

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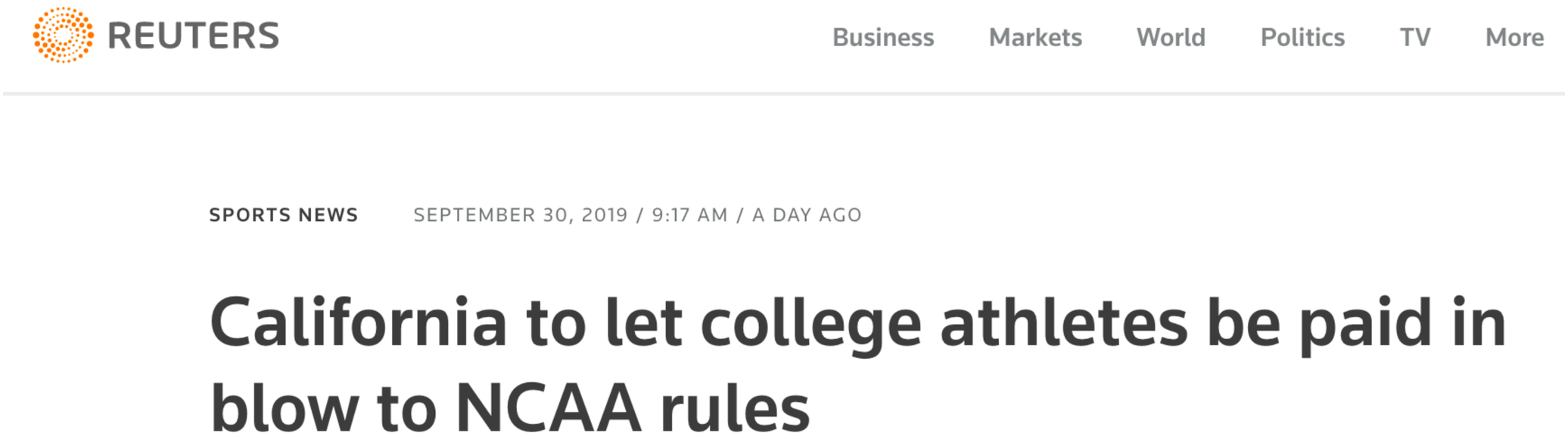

https://www.reuters.com/article/us-sport-california-education/california-to-let-college-athletes-be-paid-in-blow-to-ncaa-rules-idUSKBN1WF1SR

Disambiguation Strategy:

- Alternatively, keep all parses

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- Alternatively, keep all parses
  - *(Might even be the appropriate action for some jokes)*

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  - Globally bad parses can construct good subtrees
  - …will reconstruct along another branch
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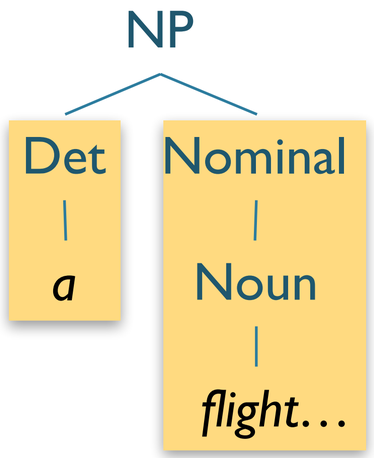
- Efficient parsing techniques require storage of partial solutions
- Example: *a flight from Indianapolis to Houston on TWA*

Shared Sub-Problems



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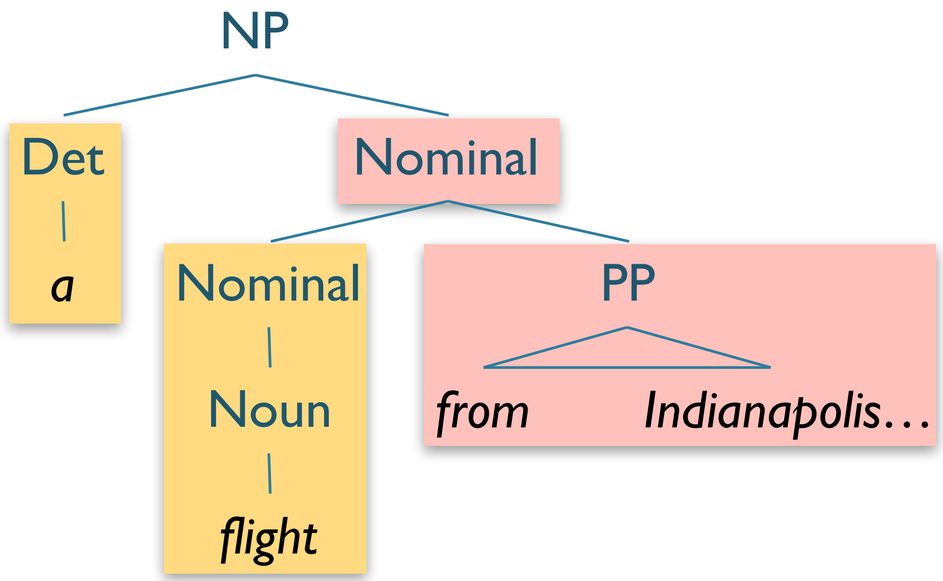

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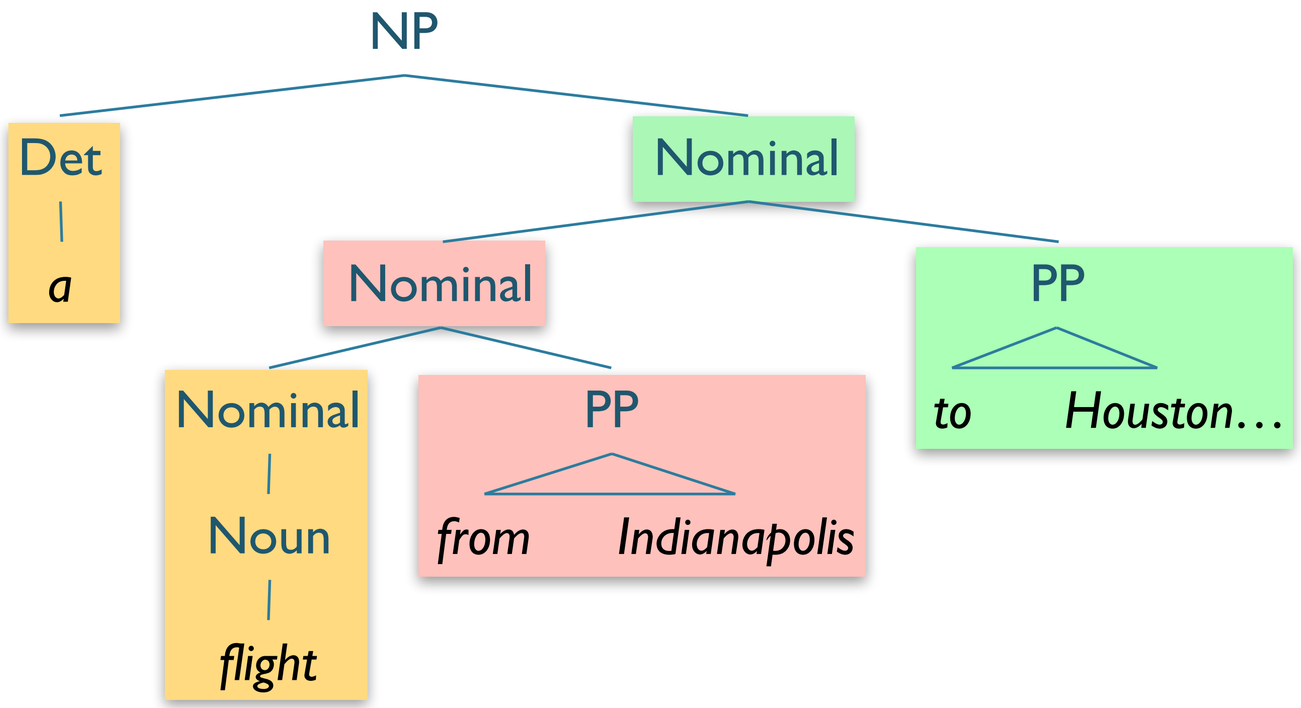

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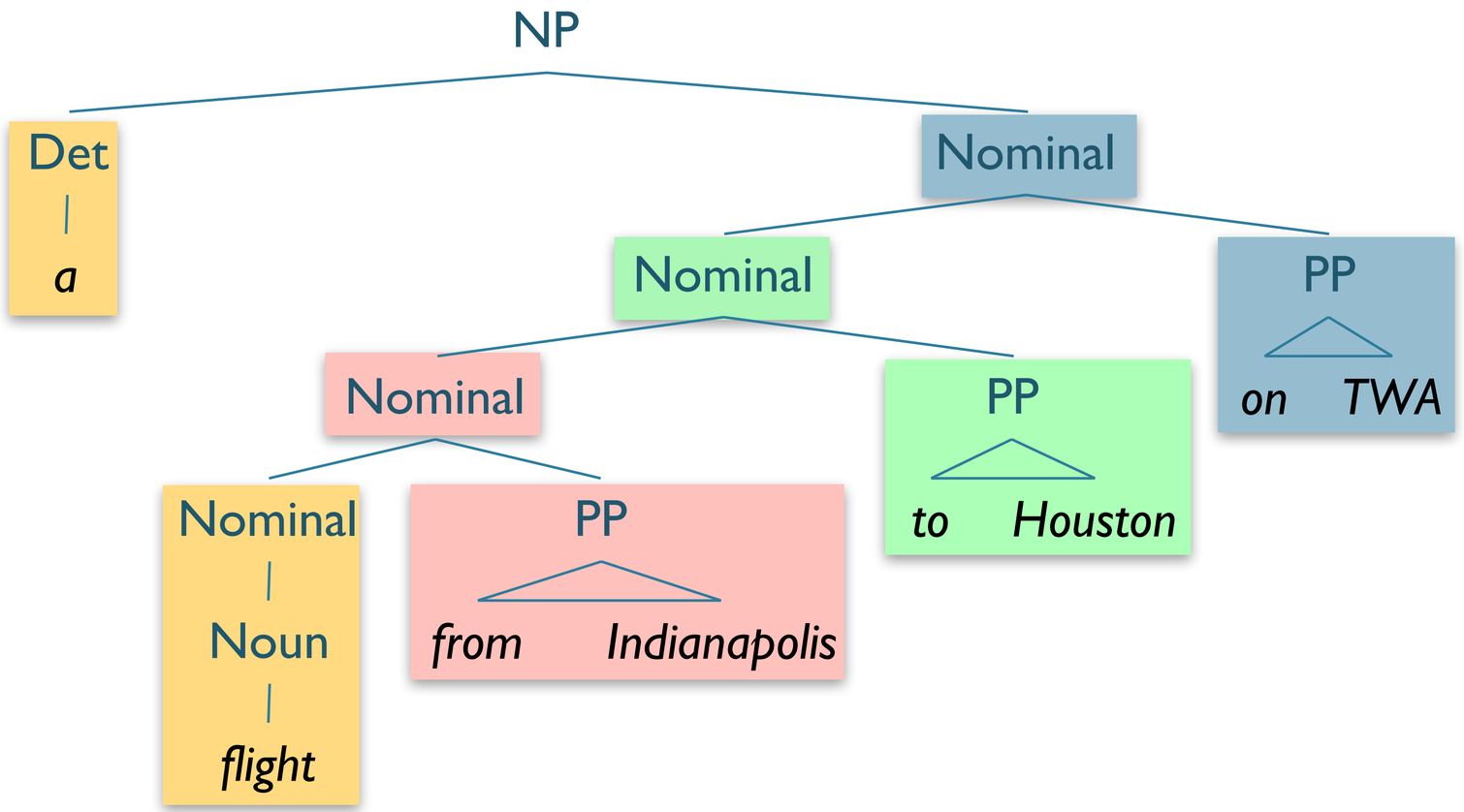

Parsing Challenges

- Parsing-as-Search
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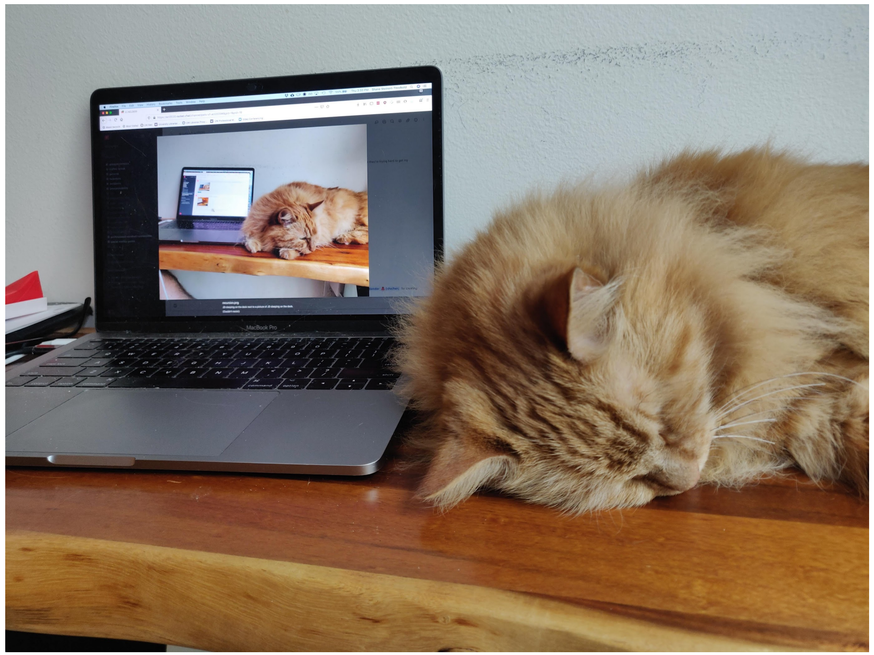
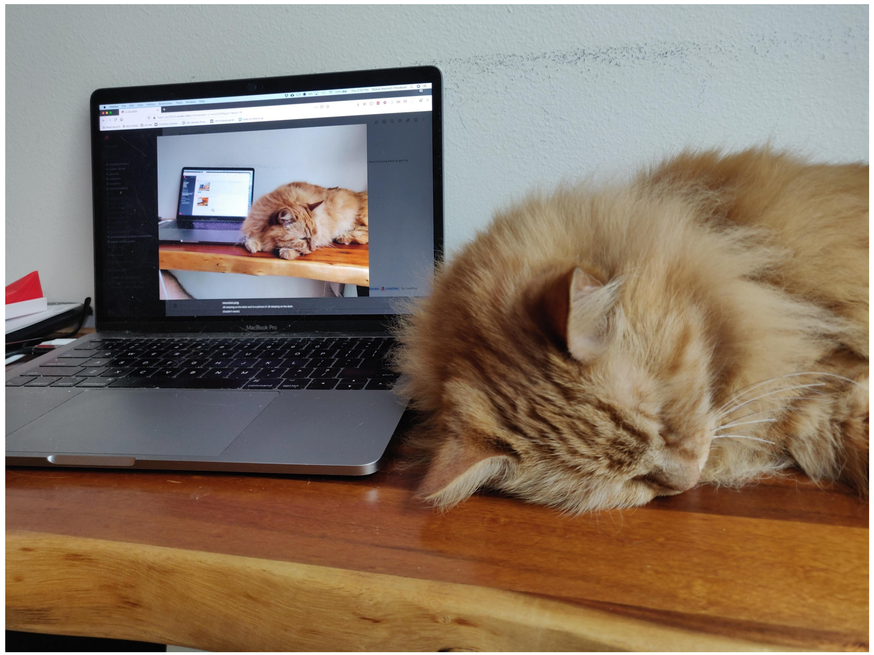
Recursion

- Many grammars have recursive rules
  - $S \rightarrow S\ Conj\ S$



Recursion

- Many grammars have recursive rules
  - $S \rightarrow S\ Conj\ S$
- In search approaches, recursion is problematic
  - Can yield infinite searches
  - Top-down especially vulnerable



Roadmap

- Parsing-as-Search
- Parsing Challenges
- **Strategy: Dynamic Programming**
- Grammar Equivalence
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Dynamic Programming

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Dynamic Programming

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- Insight:
  - Global parse composed of sub-parses
  - Can record these sub-parses and re-use
- Dynamic programming avoids repeated work by recording the subproblems
  - Here, stores subtrees

Parsing with Dynamic Programming

- Avoids repeated work
- Allows implementation of (relatively) efficient parsing algorithms
  - Polynomial time in input length
  - Typically cubic (n^3) or less

Parsing with Dynamic Programming

- Avoids repeated work
- Allows implementation of (relatively) efficient parsing algorithms
  - Polynomial time in input length
  - Typically cubic (n^3) or less
- Several different implementations
  - Cocke-Kasami-Younger (CKY) algorithm
  - Earley algorithm
  - Chart parsing

Roadmap

- Parsing-as-Search
- Parsing Challenges
- Strategy: Dynamic Programming
- **Grammar Equivalence**
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Grammar Equivalence and Form

- ***Weak*** Equivalence
  - **Accepts** same language
  - May produce **different** structures

- ***Strong*** Equivalence
  - Accepts same language
  - Produces **same** structures

Grammar Equivalence and Form

Grammar Equivalence and Form

- Reason?
  - We can create a weakly-equivalent grammar that allows for greater efficiency
  - This is required by the CKY algorithm

Chomsky Normal Form (CNF)

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- All productions are of the form:
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- Most of our grammars are not of this form:
  - $S \rightarrow \mathit{Wh\text{-}NP}\ \mathit{Aux}\ \mathit{NP}\ \mathit{VP}$
- Need a general conversion procedure

CNF Conversion

Hybrid productions:

$$INF\text{-}VP \rightarrow \textbf{to}\ VP$$

Unit productions:

$$A \rightarrow B$$

Long productions:

$$A \rightarrow B\ C\ D\ \ldots$$

CNF Conversion: Hybrid Productions

- Hybrid production:
  - Replace all terminals with dummy non-terminal
  - $INF\text{-}VP \rightarrow \textbf{to}\ VP$
    - $INF\text{-}VP \rightarrow TO\ VP$
    - $TO \rightarrow \textbf{to}$

CNF Conversion: Unit Productions

- Unit productions:
  - Rewrite RHS with RHS of all derivable, non-unit productions
  - If $A \overset{*}{\Rightarrow} B$ and $B \rightarrow \gamma$, **add** $A \rightarrow \gamma$ [where γ is any non-unit RHS]
  - **[***$A \overset{*}{\Rightarrow} B$: B is reachable from A by a sequence of unit productions***]**
- *Nominal* → *Noun*, *Noun* → **dog**
  - *Nominal* → **dog**
  - *Noun* → **dog**
- NB: this example has γ as a single terminal, but the rule applies to all non-unit RHS.

CNF Conversion: Long Productions

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- Long productions

$$S \rightarrow Aux\ NP\ VP$$

CNF Conversion: Long Productions

- Long productions

$S \rightarrow Aux\ NP\ VP$	
$S \rightarrow \boldsymbol{X1}\ VP$	$\boldsymbol{X1} \rightarrow Aux\ NP$

CNF Conversion: Long Productions

- Long productions

$S \rightarrow Aux\ NP\ VP$

$S \rightarrow \mathbf{X1}\ VP \qquad \mathbf{X1} \rightarrow Aux\ NP$

- Introduce unique nonterminals, and spread over rules

CNF Conversion

Convert terminals in hybrid rules to dummy non-terminals

Convert unit productions

Binarize long production rules

$\mathcal{L}_1$ Grammar	$\mathcal{L}_1$ in CNF
$S \rightarrow NP\ VP$	$S \rightarrow NP\ VP$
$S \rightarrow Aux\ NP\ VP$	$S \rightarrow X1\ VP$
	$X1 \rightarrow Aux\ NP$
$S \rightarrow VP$	$S \rightarrow book \mid include \mid prefer$
	$S \rightarrow Verb\ NP$
	$S \rightarrow X2\ PP$
	$S \rightarrow Verb\ PP$
	$S \rightarrow VP\ PP$
$NP \rightarrow Pronoun$	$NP \rightarrow I \mid she \mid me$
$NP \rightarrow Proper\text{-}Noun$	$NP \rightarrow TWA \mid Houston$
$NP \rightarrow Det\ Nominal$	$NP \rightarrow Det\ Nominal$
$Nominal \rightarrow Noun$	$Nominal \rightarrow book \mid flight \mid meal \mid money$
$Nominal \rightarrow Nominal\ Noun$	$Nominal \rightarrow Nominal\ Noun$
$Nominal \rightarrow Nominal\ PP$	$Nominal \rightarrow Nominal\ PP$
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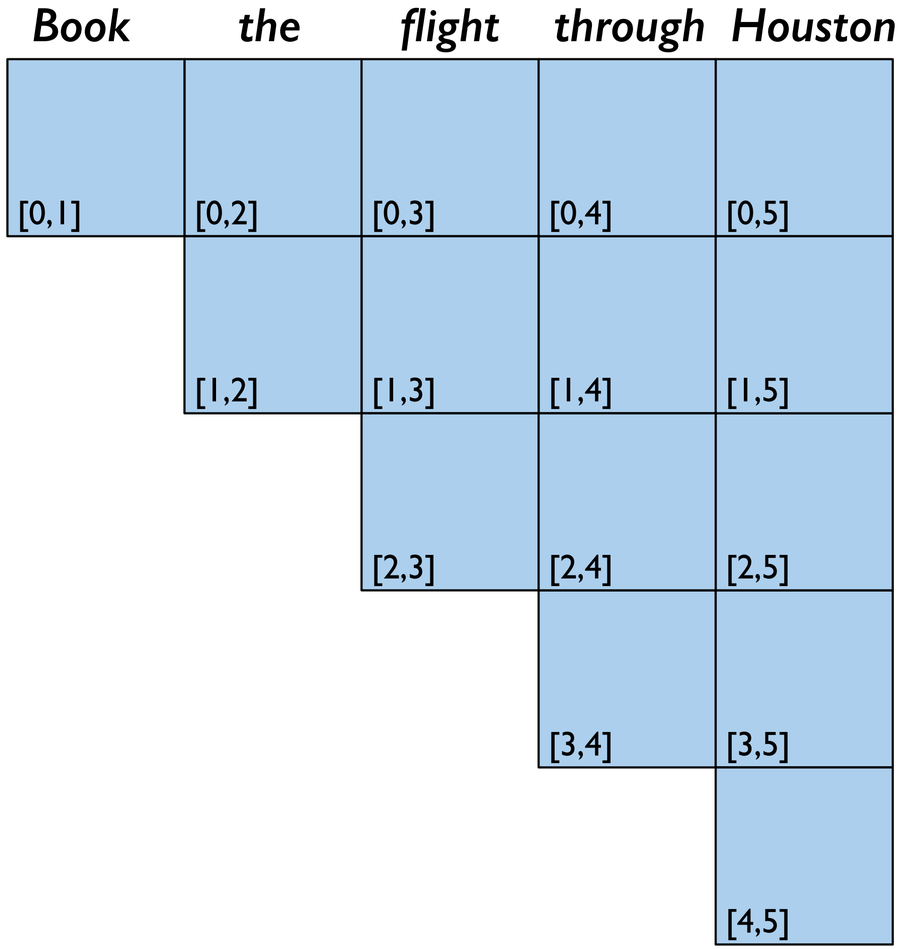
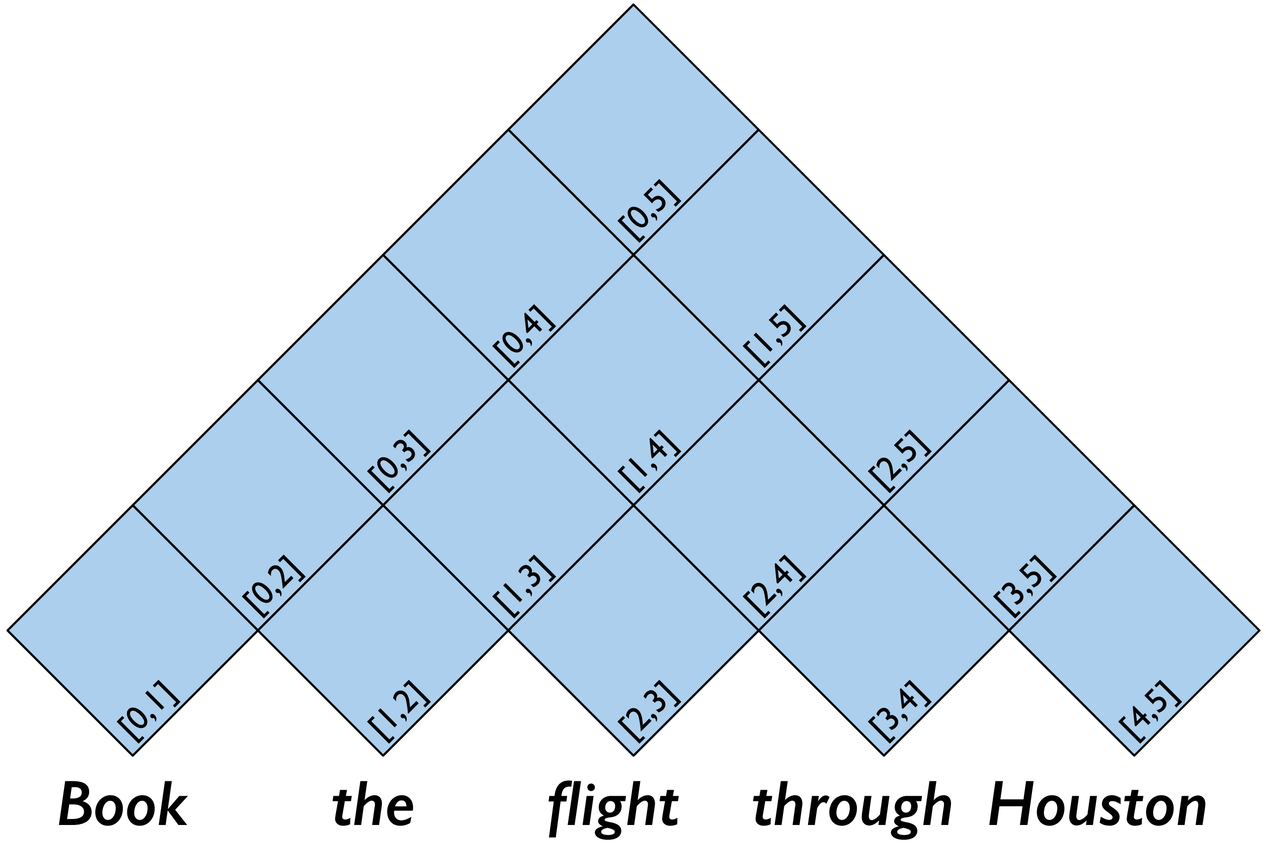
CKY Parsing

- (Relatively) efficient parsing algorithm
- Based on tabulating substring parses to avoid repeat work
- Approach:
  - Use CNF Grammar
  - Build an $(n + 1) \times (n + 1)$ matrix to store subtrees
    - Upper triangular portion
  - Incrementally build parse spanning whole input string

CKY Matrix


Book
the
flight
through
Houston
[0,1]
[0,2]
[0,3]
[0,4]
[0,5]
[1,2]
[1,3]
[1,4]
[1,5]
[2,3]
[2,4]
[2,5]
[3,4]
[3,5]
[4,5]

CKY Matrix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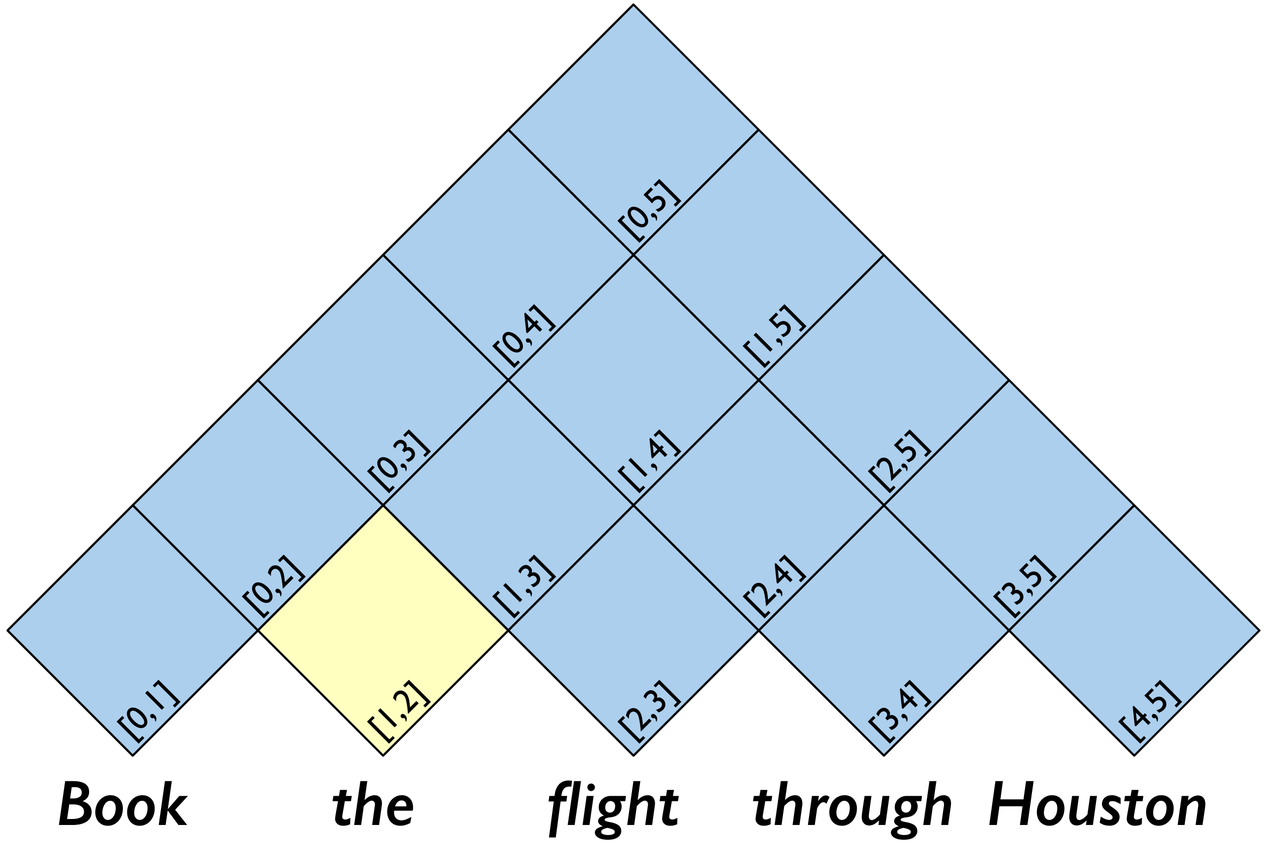
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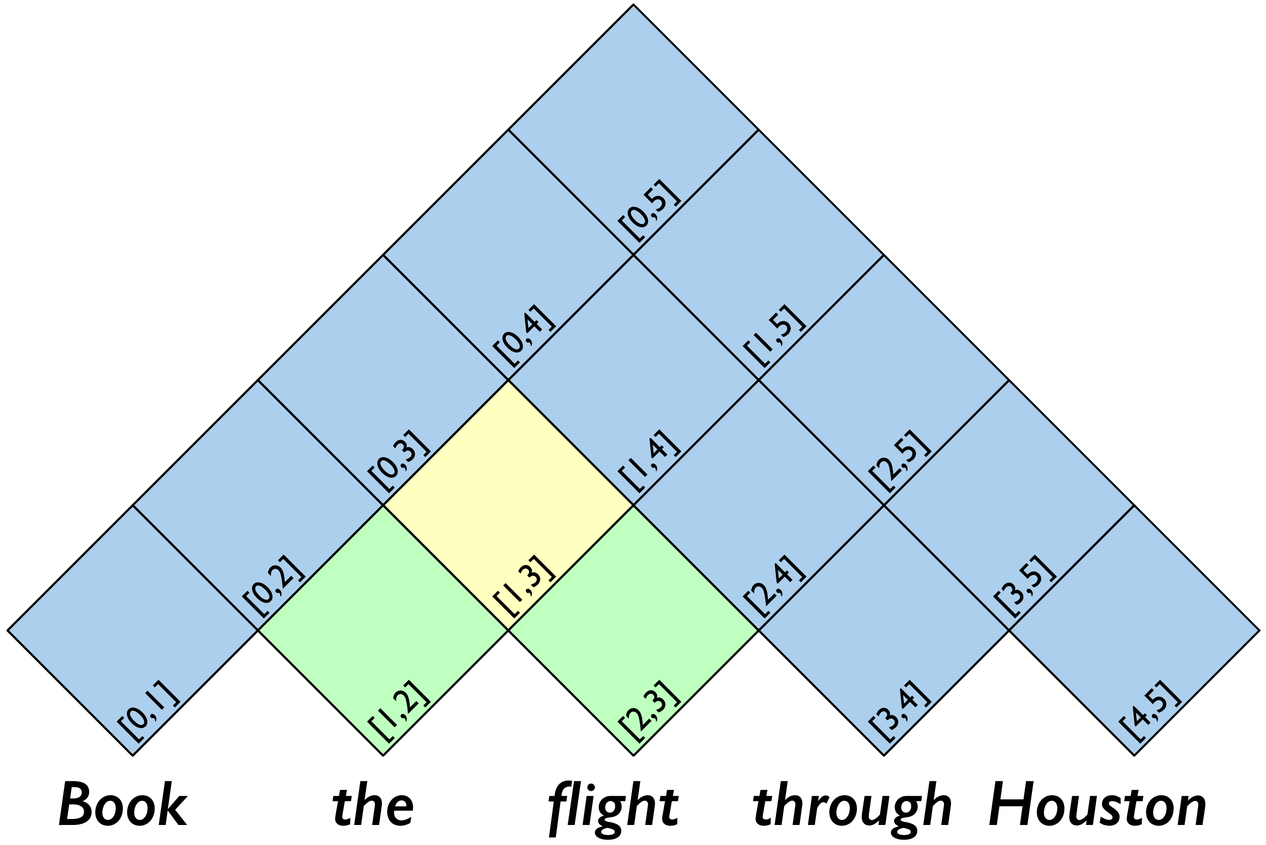
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Dynamic Programming in CKY

- Key idea:
  - for $i < k < j$
  - …and a parse spanning substring [i, j]
  - There is a k such that there are parses spanning [i, k] and [k, j]
  - We can construct parses for whole sentences by building from these partial parses
- So to have a rule $A \rightarrow B\ C$ in [i, j]
  - Must have B in [i, k] and C in [k, j] for some $i < k < j$
  - CNF forces this for all $j > i + 1$