# CKY Parsing & CNF Conversion

LING 571 — Deep Processing Techniques for NLP
October 5, 2022
Shane Steinert-Threlkeld

#### Announcements

- HW #1 due tonight at 11:59pm.
- Python on Patas: installed versions `ls /opt | grep python`. E.g., invoke by:
  - /opt/python-3.6/bin/python3
  - nltk is installed.
- [For personal projects, but not 571 HW, you can use the latest of everything via Anaconda (download with wget).]
- When in doubt, use full paths for everything (python binary, file names, etc)
- check\_hwX.sh: invoke from your local directory (for permission reasons)

# Type Hinting in Python

Supported in ≥3.6 [tutorial]

```
from typing import List
from nltk.grammar import Production

def fix_hybrid_production(hybrid_prod: Production) -> List[Production]:
```

•••

```
*always* type-annotate your Python

the cost to you is minimal (you have to type a few extra characters)

the benefits to you are great (documentation + help from your IDE / editor) *even if you never run a static type checker*

it's such a no-brainer

from typing import List

def process(xs: List[int]) -> None:

xs.

append
clear
copy
count
extend
index
index
insert
pop
remove
reverse
sort
add_
```

Joel Grus ♥ 📓
@joelgrus

# Type Hinting in Python

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```
from typing import List
from nltk.grammar import Production

def fix_hybrid_production(hybrid_prod: Production) -> List[Production]:
```

Also available in PyCharm through docstrings and/or comments:

```
def fix_hybrid_productions(hybrid_prod):
    """

This function takes a hybrid production and
    returns a list of new CNF productions
    :type hybrid_prod: Production
    :rtype: list[Production]
    """
```

#### Joke of the Week (PP Attachment Ambiguity)

#### tott @crazytott · Oct 5

A cop just knocked on my door and told me that my dogs were chasing people on bikes???? Wtf??? My dogs don't even own bikes tf

### Roadmap

- Parsing-as-Search
- Parsing Challenges
- Strategy: Dynamic Programming
- Grammar Equivalence
- CKY parsing algorithm

# Computational Parsing

- Given a body of (annotated) text, how can we derive the grammar rules of a language, and employ them in automatic parsing?
  - Treebanks & PCFGs
- Given a grammar, how can we derive the analysis of an input sentence?
  - Parsing as search
  - CKY parsing
    - Conversion to CNF

# What is Parsing?

- CFG parsing is the task of assigning trees to input strings
  - ullet For any input A and grammar G
    - ...assign  $\geq 0$  parse trees T that represent its syntactic structure, and...
    - Cover all and only the elements of A
    - Have, as root, the start symbol S of G
    - ...do not necessarily pick one single (or correct) analysis
- Subtask: Recognition
  - Given input A, G is A in language defined by G or not?

#### Motivation

- Is this sentence in the language i.e. is it "grammatical?"
  - \* I prefer United has the earliest flight.
  - FSAs accept regular languages defined by finite-state automata.
  - Our parsers accept languages defined by CFG (equiv. pushdown automata).

#### Motivation

- Is this sentence in the language i.e. is it "grammatical?"
  - \* I prefer United has the earliest flight.
  - FSAs accept regular languages defined by finite-state automata.
  - Our parsers accept languages defined by CFG (equiv. pushdown automata).
- What is the syntactic structure of this sentence?
  - What airline has the cheapest flight?
  - What airport does Southwest fly from near Boston?
  - Syntactic parse provides framework for semantic analysis
    - What is the subject? Direct object?

# Parsing as Search

 Syntactic parsing searches through possible trees to find one or more trees that derive input

# Parsing as Search

- Syntactic parsing searches through possible trees to find one or more trees that derive input
- Formally, search problems are defined by:
  - Start state S
  - Goal state *G* (with a test)
  - Set of actions that transition from one state to another
    - "Successor function"
  - A path cost function

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- Path cost:
  - ...ignored for now.

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  - Partial solution to search problem (partial parse)

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- Search start node (initial state):
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  - Start symbol of CFG
- Goal node:
  - ullet Full parse tree: covering all of, and only the input, rooted at S

# Search Algorithms

- Depth First
  - Keep expanding nonterminals until they reach words
  - If no more expansions available, back up

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  - If no more expansions available, back up
- Breadth First
  - Consider all parses that expand a single nonterminal...
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- Other alternatives, if have associated path costs.

# Parse Search Strategies

- Two constraints on parsing:
  - Must start with the start symbol
  - Must cover exactly the input string

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- Two constraints on parsing:
  - Must start with the start symbol
  - Must cover exactly the input string
- Correspond to main parsing search strategies
  - Top-down search (Goal-directed)
  - Bottom-up search (Data-driven search)

Grammar	Lexicon
$S \rightarrow NP VP$	$Det \rightarrow that \mid this \mid a$
$S \rightarrow Aux NP VP$	$Noun \rightarrow book \mid flight \mid meal \mid money$
$S \rightarrow VP$	$Verb \rightarrow book \mid include \mid prefer$
$NP \rightarrow Pronoun$	$Pronoun \rightarrow I \mid she \mid me$
$NP \rightarrow Proper-Noun$	Proper-Noun → Houston   NWA
$NP \rightarrow Det\ Nominal$	$Aux \rightarrow does$
$Nominal \rightarrow Noun$	$Preposition \rightarrow from \mid to \mid on \mid near \mid through$

```
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     VP \rightarrow Verb NP PP
       VP \rightarrow Verb PP
         VP \rightarrow VP PP
   PP \rightarrow Preposition NP
```

Jurafsky & Martin, Speech and Language Processing, p.390

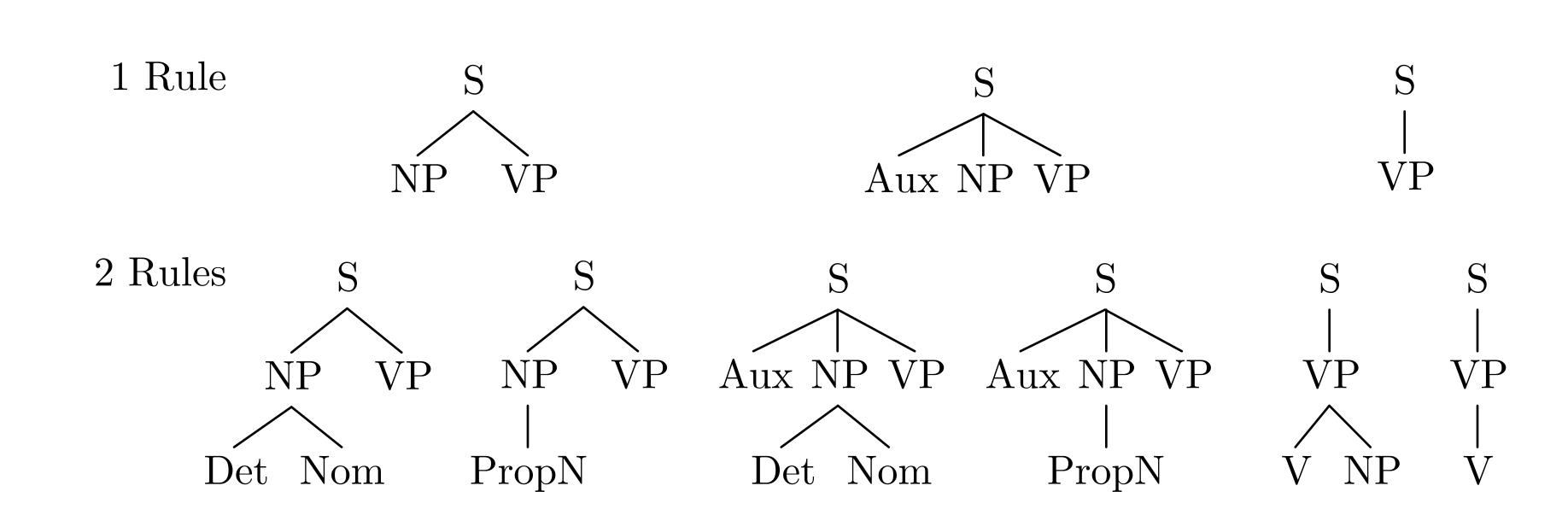
All valid parse trees must be rooted with start symbol

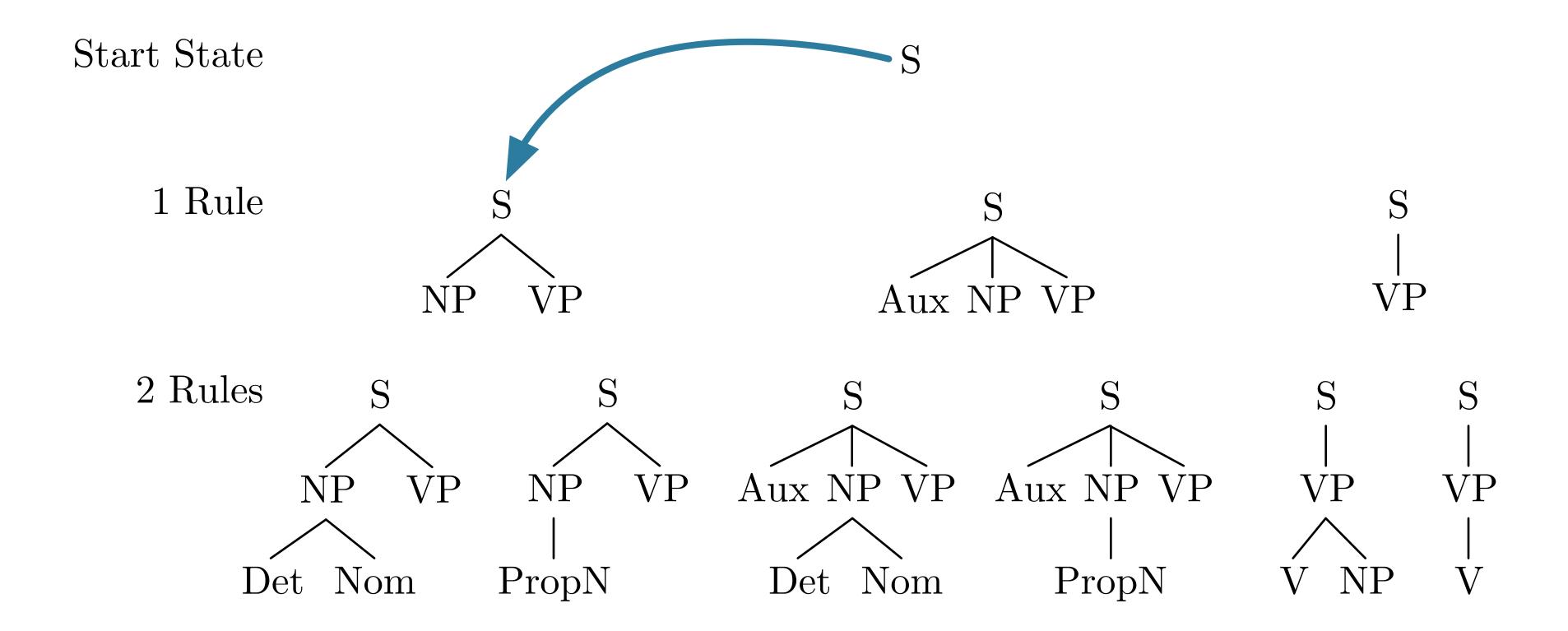
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  - e.g.  $S \rightarrow NP VP$

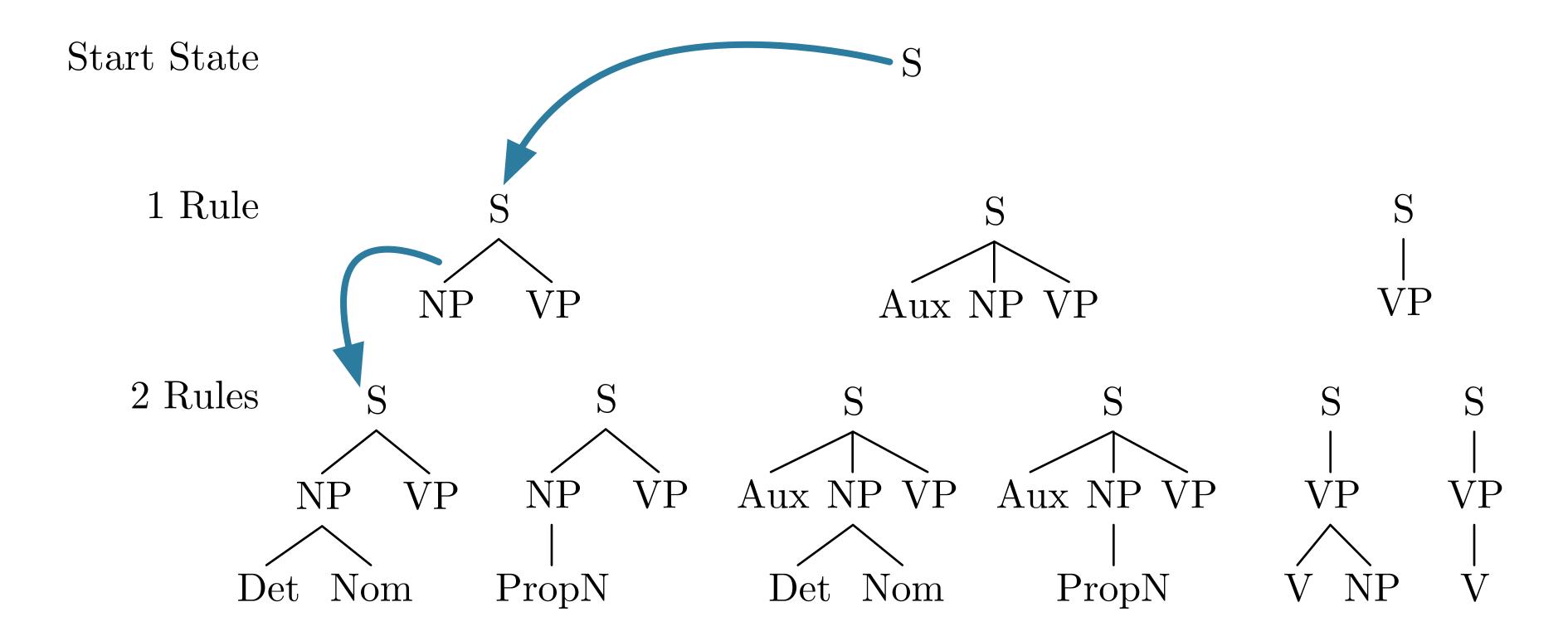
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  - e.g.  $NP \rightarrow Det\ Nominal;\ VP \rightarrow V\ NP$

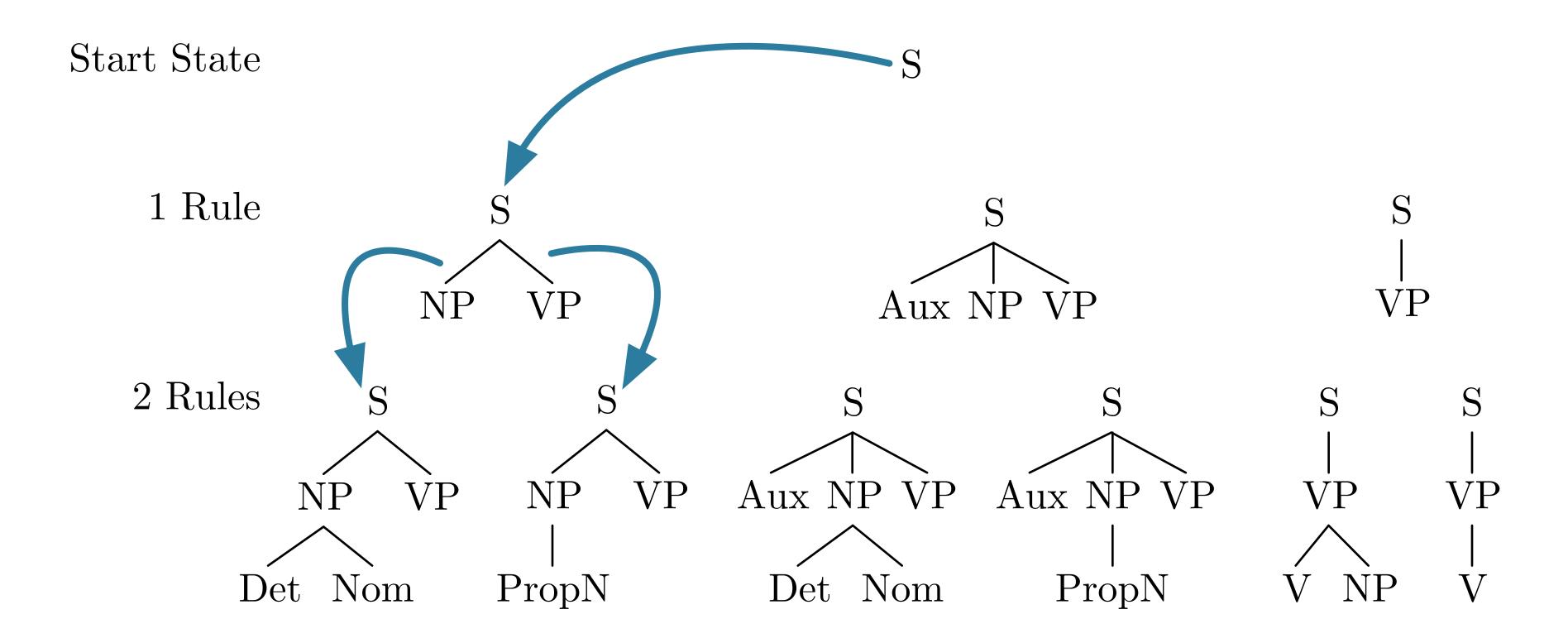
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- Terminate when all leaves are terminals

Start State

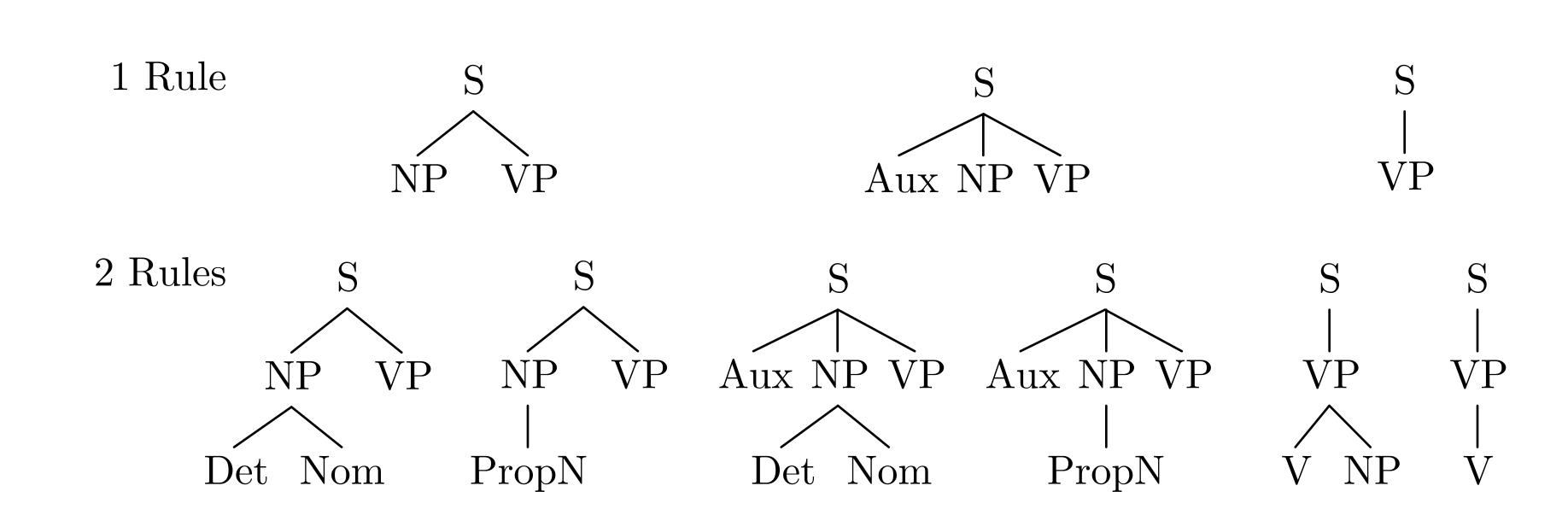


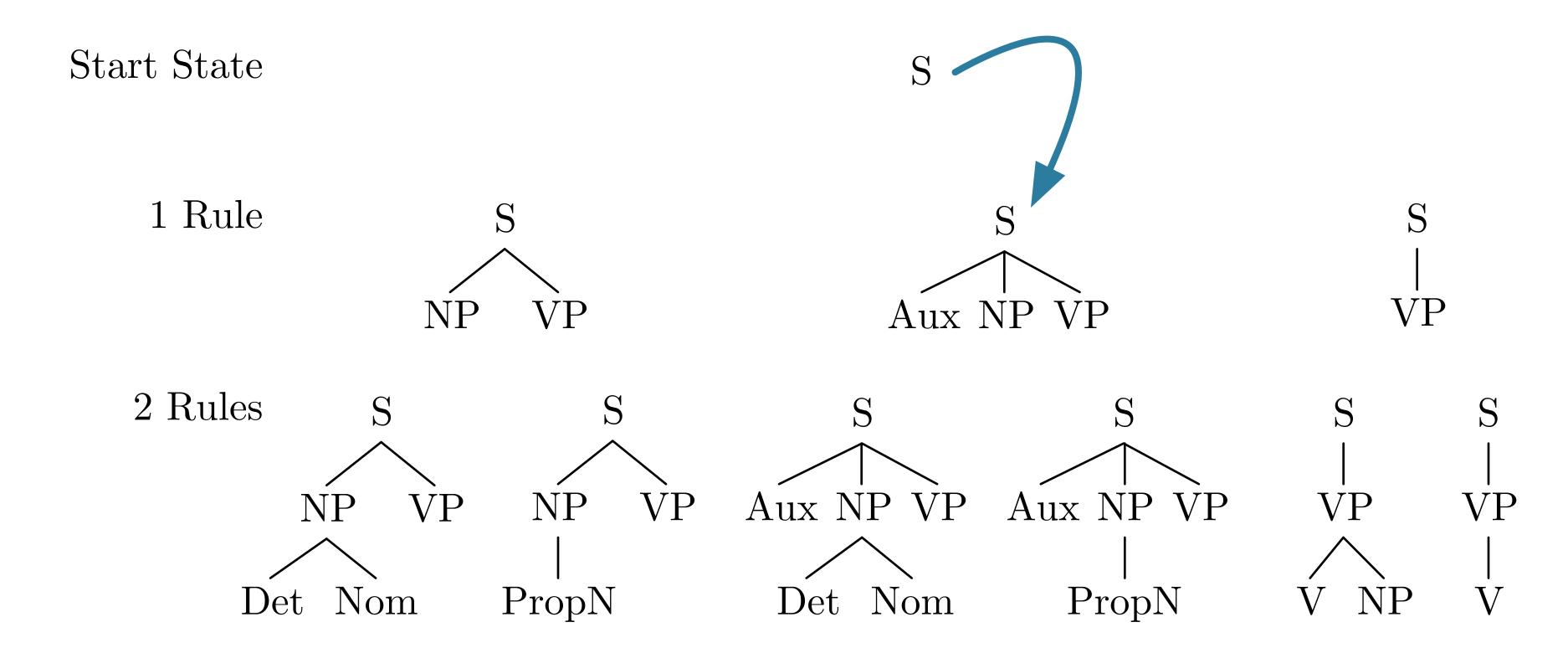


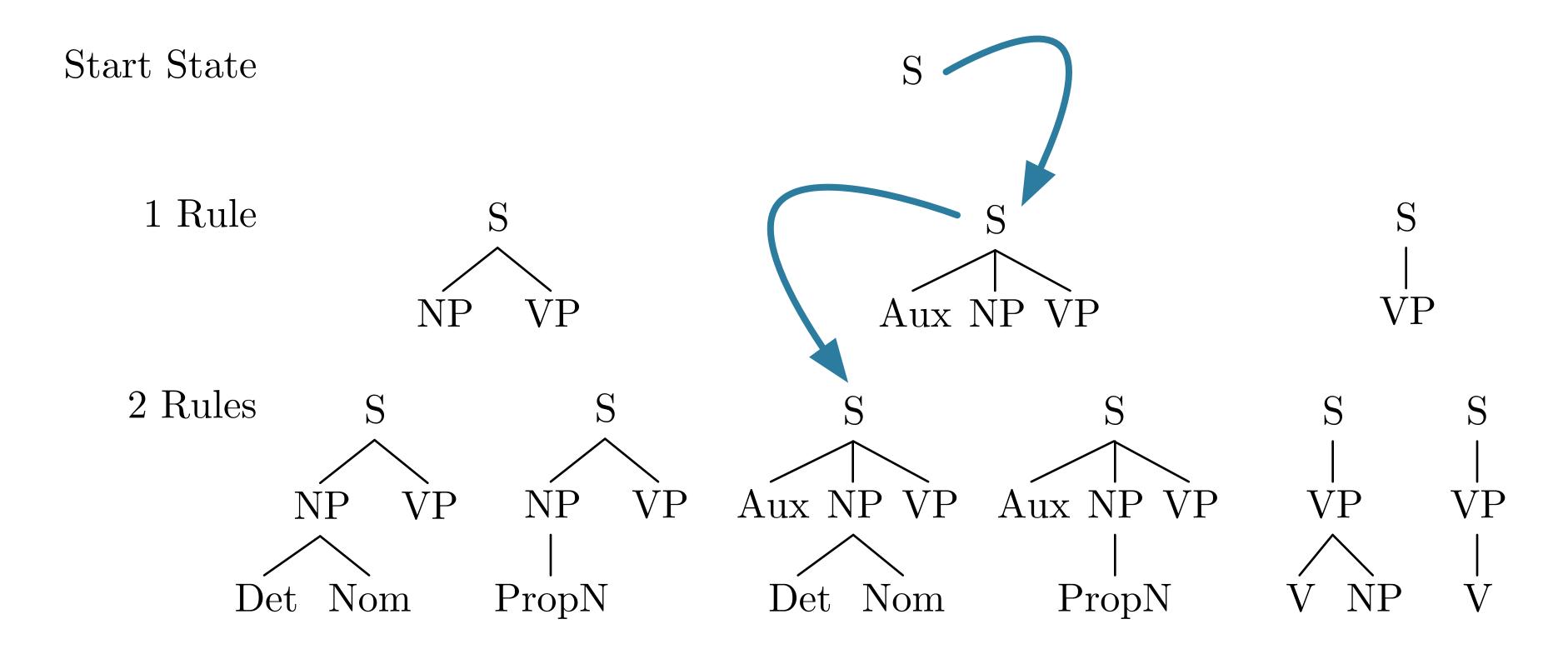


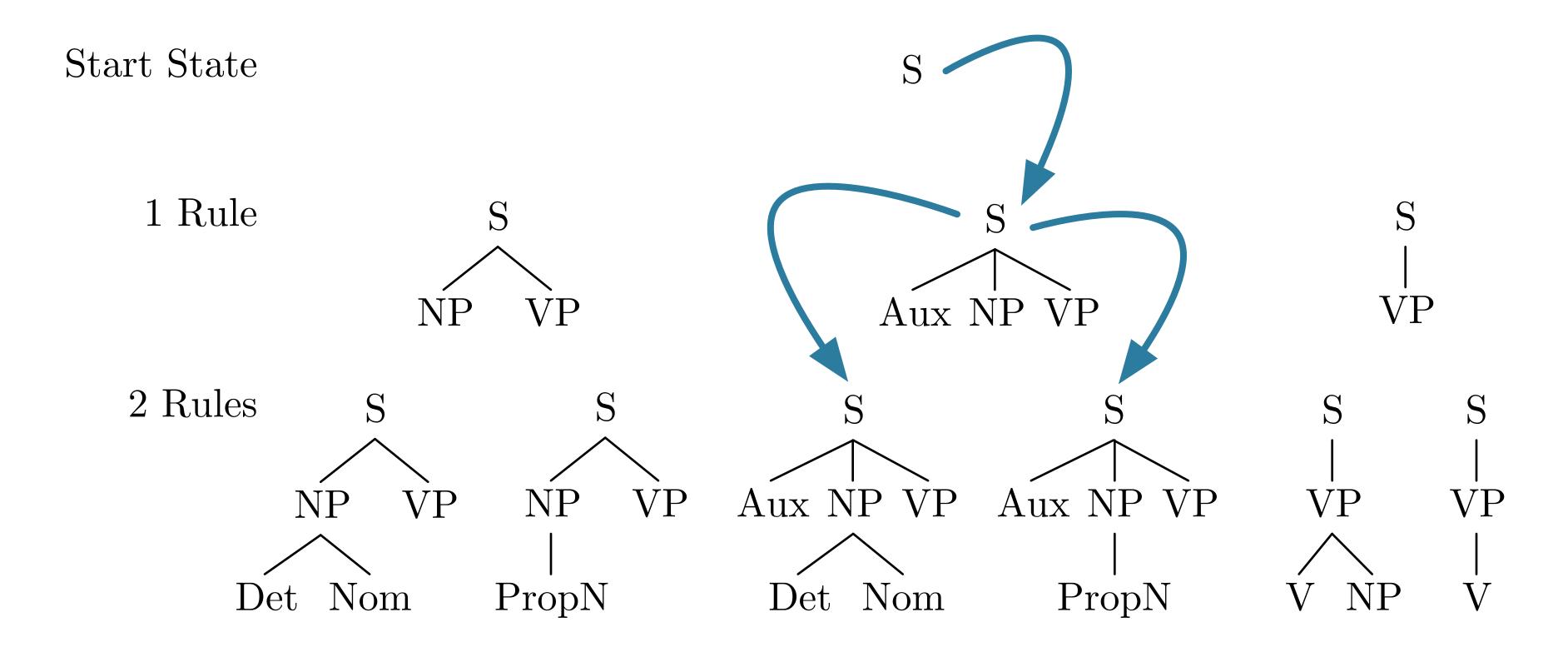


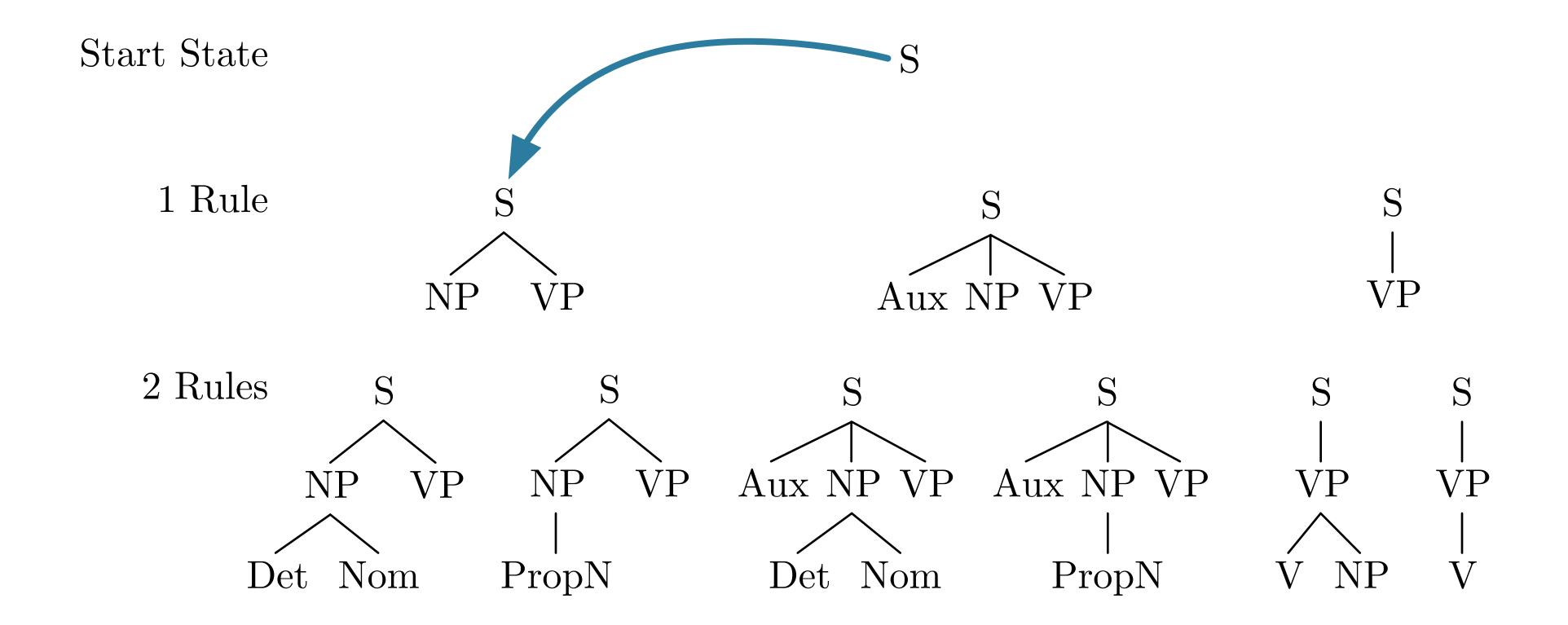
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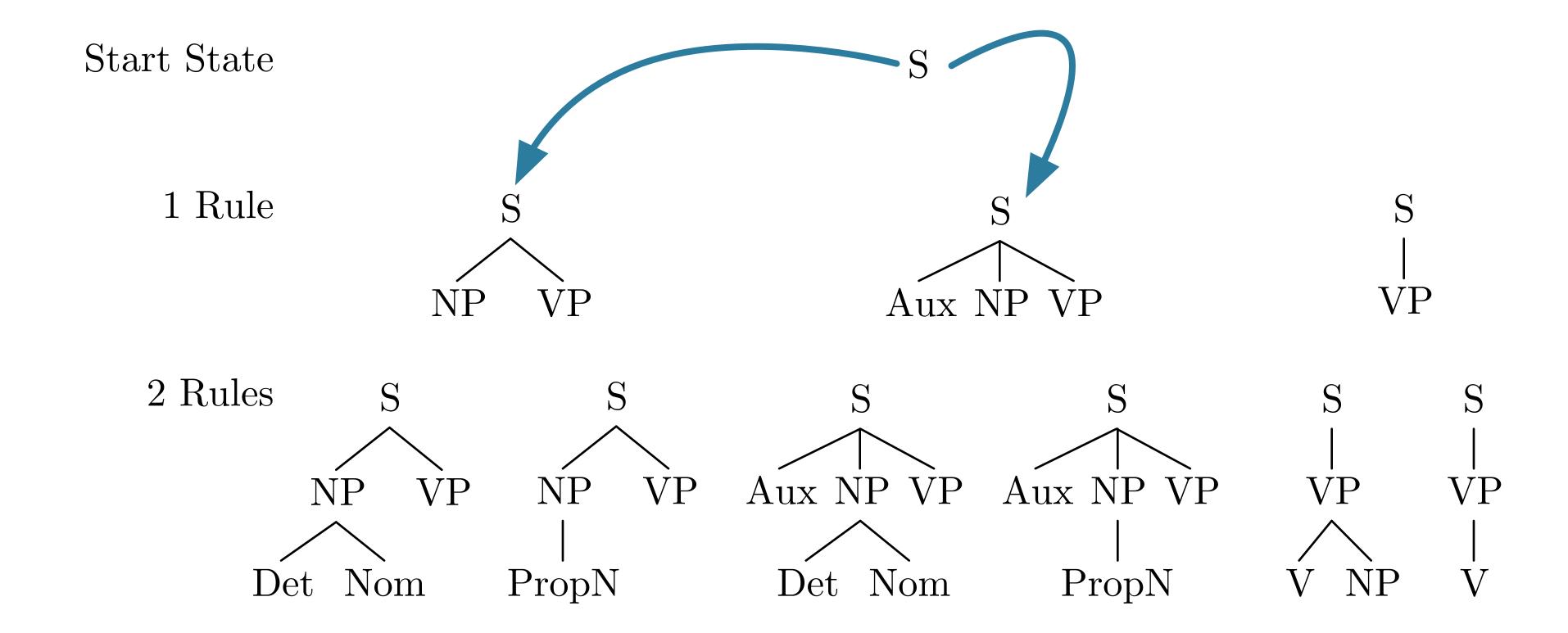












Aux NP VP

Det Nom

Aux NP VP

PropN

VP

NP

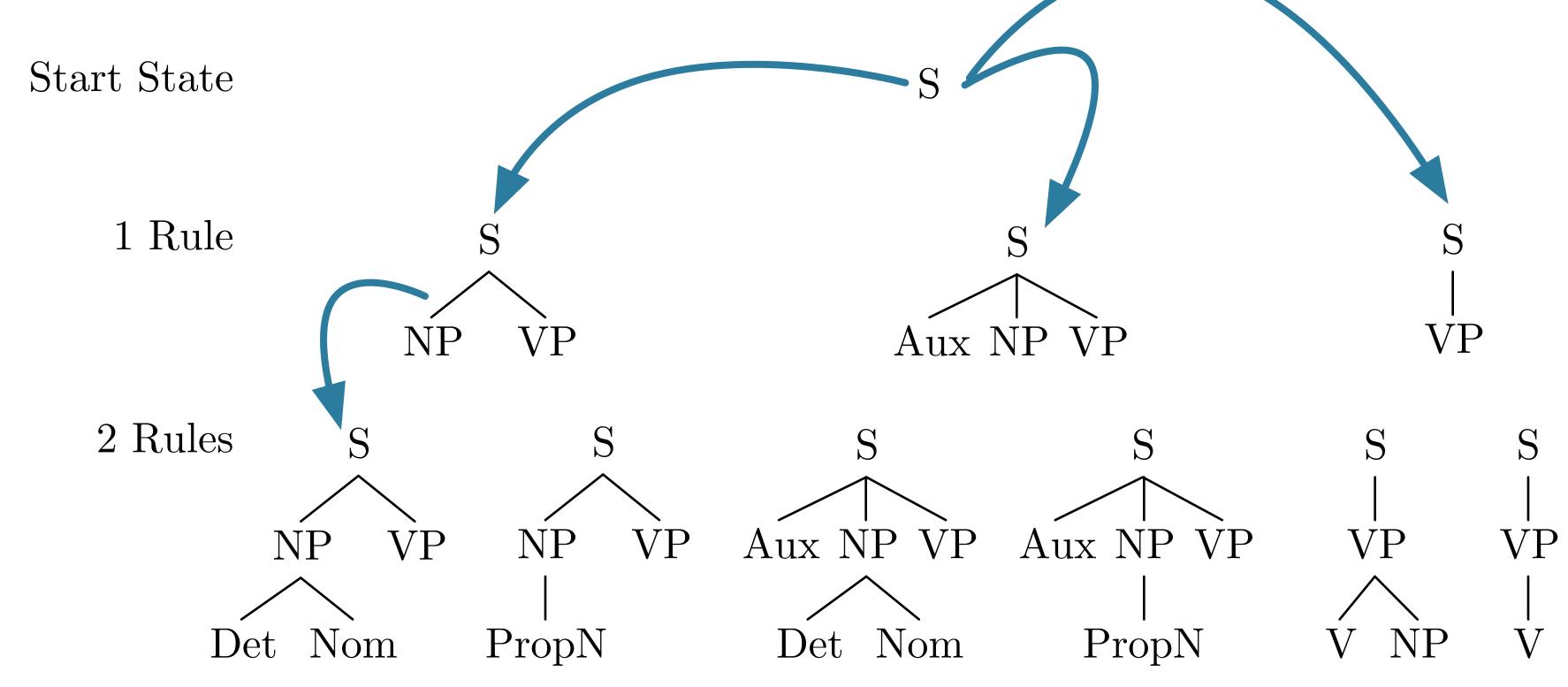
VP

NP

PropN

VP

Det Nom



Breadth-First Search Start State 1 Rule Aux NP VP NPVP 2 Rules Aux NP VP VP Aux NP VP VP NPVP

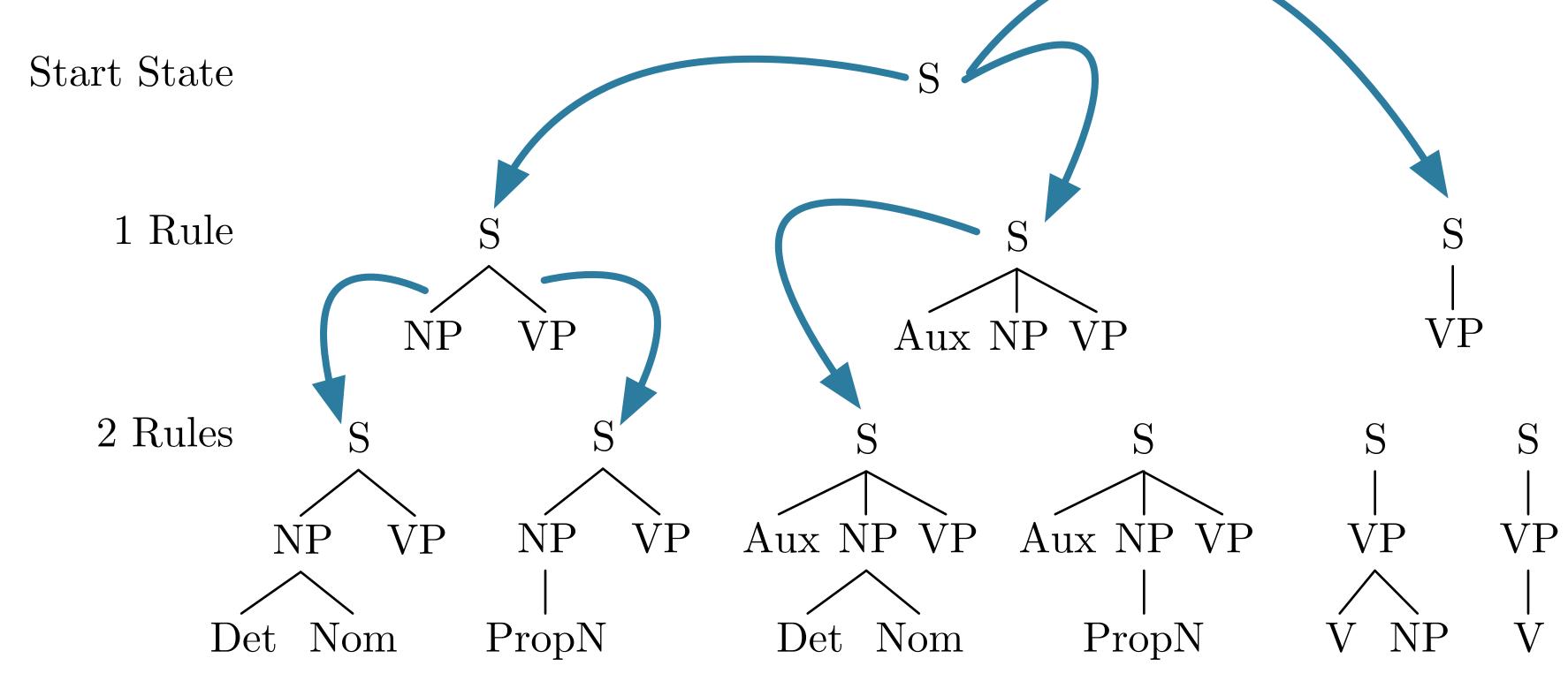
Det Nom

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# Pros and Cons of Top-down Parsing

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- Cons:
  - Produces trees that may not match input
  - May not terminate in presence of recursive rules
  - May re-derive subtrees as part of search

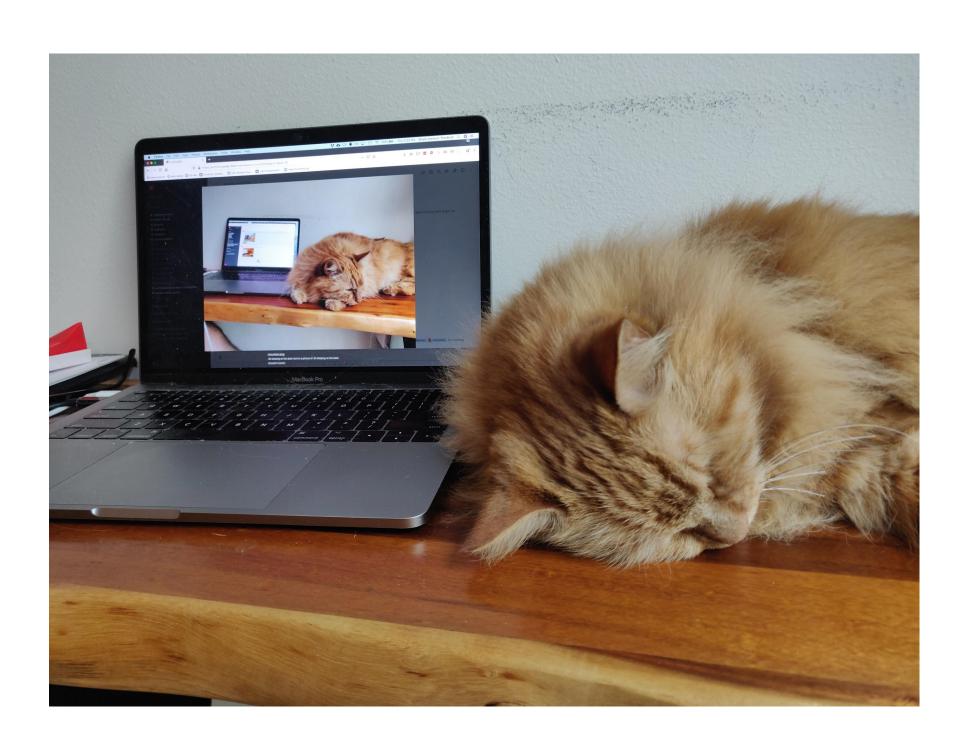
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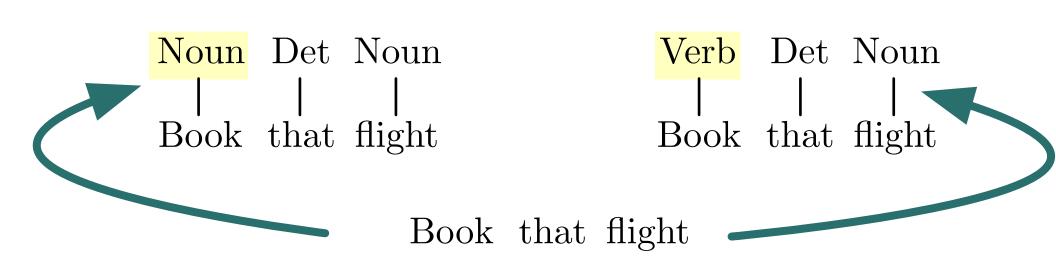
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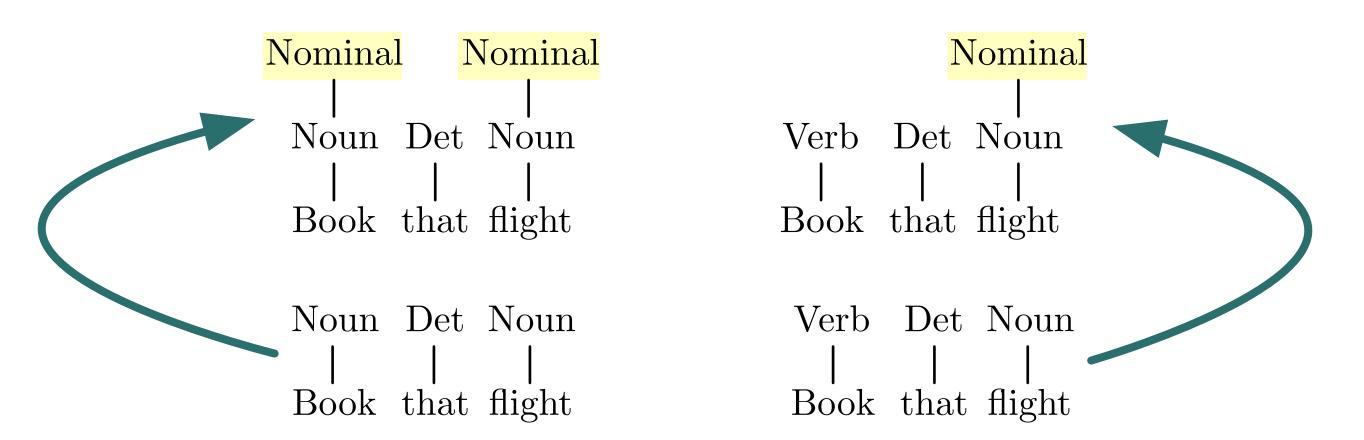


- Try to find all trees that span the input
  - Start with input string
    - Book that flight

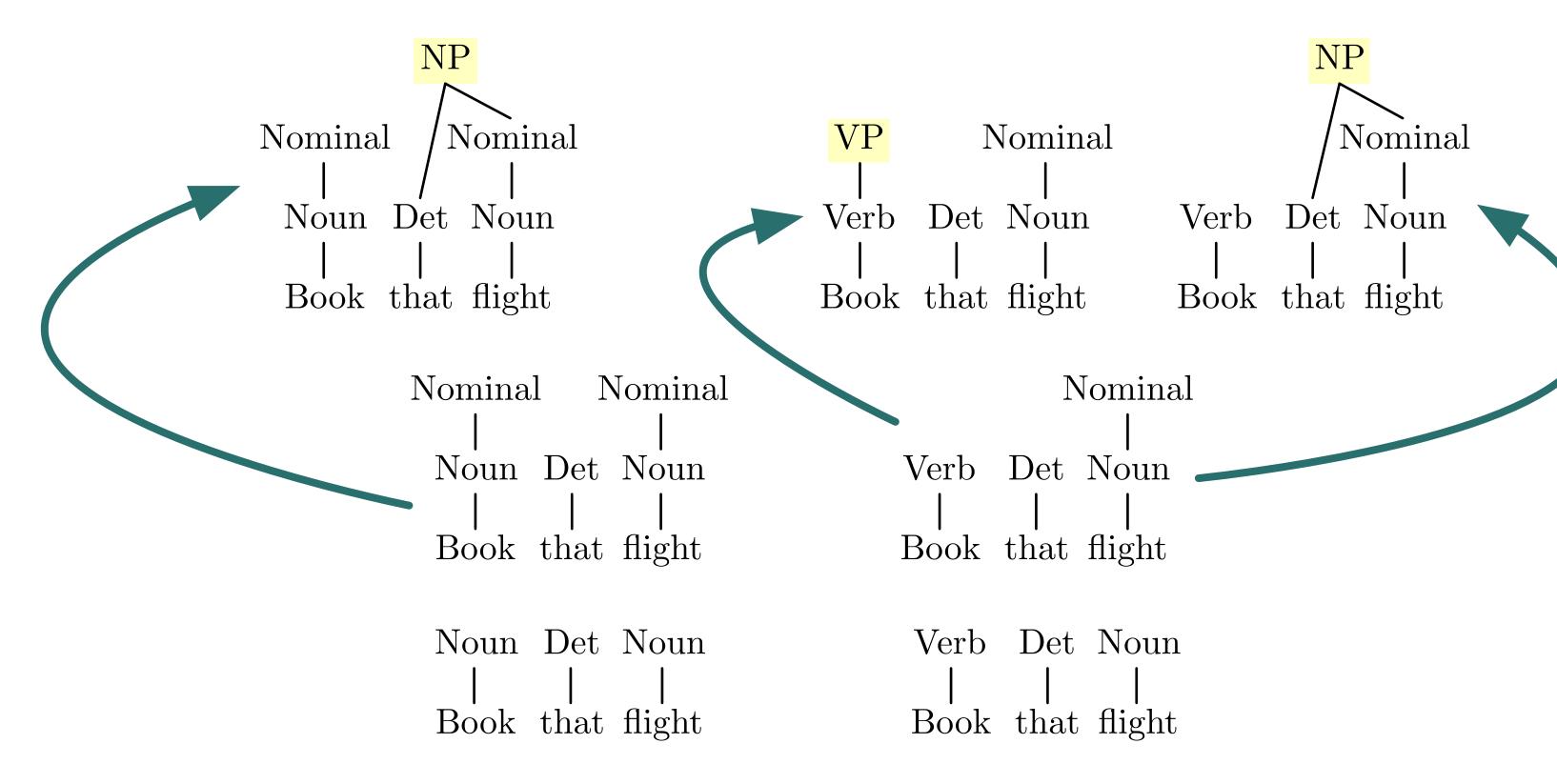
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- Stop when spanned by S, or no more rules apply

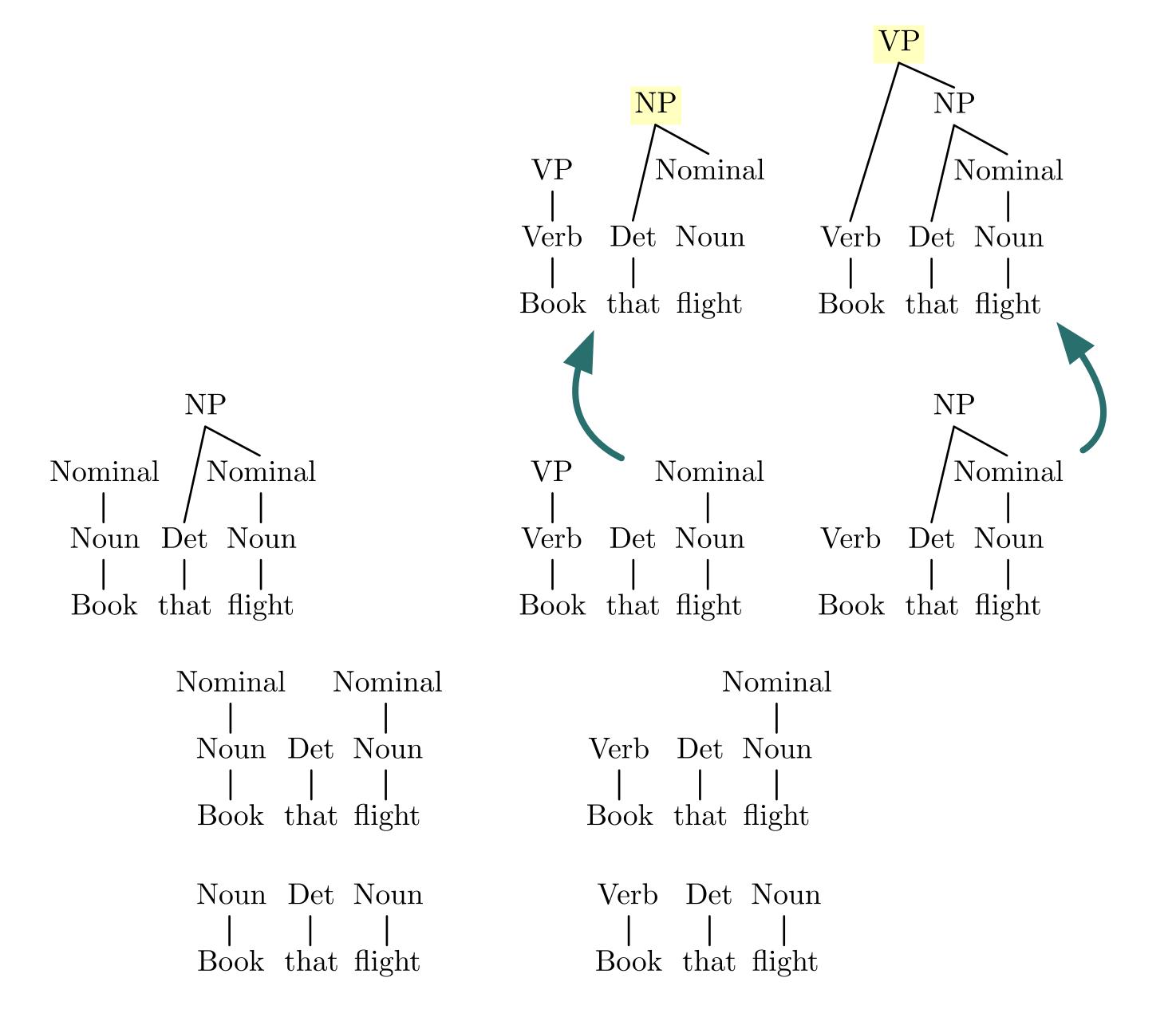




Book that flight



Book that flight



Book that flight

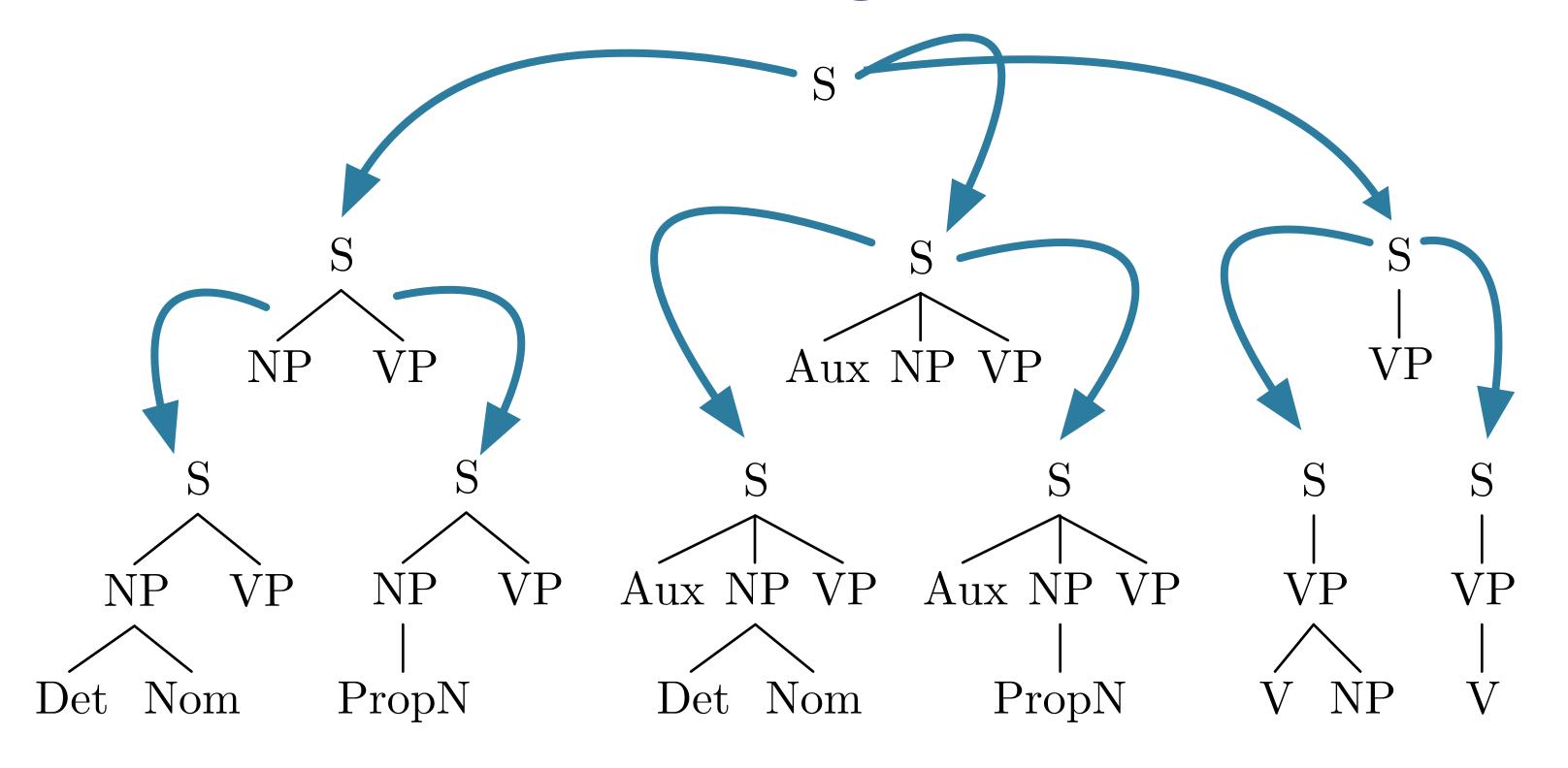
# Pros and Cons of Bottom-Up Search

- Pros:
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  - Recursive rules less problematic
  - Useful for incremental/fragment parsing

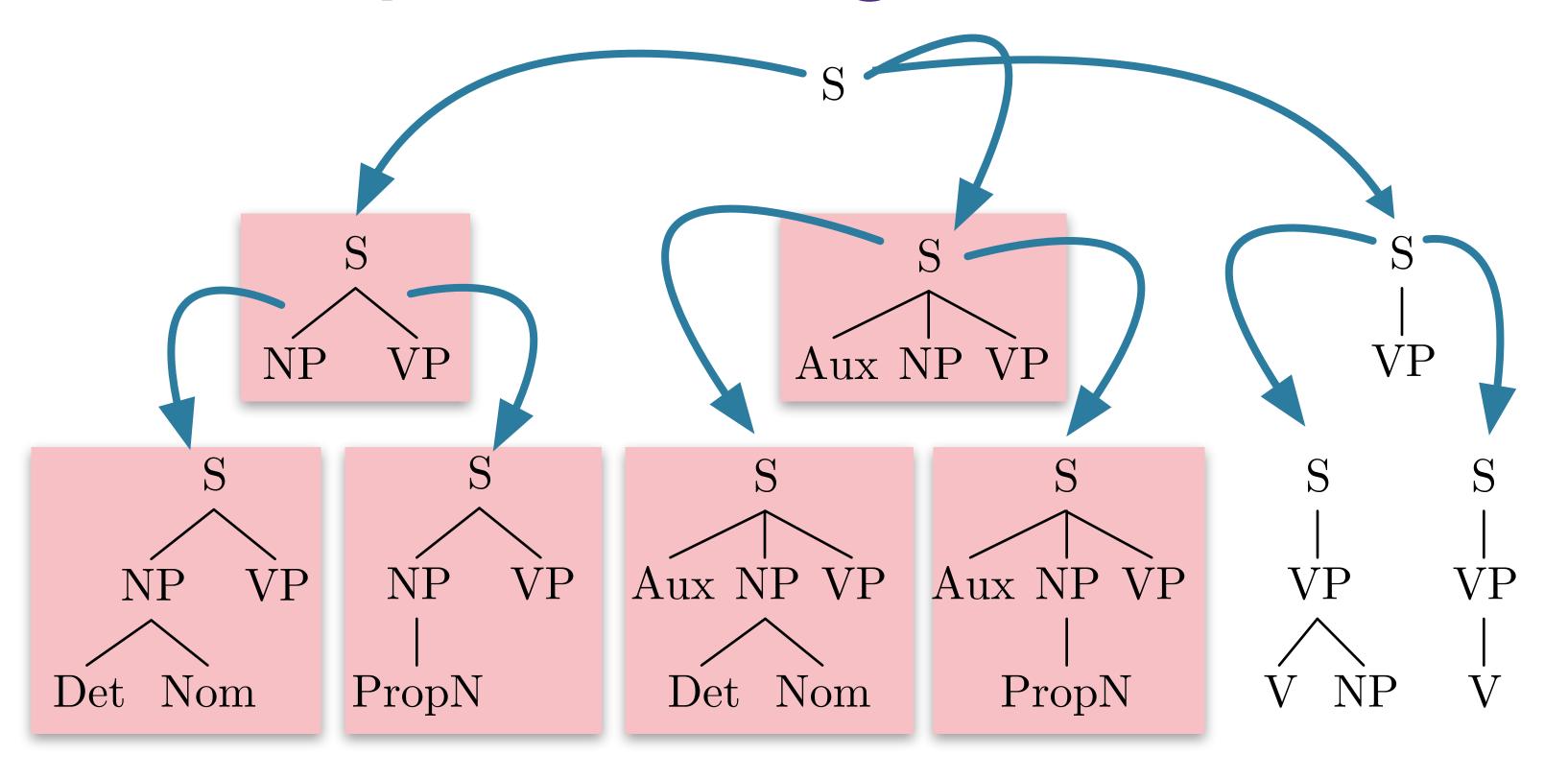
# Pros and Cons of Bottom-Up Search

- Pros:
  - Will not explore trees that don't match input
  - Recursive rules less problematic
  - Useful for incremental/fragment parsing
- Cons:
  - Explore subtrees that will not fit full input

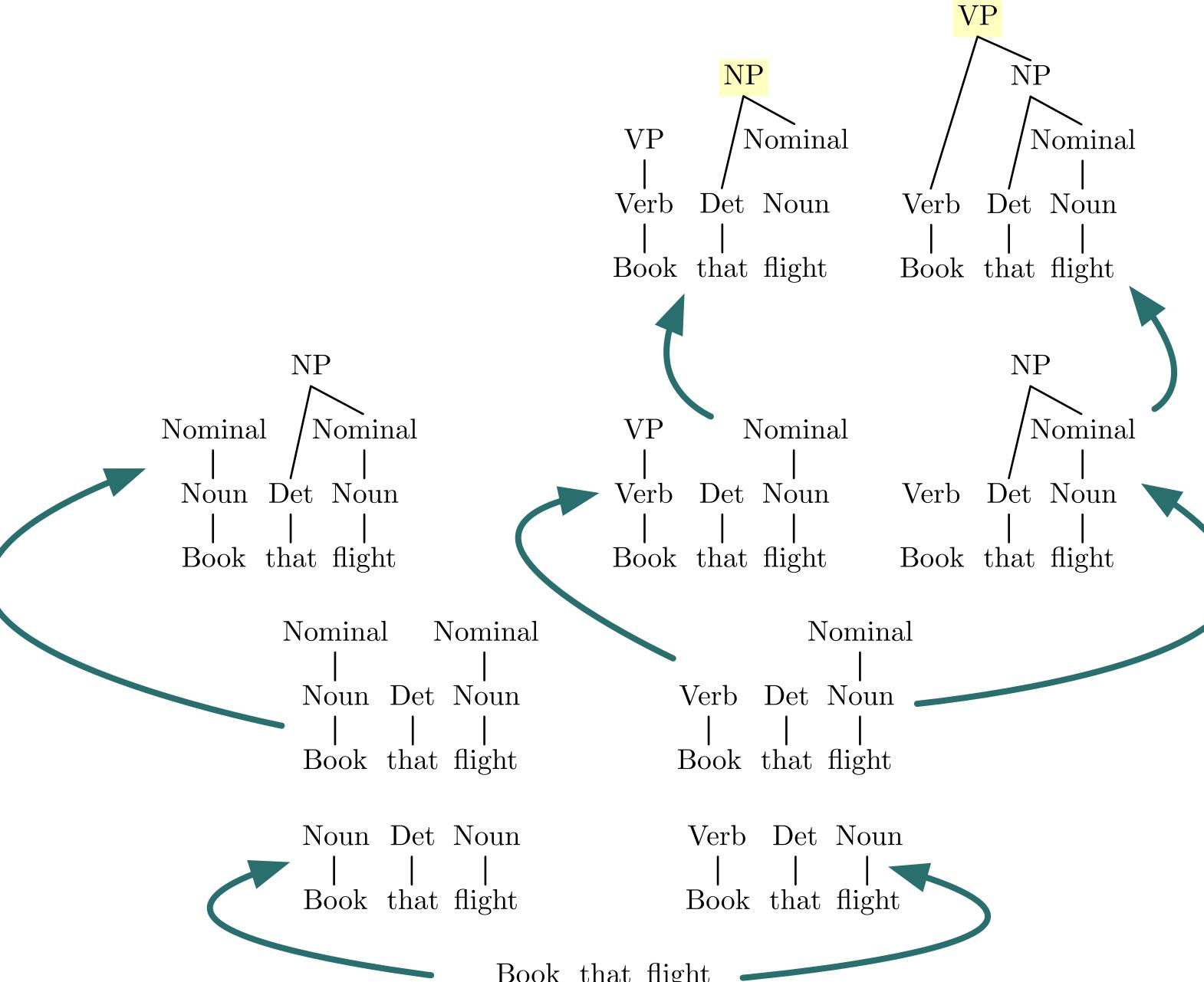
# Recap: Parsing as Search



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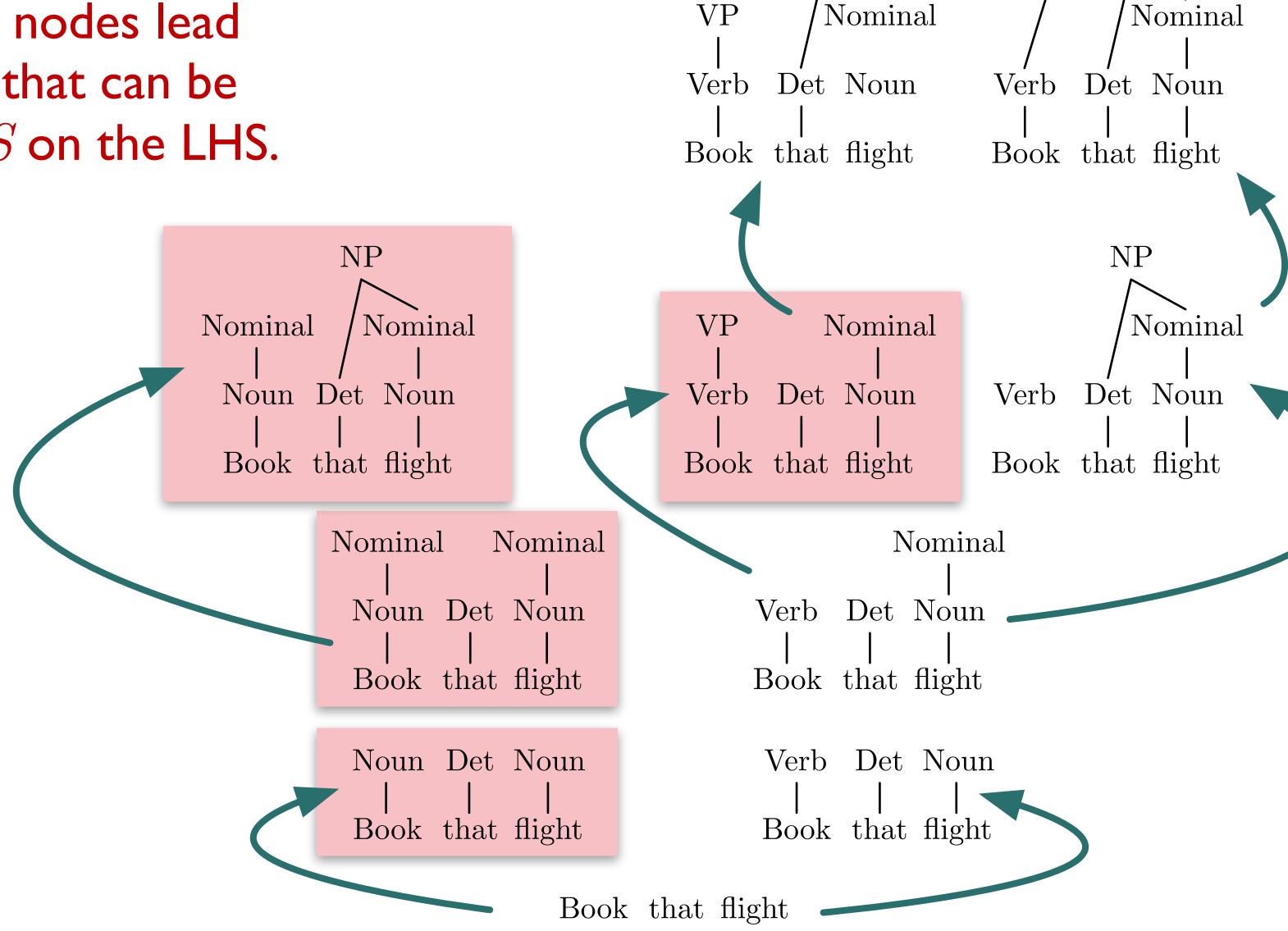


None of these nodes can produce book as first terminal



Book that flight

None of these nodes lead lead to a RHS that can be combined with S on the LHS.



VP

NP

NP

# Parsing Challenges

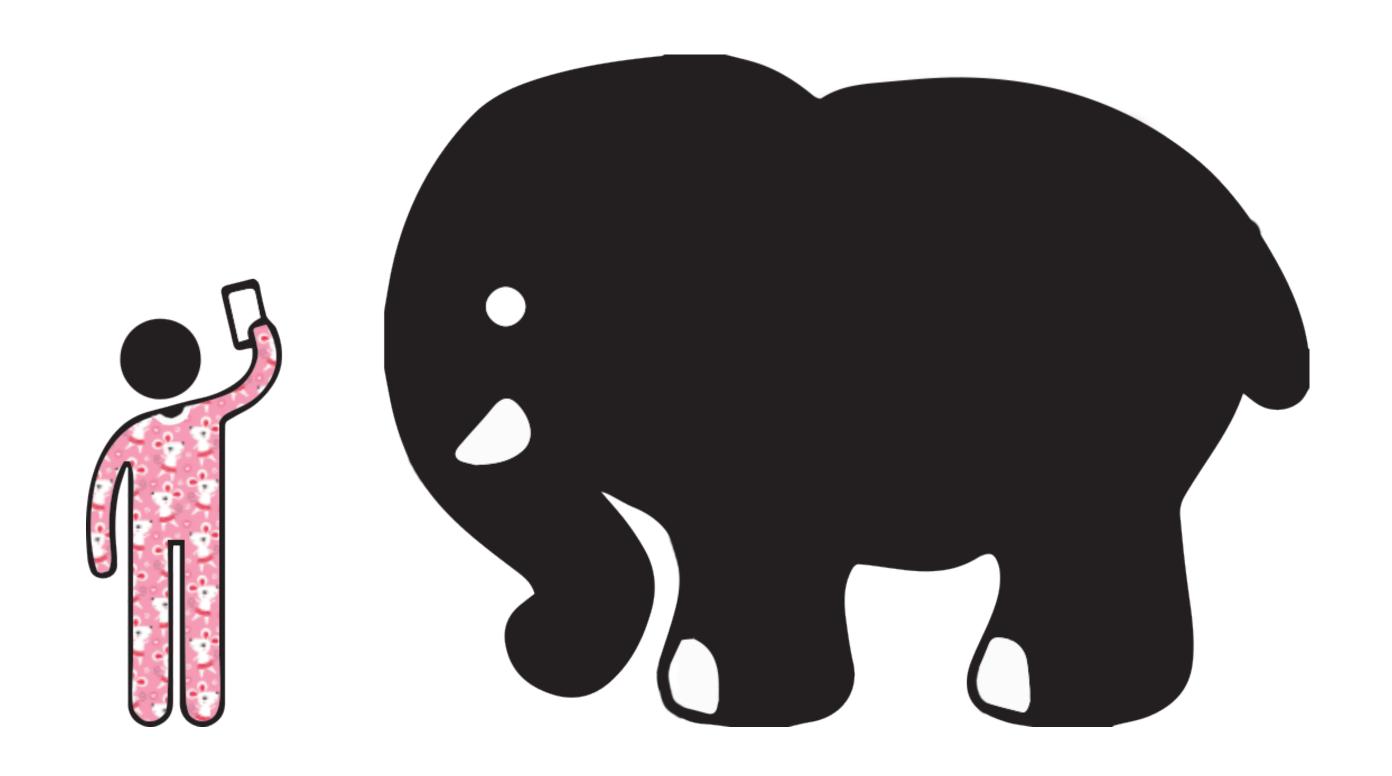
- Parsing-as-Search
- Parsing Challenges
  - Ambiguity
  - Repeated Substructure
  - Recursion
- Strategy: Dynamic Programming
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# Parsing Ambiguity

- Lexical Ambiguity:
  - Book/NN → I left a book on the table.
  - Book/VB → Book that flight.
- Structural Ambiguity

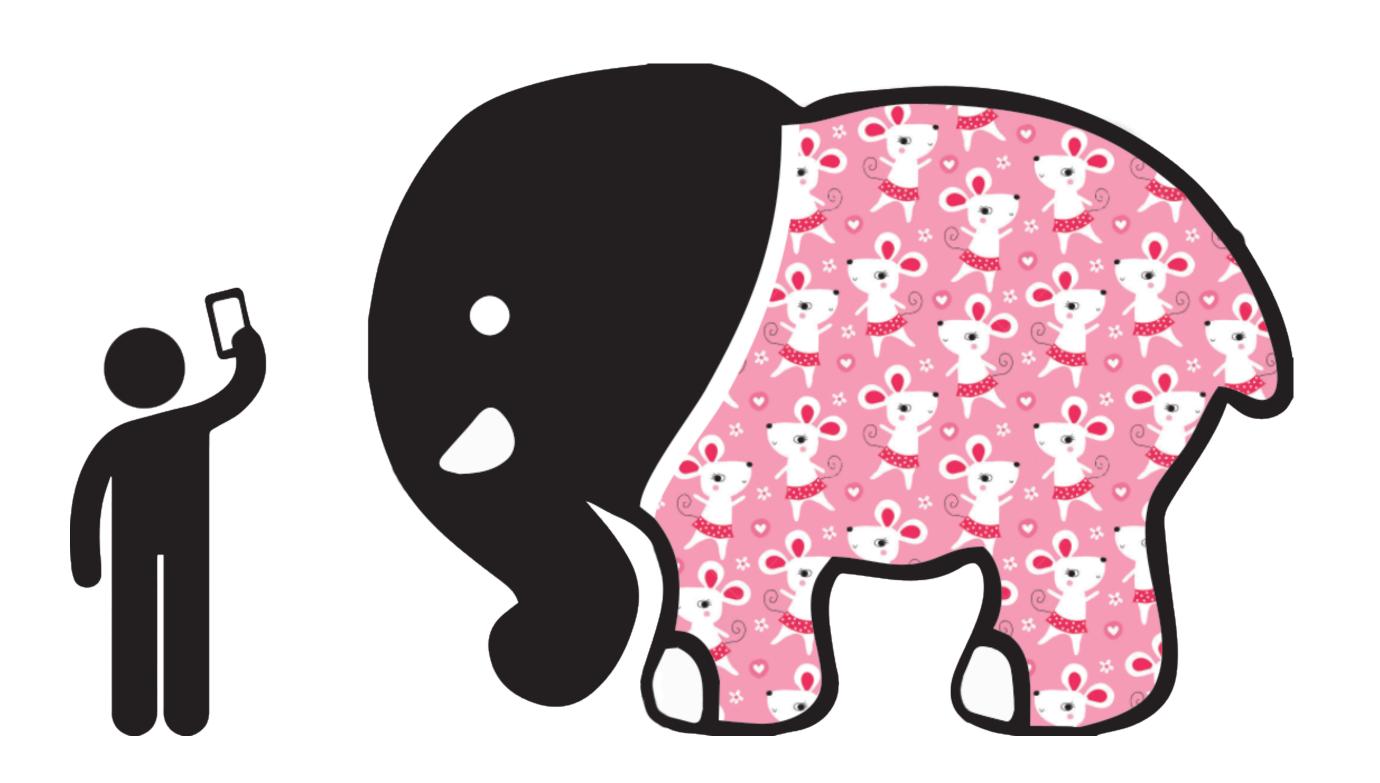
# Attachment Ambiguity

"One morning, I shot an elephant in my pajamas.

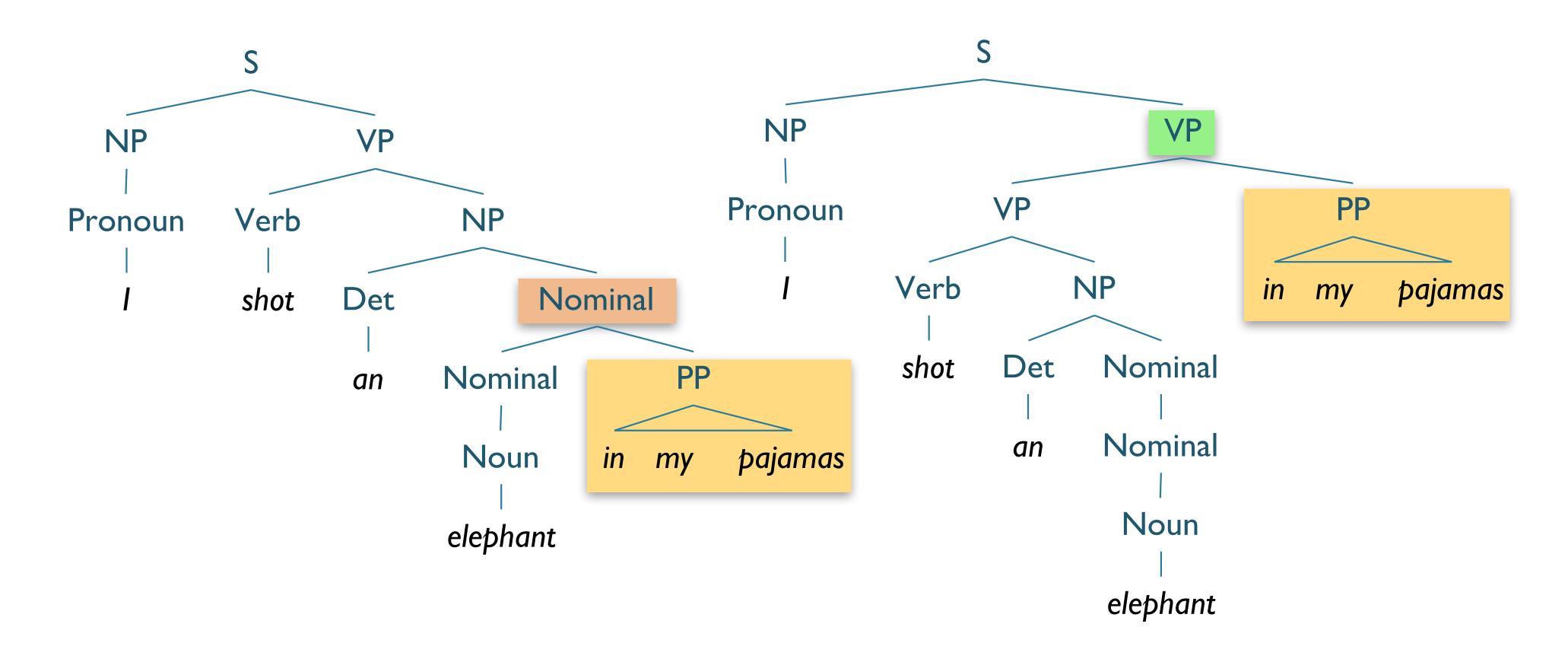


# Attachment Ambiguity

"One morning, I shot an elephant in my pajamas. How he got into my pajamas, I'll never know." — *Groucho Marx* 



### Attachment Ambiguity



### "We saw the Eiffel Tower flying to Paris"



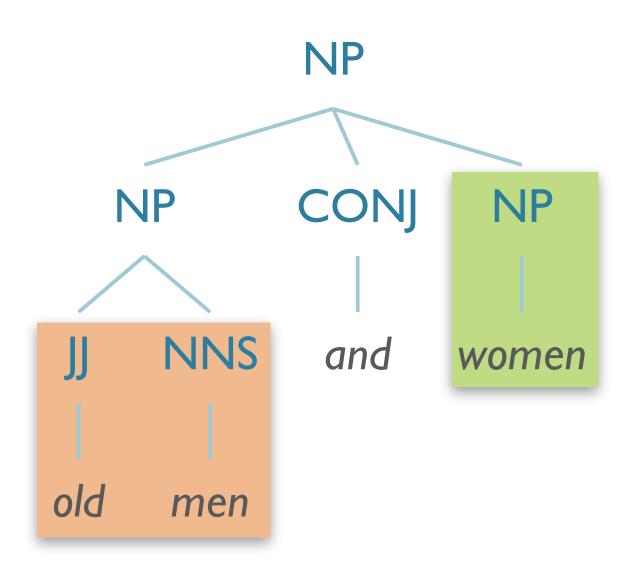
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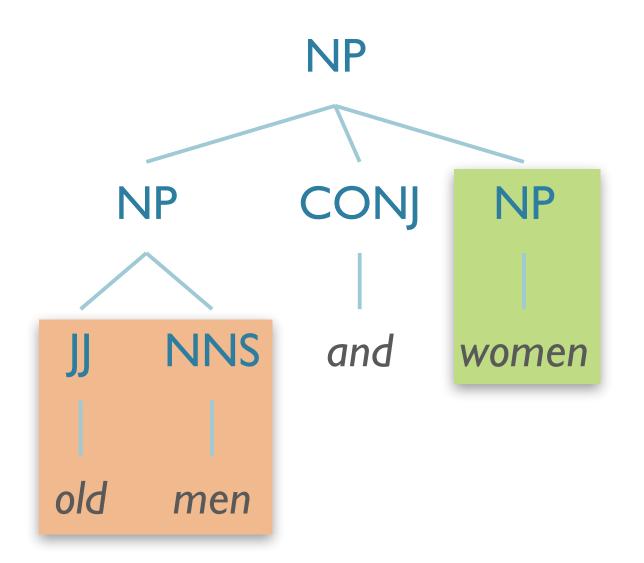
[old men] and [women]

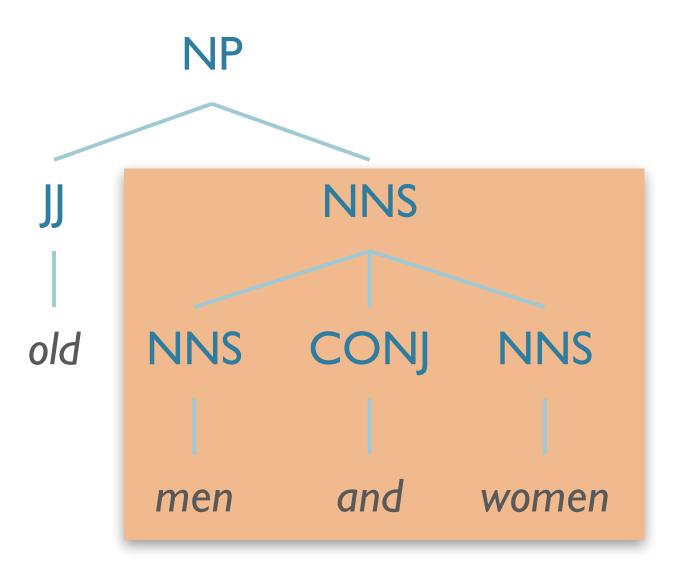


### Coordination Ambiguity:

[old men] and [women]

[old [men and women]]





### Local vs. Global Ambiguity

- Local ambiguity:
  - Ambiguity that cannot contribute to a full, valid parse
  - e.g. Book/NN in "Book that flight"

### Local vs. Global Ambiguity

- Local ambiguity:
  - Ambiguity that cannot contribute to a full, valid parse
  - e.g. Book/NN in "Book that flight"
- Global ambiguity
  - Multiple valid parses

### Why is Ambiguity a Problem?

- Local ambiguity:
  - increased processing time

- Global ambiguity:
  - Would like to yield only "reasonable" parses
  - Ideally, the one that was intended\*

### Solution to Ambiguity?

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• Disambiguation!

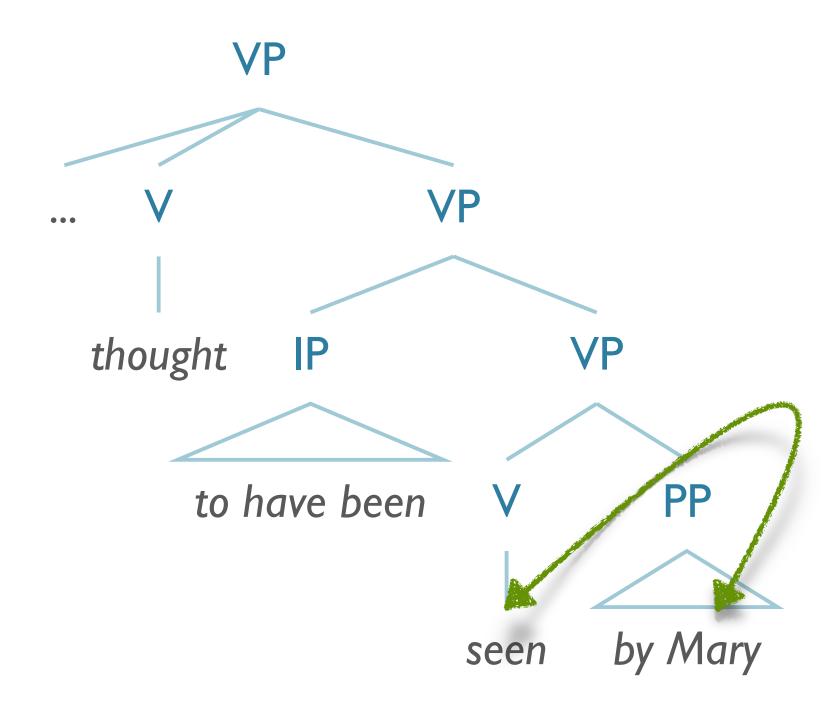
### Solution to Ambiguity?

- Disambiguation!
- Different possible strategies to select correct interpretation:

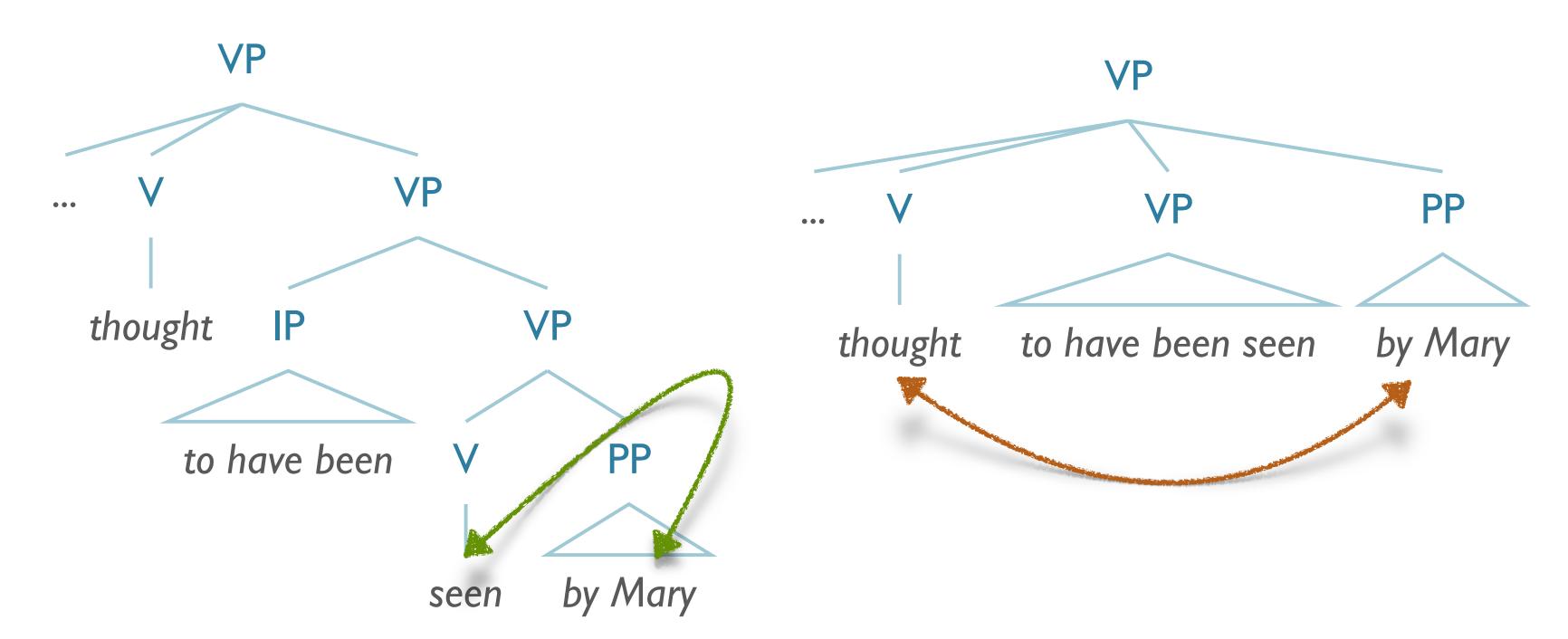
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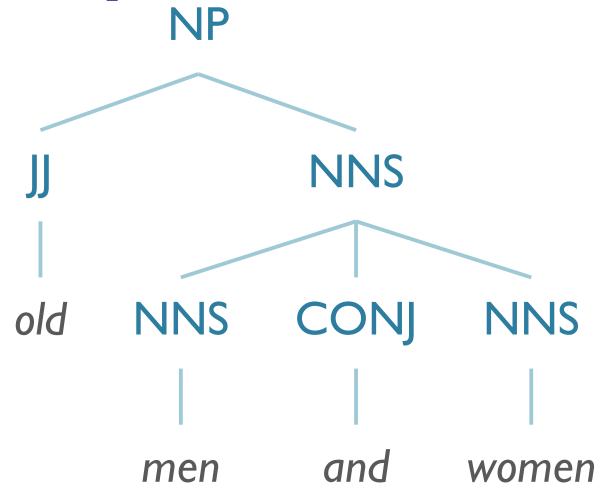


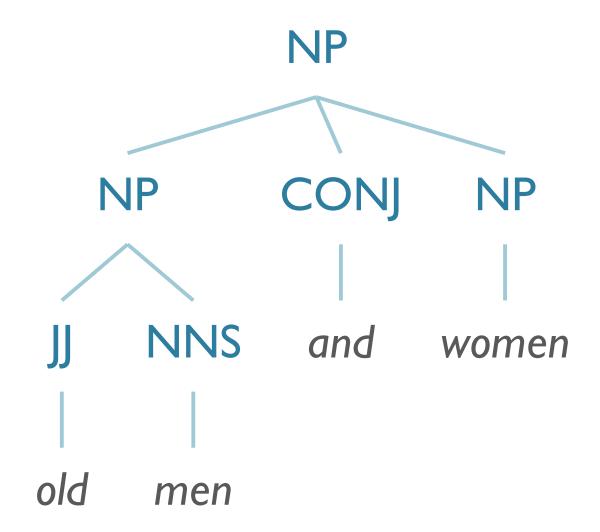
Some phrases more likely overall

Some phrases more likely overall

• [old [men and women]] is a more common construction than [old men] and

[women]





# Disambiguation Strategy: Semantic

Some interpretations we know to be semantically impossible

# Disambiguation Strategy: Semantic

- Some interpretations we know to be semantically impossible
  - Eiffel tower as subject of fly

# Disambiguation Strategy: Pragmatic

• Some interpretations are possible, unlikely given world knowledge

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  - e.g. elephants and pajamas

### Incremental Parsing and Garden Paths

- Idea: model left-to-right nature of (English) text
- Problem: "garden path" sentences

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SPORTS NEWS

SEPTEMBER 30, 2019 / 9:17 AM / A DAY AGO

## California to let college athletes be paid in blow to NCAA rules

https://www.reuters.com/article/us-sport-california-education/california-to-let-college-athletes-be-paid-in-blow-to-ncaa-rules-idUSKBN1WF1SR

#### Disambiguation Strategy:



Alternatively, keep all parses

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- Alternatively, keep all parses
  - (Might even be the appropriate action for some jokes)

### Parsing Challenges

- Parsing-as-Search
- Parsing Challenges
  - Ambiguity
  - Repeated Substructure
  - Recursion
- Strategy: Dynamic Programming
- Grammar Equivalence
- CKY parsing algorithm

### Repeated Work

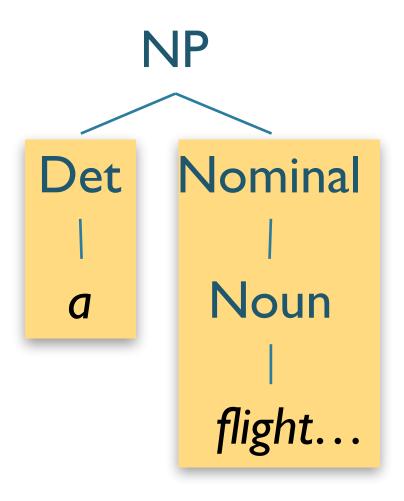
- Search (top-down/bottom-up) both lead to repeated substructures
  - Globally bad parses can construct good subtrees
  - ...will reconstruct along another branch
  - No static backtracking can avoid

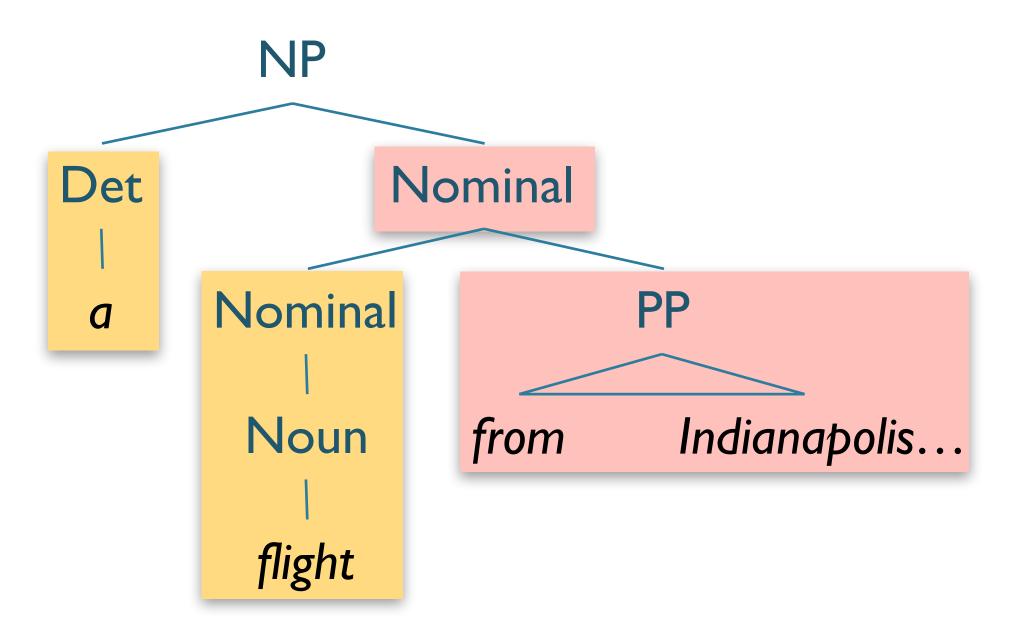
### Repeated Work

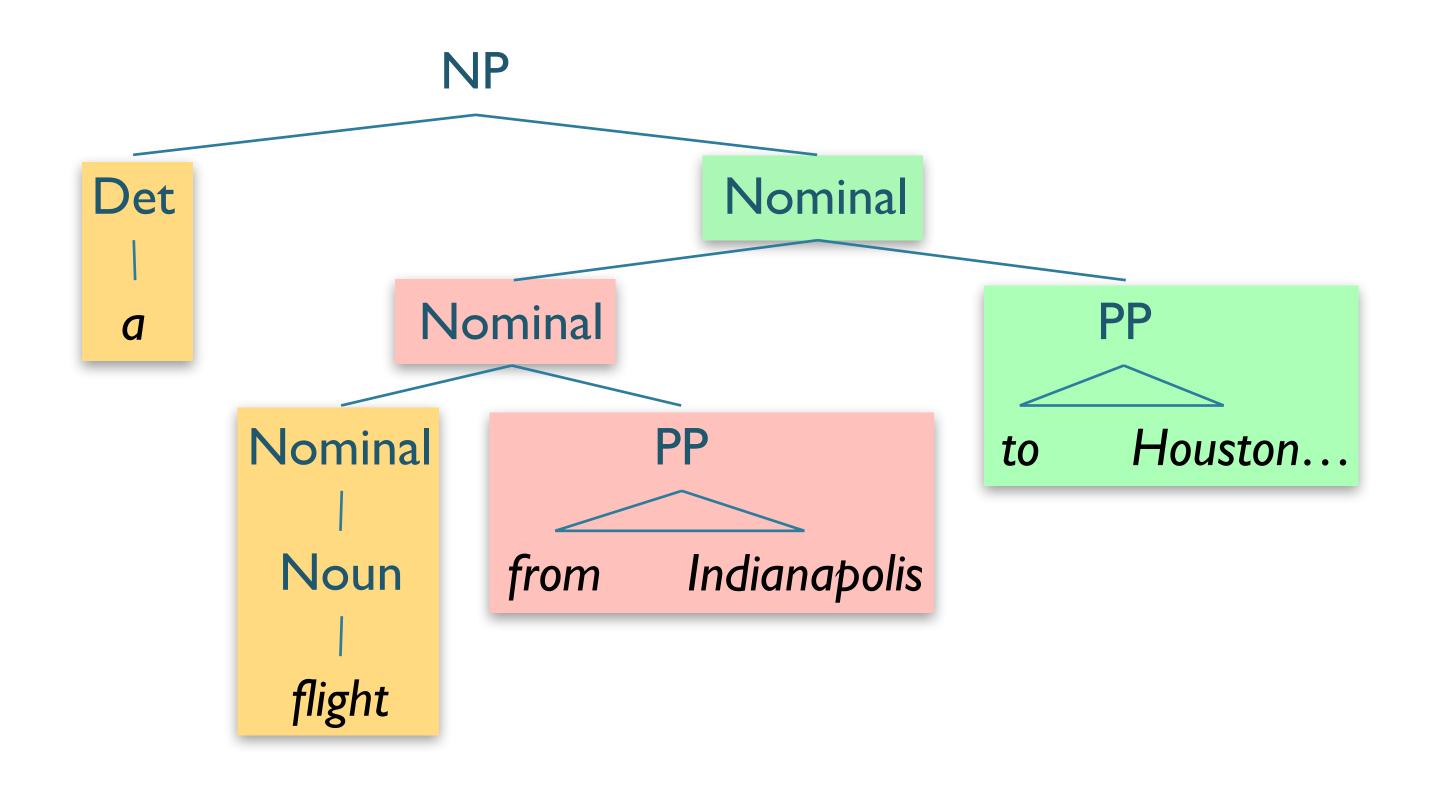
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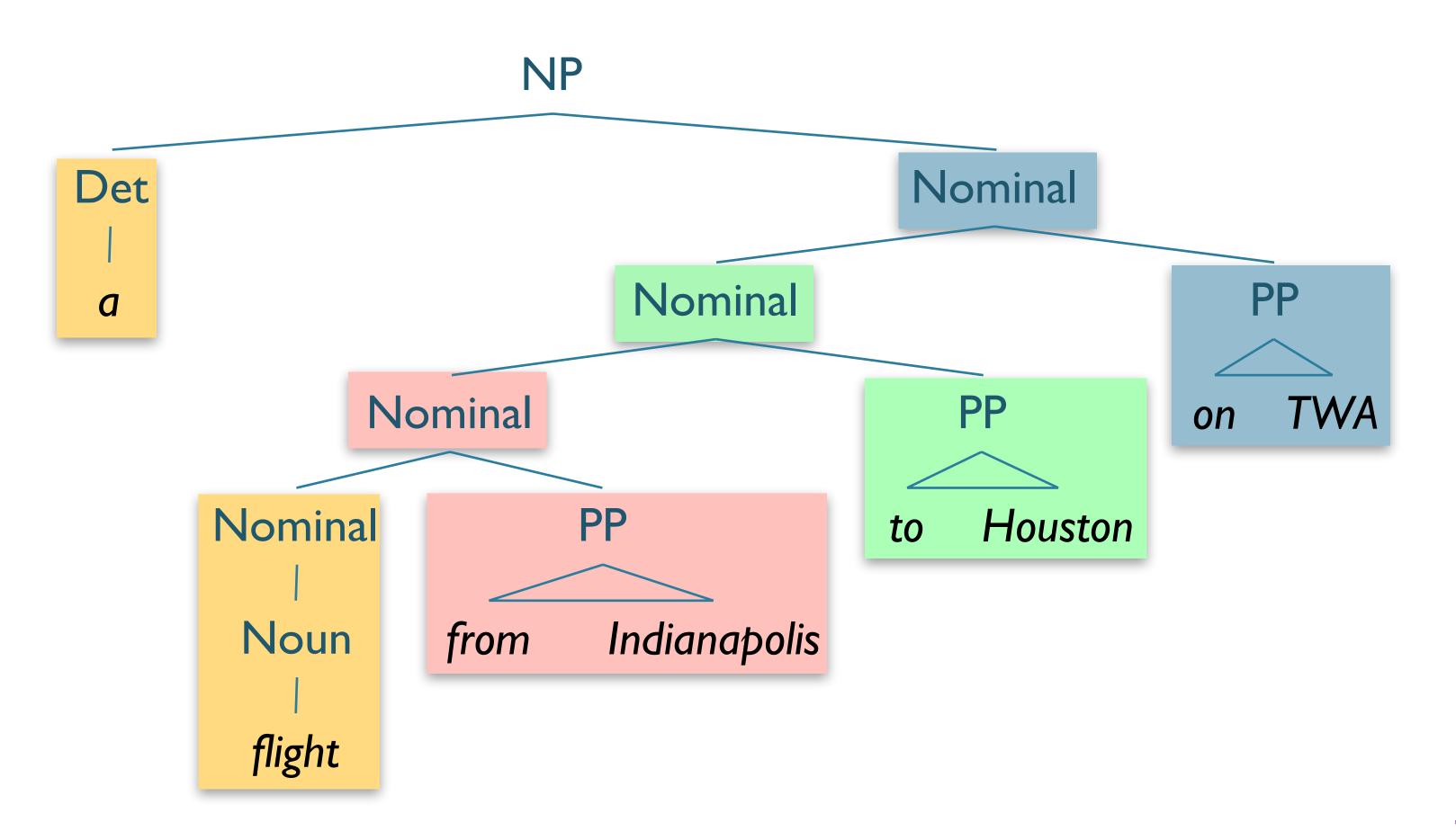
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  - No static backtracking can avoid
- Efficient parsing techniques require storage of partial solutions
- Example: a flight from Indianapolis to Houston on TWA







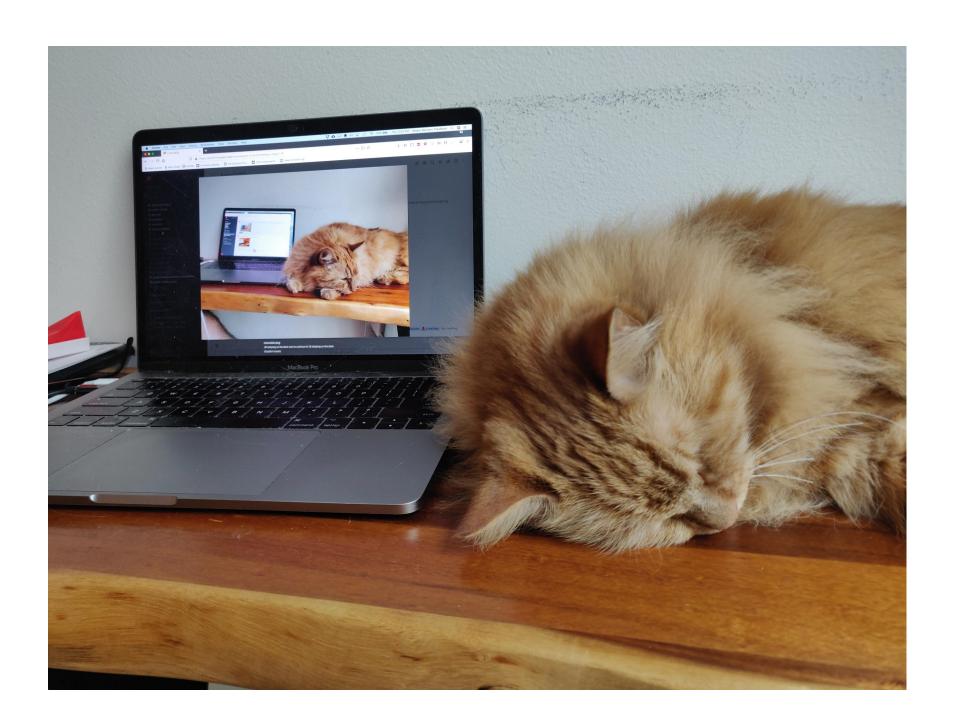


### Parsing Challenges

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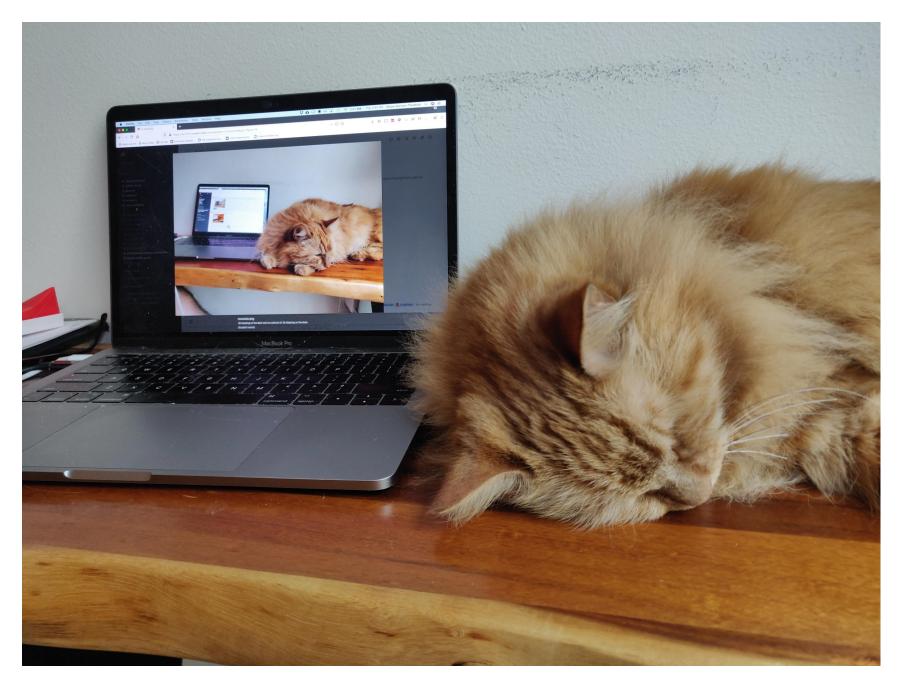
### Recursion

- Many grammars have recursive rules
  - $\bullet$   $S \rightarrow S$  Conj S



### Recursion

- Many grammars have recursive rules
  - $S \rightarrow S Conj S$
- In search approaches, recursion is problematic
  - Can yield infinite searches
  - Top-down especially vulnerable



## Roadmap

- Parsing-as-Search
- Parsing Challenges
- Strategy: Dynamic Programming
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# Dynamic Programming

- Challenge:
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- Insight:
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  - Repeated substructure → Repeated Work
- Insight:
  - Global parse composed of sub-parses
  - Can record these sub-parses and re-use
- Dynamic programming avoids repeated work by recording the subproblems
  - Here, stores subtrees

## Parsing with Dynamic Programming

- Avoids repeated work
- Allows implementation of (relatively) efficient parsing algorithms
  - Polynomial time in input length
  - Typically cubic  $(n^3)$  or less

# Parsing with Dynamic Programming

- Avoids repeated work
- Allows implementation of (relatively) efficient parsing algorithms
  - Polynomial time in input length
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- Several different implementations
  - Cocke-Kasami-Younger (CKY) algorithm
  - Earley algorithm
  - Chart parsing

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## Grammar Equivalence and Form

- Weak Equivalence
  - Accepts same language
  - May produce different structures

- Strong Equivalence
  - Accepts same language
  - Produces same structures

## Grammar Equivalence and Form

## Grammar Equivalence and Form

- Reason?
  - We can create a weakly-equivalent grammar that allows for greater efficiency
  - This is required by the CKY algorithm

Required by CKY Algorithm

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- All productions are of the form:
  - $\bullet \quad A \rightarrow B \quad C$
  - $\bullet$   $A \rightarrow a$

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- All productions are of the form:
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- Most of our grammars are not of this form:
  - $S \rightarrow Wh-NP \ Aux \ NP \ VP$

- Required by CKY Algorithm
- All productions are of the form:
  - $\bullet \quad A \rightarrow B \quad C$
  - $\bullet$   $A \rightarrow a$
- Most of our grammars are not of this form:
  - $S \rightarrow Wh-NP \ Aux \ NP \ VP$
- Need a general conversion procedure

## CNF Conversion

#### Hybrid productions:

$$INF-VP \rightarrow \mathbf{to} VP$$

#### Unit productions:

$$A \rightarrow B$$

#### Long productions:

$$A \rightarrow B C D \dots$$

# CNF Conversion: Hybrid Productions

- Hybrid production:
  - Replace all terminals with dummy non-terminal
  - $INF-VP \rightarrow \mathbf{to} VP$ 
    - $INF-VP \rightarrow TO VP$
    - $TO \rightarrow \mathbf{to}$

- Unit productions:
  - Rewrite RHS with RHS of all derivable, non-unit productions
  - If  $A \stackrel{*}{\Rightarrow} B$  and  $B \rightarrow \gamma$ , add  $A \rightarrow \gamma$  [where  $\gamma$  is any non-unit RHS]
  - [A ⇒ B: B is reachable from A by a sequence of unit productions]
- Nominal → Noun, Noun → dog
  - Nominal → dog
  - Noun → dog
- NB: this example has  $\gamma$  as a single terminal, but the rule applies to all non-unit RHS.

Long productions

 $S \rightarrow Aux NP VP$ 

Long productions

```
S \rightarrow Aux \ NP \ VP
S \rightarrow X1 \ VP \qquad X1 \rightarrow Aux \ NP
```

Long productions

```
S \rightarrow Aux \ NP \ VP
S \rightarrow X1 \ VP \qquad X1 \rightarrow Aux \ NP
```

Introduce unique nonterminals, and spread over rules

### CNF Conversion

Convert terminals in hybrid rules to dummy non-terminals

Convert unit productions

Binarize long production rules

£ <sub>1</sub> Grammar	£ <sub>1</sub> in CNF
$S \rightarrow NP VP$	$S \rightarrow NP VP$
$S \rightarrow Aux NP VP$	$S \rightarrow X1 \ VP$
	$X1 \rightarrow Aux NP$
$S \to VP$	$S \rightarrow book / include / prefer$
	$S \rightarrow Verb NP$
	$S \rightarrow X2 PP$
	$S \rightarrow Verb PP$
	$S \rightarrow VP PP$
$NP \rightarrow Pronoun$	$NP \rightarrow I / she / me$
$NP \rightarrow Proper-Noun$	$NP \rightarrow TWA / Houston$
$NP \rightarrow Det\ Nominal$	$NP \rightarrow Det\ Nominal$
$Nominal \rightarrow Noun$	$Nominal \rightarrow book / flight / meal / money$
$Nominal \rightarrow Nominal \ Noun$	$Nominal \rightarrow Nominal \ Noun$
$Nominal \rightarrow Nominal PP$	$Nominal \rightarrow Nominal PP$
$VP \rightarrow Verb$	$VP \rightarrow book / include / prefer$
$VP \rightarrow Verb NP$	$VP \rightarrow Verb NP$
$VP \rightarrow Verb NP PP$	$VP \rightarrow X2 PP$
	$X2 \rightarrow Verb NP$
$VP \rightarrow Verb PP$	$VP \rightarrow Verb PP$
$VP \rightarrow VP PP$	$VP \rightarrow VP PP$
$PP \rightarrow Preposition NP$	$PP \rightarrow Preposition NP$

£ <sub>1</sub> Grammar	$\mathcal{L}_1$ in CNF		
$S \rightarrow NP VP$	$S \rightarrow NP VP$		
$S \rightarrow Aux NP VP$	$S \rightarrow X1 \ VP$		
	$X1 \rightarrow Aux NP$		
$S \to VP$	$S \rightarrow book / include / prefer$		
	$S \rightarrow Verb NP$		
	$S \rightarrow X2 PP$		
	$S \rightarrow Verb PP$		
	$S \rightarrow VP PP$		
$NP \rightarrow Pronoun$	$NP \rightarrow I / she / me$		
$NP \rightarrow Proper-Noun$	$NP \rightarrow TWA / Houston$		
$NP \rightarrow Det\ Nominal$	$NP \rightarrow Det\ Nominal$		
$Nominal \rightarrow Noun$	$Nominal \rightarrow book / flight / meal / money$		
$Nominal \rightarrow Nominal \ Noun$	$Nominal \rightarrow Nominal \ Noun$		
$Nominal \rightarrow Nominal PP$	$Nominal \rightarrow Nominal PP$		
$VP \rightarrow Verb$	$VP \rightarrow book / include / prefer$		
$VP \rightarrow Verb NP$	$VP \rightarrow Verb NP$		
$VP \rightarrow Verb NP PP$	$VP \rightarrow X2 PP$		
	$X2 \rightarrow Verb NP$		
$VP \rightarrow Verb PP$	$VP \rightarrow Verb PP$		
$VP \rightarrow VP PP$	$VP \rightarrow VP PP$		
$PP \rightarrow Preposition NP$	$PP \rightarrow Preposition NP$		

$\mathcal{L}_1$ Grammar	$\mathcal{L}_1$ in CNF
$S \rightarrow NP VP$	$S \rightarrow NP VP$
$S \rightarrow Aux NP VP$	$S \rightarrow X1 VP$
	$X1 \rightarrow Aux NP$
$S \to VP$	$S \rightarrow book / include / prefer$
	$S \rightarrow Verb NP$
	$S \rightarrow X2 PP$
	$S \rightarrow Verb PP$
	$S \rightarrow VP PP$
$NP \rightarrow Pronoun$	$NP \rightarrow I / she / me$
$NP \rightarrow Proper-Noun$	$NP \rightarrow TWA / Houston$
$NP \rightarrow Det\ Nominal$	$NP \rightarrow Det\ Nominal$
$Nominal \rightarrow Noun$	$Nominal \rightarrow book \ / \ flight \ / \ meal \ / \ money$
$Nominal \rightarrow Nominal \ Noun$	$Nominal \rightarrow Nominal \ Noun$
$Nominal \rightarrow Nominal PP$	$Nominal \rightarrow Nominal PP$
$VP \rightarrow Verb$	$VP \rightarrow book / include / prefer$
$VP \rightarrow Verb NP$	$VP \rightarrow Verb NP$
$VP \rightarrow Verb NP PP$	$VP \rightarrow X2 PP$
	$X2 \rightarrow Verb NP$
$VP \rightarrow Verb PP$	$VP \rightarrow Verb PP$
$VP \rightarrow VP PP$	$VP \rightarrow VP PP$
$PP \rightarrow Preposition NP$	$PP \rightarrow Preposition NP$

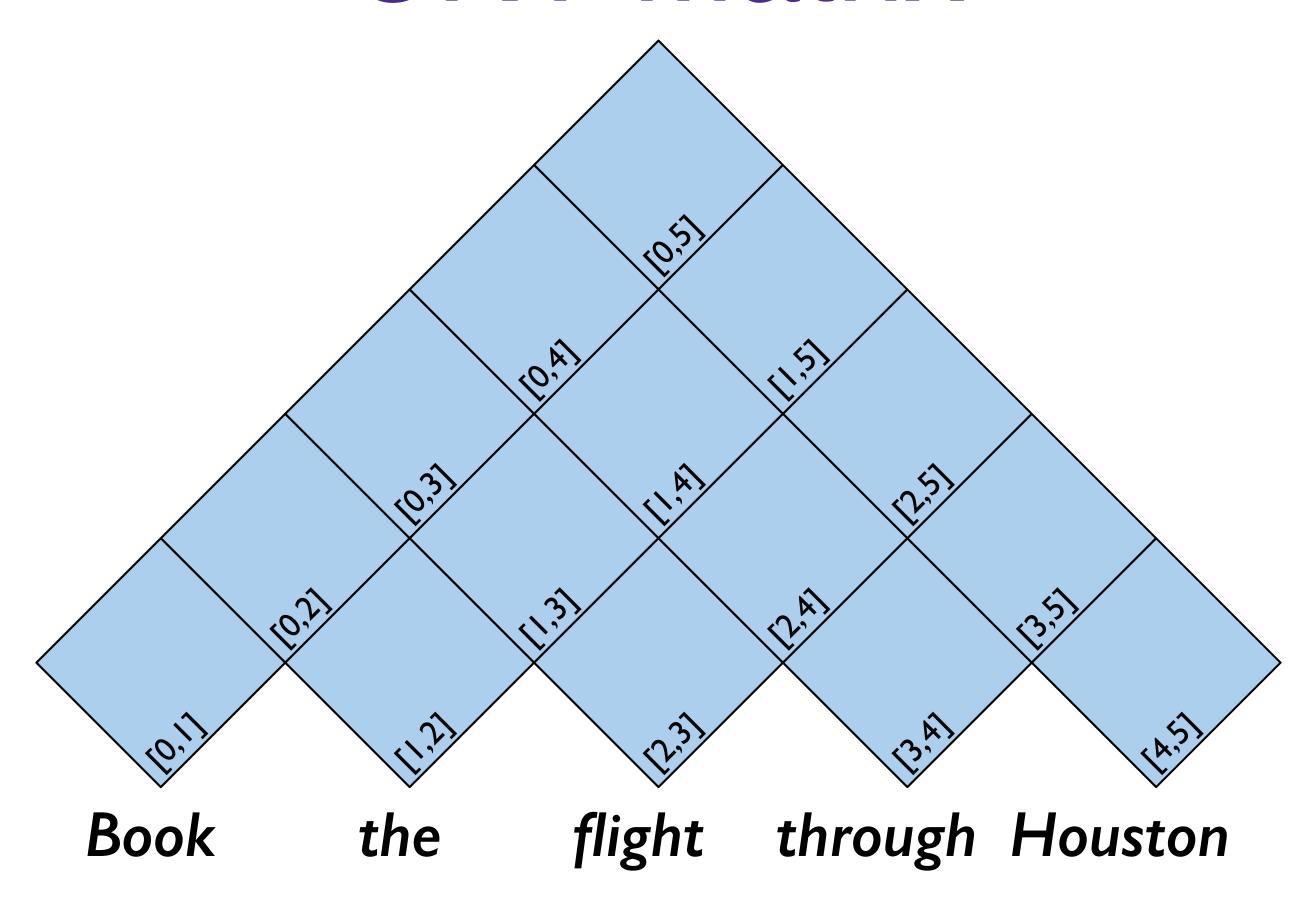
## Roadmap

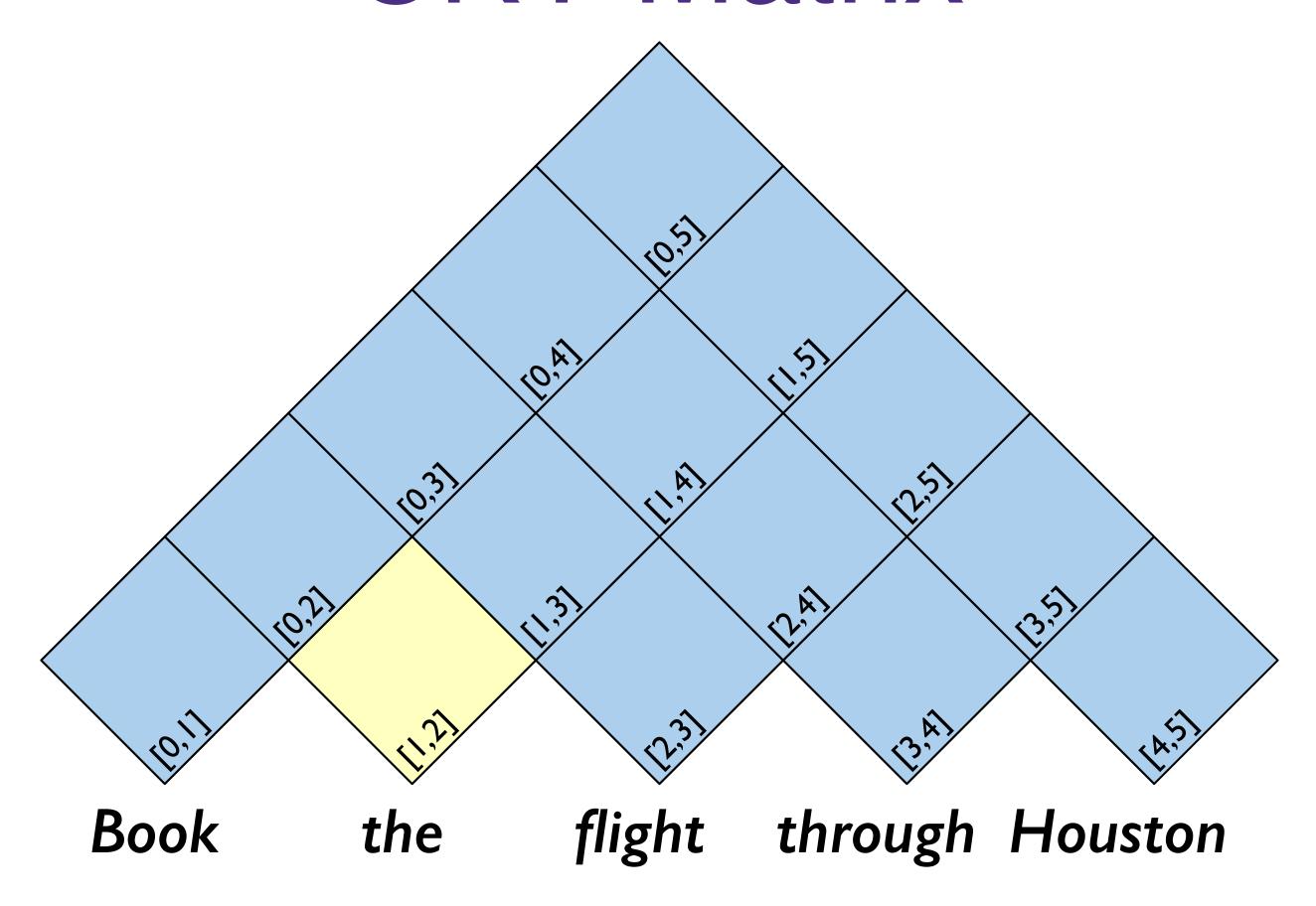
- Parsing-as-Search
- Parsing Challenges
- Strategy: Dynamic Programming
- Grammar Equivalence
- CKY parsing algorithm

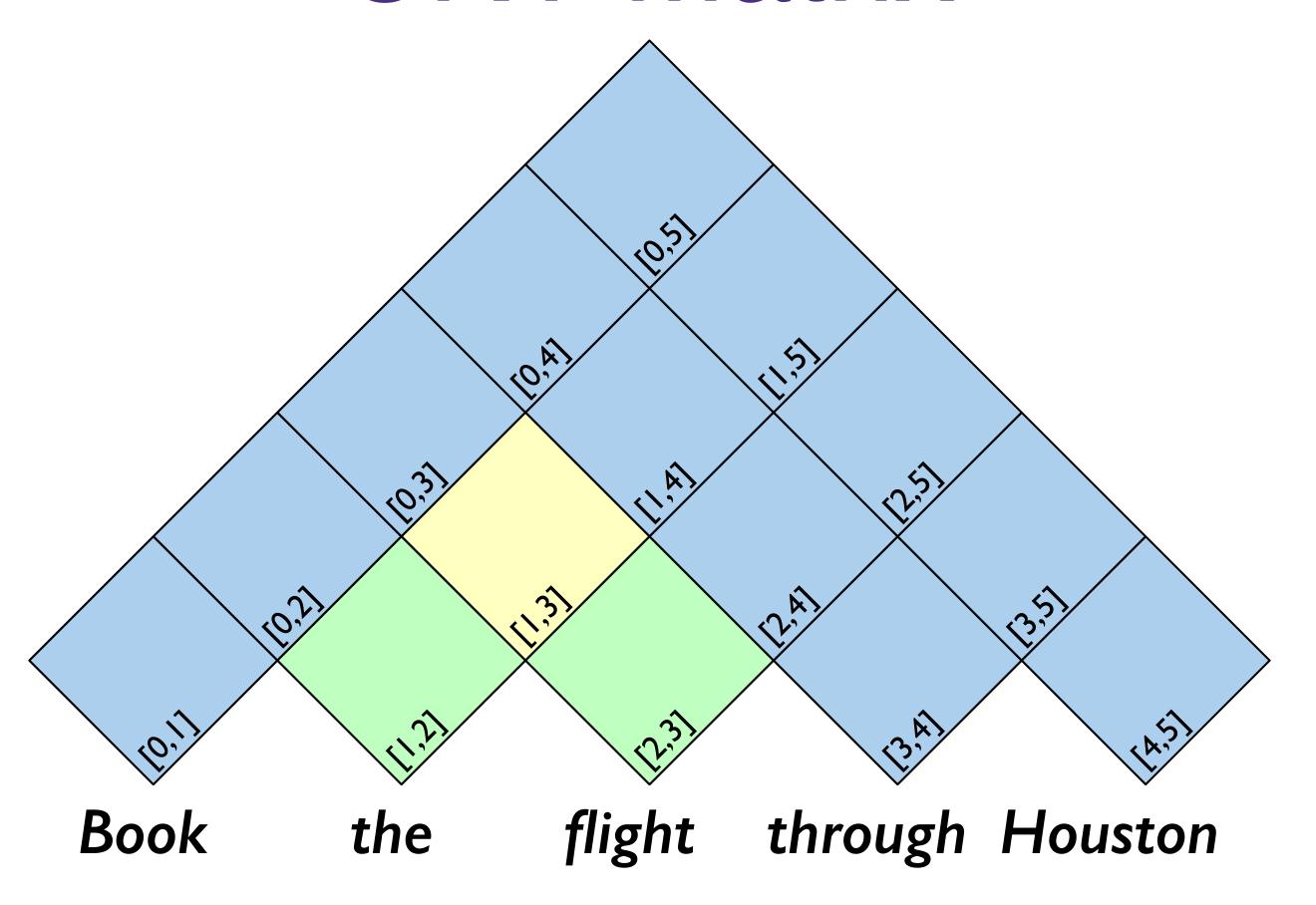
## CKY Parsing

- (Relatively) efficient parsing algorithm
- Based on tabulating substring parses to avoid repeat work
- Approach:
  - Use CNF Grammar
  - Build an  $(n + 1) \times (n + 1)$  matrix to store subtrees
    - Upper triangular portion
  - Incrementally build parse spanning whole input string

Book	the	flight	through	Houston
[0,1]	[0,2]	[0,3]	[0,4]	[0,5]
	[1,2]	[1,3]	[1,4]	[1,5]
		[2,3]	[2,4]	[2,5]
			[3,4]	[3,5]
				[4,5]







## Dynamic Programming in CKY

- Key idea:
  - ullet for i < k < j
  - ...and a parse spanning substring [ i, j ]
  - There is a k such that there are parses spanning [i, k] and [k, j]
  - We can construct parses for whole sentences by building from these partial parses
- So to have a rule  $A \rightarrow B C$  in [i, j]
  - ullet Must have  $oldsymbol{B}$  in  $[oldsymbol{i},oldsymbol{k}]$  and  $oldsymbol{C}$  in  $[oldsymbol{k},oldsymbol{j}]$  for some  $oldsymbol{i} < oldsymbol{k} < oldsymbol{j}$
  - ullet CNF forces this for all j>i+1