K nearest neighbor

LING 572 Advanced Statistical Methods for NLP

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The term “weight” in ML

- Weights of features
- Weights of instances
- Weights of classifiers

The term “binary” in ML

- Classification problem:
  - Binary: the number of classes is 2
  - Multi-class: the number is classes is > 2

- Features:
  - Binary: the number of possible feature values is 2.
  - Categorical / discrete: > 2 values
  - Real-valued / scalar / continuous: the feature values are real numbers

- File format:
  - Binary: human un-readable
  - Text: human readable

kNN

Instance-based (IB) learning

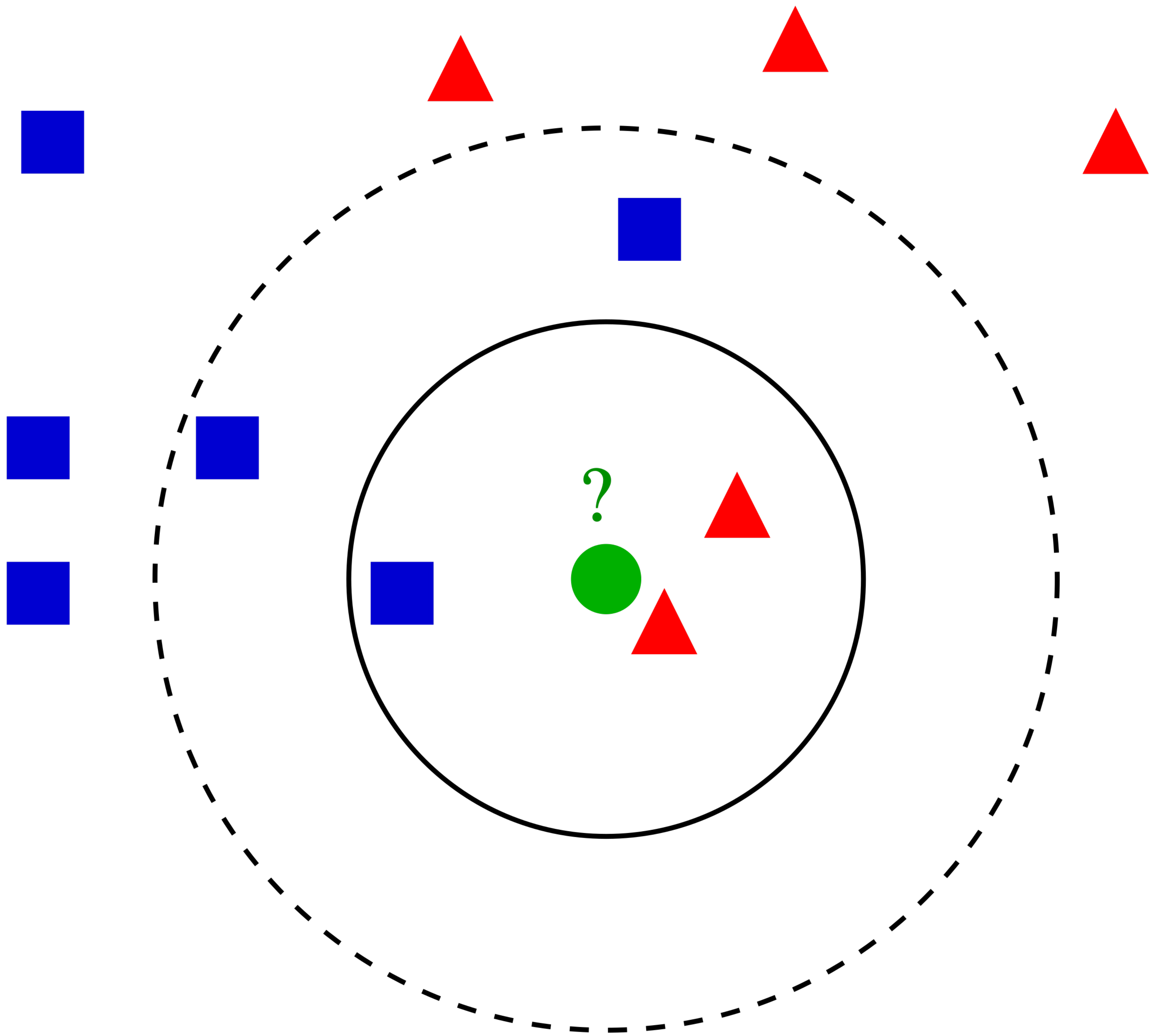
- No training: store all training instances.

  ➔ “Lazy learning”

- Examples:
  - kNN
  - Locally weighted regression
  - Case-based reasoning
  - …

- The most well-known IB method: kNN

kNN



img: Antti Ajanki, CC-by-SA 3.0

kNN

- Training: record labeled instances as feature vectors

- Test: for a new instance d,
  - find k training instances that are closest to d.
  - perform majority voting or weighted voting.

- Properties:
  - A “lazy” classifier. No learning in the training stage.
  - Feature selection and distance measure are crucial.

The algorithm

- Determine parameter K
- Calculate the distance between the test instance and all the training instances
- Sort the distances and determine K nearest neighbors
- Gather the labels of the K nearest neighbors
- Use simple majority voting or weighted voting.

Issues

- What's *K*?
- How do we weight/scale/select features?
- How do we combine instances by voting?

Picking K

- Split the data into
  - Training data
  - Dev/val data
  - Test data

- Pick k with the lowest error rate on the validation set
  - use N-fold cross validation if the training data is small

Normalizing attribute values

- Distance could be dominated by some attributes with large numbers:
  - Example: features: age, income
  - Original data: x_1=(35, 76K), x_2=(36, 80K), x_3=(70, 79K)

- Rescale: i.e., normalize to [0,1]
  - Assume: age $\in$ [0,100], income $\in$ [0, 200K]
  - After normalization: x_1=(0.35, 0.38),
    x_2=(0.36, 0.40), x_3 = (0.70, 0.395).

The Choice of Features

- Imagine there are 100 features, and only 2 of them are relevant to the target label.

- Differences in irrelevant features likely to dominate:
  - kNN is easily misled in high-dimensional space.
  - Feature weighting or feature selection is key (It will be covered next time)

Feature weighting

- Reweighting a dimension j by weight w_j
  - Can increase or decrease weight of feature on that dimension
  - Setting w_j to zero eliminates this dimension altogether.

- Use (cross-)validation to automatically choose weights $w_1, \ldots, w_{|F|}$

Some similarity measures

- Euclidean distance:

$$d(d_i, d_j) = \sqrt{\Sigma_k (a_{i,k} - a_{j,k})^2}$$

- Weighted Euclidean distance:

$$d(d_i, d_j) = \sqrt{\Sigma_k w_k (a_{i,k} - a_{j,k})^2}$$

- Cosine:

$$cos(d_i, d_j) = \frac{\sum_k a_{i,k} a_{j,k}}{\sqrt{\sum_k a_{i,k}^2}\sqrt{\sum_k a_{j,k}^2}}$$

Voting by k-nearest neighbors

- Suppose we have found the k-nearest neighbors.

- Let $f_i(x)$ be the class label for the *i*-th neighbor of *x*.

$$\delta(c, f_i(x)) = \begin{cases} 1 & f_i(x) = c \\ 0 & \text{otherwise} \end{cases}$$

$$g(c) = \sum_i \delta(c, f_i(x))$$

that is, g(c) is the number of neighbors with label c.

Voting

- Majority voting: $c^* = \arg\max_c g(c)$

- Weighted voting: weighting is on each neighbor

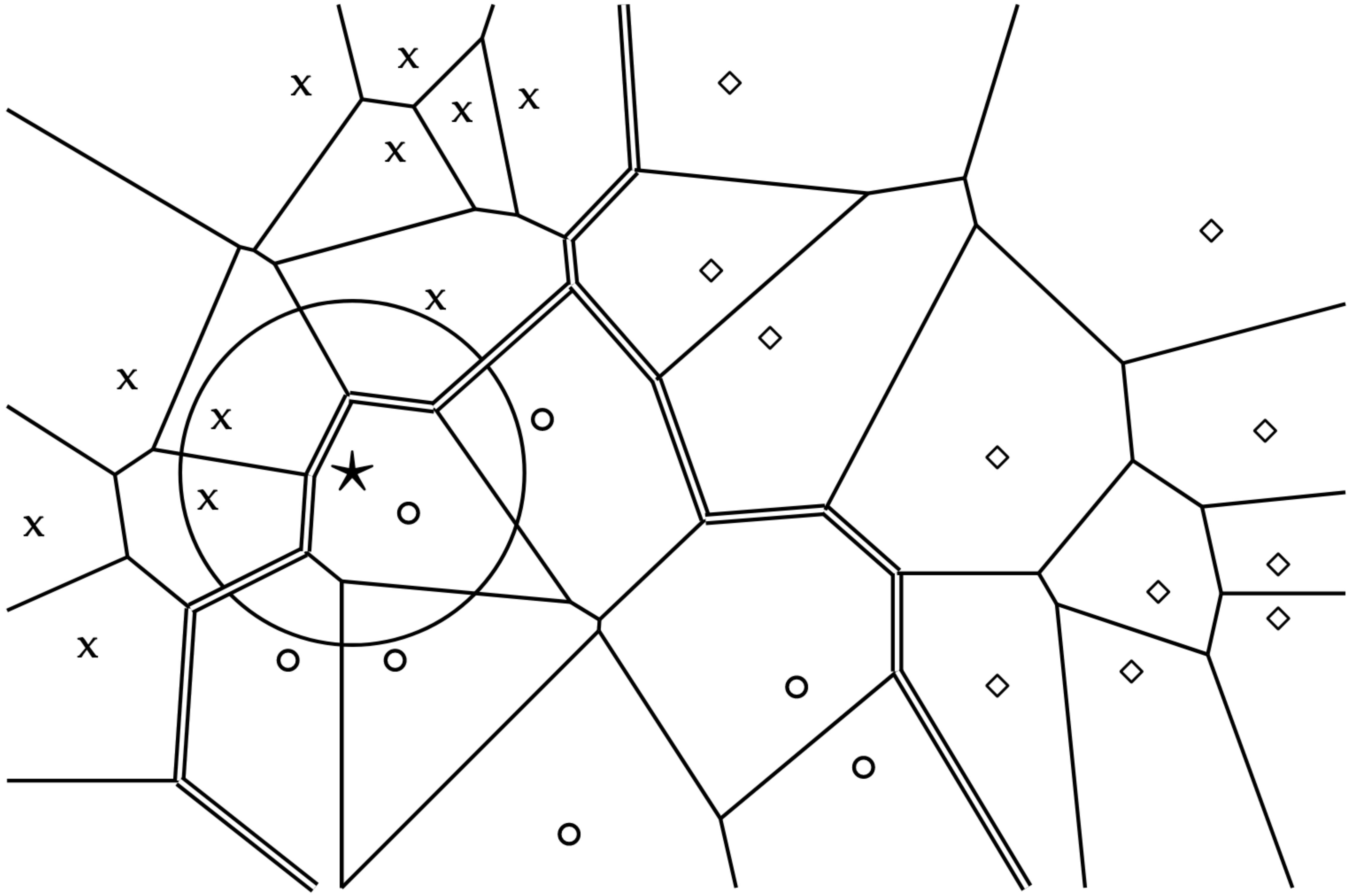
$$c^* = \arg\max_c \sum_i w_i \delta(c, f_i(x))$$

- Weighted voting allows us to use more training examples, e.g.:

$$w_i = \frac{1}{d(x, x_i)}$$

➔ We can use all the training examples.

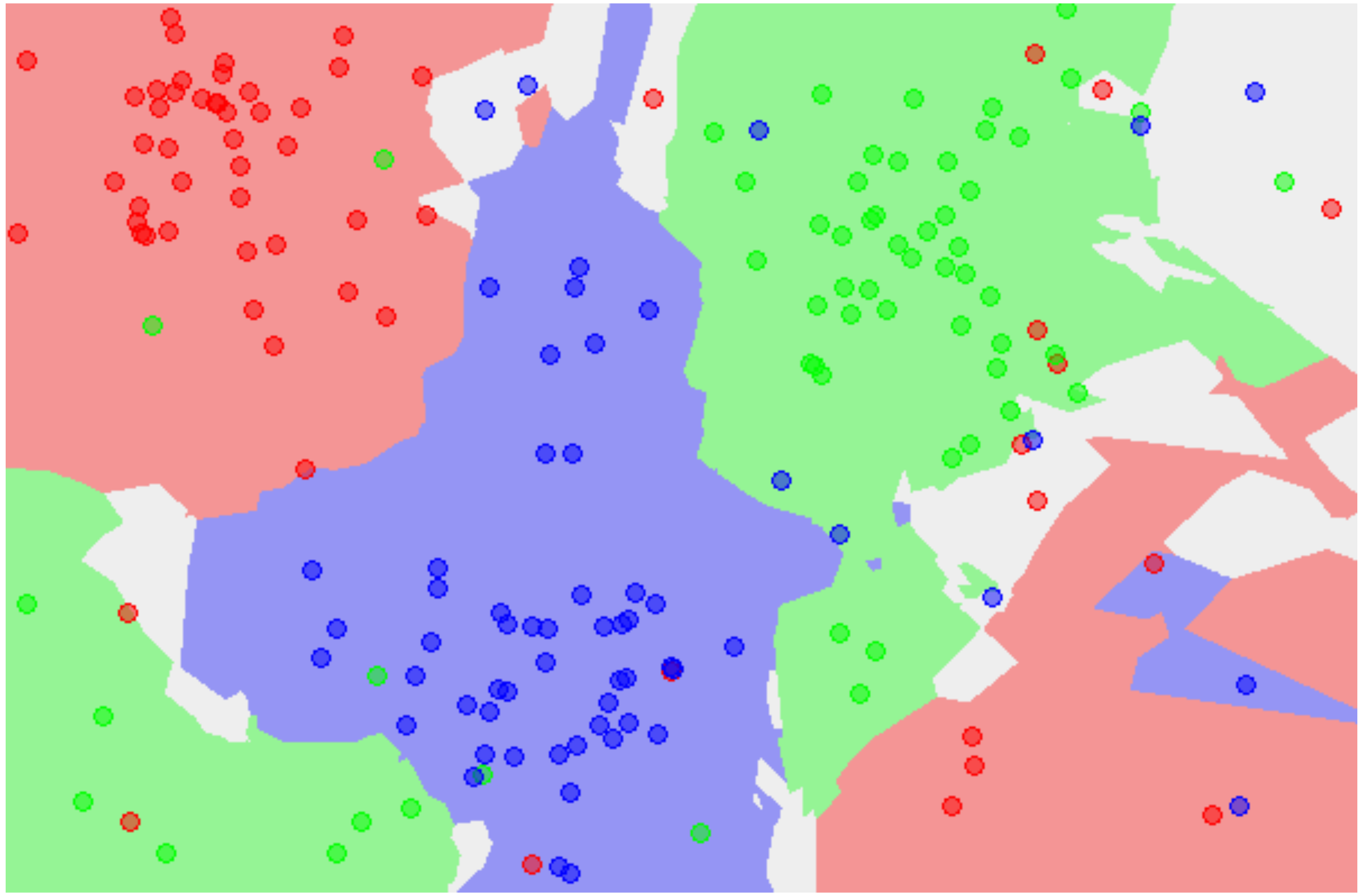
kNN Decision Boundary



IR, fig 14.6

1-NN: unions of cells of *Voronoi tessellation*

kNN Decision Boundary



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5-NN example

Summary of kNN algorithm

- Decide k, feature weights, and similarity measure

- Given a test instance x
  - Calculate the distances between x and all the training data
  - Choose the k nearest neighbors
  - Let the neighbors vote

Pros/Cons of kNN algorithm

- Strengths:
  - Simplicity (conceptual)
  - Efficiency at training: no training
  - Handling multi-class
  - Stability and robustness: averaging k neighbors
  - Predication accuracy: when the training data is large
    - Complex decision boundaries

- Weakness:
  - Efficiency at testing time: need to calculate all distances
    - Better search algorithms: e.g., use k-d trees
    - Reduce the amount of training data used at the test time: e.g., Rocchio algorithm
  - Sensitivity to irrelevant or redundant features
  - Distance metrics unclear on non-numerical/binary values