CKY Parsing & CNF Conversion

LING 571 — Deep Processing Techniques for NLP
October 5, 2022
Shane Steinert-Threlkeld

Announcements

- **HW #1** due tonight at **11:59pm**.
- Python on Patas: installed versions `ls /opt l grep python`. E.g., invoke by:
  - **`/opt/python-3.6/bin/python3`**
  - `nltk` is installed.
- [For personal projects, but not 571 HW, you can use the latest of everything via Anaconda (download with `wget`).]
- When in doubt, use *full paths* for everything (python binary, file names, etc)
- check_hwX.sh: invoke from your local directory (for permission reasons)

Joke of the Week (PP Attachment Ambiguity)

tott @crazytott · Oct 5

A cop just knocked on my door and told me that my dogs were chasing people on bikes???? Wtf??? My dogs don't even own bikes tf

Roadmap

- **Parsing-as-Search**
- Parsing Challenges
- Strategy: Dynamic Programming
- Grammar Equivalence
- CKY parsing algorithm

Computational Parsing

- Given a body of (annotated) text, how can we derive the grammar rules of a language, and employ them in automatic parsing?
  - Treebanks & PCFGs
- Given a grammar, how can we derive the analysis of an input sentence?
  - Parsing as search
  - CKY parsing
    - Conversion to CNF

What is Parsing?

- CFG parsing is the task of assigning trees to input strings
  - For any input A and grammar G
    - …assign ≥0 parse trees T that represent its syntactic structure, and…
    - Cover all and only the elements of A
    - Have, as root, the start symbol S of G
    - …do not necessarily pick one single (or correct) analysis
- Subtask: Recognition
  - Given input A, G – is A in language defined by G or not?

Motivation

- Is this sentence in the language — i.e. is it “grammatical?”
  - ** I prefer United has the earliest flight.*
  - FSAs accept regular languages defined by finite-state automata.
  - Our parsers accept languages defined by CFG (equiv. pushdown automata).

Motivation

- Is this sentence in the language — i.e. is it "grammatical?"
  - ** I prefer United has the earliest flight.*
  - FSAs accept regular languages defined by finite-state automata.
  - Our parsers accept languages defined by CFG (equiv. pushdown automata).
- What is the syntactic structure of this sentence?
  - *What airline has the cheapest flight?*
  - *What airport does Southwest fly from near Boston?*
  - Syntactic parse provides framework for semantic analysis
    - What is the subject? Direct object?

Parsing as Search

- Syntactic parsing searches through possible trees to find one or more trees that derive input

Parsing as Search

- Syntactic parsing searches through possible trees to find one or more trees that derive input
- Formally, search problems are defined by:
  - Start state S
  - Goal state G (with a test)
  - Set of actions that transition from one state to another
    - "Successor function"
  - A path cost function

Parsing as Search: One Model

- Start State S: Start Symbol

Parsing as Search: One Model

- Start State S: Start Symbol
- Goal test:
  - Does the parse tree cover all of, and only, the input?

Parsing as Search: One Model

- Start State S: Start Symbol
- Goal test:
  - Does the parse tree cover all of, and only, the input?
- Successor function:
  - Expand a nonterminal using a production where nonterminal is the LHS of the production

Parsing as Search: One Model

- Start State S: Start Symbol
- Goal test:
  - Does the parse tree cover all of, and only, the input?
- Successor function:
  - Expand a nonterminal using a production where nonterminal is the LHS of the production
- Path cost:
  - …ignored for now.

Parsing as Search: One Model

- Node:
  - Partial solution to search problem (partial parse)

Parsing as Search: One Model

- Node:
  - Partial solution to search problem (partial parse)
- Search start node (initial state):
  - Input string
  - Start symbol of CFG

Parsing as Search: One Model

- Node:
  - Partial solution to search problem (partial parse)
- Search start node (initial state):
  - Input string
  - Start symbol of CFG
- Goal node:
  - Full parse tree: covering all of, and only the input, rooted at S

Search Algorithms

- Depth First
  - Keep expanding nonterminals until they reach words
  - If no more expansions available, back up

Search Algorithms

- Depth First
  - Keep expanding nonterminals until they reach words
  - If no more expansions available, back up
- Breadth First
  - Consider all parses that expand a single nonterminal…
  - …then all with two expanded, etc…

Search Algorithms

- Depth First
  - Keep expanding nonterminals until they reach words
  - If no more expansions available, back up
- Breadth First
  - Consider all parses that expand a single nonterminal…
  - …then all with two expanded, etc…
- Other alternatives, if have associated path costs.

Parse Search Strategies

- Two constraints on parsing:
  - Must start with the start symbol
  - Must cover exactly the input string

Parse Search Strategies

- Two constraints on parsing:
  - Must start with the start symbol
  - Must cover exactly the input string
- Correspond to main parsing search strategies
  - Top-down search (Goal-directed)
  - Bottom-up search (Data-driven search)

A Grammar

Grammar	Lexicon
$S \rightarrow NP\ VP$	$Det \rightarrow that \mid this \mid a$
$S \rightarrow Aux\ NP\ VP$	$Noun \rightarrow book \mid flight \mid meal \mid money$
$S \rightarrow VP$	$Verb \rightarrow book \mid include \mid prefer$

Jurafsky & Martin, Speech and Language Processing, p.390

A Grammar

Grammar	Lexicon
$S \rightarrow NP\ VP$	$Det \rightarrow that \mid this \mid a$
$S \rightarrow Aux\ NP\ VP$	$Noun \rightarrow book \mid flight \mid meal \mid money$
$S \rightarrow VP$	$Verb \rightarrow book \mid include \mid prefer$
$NP \rightarrow Pronoun$	$Pronoun \rightarrow I \mid she \mid me$
$NP \rightarrow Proper\text{-}Noun$	$Proper\text{-}Noun \rightarrow Houston \mid NWA$
$NP \rightarrow Det\ Nominal$	$Aux \rightarrow does$
$Nominal \rightarrow Noun$	$Preposition \rightarrow from \mid to \mid on \mid near \mid through$

Jurafsky & Martin, Speech and Language Processing, p.390

A Grammar

Grammar	Lexicon
$S \rightarrow NP\ VP$	$Det \rightarrow that \mid this \mid a$
$S \rightarrow Aux\ NP\ VP$	$Noun \rightarrow book \mid flight \mid meal \mid money$
$S \rightarrow VP$	$Verb \rightarrow book \mid include \mid prefer$
$NP \rightarrow Pronoun$	$Pronoun \rightarrow I \mid she \mid me$
$NP \rightarrow Proper\text{-}Noun$	$Proper\text{-}Noun \rightarrow Houston \mid NWA$
$NP \rightarrow Det\ Nominal$	$Aux \rightarrow does$
$Nominal \rightarrow Noun$	$Preposition \rightarrow from \mid to \mid on \mid near \mid through$
$Nominal \rightarrow Nominal\ Noun$	
$Nominal \rightarrow Nominal\ PP$	
$VP \rightarrow Verb$	

Jurafsky & Martin, Speech and Language Processing, p.390

A Grammar

Grammar	Lexicon
$S \rightarrow NP\ VP$	$Det \rightarrow that \mid this \mid a$
$S \rightarrow Aux\ NP\ VP$	$Noun \rightarrow book \mid flight \mid meal \mid money$
$S \rightarrow VP$	$Verb \rightarrow book \mid include \mid prefer$
$NP \rightarrow Pronoun$	$Pronoun \rightarrow I \mid she \mid me$
$NP \rightarrow Proper\text{-}Noun$	$Proper\text{-}Noun \rightarrow Houston \mid NWA$
$NP \rightarrow Det\ Nominal$	$Aux \rightarrow does$
$Nominal \rightarrow Noun$	$Preposition \rightarrow from \mid to \mid on \mid near \mid through$
$Nominal \rightarrow Nominal\ Noun$	
$Nominal \rightarrow Nominal\ PP$	
$VP \rightarrow Verb$	
$VP \rightarrow Verb\ NP$	
$VP \rightarrow Verb\ NP\ PP$	
$VP \rightarrow Verb\ PP$	
$VP \rightarrow VP\ PP$	
$PP \rightarrow Preposition\ NP$	

Jurafsky & Martin, Speech and Language Processing, p.390

Top-down Search

- All valid parse trees must be rooted with start symbol

Top-down Search

- All valid parse trees must be rooted with start symbol
- Begin search with productions where S is on LHS
  - e.g. $S \rightarrow NP\ VP$

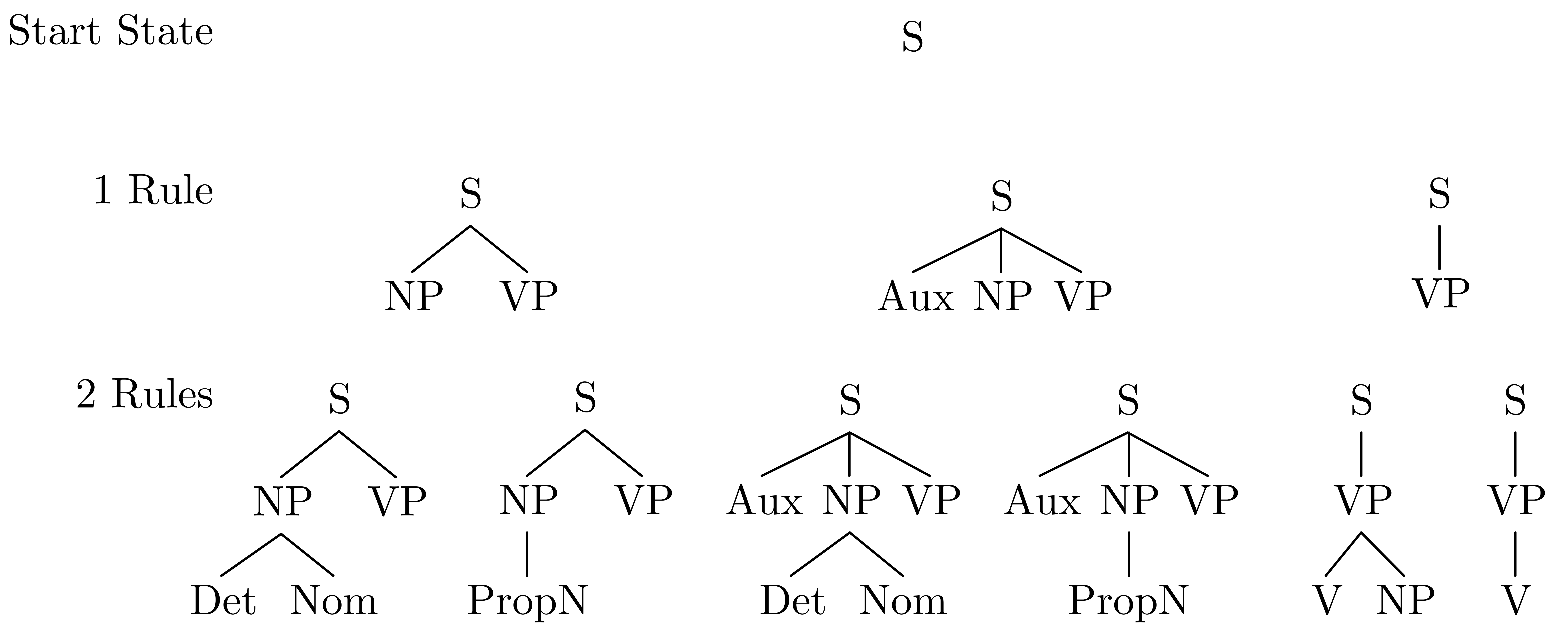
Top-down Search

- All valid parse trees must be rooted with start symbol
- Begin search with productions where S is on LHS
  - e.g. $S \rightarrow NP\ VP$
- Successively expand nonterminals
  - e.g. $NP \rightarrow Det\ Nominal$; $VP \rightarrow V\ NP$

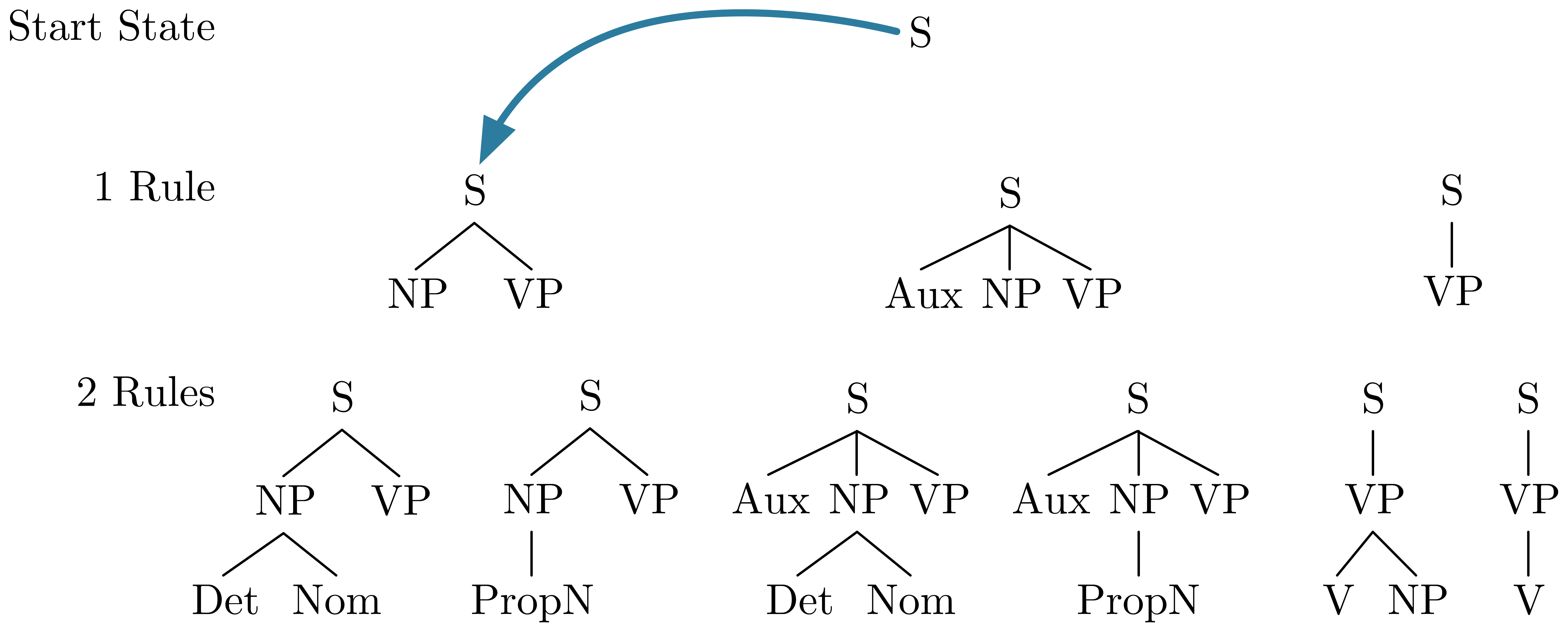
Top-down Search

- All valid parse trees must be rooted with start symbol
- Begin search with productions where S is on LHS
  - e.g. $S \rightarrow NP\ VP$
- Successively expand nonterminals
  - e.g. $NP \rightarrow Det\ Nominal;\ VP \rightarrow V\ NP$
- Terminate when all leaves are terminals

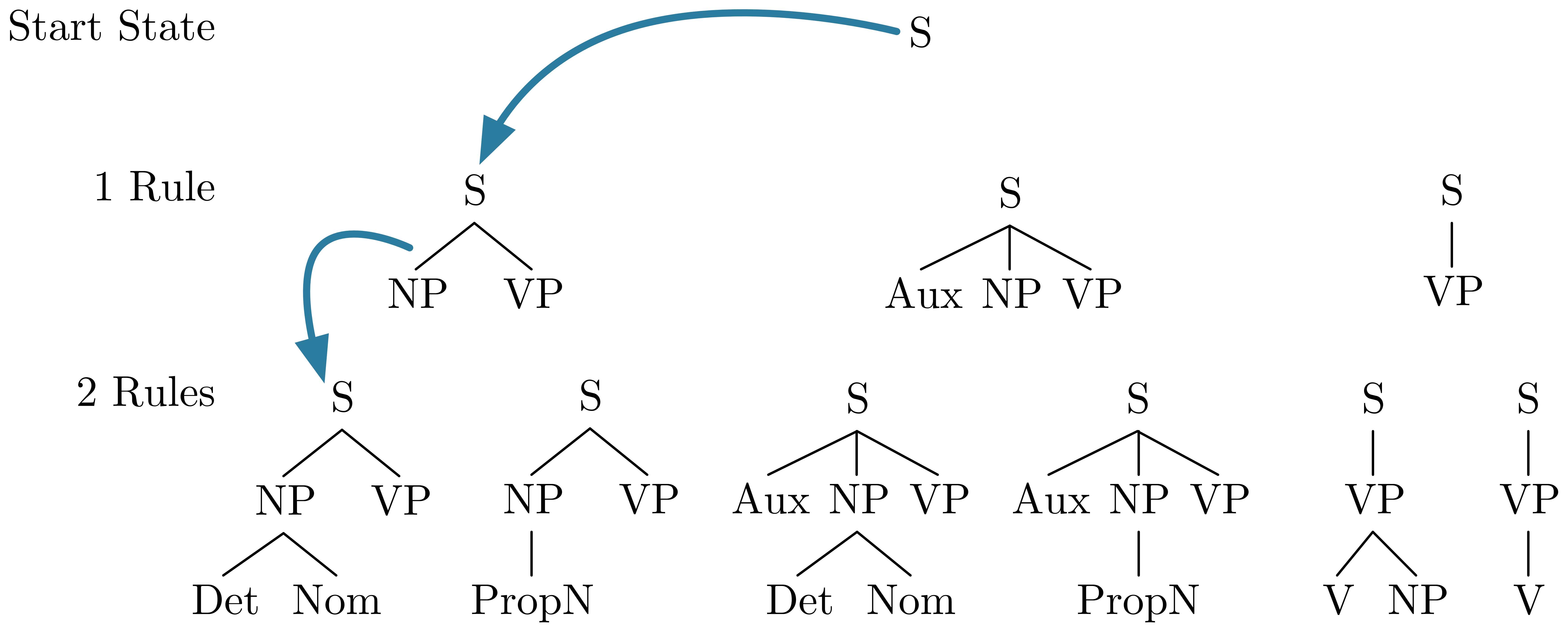
Depth-First Search



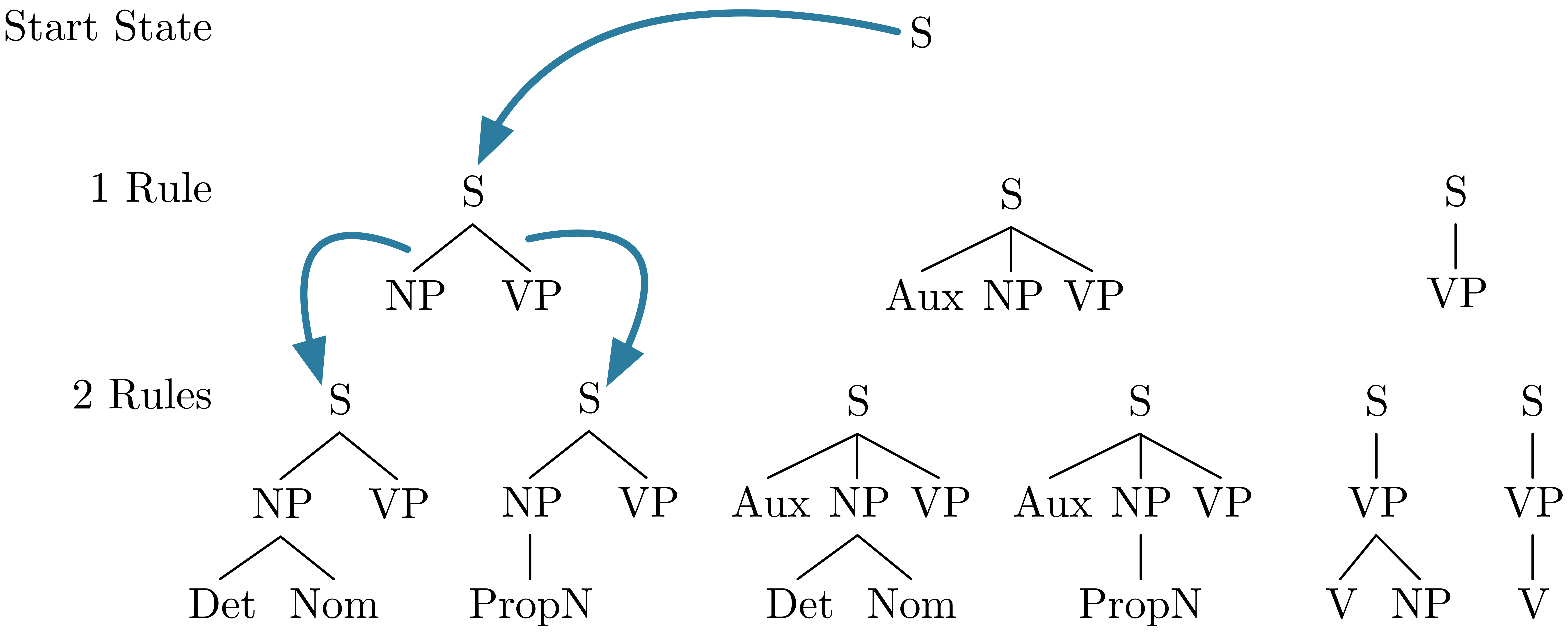
Depth-First Search



Depth-First Search



Depth-First Search

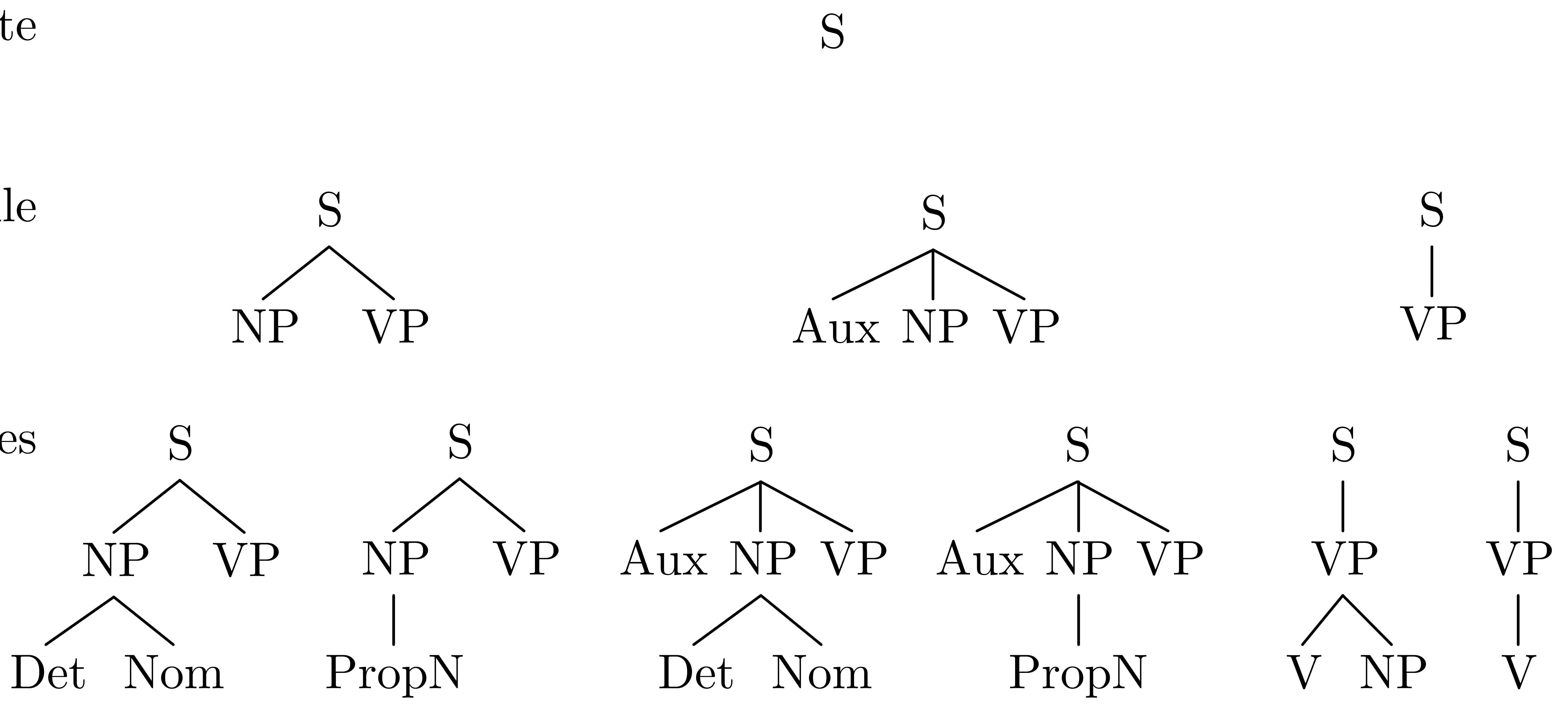


Depth-First Search

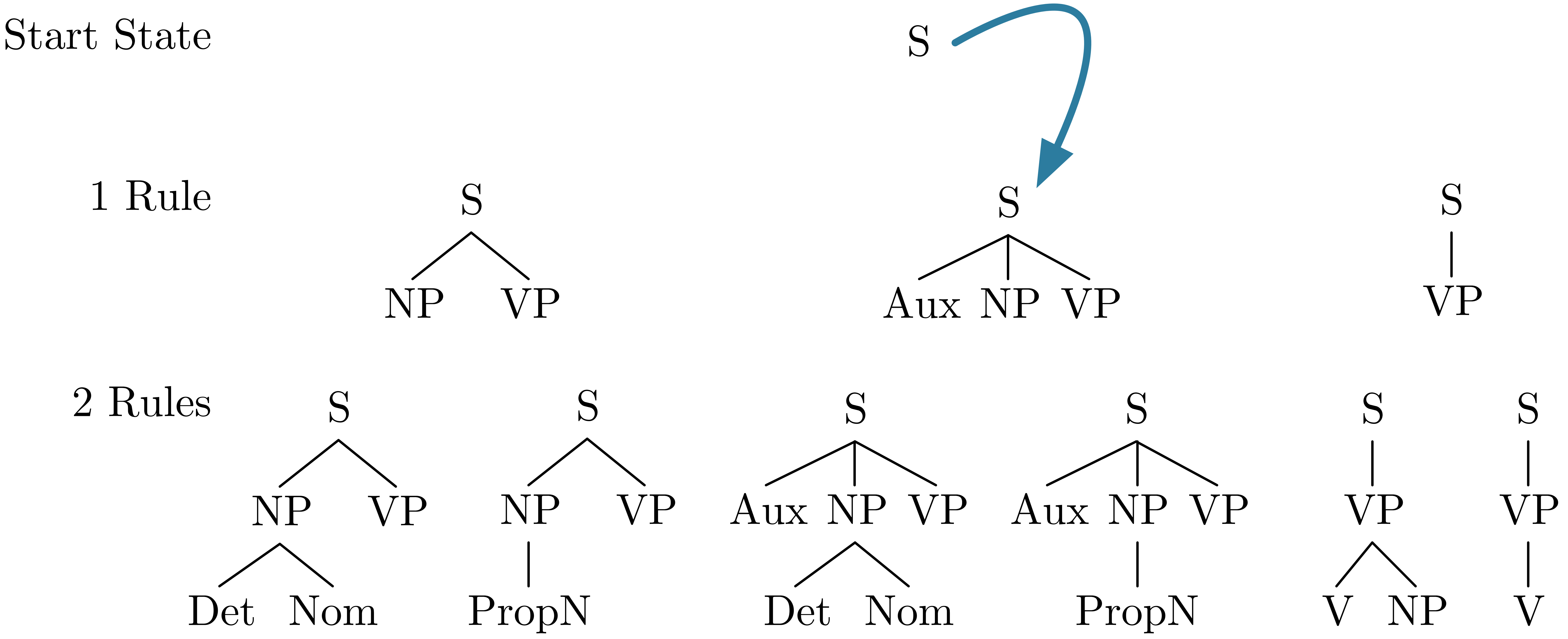
Start State

1 Rule

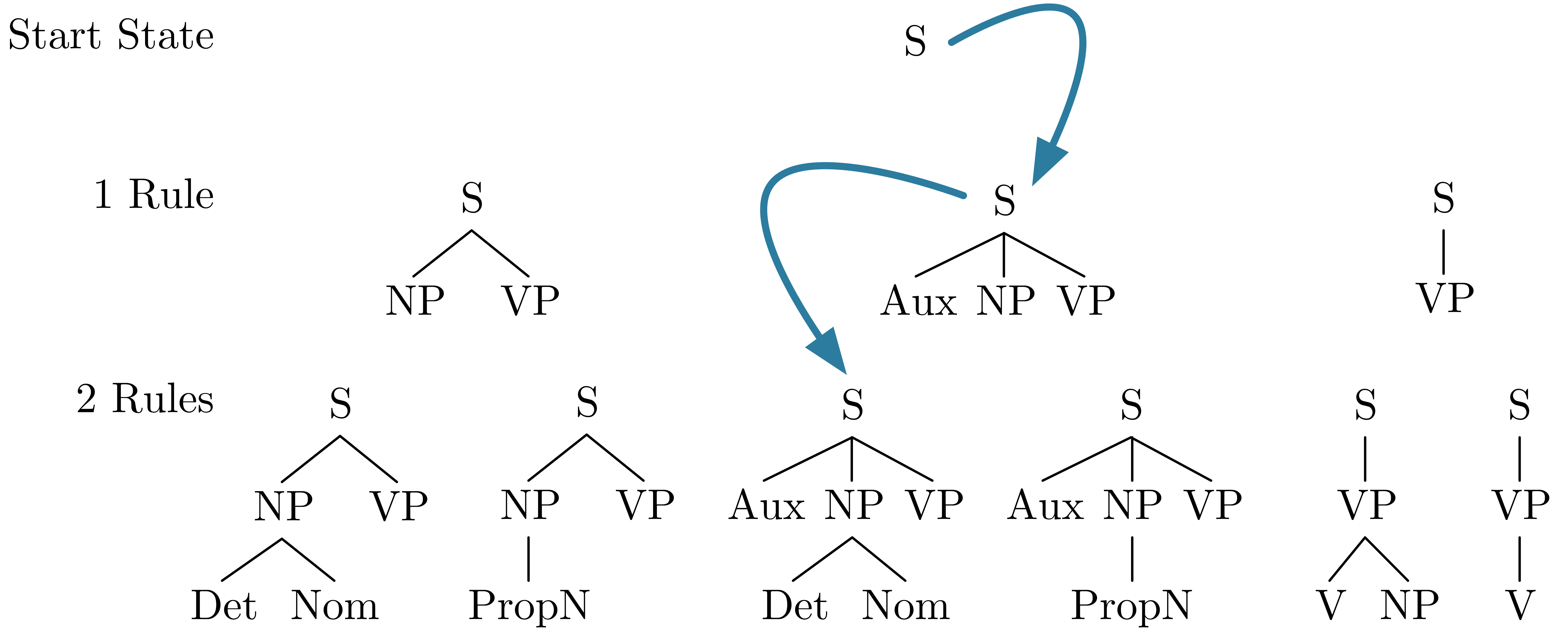
2 Rules



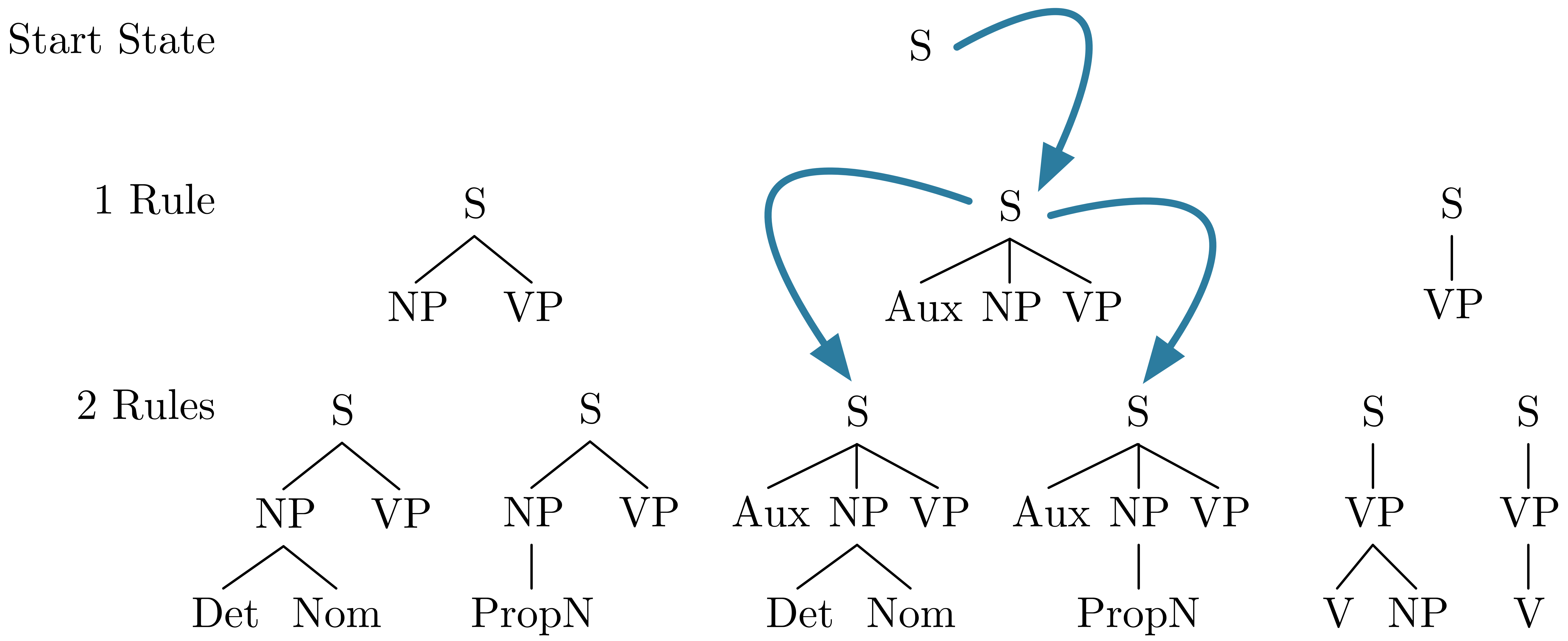
Depth-First Search



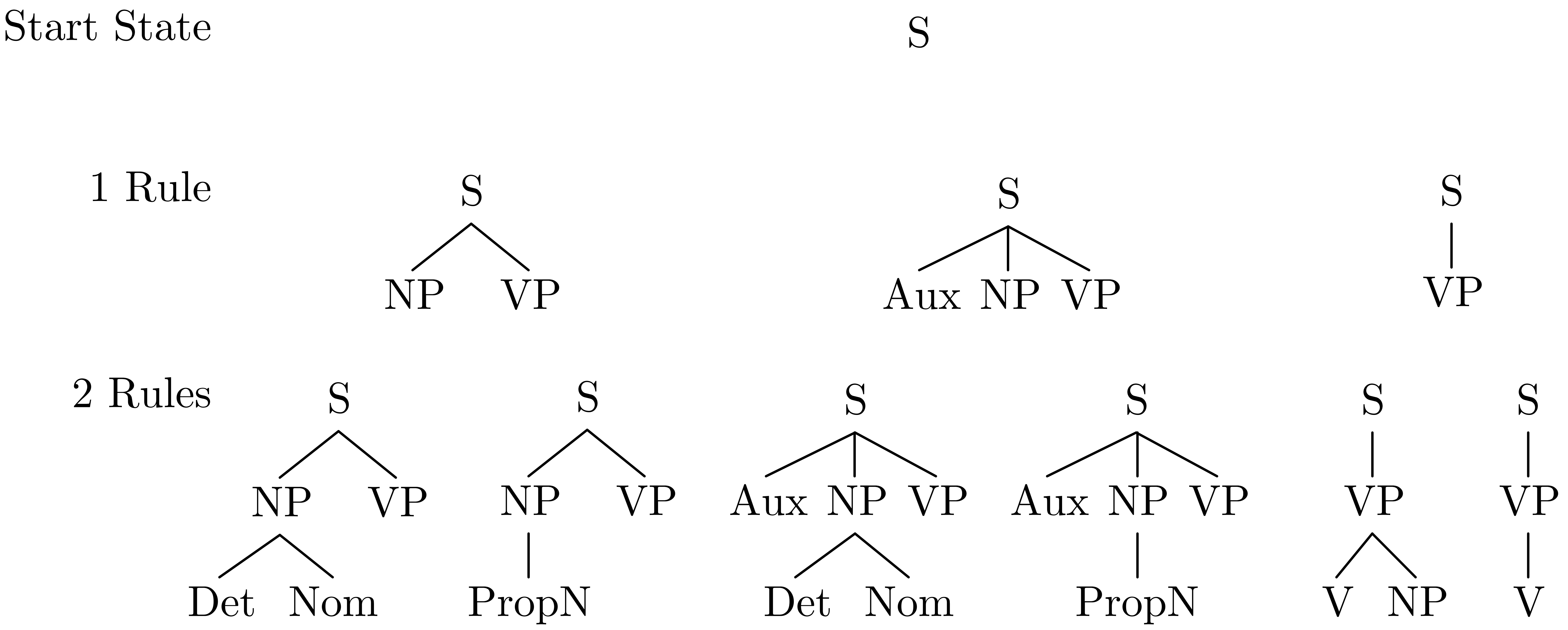
Depth-First Search



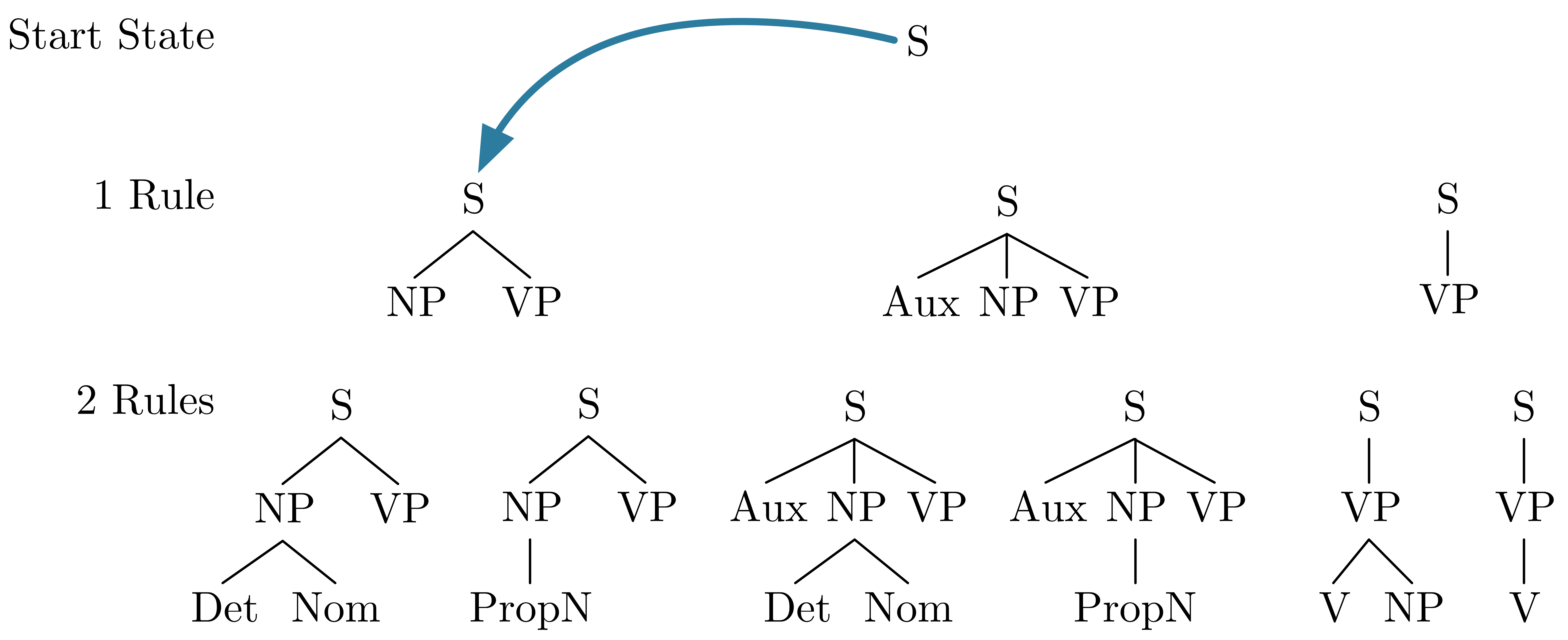
Depth-First Search



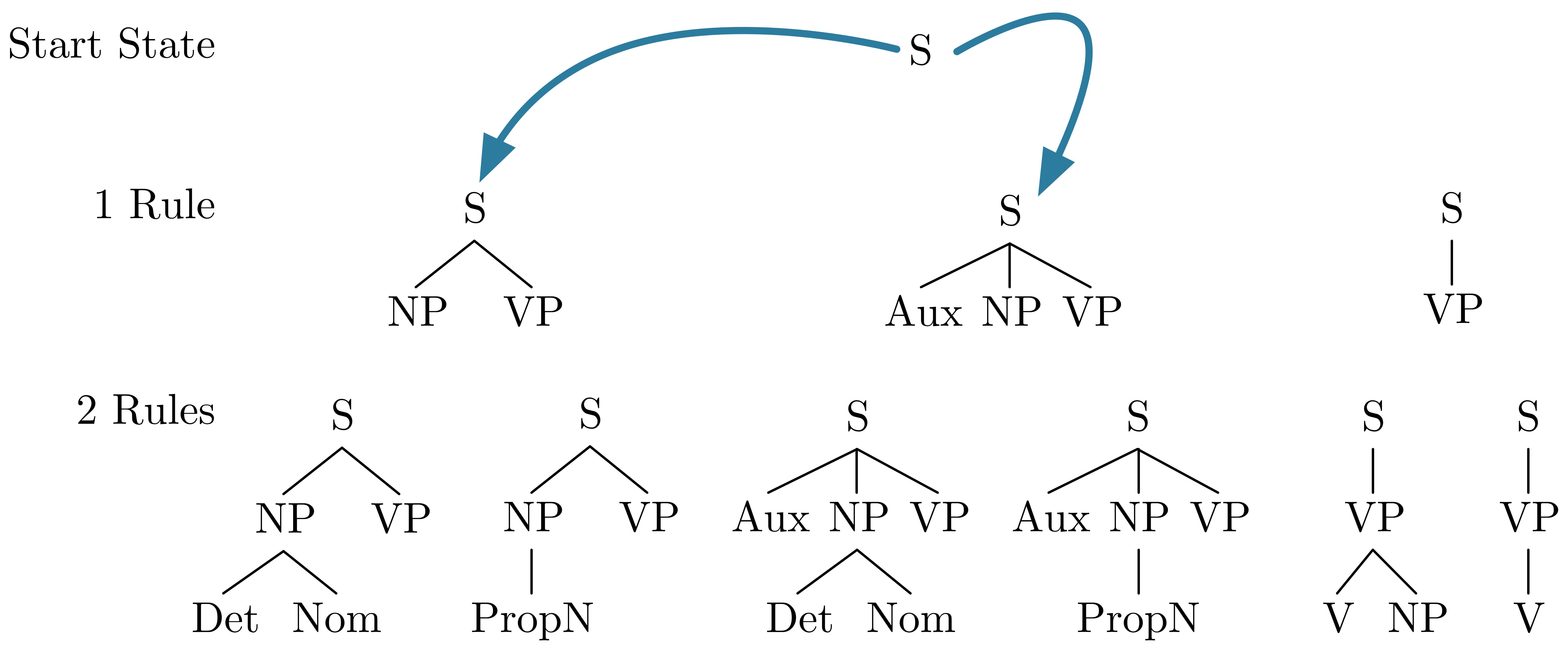
Breadth-First Search



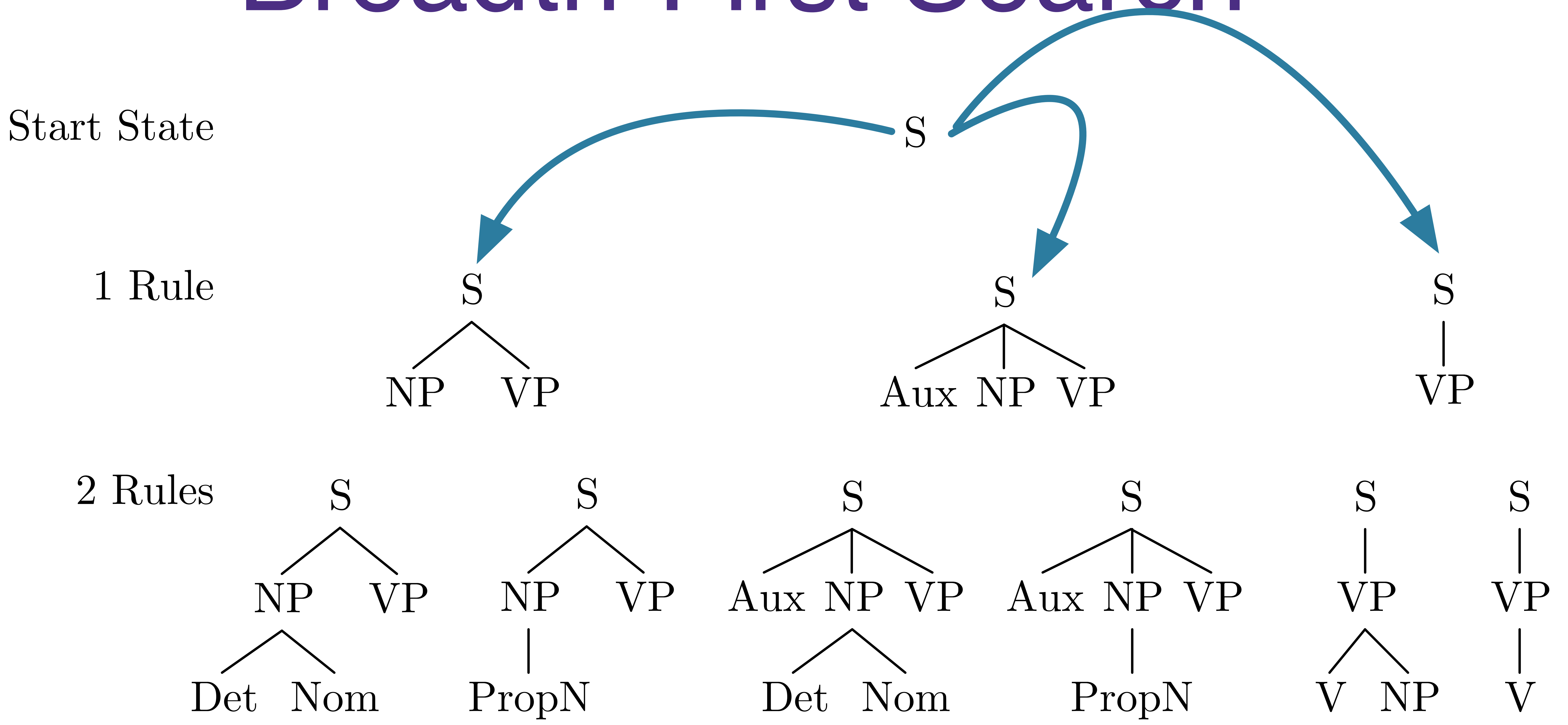
Breadth-First Search



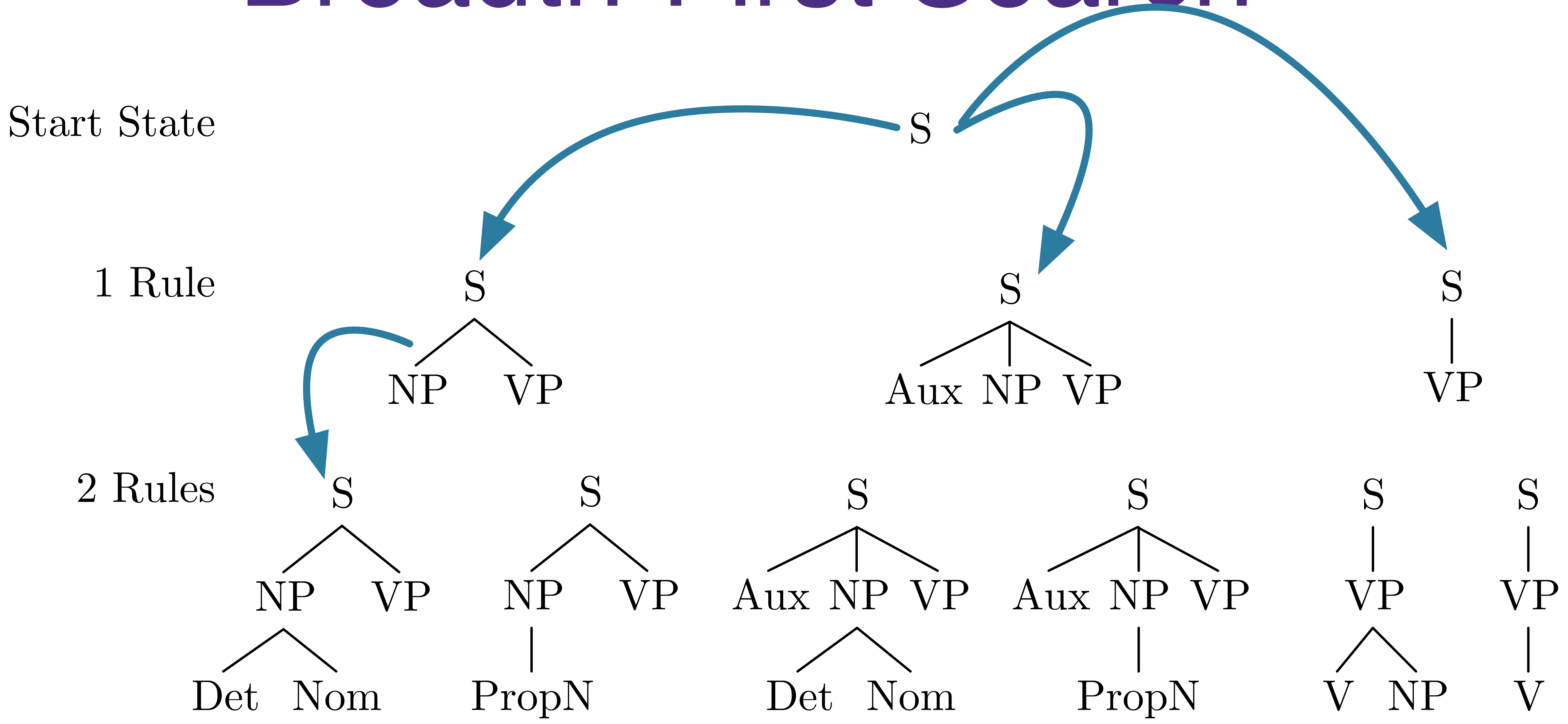
Breadth-First Search



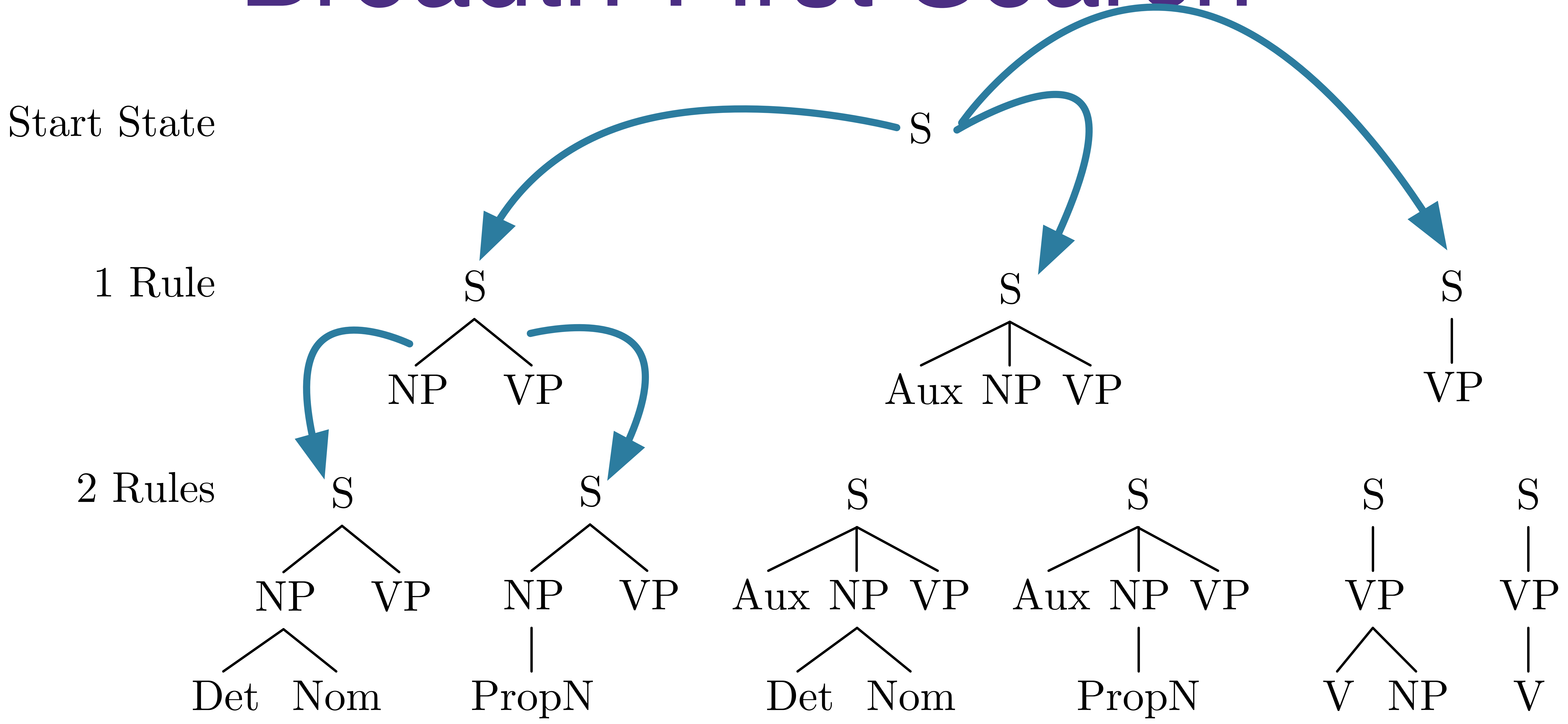
Breadth-First Search



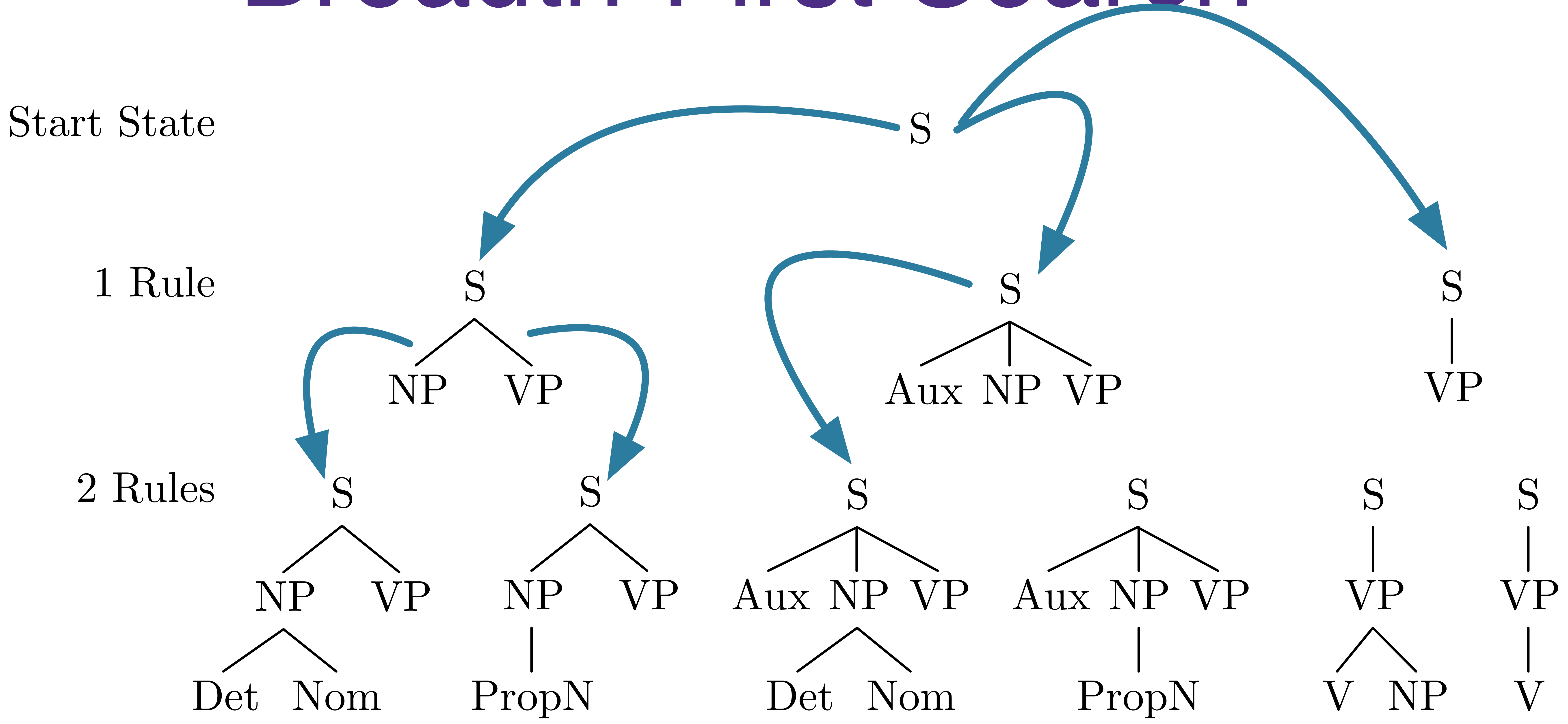
Breadth-First Search



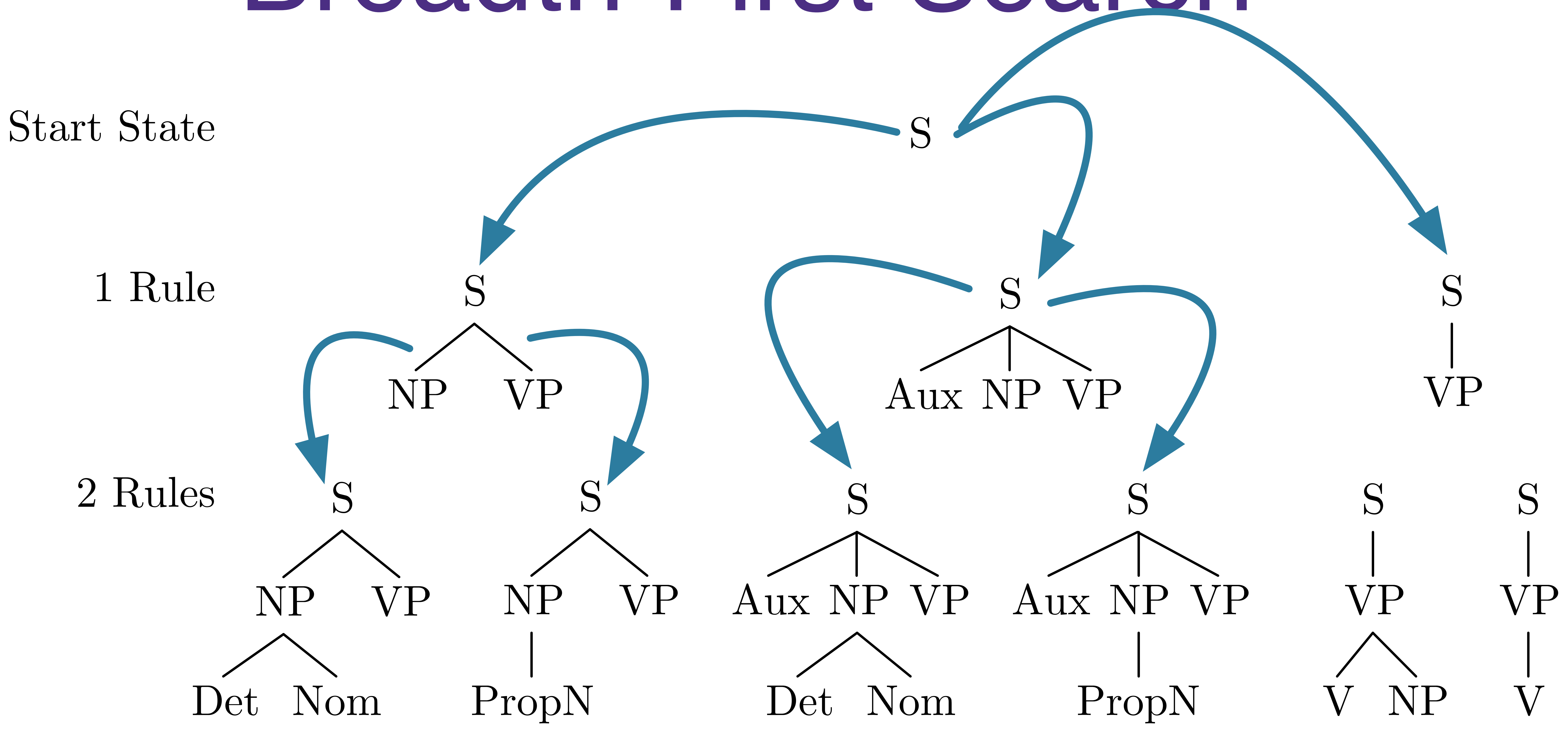
Breadth-First Search



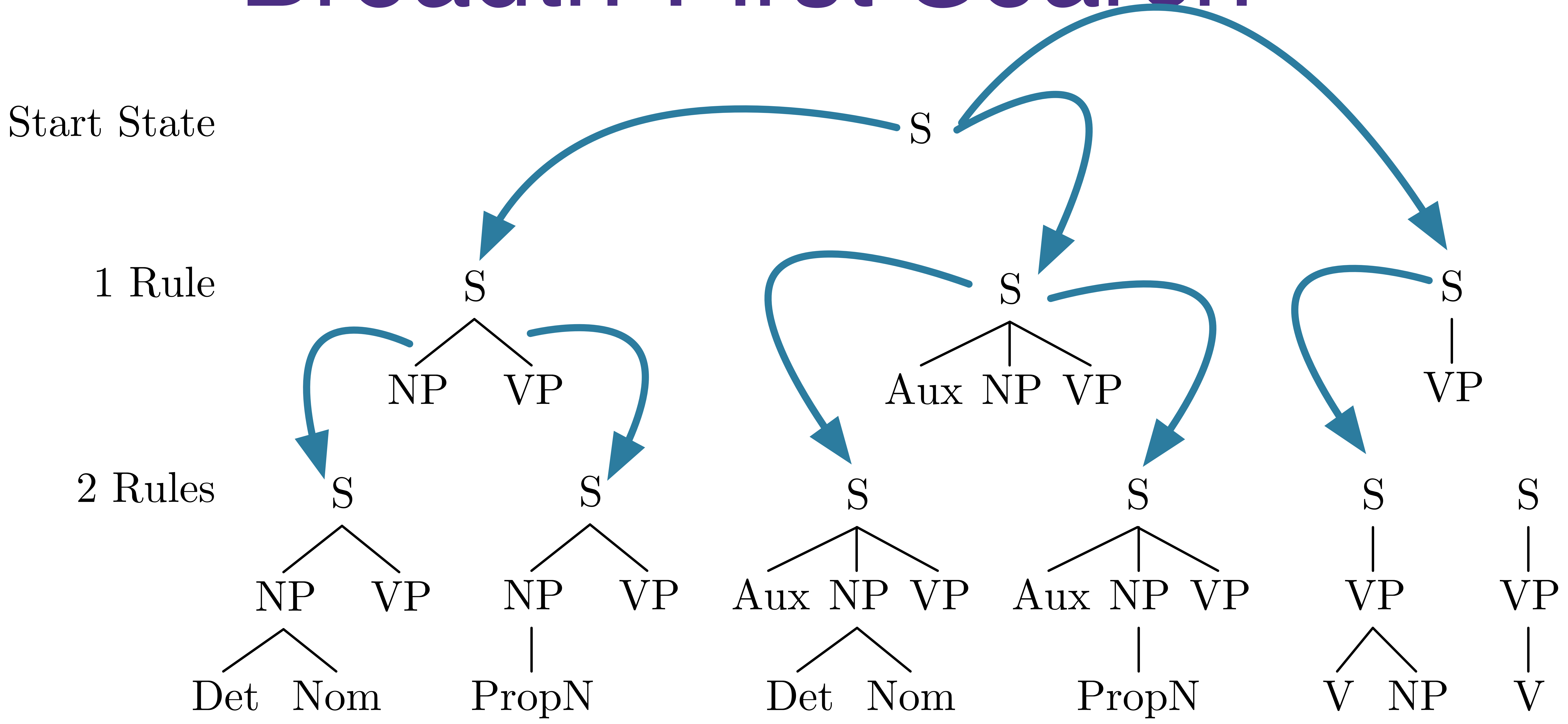
Breadth-First Search



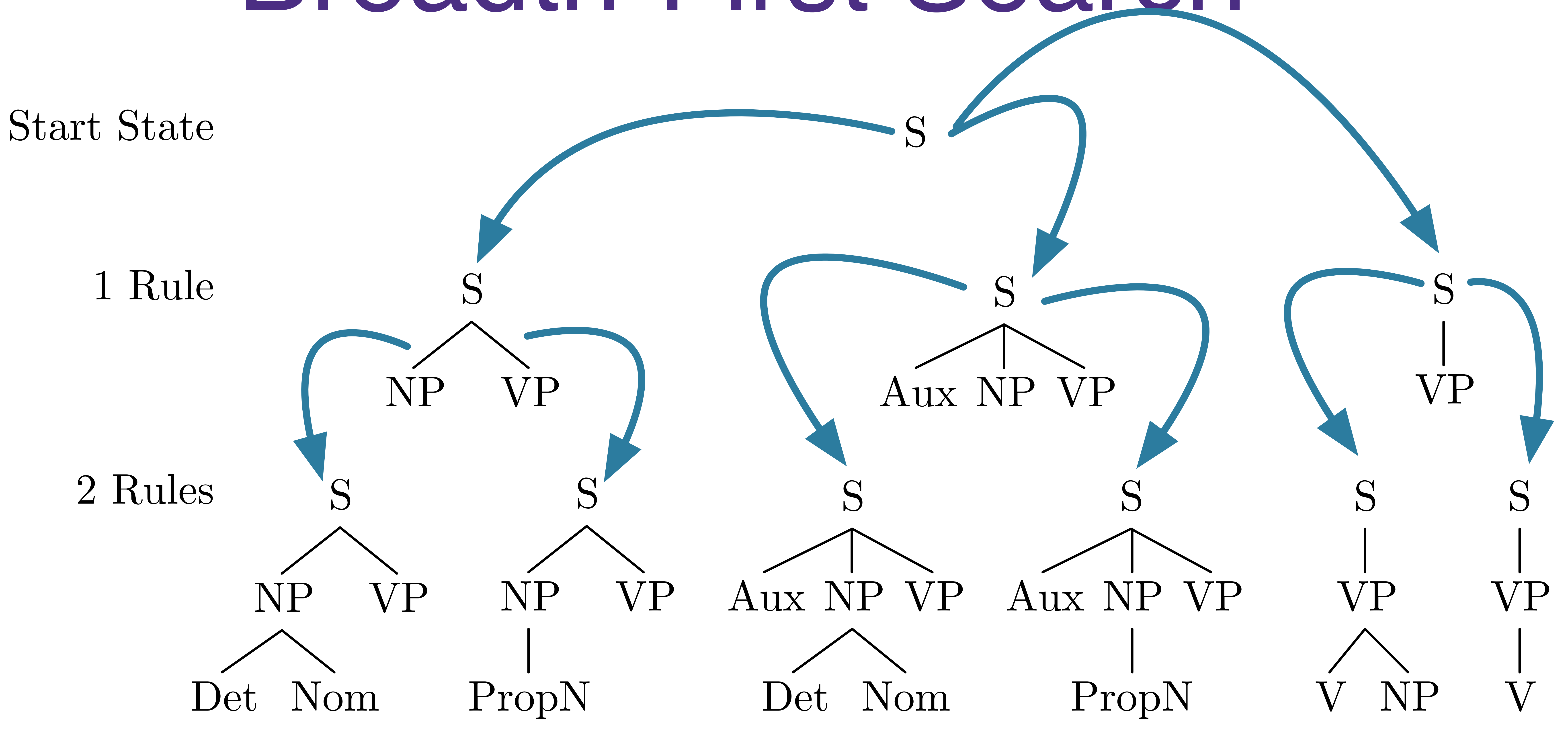
Breadth-First Search



Breadth-First Search



Breadth-First Search



Pros and Cons of Top-down Parsing

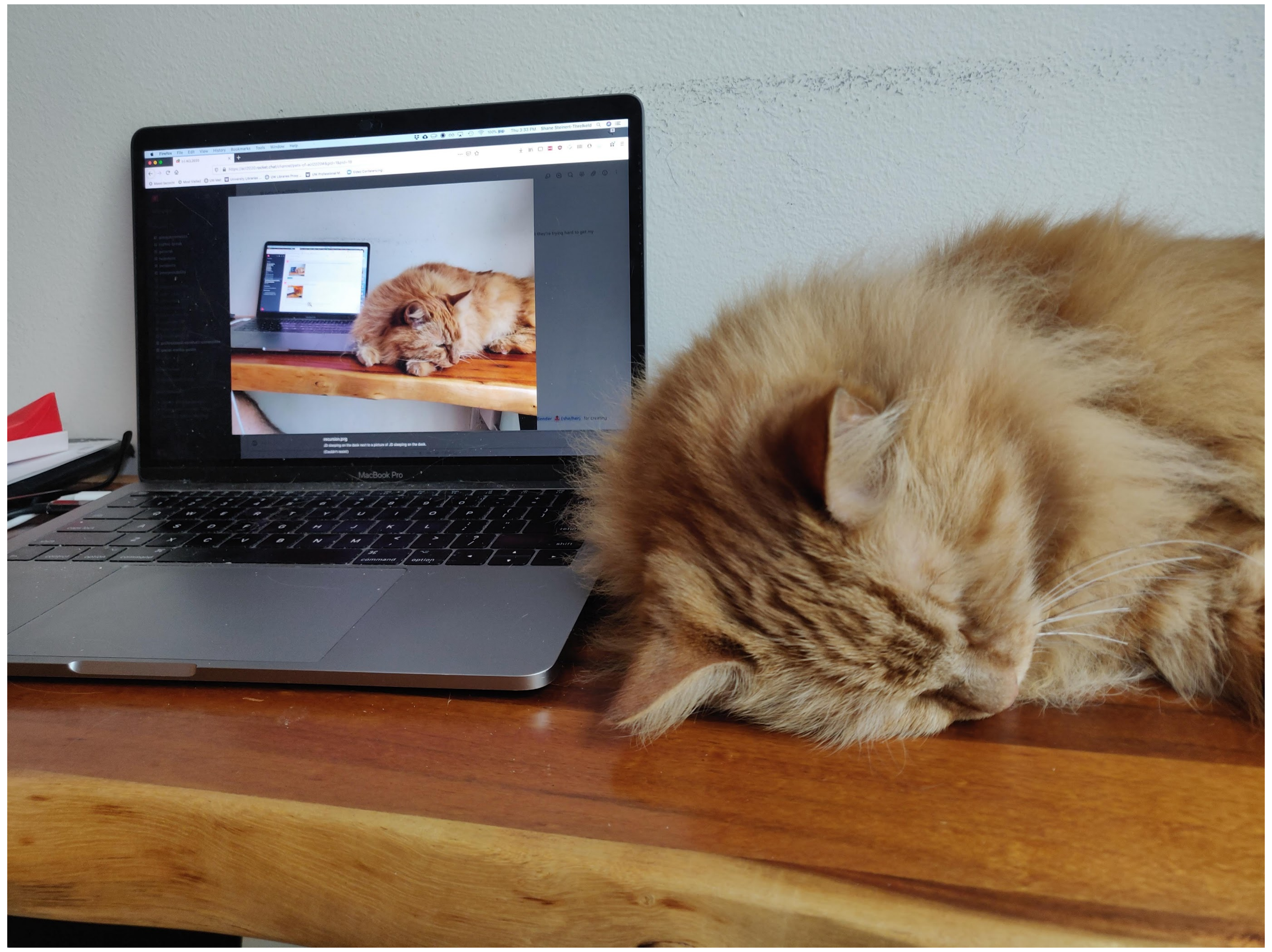
- Pros:
  - Doesn’t explore trees not rooted at S
  - Doesn’t explore subtrees that don’t fit valid trees

Pros and Cons of Top-down Parsing

- Pros:
  - Doesn't explore trees not rooted at S
  - Doesn't explore subtrees that don't fit valid trees
- Cons:
  - Produces trees that may not match input
  - May not terminate in presence of recursive rules
  - May re-derive subtrees as part of search

Pros and Cons of Top-down Parsing

- Pros:
  - Doesn't explore trees not rooted at S
  - Doesn't explore subtrees that don't fit valid trees
- Cons:
  - Produces trees that may not match input
  - May not terminate in presence of recursive rules
  - May re-derive subtrees as part of search



Bottom-Up Parsing

Bottom-Up Parsing

- Try to find all trees that span the input
  - Start with input string
    - Book that flight

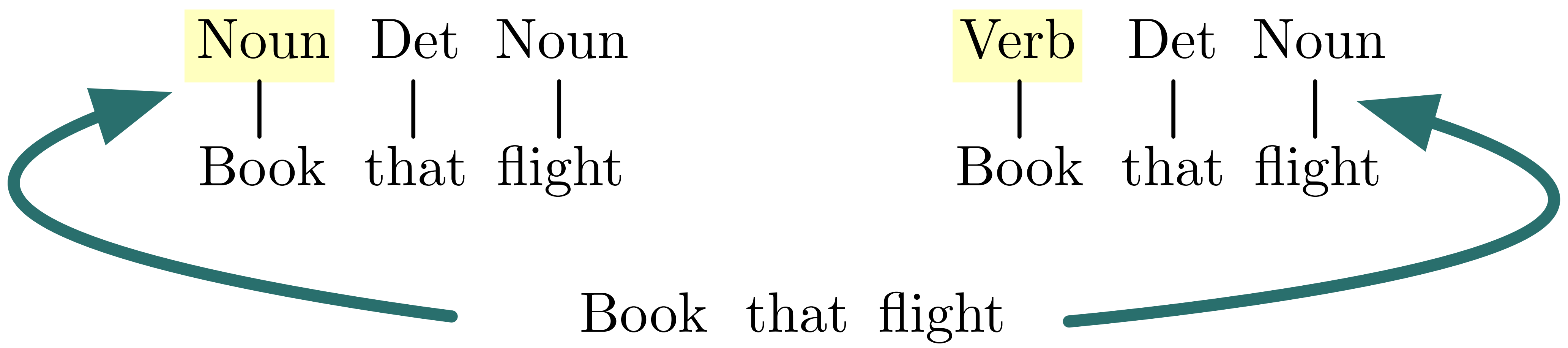
Bottom-Up Parsing

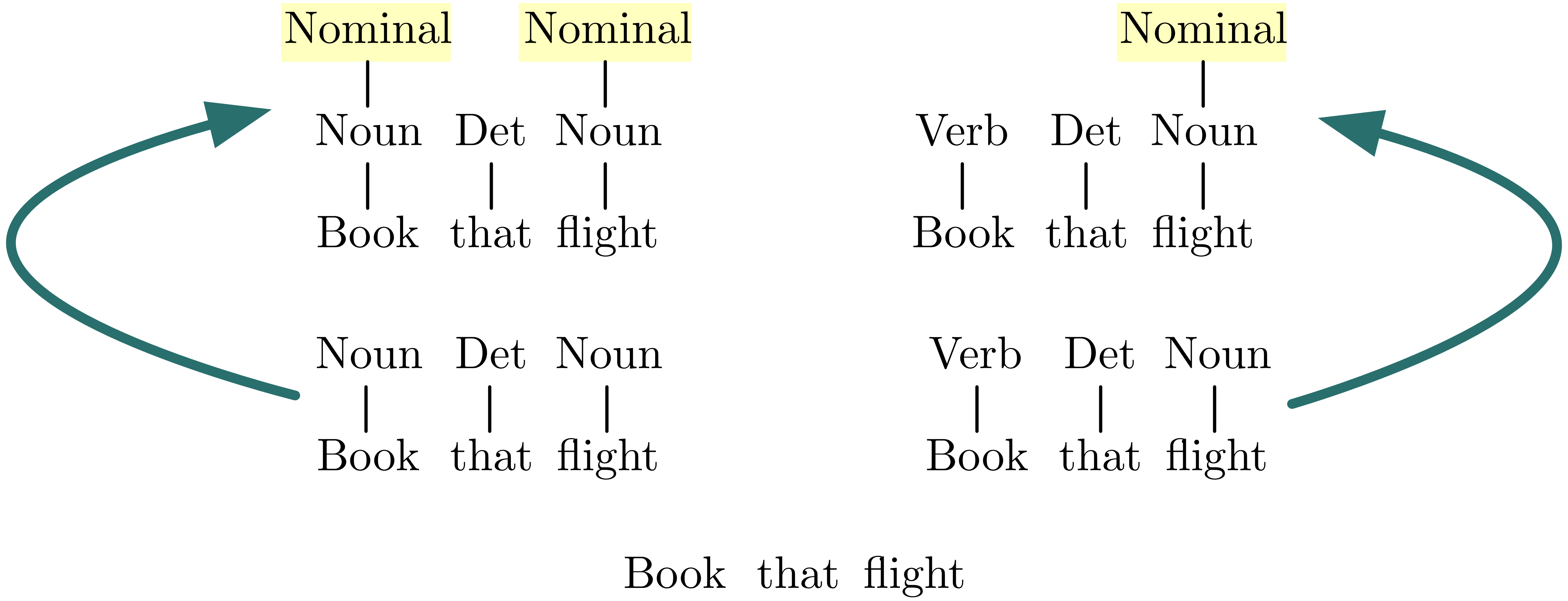
- Try to find all trees that span the input
  - Start with input string
    - Book that flight
- Use all productions with current subtree(s) on RHS
  - e.g. $\boldsymbol{N} \rightarrow \mathrm{Book};\ \boldsymbol{V} \rightarrow \mathrm{Book}$

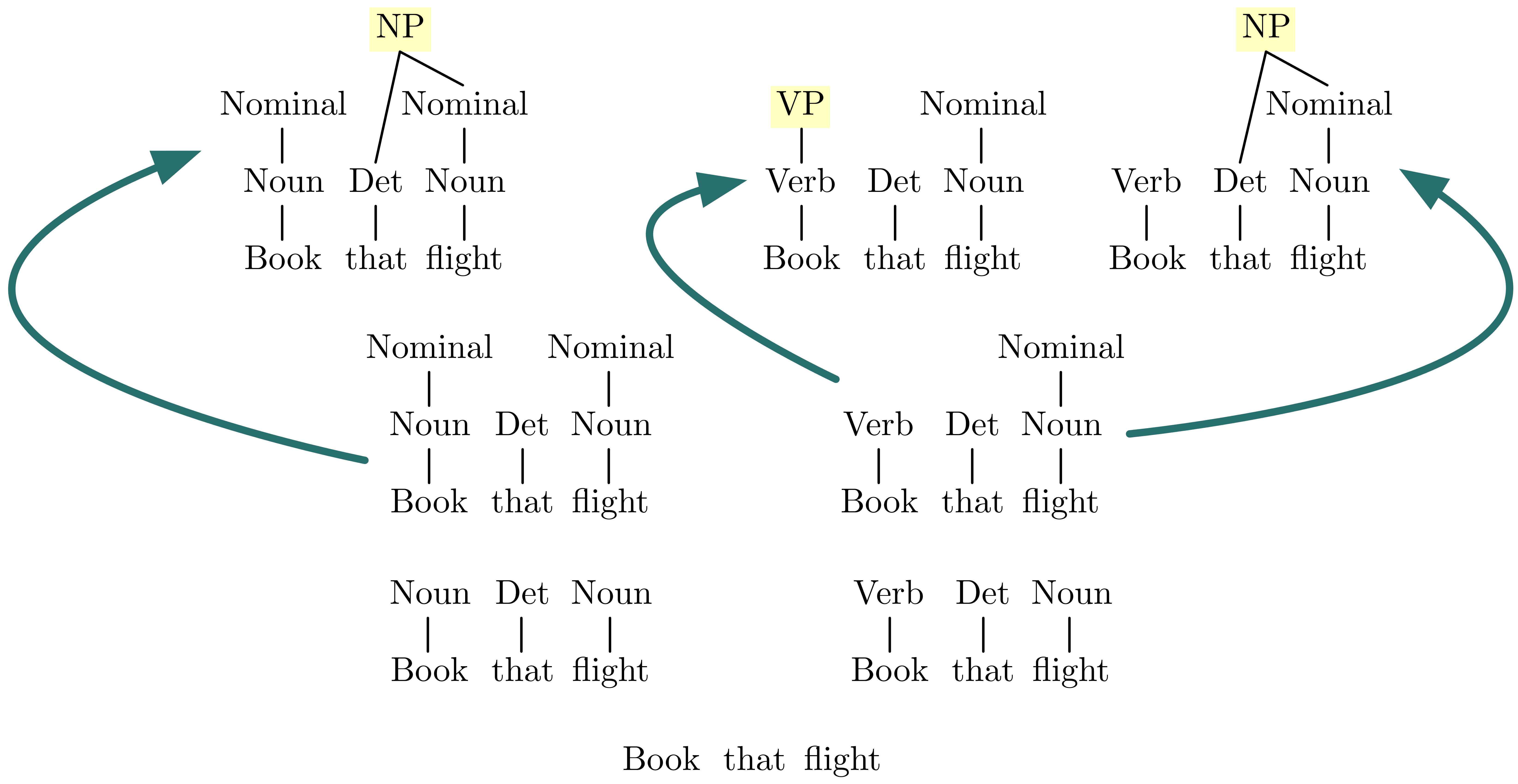
Bottom-Up Parsing

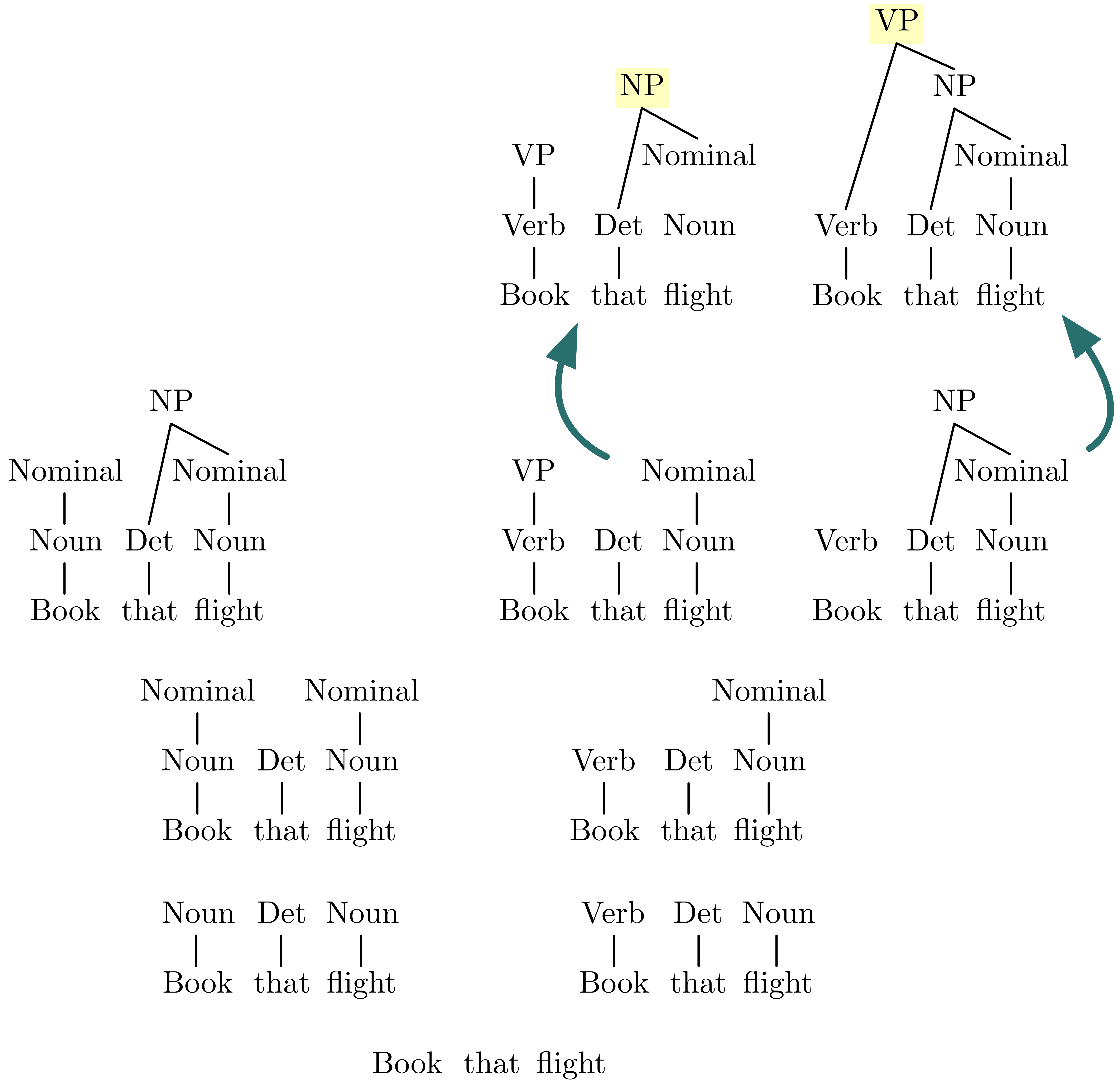
- Try to find all trees that span the input
  - Start with input string
    - Book that flight
- Use all productions with current subtree(s) on RHS
  - e.g. $N \rightarrow \text{Book};\ V \rightarrow \text{Book}$
- Stop when spanned by S, or no more rules apply

Book that flight


Noun Det Noun
Book that flight
Verb Det Noun
Book that flight
Book that flight


Nominal
Nominal
Noun Det Noun
Book that flight
Noun Det Noun
Book that flight
Nominal
Verb Det Noun
Book that flight
Verb Det Noun
Book that flight
Book that flight


NP
Nominal Nominal
Noun Det Noun
Book that flight
VP Nominal
Verb Det Noun
Book that flight
NP
Nominal
Verb Det Noun
Book that flight
Nominal Nominal
Noun Det Noun
Book that flight
Nominal
Verb Det Noun
Book that flight
Noun Det Noun
Book that flight
Verb Det Noun
Book that flight
Book that flight


VP
NP
VP
Nominal
Verb Det Noun
Book that flight
VP
NP
Nominal
Verb Det Noun
Book that flight
NP
Nominal Nominal
Noun Det Noun
Book that flight
VP Nominal
Verb Det Noun
Book that flight
NP
Nominal
Verb Det Noun
Book that flight
Nominal Nominal
Noun Det Noun
Book that flight
Nominal
Verb Det Noun
Book that flight
Noun Det Noun
Book that flight
Verb Det Noun
Book that flight
Book that flight

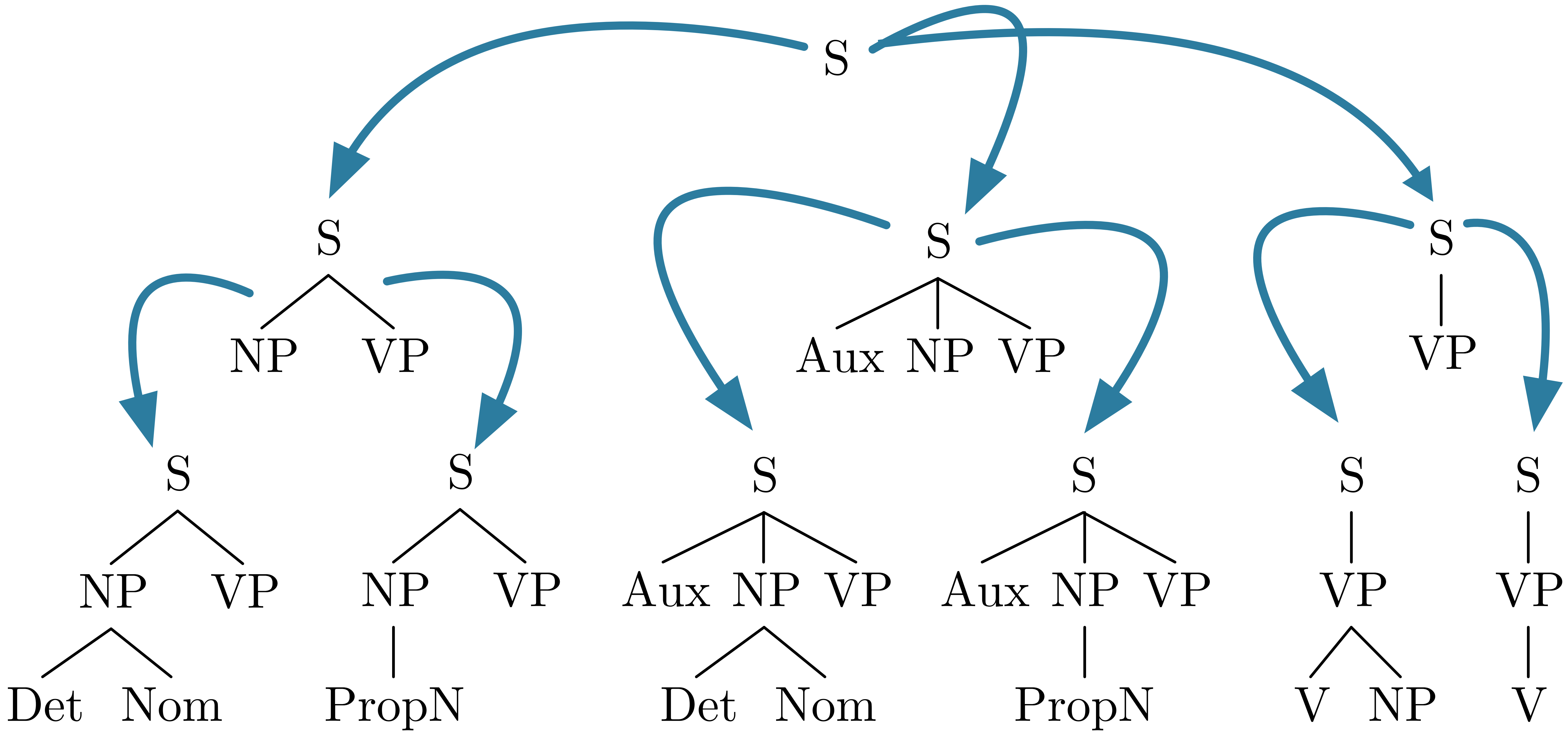
Pros and Cons of Bottom-Up Search

- Pros:
  - Will not explore trees that don’t match input
  - Recursive rules less problematic
  - Useful for incremental/fragment parsing

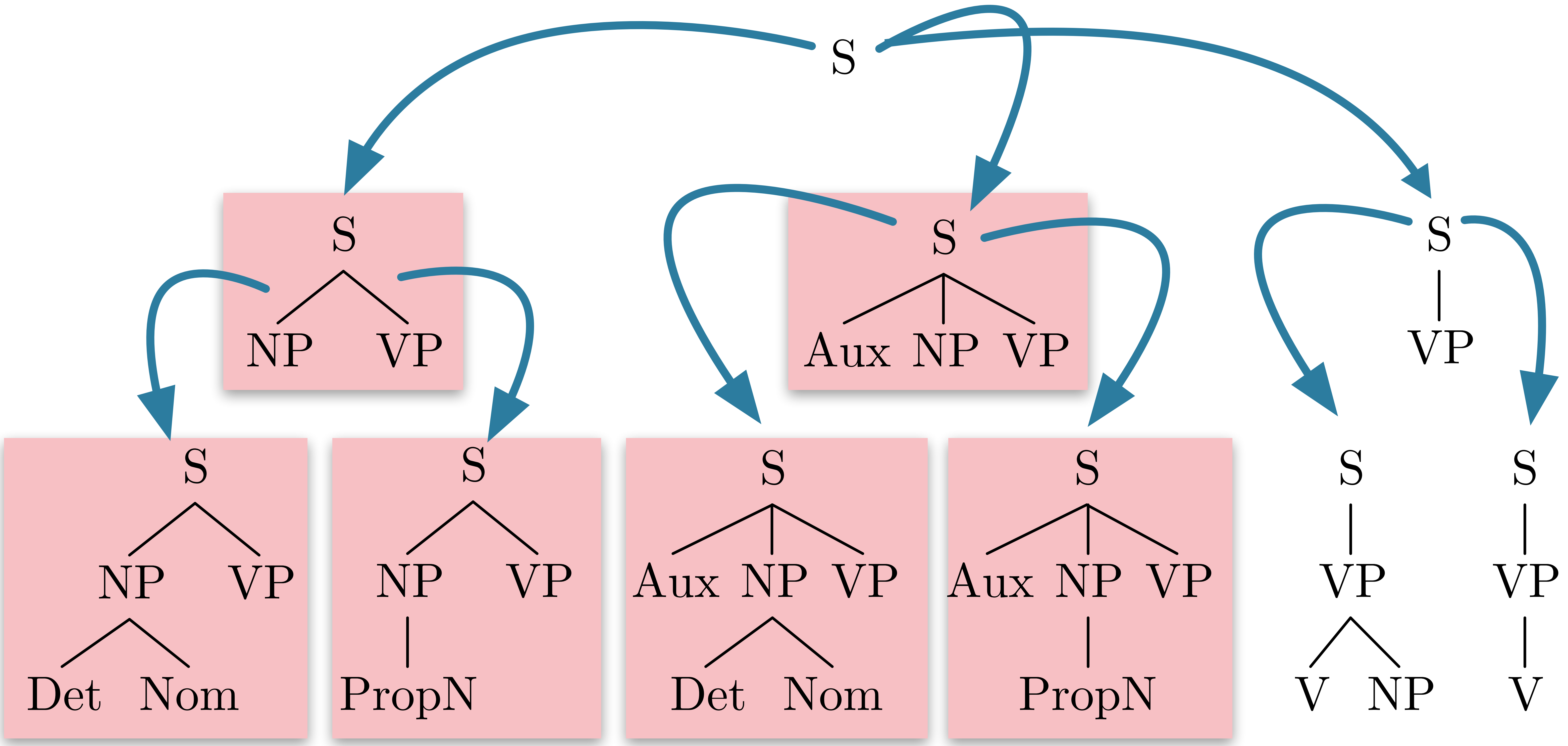
Pros and Cons of Bottom-Up Search

- Pros:
  - Will not explore trees that don’t match input
  - Recursive rules less problematic
  - Useful for incremental/fragment parsing
- Cons:
  - Explore subtrees that will not fit full input

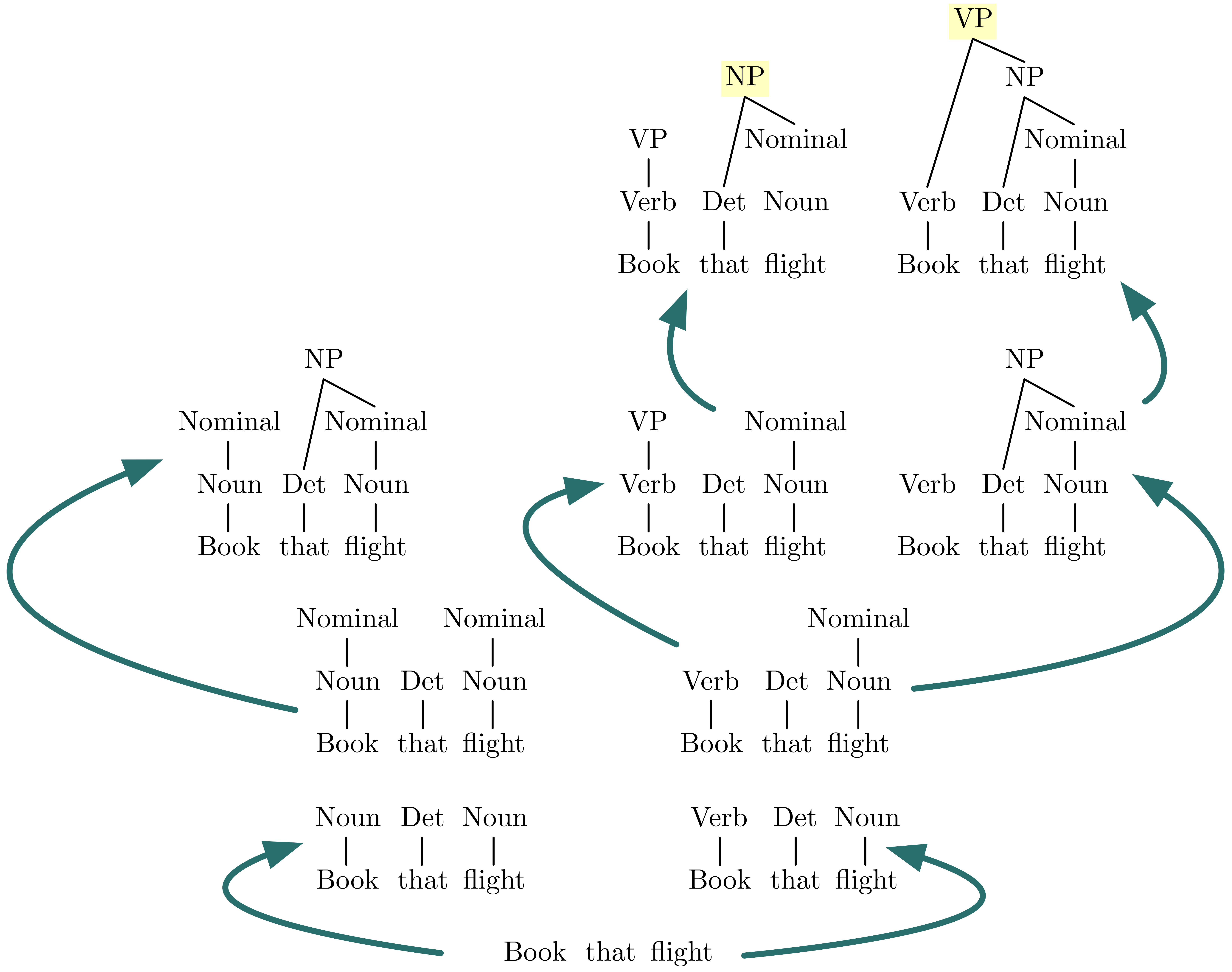
Recap: Parsing as Search



Recap: Parsing as Search

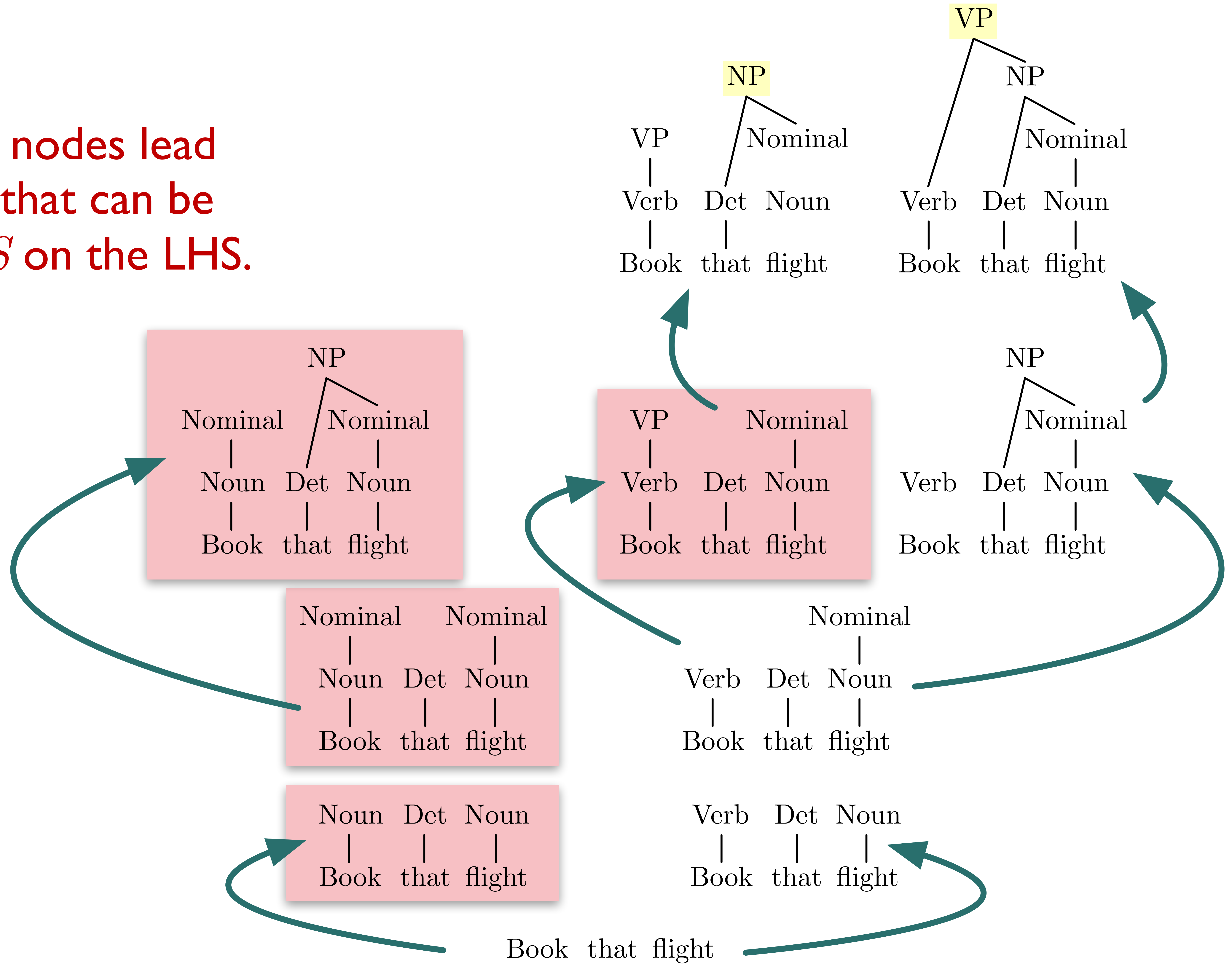


None of these nodes can produce *book* as first terminal



VP
NP
Nominal
Verb Det Noun
Book that flight
NP
VP Nominal
Verb Det Noun
Book that flight
NP
Nominal Nominal
Noun Det Noun
Book that flight
VP Nominal
Verb Det Noun
Book that flight
NP
Nominal
Verb Det Noun
Book that flight
Nominal Nominal
Noun Det Noun
Book that flight
Nominal
Verb Det Noun
Book that flight
Noun Det Noun
Book that flight
Verb Det Noun
Book that flight
Book that flight

None of these nodes lead lead to a RHS that can be combined with S on the LHS.



Parsing Challenges

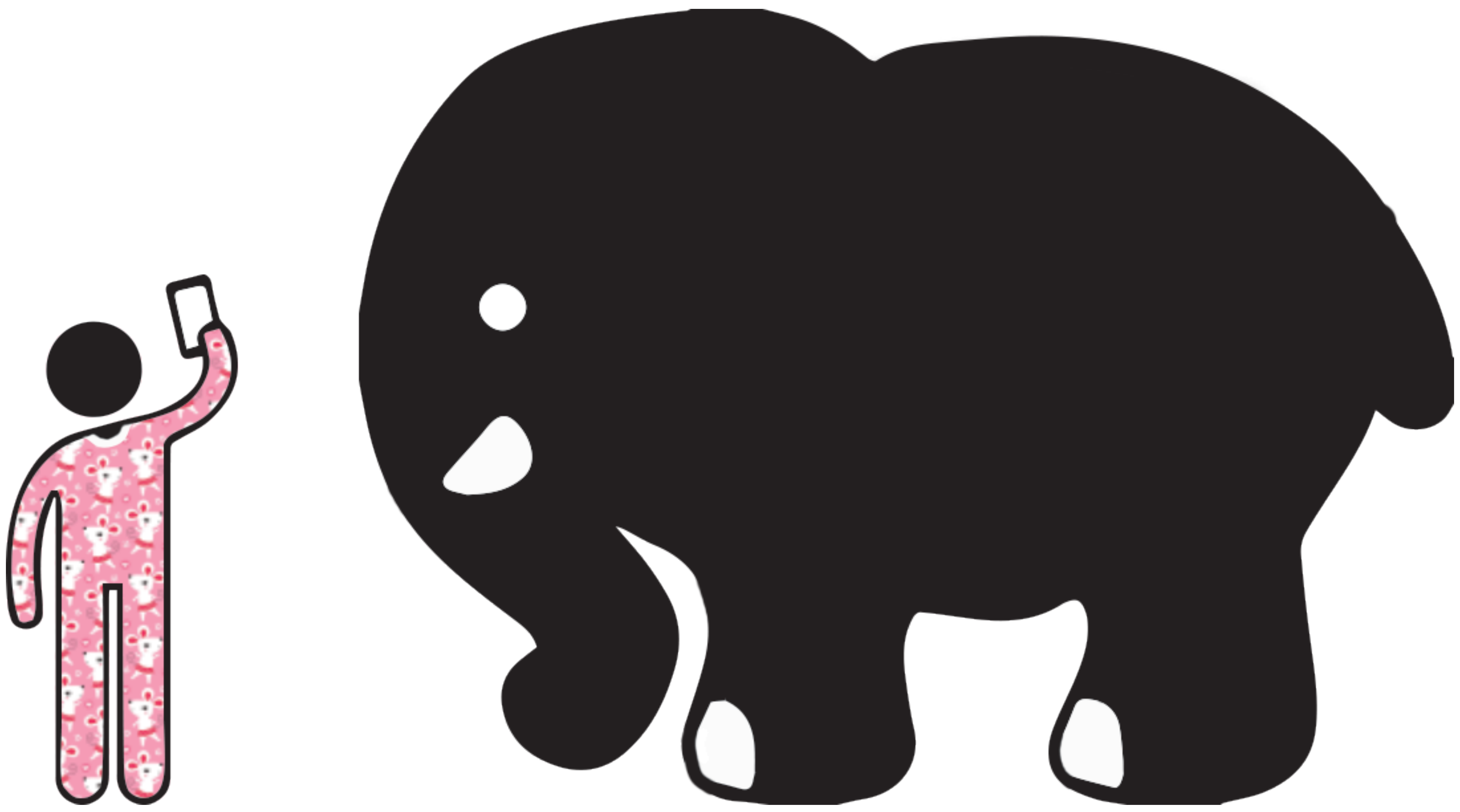
- Parsing-as-Search
- **Parsing Challenges**
  - **Ambiguity**
  - Repeated Substructure
  - Recursion
- Strategy: Dynamic Programming
- Grammar Equivalence
- CKY parsing algorithm

Parsing Ambiguity

- **Lexical Ambiguity**:
  - Book/NN → *I left a **book** on the table.*
  - Book/VB → ***Book** that flight.*
- Structural Ambiguity

Attachment Ambiguity

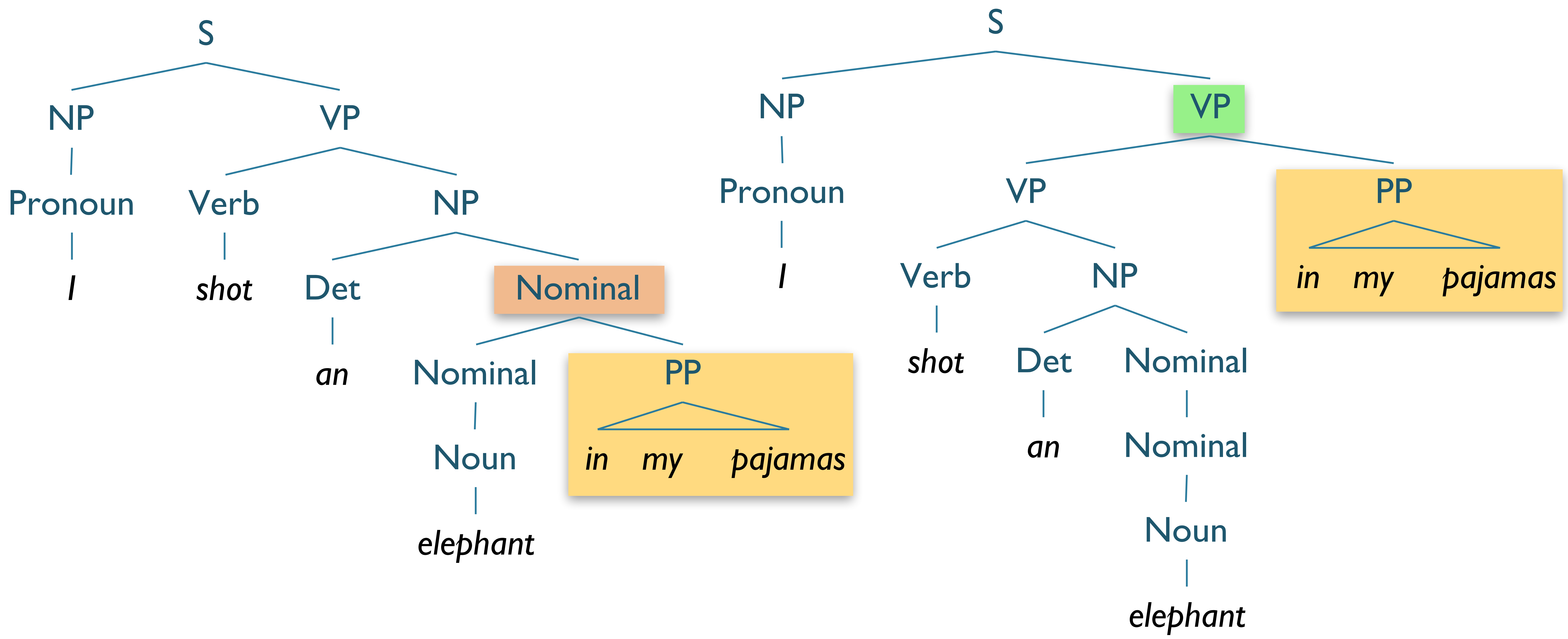
“One morning, I shot an elephant in my pajamas.



Attachment Ambiguity

"One morning, I shot an elephant in my pajamas.
How he got into my pajamas, I'll never know." — *Groucho Marx*

Attachment Ambiguity



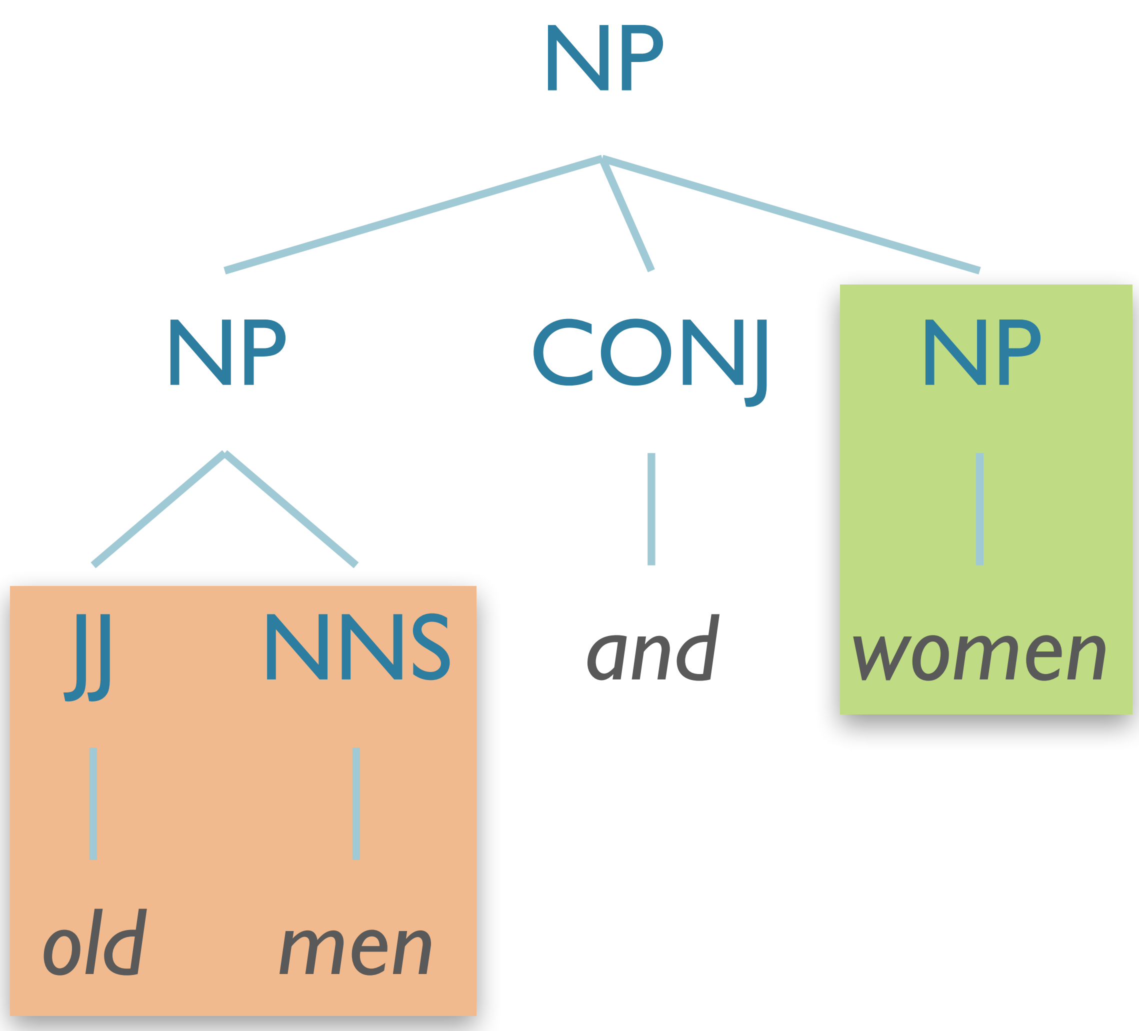
“We saw the Eiffel Tower flying to Paris”

“We saw the Eiffel Tower flying to Paris”

Coordination Ambiguity:

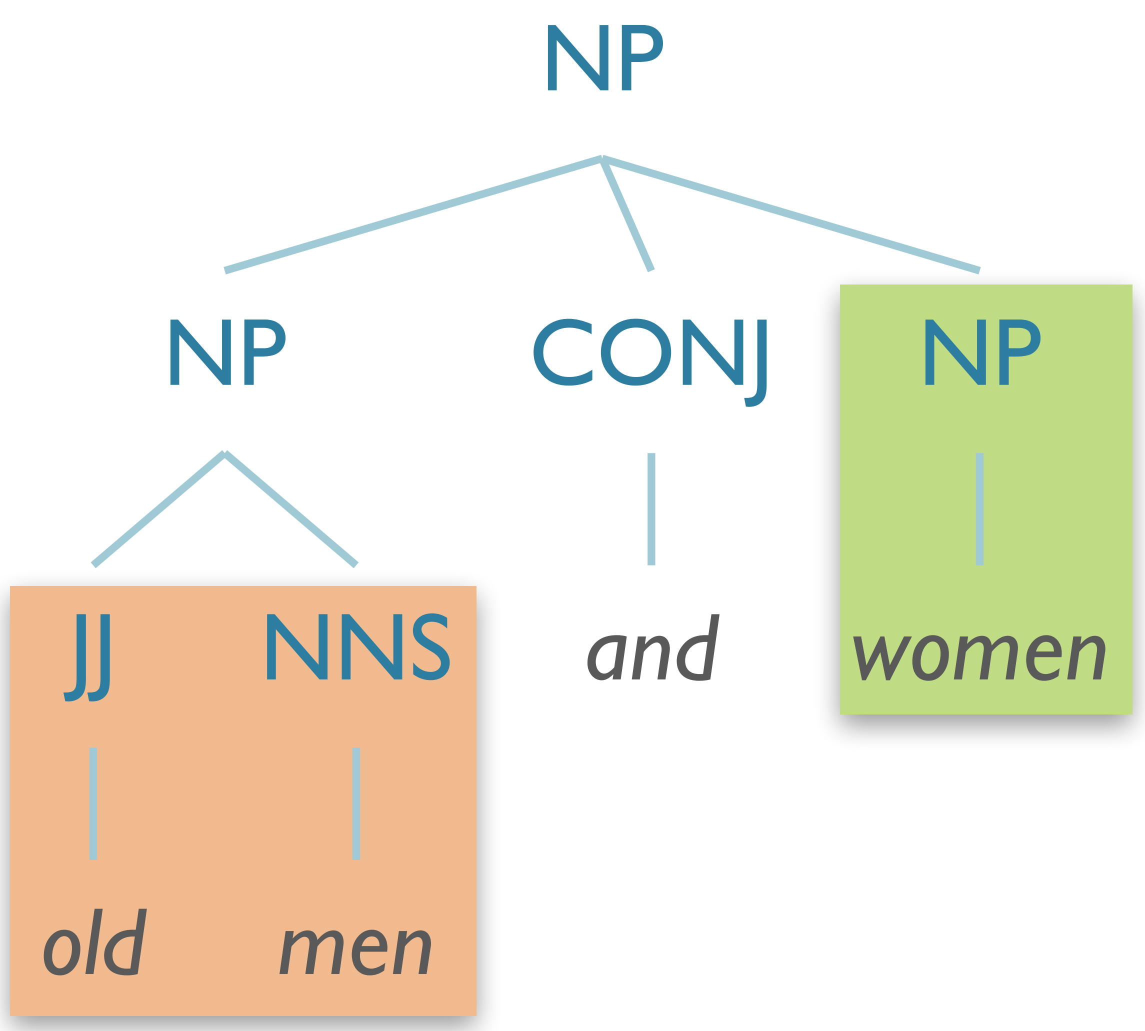
Coordination Ambiguity:

[old men] *and* ***[women]***

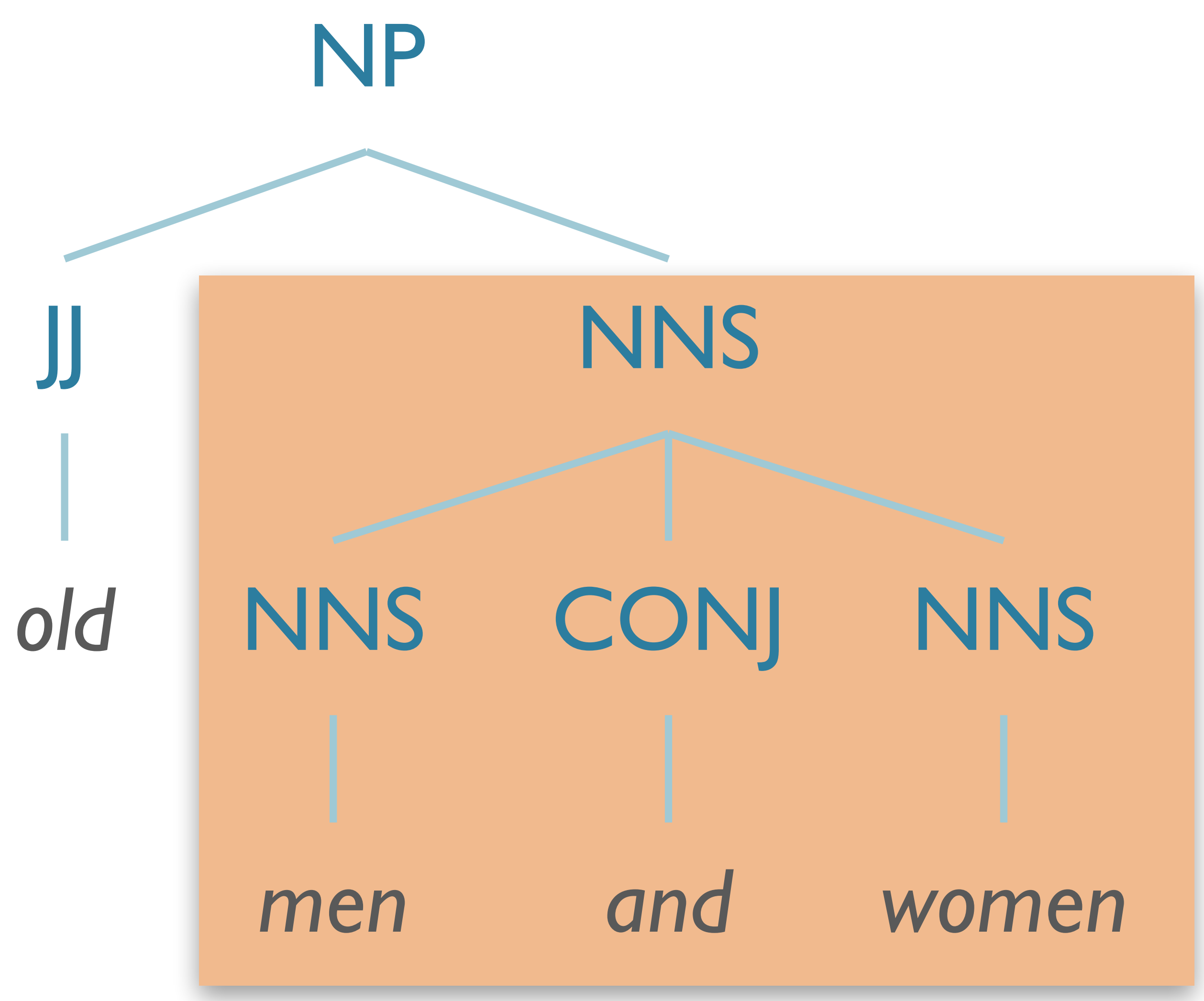


Coordination Ambiguity:

[old men] *and* ***[women]***



[old ***[men and women]****]*



Local vs. Global Ambiguity

- ***Local*** ambiguity:
  - Ambiguity that cannot contribute to a full, valid parse
  - e.g. *Book/NN* in *"Book that flight"*

Local vs. Global Ambiguity

- *Local* ambiguity:
  - Ambiguity that cannot contribute to a full, valid parse
  - e.g. *Book/NN* in *“Book that flight”*
- *Global* ambiguity
  - Multiple valid parses

Why is Ambiguity a Problem?

- *Local* ambiguity:
  - increased processing time
- *Global* ambiguity:
  - Would like to yield only “reasonable” parses
  - Ideally, the one that was intended*

Solution to Ambiguity?

Solution to Ambiguity?

- ***Dis***ambiguation!

Solution to Ambiguity?

- ***Dis*****ambiguation!**
- Different possible strategies to select correct interpretation:

Disambiguation Strategy:
Statistical

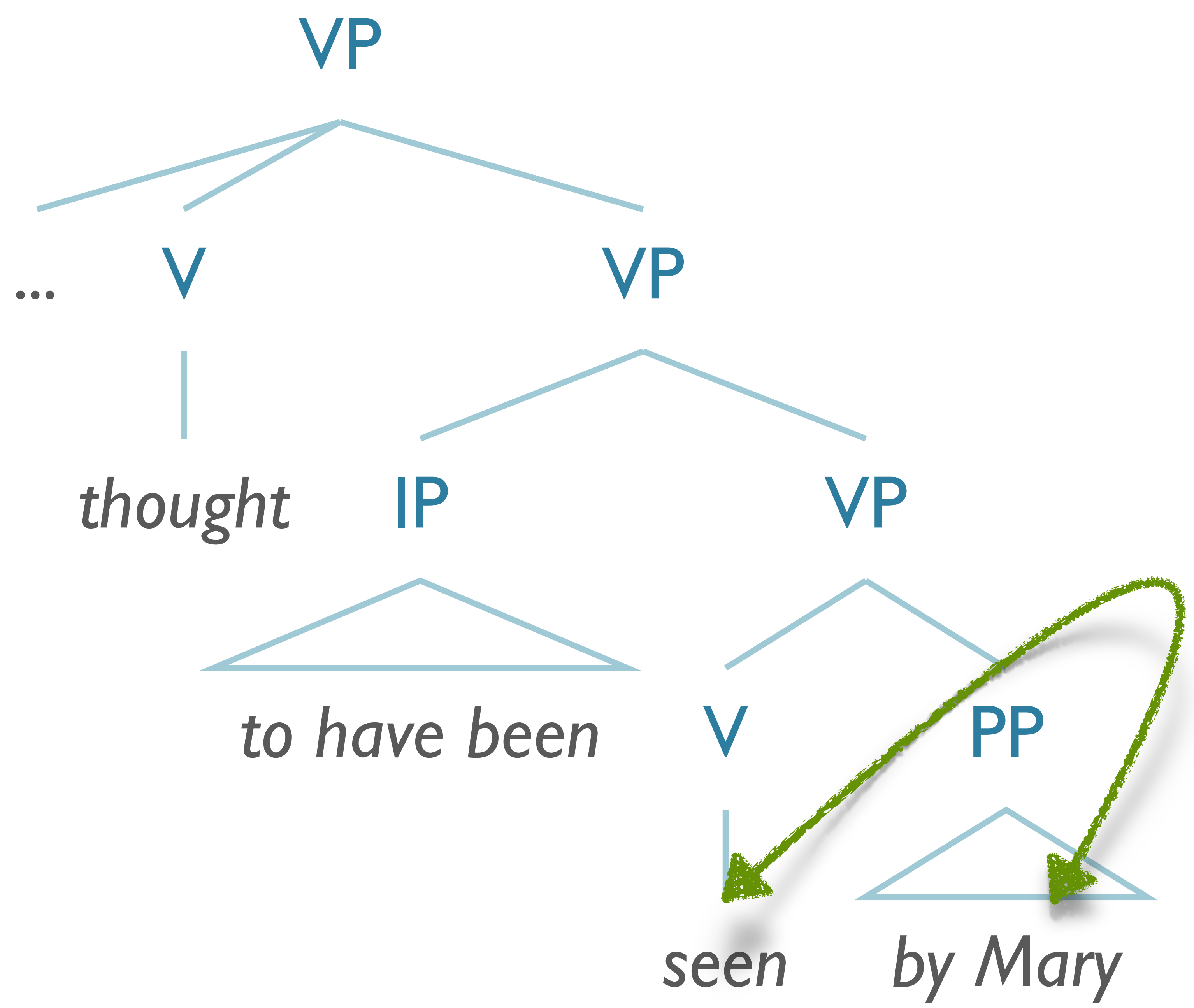
- Some prepositional structs more likely to attach high/low

Disambiguation Strategy: Statistical

- Some prepositional structs more likely to attach high/low
  - *John was thought to have been seen by Mary*
    - Mary could be doing the seeing or thinking — seeing more likely

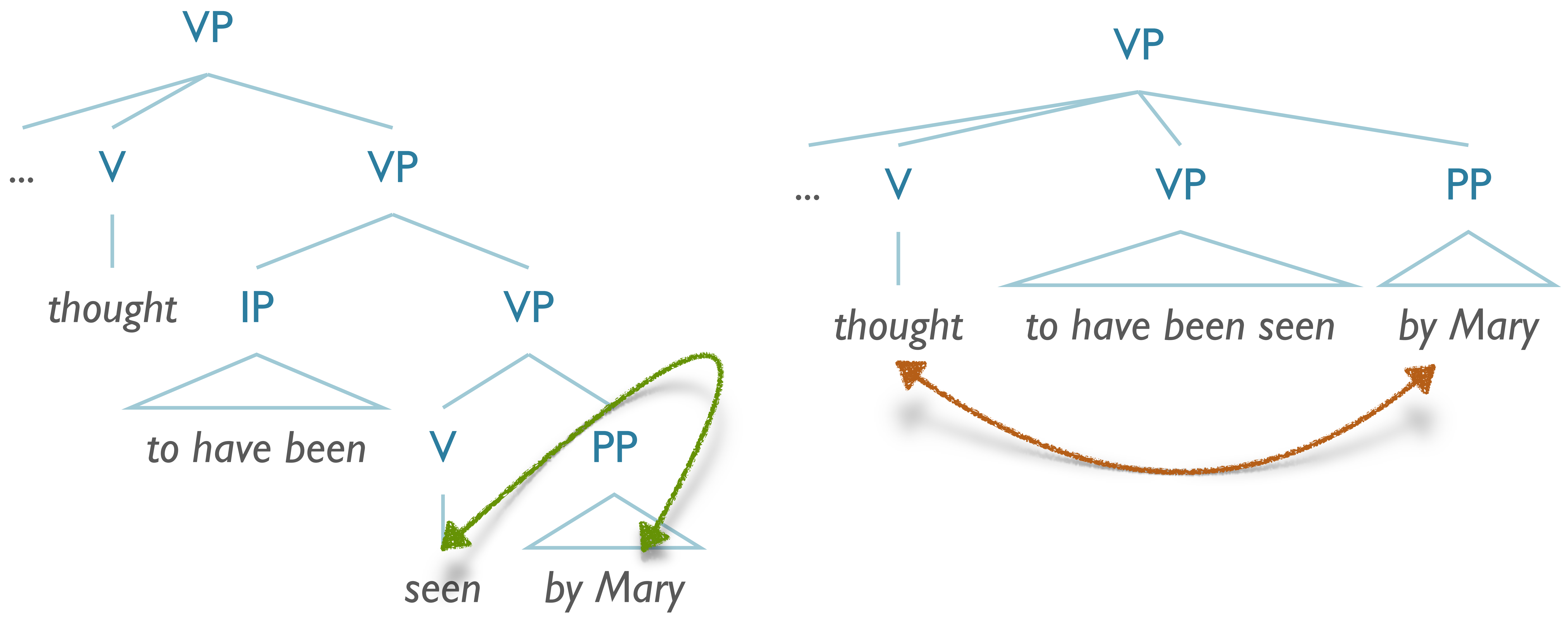
Disambiguation Strategy: Statistical

- Some prepositional structs more likely to attach high/low
  - *John was thought to have been seen by Mary*
    - Mary could be doing the seeing or thinking — seeing more likely



Disambiguation Strategy: Statistical

- Some prepositional structs more likely to attach high/low
  - *John was thought to have been seen by Mary*
    - Mary could be doing the seeing or thinking — seeing more likely

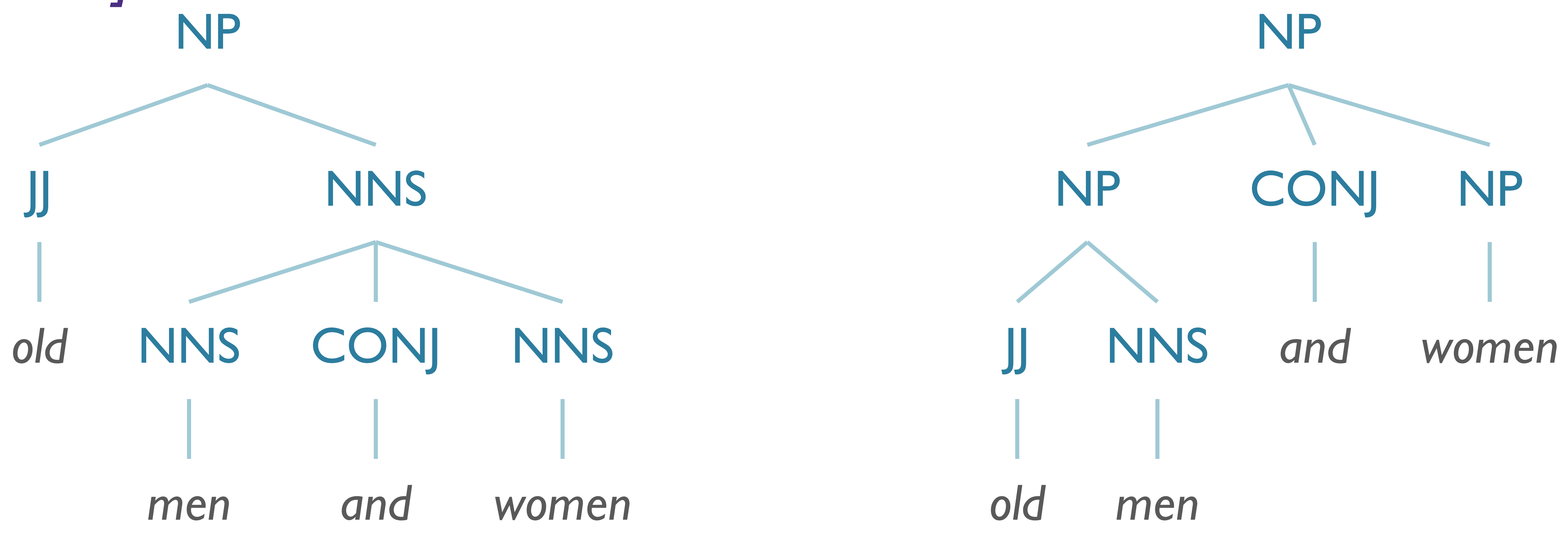


Disambiguation Strategy: Statistical

- Some phrases more likely overall

Disambiguation Strategy: Statistical

- Some phrases more likely overall
  - *[old [men and women]] is a more common construction than [old men] and [women]*



Disambiguation Strategy:
Semantic

- Some interpretations we know to be semantically impossible

Disambiguation Strategy:
Semantic

- Some interpretations we know to be semantically impossible
  - *Eiffel tower* as subject of *fly*

Disambiguation Strategy: Pragmatic

- Some interpretations are possible, unlikely given world knowledge

Disambiguation Strategy: Pragmatic

- Some interpretations are possible, unlikely given world knowledge
  - e.g. elephants and pajamas

Incremental Parsing and Garden Paths

- Idea: model *left-to-right* nature of (English) text
- Problem: "garden path" sentences

Incremental Parsing and Garden Paths

- Idea: model *left-to-right* nature of (English) text
- Problem: “garden path” sentences

REUTERS   Business   Markets   World   Politics   TV   More

SPORTS NEWS   SEPTEMBER 30, 2019 / 9:17 AM / A DAY AGO

California to let college athletes be paid in blow to NCAA rules

https://www.reuters.com/article/us-sport-california-education/california-to-let-college-athletes-be-paid-in-blow-to-ncaa-rules-idUSKBN1WF1SR

Disambiguation Strategy:

🤷‍♂️

- Alternatively, keep all parses

Disambiguation Strategy:
🤷‍♂️

- Alternatively, keep all parses
  - *(Might even be the appropriate action for some jokes)*

Parsing Challenges

- Parsing-as-Search
- **Parsing Challenges**
  - Ambiguity
  - **Repeated Substructure**
  - Recursion
- Strategy: Dynamic Programming
- Grammar Equivalence
- CKY parsing algorithm

Repeated Work

- Search (top-down/bottom-up) both lead to repeated substructures
  - Globally bad parses can construct good subtrees
  - …will reconstruct along another branch
  - No static backtracking can avoid

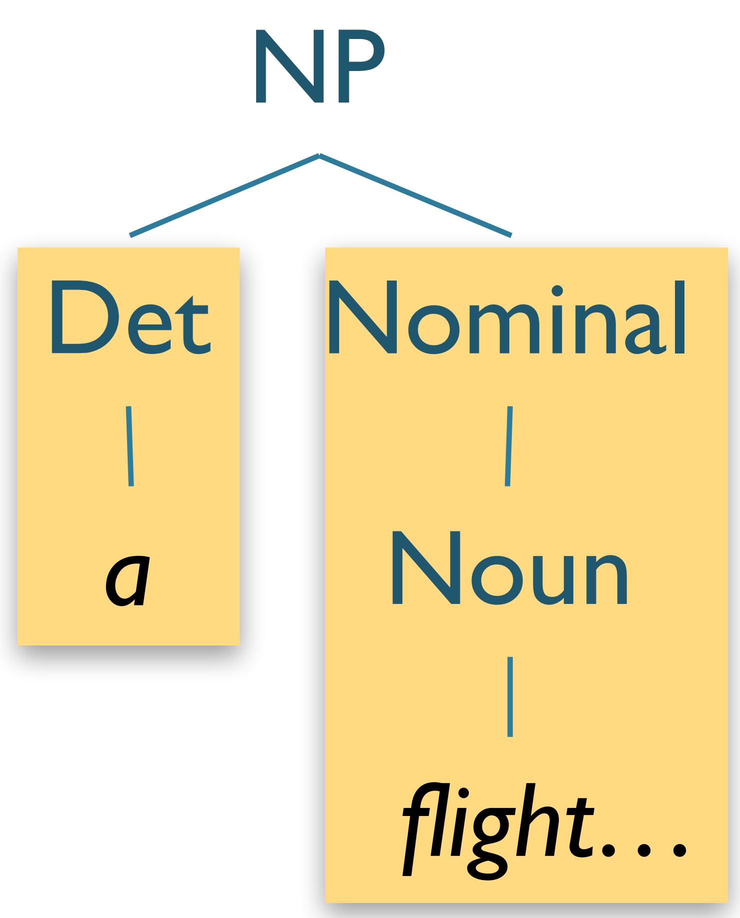
Repeated Work

- Search (top-down/bottom-up) both lead to repeated substructures
  - Globally bad parses can construct good subtrees
  - …will reconstruct along another branch
  - No static backtracking can avoid
- Efficient parsing techniques require storage of partial solutions

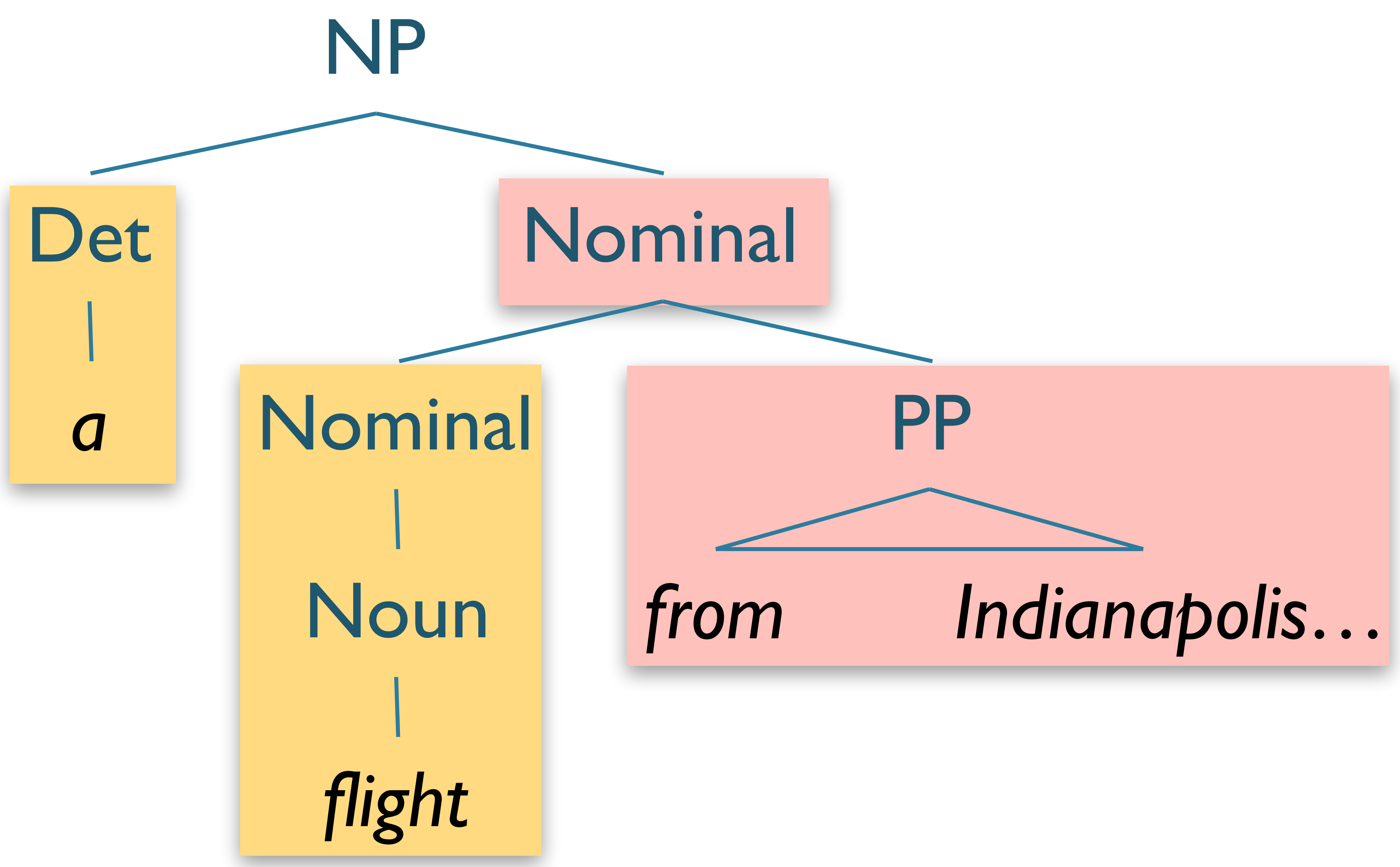
Repeated Work

- Search (top-down/bottom-up) both lead to repeated substructures
  - Globally bad parses can construct good subtrees
  - …will reconstruct along another branch
  - No static backtracking can avoid
- Efficient parsing techniques require storage of partial solutions
- Example: *a flight from Indianapolis to Houston on TWA*

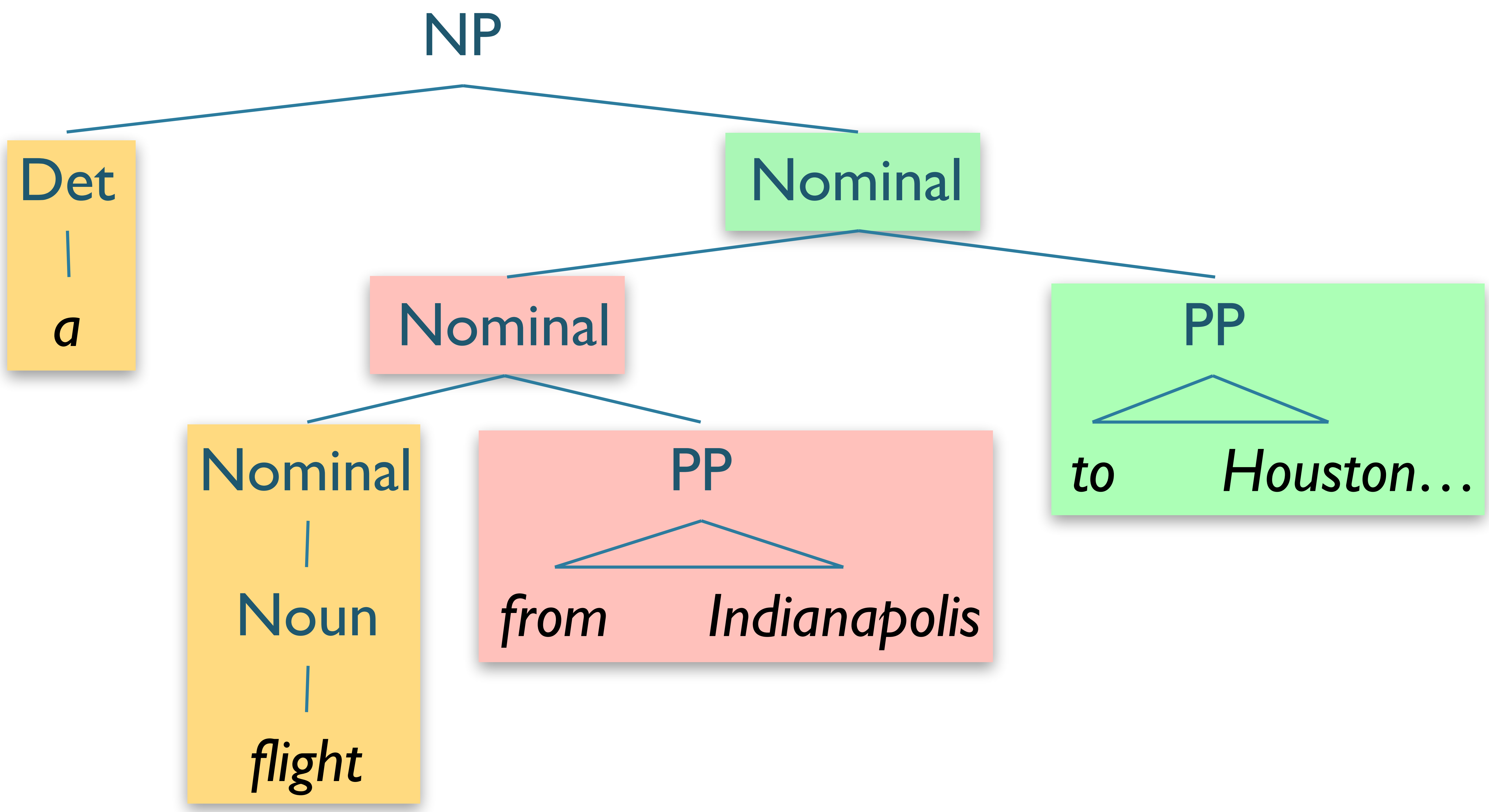
Shared Sub-Problems



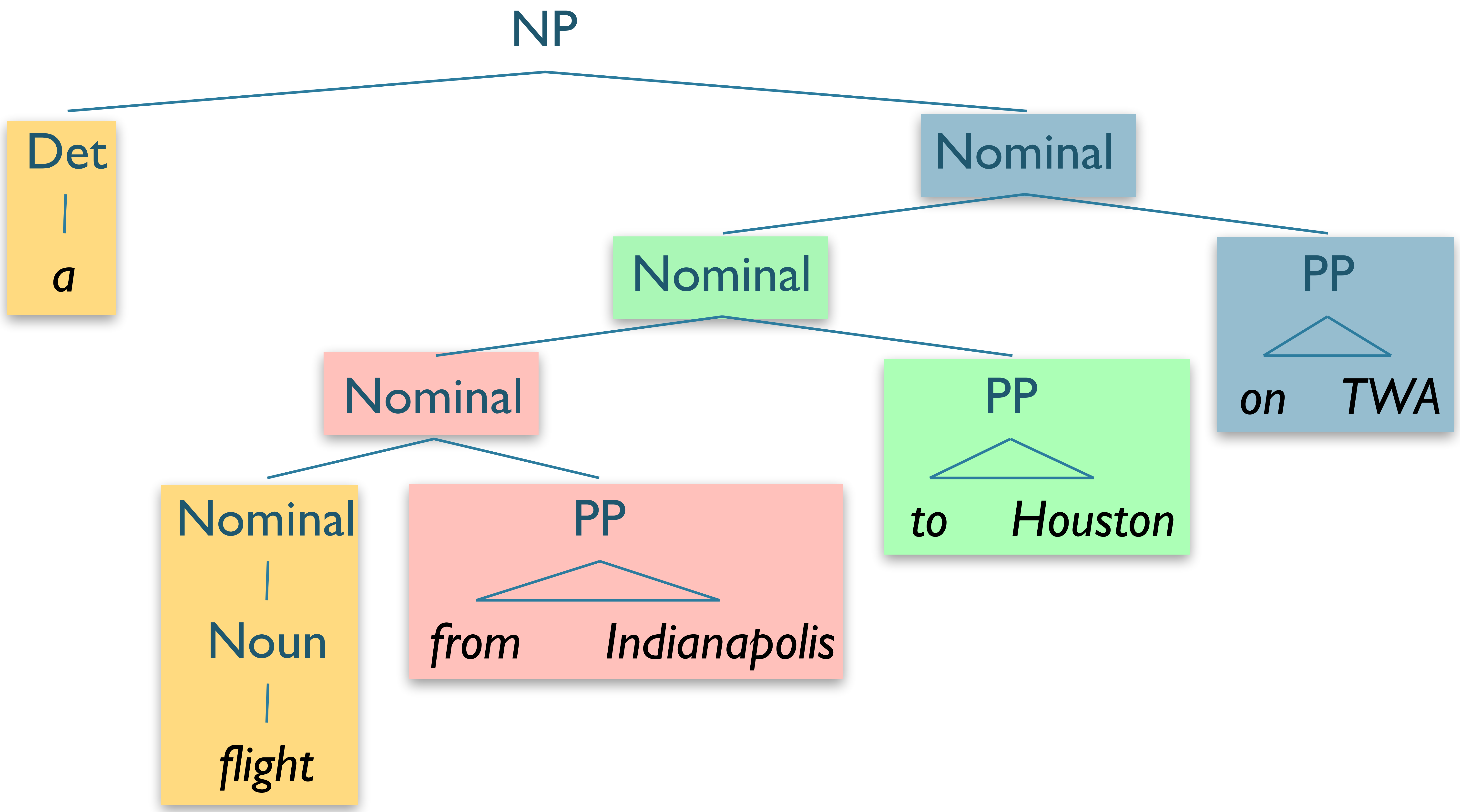
Shared Sub-Problems



Shared Sub-Problems



Shared Sub-Problems

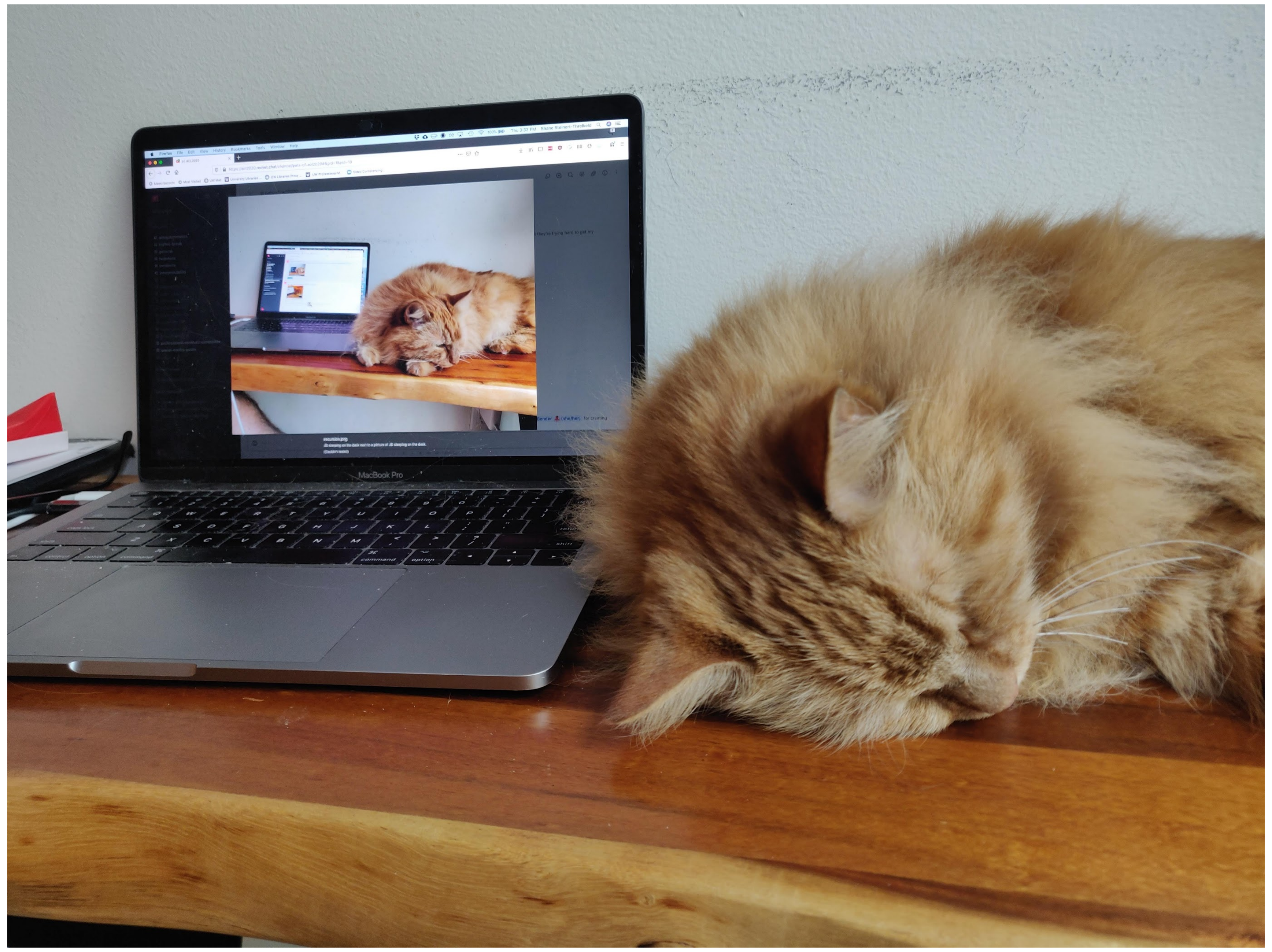


Parsing Challenges

- Parsing-as-Search
- **Parsing Challenges**
  - Ambiguity
  - Repeated Substructure
  - **Recursion**
- Strategy: Dynamic Programming
- Grammar Equivalence
- CKY parsing algorithm

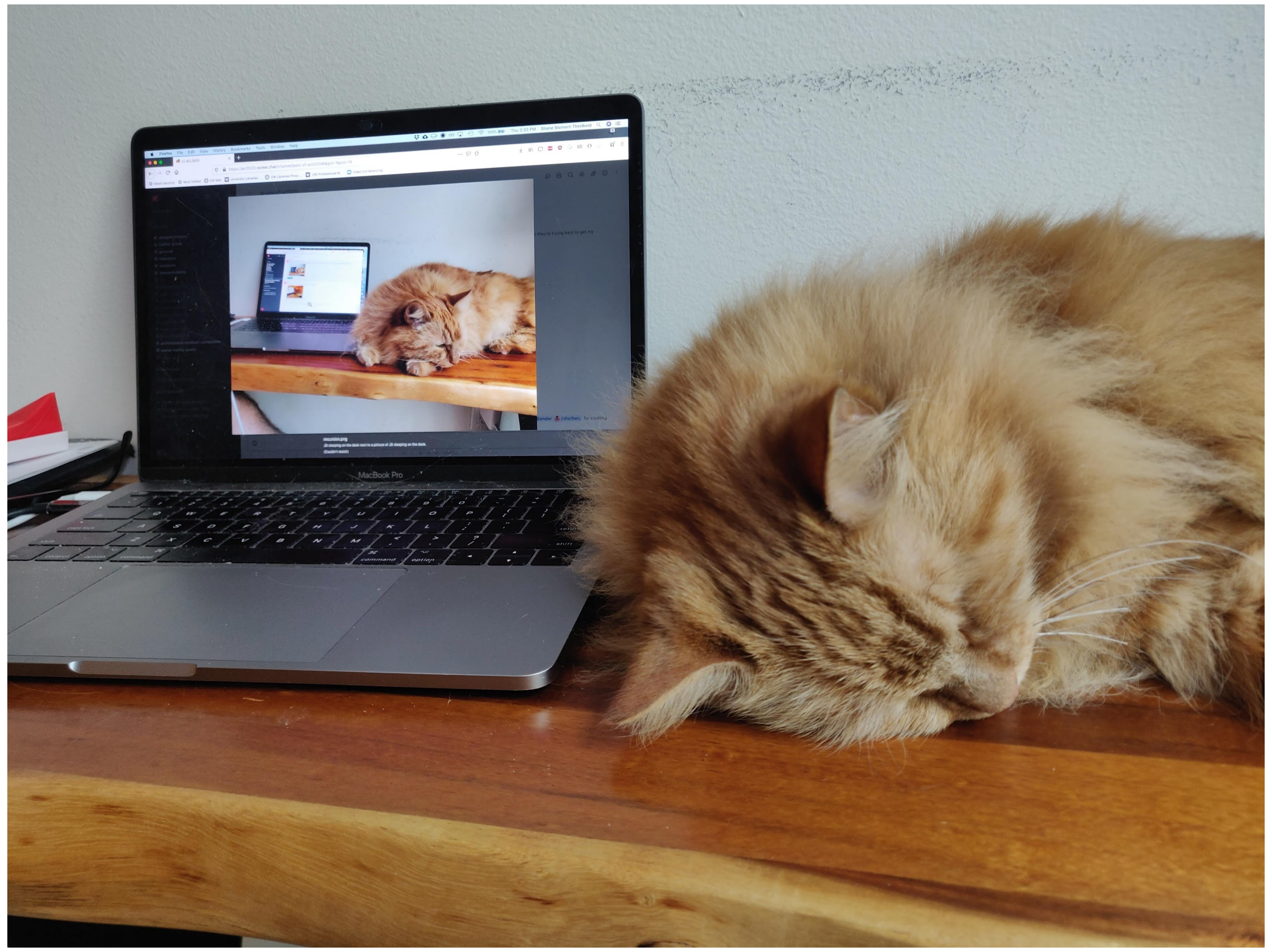
Recursion

- Many grammars have recursive rules
  - $S \rightarrow S\ Conj\ S$



Recursion

- Many grammars have recursive rules
  - $S \rightarrow S\ Conj\ S$
- In search approaches, recursion is problematic
  - Can yield infinite searches
  - Top-down especially vulnerable



Roadmap

- Parsing-as-Search
- Parsing Challenges
- **Strategy: Dynamic Programming**
- Grammar Equivalence
- CKY parsing algorithm

Dynamic Programming

- Challenge:
  - Repeated substructure → Repeated Work

Dynamic Programming

- Challenge:
  - Repeated substructure → Repeated Work
- Insight:
  - Global parse composed of sub-parses
  - Can record these sub-parses and re-use

Dynamic Programming

- Challenge:
  - Repeated substructure → Repeated Work
- Insight:
  - Global parse composed of sub-parses
  - Can record these sub-parses and re-use
- Dynamic programming avoids repeated work by recording the subproblems
  - Here, stores subtrees

Parsing with Dynamic Programming

- Avoids repeated work
- Allows implementation of (relatively) efficient parsing algorithms
  - Polynomial time in input length
  - Typically cubic (n^3) or less

Parsing with Dynamic Programming

- Avoids repeated work
- Allows implementation of (relatively) efficient parsing algorithms
  - Polynomial time in input length
  - Typically cubic (n^3) or less
- Several different implementations
  - Cocke-Kasami-Younger (CKY) algorithm
  - Earley algorithm
  - Chart parsing

Roadmap

- Parsing-as-Search
- Parsing Challenges
- Strategy: Dynamic Programming
- **Grammar Equivalence**
- CKY parsing algorithm

Grammar Equivalence and Form

- *Weak* Equivalence
  - **Accepts** same language
  - May produce **different** structures

- *Strong* Equivalence
  - Accepts same language
  - Produces **same** structures

Grammar Equivalence and Form

Grammar Equivalence and Form

- Reason?
  - We can create a weakly-equivalent grammar that allows for greater efficiency
  - This is required by the CKY algorithm

Chomsky Normal Form (CNF)

- Required by CKY Algorithm

Chomsky Normal Form (CNF)

- Required by CKY Algorithm
- All productions are of the form:
  - $A \rightarrow B\ C$
  - $A \rightarrow \mathbf{a}$

Chomsky Normal Form (CNF)

- Required by CKY Algorithm
- All productions are of the form:
  - $A \rightarrow B\ C$
  - $A \rightarrow \mathbf{a}$
- Most of our grammars are not of this form:
  - $S \rightarrow \textit{Wh-NP}\ Aux\ NP\ VP$

Chomsky Normal Form (CNF)

- Required by CKY Algorithm
- All productions are of the form:
  - $A \rightarrow B\ C$
  - $A \rightarrow \mathbf{a}$
- Most of our grammars are not of this form:
  - $S \rightarrow \textit{Wh-NP}\ \textit{Aux}\ \textit{NP}\ \textit{VP}$
- Need a general conversion procedure

CNF Conversion

Hybrid productions:

$INF\text{-}VP \rightarrow \textbf{to}\ VP$

Unit productions:

$A \rightarrow B$

Long productions:

$A \rightarrow B\ C\ D \ldots$

CNF Conversion: Hybrid Productions

- Hybrid production:
  - Replace all terminals with dummy non-terminal
  - $\mathit{INF}\text{-}\mathit{VP} \rightarrow \textbf{to}\ \mathit{VP}$
    - $\mathit{INF}\text{-}\mathit{VP} \rightarrow \mathit{TO}\ \mathit{VP}$
    - $\mathit{TO} \rightarrow \textbf{to}$

CNF Conversion: Unit Productions

- Unit productions:
  - Rewrite RHS with RHS of all derivable, non-unit productions
  - If $A \overset{*}{\Rightarrow} B$ and $B \rightarrow \gamma$, **add** $A \rightarrow \gamma$ [where γ is any non-unit RHS]
  - [*$A \overset{*}{\Rightarrow} B$: B is reachable from A by a sequence of unit productions]*
- *Nominal* → *Noun*, *Noun* → **dog**
  - *Nominal* → **dog**
  - *Noun* → **dog**
- NB: this example has γ as a single terminal, but the rule applies to all non-unit RHS.

CNF Conversion: Long Productions

CNF Conversion: Long Productions

- Long productions

$$S \rightarrow Aux\ NP\ VP$$

CNF Conversion: Long Productions

- Long productions

$S \rightarrow Aux\ NP\ VP$	
$S \rightarrow \boldsymbol{X1}\ VP$	$\boldsymbol{X1} \rightarrow Aux\ NP$

CNF Conversion: Long Productions

- Long productions

$S \rightarrow Aux\ NP\ VP$
$S \rightarrow \boldsymbol{X1}\ VP \qquad \boldsymbol{X1} \rightarrow Aux\ NP$

- Introduce unique nonterminals, and spread over rules

CNF Conversion

Convert terminals in hybrid rules to dummy non-terminals

Convert unit productions

Binarize long production rules

$\mathcal{L}_1$ **Grammar**	$\mathcal{L}_1$ **in CNF**
$S \rightarrow NP\ VP$	$S \rightarrow NP\ VP$
$S \rightarrow Aux\ NP\ VP$	$S \rightarrow X1\ VP$
	$X1 \rightarrow Aux\ NP$
$S \rightarrow VP$	$S \rightarrow book \mid include \mid prefer$
	$S \rightarrow Verb\ NP$
	$S \rightarrow X2\ PP$
	$S \rightarrow Verb\ PP$
	$S \rightarrow VP\ PP$
$NP \rightarrow Pronoun$	$NP \rightarrow I \mid she \mid me$
$NP \rightarrow Proper\text{-}Noun$	$NP \rightarrow TWA \mid Houston$
$NP \rightarrow Det\ Nominal$	$NP \rightarrow Det\ Nominal$
$Nominal \rightarrow Noun$	$Nominal \rightarrow book \mid flight \mid meal \mid money$
$Nominal \rightarrow Nominal\ Noun$	$Nominal \rightarrow Nominal\ Noun$
$Nominal \rightarrow Nominal\ PP$	$Nominal \rightarrow Nominal\ PP$
$VP \rightarrow Verb$	$VP \rightarrow book \mid include \mid prefer$
$VP \rightarrow Verb\ NP$	$VP \rightarrow Verb\ NP$
$VP \rightarrow Verb\ NP\ PP$	$VP \rightarrow X2\ PP$
	$X2 \rightarrow Verb\ NP$
$VP \rightarrow Verb\ PP$	$VP \rightarrow Verb\ PP$
$VP \rightarrow VP\ PP$	$VP \rightarrow VP\ PP$
$PP \rightarrow Preposition\ NP$	$PP \rightarrow Preposition\ NP$

$\mathcal{L}_1$ **Grammar**	$\mathcal{L}_1$ **in CNF**
$S \rightarrow NP\ VP$	$S \rightarrow NP\ VP$
$S \rightarrow Aux\ NP\ VP$	$S \rightarrow X1\ VP$
	$X1 \rightarrow Aux\ NP$
$S \rightarrow VP$	$S \rightarrow book \mid include \mid prefer$
	$S \rightarrow Verb\ NP$
	$S \rightarrow X2\ PP$
	$S \rightarrow Verb\ PP$
	$S \rightarrow VP\ PP$
$NP \rightarrow Pronoun$	$NP \rightarrow I \mid she \mid me$
$NP \rightarrow Proper\text{-}Noun$	$NP \rightarrow TWA \mid Houston$
$NP \rightarrow Det\ Nominal$	$NP \rightarrow Det\ Nominal$
$Nominal \rightarrow Noun$	$Nominal \rightarrow book \mid flight \mid meal \mid money$
$Nominal \rightarrow Nominal\ Noun$	$Nominal \rightarrow Nominal\ Noun$
$Nominal \rightarrow Nominal\ PP$	$Nominal \rightarrow Nominal\ PP$
$VP \rightarrow Verb$	$VP \rightarrow book \mid include \mid prefer$
$VP \rightarrow Verb\ NP$	$VP \rightarrow Verb\ NP$
$VP \rightarrow Verb\ NP\ PP$	$VP \rightarrow X2\ PP$
	$X2 \rightarrow Verb\ NP$
$VP \rightarrow Verb\ PP$	$VP \rightarrow Verb\ PP$
$VP \rightarrow VP\ PP$	$VP \rightarrow VP\ PP$
$PP \rightarrow Preposition\ NP$	$PP \rightarrow Preposition\ NP$

$\mathscr{L}_1$ **Grammar**	$\mathscr{L}_1$ **in CNF**
$S \rightarrow NP\ VP$	$S \rightarrow NP\ VP$
$S \rightarrow Aux\ NP\ VP$	$S \rightarrow X1\ VP$
	$X1 \rightarrow Aux\ NP$
$S \rightarrow VP$	$S \rightarrow book \mid include \mid prefer$
	$S \rightarrow Verb\ NP$
	$S \rightarrow X2\ PP$
	$S \rightarrow Verb\ PP$
	$S \rightarrow VP\ PP$
$NP \rightarrow Pronoun$	$NP \rightarrow I \mid she \mid me$
$NP \rightarrow Proper\text{-}Noun$	$NP \rightarrow TWA \mid Houston$
$NP \rightarrow Det\ Nominal$	$NP \rightarrow Det\ Nominal$
$Nominal \rightarrow Noun$	$Nominal \rightarrow book \mid flight \mid meal \mid money$
$Nominal \rightarrow Nominal\ Noun$	$Nominal \rightarrow Nominal\ Noun$
$Nominal \rightarrow Nominal\ PP$	$Nominal \rightarrow Nominal\ PP$
$VP \rightarrow Verb$	$VP \rightarrow book \mid include \mid prefer$
$VP \rightarrow Verb\ NP$	$VP \rightarrow Verb\ NP$
$VP \rightarrow Verb\ NP\ PP$	$VP \rightarrow X2\ PP$
	$X2 \rightarrow Verb\ NP$
$VP \rightarrow Verb\ PP$	$VP \rightarrow Verb\ PP$
$VP \rightarrow VP\ PP$	$VP \rightarrow VP\ PP$
$PP \rightarrow Preposition\ NP$	$PP \rightarrow Preposition\ NP$

Roadmap

- Parsing-as-Search
- Parsing Challenges
- Strategy: Dynamic Programming
- Grammar Equivalence
- **CKY parsing algorithm**

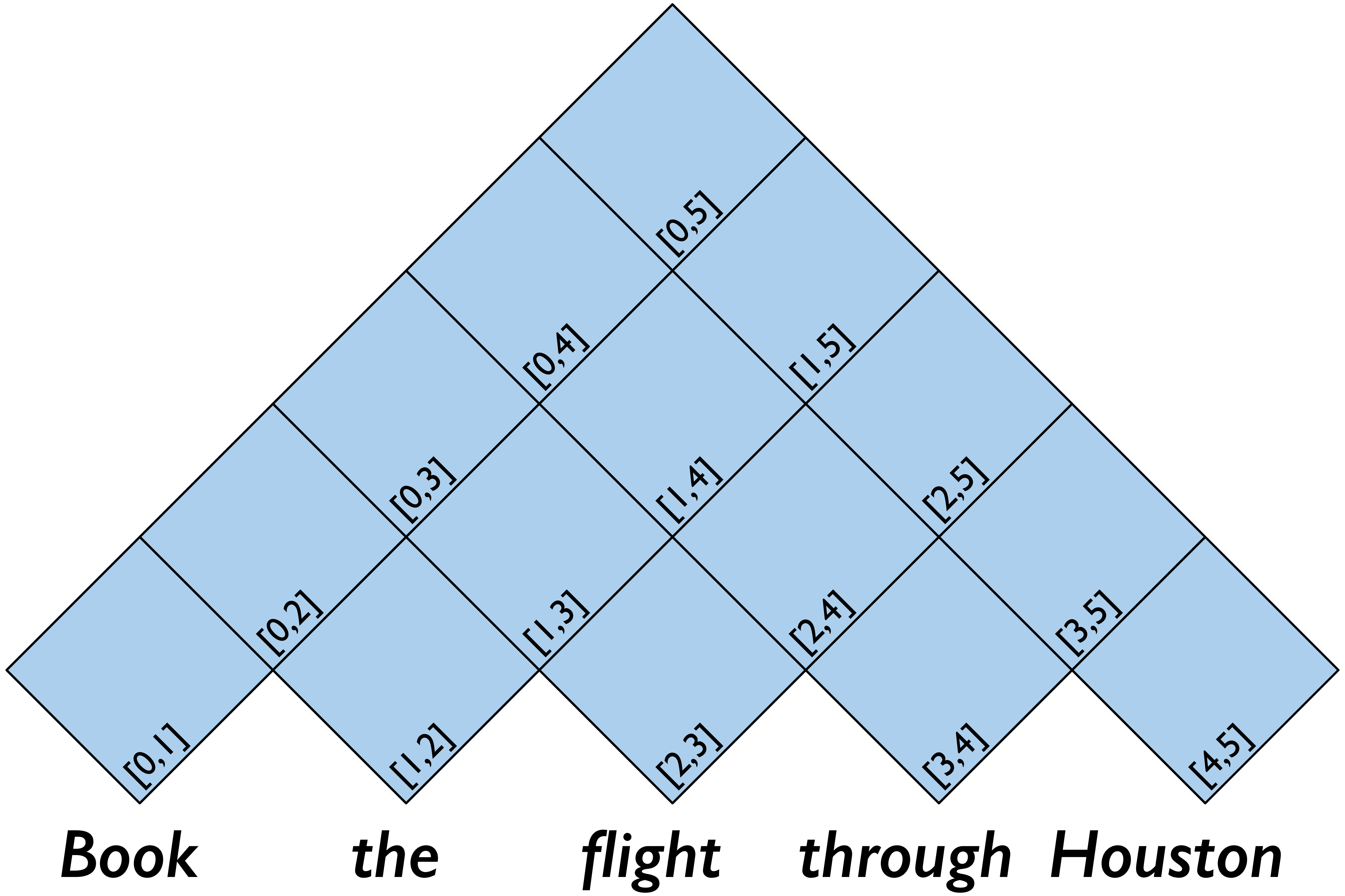
CKY Parsing

- (Relatively) efficient parsing algorithm
- Based on tabulating substring parses to avoid repeat work
- Approach:
  - Use CNF Grammar
  - Build an $(n + 1) \times (n + 1)$ matrix to store subtrees
    - Upper triangular portion
  - Incrementally build parse spanning whole input string

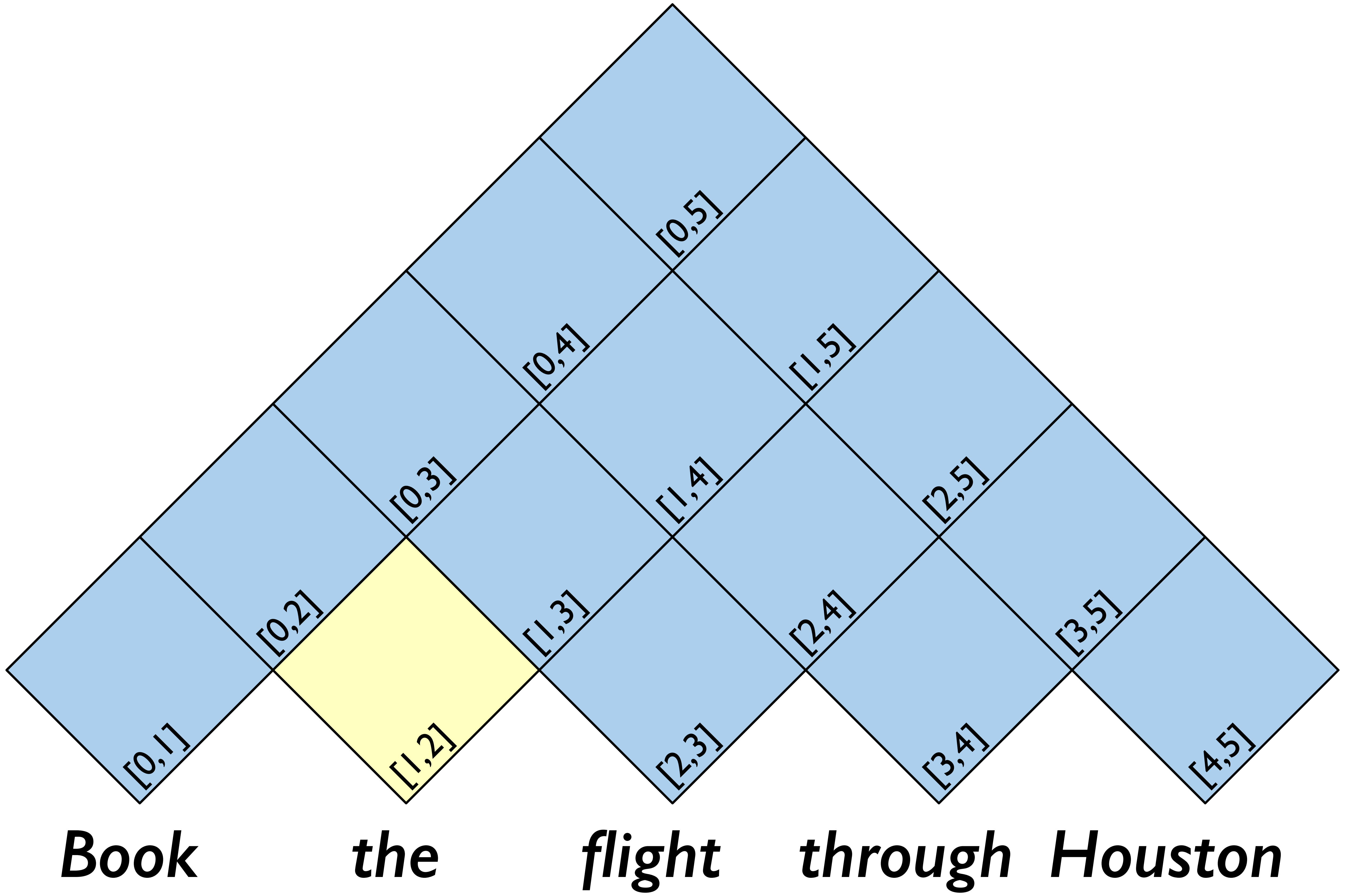
CKY Matrix

Book	*the*	*flight*	*through*	*Houston*
[0,1]	[0,2]	[0,3]	[0,4]	[0,5]
	[1,2]	[1,3]	[1,4]	[1,5]
		[2,3]	[2,4]	[2,5]
			[3,4]	[3,5]
				[4,5]

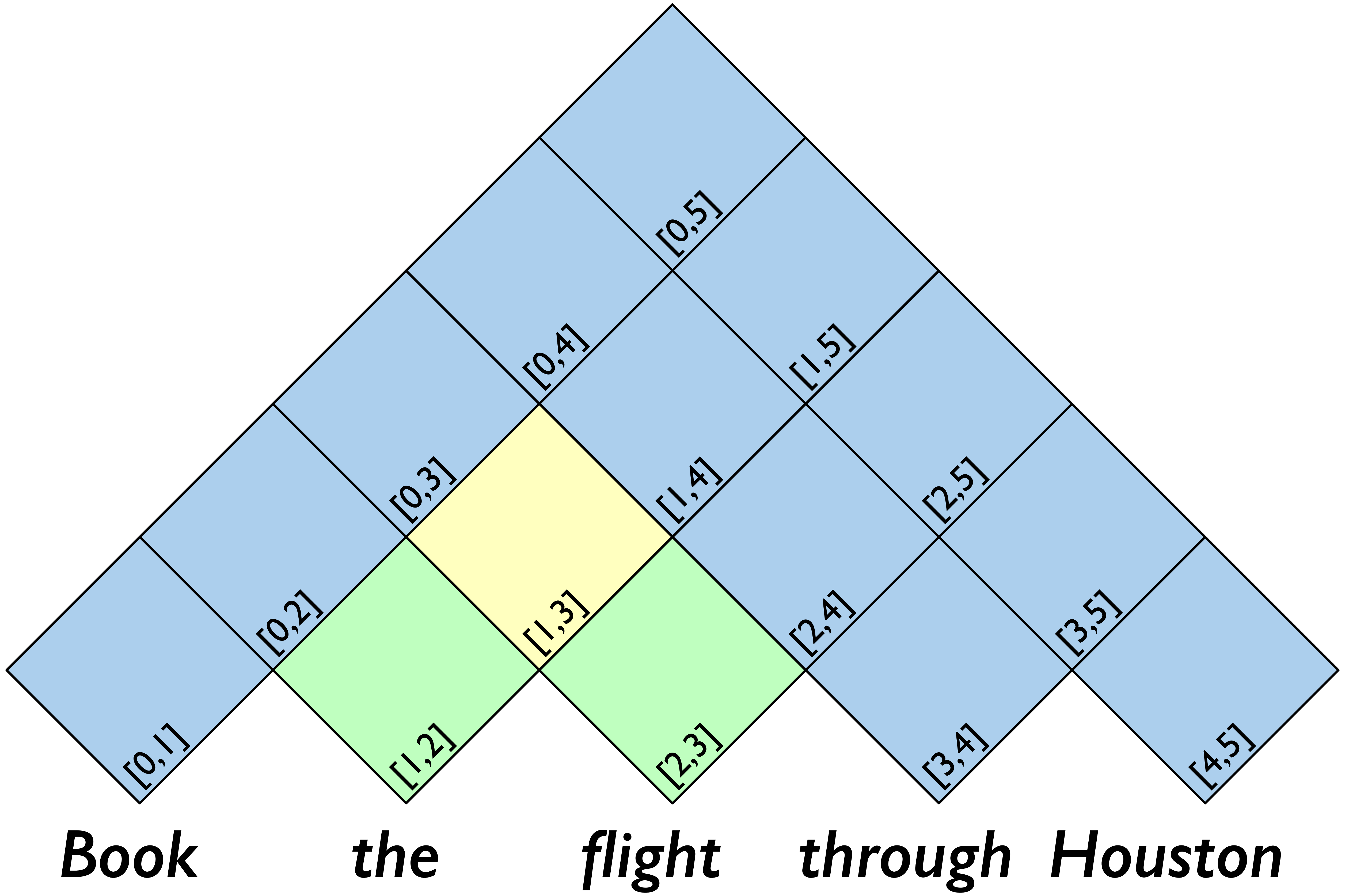
CKY Matrix


[0,5]
[0,4]
[1,5]
[0,3]
[1,4]
[2,5]
[0,2]
[1,3]
[2,4]
[3,5]
[0,1]
[1,2]
[2,3]
[3,4]
[4,5]
Book *the* *flight* *through* *Houston*

CKY Matrix


[0,5]
[0,4]
[1,5]
[0,3]
[1,4]
[2,5]
[0,2]
[1,3]
[2,4]
[3,5]
[0,1]
[1,2]
[2,3]
[3,4]
[4,5]
Book
the
flight
through
Houston

CKY Matrix


[0,5]
[0,4]
[1,5]
[0,3]
[1,4]
[2,5]
[0,2]
[1,3]
[2,4]
[3,5]
[0,1]
[1,2]
[2,3]
[3,4]
[4,5]
Book
the
flight
through
Houston

Dynamic Programming in CKY

- Key idea:
  - for $i < k < j$
  - ...and a parse spanning substring [i, j]
  - There is a k such that there are parses spanning [i, k] and [k, j]
  - We can construct parses for whole sentences by building from these partial parses
- So to have a rule $A \rightarrow B\ C$ in [i, j]
  - Must have B in [i, k] and C in [k, j] for some $i < k < j$
  - CNF forces this for all $j > i + 1$