Practice: Introduction to R

Jay Wei

2023-09-10

Contents

1 Exercise 1

1.1 Question a

Suppose x has a normal distribution with a mean of 6 and a variance of 10. Find the 0.9 quantile from the probability distribution.

$\Pr(x < 10.05) = 0.9$

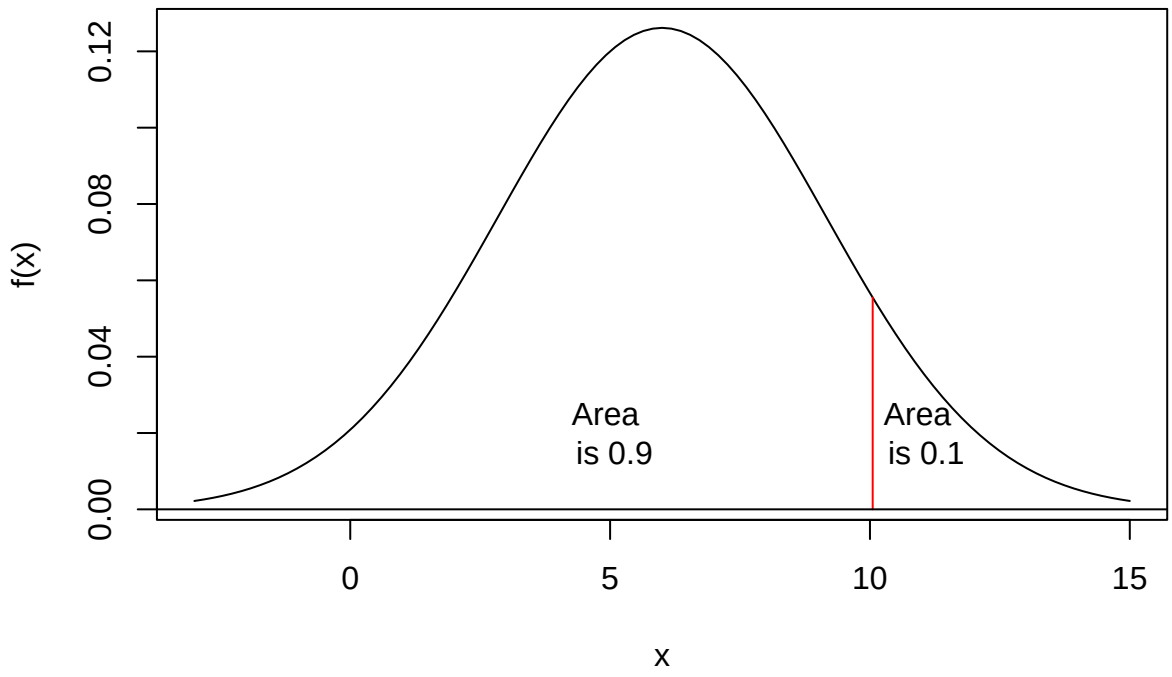
```
qnorm(p=0.9, mean=6, sd=sqrt(10))
```

```
## [1] 10.05262
```

```
dnorm(x = 10.05, mean = 6, sd = sqrt(10))  #f(x) where x = 10.05
```

```
## [1] 0.05555643
```

```
curve(expr = dnorm(x = x, mean = 6, sd = sqrt(10)),
      xlim = c(-3, 15),
      ylab = "f(x)", xlab = "x")

segments(x0 = qnorm(p = 0.9, mean = 6, sd = sqrt(10)),
         x1 = qnorm(p = 0.9, mean = 6, sd = sqrt(10)),
         y0 = 0,
         y1 = dnorm(x = qnorm(p = 0.9, mean = 6, sd = sqrt(10)),
                    mean = 6, sd = sqrt(10)),
         col = "red") #Line from (10.05, 0) to (10.05, 0.0556)

abline(h = 0) #Horizontal line at 0
text(x = 5, y = 0.02, labels = "Area \n is 0.9")
text(x = 11, y = 0.02, labels = "Area \n is 0.1")
```



1.2 Question b

Suppose x has a normal distribution with a mean of 6 and a variance of 10. Find the probability that x is less than 10.05.

```
pnorm(q=10.05, mean = 6, sd = sqrt(10))
```

```
## [1] 0.8998544
```

1.3 Question c

Suppose x has a chi-square distribution with a degrees of freedom of 10. Find the 0.1 quantile from the probability distribution.

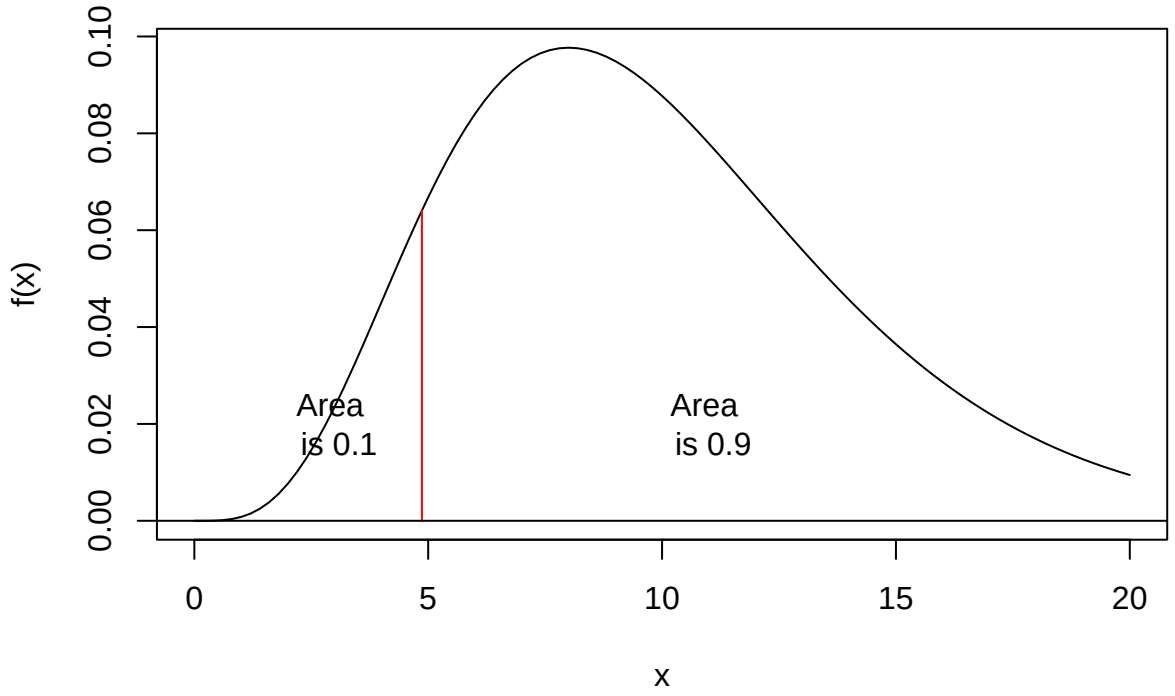
$\Pr(x < 4.87) = 0.1$

```
qchisq(p= 0.1, df=10)
```

```
## [1] 4.865182
```

```
curve(expr = dchisq(x = x, df=10),
      xlim = c(0, 20),
      ylab = "f(x)", xlab = "x")

segments(x0 = qchisq(p= 0.1, df=10),
         x1 = qchisq(p= 0.1, df=10),
```

```
         y0 = 0,
         y1 = dchisq(x = qchisq(p= 0.1, df=10), df=10),
         col = "red")

abline(h = 0) #Horizontal line at 0

text(x = 3, y = 0.02, labels = "Area \n is 0.1")
text(x = 11, y = 0.02, labels = "Area \n is 0.9")
```



1.4 Question d

Suppose x has a t-distribution with a degrees of freedom of 1000. Find the 0.9 quantile from the probability distribution.

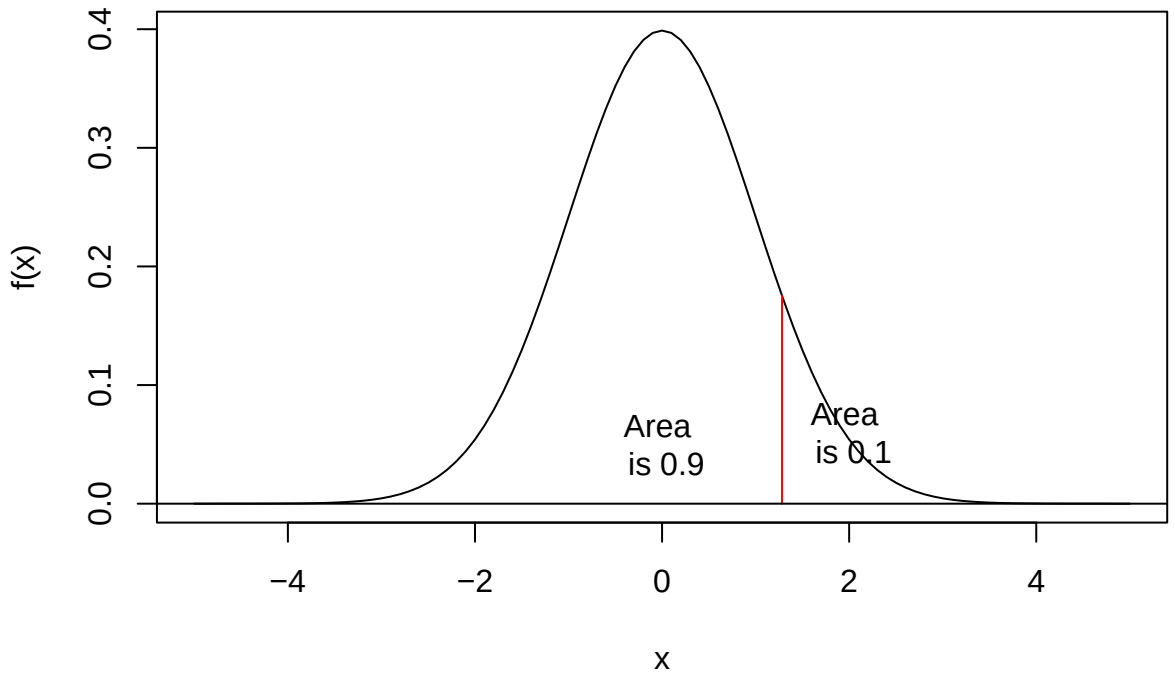
$\Pr(x < 1.28 = 0.9)$

```
qt(p=0.9, df=1000)
```

```
## [1] 1.282399
```

```
curve(expr = dt(x = x, df=1000),
      xlim = c(-5, 5),
      ylab = "f(x)", xlab = "x")

segments(x0 = qt(p= 0.9, df=1000),
         x1 = qt(p= 0.9, df=1000),
         y0 = 0,
```

```
          y1 = dt(x = qt(p= 0.9, df=1000), df=1000),
          col = "red")

abline(h = 0) #Horizontal line at 0

text(x = 0, y = 0.05, labels = "Area \n is 0.9")
text(x = 2, y = 0.06, labels = "Area \n is 0.1")
```



2 Exercise 2

What factors help determine a diamond's price? The purpose of this project is to determine which factors and to predict price. We have a data set that contains the price, carats, color, clarity, and certification body of 308 round cut diamonds sampled from a publication in the year 2000.

2.1 Question a

What is the population from which the sample is taken? Describe possible implications of this population with regards to making inferences to round cut diamonds sold in Lincoln jewelry stores currently.

Most likely, the year the data was collected would cause problems with using the regression model for Lincoln jewelry stores now. Also, one would need to know if the prices in the publication were only for a particular location or if they would be representative of the entire world's diamond market.

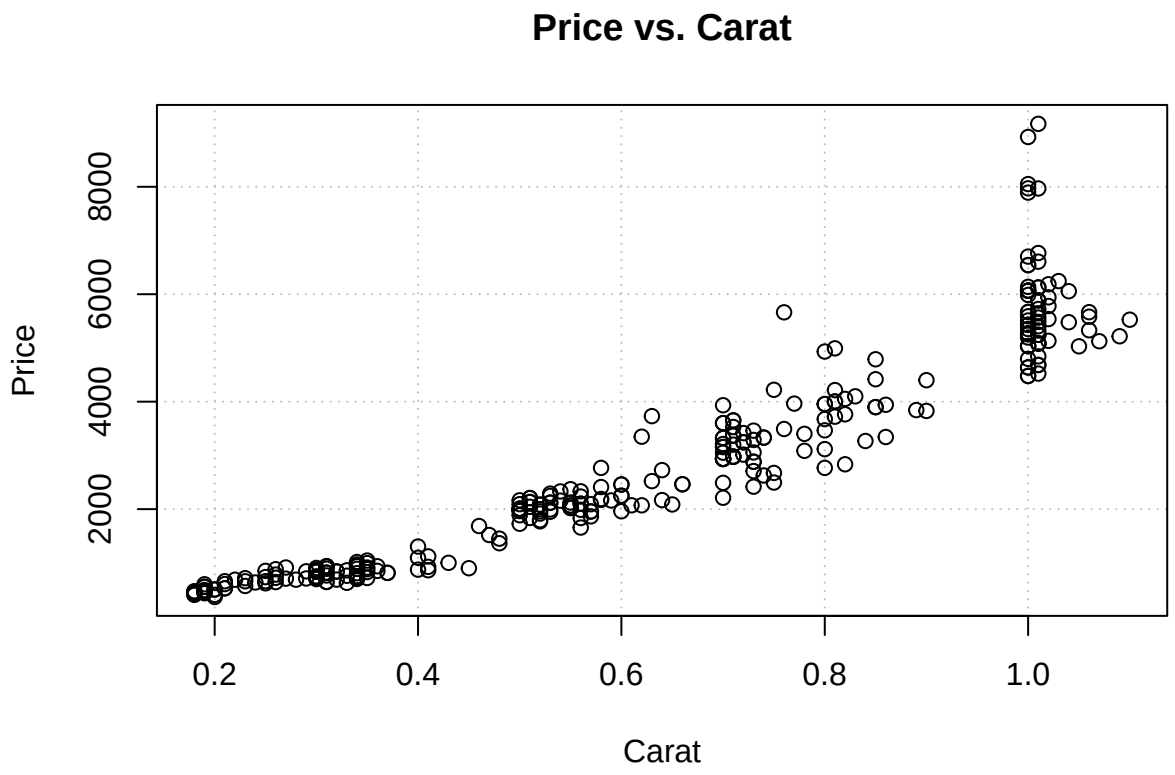
2.2 Question b

Construct a scatter plot for price (y-axis) vs. carat (x-axis). Does there appear to be a relationship between price and carat? Explain.

```
diamonds <- read.csv("DiamondPrices.csv")
head(diamonds)
```

```
##   carat color clarity certify    price
## 1  0.30     D     VS2     GIA 745.9184
## 2  0.30     E     VS1     GIA 865.0820
## 3  0.30     G    VVS1     GIA 865.0820
## 4  0.30     G     VS1     GIA 721.8565
## 5  0.31     D     VS1     GIA 940.1322
## 6  0.31     E     VS1     GIA 890.8626
```

```
# scatter plot

plot(x = diamonds$carat, y = diamonds$price,
     xlab = "Carat", ylab = "Price",
     main  = "Price vs. Carat", col = "black",
     pch = 1, lwd = 1,
     panel.first = grid(col = "gray", lty = "dotted"))
```



Yes, there appears to be a positive relationship because as carat increases the price tends to increase as well. There may be a quadratic relationship too.

2.3 Question c

Find the sample regression models individually for carat, color, clarity, and certify (explanatory variables) with the price (response variable). State the four models.

```
#Find models for each explanatory variable
mod.fit.carat <- lm(formula = price ~ carat, data = diamonds)
summary(mod.fit.carat)
```

```
##
## Call:
## lm(formula = price ~ carat, data = diamonds)
##
## Residuals:
##     Min      1Q  Median      3Q     Max
## -1297.5  -346.2   -66.5   249.3  3776.3
##
## Coefficients:
##             Estimate Std. Error t value Pr(>|t|)
## (Intercept) -1316.73      90.82  -14.50   <2e-16 ***
## carat        6645.02     131.83   50.41   <2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 640.3 on 306 degrees of freedom
```

```
## Multiple R-squared:  0.8925, Adjusted R-squared:  0.8922
## F-statistic:  2541 on 1 and 306 DF,  p-value: < 2.2e-16
```

$$\widehat{\text{Price}} = -1316.73 + 6645.02\text{Carat}$$

```
mod.fit.color <- lm(formula = price ~ color, data = diamonds)
summary(mod.fit.color)
```

```
##
## Call:
## lm(formula = price ~ color, data = diamonds)
##
## Residuals:
##     Min      1Q  Median      3Q     Max
## -3563.4 -1806.3  -400.3  1514.2  5228.6
##
## Coefficients:
##             Estimate Std. Error t value Pr(>|t|)
## (Intercept)   4067.5      483.9   8.406  1.7e-15 ***
## colorE        -912.0      565.1  -1.614  0.10757
## colorF       -1325.4      529.0  -2.505  0.01276 *
## colorG       -1531.8      540.2  -2.836  0.00488 **
## colorH       -1223.1      543.7  -2.250  0.02519 *
## colorI       -1102.8      572.6  -1.926  0.05504 .
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 1936 on 302 degrees of freedom
## Multiple R-squared:  0.03044,    Adjusted R-squared:  0.01439
## F-statistic: 1.896 on 5 and 302 DF,  p-value: 0.09476
```

$$\widehat{\text{Price}} = 4067.5 - 912.0\text{E} - 1325.4\text{F} - 1531.8\text{G} - 1223.1\text{H} - 1102.8\text{I}$$

```
mod.fit.clarity <- lm(formula = price ~ clarity, data = diamonds)
summary(mod.fit.clarity)
```

```
##
## Call:
## lm(formula = price ~ clarity, data = diamonds)
##
## Residuals:
##     Min      1Q  Median      3Q     Max
## -2990.6 -1111.4  -567.7  1182.2  6426.9
##
## Coefficients:
##             Estimate Std. Error t value Pr(>|t|)
## (Intercept)   1543.8      283.1   5.453 1.03e-07 ***
## clarityVS1    1353.3      351.7   3.848 0.000145 ***
## clarityVS2    1812.3      383.0   4.732 3.42e-06 ***
## clarityVVS1   1645.9      384.7   4.279 2.52e-05 ***
```

```
## clarityVVS2   1524.9      354.1   4.307 2.24e-05 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 1878 on 303 degrees of freedom
## Multiple R-squared:  0.08428,    Adjusted R-squared:  0.0722
## F-statistic: 6.972 on 4 and 303 DF,  p-value: 2.216e-05
```

$$\widehat{\text{Price}} = 1543.8 + 1353.3\text{VS1} + 1812.3\text{VS2} + 1645.9\text{VVS1} + 1524.9\text{VVS2}$$

```r
mod.fit.certify<-lm(formula = price ~ certify, data = diamonds)
summary(mod.fit.certify)
```

```
##
## Call:
## lm(formula = price ~ certify, data = diamonds)
##
## Residuals:
##     Min      1Q  Median      3Q     Max
## -2413.3  -964.1  -416.1   933.5  6128.7
##
## Coefficients:
##             Estimate Std. Error t value Pr(>|t|)
## (Intercept)   3042.3      135.7  22.425  < 2e-16 ***
## certifyHRD    1071.6      231.5   4.629 5.44e-06 ***
## certifyIGI   -1743.5      232.5  -7.500 7.00e-13 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 1667 on 305 degrees of freedom
## Multiple R-squared:  0.2736, Adjusted R-squared:  0.2688
## F-statistic: 57.44 on 2 and 305 DF,  p-value: < 2.2e-16
```

$$\widehat{\text{Price}} = 3042.3 + 1071.6\text{HRD} - 1743.5\text{IGI}$$

2.4 Quesiton d

Is there a linear relationship between carat and price? Perform both a t-test and a F-test to answer this question. Make sure to include all 5 steps for each test. I recommend using the p-value method to make the test easier.

The answer below is the same for both tests.

i) $H_0 : \beta_{\text{Carat}} = 0$ vs. $H_a : \beta_{\text{Carat}} \neq 0$
ii) p-value $< 2 * 10^{-16}$
iii) $\alpha = 0.05$
iv) Since p-value < 0.05, reject H_0.
v) There is a linear relationship between carat and price.

2.5 Quesiton e

Is there a linear relationship between color and price, clarity and price, and certify and price? Explain your answers using the appropriate hypothesis tests. Note that you do not need to repeat all 5 steps for a hypothesis test here. I am only interested in determining if you can correctly interpret the hypothesis test results.

Variable	F-test p-value
Color	0.09476
Clarity	$2.216 * 10^{-5}$
Certify	$2.2 * 10^{-16}$

There is a linear relationship for clarity and certify since the p-values are small. The p-value for color is only a little above the chosen $\alpha = 0.05$ for this project. Thus, it is difficult to conclude if there is or is not a relationship.

2.6 Question f

Which explanatory variable (carat, color, clarity, or certify) is doing the best job of estimating the price? Explain your answer using R^2.

Variable	R^2
Carat	89.25%
Color	3.04%
Clarity	8.42%
Certify	27.36%

Carat is the best since it has the largest R^2.

2.7 Question g

What characteristics of a diamond tend to cause it to be highly priced? Specifically state these characteristics and justify them with the use of your sample models.

Check solutions.

2.8 Quesiton h

Suppose two people are about to get engaged and they plan to purchase a diamond engagement ring. You offer your statistical expertise to aid them in pricing diamonds. Suppose the couple is interested in a 0.5 carat diamond. Find the estimated price of a 0.5 carat diamond using the appropriate sample regression model. Show how you can get the answer using by-hand calculations and through the use of R.

```
predict(object = mod.fit.carat, newdata = data.frame(carat = 0.5),
        interval = "prediction", level = 0.95)
```

```
##        fit      lwr      upr
## 1 2005.778 743.4186 3268.138
```

$\hat{\text{Price}} = -1316.73 + 6645.02\text{Carat} = -1316.73 + 6645.02 * 0.5 = 2005.78$

2.9 Question i

which is the more appropriate interval to use with estimating the price: confidence or prediction? Explain your answer and find the interval.

Prediction interval since we want to price the diamond for one couple's engagement ring. The interval from the output is $ 743.42 < Price < $ 3,268.14.

2.10 Question j

When shopping for a diamond engagement ring, a local jewelry store quotes the couple a price of $2,500 for a 0.5 carat, round cut diamond that can be placed in a ring. Is this price fair? Explain your answer using the correct information found in this project. Assume your regression model can be used to estimate prices of diamonds in Lincoln.

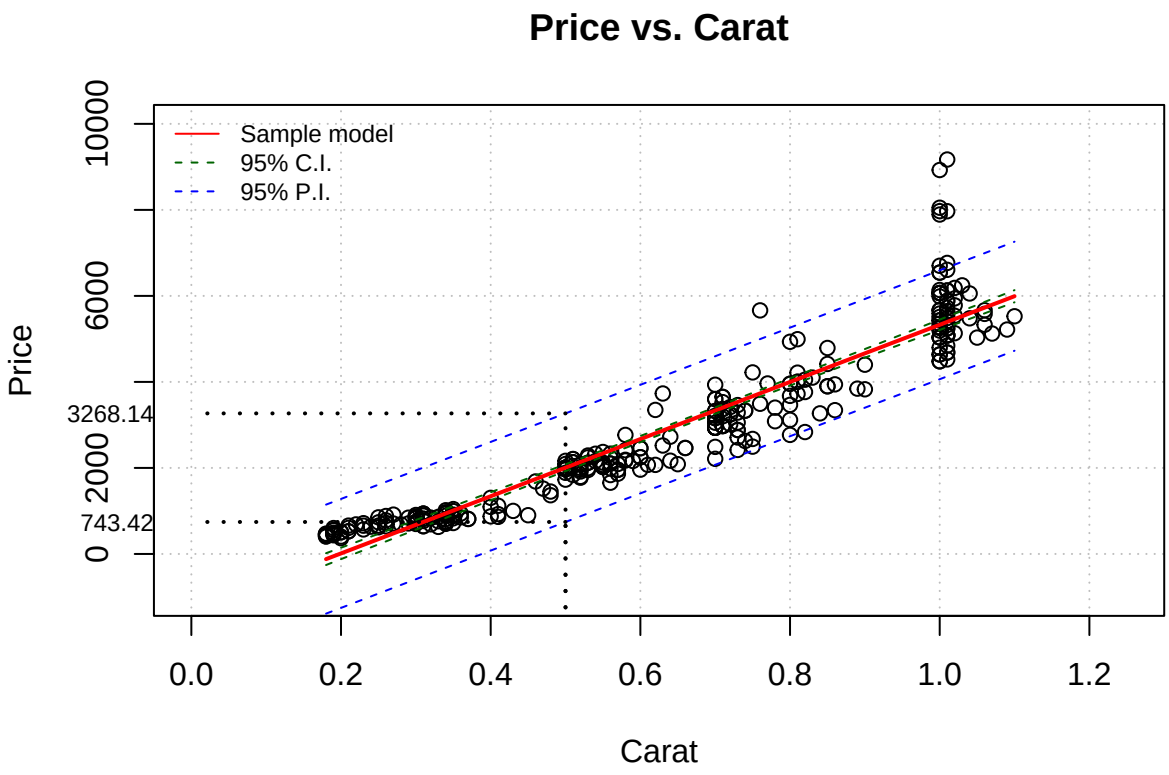
Yes, since the $ 2,500 price is inside the prediction interval.

2.11 Question k

Using carat as the explanatory variable, construct a scatter plot with the sample model, 95% confidence interval bands, and 95% prediction interval bands plotted upon it. Show your interval on the plot at carat = 0.5.

```
plot(x = diamonds$carat, y = diamonds$price,
     xlab = "Carat", ylab = "Price",
     main = "Price vs. Carat", col = "black", pch = 1, lwd = 1,
     panel.first = grid(col = "gray", lty = "dotted"),
     xlim = c(0,1.25), ylim = c(-1000, 10000))

curve(expr = predict(object = mod.fit.carat,
                     newdata = data.frame(carat = x)),
      col  = "red", lty = "solid", lwd = 2, add = TRUE,
      from = min(diamonds$carat), to = max(diamonds$carat))

curve(expr =  predict(object = mod.fit.carat,
                      newdata = data.frame(carat = x),
                      interval = "confidence", level = 0.95)[,2],
      col = "darkgreen", lty = "dashed", lwd = 1, add = TRUE,
      from = min(diamonds$carat), to = max(diamonds$carat))

curve(expr =  predict(object = mod.fit.carat,
                      newdata = data.frame(carat = x),
                      interval = "confidence", level = 0.95)[,3],
      col = "darkgreen", lty = "dashed", lwd = 1, add = TRUE,
      from = min(diamonds$carat), to = max(diamonds$carat))

curve(expr =  predict(object = mod.fit.carat,
                      newdata = data.frame(carat = x),
                      interval = "prediction", level = 0.95)[,2],
      col = "blue", lty = "dashed", lwd = 1, add = TRUE,
      from = min(diamonds$carat), to = max(diamonds$carat))
```

```r
curve(expr =  predict(object = mod.fit.carat,
                      newdata = data.frame(carat = x),
                      interval = "prediction", level = 0.95)[,3],
      col = "blue", lty = "dashed", lwd = 1, add = TRUE,
      from = min(diamonds$carat), to = max(diamonds$carat))

legend("topleft", legend = c("Sample model", "95% C.I.", "95% P.I."),
       col = c("red", "darkgreen", "blue"),
       lty = c("solid", "dashed", "dashed"),
       bty = "n", cex = 0.75)

save.pred<-predict(object = mod.fit.carat,
                   newdata = data.frame(carat = 0.5),
                   interval = "prediction", level = 0.95)

# Lower
segments(x0 = 0.5, y0 = -2000, x1 = 0.5, y1 = save.pred[1,2],
         lty = "dotted", col = "black", lwd = 2)

segments(x0 = 0.5, y0 = save.pred[1,2], x1 = 0, y1 =save.pred[1,2] ,
         lty = "dotted", col = "black", lwd = 2)

# Upper
segments(x0 = 0.5, y0 = -2000, x1 = 0.5, y1 = save.pred[1,3],
         lty = "dotted", col = "black", lwd = 2)

segments(x0 = 0.5, y0 = save.pred[1,3], x1 = 0, y1 =save.pred[1,3] ,
         lty = "dotted", col = "black", lwd = 2)

mtext(text = round(save.pred[2],2), side=2, cex = 0.75,
      at = save.pred[2], las = 2)  #las makes perpendicular to axis
mtext(text = round(save.pred[3],2), side=2, cex = 0.75,
      at = save.pred[3], las = 2)
```


Price vs. Carat
Sample model
95% C.I.
95% P.I.
Price
Carat
10000
6000
2000
0
3268.14
743.42
0.0
0.2
0.4
0.6
0.8
1.0
1.2