Chpater 1 Selected Computer Exercises

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目錄

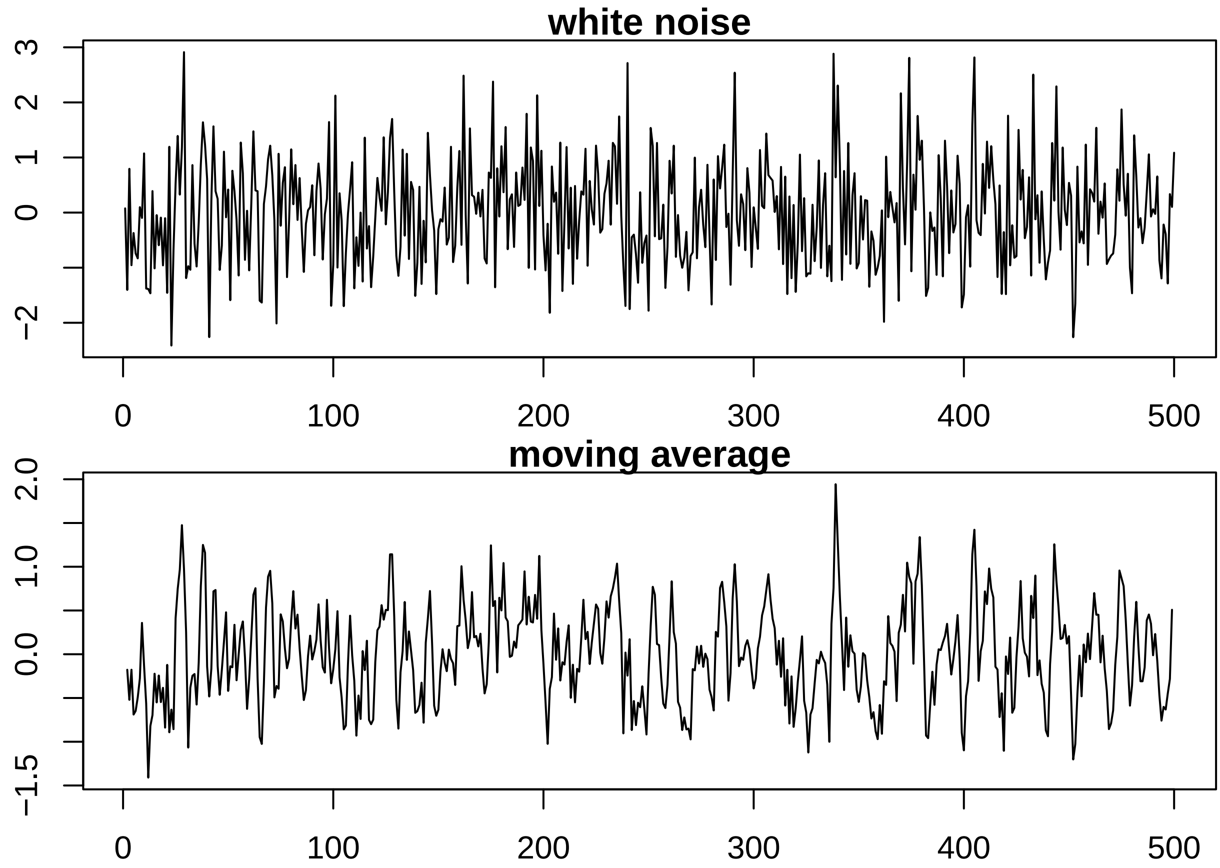

```
library(astsa)
```

1 Example 1.9

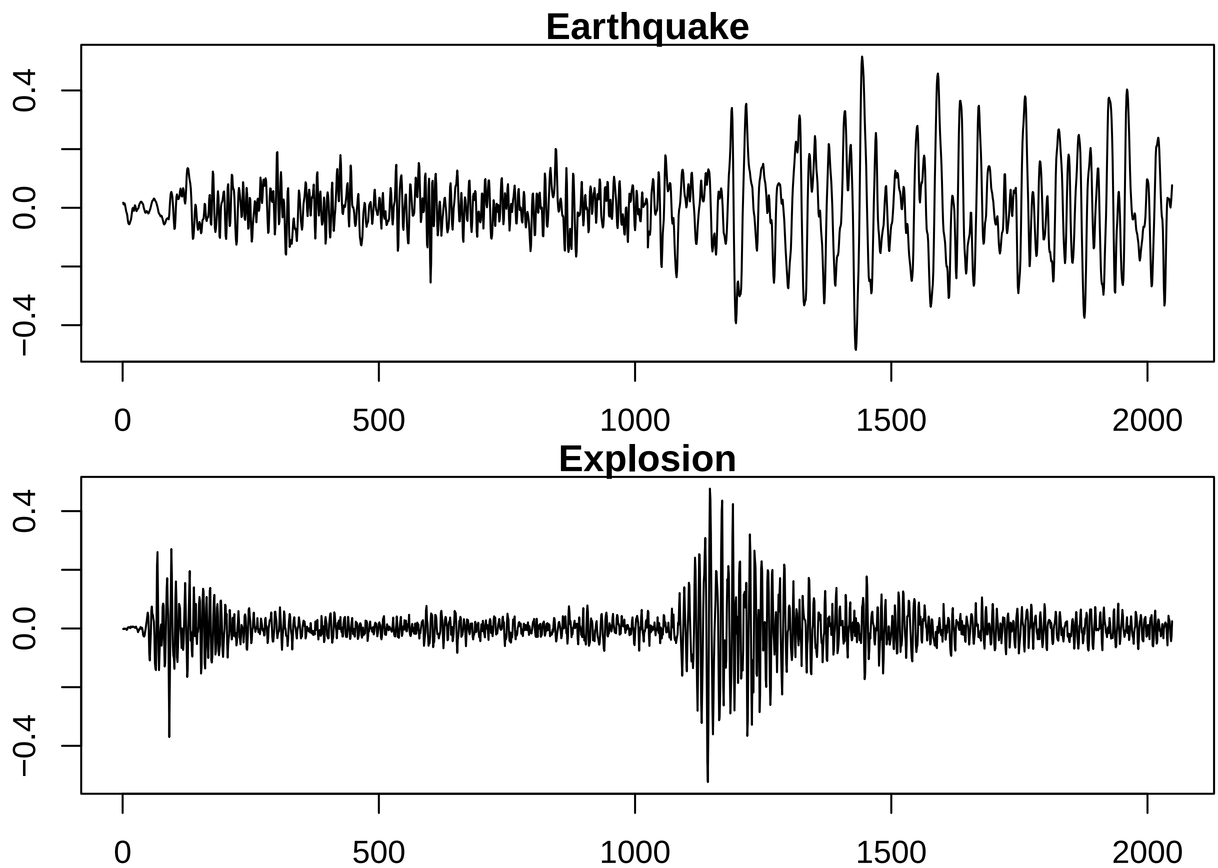
$v_t = \frac{1}{3}(w_{t-1} + w_t + w_{t+1})$

```
# Moving Average
w <- rnorm(500,0,1)
v <- filter(w, sides = 2, rep(1/3,3))
par(mfrow=c(2,1))
par(mar=c(2,2,1,1))
```

```
plot.ts(w, main="white noise")
plot.ts(v, main="moving average")
```

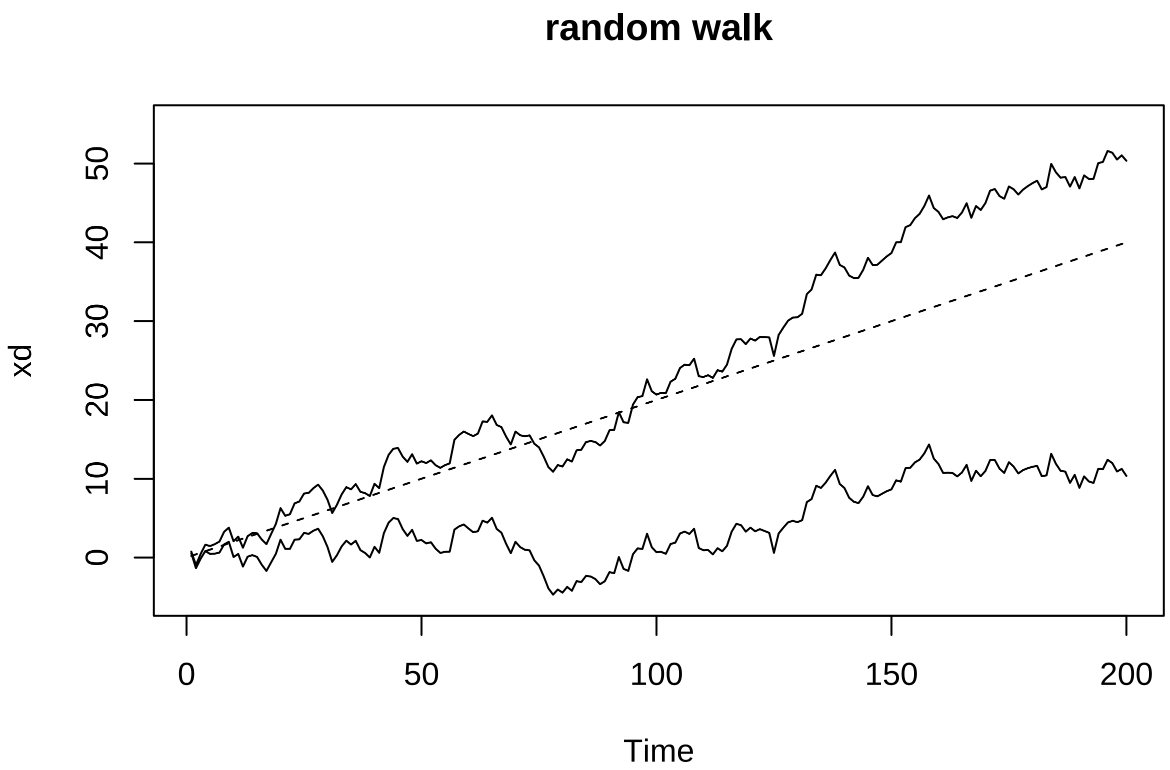



```
# Figure 1.7
par(mfrow=c(2,1))
par(mar=c(2,2,1,1))
plot(EQ5, main = "Earthquake")
plot(EXP6, main = "Explosion")
```



2 Example 1.11

```
set.seed(154)
w <- rnorm(200, 0, 1); x = cumsum(w)
wd <- w +.2; xd <- cumsum(wd)
plot.ts(xd, ylim=c(-5, 55), main = "random walk")
lines(x); lines( .2*(1:200), lty="dashed")
```



$$x_t = \delta + x_{t-1} + w_t \tag{1}$$

$$\iff x_t = \delta t + \sum_{j=1}^{t} w_j \tag{2}$$

Random walk, $\sigma_w = 1$, with drift $\delta = 0.2$ (upper jagged lines), w/o drift, $\delta = 0$ (lower jagged line), and a straight line with slope .2 (dashed line)

```r
# the use of cumsum()

x <- c(1,2,3,4,5)
cumsum(x)
```

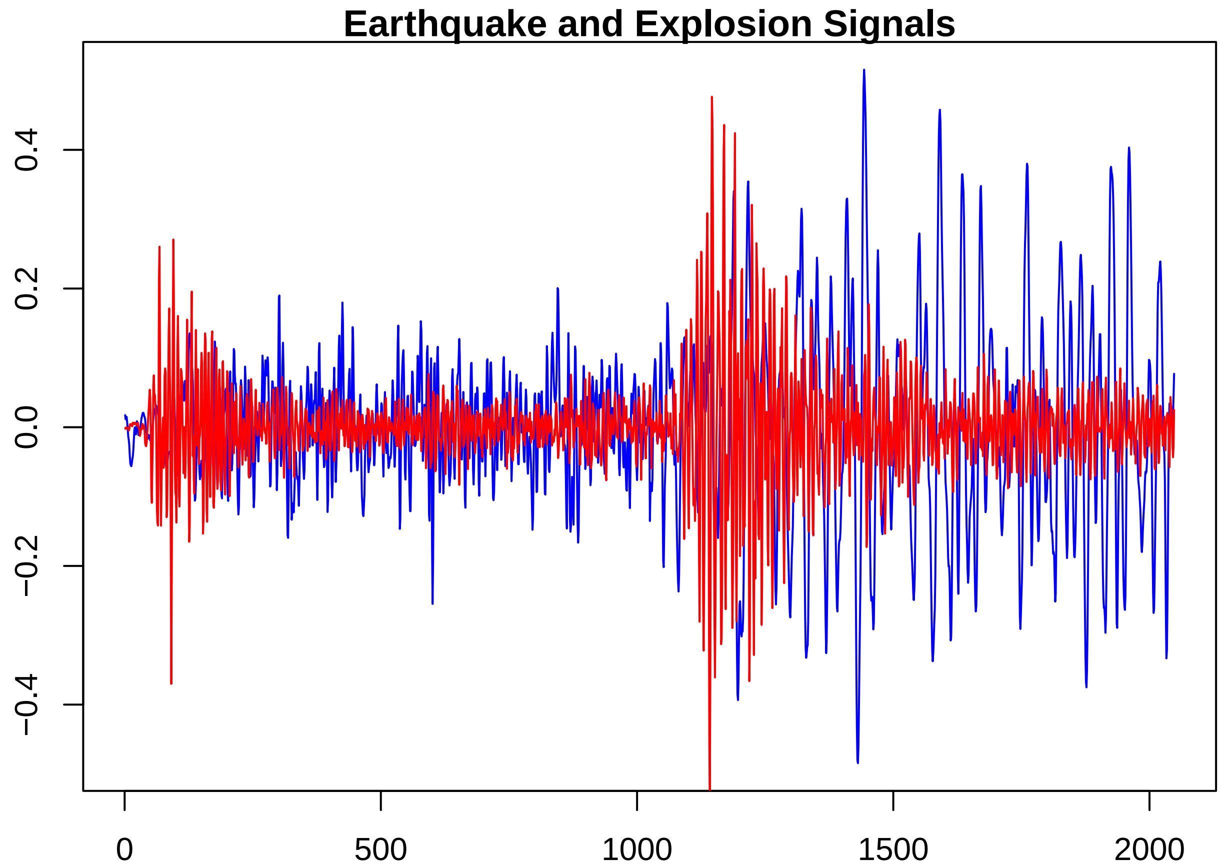
```
## [1]  1  3  6 10 15
```

3 Problem 1.1

To compare the earthquake and explosion signals, plot the data displayed in Figure 1.7 on the same graph using different colors or different line types and comment on the results. (The R code in Example 1.11 may be of help on how to add lines to existing plots.)

The `mar` argument of the `par()` function is a vector of 4 values that specifies the margins of the plot in lines of text. The values are in the order of bottom, left, top, and right margins.

```
par(mar=c(2,2,1,1))
plot(EQ5, main = "Earthquake and Explosion Signals", col="blue", lty="solid")
lines(EXP6, col= "red", lty="solid")
```



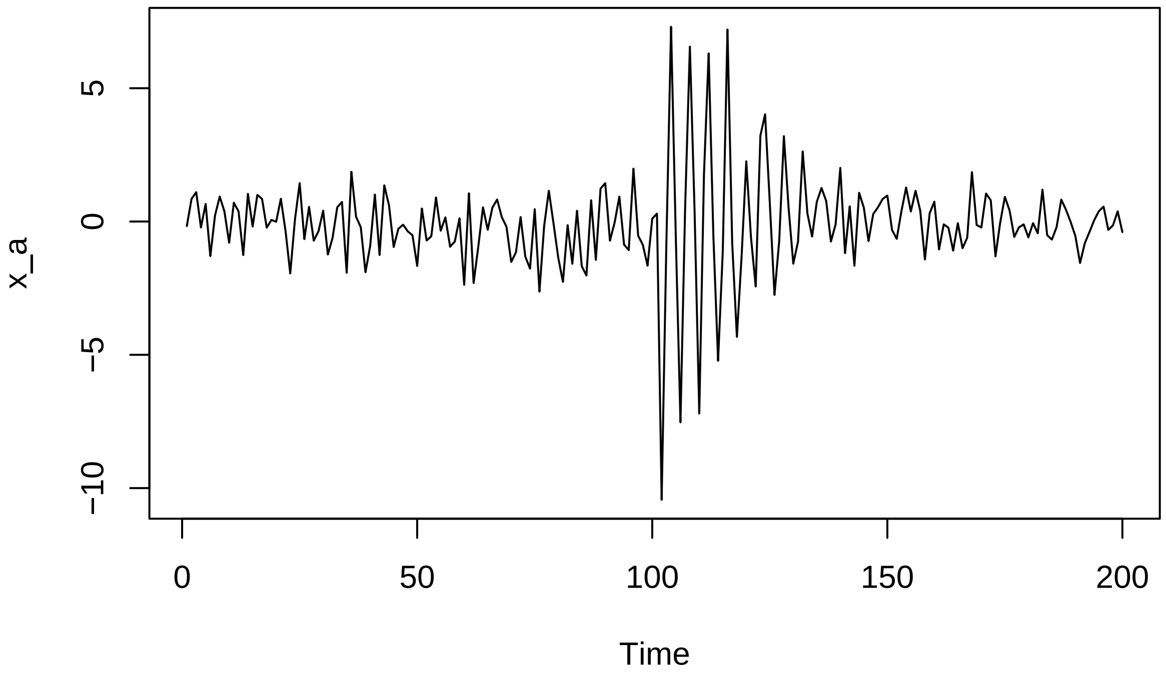
4 Problem 1.2

Consider a signal-plus-noise model of the general form $x_t = s_t + w_t$, where w_t is Gaussian white noise with $\sigma_w^2 = 1$. Simulate and plot n = 200 observations from each of the following two models (Save the data or your code for use in Problem 1.22):

(a) $x_t = s_t + w_t$

$$s_t = \begin{cases} 0 & t = 1, ..., 100 \\ 10exp(-\frac{(t-100)}{20})cos(2\pi t/4) & t = 101, ..., 200 \end{cases}$$

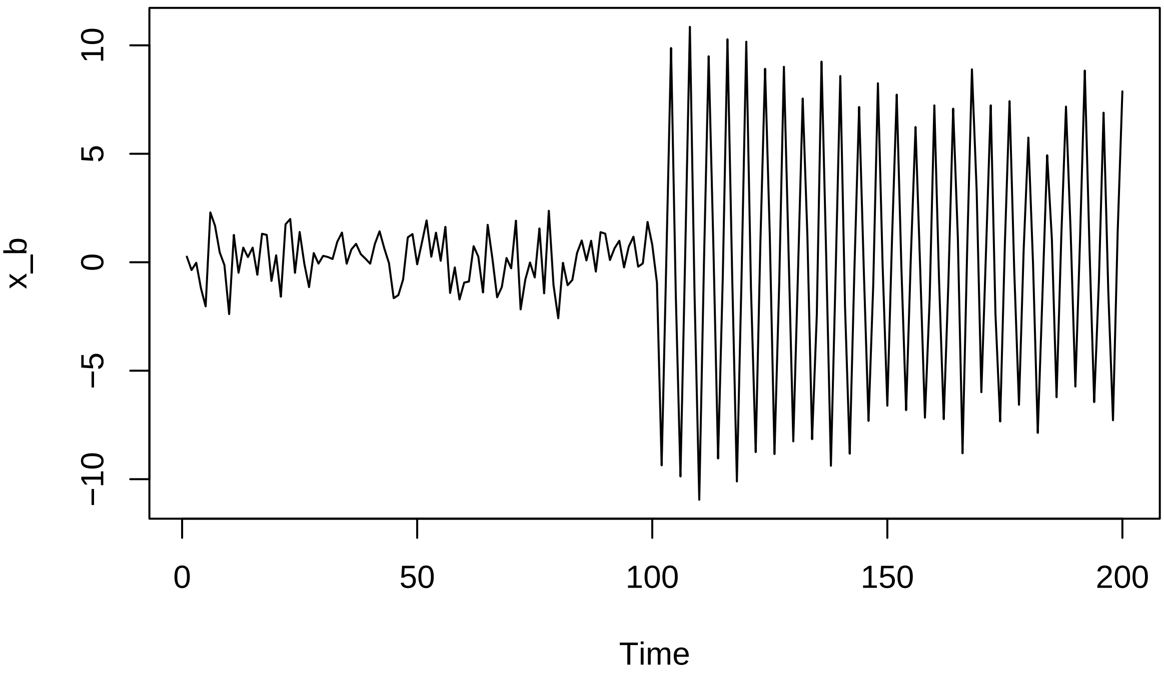
```
w_a <- rnorm(200, 0, 1)
s_a <- c(rep(0,100), 10*exp(-(1:100)/20)*cos(2*pi*101:200/4))
x_a <- ts(s_a+w_a)
plot(x_a)
```



(b) $x_t = s_t + w_t$

$$s_t = \begin{cases} 0 & t = 1, ..., 100 \\ 10exp(-\frac{(t-100)}{200})cos(2\pi t/4) & t = 101, ..., 200 \end{cases}$$

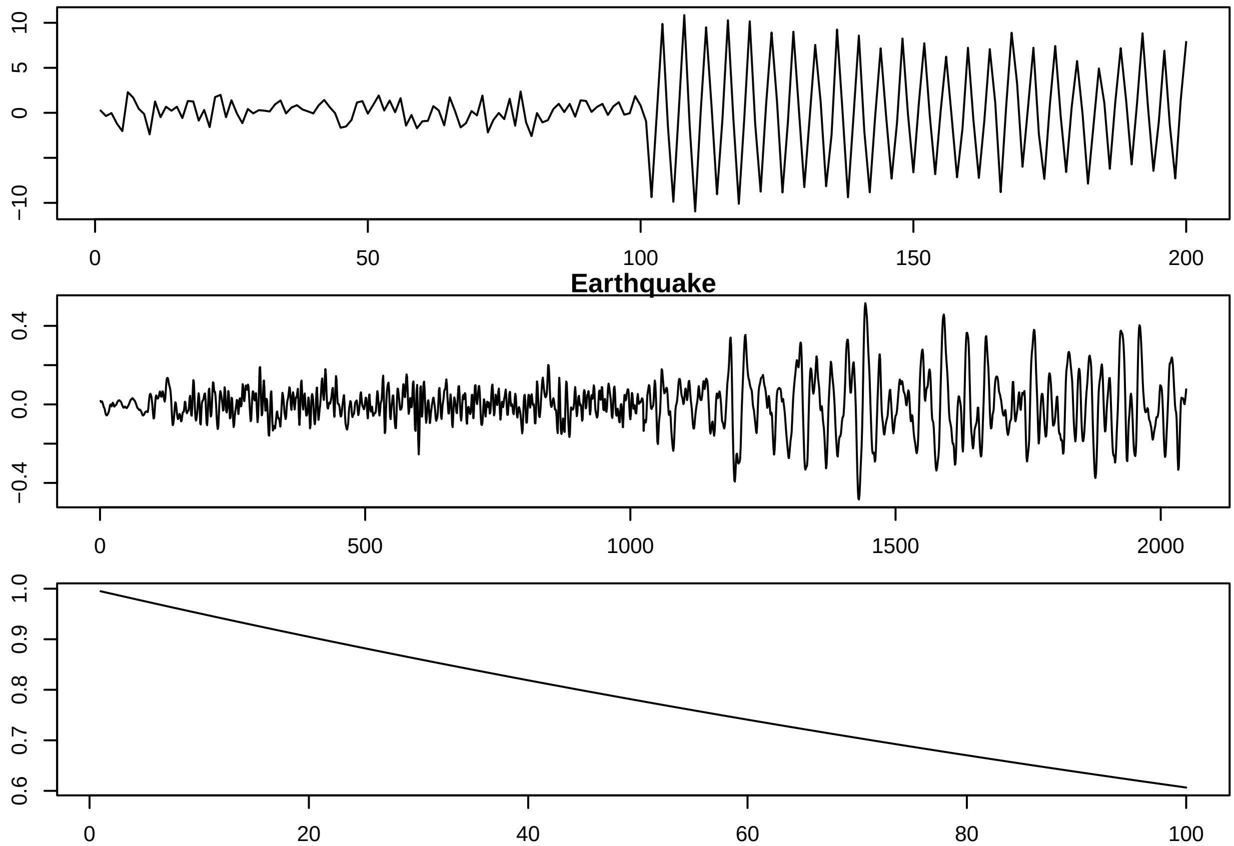
```
w_b <- rnorm(200, 0, 1)
s_b <- c(rep(0, 100), 10*exp(-(1:100)/200))*cos(2*pi*101:200/4)
x_b <- ts(s_b+w_b)
plot(x_b)
```



(c) Compare the general appearance of the series (a) and (b) with the earthquake series and the explosion series shown in Figure 1.7. In addition, plot (or sketch) and compare the signal modulators (a) $\exp\{-t/20\}$ and (b) $\exp\{-t/200\}$, for $t = 1, 2, \ldots, 100$.

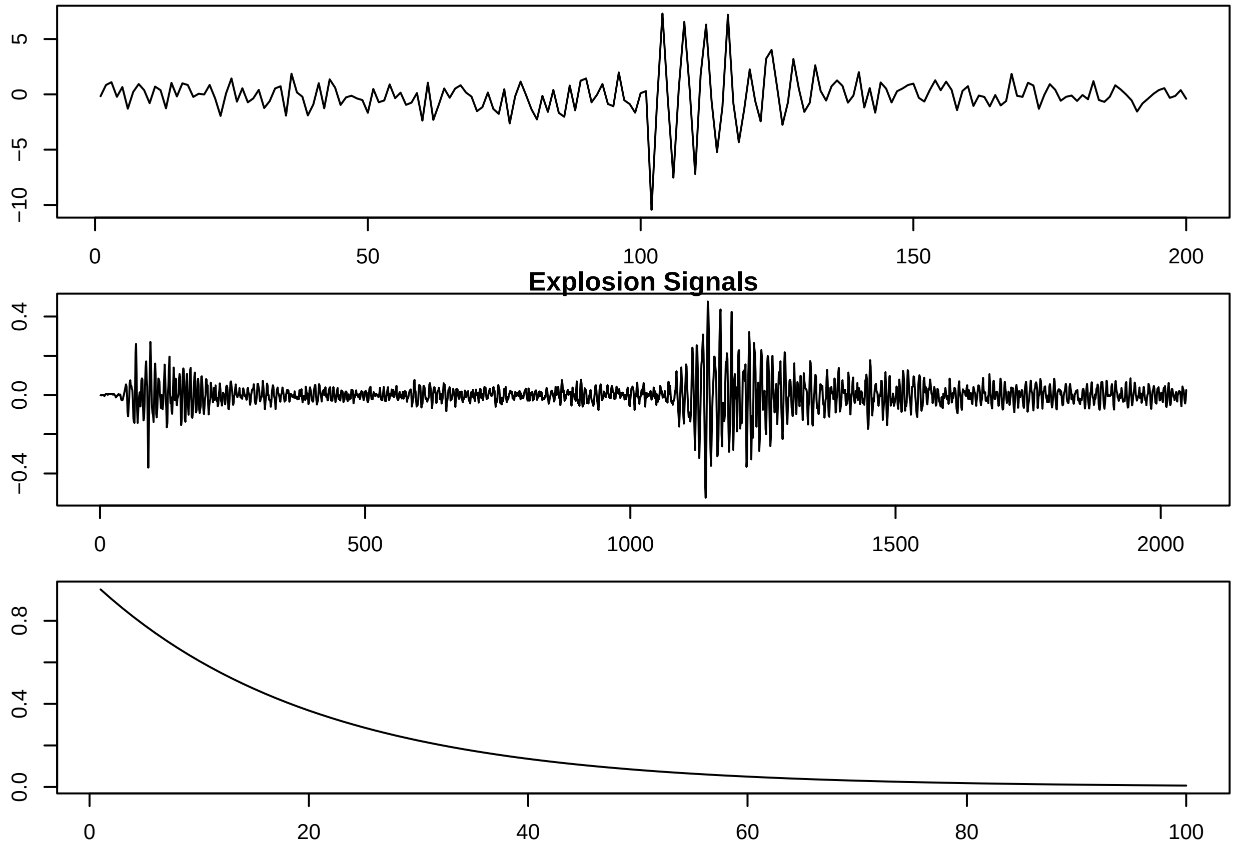
Notice the signal modulators exp{-t/200} is most similar to the Earthquake series in the fact that there is fairly little noise from time 1 to 100 and then there is a jolt to the system after which the noise slowly decreases. The signal modulator shows the fact that the noise will die down slowly as its decrease is slow.

```
par(mar=c(2,2,1,1))
par(mfrow=c(3,1))
plot(x_b)
plot(EQ5, main = "Earthquake")
x_c2 <- ts(exp(-(1:100)/200))
plot(x_c2)
```



Notice the signal modulators exp{-t/20} is most similar to the Explosion series in the fact that there is fairly little noise from time 1 to 100 and then there is a jolt to the system after which the noise quickly dies down after that. The signal modulator shows the fact that the noise will die down quickly. It decreases fairly rapidly.

```
par(mar=c(2,2,1,1))
par(mfrow=c(3,1))
plot(x_a)
plot(EXP6, main = "Explosion Signals")
x_c1 <- ts(exp(-(1:100)/20))
plot(x_c1)
```

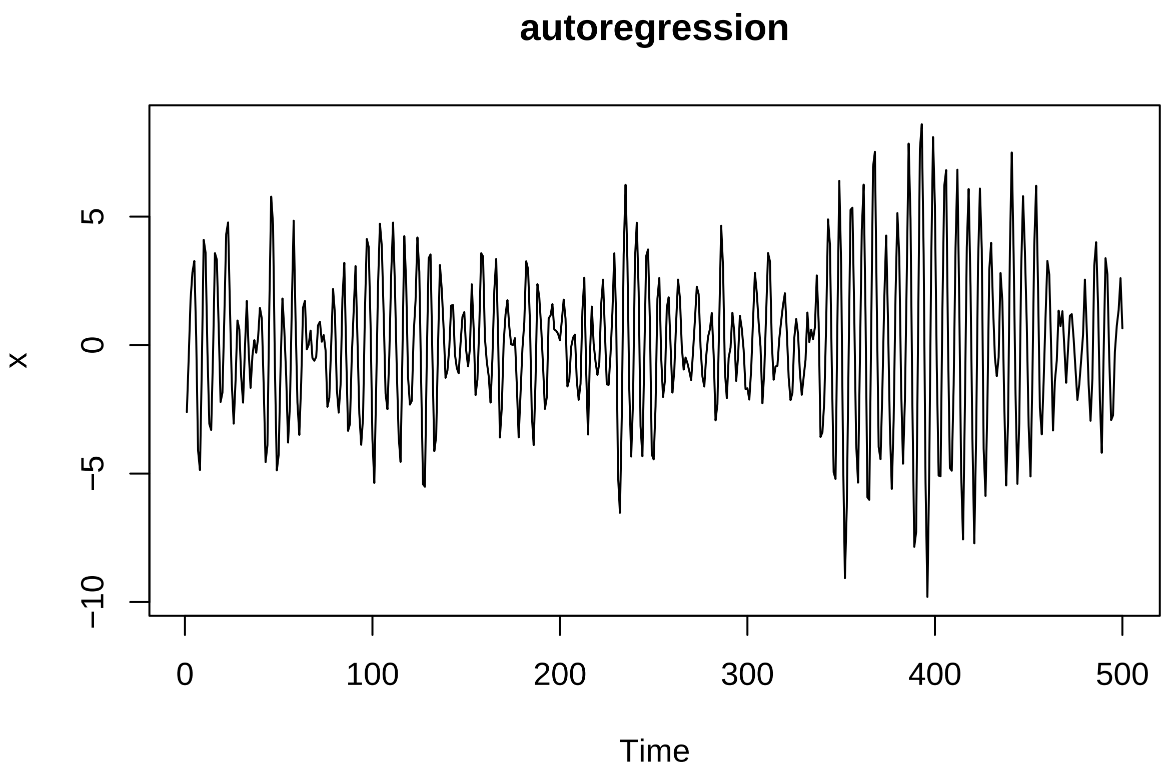


5 Problem 1.3

Generate n = 100 observations from the autoregression $x_t = -.9x_{t-2} + w_t$ with $\sigma_w = 1$, using the method described in Example 1.10, page 13. Next, apply the moving average filter $v_t = (x_t + x_{t-1} + x_{t-2} + x_{t-3})/4$ to x_t, the data you generated. Now plot x_t as a line and superimpose v_t as a dashed line. Comment on the behavior of x_t and how applying the moving average filter changes that behavior. [Hints: Use `v = filter(x, rep(1/4, 4), sides = 1)` for the filter and note that the R code in Example 1.11 may be of help on how to add lines to existing plots.]

$x_t = x_{t-1} - 0.9x_{t-2} + w_t$

```
# Example 1.10
w <- rnorm(550, 0, 1) # 50 extra to avoid startup problems
x <- filter(w, filter = c(1, -.9), method = "recursive")[-(1:50)]
plot.ts(x, main ="autoregression")
```



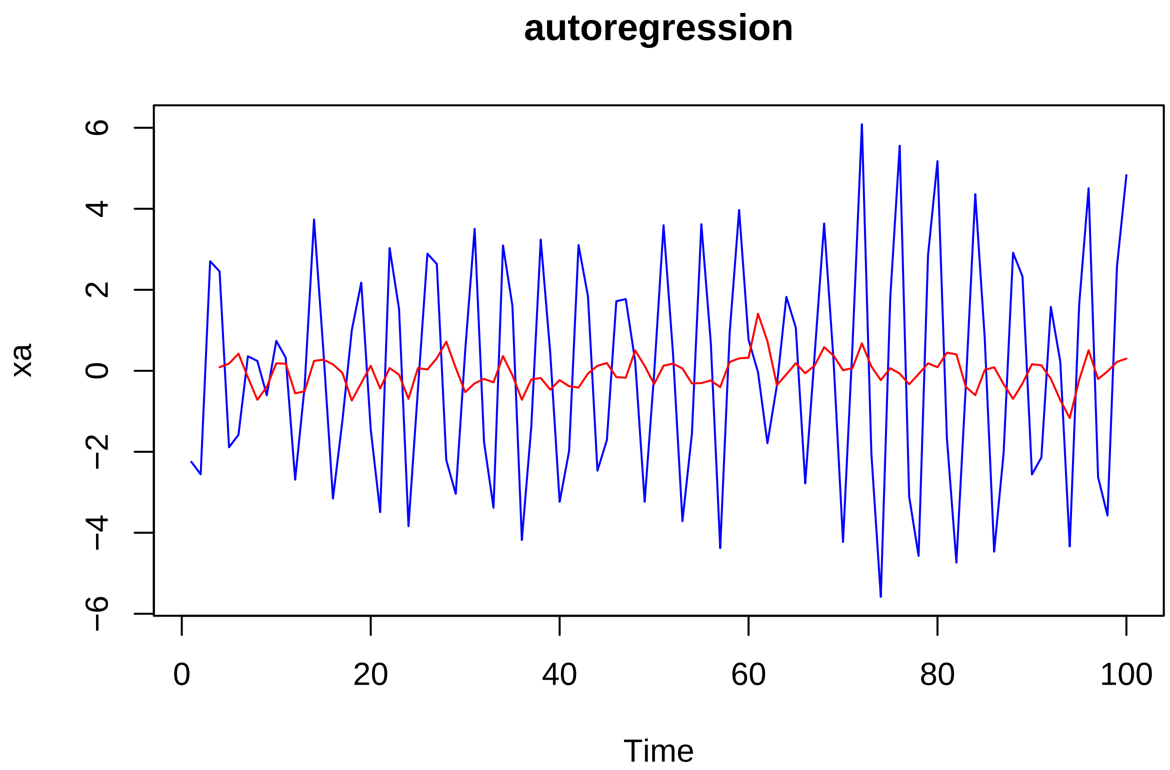
(a) $v_t = (x_t + x_{t-1} + x_{t-2} + x_{t-3})/4$

Below is a plot of the series, generated by the model above, with the filter superimposed on the series. In general, it looks as though the variability in the data increases over time, implying that the series may not be stationary. However, when the filter is added, it decreases the noise (which should happen with averages), but it also looks like the moving average is stationary. The mean did not change after applying the MA filter (which should also happen with averages).

```
w <- rnorm(150, 0, 1)
xa <- filter(w, filter = c(0, -.9), method = "recursive")[-(1:50)]


va <- filter(xa, rep(1/4, 4), method = "convolution" , sides = 1)
# sides = 1 the filter coefficients are for past values only
# method="convolution" for moving average


plot.ts(xa, main ="autoregression", col="blue")
lines(va, col="red")
```



(b) Repeat (a) but with $x_t = cos(2\pi t/4)$

Below is a plot of the series, generated by the model above, with the filter superimposed on the series. Since this series has no random component, and it is based on cosine, the plot looks perfectly cyclical. When the filter is added, it does not change the mean or but changes the variability to 0 in the data.

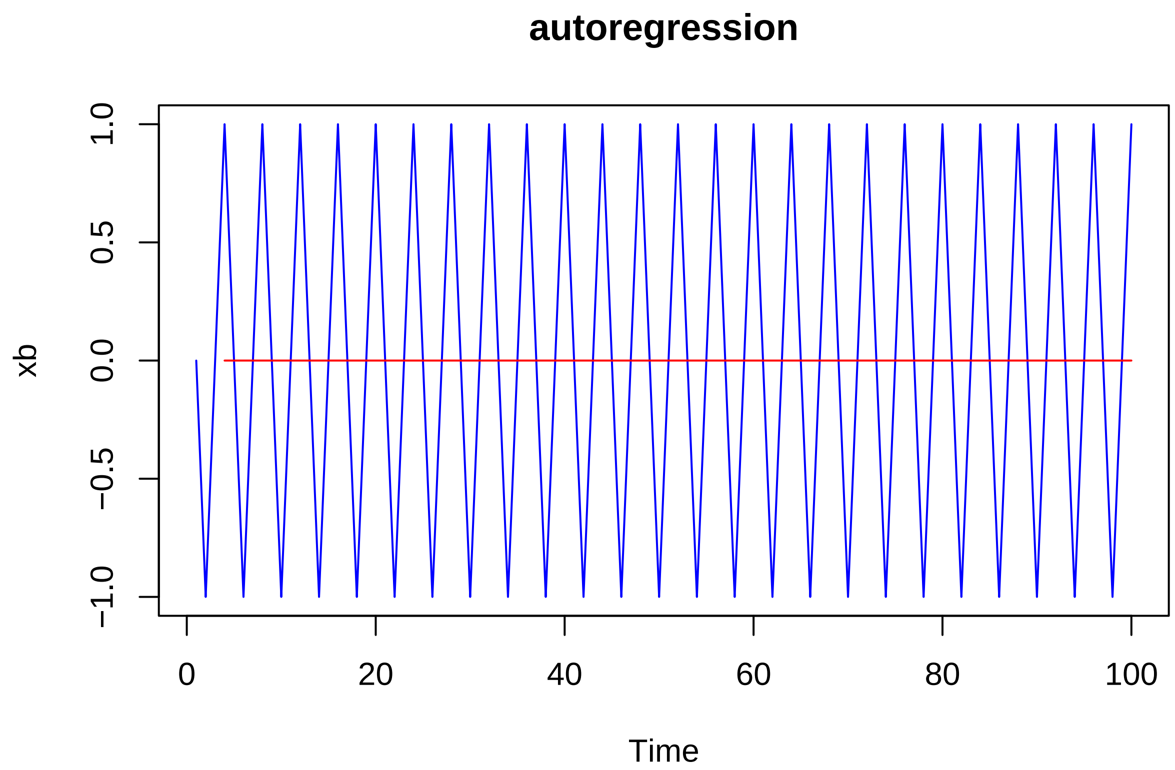
```
xb <- cos(2*pi*1:100/4)



vb <- filter(xb, rep(1/4, 4), method = "convolution" , sides = 1)
# sides = 1 the filter coefficients are for past values only



plot.ts(xb, main ="autoregression", col="blue")
lines(vb, col="red")
```



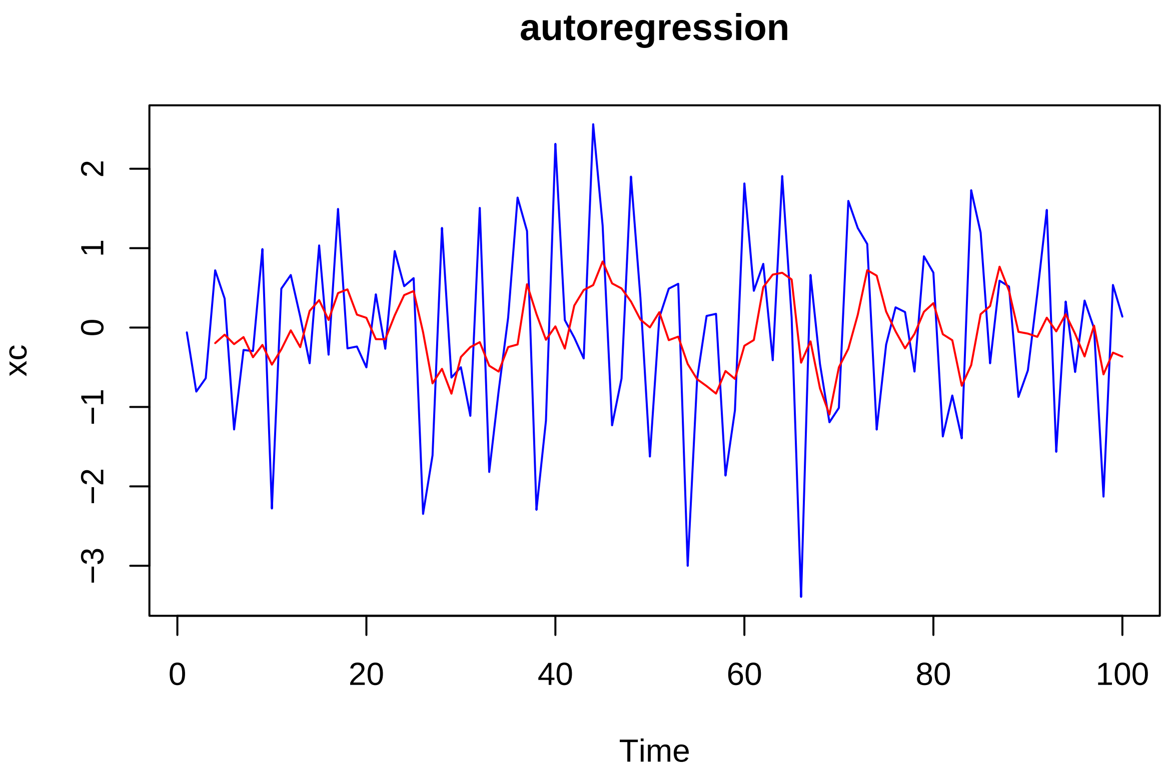
(c) Repeat (b) but with added N(0,1) noise, $x_t = cos(2\pi t/4) + w_t$

Below is a plot of the series, generated by the model above, with the filter superimposed on the series. This series has a random component, and therefore, is not deterministic like the series in (b). The series may be stationary as the mean is constant over time and the variability does not seem to increase or decrease over time. When the filter is added, it does not change the mean, but does decrease the variability in the data slightly.

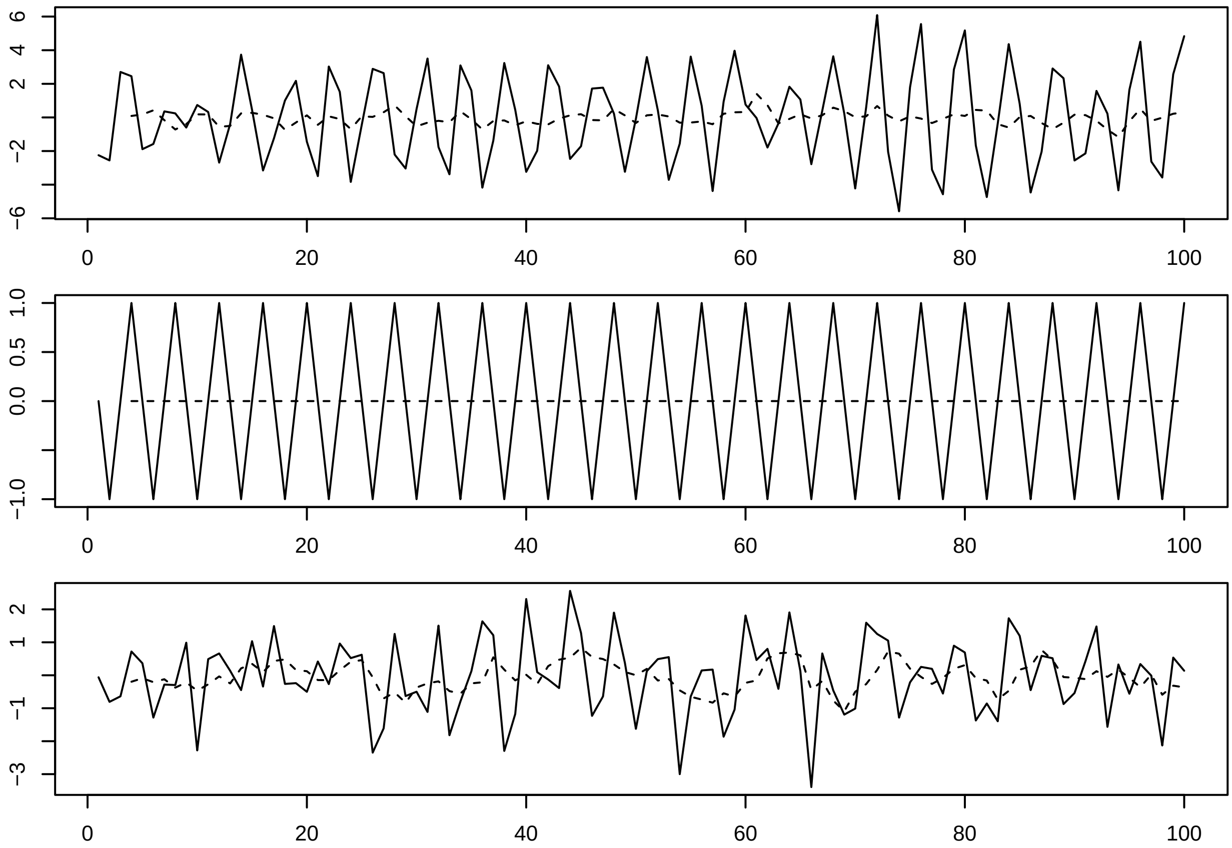
```
w <- rnorm(100, 0, 1)
xc <- cos(2*pi*1:100/4)+w


vc <- filter(xc, rep(1/4, 4), method="convolution", sides = 1)
# sides = 1 the filter coefficients are for past values only


plot.ts(xc, main ="autoregression", col="blue")
lines(vc, col="red")
```




```
par(mfcol=c(3,1))
par(mar=c(2,2,1,1))
plot(cbind(xa, va), plot.type = "single", lty=1:2)
plot(cbind(xb, vb), plot.type = "single", lty=1:2)
plot(cbind(xc, vc), plot.type = "single", lty=1:2)
```



(d) In series (a), we see an increase in the variability of the data toward the end of the series (ranging from –6 to 6), and the MA filter reduces that variability quite a bit in the second half of the series. In contrast, (c) has smaller variability to start with (ranging from –2 to 2), so the filter does not smooth very much. The MA filter smoothes the data slightly, but not as significantly as when the MA is applied to a series with much larger variability. In (b), the MA filter basically removes the variability as the series is completely deterministic.

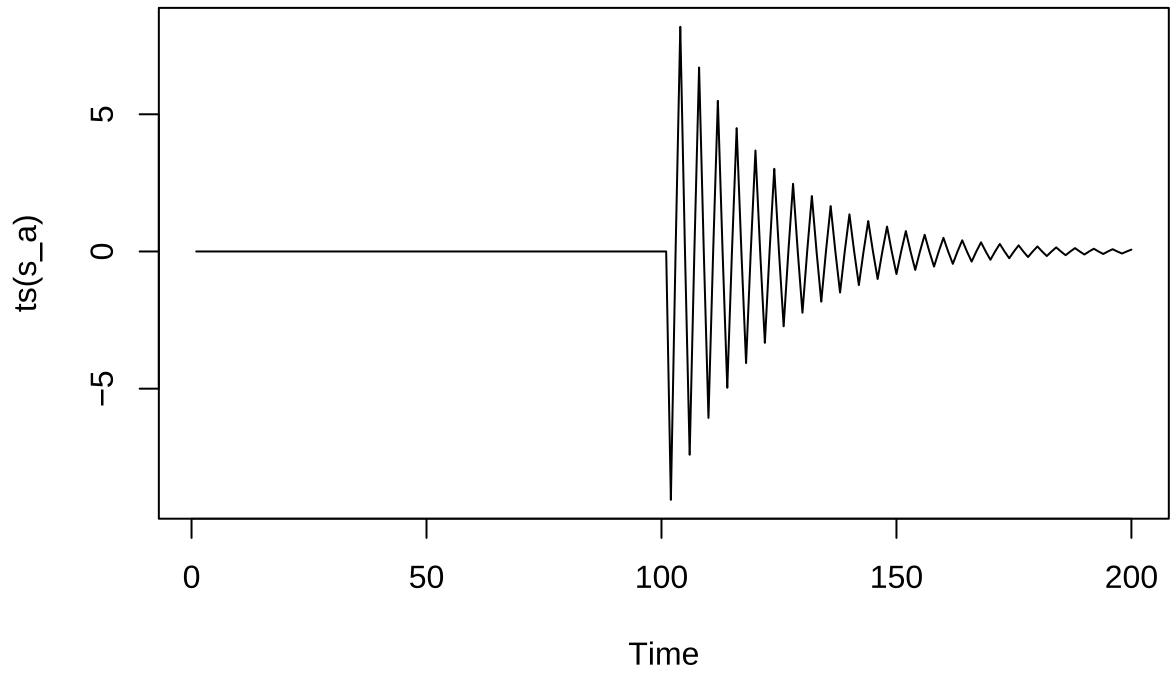
6 Problem 1.5

For the two series, x_t, in Problem 1.2 (a) and (b):

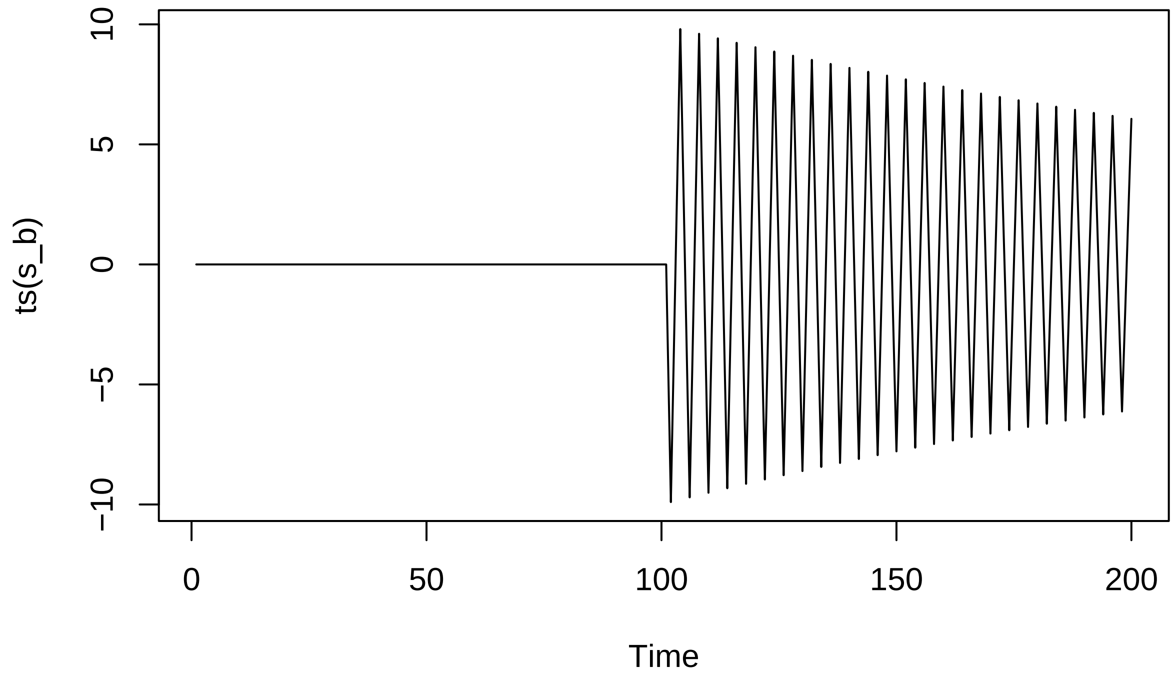
Compute and plot the mean functions $\mu_x(t)$, for t =1,…,200.

$E(x_t) = E(s_t) + E(w_t) = E(s_t) = s_t$

```
s_a <- c(rep(0,100), 10*exp(-(1:100)/20)*cos(2*pi*101:200/4))
plot(ts(s_a))
```



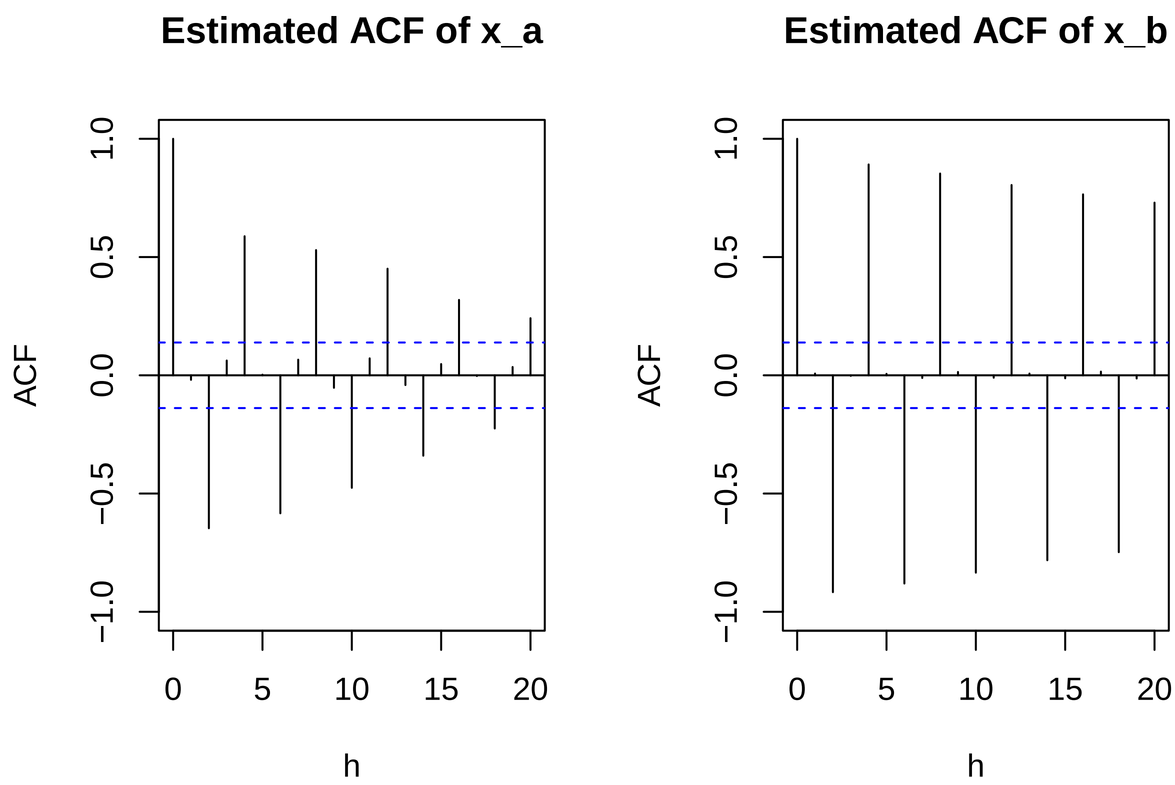

```
s_b <- c(rep(0, 100), 10*exp(-(1:100)/200))*cos(2*pi*101:200/4)
plot(ts(s_b))
```



7 Problem 1.22

Although the model in Problem 1.2(a) is not stationary (Why?), the sample ACF can be informative. For the data you generated in that problem, calculate and plot the sample ACF, and then comment.

```
par(mfrow=c(1,2))
acf(x_a, type = "correlation", lag.max = 20, ylim=c(-1,1),
    xlab="h", main= "Estimated ACF of x_a")
acf(x_b, type = "correlation", lag.max = 20, ylim=c(-1,1),
    xlab="h", main= "Estimated ACF of x_b")
```

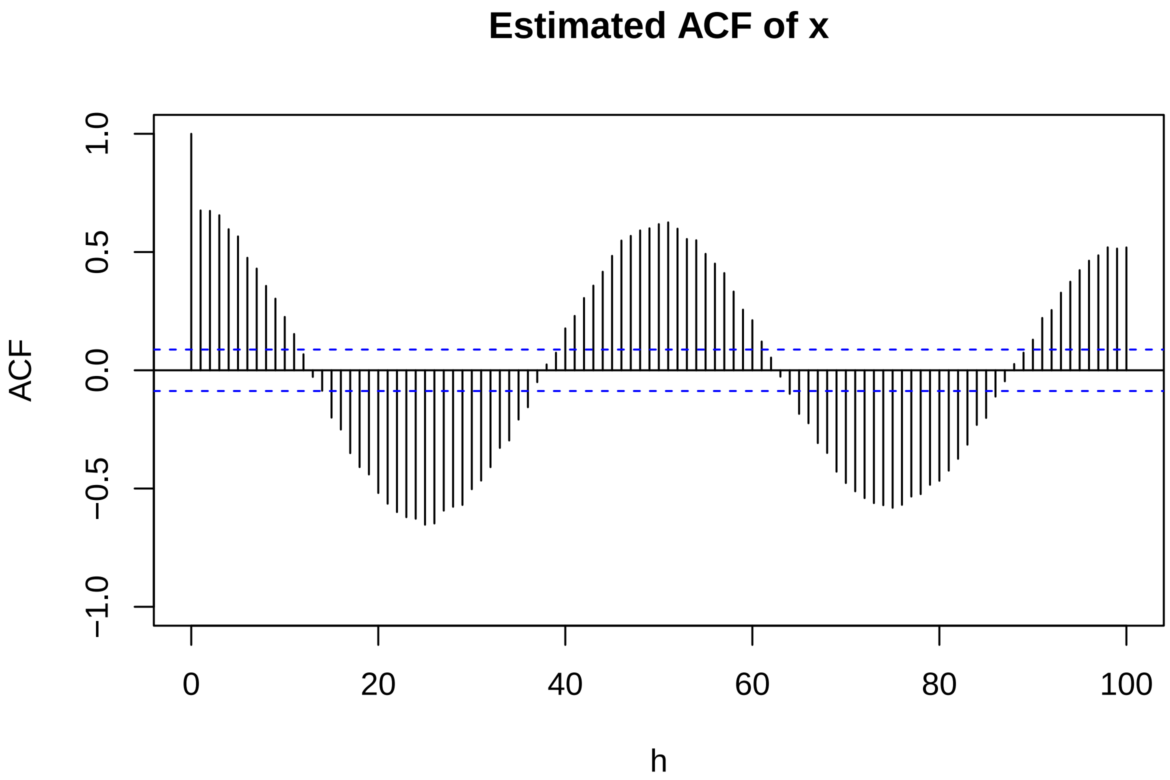


8 Problem 1.23

Simulate a series of n = 500 observations from the signal-plus-noise model presented in Example 1.12 with $\sigma_w^2 = 1$. Compute the sample ACF to lag 100 of the data you generated and comment.

$x_t = 2cos(2\pi t/50 + 0.6\pi) + w_t$

```
w <- rnorm(500, 0, 1)
x <- 2*cos(2*pi*1:500/50+.6*pi)+w
acf(x, type = "correlation", lag.max = 100, ylim=c(-1,1),
    xlab="h", main= "Estimated ACF of x")
```


Estimated ACF of x
ACF
1.0
0.5
0.0
−0.5
−1.0
0
20
40
60
80
100
h