# Empirical Macroeconomics and Finance

## Structural Vector Autoregressive Models

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Section 1

# Introduction

# VAR and Structural Econometric Models

- The reduced-form VAR model is simply a statistical description of the dynamic interrelations between $k$ different variables contained in the vector $y_t$.
- No economic prior has been imposed on the econometric models.

# Structural Vector Autoregressions

- Structural vector autoregressions can be used to address the following type of question in macroeconomics:
  - How does the economy respond to different shocks such as monetary shocks, fiscal shocks, and oil price shocks?
  - What is the contribution of the different shocks to the business cycle?
- The answers to these type of questions are key in business cycle analysis, where the purpose is to study impulses and propagations.

# Structural Vector Autoregressions

- More recently, the answers provided have been very useful in the construction and evaluation of dynamic stochastic general equilibrium (DSGE) models
- Discriminate between economic theories
    - Does RBC (or New Keynesian) fit the facts?
    - Contributions of demand vs. supply shocks (real vs. nominal shocks)?
    - Response of hour to technology shocks?

Section 2

# Motivation: A Recap

# Autoregressions

- From AR models to VAR models
- From VAR models to SVAR models

Section 3

# Structural VAR (SVAR) Models

# Structural VAR (SVAR)

- The SVAR is

$$y_t = D_0 y_t + D_1 y_{t-1} + \cdots + D_p y_{t-p} + B u_t$$

  - where $u_t \sim^{i.i.d.} (0, I)$, and $Bu_t \sim^{i.i.d.} (0, BB')$
  - $Bu_t$ is called the structural shock. B u_t = e_t

- We can rewrite it as:

$$y_t = (I - D_0)^{-1} D_1 y_{t-1} + \cdots + (I - D_0)^{-1} D_p y_{t-p} + (I - D_0)^{-1} B u_t$$

- That is,

$$y_t = \Phi_1 y_{t-1} + \cdots + \Phi_p y_{t-p} + \varepsilon_t$$

  - where $\Phi_j = (I - D_0)^{-1} D_j$, and $\varepsilon_t = (I - D_0)^{-1} B u_t$

# Identification

- As $D_j = (I - D_0)\Phi_j$, we can obtain $D_j$ from $D_0$.
- Hence, according to

$$\varepsilon_t = (I - D_0)^{-1} B u_t$$

the identification can be achieved by

$$E(\varepsilon\varepsilon') = \Sigma_\varepsilon = (I - D_0)^{-1} BB'(I - D_0)^{-1'}$$

  - $\frac{k(k-1)}{2} + k$ parameters can be identified from $\Sigma_\varepsilon$.
  - On the other hand, we need to identify $2k^2$ parameters in $D_0$ and B.
  - Thus, the difference is $\frac{k(3k-1)}{2}$.

# Standard Assumptions

(a) $B$ is diagonal. (Structural shocks are uncorrelated to each other.)

(b) Standardization: $D_{jj,0} = 0, \;\; j = 1, ...k$ or $[D_0]_{jj} = 0$

- These conditions imply

$$\underbrace{k^2 - k}_{\text{by}(a)} + \underbrace{k}_{\text{by}(b)}$$

- We still need to identify

$$\frac{k(3k-1)}{2} - (k^2 - k) - k = \frac{k(k-1)}{2}$$

# Identification

- How to obtain $\frac{k(k-1)}{2}$ conditions?
  - Short-run restriction
    - ◇ Recursive (semi-structural)
    - ◇ Economic theory (structural)
  - Long-run restriction
- See Sims (1980), Bernanke (1986), and Blanchard and Quah (1989).