Financial Econometrics

Difference-in-Difference Design

Tim C.C. Hung 洪志清

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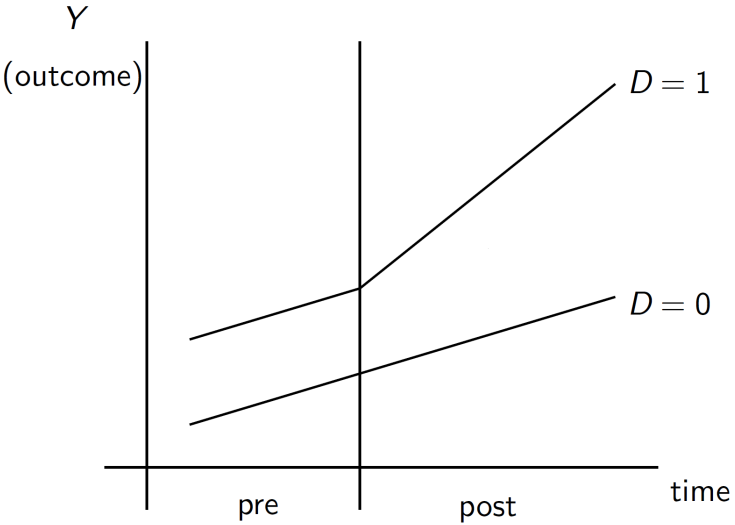
- If we can observe **group-level** outcomes several times (at least before and after treatment).

- Assume **in the absence of treatment**, outcomes of treatment and control group **move in a parallel way**.

- Then, we can construct the **counterfactual trend in outcomes of treatment group by** using:
  - Trend in outcomes of control group!

- Comparing observed trend with counterfactual trend in outcome of treatment group, we can get causal effect of treatment.

Main Idea of Difference-in-Differences



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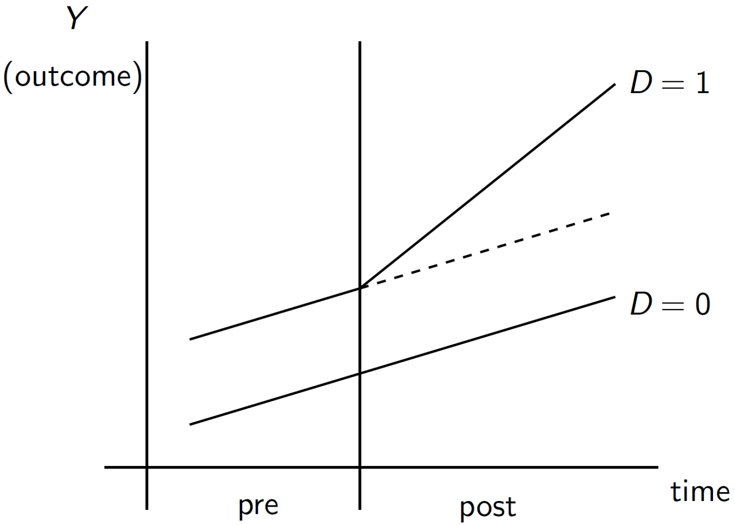

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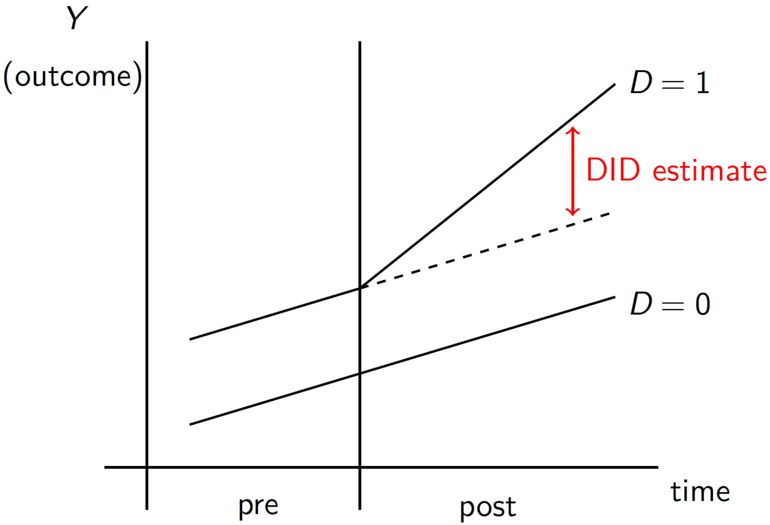

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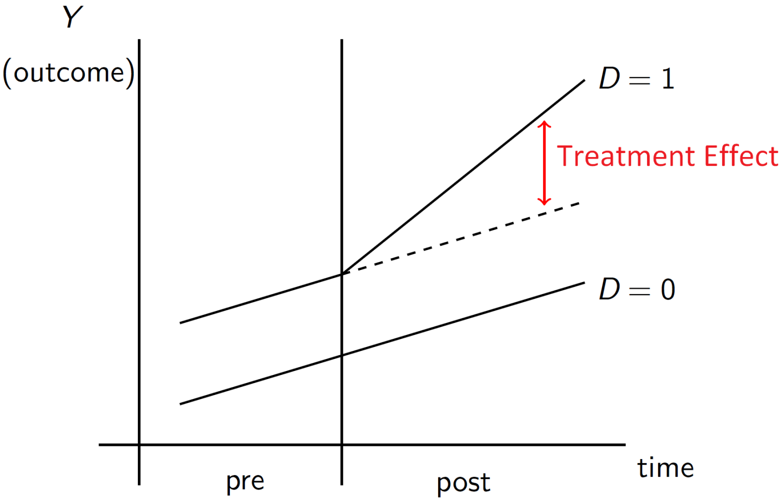

Example for Difference-in-Differences

- David Card and Alan Krueger (1994) "**Minimum Wages and Employment: A Case Study of the Fast-Food Industry in New Jersey and Pennsylvania**", *AER*
  - They want to estimate the causal effect of raising minimum wage on employment of low-skilled workers.

Card and Krueger (1994)

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- What is the effect of increasing the minimum wage on employment?

- Minimum wage is effective only in certain jobs:
    - Low-skilled jobs

- How much does an increase in the minimum wage reduce demand for low-skilled workers?
    - In a competitive labor market, increases in the minimum wage would move up a downward-sloping labor demand curve.

    - Employment would fall.

Card and Krueger (1994)

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- Card and Krueger (1994) analyse the effect of a minimum wage increase in New Jersey (NJ) using a DID methodology.
- 1992 Feb., NJ increased the state minimum wage from \$4.25 to \$5.05.
- Pennsylvania (PA)'s minimum wage stayed at \$4.25.

- They surveyed about 400 fast food stores both in NJ and in PA both before and after the minimum wage increase in NJ.

Card and Krueger (1994)

- Two Groups:
  - Treatment Group: NJ
  - Control Group: PA
- Two Periods:
  - Pre-treatment period: February 1992
  - Post-treatment period: November 1992
- Let Y_{st} denote the average employment in state s at time t.

Card and Krueger (1994)

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- To estimate the effect of minimum wage on employment in NJ, we would like to know the following counterfactual:
  - **In absence of raising minimum wage to \$5.05**, what the average employment level in NJ would be ?
- DID method suggests us construct the counterfactual employment in NJ by using:
  - Average employment level in NJ before reform +
  - The trend in average employment level in PA (control group)

$$Y_{NJ,Feb} + (Y_{PA,Nov} - Y_{PA,Feb})$$

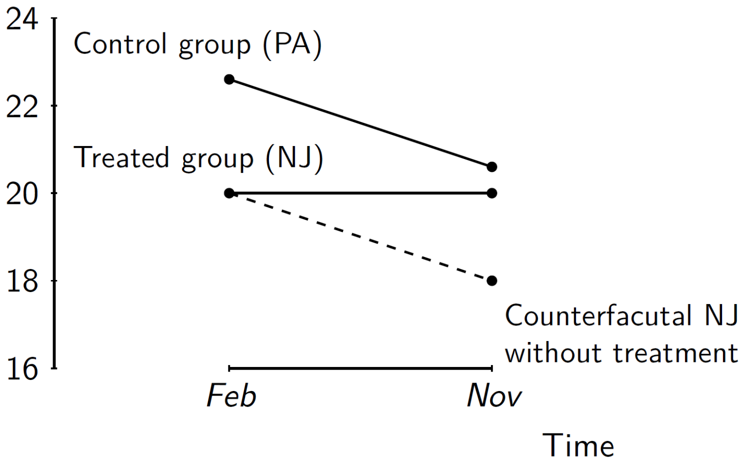
- We can identify the effect of minimum wage on employment in NJ by taking difference in **realized employment and counterfactual employment** in NJ:

$$\begin{aligned}\alpha_{DID} &= Y_{NJ,Nov} - [Y_{NJ,Feb} + (Y_{PA,Nov} - Y_{PA,Feb})] \\ &= (Y_{NJ,Nov} - Y_{NJ,Feb}) - (Y_{PA,Nov} - Y_{PA,Feb})\end{aligned}$$

- If PA is a good control group.

- The trend in employment rate of PA should absorb any other changes in employment that are unrelated to increase minimum wage.

Employment


24
22
20
18
16
Control group (PA)
Treated group (NJ)
Counterfacutal NJ
without treatment
Feb
Nov
Time

$$
\begin{aligned}
\alpha_{DID} &= (Y_{NJ,Nov} - Y_{NJ,Feb}) - (Y_{PA,Nov} - Y_{PA,Feb}) \\
&= (21.03 - 20.44) - (21.17 - 23.33) \\
&= 0.59 - (-2.16) = 2.76
\end{aligned}
$$

- Instead of comparing the employment of NJ in February (before reform) and November (after reform),
- DID suggests we need to adjust for change (trend) in labor demand when there was no increase in minimum wage.

$$\begin{aligned}\alpha_{DID} &= (Y_{NJ,Nov} - Y_{PA,Nov}) - (Y_{NJ,Feb} - Y_{PA,Feb}) \\ &= (21.03 - 21.17) - (20.44 - 23.33) \\ &= (-0.14) - (-2.89) = 2.76\end{aligned}$$

- Instead of comparing the employment of NJ and PA after reform,
- DID suggests we need to adjust for the fact that in the pre-reform period, NJ had less employment than PA.

Card and Krueger (1994)

	Store by State		
			Difference
	PA	NJ	NJ - PA
Variables	(1)	(2)	(3)
1. Mean Employment at Feburary 1992	23.33	20.44	-2.89
	(1.35)	(0.51)	(1.44)
2. Mean Employment at November 1992	21.17	21.03	-0.14
	(0.94)	(0.52)	(1.07)
3. Change in Mean Employment between Feb. and Nov.	-2.16	0.59	2.76
	(1.25)	(0.54)	(1.44)

- Surprisingly, employment $\nearrow$ in NJ relative to PA after min. wage $\nearrow$.
- **Extensions**: 白經濟
  調漲基本工資—少數贏家或全民勝利？
  基本工資害死你？

DID and Potential Outcomes

- Basic setup: two time periods, two groups
- Two Periods:
  - In period $t = 0$: neither group is treated (pre-period)
  - In period $t = 1$: one of the groups is treated (post-period)
- Two Groups:
  - $D_i = 1$: those that are treated at $t = 1$ (treatment group)
  - $D_i = 0$: those that are always untreated (control group)

DID and Potential Outcomes

- Potential Outcomes:
  - Y_{it}^1: the potential outcome for i if she would receive treatment at time t.
  - Y_{it}^0: the potential outcome for i if she would NOT receive treatment.
- Realized Outcomes:
  - Y_{it} is the observed outcome for unit i at time t
  - Observed outcomes Y_{it} are realized as

$$Y_{it} = Y_{it}^0(1 - D_i) + Y_{it}^1 D_i$$

  - Observed outcomes at $t = 0$:

$$Y_{i0} = Y_{i0}^0$$

  - Observed outcomes at $t = 1$:

$$Y_{i1} = Y_{i1}^0(1 - D_i) + Y_{i1}^1 D_i$$

Identification Results for DID

- Our main interest is average treatment effect on treated (ATT):
- DID can help us identify ATT

$$\alpha_{ATT} = E[Y_{i1}^1 - Y_{i1}^0 | D_i = 1]$$

- Missing data problem: $E[Y_{i1}^0 | D_i = 1]$ is unknown!

Identification Results for DID

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Assumption (Common Trend Assumption)

$$\begin{aligned} E[Y_{i1}^0 - Y_{i0}^0 | D_i = 1] &= E[Y_{i1}^0 - Y_{i0}^0 | D_i = 0] \\ &= E[Y_{i1} - Y_{i0} | D_i = 0] \end{aligned}$$

- The treatment group and control group would have exhibited the same trend in the absence of the treatment.

- We can use common trend assumption to construct a counterfactual for treatment group at $t = 1$.

$$\begin{aligned} E[Y_{i1}^0 | D_i = 1] &= E[Y_{i0}^0 | D_i = 1] + E[Y_{i1}^0 - Y_{i0}^0 | D_i = 0] \\ &= E[Y_{i0} | D_i = 1] + E[Y_{i1}^0 - Y_{i0} | D_i = 0] \end{aligned}$$

- We can use oberved outcomes to represent unobserved $E[Y_{i1}^0 | D_i = 1]$.

Identification Results for DID

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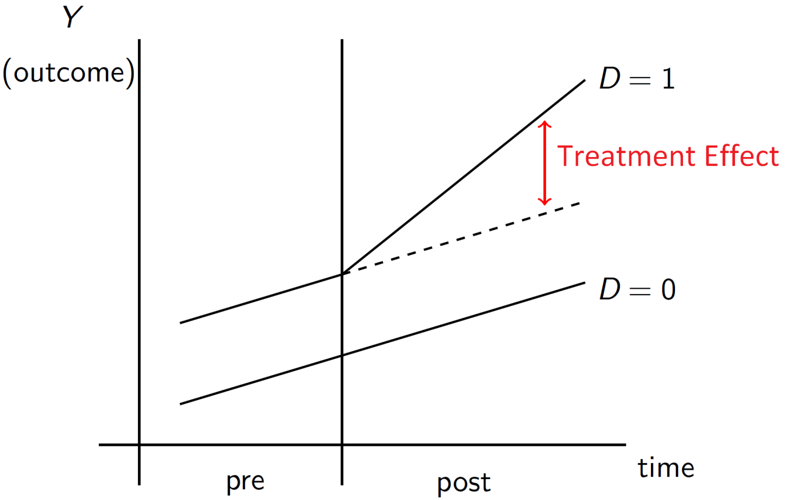
- Apply common trend assumptions:

$$
\begin{aligned}
\alpha_{ATT} &= E[Y_{i1}^1 - Y_{i1}^0 | D_i = 1] \\
&= E[Y_{i1}^1 | D_i = 1] - E[Y_{i1}^0 | D_i = 1] \\
&= E[Y_{i1}^1 | D_i = 1] - E[Y_{i0}^0 | D_i = 1] - E[Y_{i1}^0 - Y_{i0}^0 | D_i = 0] \\
&= E[Y_{i1}^1 - Y_{i0}^0 | D_i = 1] - E[Y_{i1}^0 - Y_{i0}^0 | D_i = 0] \\
&= E[Y_{i1} - Y_{i0} | D_i = 1] - E[Y_{i1} - Y_{i0} | D_i = 0] = \alpha_{DID}
\end{aligned}
$$

- The **average treatment effect on treated (ATT)** can be identified by difference in trend of outcome between treatment and control groups.

Main Idea of Difference-in-Differences



DID Estimation

- We can estimate the DID estimator in a regression framework
- Advantages:
  - It is easy to calculate standard errors.
  - We can control for other variables which may reduce the residual variance (lead to smaller standard errors).
  - It is easy to include multiple periods.
  - We can study treatments with different treatment intensity (e.g. varying increases in the minimum wage for different states): continous DID.

DID Estimation

- Basic case: two groups and two periods.

- To implement DID method in a regression framework, we estimate:

$$Y_{ist} = \mu + \gamma D_s + \delta Post_t + \alpha_{DID} D_s \cdot Post_t + \epsilon_{ist}$$

  - D_s is a dummy indicating treatment group.
  - $Post_t$ is a dummy indicating post-treatment period.
  - γ captures differences across groups that are constant over time.
  - δ captures differences over time that are common to all groups.
  - α_{DID} is the coefficient of interest (the causal effect of treatment).

DID Estimation

$$Y_{ist} = \mu + \gamma D_s + \delta Post_t + \alpha_{DID} D_s \cdot Post_t + \epsilon_{ist}$$

- If $E[\epsilon_{ist}|D_s, Post_t] = 0$, we can show that
  - **Pre-treatment mean of outcome for control group:** $E[\epsilon_{ist}|D_s = 0, Post_t = 0] = \mu$
  - **Post-treatment mean of outcome for control group:** $E[\epsilon_{ist}|D_s = 0, Post_t = 1] = \mu + \delta$
  - **Pre-treatment mean of outcome for treatment group:** $E[\epsilon_{ist}|D_s = 1, Post_t = 0] = \mu + \gamma$
  - **Post-treatment mean of outcome for treatment group:** $E[\epsilon_{ist}|D_s = 1, Post_t = 1] = \mu + \gamma + \delta + \alpha_{DID}$

DID Estimation

	Pre	Post	Pre/Post Difference
Control Group	μ	$\mu + \delta$	δ
Treatment Group	$\mu + \gamma$	$\mu + \gamma + \delta + \alpha_{DID}$	$\delta + \alpha_{DID}$
DID			α_{DID}

DID Estimation

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- α_{DID} can represent treatment effect identified by DID method:

$$\begin{aligned}
\alpha_{DID} &= \{E[\epsilon_{ist}|D_s = 1, Post_t = 1] - E[\epsilon_{ist}|D_s = 1, Post_t = 0]\} \\
&\quad - \{E[\epsilon_{ist}|D_s = 0, Post_t = 1] - E[\epsilon_{ist}|D_s = 0, Post_t = 0]\} \\
&= \{(\mu + \gamma + \delta + \alpha_{DID}) - (\mu + \gamma)\} - \{(\mu + \delta) - \mu\}
\end{aligned}$$

Test Common Trend Assumption

- The key assumption for any DID strategy is common trend assumption.
- The outcome in treatment and control group would follow **the same time trend in the absence of the treatment**.
  - This does not mean they have the same mean of the outcome!
  - Common trend assumption is difficult to verify.
  - We can use pre-treatment data to show that the trends are the same: 1. Graphical evidence; 2. DID event-study design
- We can include leads and lags into the DID design:
  1. Examine common trend assumption.
  2. Analyze whether the treatment effect changes over time.
- It is so-called DID event-study design.

Example of Tests on Parallel Trends

- A recent debate of massive lockdowns on future economic outputs.
- Correia, Luck, and Verner (2020): regions with stricter lockdowns experience stronger economic bounce backs.
- Lilley, Lilley, and Rinaldi (2020): parallel trend does not exist!

Figure 4: Effect of NPI intensity on manufacturing employment and output