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Financial Econometrics

Potential Outcomes Framework

Tim C.C. Hung 洪志清

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Causal Effect and Potential Outcomes

- LSE motto: *Rerum cognoscere causas*
  $\rightarrow$ *to know the causes of things*

- Estimating Causal effect of treatment is a challenging task.
  - Because **we can NOT observe counterfactual outcomes** if one had chosen different treatments.

- In order to obtain causal effect, we need to compare **observed outcomes** with **counterfactual outcomes**.

- The **potential outcomes framework** provides a way to quantify causal effects.

Causal Effect and Potential Outcomes

Unobservable Counterfactuals

Causal Effect and Potential Outcomes

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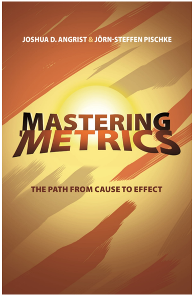
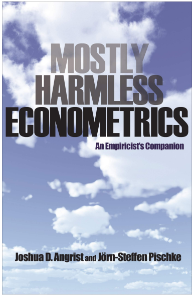
- Textbook resources:





by Joahua Angrist (MIT) and Jorn-steffen Pischke (LSE)

- Slides revised from 楊子霆 (IEAS)

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- Treatment: An intervention, whose effect(s) we wish to assess relative to some other (non-)interventions.

- D_i: a dummy that indicates whether individual i receive treatment.

$$D_i = \begin{cases} 1, & \text{if individual } i \text{ received the treatment} \\ 0, & \text{otherwise} \end{cases}$$

- Examples:
    - Attend graduate school or not
    - Have health insurance or not
    - Win a lottery or not
    - Increase corporate tax rate or not
    - Democracy v.s. Dictatorship

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- D_i can be a multiple valued (continuous) variable.

$$D_i = s$$

- Examples:
  - Schooling years
  - Number of children
  - Number of polices
  - Number of advertisements
  - Money supply
  - Income tax rate

- In the following slides, we assume treatment variable D_i is a dummy.

Potential Outcomes Framework

- A potential outcome is the outcome that would be realized if the individual received a specific value of the treatment.
- Suppose there are two treatments for each individual:
  - $D_i = 1$
  - $D_i = 0$
- Thus, each individual i has two potnetial outcomes and one for each value of the treatment:
  - Y_i^1: Potential outcome for an individual i if getting treatment
  - Y_i^0: Potential outcome for an individual i if NOT getting treatment
- Example:
  - Annual earnings if attending graduate school
  - Annual earnings if NOT attending graduate school

Causal Effect

- Causal Effect: the comparisons between the potential outcomes under each treatment.
- The differences between observed (potential) outcome and counterfactual (potential) outcome!
- Individual Treatment Effect (ITE):

$$\tau_i = Y_i^1 - Y_i^0$$

  - Also called Individual Causal Effect
  - The difference between an individual i's outcome under treatment v.s. without treatment
- Example:
  - The difference in individual i's earnings if he/she attends graduate school v.s. not attending graduate school.
- Almost always unidentified without strong assumptions.

Individual Causal Effect

- Imagine a population with 4 people.

i	Y_i^1	Y_i^0	Y_i	D_i	$Y_i^1 - Y_i^0$
David	3	?	3	1	?
Tina	2	?	2	1	?
Mary	?	1	1	0	?
Bill	?	1	1	0	?

- We want to evaluate the effect of attending graduate school on the annual earnings.
  - Y_i: observed annual earnings for individual i
  - D_i: Attending graduate school, $D_i = 1$; otherwise, $D_i = 0$
  - Y_i^1: (Potential) annual earnings if individual i attend graduate school
  - Y_i^0: (Potential) annual earnings if individual i do not attend grad school

Individual Causal Effect

- What is Individual causal effect of attending graduate school for David?
  - We only observe earnings for David who attended graduate school
  - Only observe Y^1
- What is Individual causal effect of attending graduate school for Bill?
  - We only observe earnings for Bill who did not attend graduate school
  - Only observe Y^0
- Suppose **we can observe counterfactual outcomes**.

i	Y_i^1	Y_i^0	Y_i	D_i	$Y_i^1 - Y_i^0$
David	3	2	3	1	1
Tina	2	1	2	1	1
Mary	1	1	1	0	0
Bill	1	1	1	0	0

- The ITE for David: $\alpha_{David} = 1$
- The ITE for Bill: $\alpha_{Bill} = 0$

Causal Effect for General Population

- People might be more interested in the **causal effect for general population**
- We usually cannot rule out that the ITE differs across individuals. $\rightarrow$ effect heterogeneity!
- Thus, ITE might not represent causal effect for general population.
- Understand the treatment effect (causal effect) for general population:
  - Estimate the **population average of the individual treatment effects**.
- We usually use $E[Y_i]$ to denote **population average** if Y_i
- Suppose we have a population with N individuals:

$$E[Y_i] = \frac{1}{N}\sum_{i=1}^{N} Y_i$$

Causal Effect for General Population

- Average Treatment Effect (ATE):

$$\alpha_{ATE} = E[\tau_i] = E[Y_i^1 - Y_i^0] = \frac{1}{N}\sum_{i=1}^{N}[Y_i^1 - Y_i^0]$$

- Average of ITEs over the population.
  - E.g. Average effect of attending graduate school on annual earnings for the whole population.
    $\rightarrow$ Average difference between the earnings of the same individuals if they attend graduate schools v.s. if not attending graduate schools!

- We'll spend a lot time trying to identify/estimate ATE!

Average Treatment Effect (ATE)

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- Missing data problem also arises when we estimate ATE.

i	Y_i^1	Y_i^0	Y_i	D_i	$Y_i^1 - Y_i^0$
David	3	?	3	1	?
Tina	2	?	2	1	?
Mary	?	1	1	0	?
Bill	?	1	1	0	?
$E[Y_i^1]$	?				
$E[Y_i^0]$		?			
$E[Y_i^1 - Y_i^0]$					?

- What is the effect of attending graduate school on average annual earnings of the whole population (ATE)?

- $\alpha_{ATE} = E[Y_i^1 - Y_i^0] = ?$

Average Treatment Effect (ATE)

- Suppose **we can observe counterfactual outcomes**.

i	Y_i^1	Y_i^0	Y_i	D_i	$Y_i^1 - Y_i^0$
David	3	2	3	1	1
Tina	2	1	2	1	1
Mary	1	1	1	0	0
Bill	1	1	1	0	0
$E[Y_i^1]$	1.75				
$E[Y_i^0]$		1.25			
$E[Y_i^1 - Y_i^0]$					0.5

- What is the effect of attending graduate school on average annual earnings of the whole population (ATE)?

- $\alpha_{ATE} = \frac{1+1+0+0}{4} = 0.5$

Causal Effect for a Specific Sub-population

- Conditional Average Treatment Effect (CATE):

$$\alpha_{CATE} = E[\tau_i | X_i = f] = E[Y_i^1 - Y_i^0 | X_i = f] = \frac{1}{N_f} \sum_{i:X_i=f} [Y_i^1 - Y_i^0]$$

  - N_f is the number of units in the sub-population.
    E.g. Average effect of attending graduate school on annual earnings for **females**.

- More formally, the average difference between the earnings of **females** if they attend graduate schools v.s. if not attending graduate schools.

Average Treatment Effect on Treated (ATT)

- Average Treatment Effect on the Treated (ATT):

$$\alpha_{ATT} = E[\tau_i | D_i = 1] = E[Y_i^1 - Y_i^0 | D_i = 1] = \frac{1}{N_1} \sum_{i:D_i=1} [Y_i^1 - Y_i^0]$$

  - $N_1 = \sum_i D_i$
  - Note that ATT is a special case of CATE.
  - Average of ITEs over the treated population.
    $\rightarrow$ Average effect of attending graduate school on annual earnings for those attending graduate school ($D_i = 1$)
- More formally, the average difference between the earnings of those attending graduate schools v.s. earnings if they had not attended graduate schools.
- We'll also spend a lot time trying to identify/estimate ATT!

- Missing data problem also arises when we estimate ATT.

i	Y_i^1	Y_i^0	Y_i	D_i	$Y_i^1 - Y_i^0$
David	3	?	3	1	?
Tina	2	?	2	1	?
Mary	?	1	1	0	?
Bill	?	1	1	0	?
$E[Y_i^1 \mid D_i = 1]$	2.5				
$E[Y_i^0 \mid D_i = 1]$		?			
$E[Y_i^1 - Y_i^0 \mid D_i = 1]$					?

- What is the effect of attending graduate school on average annual earnings for those who choose to attend graduate school (ATT)?

- $\alpha_{ATT} = E[Y_i^1 - Y_i^0 \mid D_i = 1] = ?$

Average Treatment Effect on Treated (ATT)

- Suppose **we can observe counterfactual outcomes**.

i	Y_i^1	Y_i^0	Y_i	D_i	$Y_i^1 - Y_i^0$
David	3	2	3	1	1
Tina	2	1	2	1	1
Mary	1	1	1	0	0
Bill	1	1	1	0	0
$E[Y_i^1 \mid D_i = 1]$	2.5				
$E[Y_i^0 \mid D_i = 1]$		1.5			
$E[Y_i^1 - Y_i^0 \mid D_i = 1]$					1

- What is the effect of attending graduate school on average annual earnings for those who choose to attend graduate school (ATT)?

- $\alpha_{ATT} = \frac{1+1}{2} = 1$

Fundamental Problem of Causal Inference

- We can never **directly observe** causal effects (ITE, ATE, or ATT).

- $\because$ we can't observe both potential outcomes (Y_i^1, Y_i^0) for any individual.

- For someone receiving the treatment $(D_i = 1)$,
  - Y_i^1 is observed
  - But Y_i^0 is the **unobserved** counterfactual outcome.
    $\rightarrow$ This represents what would have happened to an individual i if assigned to the control group.

- We need to compare **potential outcomes**, but we only have **observed outcomes**.

- Causal inference is a **missing data problem**.

Stable Unit Treatment Value Assumption

Assumption

Observed outcomes are realized as:

$$Y_i = Y_i^1 D_i + Y_i^0 (1 - D_i)$$

- Implies that observed outcomes for an individual i are **unaffected** by the treatment status of other individual j.
- Individual i's observed outcomes are only affected by his/her own treatment.
- Rules out possible treatment effect from other individuals (spillover effect/externality).

Selection Bias

- **Causality is defined by potential outcomes**, not by realized (observed) outcomes.

- In fact, we can **NOT** observe all potential outcomes.

- By using observed data, we can only establish association (correlation).

- That is, the observed difference in average outcome between those getting treatment and those not getting treatment.

$$\alpha_{corr} = \underbrace{E[Y_i|D_i = 1] - E[Y_i|D_i = 0]}_{\text{observed difference in average outcome}}$$

Selection Bias

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- Note that the values of **observed outcomes** Y depend on either treatment status D or the value of potential outcomes (Y^1, Y^0).

$$Y_i = Y_i^1 D_i + Y_i^0 (1 - D_i) \text{ or}$$

$$Y_i = \begin{cases} Y_i^1, & \text{if } D_i = 1 \\ Y_i^0, & \text{if } D_i = 0 \end{cases}$$

- If we find two individuals (groups) have different **observed outcomes** Y, it could be due to:
  1. They receive different treatment D (causal effect of treatment).
     $\rightarrow D_i \neq D_j$
  2. Given they receive the same treatment, their value of potential outcomes (Y^1, Y^0) are different (selection bias).
     $\rightarrow$ Both receive treatment, $D = 1$, but $Y_i^1 \neq Y_j^1$
     $\rightarrow$ Both do not receive treatment, $D = 0$, but $Y_i^0 \neq Y_j^0$

Selection Bias

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- The observed association usually mix up causal effect (ATT) and selection bias.

$$
\begin{aligned}
\alpha_{corr} &= \underbrace{E[Y_i|D_i=1]-E[Y_i|D_i=0]}_{\text{observed difference in average outcome}} \\
&= E[Y_i^1|D_i=1] - E[Y_i^0|D_i=1] + E[Y_i^0|D_i=1] - E[Y_i^0|D_i=0] \\
&= \underbrace{E[Y_i^1 - Y_i^0|D_i=1]}_{\text{ATT}} + \underbrace{E[Y_i^0|D_i=1] - E[Y_i^0|D_i=0]}_{\text{Selection Bias}}
\end{aligned}
$$

- **Selection Bias** implies:
  - The value of potential outcomes for treatment and control groups are different **even if both groups receive the same treatment** (e.g. Both are Y_i^0).
  - This means two groups could be quite different in other dimensions: **other things are NOT equal!**

Observed Association

- Observed association is neither necessary nor sufficient for causality.

- In graduate school example, the observed difference in average earnings between those attending graduate school v.s. those not attending:

$$\alpha_{corr} = E[Y_i|D_i = 1] - E[Y_i|D_i = 0] = 1.5$$

i	Y_i^1	Y_i^0	Y_i	D_i	$Y_i^1 - Y_i^0$
David	3	?	3	1	?
Tina	2	?	2	1	?
Mary	?	1	1	0	?
Bill	?	1	1	0	?
$E[Y_i\|D_i = 1]$			2.5		
$E[Y_i\|D_i = 0]$			1		

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- But we are interested in the causal effect (ATT).

$$\alpha_{ATT} = E[Y_i^1 | D_i = 1] - E[Y_i^0 | D_i = 1] = 1$$

  - suppose we can observe the counterfactual outcomes

i	Y_i^1	Y_i^0	Y_i	D_i	$Y_i^1 - Y_i^0$
David	3	2	3	1	1
Tina	2	1	2	1	1
Mary	1	1	1	0	0
Bill	1	1	1	0	0
$E[Y_i^1 \| D_i = 1]$	2.5				
$E[Y_i^0 \| D_i = 1]$		1.5			

Selection Bias

$$\underbrace{E[Y_i|D_i = 1] - E[Y_i|D_i = 0]}_{\text{observed difference in average outcome (1.5)}}$$

$$= \underbrace{E[Y_i^1 - Y_i^0|D_i = 1]}_{\text{ATT (1)}} + \underbrace{E[Y_i^0|D_i = 1] - E[Y_i^0|D_i = 0]}_{\text{Selection Bias}}$$

- $\alpha_{corr} \neq \alpha_{ATT}$

Selection Bias

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- Selection Bias $= E[Y_i^0|D_i = 1] - E[Y_i^0|D_i = 0] = 0.5$

i	Y_i^1	Y_i^0	Y_i	D_i	$Y_i^1 - Y_i^0$
David	3	2	3	1	1
Tina	2	1	2	1	1
Mary	1	1	1	0	0
Bill	1	1	1	0	0
$E[Y_i^0\|D_i = 1]$		1.5			
$E[Y_i^0\|D_i = 0]$		1			

- Here, selection bias is positive (0.5 million NT$).

- Those who attend graduate school could be more intelligent so they can earn more even if they did not attend graduate school.

Causal Effect and Identification Strategy

- **Identification Strategy** tells us what we can learn about a **causal effect** from the **observed data**.
- The goal of identification strategy is to eliminate the **selection bias**.
- Identification depends on **assumptions**, not on estimation strategies!
- If an effect is not identified, no estimation method will recover it.
- **"What's your identification strategy?"** = what are the assumptions that allow you to claim you've estimated a causal effect?

Regression and Potential Outcome

- Assume **potential outcomes** are determined by the following equations:

$$Y_i^1 = \delta + \alpha + \beta X_i + \epsilon_i$$
$$Y_i^0 = \delta + \beta X_i + \epsilon_i$$

- Assume $E[\epsilon_i | X_i] = 0$

- α is the causal effect of the treatment:
$\rightarrow$ constant effects: $Y_i^1 - Y_i^0 = \alpha$ for all individuals.

- **Observed outcome** can be represented by the following equation:

$$\begin{aligned} Y_i &= Y_i^1 D_i + Y_i^0 (1 - D_i) \\ &= Y_i^0 + (Y_i^1 - Y_i^0) D_i \\ &= \delta + \alpha D_i + \beta X_i + \epsilon_i \end{aligned}$$

Identification Results for Regression

- What if we don't include an observed characteristics X in the regression:

$$Y_i = \delta + \alpha D_i + u_i, \text{ and}$$

$$u_i = \beta X_i + \epsilon_i$$

- The causal effect from the above regression $\hat{\alpha}$:

$$\begin{aligned}
\hat{\alpha} = & \underbrace{E[Y_i|D_i = 1] - E[Y_i|D_i = 0]}_{\text{observed difference in average outcome}} \\
&= \underbrace{E[Y_i^1 - Y_i^0|D_i = 1]}_{\text{ATT}} + \underbrace{E[Y_i^0|D_i = 1] - E[Y_i^0|D_i = 0]}_{\text{Selection Bias}} \\
&= \alpha + E[u_i|D_i = 1] - E[u_i|D_i = 0] \\
&= \alpha + \underbrace{\beta E[X_i|D_i = 1] - \beta E[X_i|D_i = 0]}_{\text{Selection Bias}}
\end{aligned}$$

Identification Results for Regression

- To identify causal effect of D, we need to include an observed characteristics X into the regression model:

$$Y_i = \delta + \alpha_{reg} D_i + \beta X_i + \epsilon_i$$

- Based on CIA, including key observed covariates into regression can help us eliminate selection bias.

- Therefore, we can get causal effect of D by running the above regression:

$$\alpha_{reg}(X) = \underbrace{E[Y_i|X_i, D_i = 1] - E[Y_i|X_i, D_i = 0]}_{\text{observed difference in average outcome at given } X_i}$$

Conditional Independence Assumption (CIA)

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Assumption (Conditional Independence Assumption)

$$(Y_i^1, Y_i^0) \perp D_i | X_i$$

- CIA asserts that conditional on observable characteristics X, potential outcomes are independent of treatment assigned D.
- In other words, observed covariates X can fully explain the difference in potential outcome between treatment and control groups.
- CIA ensures:
  - $E[Y_i^0 | X_i, D_i = 1] = E[Y_i^0 | X_i, D_i = 0]$
  - $E[Y_i^1 | X_i, D_i = 1] = E[Y_i^1 | X_i, D_i = 0]$

Identification Results for Regression

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$$
\begin{aligned}
\alpha_{reg}(X) = \; & \underbrace{E[Y_i|X_i, D_i = 1] - E[Y_i|X_i, D_i = 0]}_{\text{observed difference in average outcome at given } X_i} \\
& = E[Y_i^1|X_i, D_i = 1] - E[Y_i^0|X_i, D_i = 1] \\
& + E[Y_i^0|X_i, D_i = 1] - E[Y_i^0|X_i, D_i = 0] \\
& = \underbrace{E[Y_i^1 - Y_i^0|X_i, D_i = 1]}_{\text{Causal Effect (CATT)}} + \underbrace{E[Y_i^0|X_i, D_i = 1] - E[Y_i^0|X_i, D_i = 0]}_{\text{Selection Bias}} \\
& = \underbrace{E[Y_i^1 - Y_i^0|X_i, D_i = 1]}_{\text{Causal Effect (CATT)}} \\
& + \underbrace{\beta E[X_i|X_i, D_i = 1] - \beta E[X_i|X_i, D_i = 0]}_{\text{Selection Bias}} \\
& = \underbrace{E[Y_i^1 - Y_i^0|X_i, D_i = 1]}_{\text{Causal Effect (CATT)}} + \underbrace{0}_{\text{Selection Bias}} = \underbrace{E[Y_i^1 - Y_i^0|X_i]}_{\text{Causal Effect (CATE)}}
\end{aligned}
$$

Identification Results for Regression

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- Note that there are as many causal effects (CATE or CATT) as the number of value in X_i
- People might find it useful to boil a set of estimates down to a single summary measure.
  - e.g. population average treatment effect
- Again, applying the law of iterated expectations (LIE), we can identify **average treatment effect** (ATE) or **average treatment effect on the treated** (ATT).
  - Take average of CATE or CATT over all subgroups (all possible X-values).
- Lastly, think again, what are the asumptions we have made so far?