Financial Econometrics

Instrumental Variables

Tim C.C. Hung 洪志清

May 16^{th}, 2022

Unobservable Omitted Variable

- Even if we can control all observed variable, selection bias might still exist due to **unobservable omitted variables**.

- That is, the treatment and control group may be very different in some characteristics that we **CANNOT observe**!

- In other words, CIA (**selection on observables**) is not valid!

- Thus, we CANNOT eliminate selection bias by including more covariates into regression.

Unobservable Omitted Variable

- Suppose the true model is:

$$Y_i = \delta + \alpha D_i + \beta X_i + \epsilon_i$$

- But now X_i is an unobserved characteristics (e.g. ability, preference, health).

- So we cannot include it into our regression and estimate the following model:

$$Y_i = \delta + \alpha D_i + u_i$$

where $u_i = \beta X_i + \epsilon_i$

Unobservable Omitted Variable

國立臺灣大學 National Taiwan University

- As mentioned before, failure to include key covaraites will lead to omitted variables bias:

$$\hat{\alpha} \xrightarrow{p} \alpha + \frac{Cov(u_i, D_i)}{Var(D_i)}$$
$$= \alpha + \beta \frac{Cov(X_i, D_i)}{Var(D_i)}$$

- Remember there is **NO** omitted variable bias if D_i is unrelated to u_i (X_i).
  1. X_i is unrelated to Y_i: $\beta = 0$
  2. X_i is unrelated to D_i: $Cov(X_i, D_i) = 0$

- To obtain causal effect (eliminate OVB), we need a variation in D_i that is unrelated to unobserved confounding factor u_i (X_i).

Main Idea of Instrumental Variables

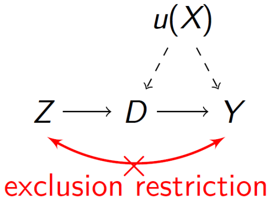
- The Instrumental Variable (IV) is an exogenous sources of variation that drives the treatment D_i but unrelated to other confounding factors u_i (X_i) that affect outcome Y_i.

- Intuitively, IV breaks variation of the treatment D_i into two parts:
  1. A part that might be correlated with other confounding factors u_i (X_i).
  2. A part that is not.

- We use the variation in D_i that is not correlated with u_i (X_i) to estimate causal effect of the treatment.

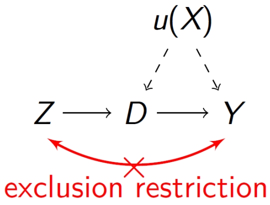
Main Idea of Instrumental Variables

國立臺灣大學 National Taiwan University



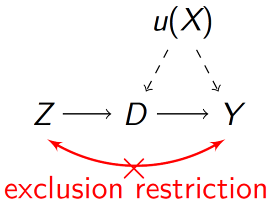
- Y is an outcome. (e.g. earnings)
- Z is the instrument.
- D is the treatment. (e.g. college degree)
- $u(X)$ is the unobserved confounding factor. (e.g. ability)

Main Idea of Instrumental Variables



- **Unobserved ability** $u(X)$ might confound with the effect of college degree D.
- We need to find an IV that generate a variation in getting college degree D that is unrelated to ability $u(X)$.
- IV initiates a causal chain: the instrument Z affects D, which in turn affects Y.

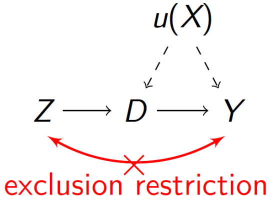
Main Idea of Instrumental Variables



- A valid IV needs to satisfy the following conditions:
  1. First-stage relationship (Instrument relevance): Z affects D.

  2. Exclusion restriction (Instrument exogeneity):
     - ★ **No direct or indirect effect** of the instrument Z on the outcome Y NOT through the treatment variable D.
     - ★ The instrument Z affects the outcome Y **only through the treatment variable** D.

Main Idea of Instrumental Variables



- We can test whether the **instrument relevance** is satisfied.
- But the **instrument exogeneity** cannot be tested.
- You have to try to convince your audience that it is satisfied.

Example of Instrumental Variables

Joshua D. Angrist (1990)
"**Lifetime Earnings and the Vietnam Era Draft Lottery: Evidence from Social Security Administrative Records**", *AER*

- He wanted to examine the effect of military service on lifetime income.
- We will use Angrist's paper on the effects of military service (D_i) on earnings (Y_i) as an example to go through key concept of IV methods.

Example of Instrumental Variables

- Joining military service is a personal choice.

- Is there any selection bias due to unobservable confounding factors in this example?

- **Time preference**:
  - Less patient people may voluntarily join military service early.
  - This myopic thinking may have negative impact on their earnings (e.g. less human capital investment)

- **Health condition**:
  - Better health people can join military service.
  - Better health condition also have positive impact on their earnings

- We need a IV for the treatment variable of joining military service!

Example of Instrumental Variables

- Angrist (1990) uses the Vietnam draft lottery (Z_i) as in IV for military service!
- In the 1960s and early 1970s, young American men were draft for military service to serve in Vietnam.
- Concerns about the fairness of the conscription policy lead to the introduction of a draft lottery in 1970.

Example of Instrumental Variables

- From 1970 to 1972 random sequence numbers were assigned to each birth date in cohorts of 19-year-olds.
- Men with lottery numbers below a cutoff were drafted while men with numbers above the cutoff could not be drafted.
- The draft did NOT perfectly determinate military service:
  - Many draft-eligible men were exempted for health and other reasons.
  - Draft-ineligible men volunteered for service.
- Next, let's briefly discuss whether draft eligibility induced by lottery is a good IV or not.

Example of Instrumental Variables

- **First-stage relationship (Instrument Relevance)**: Z_i affects D_i
  - Vietnam veteran status (joining military service) was not completely determined by randomized draft eligibility.
  - But draft eligibility is highly correlated with Vietnam veteran status.
- **Exclusion restriction (Instrument Exogeneity)**:
  - The draft eligibility is determined by random numbers.
  - This should not affect one's earnings directly.

IV and Potential Outcomes

- Treatment Assignment

$$Z_i = \begin{cases} 1 & \text{if an individual } i \text{ is eligible for a treatment} \\ 0 & \text{if an individual } i \text{ is not eligible for a treatment} \end{cases}$$

  - $Z_i = 1$: those who get draft eligibility (due to lottery results).
  - $Z_i = 0$: those who do not get draft eligibility (due to lottery results).

IV and Potential Outcomes

- Potential Treatments
  - D_i^z: treatment status given the value of Z.
  - D_i^1: treatment status if eligible for a treatment.
  - D_i^0: treatment status if not eligible for a treatment.

- Observed Treatments

$$D_i = \begin{cases} D_i^1 & \text{if } Z_i = 1 \\ D_i^0 & \text{if } Z_i = 0 \end{cases}$$

- or in a more compact notation: $D_i = Z_i D_i^1 + (1 - Z_i) D_i^0$

IV and Potential Outcomes

- **Exclusion Restriction**: the instrument has no direct effect on the outcome, once we fix the value of the treatment.
- Given the exclusion restriction, we know that the potential outcomes for each treatment status only depend on the treatment D_i, not the instrument Z_i.

IV and Potential Outcomes

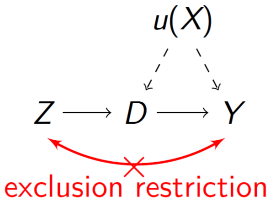
National Taiwan University

- Potential Outcomes
  - Y_i^1: outcome if an individual i get treatment (either $D_i^1 = 1$ or $D_i^0 = 1$).
  - Y_i^0: outcome if an individual i does not get treatment (either $D_i^1 = 0$ or $D_i^0 = 0$)
- Observed Outcomes

$$Y_i = \begin{cases} Y_i^1 & \text{if } D_i^1 = 1 \text{ or } D_i^0 = 1 \\ Y_i^0 & \text{if } D_i^1 = 0 \text{ or } D_i^0 = 0 \end{cases}$$

- or in a more compact notation: $Y_i = D_i^z Y_i^1 + (1 - D_i^z) Y_i^0$

Identification Results for IV



- The IV method characterize a causal chain reaction leading from the instrument Z_i (draft eligibility) to outcome Y_i (earnings).

- Intuitively:

$$
\begin{aligned}
&\text{Effect of instrument on outcome} \\
&= (\text{Effect of instrument on treatment}) \\
&\times (\text{Effect of treatment on outcome})
\end{aligned}
$$

Identification Results for IV

國立臺灣大學 National Taiwan University

- Rearranging:

$$\begin{aligned}&\text{Effect of treatment on outcome}\\&=\frac{\text{Effect of instrument on outcome}}{\text{Effect of instrument on treatment}}\end{aligned}$$

- Formal representation:

$$\alpha_{IV}=\frac{E[Y_i|Z_i=1]-E[Y_i|Z_i=0]}{E[D_i|Z_i=1]-E[D_i|Z_i=0]}$$

Identification Results for IV

- By using the following identification assumptions, we can prove the causal effect that IV identify is a **local average treatment effect (LATE)**.

Assumption (First-Stage Relationship)

$$E[D_i|Z_i = 1] - E[D_i|Z_i = 0] \neq 0$$

- We need the instrument to have a significant effect on the treatment.

Identification Results for IV

National Taiwan University

Assumption (Independent Assumption)

$$(Y_i^1, Y_i^0, D_i^1, D_i^0) \perp Z_i$$

- The IV is independent of potential outcomes and potential treatment (i.e. as good as randomly assigned).
- The independence assumption is sufficient for a causal interpretation of the **reduced form**:

$$\begin{aligned} E[Y_i|Z_i = 1] - E[Y_i|Z_i = 0] &= E[Y_i(D_i^1)|Z_i = 1] - E[Y_i(D_i^0)|Z_i = 0] \\ &= E[Y_i(D_i^1)] - E[Y_i(D_i^0)] \end{aligned}$$

- Independence also means that the first stage captures the causal effect of Z_i on D_i:

$$\begin{aligned} E[D_i|Z_i = 1] - E[D_i|Z_i = 0] &= E[D_i^1|Z_i = 1] - E[D_i^0|Z_i = 0] \\ &= E[D_i^1 - D_i^0] \end{aligned}$$

Identification Results for IV

National Taiwan University

Assumption (Exclusion Restriction)

$$Y_i^d(Z=1) = Y_i^d(Z=0) = Y_i^d$$

- The potential outcomes for each treatment status only depend on the treatment D_i, not the instrument Z_i.
- **Vietnam draft lottery example**:
  - An individual's earnings potential as a veteran (join military service) or non-veteran (not join military service) are assumed to be unchanged by draft eligibility status.

Identification Results for IV

National Taiwan University

Assumption (Exclusion Restriction)

$$Y_i^d(Z=1) = Y_i^d(Z=0) = Y_i^d$$

- The exclusion restriction would be violated if low lottery numbers may have affected schooling (e.g. to avoid the draft).
- If this was the case the lottery number would be correlated with earnings for at least two cases:
  1. through its effect on military service.
  2. through its effect on educational attainment.
- The fact that the lottery number is randomly assigned (and therefore satisfies the independence assumption) does NOT ensure that the exclusion restriction is satisfies!

IV and Compliers

- The variation in treatment D_i (veteran status) was not entirely from the draft eligibility Z_i but also from individual choice.

- Thus, $D_i^1 = 1$ or $D_i^1 = 0$?
  - $D_i^1 = 1$: Those who get draft eligibility choose to join military service.
  - $D_i^1 = 0$: Those who get draft eligibility choose NOT to join.

- Similarly, $D_i^0 = 1$ or $D_i^0 = 0$?
  - $D_i^0 = 1$: Those who did NOT get draft eligibility choose to join.
  - $D_i^0 = 0$: Those who did NOT get draft eligibility choose NOT to join.

IV and Compliers

National Taiwan University

- We can define four types of individuals based on whether they fellow the draft eligibility results:

1. **Compliers**: $D_i^1 > D_i^0$ $(D_i^1 = 1,\ D_i^0 = 0)$
   - David got draft eligibility and joined military service.
   - John did not get draft eligibility and did not join military service.
2. **Always Takers**: $D_i^1 = D_i^0 = 1$
   - Steve always joined military service no matter the lottery results (whether he got draft eligibility).
3. **Never Takers**: $D_i^1 = D_i^0 = 0$
   - Trump never joined military service no matter the lottery results (whether he got draft eligibility).
4. **Defiers**: $D_i^1 < D_i^0$ $(D_i^1 = 0,\ D_i^0 = 1)$
   - Jimmy got draft eligibility but did NOT join military service.
   - Jonson did NOT get draft eligibility but joined military service.

Identification Results for IV

國立臺灣大學 National Taiwan University

Assumption (Monotonicity Assumption)

$$D_i^1 > D_i^0$$

- Lastly, we need to make another assumption about the relationship between the instrument and the treatment.
- Monotonicity says that the presence of the instrument never dissuades someone from taking the treatment.
- This is sometimes called **no defiers**!
- In the draft lottery example: draft eligibility should encourage people to join military service.

Identification Results for IV

國立臺灣大學 National Taiwan University

Theorem (IV Identify LATE)

$$\alpha_{IV} = \frac{E[Y_i|Z_i = 1] - E[Y_i|Z_i = 0]}{E[D_i|Z_i = 1] - E[D_i|Z_i = 0]} = E[Y_i^1 - Y_i^0|D_i^1 > D_i^0]$$

- IV estimator represents the causal effect for **compliers**!
- Lottery IV can identify the causal effect of military service on lifetime earnings for those who obey the lottery results (e.g. David and John).

Identification Results for IV

Proof:

$$
\begin{aligned}
\alpha_{IV} &= \frac{E[Y_i|Z_i=1]-E[Y_i|Z_i=0]}{E[D_i|Z_i=1]-E[D_i|Z_i=0]} \\
&= \frac{E[D_i^1Y_i^1+(1-D_i^1)Y_i^0|Z_i=1]-E[D_i^0Y_i^1+(1-D_i^0)Y_i^0|Z_i=0]}{E[D_i^1|Z_i=1]-E[D_i^0|Z_i=0]} \\
&= \frac{E[D_i^1Y_i^1+(1-D_i^1)Y_i^0]-E[D_i^0Y_i^1+(1-D_i^0)Y_i^0]}{E[D_i^1]-E[D_i^0]} \\
&= \frac{E[(Y_i^1-Y_i^0)(D_i^1-D_i^0)]}{E[D_i^1]-E[D_i^0]}
\end{aligned}
$$

Identification Results for IV

- Note that since D_i^z is a dummy,

- IV estimates cannot say anything about causal effect for **always takers** or **never takers**: $D_i^1 - D_i^0 = 0$.

- $D_i^1 - D_i^0 = 1$: **compliers** or $D_i^1 - D_i^0 = -1$: **defiers**?

- Using Monotonicity Assumption, only $D_i^1 - D_i^0 = 1$ exists.

- Therefore, $E[(Y_i^1 - Y_i^0)(D_i^1 - D_i^0)]$ can become the following terms:

$$
\begin{aligned}
& E[(Y_i^1 - Y_i^0)(D_i^1 - D_i^0)] \\
& = E[(Y_i^1 - Y_i^0) \times (1) | D_i^1 - D_i^0 = 1] Pr(D_i^1 - D_i^0 = 1) \\
& + E[(Y_i^1 - Y_i^0) \times (-1) | D_i^1 - D_i^0 = -1] Pr(D_i^1 - D_i^0 = -1) \\
& = E[(Y_i^1 - Y_i^0) \times (1) | D_i^1 - D_i^0 = 1] Pr(D_i^1 - D_i^0 = 1)
\end{aligned}
$$

- Note: $E[D_i^1] - E[D_i^0] = Pr(D_i^1 - D_i^0 = 1)$

Identification Results for IV

Continue the Proof:

$$
\begin{aligned}
\alpha_{IV} &= \frac{E[Y_i|Z_i=1]-E[Y_i|Z_i=0]}{E[D_i|Z_i=1]-E[D_i|Z_i=0]} \\
&= \frac{E[D_i^1Y_i^1+(1-D_i^1)Y_i^0|Z_i=1]-E[D_i^0Y_i^1+(1-D_i^0)Y_i^0|Z_i=0]}{E[D_i^1|Z_i=1]-E[D_i^0|Z_i=0]} \\
&= \frac{E[D_i^1Y_i^1+(1-D_i^1)Y_i^0]-E[D_i^0Y_i^1+(1-D_i^0)Y_i^0]}{E[D_i^1]-E[D_i^0]} \\
&= \frac{E[(Y_i^1-Y_i^0)(D_i^1-D_i^0)]}{E[D_i^1]-E[D_i^0]} \\
&= \frac{E[(Y_i^1-Y_i^0)\times(1)|D_i^1-D_i^0=1]Pr(D_i^1-D_i^0=1)}{Pr(D_i^1-D_i^0=1)} \\
&= E[(Y_i^1-Y_i^0)|D_i^1>D_i^0] = \alpha_{LATE}
\end{aligned}
$$

Identification Results for IV

- **Never takers** and **always takers** do NOT change their treatment status when the instrument gets switched on and off.
  - So only **defiers** and **compliers** contribute to IV estimate.
  - IV estimate is the sum of those two effects.

- By using monotonicity assumption, we rule out the effect from **defiers**.

- Therefore, IV estimates the **average treatment effect for compliers**.

Identification Results for IV

LATE

- $\alpha_{LATE} = E[Y_i^1 - Y_i^0 | D_i^1 > D_i^0]$, the Local Average Treatment Effect for **compliers**.

- ATE for the individuals whose treatment status (join military service) are changed by the instrument (lottery draft).

- LATE (α_{LATE}) is different when using different instruments, Z_i.

- Whether LATE is interesting or not depends on the instrument.

Identification Results for IV

- Without further assumptions (e.g. constant causal effects), LATE is not informative about effects on never-takers or always-takers.

  $\rightarrow$ because the instrument does not affect their treatment status.

- In most applications we would be mostly interested in estimating the average treatment effect on the whole population (ATE).

$$= E[Y_i^1 - Y_i^0]$$

- This is usually not possible with IV.

Identification Results for IV

Special Case

- If D_i is randomized (e.g. RCT) and everybody is a complier, then $Z_i = D_i$.

- That is, no never taker or always taker.

- One-sided noncompliance, $D_i^0 = 0$, then:

$$
\begin{aligned}
E[Y_i^1 - Y_i^0 | D_i^1 > D_i^0] &= E[Y_i^1 - Y_i^0 | D_i^1 = 1] \\
&= E[Y_i^1 - Y_i^0 | Z_i = 1, D_i^1 = 1] \\
&= E[Y_i^1 - Y_i^0 | D_i = 1] \\
&= E[Y_i^1 - Y_i^0]
\end{aligned}
$$

- Thus, $\alpha_{LATE} = \alpha_{ATT} = \alpha_{ATE}$!

IV Estimation

- Causal relationship of interest: the effect of military service on earnings

$$Y_i = \delta + \alpha_{IV} D_i + u_i$$

- Remember, we just derive:

$$\text{Effect of treatment on outcome} = \frac{\text{Effect of instrument on outcome}}{\text{Effect of instrument on treatment}}$$

IV Estimation: Intuition

National Taiwan University

- We can estimate α_{IV} by running the following two regressions:

1 Reduced Form regression: the effect of lottery draft on earnings

$$Y_i = \mu + \alpha_{RF} Z_i + \epsilon_i$$

$$\alpha_{RF} = \frac{Cov(Y_i, Z_i)}{Var(Z_i)}$$

2 First-Stage regression: the effect of lottery draft on military service

$$D_i = \tau + \alpha_{FS} Z_i + \eta_i$$

$$\alpha_{FS} = \frac{Cov(D_i, Z_i)}{Var(Z_i)}$$

- The IV estimator is:

$$\widehat{\alpha_{IV}} = \frac{\widehat{\alpha_{RF}}}{\widehat{\alpha_{FS}}} = \frac{\widehat{Cov}(Y_i, Z_i)}{\widehat{Cov}(D_i, Z_i)}$$

IV Estimation: 2-Stage Least Square

- In practice we often estimate IV using Two stage least squares estimation (2SLS).

- If identification assumptions only hold after conditioning on X, covariates are often introduced using TSLS regression.

- It is called 2SLS because you could estimate it as follows:
  1. Obtain the first stage fitted values:

$$\widehat{D_i} = \widehat{\tau} + \widehat{\alpha_{FS}} Z_i + X_i' \hat{\beta}$$

  2. Plug the first stage fitted values into the second-stage equation

$$Y_i = X_i' \gamma + \alpha_{2SLS} \widehat{D_i} + u_i^*$$

- The intuition: 2SLS only retains the variation in D_i that is generated by quasi-experimental variation Z_i (and thus hopefully exogenous).

IV Remarks

1 Check IV relevance!

- Does this IV make sense?
- Do the coefficients have the right magnitude and sign?
- Report the F-statistic in the first stage regression.
  - ★ If $F > 10$, instruments are strong - use 2SLS
  - ★ If $F < 10$, weak instruments - find better IV
- If instruments are weak, then the 2SLS estimator is biased and the t-statistic has a non-normal distribution.

Weak IV Problem

National Taiwan University

- What are the consequences of weak instruments?

$$\widehat{\alpha_{2SLS}} = \frac{\widehat{Cov}(Y_i, Z_i)}{\widehat{Cov}(D_i, Z_i)}$$

- If $Cov(D_i, Z_i)$ is small (close to zero), a small change in $\widehat{Cov}(Y_i, Z_i)$ due to finite sample perturbation can induce a huge change in $\widehat{\alpha_{2SLS}}$!

- Under this situation, the normal distribution is a poor approximation to the sampling distribution of $\widehat{\alpha_{2SLS}}$.

- If instruments are weak, the usual methods of inference are unreliable.

Weak IV Problem

- Weak IV Bias

$$plim\ \widehat{\alpha_{OLS}} = \alpha + \frac{\sigma_{D,u}}{\sigma_D^2}$$

$$plim\ \widehat{\alpha_{2SLS}} = \alpha + \frac{\sigma_{\hat{D},u}}{\sigma_{\hat{D}}^2}$$

- Thus,

$$\frac{plim\ \widehat{\alpha_{2SLS}} - \alpha}{plim\ \widehat{\alpha_{OLS}} - \alpha} = \frac{\sigma_{D,u}/\sigma_{\hat{D},u}}{R_{D,Z}^2} = \frac{\rho_{Z,u}/\rho_{D,u}}{\rho_{D,Z}}$$

- If the instrument is weak, the bias for the 2SLS estimate may be larger than that for the OLS estimate.

IV Remarks

2 Check exclusion restriction!

- The exclusion restriction cannot be tested directly, but it can be falsified.
- **Placebo test**:
  - Test the reduced form effect of Z_i on Y_i in situations where it is impossible or extremely unlikely that Z_i could affect D_i.
  - Because Z_i cannot affect D_i, then the exclusion restriction implies that this placebo test should have zero effect.

3 If you have many IVs pick your best instrument and report the *just-identified* model.

4 Study at the reduced form.

- Directly estimate the effect of instrument Z on outcome Y
- If you cannot see the causal relationship of interest in the reduced form it is probably not there.

Summary Findings on Vietnam Draft Lottery

National Taiwan University

1. **First stage results:**
    - Having a low lottery number (being eligible for the draft) increases veteran status by about 16 percentage points.
    - Note that the mean of veteran status is about 27 percent.
2. **Second stage results:**
    - Serving in the army lowers earnings by $2,050 - $2,741 per year.
3. **Placebo test:**
    - There is no evidence of an association between draft eligibility (having a low lottery number) and earnings in 1969.
    - Note that 1969 earnings are realized before the 1970 draft lottery.