Bayesian Statistics- HW4

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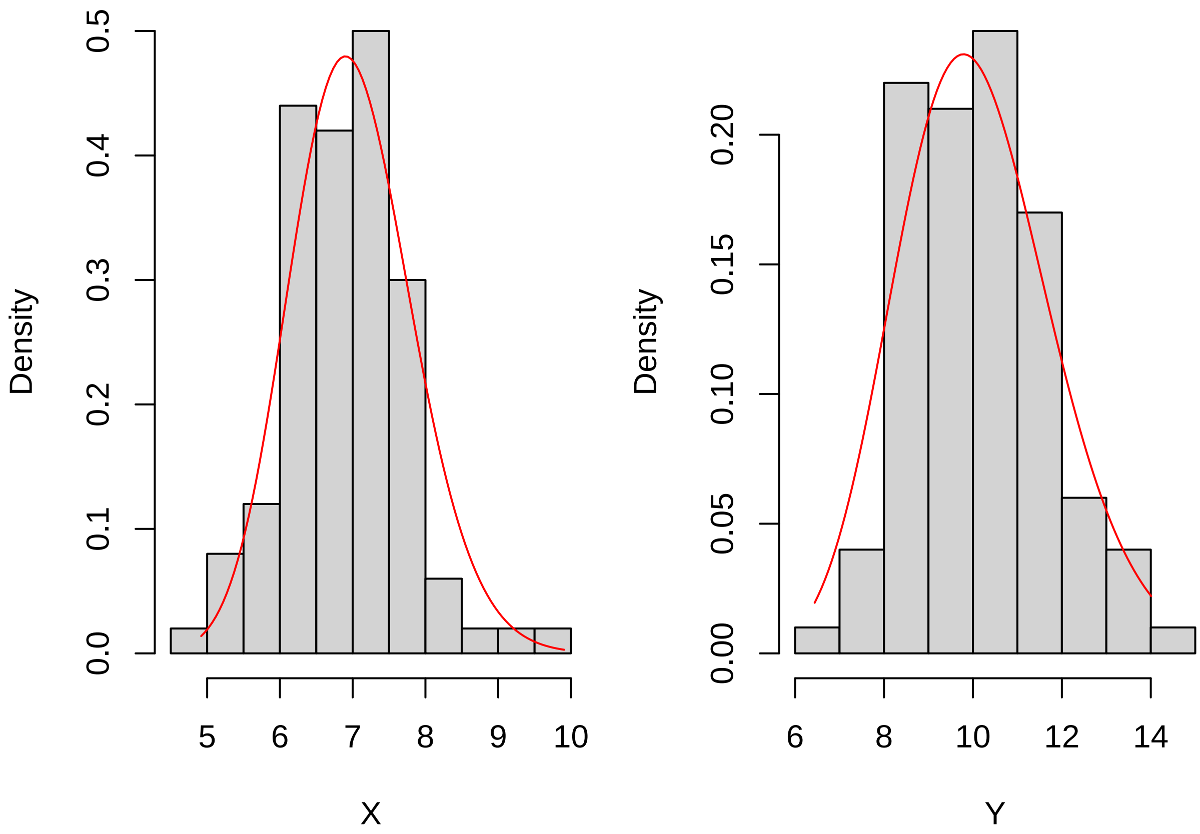
1.

(a).

```
par(mfrow=c(1, 2))
par(mai=c(0.9,0.9,0.1,0.1))

# Chrystal's prior
X <- rgamma(n = 100, shape = 70, scale = 1/10)
hist(X, prob=TRUE, breaks=10, main=NULL)
x <- seq(min(X), max(X), length.out=100)
lines(x, dgamma(x, shape=70, scale=1/10), col="red")

# Danny's prior
Y <- rgamma(n=100, shape= 33.3, scale = 1/3.3)
hist(Y, prob=TRUE, breaks=10, main=NULL)
y <- seq(min(Y), max(Y), length.out=100)
lines(y, dgamma(y, shape=33.3, scale=1/3.3), col="red")
```



(b).

```
cat("Chrystal's belief on the average ER visits is",mean(X),"\n")
```

```
## Chrystal's belief on the average ER visits is 6.880711
```

```
cat("Danny's belief on the average ER visits is", mean(Y),"\n")
```

```
## Danny's belief on the average ER visits is 10.15284
```

```
var(X)
```

```
## [1] 0.6837035
```

```
var(Y)
```

```
## [1] 2.449607
```

```
cat("Chrystal is more confident of her best guess at the average number of ER visits")
```

```
## Chrystal is more confident of her best guess at the average number of ER visits
```

(c).

```
# Chrystal's 90% credible interval
qgamma(c(0.05, 0.95), shape = 70, scale = 1/10)
```

```
## [1] 5.682967 8.430648
```

```r
# Danny's 90% credible interval
qgamma(c(0.05, 0.95), shape= 33.3, scale=1/3.3)
```

```
## [1]  7.396876 13.128930
```

(d).

Need to increase the variance.

2.

(a).

```r
n <- 7
y.sum <- 8+6+6+9+8+9+7

#Chrystal's 95% posterior credible interval
qgamma(c(0.025, 0.975), shape = 70+y.sum, scale = 1/(10+n))
```

```
## [1] 6.013237 8.568716
```

```r
#Danny's 95% posterior credible interval
qgamma(c(0.025, 0.975), shape = 33.3+y.sum, scale = 1/(3.3+n))
```

```
## [1]  6.704582 10.236459
```

(b).

To assess if the statement "the average number of ER visits during any evening hour does not exceed 6" is reasonable, one simply computes its posterior probability, Pr$(\lambda \geq 6|\alpha_n, \beta_n)$

```r
1-pgamma(6, shape = 70+y.sum, scale = 1/(10+n))
```

```
## [1] 0.9763136
```

```r
1-pgamma(6, shape = 33.3+y.sum, scale = 1/(3.3+n))
```

```
## [1] 0.998145
```

The probability is very high, so one would conclude that this statement is unlikely to be true.

(c).

```r
# use Chrystal's posterior to predict
S <- 7
pred_mu_sim <- rgamma(S, shape = 70+y.sum, rate = 10+n) # sample mu from posterior
pred_y_sim <- rpois(S, lambda = pred_mu_sim)
pred_y_sim
```

```
## [1]  5 10  7 13  6 11  9
```

```
# use Danny's posterior to predict
pred_mu_sim <- rgamma(S, shape = 33.3+y.sum, rate = 3.3+n)
pred_y_sim <- rpois(S, lambda = pred_mu_sim)
pred_y_sim
```

```
## [1] 13 11  7  3 16  5  5
```

3.

Theoretical exercises. Do on your own.

4.

Interpret on your own.