(2) c) If our data is the single value $y = 5.2$, then the likelihood is $L(\theta | y) = \theta e^{-5.2\theta}$

My Beta(3,27) prior was $p(\theta) \propto \theta^{3-1}(1-\theta)^{27-1}$

So the posterior for θ given y is

$$p(\theta | y) \propto \theta^3 (1-\theta)^{26} e^{-5.2\theta}$$

Metropolis – Hastings:

If our current value of the chain is $\theta^{[t]}$, then we

* sample a candidate value θ^* from the $\text{Beta}(a_{\text{proposed}}, b_{\text{proposed}})$ proposal density
* Let the $(t+1)$-st value in the chain be

$$\theta^{[t+1]} = \begin{cases} \theta^* & \text{with probability } \min\{a(\theta^*, \theta^{[t]}), 1\} \\ \theta^{[t]} & \text{with probability } 1 - \min\{a(\theta^*, \theta^{[t]}), 1\} \end{cases}$$

where the acceptance ratio here is

$a(\theta^*, \theta^{[t]}) =$

$$\frac{(\theta^*)^3 (1-\theta^*)^{26} e^{-5.2\theta^*}}{(\theta^{[t]})^3 (1-\theta^{[t]})^{26} e^{-5.2\theta^{[t]}}} \cdot \frac{\frac{\Gamma(a_{prop}+b_{prop})}{\Gamma(a_{prop})\Gamma(b_{prop})} \left(\theta^{[t]}\right)^{a_{prop}-1} \left(1-\theta^{[t]}\right)^{b_{prop}-1}}{\frac{\Gamma(a_{curr}+b_{curr})}{\Gamma(a_{curr})\Gamma(b_{curr})} \left(\theta^*\right)^{a_{curr}-1} \left(1-\theta^*\right)^{b_{curr}-1}}$$