Animating Human Locomotion

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1 Summary

Simulating realistic human locomotion on a computer is one of the challenges of computer graphics. The most basic form of human locomotion is walking, a skill that comes to most of us naturally. When we walk, we hardly ever consider the mechanics of walking. Once we move onto a computer, however, walking suddenly becomes a complicated process that involves many mechanisms.

In this project we do not intend to simulate a model of "real" human walking with all the muscles and joints. Instead, we will simulate an abstracted version that preserves the essence of walking and produces a reasonably realistic animation. We will use a combination of inverse kinematics and forward dynamics to animate a walking human-like figure. This figure will be created in an incremental fashion. Initially it will be a monster with two legs but only one knee and a spherical body-head. As the project progresses, we will add features such as another knee, a more human-like torso, and arms to form a more realistic human.

2 Abstraction

The essence of walking is the movement of one leg while the entire body falls forward. To clearly illustrate these basic mechanisms of walking, we can represent the legs with two sticks and two hinges. The upper leg (a stick) is is attached to the body by a hinge called the "hip joint" and the lower leg (another stick) is attached to the upper leg by a hinge called the "knee joint." The hip joint allows the upper leg to rotate forward and backward around its vertical position, but the knee joint only allows the lower leg to rotate backward with regard to the upper leg.

This project will be implemented in three phases. In Phase 1, to avoid being distracted by nonessential elements, we approximate the entire upper body as a sphere. Furthermore, only the right leg is assumed to have a knee joint to illuminate how one-legged movement proceeds. Thus, in Phase 1 the resulting creature is aptly nicknamed a "monster". The monster will walk by lifting its right leg, putting it down, and dragging its left leg. Only in Phase 2 is the knee joint added to the left leg to give the monster a more natural gait. In Phase 3 hands and a head are added to the body, simulating a walking snowman. The snowman can be modified to the stick person traditionally used in simulations of a walking figure.

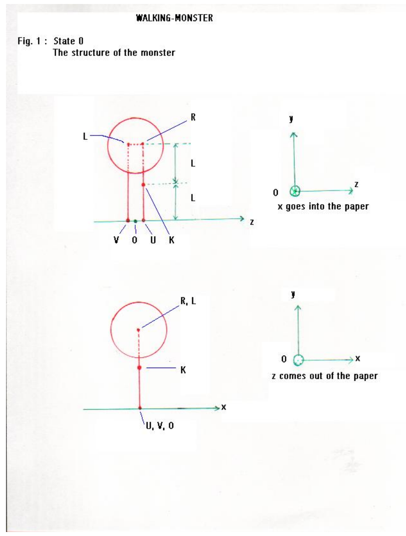

Figure 1: Structure of the monster.

3 Mathematical Representation

The details of Phase 1 will be covered here, since it covers the most basic mechanisms of walking. Phase 2 is an extension of Phase 1, and Phase 3 adds some non-essential items to Phase 2 to make the monster look human.

3.1 Phase 1

Each configuration of the monster's motion can be represented by a state.

State 1 In State 0 the monster is at rest. Fig. 1 describes the structure of the monster. The body of the monster is a sphere of radius w. The right leg is hinged to the body at a point R which is inside the body, and the left leg is hinged at point L. The separation between R and L is a. The center of the sphere is C, and points L, C, and R form a horizontal line segment. The right leg has a knee joint K and a foot, represented by a point, U. The left leg does not have a knee joint. A foot is at point V. The y-z view is presented at the top and the y-x view is presented at the bottom of the figure.

State 2 Fig. 2 describes the lifting of the right leg. The initial position of the tip of the right leg, point U (x_0, y_0), in Fig. 2(a) is obviously

$$x_0 = 0, y_0 = 0. \tag{1}$$

The final position of the tip of the right leg, point U $(x1, x2)$ in Fig. 2(c), is given by

$$x_1 = L\sin\theta_1, y_1 = 2L - L\cos\theta_1 - L, \tag{2}$$

where θ_1 is an input parameter to specify how high the upper right leg should be lifted. The position of the tip of the right leg, point U (x, y) in Fig. 2(b) can be expressed in terms of angles θ and ζ as

$$x = L\sin\theta + L\sin(\theta - \zeta), y = 2L - L\cos\theta - L\cos(\theta - \zeta). \tag{3}$$

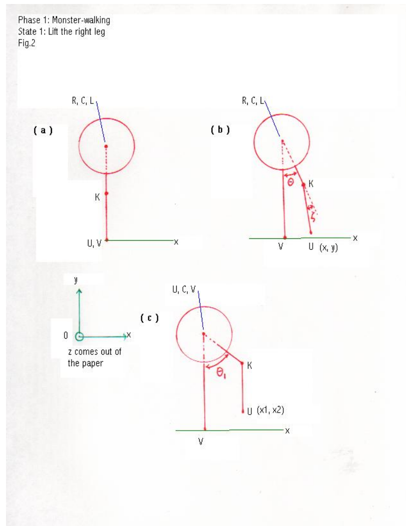

Figure 2: The monster walking.

Writng a row matrix of two components a, b as (a, b) and the corresponding column matrix as $(a, b)^T$, the above transformation from angles θ and ζ can be written as

$$(x, y)^T = J(\theta, \zeta)^T, \tag{4}$$

where J is the Jacobian of the transformation and is a 2 by 2 matrix of four elements:

$$\begin{pmatrix} \frac{\delta x}{\delta \theta} & \frac{\delta x}{\delta \zeta} \\ \frac{\delta y}{\delta \theta} & \frac{\delta y}{\delta \zeta} \end{pmatrix}.$$

Denote the inverse matrix of J as J^{-1}. We can write

$$(\delta\theta, \delta\zeta)^T = J^{-1}(\delta x, \delta y)^T. \tag{5}$$

Thus by setting $\delta x = (x_1 - x_0)/N$, $\delta y = (y_1 - y_0)/N$, at each given value of θ and ζ, where N is an input integer to specify how fine the animation between one frame to the next should be, $\delta\theta$ and $\delta zeta$ can be calculated from (5), and the new θ and ζ can be obtained as

$$\theta = \theta + \delta\theta, \zeta = \zeta + \delta\zeta. \tag{6}$$

State 3 Fig. 3 depicts how the right leg makes a step. 3(a) shows the original position of this state, which is also the final state of State 2, Fig. 2(c). The initial position of the tip of the right leg, point U, is at the position of x_1 and x_2 as given by (2). The final position of point U $(x2, y2)$ as in Fig. 3(c) is given as

$$x_2 = s, y_2 = 0, \tag{7}$$

where s is a given parameter to specify how long a step should be. The angle α in Fig. 3(b) describes the rate of falling of the monster's body and can be

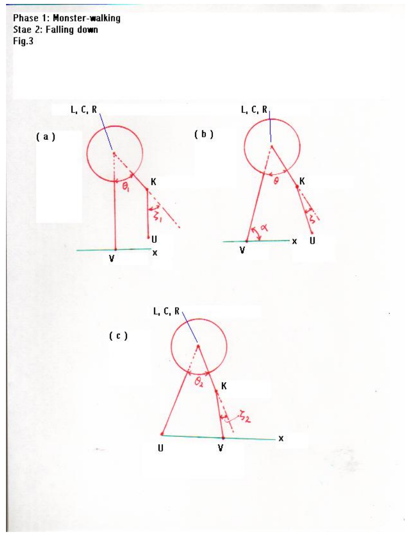

Figure 3: The monster's leg falling down.

calculated dynamically using torque from gravity and Newton's equation for an extended rigid body with the moment of inertia. In general it takes the form

$$\alpha = c_2 t^2 + c_1 t + c_0, \tag{8}$$

where c_2, c_1, c_0 are input parameters equivalent to specifying the weight and the size of the whole body, and t is time but can be considered just as a parameter. We can cut t into small increments δt (a given input), and compute $t = t + \delta t$ starting from $t = 0$ for each frame. Thus each frame will have a given angle α. The tip of the right leg, point U (x, y), in 3(b) is

$$x = 2L\cos\alpha + L\sin(\theta - (90 - \alpha)) + L\sin(\theta - (90 - \alpha) - \zeta). \tag{9}$$

Again using inverse kinetics, we can obtain $(\delta\theta, \delta\zeta)^T$ as

$$(\delta\theta, \delta\zeta)^T = J^{-1}(\delta x, \delta y)^T \tag{10}$$

where $\delta x = (x2 - x1)/N$, and $\delta y = (y2 - y1)/N$. The Jacobian J is again a 2 by 2 matrix of 4 elements

$$\begin{pmatrix} \frac{\delta x}{\delta\theta} & \frac{\delta x}{\delta\zeta} \\ \frac{\delta y}{\delta\theta} & \frac{\delta y}{\delta\zeta} \end{pmatrix}.$$

The 4 elements of partial differentiation, of course, must be calculated from (9) here.

State 4 Fig. 4 describes the process of pulling up. The tip of the right leg, U, is now fixed at x_2, y_2 of (7). The initial position of the hinge of the right upper leg, point R at x_3 and y_3, are

$$x_3 = 2L\cos\alpha_1, y_3 = 2L\sin\alpha_1, \tag{11}$$

where α_1 is the final value of the angle α in 3(c). The final position of the point R, x_4 and y_4, are

$$x_4 = s, y_4 = 2L. \tag{12}$$

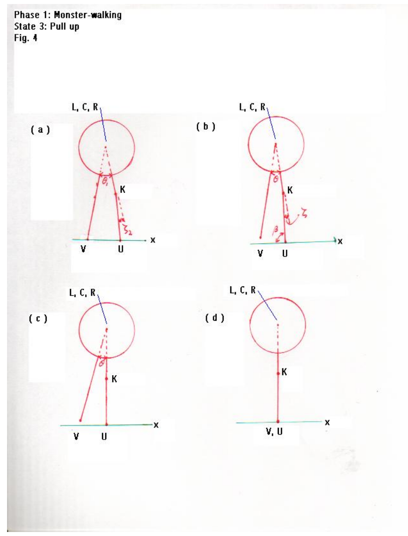

Figure 4: The monster pulls up its leg.

The general position of R as in Fig. 4(b) are now determined by two angles ζ and β as

$$x = s - L\cos\beta - L\cos(\beta - \zeta), y = L\sin\beta + L\sin(\beta - \zeta). \tag{13}$$

Thus defining $\delta x = (x_4 - x_3)/N$, and $\delta y = (y4 - y_3)/N$, and using inverse kinetics, we get

$$(\delta\beta, \delta\zeta)^T = J^{-1}(\delta x, \delta y)^T. \tag{14}$$

Now the partial differentials of the Jacobian must be calculated from (14). The angle θ in this state just decreases by an increment of $\delta\theta_4$, which is again an input parameter to specify how fast the left leg is been pulled in. When R reaches the upright position of x_4, y_4, if θ is not yet zero, then the decrease of θ by $\delta\theta_4$ should continue until θ becomes zero. If θ becomes zero before R reaches the upright position, then θ needs to be kept as zero for the remaining process until the whole monster regains the initial posture of Fig. 4(d). Note that in Fig. 4(d) the position of the monster is shifted by a distance s in the x-direction compared to the first picture in Fig. 1 – that is, it has walked a distance s.

3.2 Phase 2

Insert a knee to the left leg of the monster and let it make a more natural walk by alternatively use the right leg and the left leg to walk. The pull-up process in State 4 needs to be changed somewhat because the left leg can now be bent and then stretched to reach the final posture of Fig. 4(d). The overall picture of this phase is illustrated in Fig. 5(a).

3.3 Phase 3

In this phase, a head and two arms will be added to the monster. The arms will swing in synchronization with the leg movement to generate a natural image of walking, as in Fig. 5(b). A stick person version will also be animated, as in Fig. 5(c).

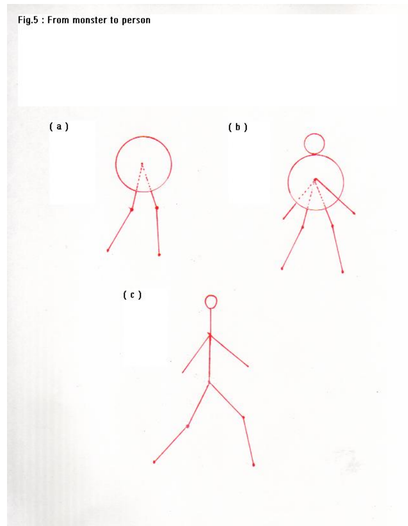

Figure 5: From a monster to a human.

4 Algorithms

This project use GLUT and OpenGL to draw and perform frame by frame animation. The calculations will follow the steps outlined in Section 3: Start from State 1, apply State 2 to the right leg (lift the right leg), state 3 to the right leg (the body falling down and the right leg stepping forward), state 4 for the right leg (pull up the whole body by stretching out the right leg), return to State 1 with the whole body standing at the shifted position, apply State 1 to the left leg (lift the left leg), and so on.

Each state will be an instance of a `State` class. We will need to implement a state machine to handle state transitions. Since the inverse of a matrix may contain nonsense values, we will also need to find pseudo-inverses. This can be done by decomposing a matrix M as $M = UDV^T$. The pseudo-inverse is found as $M^+ = VD^+U^T$. The pseudo-inverse of D is found by approximating nonsense values as 0.

5 Results

The final implementation is expected to display a stick person walking. The user will also be able to choose to view a monster walking, as in Phase 1, or a snowman walking, as in Phase 2.

6 Fall-Back Position

The magnitude of the calculations and implementations necessary to animate a human with a torso, legs, and arms may prove difficult. In case it is impossible to complete the full implementation, a preliminary animation will be submitted instead. This could be the armless snowman, or in the worst case, the one-kneed monster.