14.27/270 Ecommerce

A Primer on Auction Theory

S. Shang

Auctions: two main settings

willingness to pay

- Independent private values (IPV)
  - N bidders, each has valuation for good v_i, an independent draw from U[0,1]
  - Bidder utility is v_i-p if wins, 0 otherwise
  - v_i known only to bidder

willingness to bid

- Common values (CV)
  - N bidders, good has value v to winner of auction, v is drawn from N(0,1)
  - no one observes v but every bidder receives a signal s_i of its value (say $s_i = v + \varepsilon_i$ where ε has independent normal distribution with variance σ^2)
  - Every bidder can calculate $E(v|s_i) = w_i$ using Bayes Rule ($s_i/(1+\sigma^2)$ if above)

standard Normal distribution

$s_i = v + \varepsilon_i$

$E(s_i | v) = v \Rightarrow ?\ E(v|s_i)$

Auctions: two main settings

We often base analyses on this first setting, not necessarily because it's more common or realistic but because it's more straightforward to analyze.

- Independent private values (IPV)
  - N bidders, each has valuation for good v_i, an independent draw from U[0,1]
  - Bidder utility is v_i-p if wins, 0 otherwise
  - v_i known only to bidder
- Common values (CV)
  - N bidders, good has value v to winner of auction, v is drawn from N(0,1)
  - no one observes v but every bidder receives a signal s_i of its value (say $s_i = v + \varepsilon_i$ where ε has independent normal distribution with variance σ^2)
  - Every bidder can calculate $E(v|s_i) = w_i$ using Bayes Rule ($s_i/(1+\sigma^2)$ if above)

Auctions: two main settings

Standard example: bottle of wine

- Independent private values (IPV)
  - N bidders, each has valuation for good v_i, an independent draw from U[0,1]
  - Bidder utility is v_i-p if wins, 0 otherwise
  - v_i known only to bidder
- Common values (CV)
  - N bidders, good has value v to winner of auction, v is drawn from N(0,1)
  - no one observes v but every bidder receives a signal s_i of its value (say $s_i = v + \varepsilon_i$ where ε has independent normal distribution with variance σ^2)
  - Every bidder can calculate $E(v|s_i) = w_i$ using Bayes Rule ($s_i/(1+\sigma^2)$ if above)

Standard example: jar of quarters, no one k
how many

Auctions: four main types

- English auction
  - Bid b increases from 0 to 1 in continuous time, each bidder keeps hand up until no longer willing to bid, last remaining bidder wins at $P=b_i$.
  - In IPV, the equilibrium strategy is for bidder i to stay in until $b_i=v_i$ then drop out
  - In IPV, expected revenue is (N-1)/(N+1) (recall our previous calculation using order statistics)
  - In CV, do not want to bid w_i
    - Everyone has independent information about the value of the good---if you win by bidding your best guess of its value based on your information, the fact that you won is bad news about the true value. This phenomenon is referred to as the “Winner’s Curse” or “Caveat Emptor.”
    - Should shade bids down to take winner’s curse into account

Auctions: four main types

truthtelling mechanism

- English auction
  - Bid b increases from 0 to 1 in continuous time, each bidder keeps hand up until no longer willing to bid, last remaining bidder wins at $P=b_i$.
  - In IPV, the equilibrium strategy is for bidder i to stay in until $b_i=v_i$ then drop out
  - In IPV, expected revenue is (N-1)/(N+1) (recall our previous calculation using order statistics)
  - In CV, do not want to bid w_i
    - Everyone has independent information about the value of the good---if you win by bidding your best guess of its value based on your information, the fact that you won is bad news about the true value. This phenomenon is referred to as the "Winner's Curse" or "Caveat Emptor."
    - Should shade bids down to take winner's curse into account

Auctions: four main types

- Second price sealed bid
  - Bidders submit secret bids, winner is one with highest bid but pays second highest bid.
  - In IPV, the unique equilibrium strategy is for bidder i to bid $b_i=v_i$
    - Higher bid means your payoff changes only if you would lose at $b=v_i$ but win at higher, but that's bad because you pay more than your valuation
    - Lower bid means your payoff changes only if win at $b=v_i$ and lose at lower, but that's bad because you would have been willing to pay more to win
  - In IPV, expected revenue is (N-1)/(N+1) again
  - In CV, same as before

Auctions: four main types

truthtelling mechanism

- Second price sealed bid
  - Bidders submit secret bids, winner is one with highest bid but pays second highest bid.
  - In IPV, the unique equilibrium strategy is for bidder i to bid $b_i = v_i$
    - Higher bid means your payoff changes only if you would lose at $b = v_i$ but win at higher, but that's bad because you pay more than your valuation
    - Lower bid means your payoff changes only if win at $b = v_i$ and lose at lower, but that's bad because you would have been willing to pay more to win
  - In IPV, expected revenue is (N-1)/(N+1) again
  - In CV, same as before

Auctions: four main types

- First price sealed bid
  - Bidders submit secret bids, winner is one with highest bid but pays his bid.
  - In IPV, the unique equilibrium strategy is for bidder i to bid $b_i=((N-1)/N)v_i$
    - Equilibrium bidding strategy really has to be different from 2PSB because what you pay if you win is different
    - Bidders shade bid down, trading off bigger surplus if win v. chance of losing
    - Makes sense that amount of shading down would be a function of N
  - In IPV, expected revenue is (N-1)/(N+1) again
    - Bidders shading down ends up exactly cancelling out effect of paying top bid instead of 2^{nd} highest

Auctions: four main types

- Dutch auction
  - Bid b decreases from 1 to 0 in continuous time, with bidders raising their hands as soon as they are willing to bid
  - In IPV, the unique equilibrium strategy is for bidder i to raise hand at $b_i=((N-1)/N)v_i$
  - In IPV, expected revenue is (N-1)/(N+1) again

Auctions: revenue equivalence

N bidder, IPV with valuations $v_i \sim F$ on $[v_0, v_1]$

If object must be sold and all bidders must receive non-negative surplus, then any auction satisfying the following properties maximizes the seller's expected revenue:

- In equilibrium, the winning bidder is the highest valuation bidder
- The v_0 type gets zero expected surplus

- English, Dutch, 1PSB, 2PSB are revenue equivalent
- Sellers can often do better by setting and announcing a reservation price
- If bidders are risk-averse, 1PSB auction produces more revenue than 2PSB
- If bidders' valuations are positively correlated (instead of independent), 2PSB auction produces more revenue than 1PSB

Auctions: two main settings

- In reality most auctions have some private value and some common value character to them:
  - Van Gogh painting---I will enjoy seeing it on my wall but also care about the prestige associated with owning it and the resale value if I want to eventually sell it.
  - Highway paving contract---There is relevant information for me in my rivals' bids because we're all just estimating how much it's going to cost, but I have a cost advantage because I'm currently working on a job close by.
  - Timber tract---I know market value of milled lumber, but it's impossible to count exact number and types of trees. I can "cruise" a tract to estimate (and so can others). Furthermore, my mill is currently running at capacity and my loggers are busy, so I wouldn't be able to log the tract for several months.
  - Jar of Gummi Bears---No one knows how many so everyone has to estimate, but some people are not so excited about Gummi Bears and would rather have Junior Mints.

Not a coincidence that some of the most important economists working in high tech have a background in auction theory and empirics

- Hal Varian, chief economist at Google
- David Reiley, chief economist at Pandora
- Patrick Bajari, chief economist at Amazon
- Susan Athey, former chief economist at Microsoft
- Preston McAfee, former chief economist at Microsoft
- Michael Schwarz, chief economist at Microsoft