Ecommerce Foundational Lectures

The Basics: Differentiation and Entry

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Bertrand

- Last time we saw a model of competition between firms that seemed unrealistic and extreme.
- Demand had this knife-edge quality, willing to flip entirely for a change of a fraction of a penny.
- We learned a lot from the model, though.
- We scrutinized the assumptions and gained some insight into why we don’t usually see this cut-throat marginal-cost pricing and how firms might change market conditions to escape it.
- Today, we’re going to formalize a couple of these ideas.

Bertrand paradox

- Here was our list of assumptions for the Bertrand model.
  - Products are identical.
  - Consumers are assumed to know both prices.
  - And switch immediately as soon as one is a tiny bit lower.
  - Firms only set price once.
  - Firms can meet all demand immediately.
  - Firms’ marginal costs are constant and identical.

Hotelling model

- Let’s start with the first one---products are identical---and explore the effects of product differentiation in more detail.
- This model is due to Harold Hotelling, a mathematical statistician. It is based on his 1929 paper “Stability in Competition.”
- Interesting facts: Harold Hotelling taught statistics to Milton Friedman (’76 Nobel) and persuaded Ken Arrow (’72 Nobel) to become an economist. So his contribution to the field of economics goes far beyond this very fundamental model.

Hotelling model

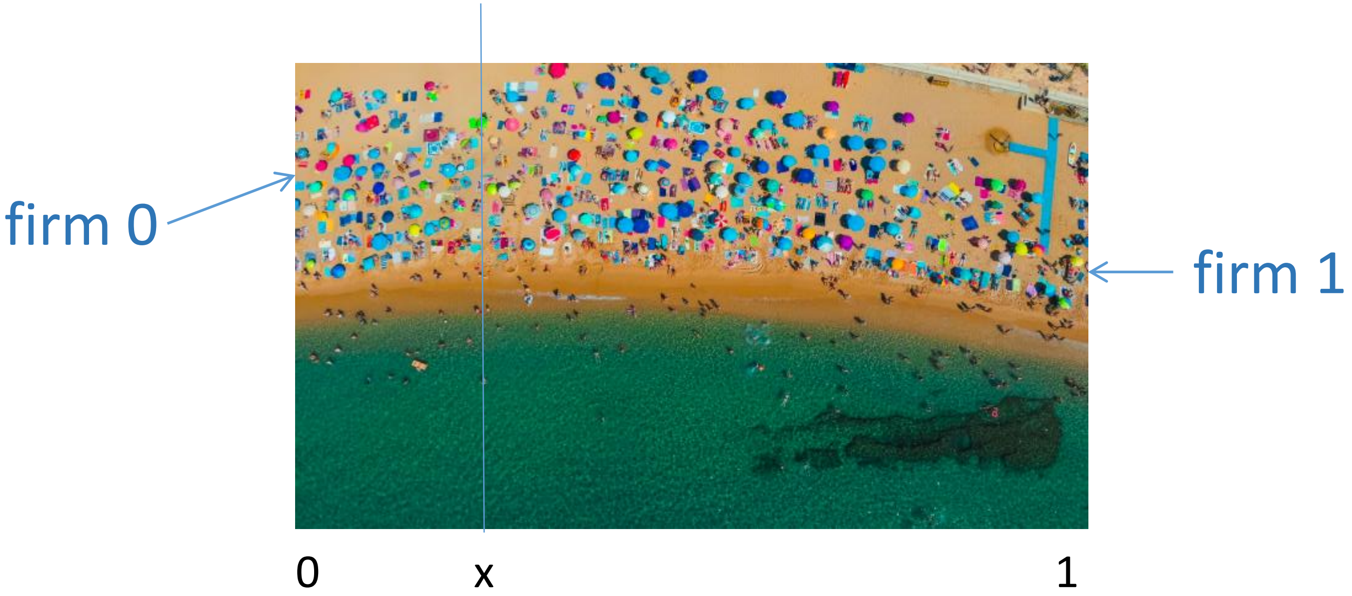
- Suppose two firms are selling (physically) identical objects.
- Customers are distributed uniformly on a unit interval, with one firm located at 0 and the other firm located at 1.

ice cream
vendor here

another
vendor here

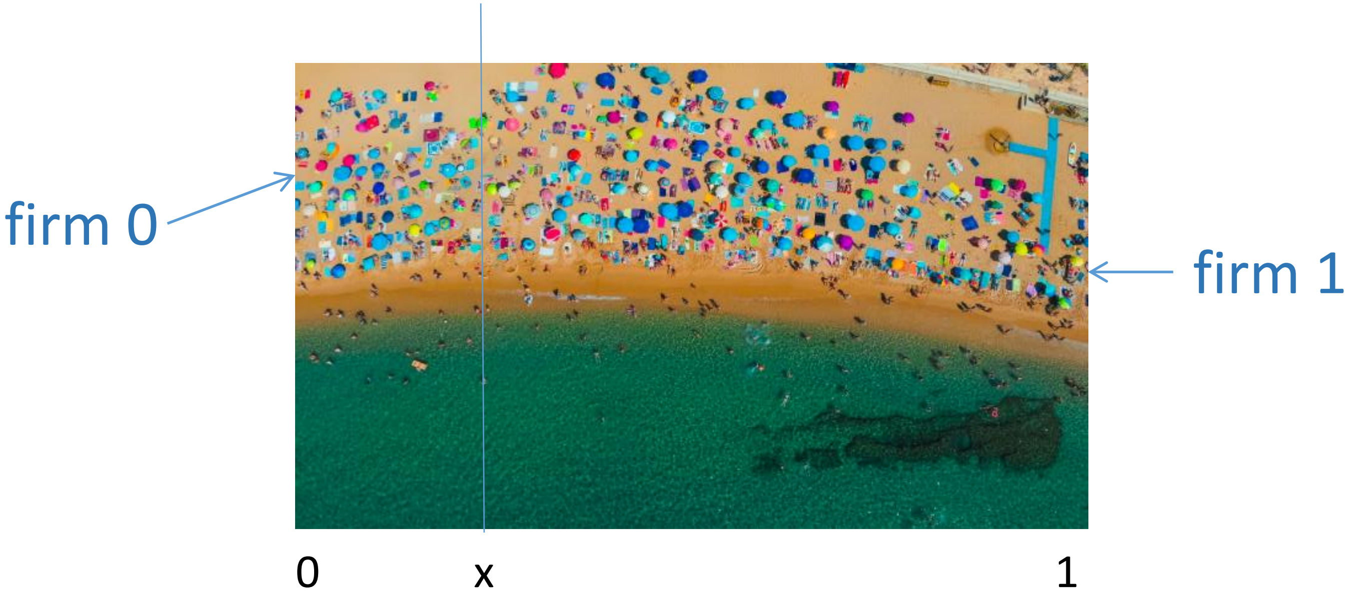
Hotelling model

- Both firms have marginal cost c.
- Customers incur a cost *t* to travel a unit distance (from one end of the beach to the other), so a consumer at location *x* in [0,1] incurs cost *xt* to travel to firm 0 and *(1-x)t* to travel to firm 1.



Hotelling model

- Customers have unit demands (0 or 1 ice cream cones) and enjoy utility $V - xt - P_0$ if they are located at x and purchase one unit from firm 0.



Their value for an ice cream cone if someone just hands them one.

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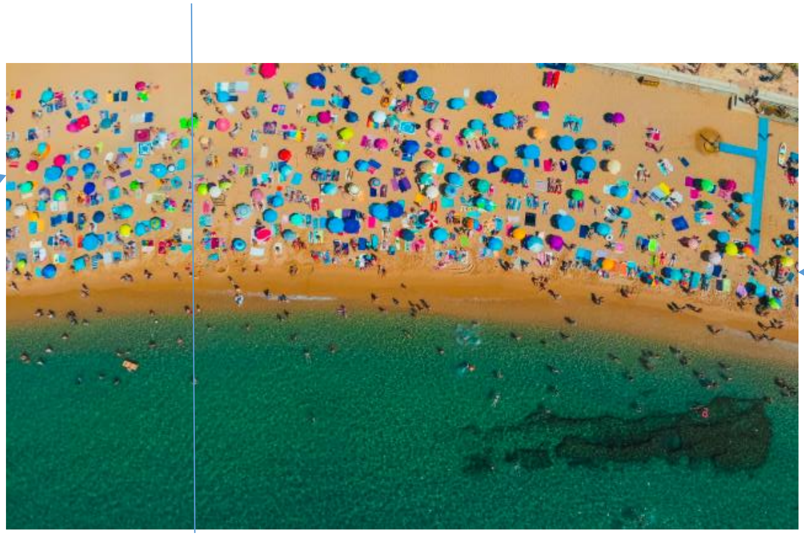
price of cone

travel costs

firm 0



firm 1

0 x 1

Their value for an ice cream cone if someone just hands them one.

Hotelling model

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price of cone

It's similar for firm 1.

travel costs

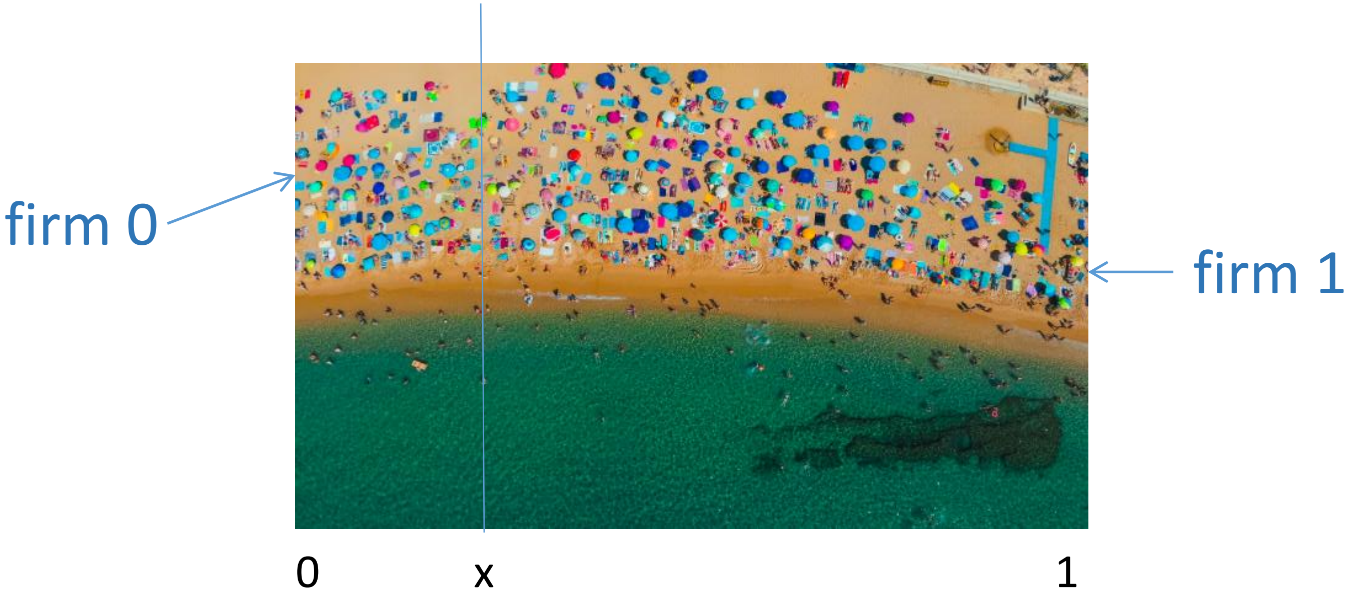
firm 0

firm 1

0 x 1

Hotelling model

- The two firms simultaneously set their prices. (We'll only consider the case where the prices are 1) close enough so that both firms have positive sales and 2) not so high relative to V that there are consumers in the middle who do not buy anything.)



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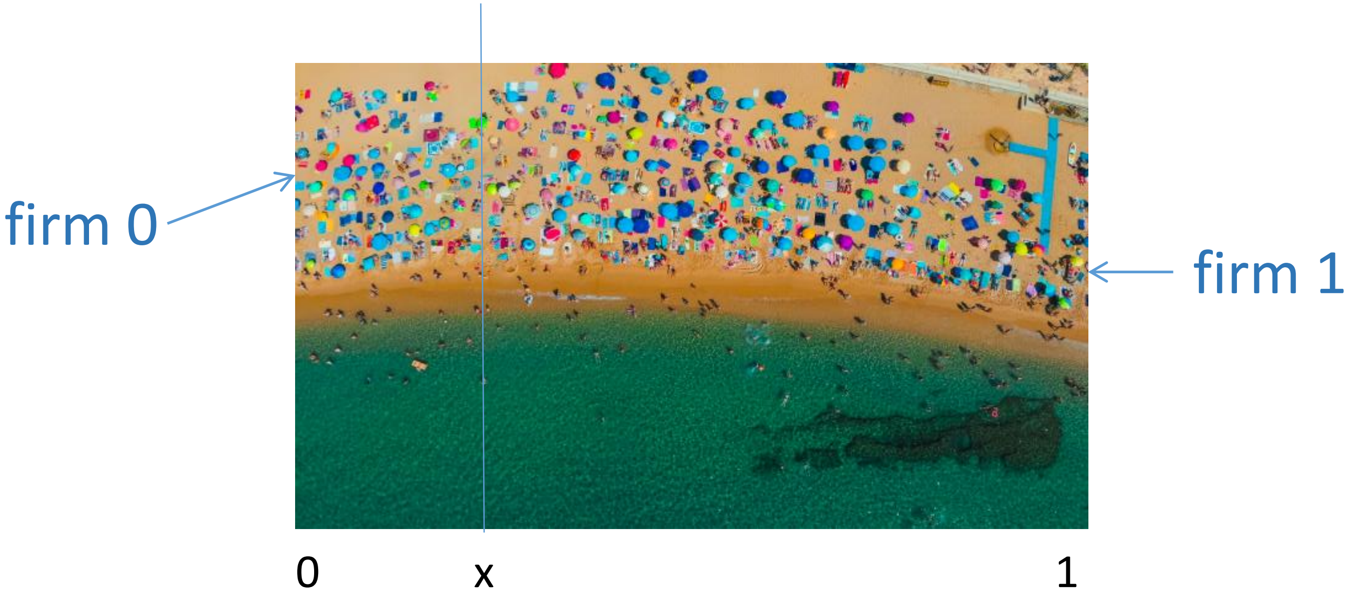
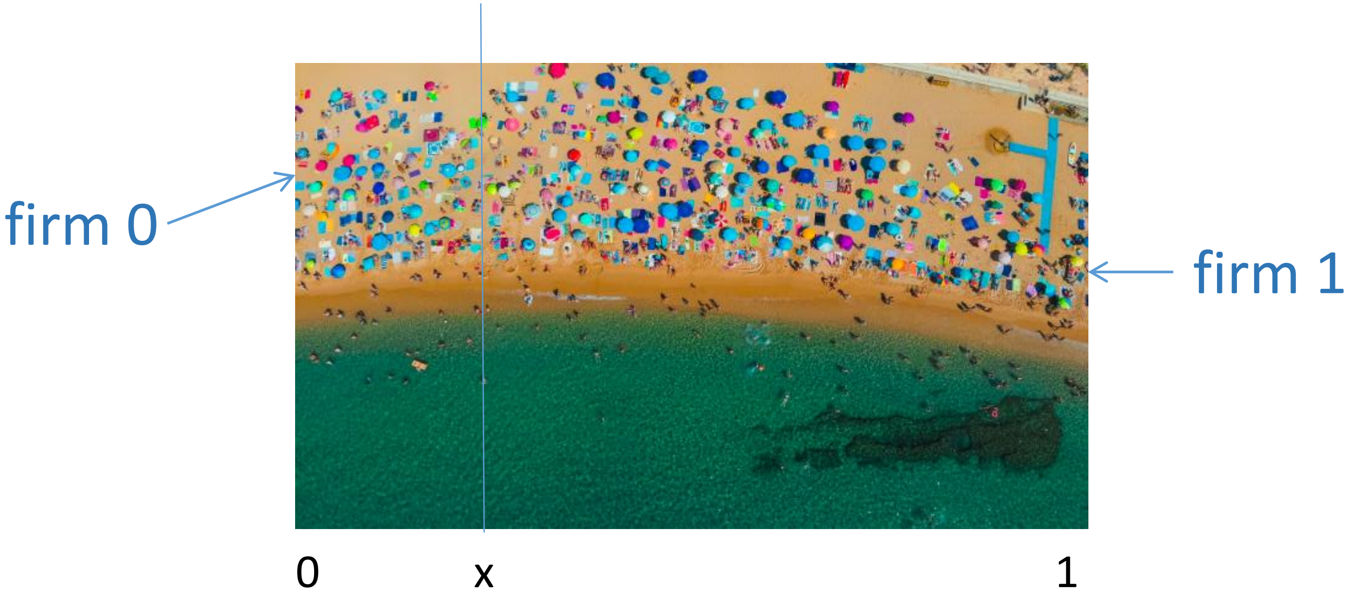


We're not saying that these other cas
are impossible or not important. It's
just that they revert back to monopol
and we want to focus on competition
and firm interaction.

Hotelling model

- How do we find P_0 and P_1?
- Well, we're going to proceed by identifying the consumer indifferent between the two firms. (This will be a function of P_0 and P_1, of course.)
- Then we will know each firm's demand and can write down profit functions. And we can then maximize profits over prices.



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firm 0

firm 1

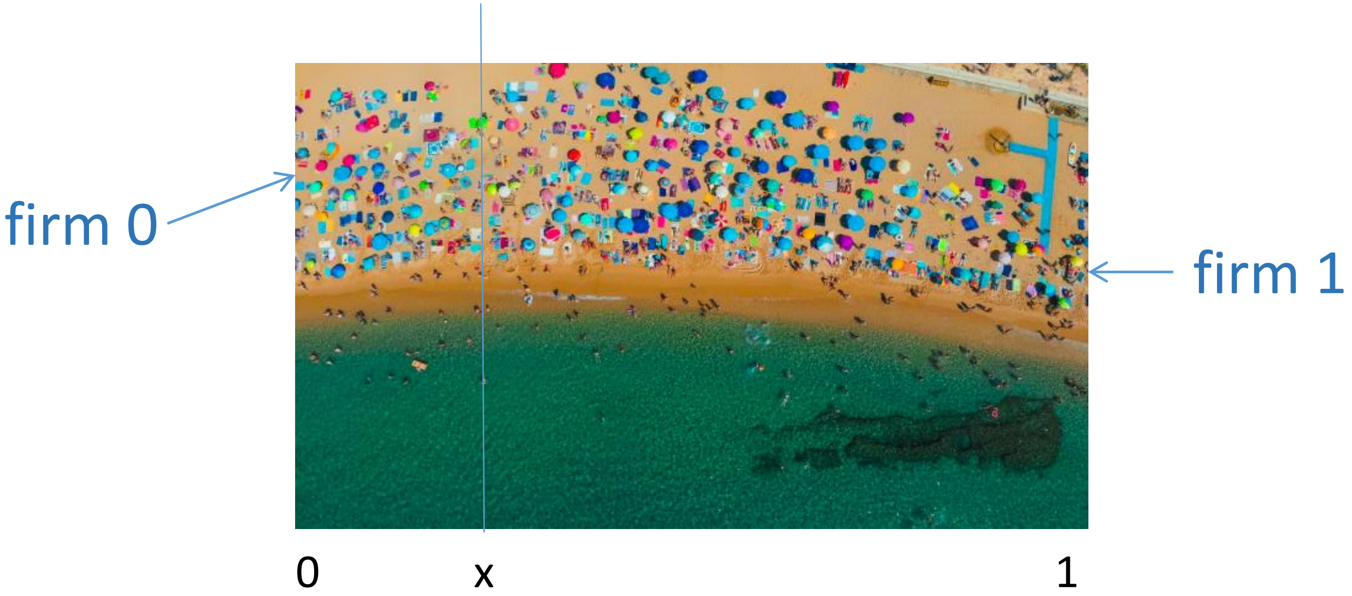
0 x 1

Firm 0 gets everyone to the left
of x, if that's the indifferent consumer; firm 1 gets everyone to the right.

Hotelling model

- A consumer indifferent between firms 0 and 1 is located at *x*, given by the following relation: $P_0 + tx = P_1 + t(1-x)$

- And then we can solve for *x*: $x = \dfrac{P_1 - P_0 - t}{2t}$

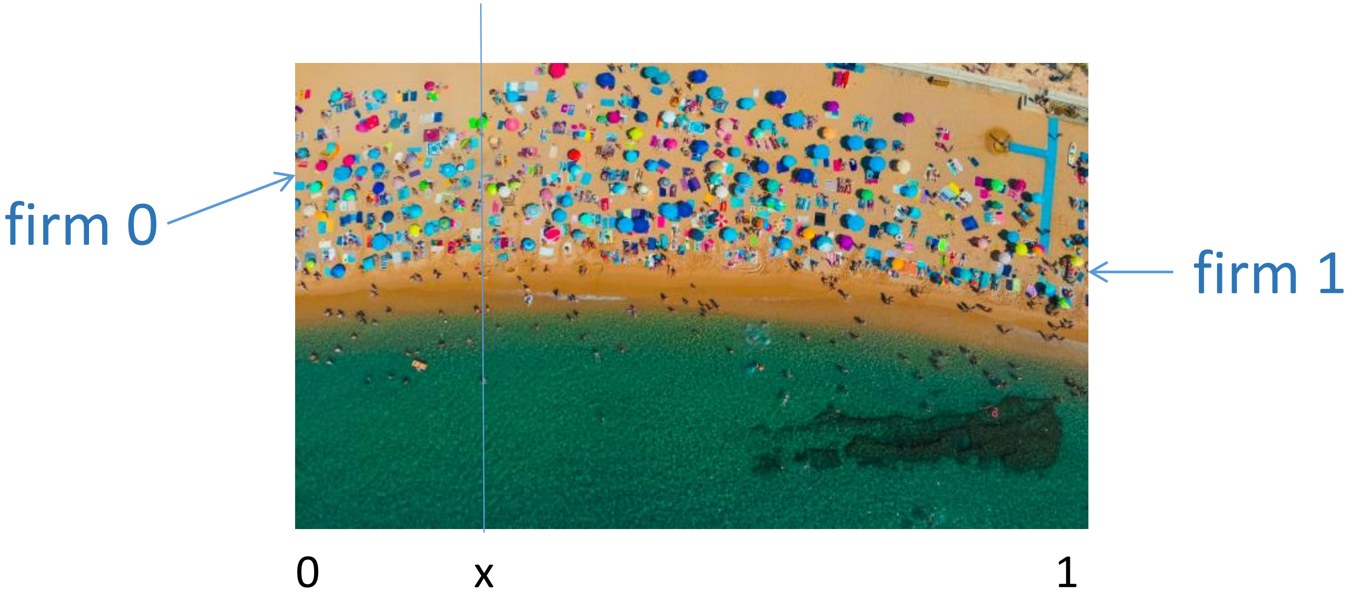


This is firm 0's demand. How do we know? Firm 0 gets everyone between 0 and *x*, and given our normalization that there is uniform distribution of consumers between 0 and 1, that's *(x-0)*, or *x*.

Hotelling model

- And here is firm 1's demand: $$(1-x) = \frac{P_0 - P_1 + t}{2t}$$

It's the rest of the unit interval.



Hotelling model

- Now that we have the demand functions that each firm faces, we can write down their profit functions:

$$\Pi^i(P_0, P_1) = \frac{1}{2t}(P_i - c)(P_j - P_i + t) \qquad \text{or}$$

$$\Pi = \frac{1}{2t}(P_i P_j - P_i^2 + P_i t - cP_j + cP_i - ct)$$

- Note that profits are a function of each firm's own price as well as the other firm's price.

demand firm i faces

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profit per sale---price minus marginal cost

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Hotelling model

- Take the derivative with respect to P_i, set equal to zero, and solve for P_i^*.

$$\frac{\partial \Pi}{\partial P_i} = \frac{1}{2t}(P_j + c + t - 2P_i)$$

$$P_j + c + t - 2P_i = 0$$

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$$\left.\begin{aligned} P_0^* &= \frac{P_1^* + c + t}{2} \\ P_1^* &= \frac{P_0^* + c + t}{2} \end{aligned}\right\}$$

Solve these simultaneously to obtain

$$P_0^* = P_1^* = c + t$$

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$$\Pi = \frac{1}{2t}(P_iP_j - P_i^2 + P_it - cP_j + cP_i - ct)$$

Hotelling model

- We can also plug those equilibrium prices back into the profit functions to obtain equilibrium profits:

$$\Pi^{0^*} = \Pi^{1^*} = \frac{t}{2}$$

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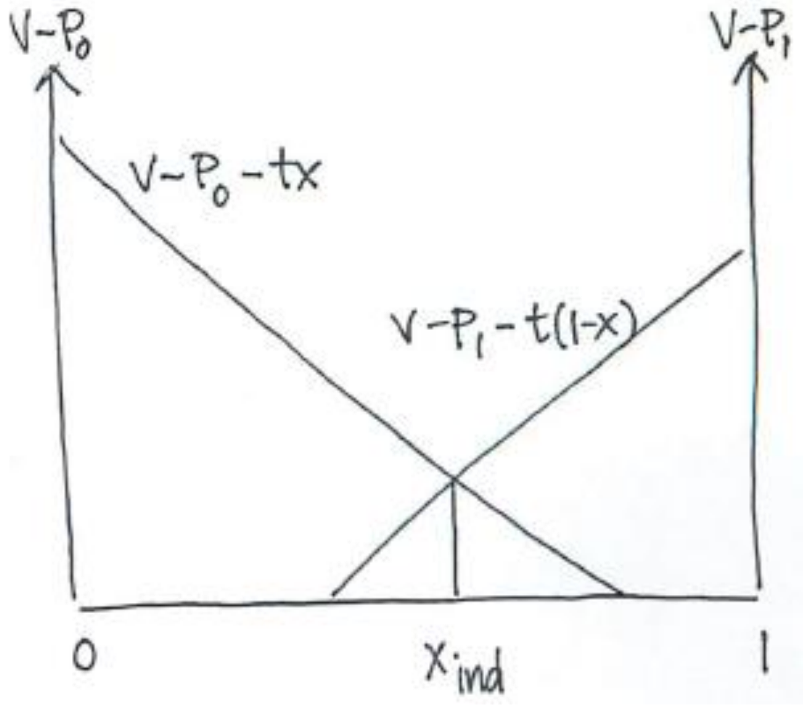

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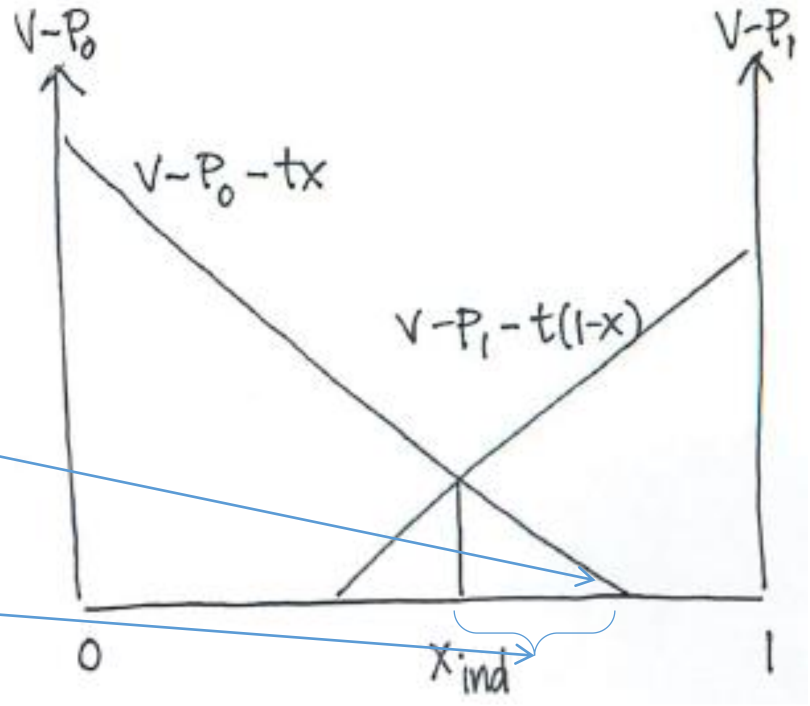
Consumers over to here would be willing to buy from firm 0.

These guys all have a better choice, though, firm 1.



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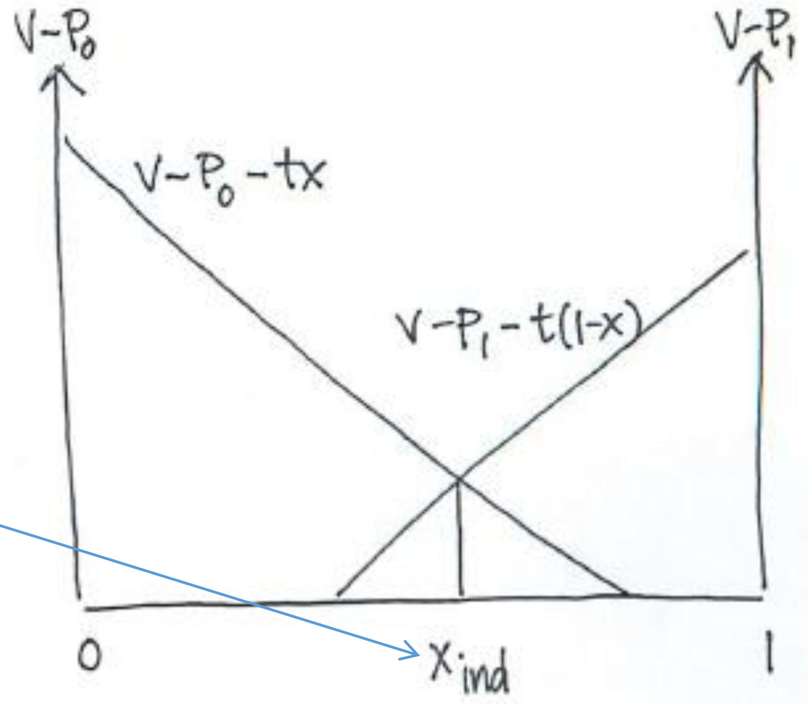
And here is the consumer indifferent between firms 0 and 1.



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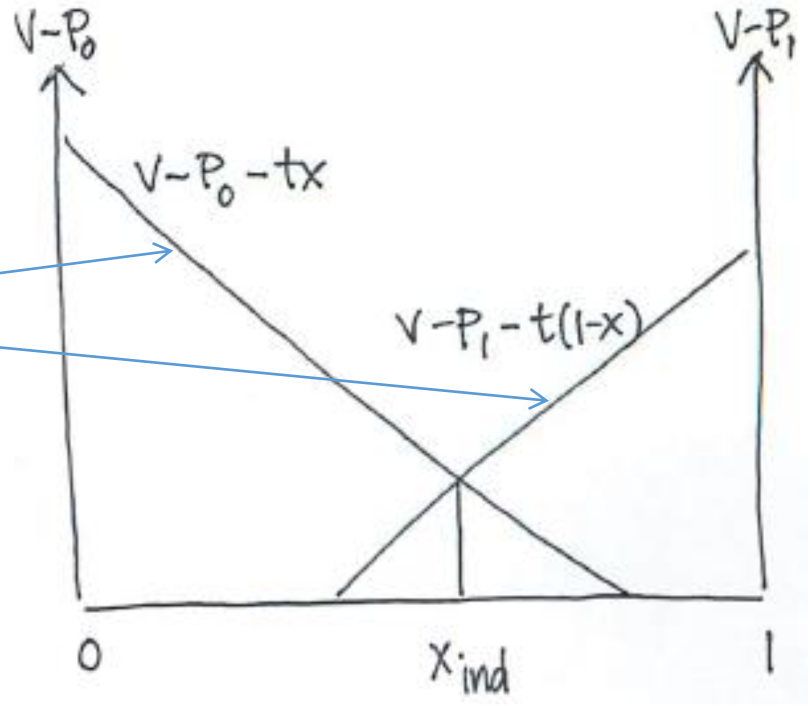
Note that the slopes of these two lines depend on t. As t increases, they become steeper.



Hotelling model

- Let's make a number of observations.
- First, we have physically identical goods and firms competing in price, just like Bertrand, but they price above marginal cost and make positive profits because they are differentiated.
- So, in other words, this differentiation allows firms to escape Bertrand.
- We used physical location as the dimension on which firms are differentiated. This is a *metaphor*, though, for any type of product differentiation that you could model horizontally. (By horizontally, we mean a type where some consumers would prefer 0 and some 1 at equal prices.)

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To lean into the food theme, here are some examples: white chocolate at 0
chocolate at 1; sweet champagne at 0 and dry champagne at 1; NY-style thi
0 and Chicago-style deep dish at 1.

Hotelling model

- Firms' markups are a function of consumer "transportation cost." If the differentiation is spatial, transportation cost can be the actual cost for consumers to travel from one firm to the other. More generally, it just means the cost for a consumer to move away from her preferred location in product space.
- As t increases, competition becomes less intense, as consumers close to each firm become more captive, giving the firm increased market power, conferring monopoly power on both firms if t is large enough.
- As t decreases, competition intensifies and approaches Bertrand at t goes to 0.

Hotelling model

- In any case, the markups allow firms who have fixed costs to be able to cover them (potentially) and, therefore, avoids the “paradox” part.
- As we have observed previously, firms want to escape Bertrand so that they are not forced to engage in cutthroat marginal cost pricing.
- In light of this model of product differentiation, one would imagine that firms would want to operate in markets where products were differentiated and they could charge above marginal cost.
- But do firms always have to accept the intrinsic degree of product differentiation in markets?

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- In light of this model of product differentiation, one would imagine that firms would want to operate in markets where products were differentiated and they could charge above marginal cost.
- But do firms always have to accept the intrinsic degree of product differentiation in markets? No, we saw an example last time (involving a ridiculous bathrobe) of firms trying to induce differentiation through branding.

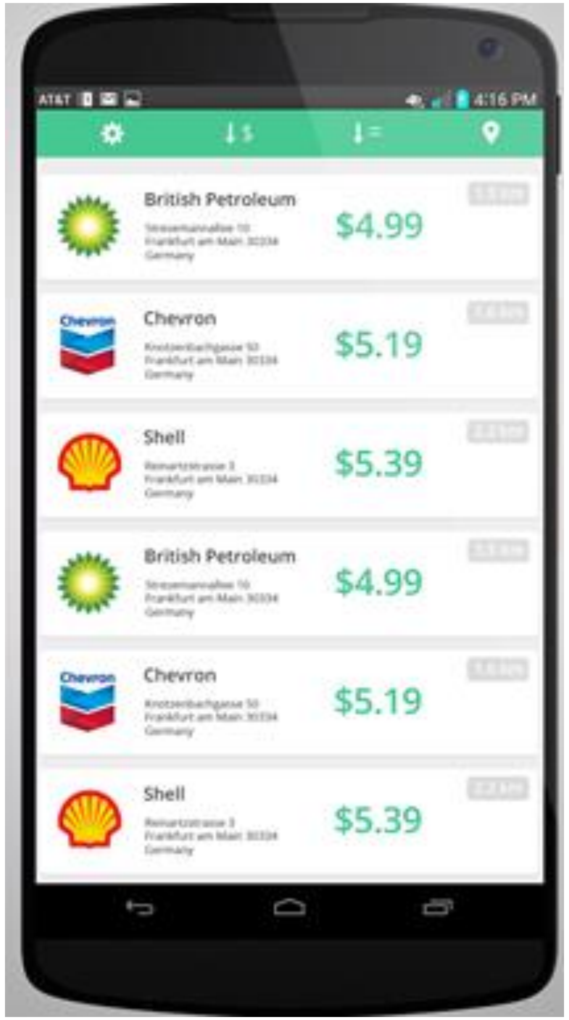
Fashion

- Clothing manufacturers are happy to indulge this desire for differentiation.
- It allows them to escape Bertrand and price their products well above marginal cost.

ridiculous bathrobe

And recall these comments as well.

Other types of differentiation



- Differentiation between goods need not be only in terms of physical attributes.
  - They can be spatially differentiated---you are unlikely to drive 15 minutes out of your way to save a few cents a gallon on gasoline.
  - They can be differentiated in terms of service quality or shopping experience---Skippy Peanut Butter is exactly the same product whether you buy it from *Shaws* or *Stop & Shop*, but you just like *Stop & Shop* better.

Escaping Bertrand

- The Hotelling model seems to suggest that escaping Bertrand can be pretty straightforward: if you differentiate yourself enough, you can achieve or approximate a monopoly position.
- That's not the whole story, of course. There are strong forces---entry and potential entry---that discipline attempts to differentiate and price at monopoly levels.