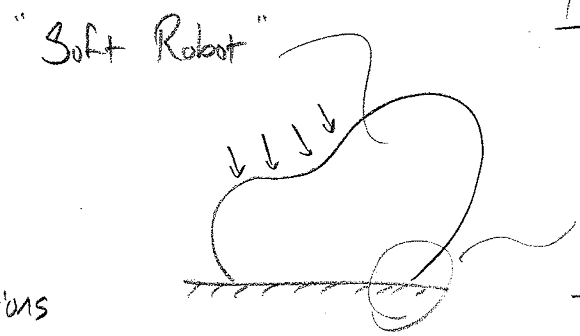
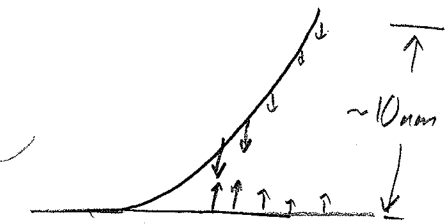
Work of Adhesion

C. Majidi 6/13/12





Potential Energy

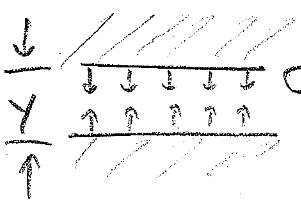
- Elastic Strain
- Force
- Pressure
- Adhesion

Surface Tractions
Mechanical Work

Surface/Interfacial Forces

- Electrostatic
- Permanent Polar (Capillary)
- Induced/Fluctuating Polarity (van der Waals)

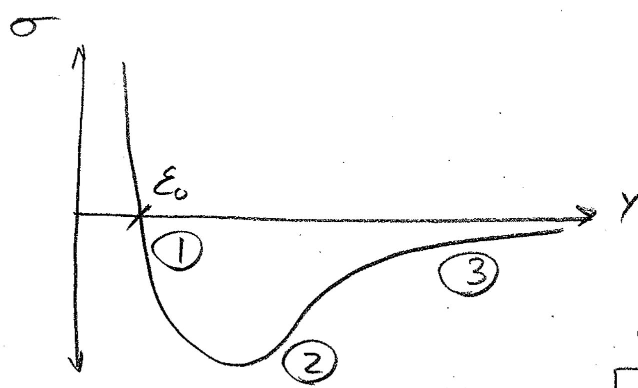
Lennard-Jones Potential

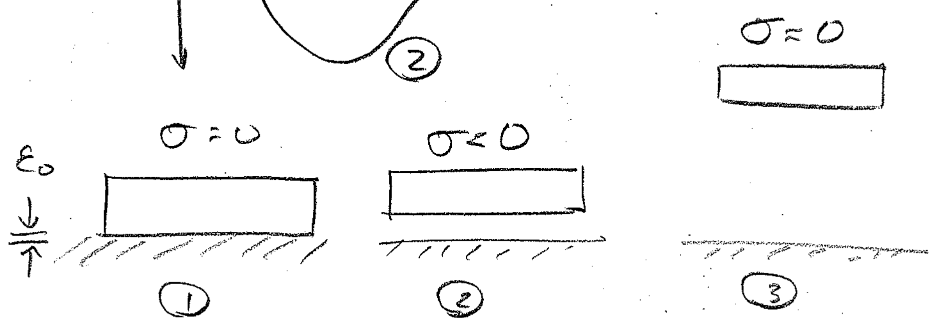


$$\sigma = \frac{\alpha}{y^{13}} - \frac{\beta}{y^{7}}$$

Nuclear Repulsion

v.d. Waals attraction





Work of Adhesion, $W_{ad} = \int_{\varepsilon}^{\infty} -\sigma \, dy$

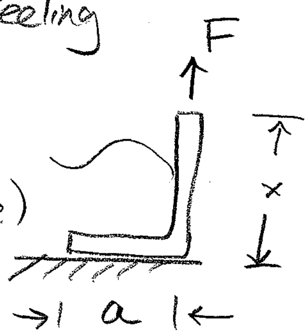
$\sim 10-100 \ mJ/m^2$

Potential Energy, $\Pi_{ad} = -[W_{ad}] \times [\text{Contact Area}]$

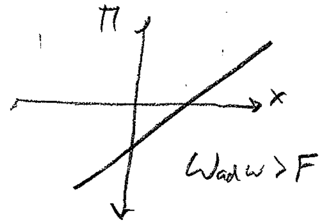
or $\Pi_{ad} = [W_{ad}] \times \underbrace{[\text{Non-Contact Area}]}_{\text{"Eulerian Coordinates"}}$

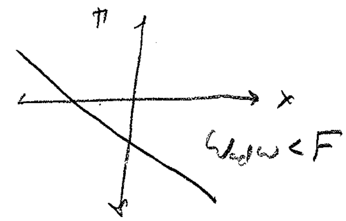
Example) Tape Peeling

Length, L (inextensible)



$$\Pi = -Fx - W_{ad} w a$$
$$= (W_{ad} w - F)x - W_{ad} w L \quad) \ a = L - x$$





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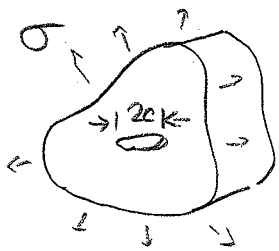
Griffith Balance



"Poisson's Ratio", $\nu = 1/2$ (incompressible)

$$\Pi = \underbrace{-\frac{\pi \sigma^2 c^2 (1-\nu^2)}{E}}_{\substack{\text{Mechanical} \\ \text{(Inglis 1913)}}} + \underbrace{2 W_{ad} c}_{\substack{\text{Adhesion} \\ \text{(Griffith 1920)}}}$$

At equilib., $\frac{d\Pi}{dc} = 0 \Rightarrow \boxed{\sigma = \sqrt{\frac{W_{ad} E}{\pi c (1-\nu^2)}}}$ Fracture strength

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Linear Elastic Fracture Mechanics (LEFM)

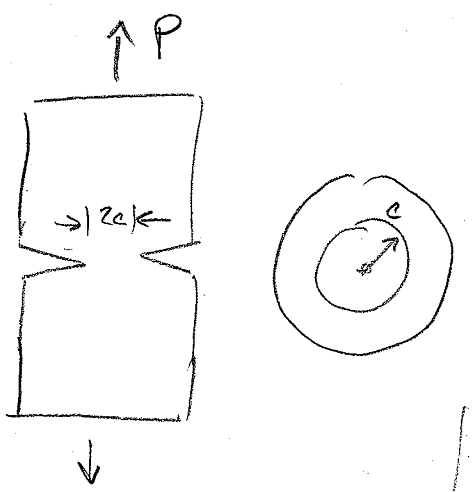
$$\boxed{G_c = W_{ad}}$$

$$G_c = \frac{(1-\nu^2)K_I^2}{E}$$, "Strain Energy Release Rate"

K_I : Mode I stress intensity factor

Griffith Crack : $\begin{cases} \Pi_{ad} = 2W_{ad}c \\ K_I = \sigma\sqrt{2\pi c} \end{cases}$

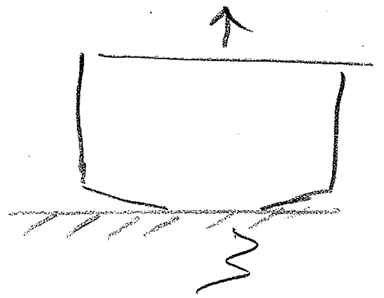
Circumferential Crack



$$\left.\begin{aligned} K_I &= \frac{P}{2c\sqrt{\pi c}} \\ G_c &= \frac{(1-\nu^2)P^2}{4\pi E c^3} \end{aligned}\right\} \Rightarrow P = \sqrt{\frac{4\pi E W_{ad} c^3}{1-\nu^2}}$$

$$\boxed{P_0 = \sqrt{\frac{4\pi E W_{ad} R^3}{1-\nu^2}}}$$

Contact Mechanics



$$G = \frac{(1-\nu^2)K_I^2}{2E}$$

Flat Punch: $P_0 = \sqrt{\frac{8\pi E W_{ad} R^3}{1-\nu^2}}$

Hemisphere (JKR theory): $P_0 = \frac{3}{2}\pi W_{ad} R$

"Effects of contact shape on the scaling of biological attachments"
Spolenak, Gorb, Gao, Arzt
Proc. Roy. Soc. A v461, 305-319 (2005)

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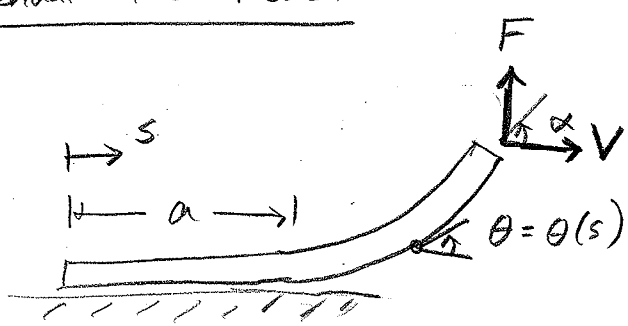
Kendall Peel Model



$\theta(a) = 0$

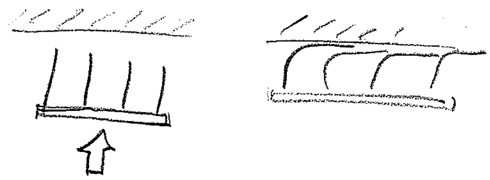
$\theta(L) = \alpha$ (if constrained)

$$\Pi = \int_a^L \Big\{ \underbrace{\tfrac{1}{2} D\left(\tfrac{d\theta}{ds}\right)^2}_{\text{Elastic Strain}} - \underbrace{F\sin\theta - V\cos\theta}_{\text{Mechanical Load}} \Big\} ds - \underbrace{\overset{W_{ad} \times [\text{width}]}{\omega}\, a}_{\text{Adhesion}}$$

Minimize Π w.r.t. θ and a:

$$\left.\begin{aligned} \theta'' &= \frac{V}{D}\sin\theta - \frac{F}{D}\cos\theta \\ \theta'(a) &= \sqrt{\frac{2w}{D}} \\ \theta(a) &= 0 \\ \theta(L) &= \alpha \;\; \underline{\underline{\text{or}}} \;\; \theta'(L) = \frac{M}{D} \end{aligned}\right\}$$

Microfiber Adhesion



$V = F = 0$

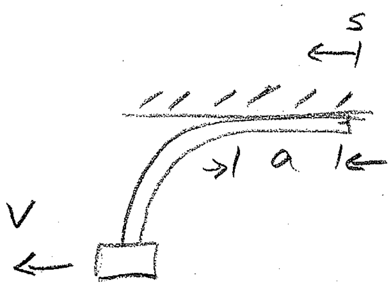
$\alpha = \frac{\pi}{2}$

$$\boxed{a = L - \frac{\pi}{2}\sqrt{\frac{D}{2w}}}$$

Stable Side Contact $\Leftrightarrow$ $L > \frac{\pi}{2}\sqrt{\frac{D}{2w}} = L_{cr}$

<u>Shear-Activated Adhesion</u>

$F = 0,\ \alpha = \frac{\pi}{2},\ V \geq 0$



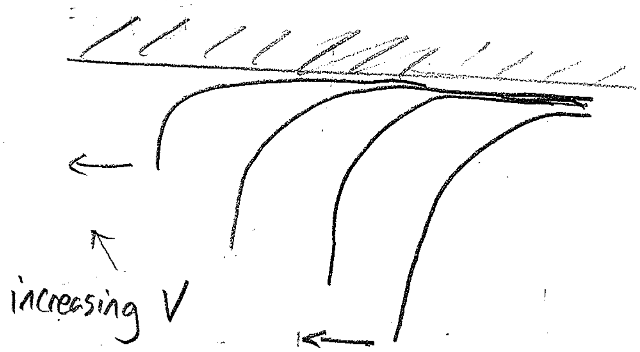
$$a = \frac{c}{k}\left\{K(c^2) - F\left(\frac{\pi}{4}, c^2\right)\right\}$$

$$c = \sqrt{\frac{1}{1 + w/2V}}$$

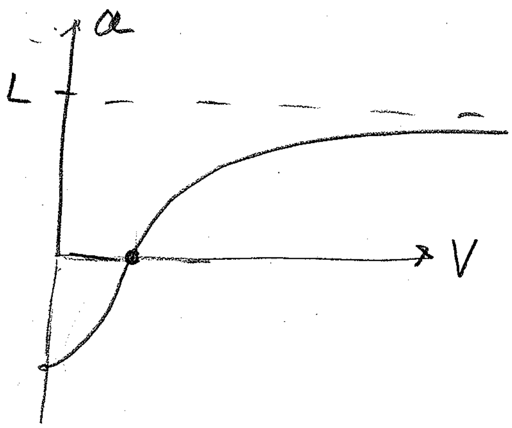
$$k = \sqrt{\frac{V}{D}}$$

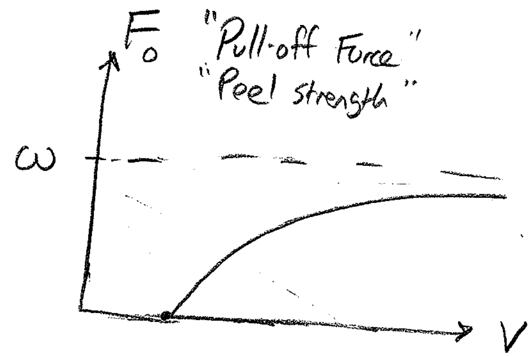
$F(\phi, m)$: "Elliptic Integral of the First Kind"

$K(m) = F\left(\frac{\pi}{2}, m\right)$ = "Complete Elliptic Integral of the First Kind"

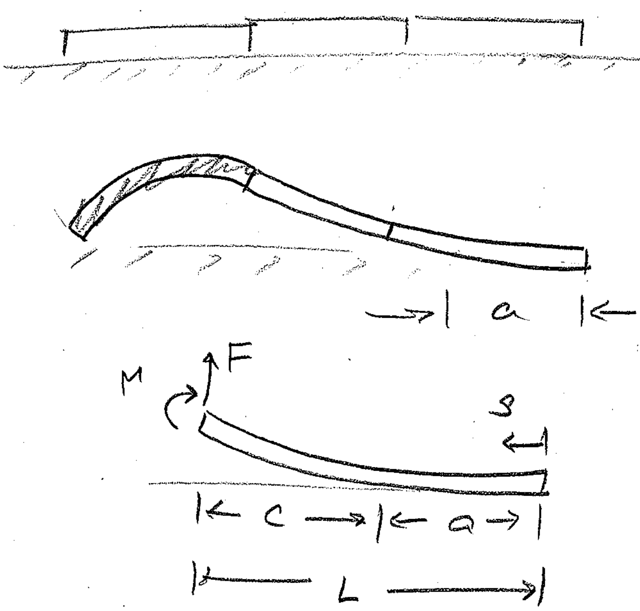


$V = 0 \Rightarrow a \leq 0$ (no contact)





Soft Robot Undulation



$$\theta'' = -\frac{F}{D}\cos\theta \approx -\frac{F}{D}$$

$$\theta' = -\frac{F}{D}s + C_1$$

$$\theta = -\frac{F}{2D}s^2 + C_1 s + C_2$$

$$\theta(a) = 0, \quad \theta'(a) = \sqrt{\frac{2\omega}{D}}, \quad \theta'(L) = \frac{M}{D}$$

$$C_1 = \frac{M}{D} + \frac{FL}{D} \equiv \sqrt{\frac{2\omega}{D}} + \frac{Fa}{D}$$

$$\therefore \boxed{M + F(L-a) = \sqrt{2\omega D}}$$

"Moment Discontinuity"

$$M_{cd} = \sqrt{2\omega D}$$