Towers Watson University

Loss Reserving: Bootstrapping Basics and Case Studies

Presented by Dave Otto and Jamie Mackay

March 29th 2011

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Loss Reserving: Bootstrapping Basics and Case Studies

It is becoming increasingly important to understand the variability inherent in a reserve estimate. However, many actuaries lack experience and sufficient knowledge about the techniques used to estimate reserve uncertainty.

The Towers Watson University Loss Reserving webcast will provide property & casualty (P&C) insurers with an opportunity to explore relevant issues, including:

- A high-level overview of the bootstrapping methodology
- Diagnostics and adjustments that are commonly performed to produce reasonable estimates of reserve uncertainty
- Practical case studies
- The bootstrapping feature on Towers Watson ResQ reserving software

Agenda

Overview

- Introduction to Reserve Uncertainty
- Brief summary of the Bootstrapping process

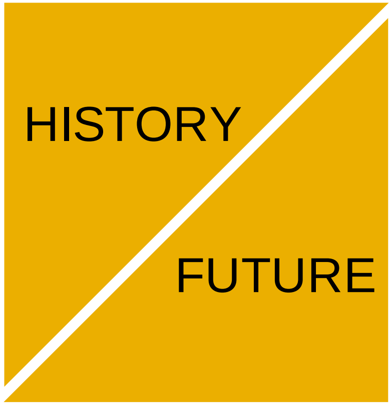
Dissection of Bootstrapping with Case Studies

- Selecting LDFs
  - Using Benchmark Development Patterns
  - Dealing with Outliers
  - Changing Development Patterns
  - Average Factor Selection
- Dealing with Sparse Data
  - Using Bootstrap Consolidation
  - Using Parametric Bootstrap

Introduction

Actuarial Reserving in a Nutshell

- Actuarial reserving has been conceptually described as a process of squaring up a triangle
- Important to note that the actuary's forecast of the future is an ESTIMATE, and as such, is subject to uncertainty



Actuarial Reserving in a Nutshell

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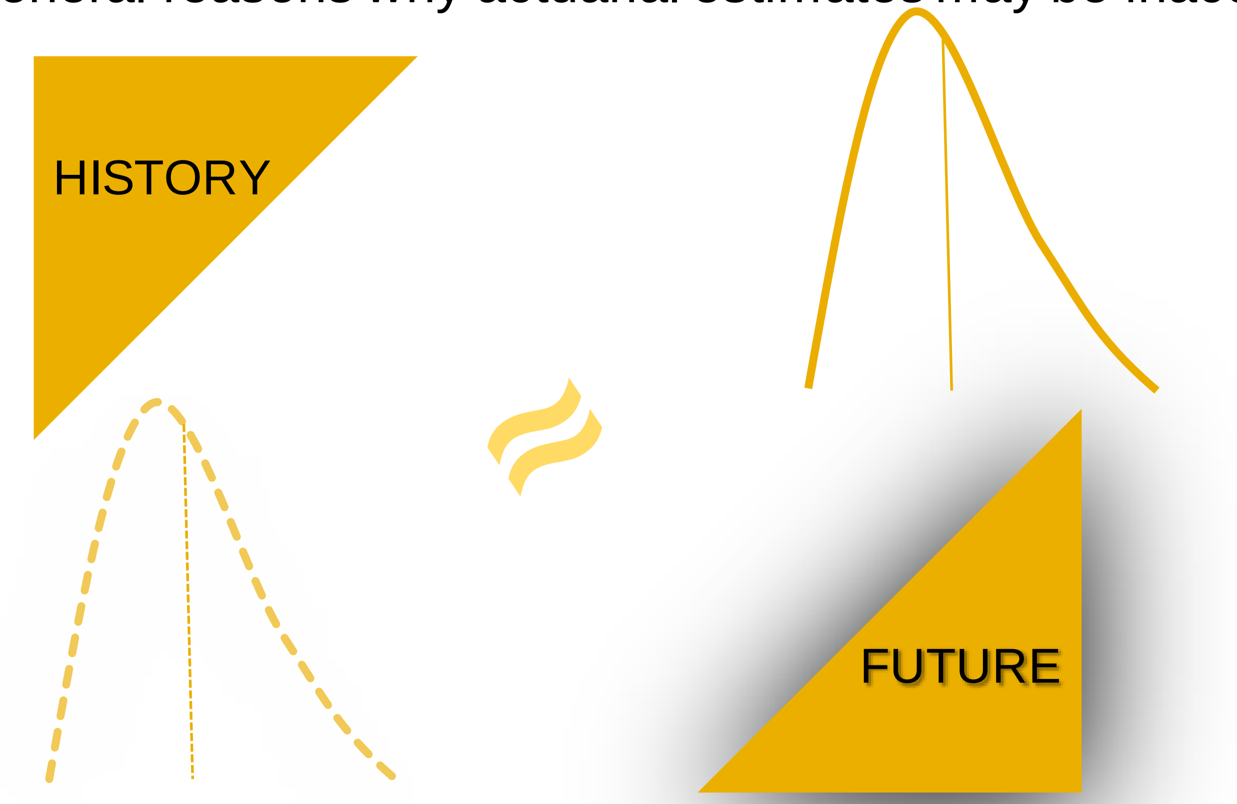
Why Are Estimates Subject to Uncertainty?

- Two general reasons why actuarial estimates may be inaccurate



1. Inability to accurately define the distribution from which past events have been generated = Estimation Variance

Why Are Estimates Subject to Uncertainty?

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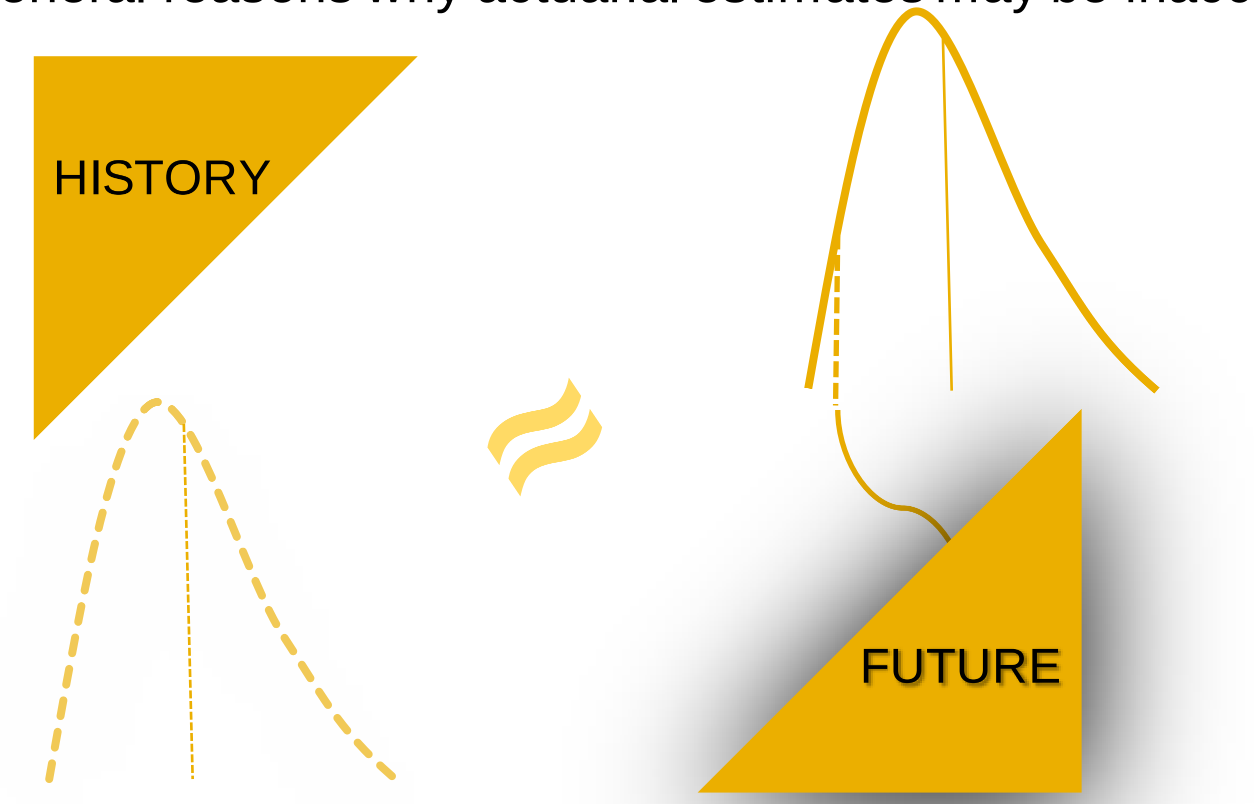


2. Inability to accurately predict which single outcome from the distribution will occur at a given time = Process Variance

Mathematical Representation of Uncertainty?

- How do we mathematically combine these two uncertainties to produce the overall uncertainty in our prediction?

1. Estimation Variance = Inability to accurately define the distribution from which past events have been generated
2. Process Variance = Inability to accurately predict which single outcome from the distribution will occur at a given time

Prediction Variance = Estimation Variance + Process Variance

Mathematical Representation of Uncertainty?

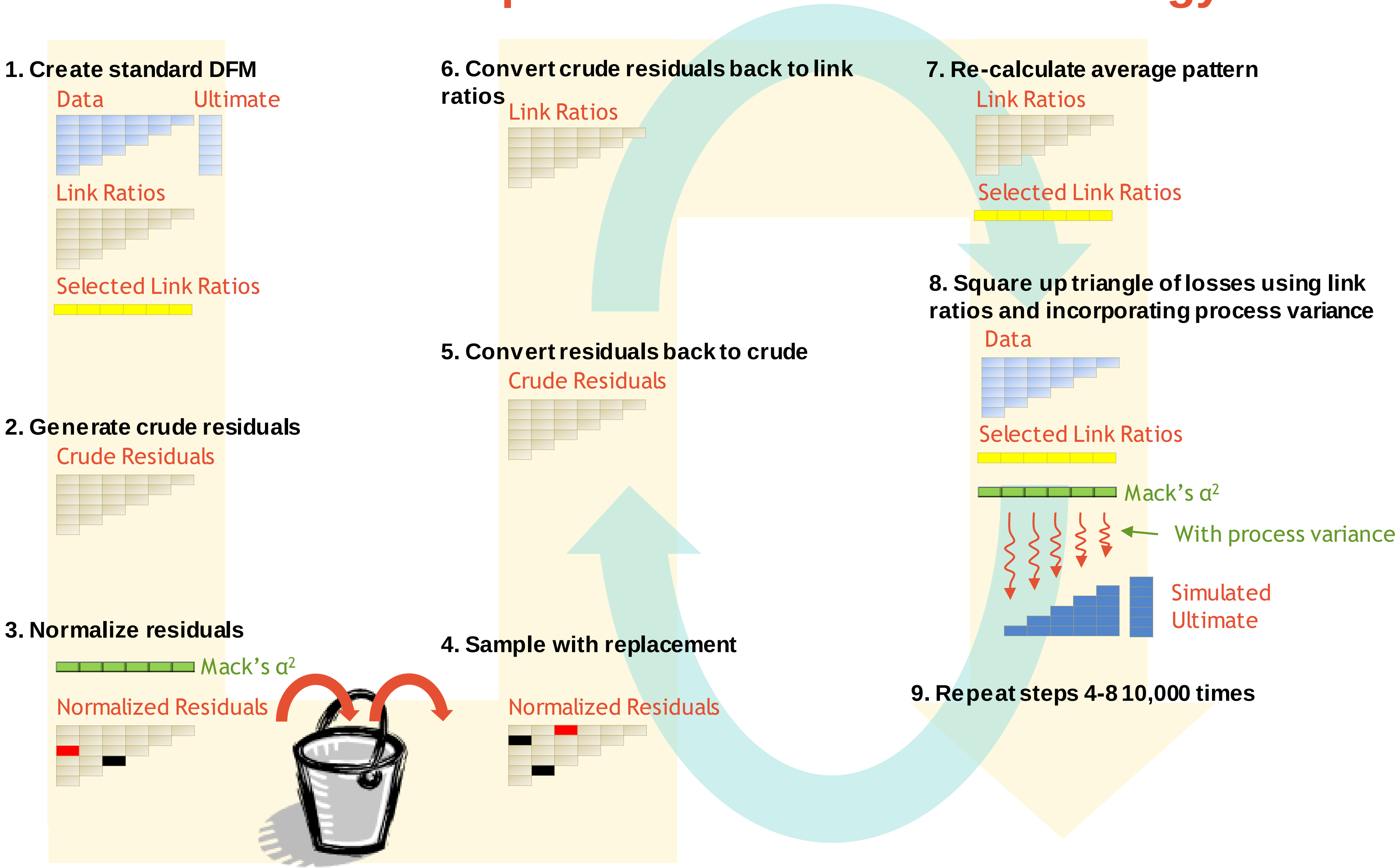
- We could calculate the standard deviation of the forecast ("prediction error") analytically, taking account of estimation uncertainty
- Bootstrapping gives a distribution of parameters, hence an estimate of the estimation error, without the hard math
- When supplemented by a second simulation step incorporating the process error, a distribution of the forecast is generated

However...

- as in all actuarial approaches, the blind application of the method, without understanding the nuances and mechanics of the calculations and assumptions, will lead to trouble
- In order to produce a valid and meaningful model, we must understand the impact of both the data and also our decision-making process during the creation of our uncertainty analysis

Overview of Bootstrapping Process

Overview of Bootstrap Process – Mack Methodology



Overview of Bootstrap Process Estimation Uncertainty

1. Create standard DFM

Data Ultimate

Link Ratios

Selected Link Ratios

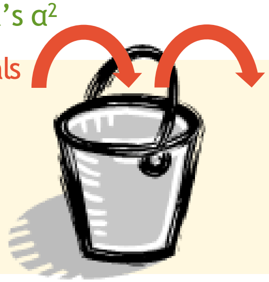
2. Generate crude residuals

Crude Residuals

3. Normalize residuals

Mack's α^2

Normalized Residuals



4. Sample with replacement

Normalized Residuals

5. Convert residuals back to crude

Crude Residuals

6. Convert crude residuals back to link ratios

Link Ratios

7. Re-calculate average pattern

Link Ratios

Selected Link Ratios

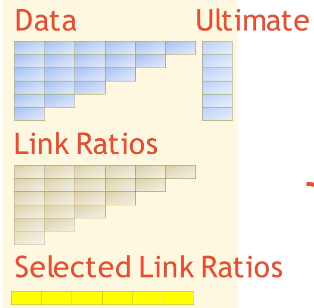
8. Square up triangle of losses using link

- Estimation uncertainty is introduced in the selection of our link ratios and is incorporated in steps 1 to 7
- Note that Mack's α^2 is used during the standardization of the residuals prior to re-sampling

9. Repeat steps 4-8 10,000 times

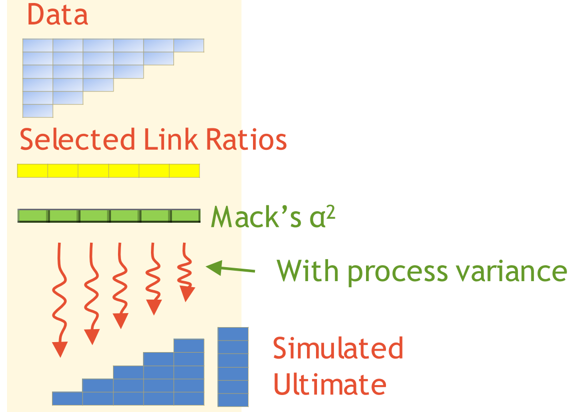
Overview of Bootstrap Process Process Uncertainty

1. Create standard DFM



- Though process variance is influenced by the selected link ratios and their residuals, it is not introduced until step 8
- The process variance is influenced by the same Mack's α^2 value as used in the standardization of the residuals

8. Square up triangle of losses using link ratios and incorporating process variance



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Overview of Bootstrap Process – Mack Methodology

- Though process variance is influenced by the selected link ratios and their residuals, it is not introduced until step 8
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1. Create standard DFM

Data

Ultimate

Link Ratios

Selected Link Ratios

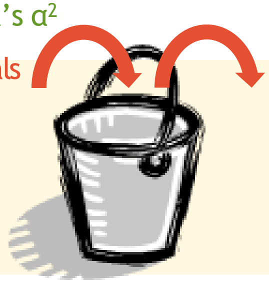
2. Generate crude residuals

Crude Residuals

3. Normalize residuals

Mack's α^2

Normalized Residuals



4. Sample with replacement

Normalized Residuals

7. Re-calculate average pattern

Link Ratios

Selected Link Ratios

8. Square up triangle of losses using link ratios and incorporating process variance

Data

Selected Link Ratios

Mack's α^2

With process variance

Simulated Ultimate

9. Repeat steps 4-8 10,000 times

Selecting LDFs

Selecting LDFs

- The assumed model underlying Mack's analytical approach depends on a chain-ladder projection using a volume-weighted average of all LDF's for each development period

  Why volume-weighted average?

- unbiased (but so is the simple average)
- volume weighted average has the smallest variance
  - One of the principles of theory of point estimation: among several unbiased estimators preference should be given to the one with the smallest variance

Selecting LDFs

- A Bootstrapping approach does allow for models that differ from this assumed ideal and will tend to provide a more robust model than an analytical application
- However, the mechanics of such changes to your underlying model require consideration
- In this section, we are going to look at just a few examples of the potential impact of data and model characteristics:
  - Using benchmark development patterns
  - “Outliers” in your model
  - Changing development patterns
- Before starting, it’s worth remembering the following:

Selecting LDFs

- The results, and validity, of a Bootstrap model is entirely dependent on the chain-ladder model (or Development Factor Model, DFM) underlying the method
- Though it may seem obvious, when creating an DFM for the purpose of Bootstrapping, you need to bear in mind that you are not only the projecting the future development and ultimate, but also the variability around it
- This means you need to pay attention to, not only what's <u>in</u> your model, but also what's being <u>excluded</u>
  - Bootstrapping will try to utilize all information in the triangle

Selecting LDFs

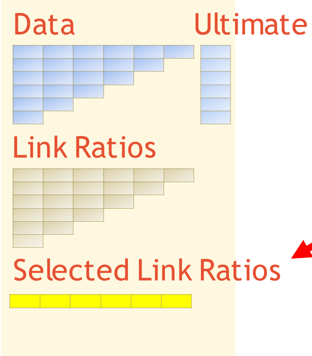
Using Benchmark Development Patterns

Selecting LDFs Using Benchmark Development Patterns

- Selecting a loss development factor that is not calculated on the underlying data will mean that that development period will not produce any estimation variance, and only process variance will be calculated
  - Industry patterns
  - Hard-coded selections
- This is because, although that data point *will* produce a residual, during the re-fitting stage of the Bootstrap, it will always produce the same LDF…

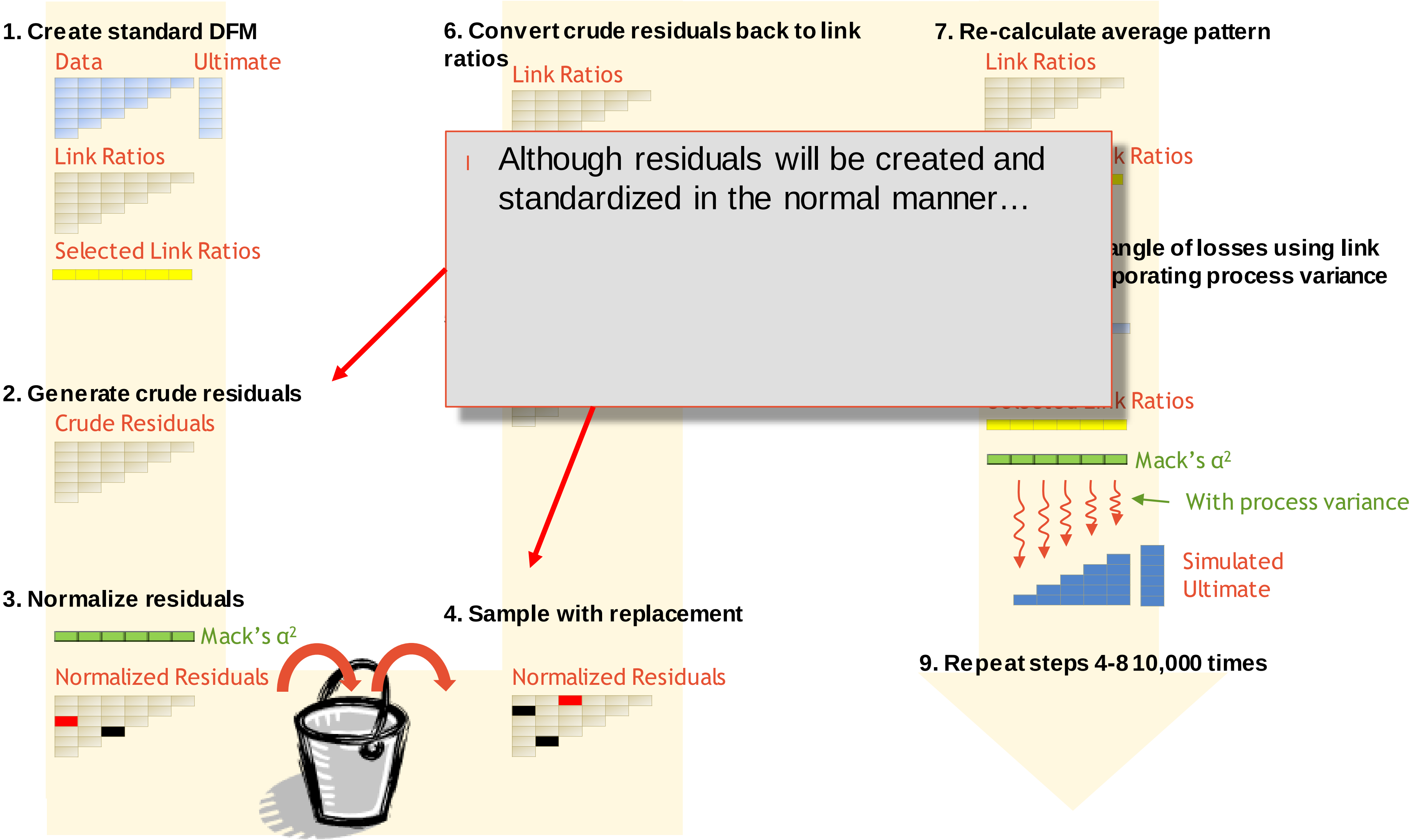
Selecting LDFs Using Benchmark Development Patterns

1. Create standard DFM

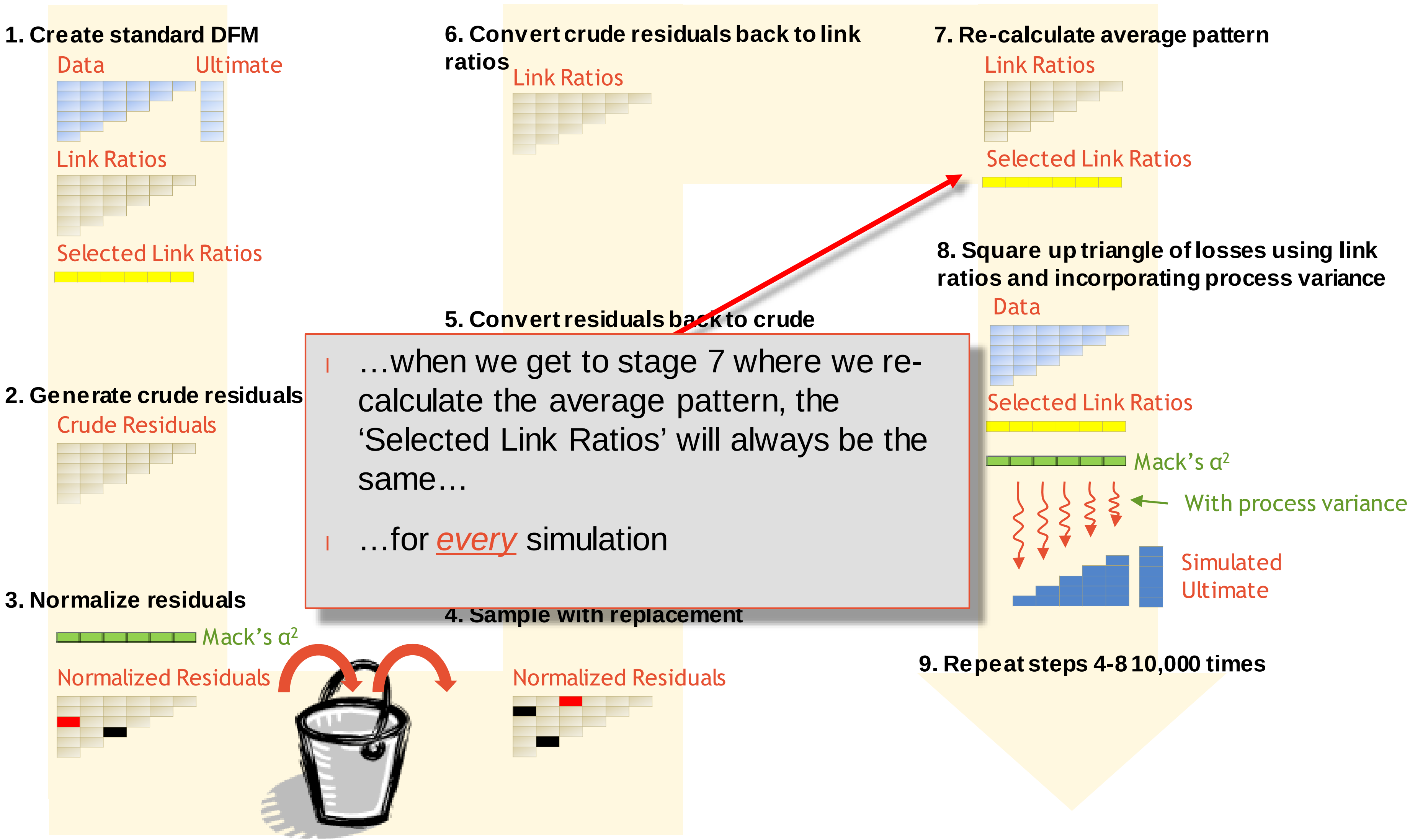


- Imagine our ‘Selected Link Ratios’ are input as values-only, i.e. they are not based on the data underlying the DFM
- This may be the case if a benchmark development pattern is used

Selecting LDFs Using Benchmark Development Patterns

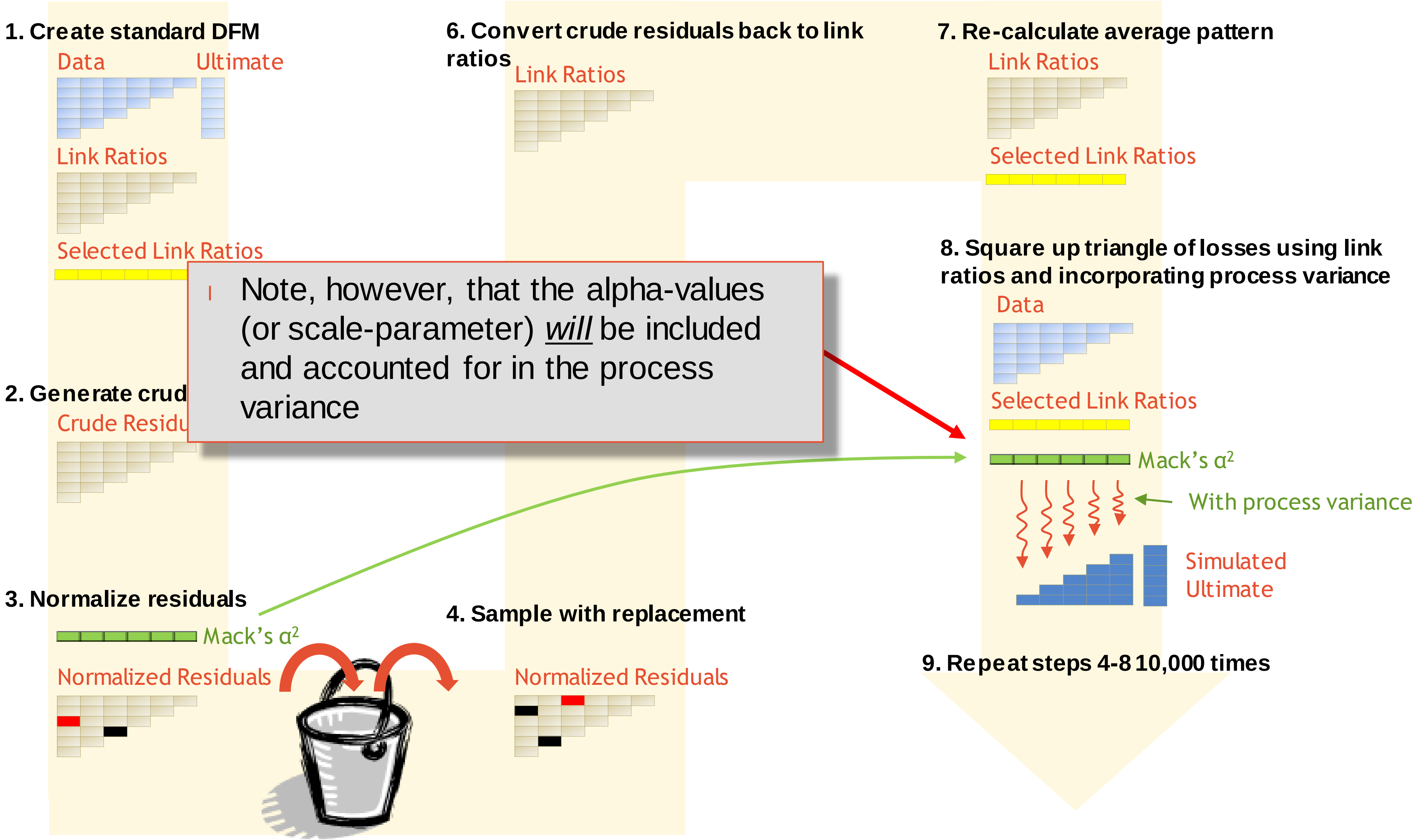


Selecting LDFs Using Benchmark Development Patterns



- …when we get to stage 7 where we re-calculate the average pattern, the 'Selected Link Ratios' will always be the same…
- …for *every* simulation

Selecting LDFs Using Benchmark Development Patterns



- Note, however, that the alpha-values (or scale-parameter) *will* be included and accounted for in the process variance

Selecting LDFs Using Benchmark Development Patterns

- Each time a ‘values-only’, or benchmark development ratio is used in a Mack model, it essentially removes the estimation variance from that entire development period
- If a benchmark pattern is used for the entire development, estimation variance is effectively removed from your model altogether, leaving only the process variance generated by the residuals

  (note that the above statement, i.e. that using a benchmark pattern removes the estimation variance, is not true for an ODP model, though the effect on the resulting prediction error is similar between the two models).

Selecting LDFs Using Benchmark Development Patterns

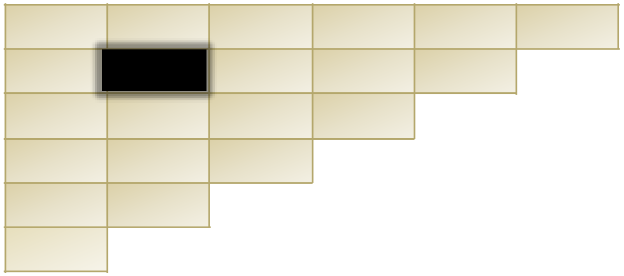
Case Study (1)

Selecting LDFs

Dealing with Outliers

Selecting LDFs Dealing with Outliers

- “Outliers” (and their impact) can be identified and dealt with in a number of different ways. The decision to either include or exclude a data point or to specifically adjust for it in the uncertainty model is entirely dependent on the line of business and the actuary’s knowledge of how it affects the results and if this adjustment (or lack thereof) is reasonable.
  - Is the outlier representative of a type of occurrence that could be expected to occur again in the future?
  - If so, is the expectation that this type of occurrence will occur only during the same lag or possibly at some other development period?
- However, understanding the influence of development points in each part of the Bootstrapping process is an important step toward making an informed decision about the ‘correct’ course of action in each instance.



Selecting LDFs Dealing with Outliers

1. Create standard DFM

Data

Ultimate

Link Ratios

Selected Link Ratios

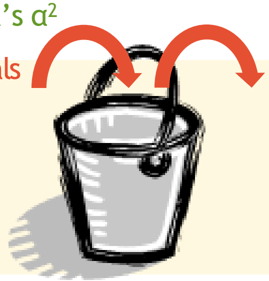
2. Generate crude residuals

Crude Residuals

3. Normalize residuals

Mack's α^2

Normalized Residuals



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Selected Link Ratios

Mack's α^2

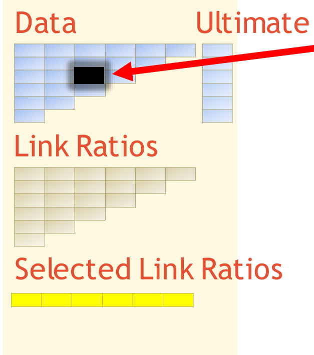
With process variance

Simulated Ultimate

9. Repeat steps 4-8 10,000 times

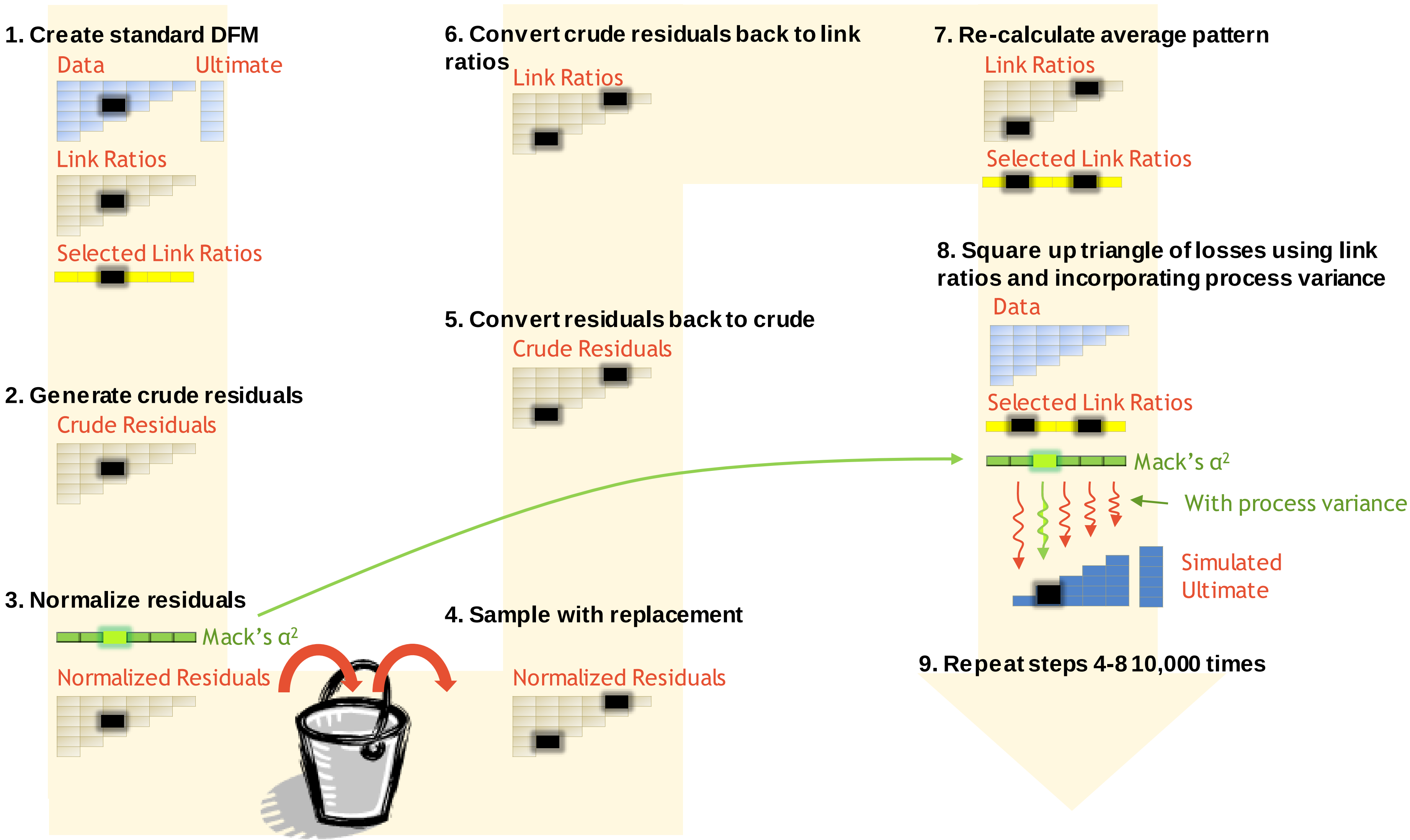
Selecting LDFs Dealing with Outliers

1. Create standard DFM



- Imagine we have a large, ‘extraordinary’ payment within our triangle of losses
- This data point will feed all the way through the projection and Bootstrap analysis…

Selecting LDFs Dealing with Outliers



Selecting LDFs Dealing with Outliers

- A large outlier will do the following things:
  - Affect underlying development pattern at the lag at which it occurs
  - Affect projected mean reserve
  - Produce a large residual that can be re-sampled anywhere in the triangle, affecting the estimation variance throughout the triangle
  - Will affect the process variance projected for the development period at which it occurs
- Some courses of action to address the impact of the outlier:
  - Alter the underlying DFM
  - Alter the Bootstrapping model

  Note: there are many ways to adjust your models to deal with outliers and we will address just a few of these today.

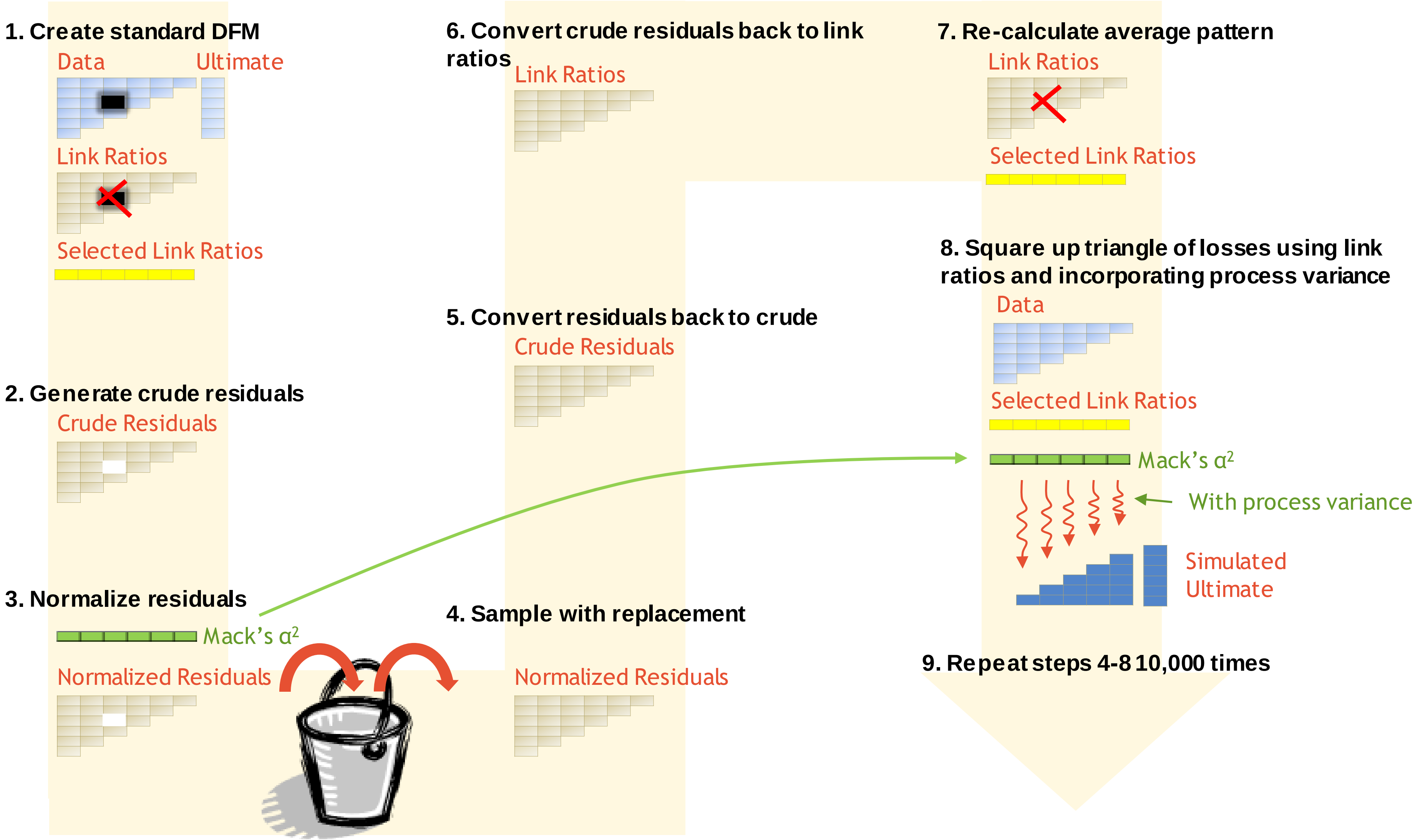
Selecting LDFs Dealing with Outliers

Excluding the 'extraordinary' LDF in the DFM

- Removing a data point from the underlying chain ladder model (in the DFM module) will:
  - Remove that data point from the calculation of the average development pattern
  - Remove that data point from the calculation of the mean reserve
  - Remove the residual produced from the set sampled
  - Remove the data point from the calculation of Mack's α^2 value

  i.e. it will be ignored by the model in it's entirety
- The decision-making process regarding whether or not to exclude a development ratio from an uncertainty model is very similar to that used when performing a chain-ladder method for the purposes of reaching a best-estimate, except we need to bear in mind that our chief concern here is ensuring that the historical development represents what we believe to be a good guide to future volatility (as opposed to simply it's impact on the average development pattern and/or ultimate)

Selecting LDFs Dealing with Outliers

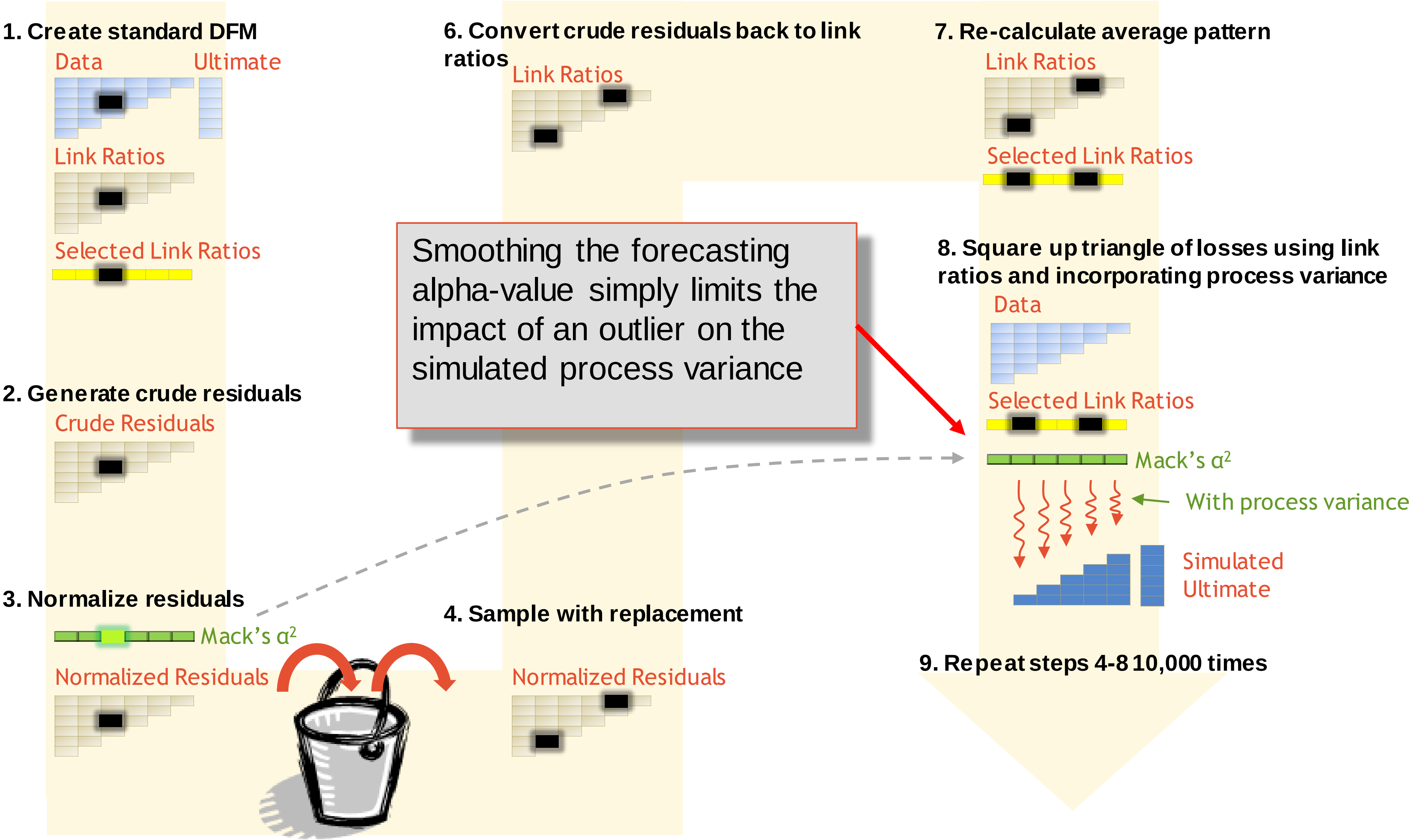


Selecting LDFs Dealing with Outliers

Manually adjusting the α value or scale parameter in Bootstrapping

- Within ResQ, the user is able to manually adjust either the alpha values (Mack models) or the scale parameter (for ODP models):
  - for calculating the residuals, or
  - for use in forecasting

Selecting LDFs Dealing with Outliers



Selecting LDFs Dealing with Outliers

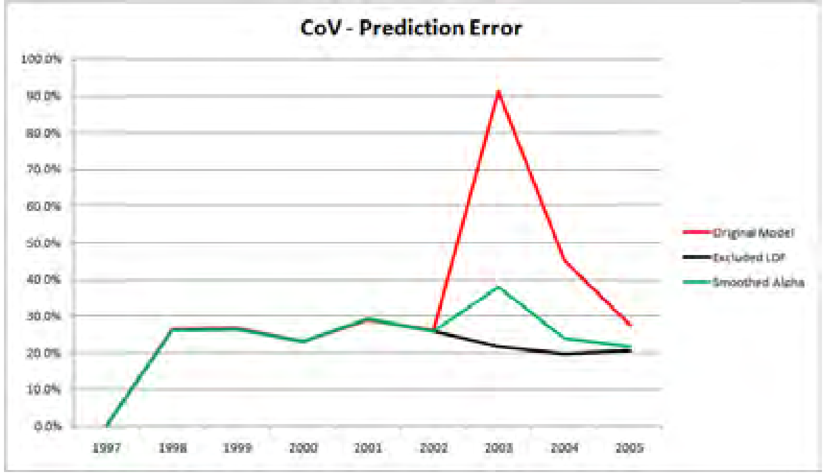
Manually adjusting the α value or scale parameter in Bootstrapping

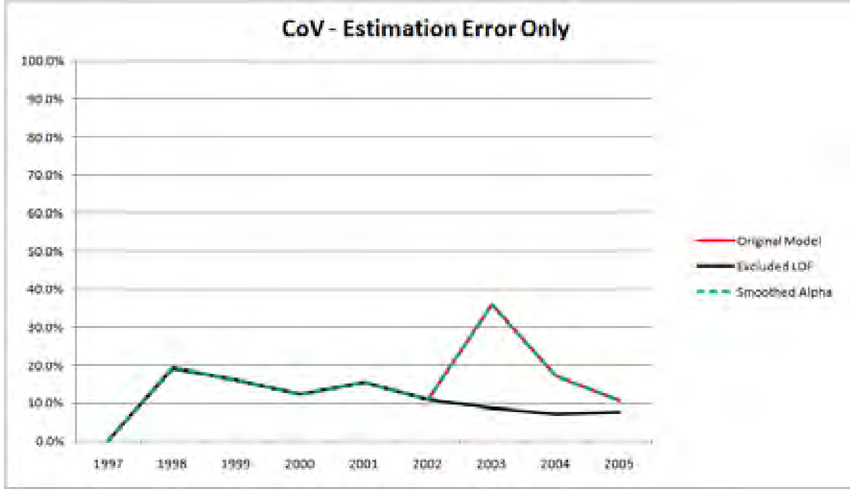
- We would normally only recommend this approach for advanced users and the following should be borne in mind:
  - Manually adjusting α or φ for use in <u>forecasting</u> will suppress (or increase) the process variance incorporated, but will not affect the calculation of standardized residuals or the underlying pattern that will influence the mean of your simulated projections
  - Manually adjusting the α or φ for standardizing <u>residuals</u> at a certain period will impact the calculation of all standardized residuals at that lag, which could be re-sampled anywhere else throughout the triangle. Reducing the variance term, α^2, will create larger standardized residuals which could potentially increase the variability elsewhere. We do not recommend adjusting these values.

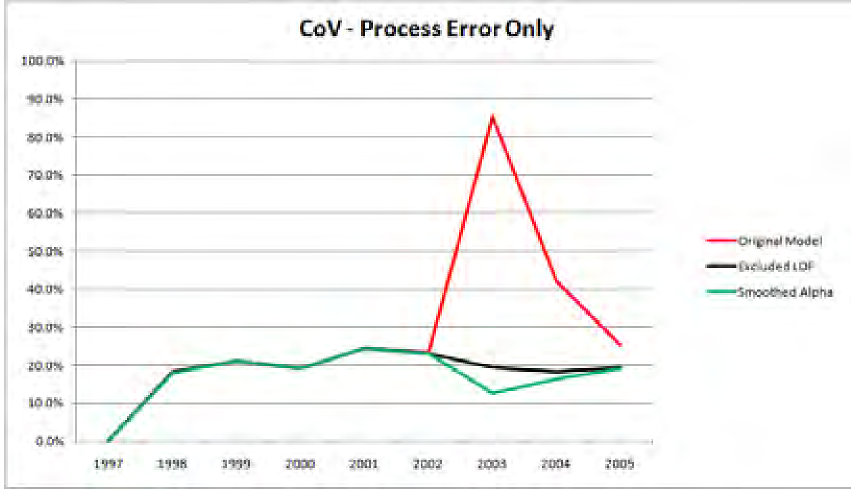
Selecting LDFs Dealing with Outliers

Case Study (2)

Selecting LDFs Dealing with Outliers

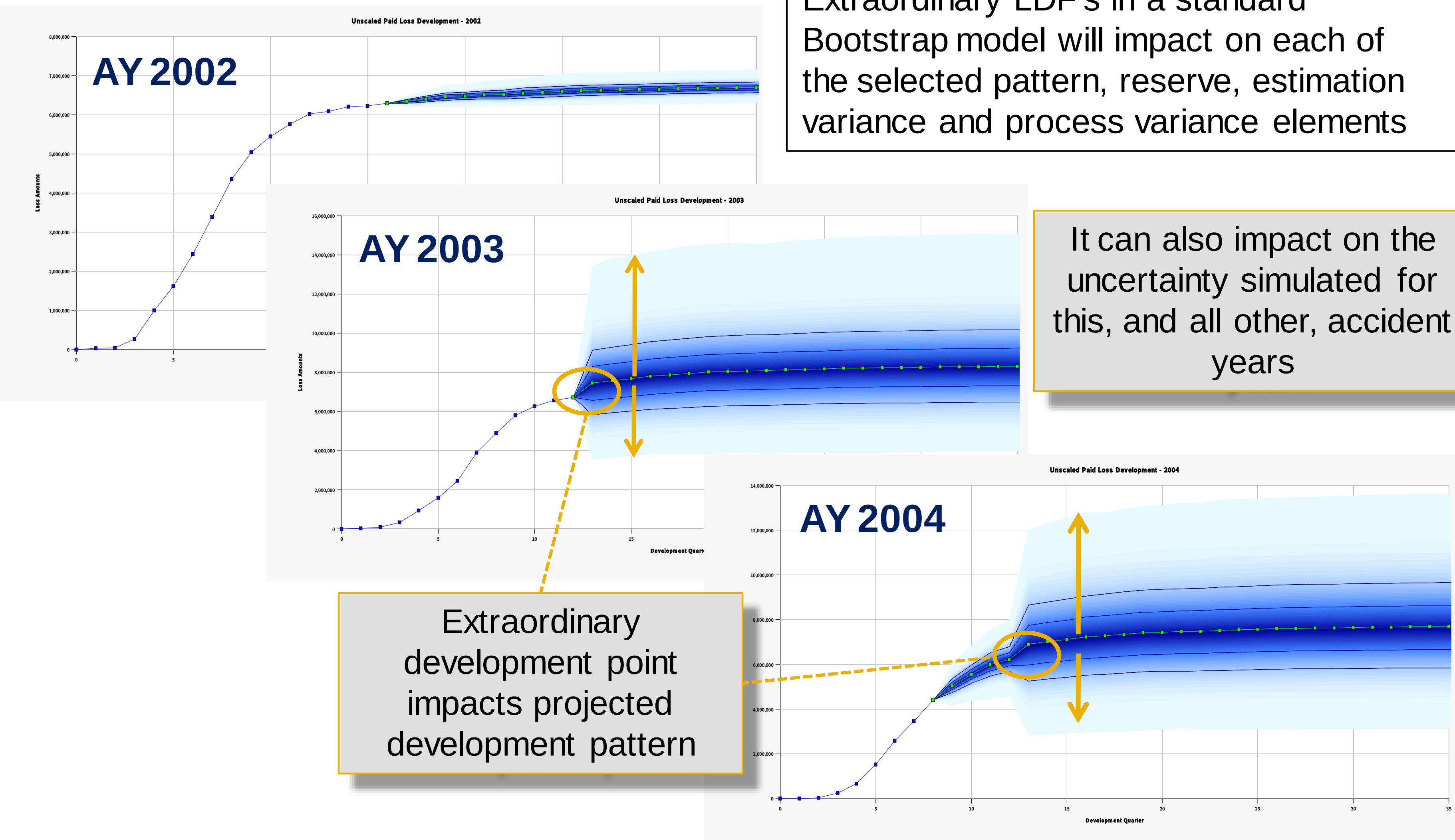






Selecting LDFs Dealing with Outliers Original Model

Extraordinary LDF’s in a standard Bootstrap model will impact on each of the selected pattern, reserve, estimation variance and process variance elements

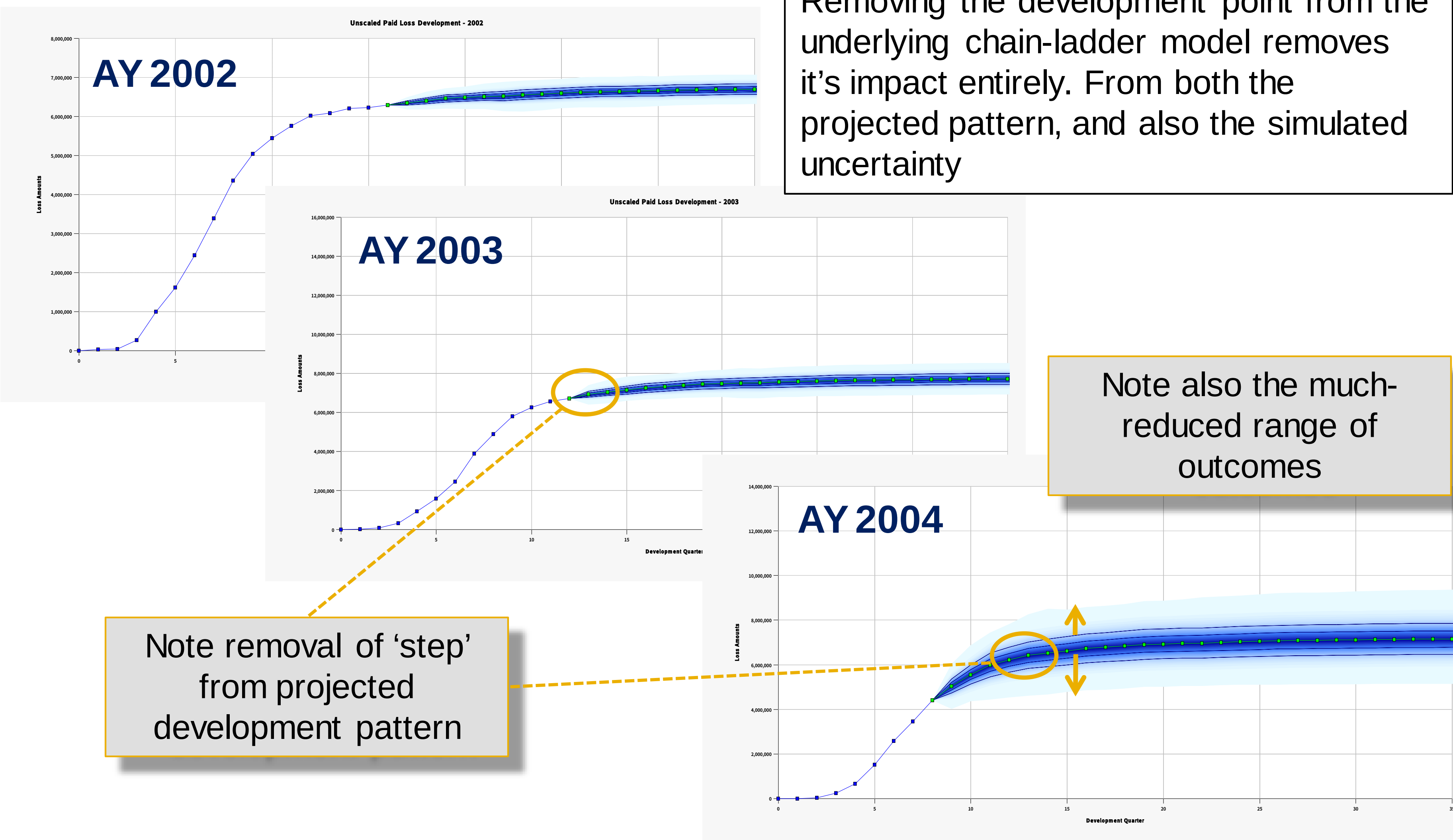


It can also impact on the uncertainty simulated for this, and all other, accident years

Extraordinary development point impacts projected development pattern

Selecting LDFs Dealing with Outliers Excluded LDF

Removing the development point from the underlying chain-ladder model removes it's impact entirely. From both the projected pattern, and also the simulated uncertainty

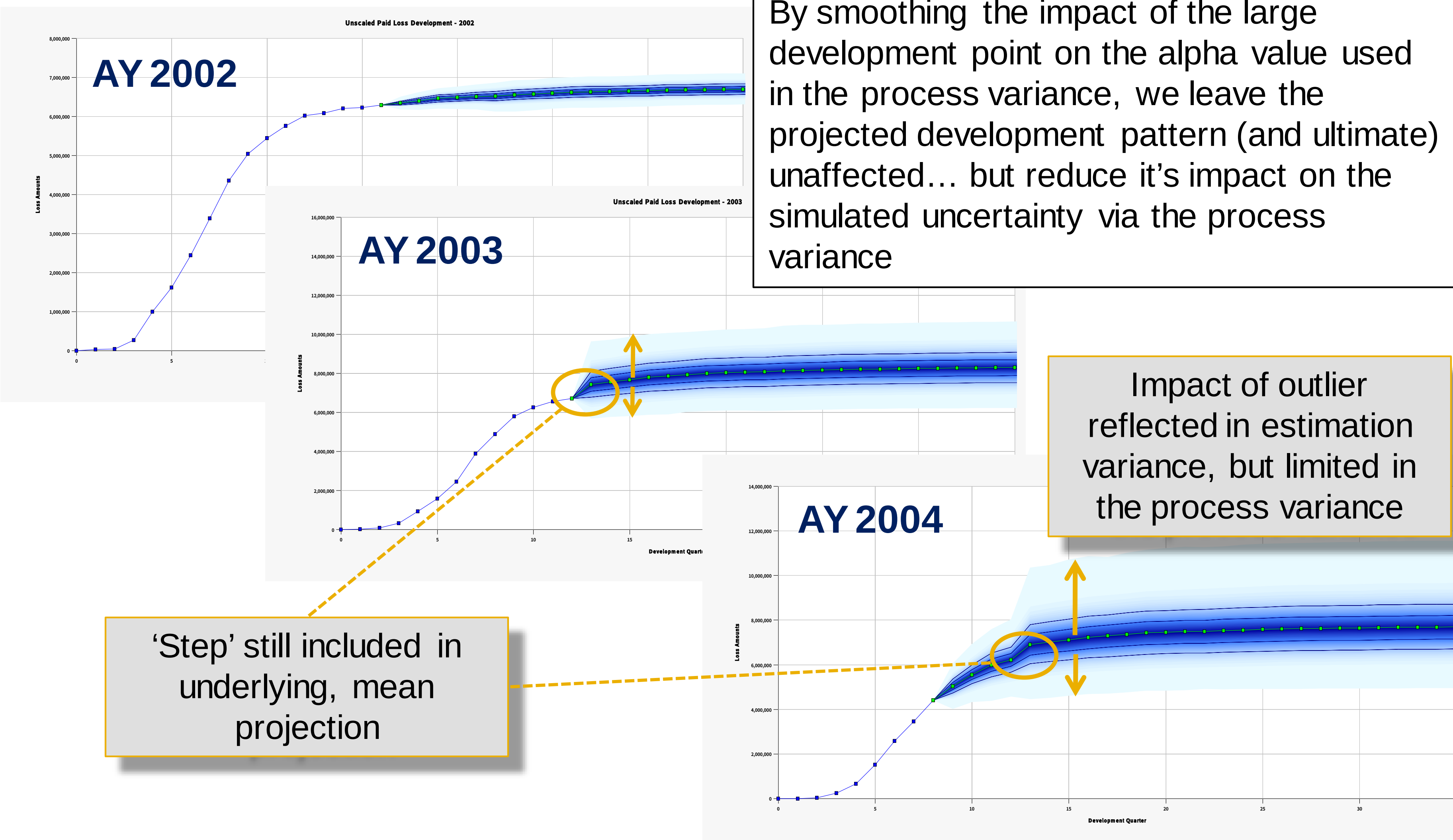


Note also the much-reduced range of outcomes

Note removal of 'step' from projected development pattern

Selecting LDFs Dealing with Outliers Suppressed Process α

By smoothing the impact of the large development point on the alpha value used in the process variance, we leave the projected development pattern (and ultimate) unaffected… but reduce it's impact on the simulated uncertainty via the process variance



Impact of outlier reflected in estimation variance, but limited in the process variance

'Step' still included in underlying, mean projection

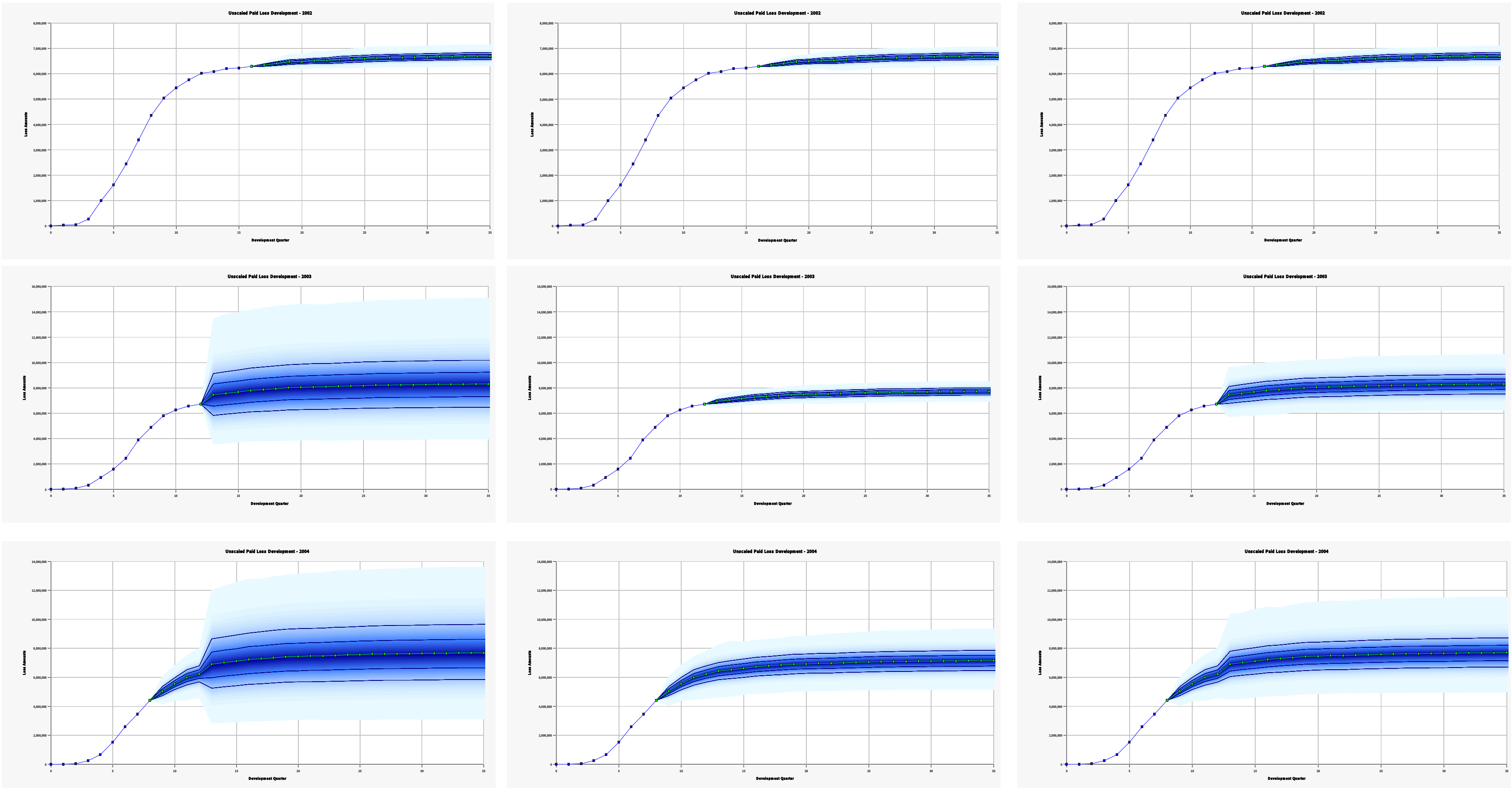
Selecting LDFs Dealing with Outliers Comparison

Original Model | Excluding LDF from DFM | Suppressed Alpha for Process Variance

AY 2002

AY 2003

AY 2004



Selecting LDFs Dealing with Outliers

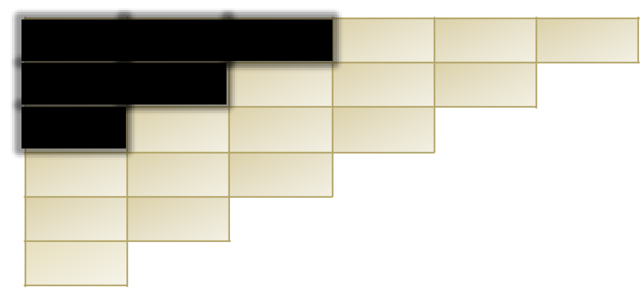
- Another option:
- From a theoretical perspective, if an outlier creates a distortion in the smoothness of a development pattern, then you can argue that it shouldn't affect the estimation variance but instead should affect the process variance
- Therefore, one option would be to exclude the outlier outright so it doesn't affect estimation variance and then adjust the process variance accordingly

Selecting LDFs

Changing Development Patterns

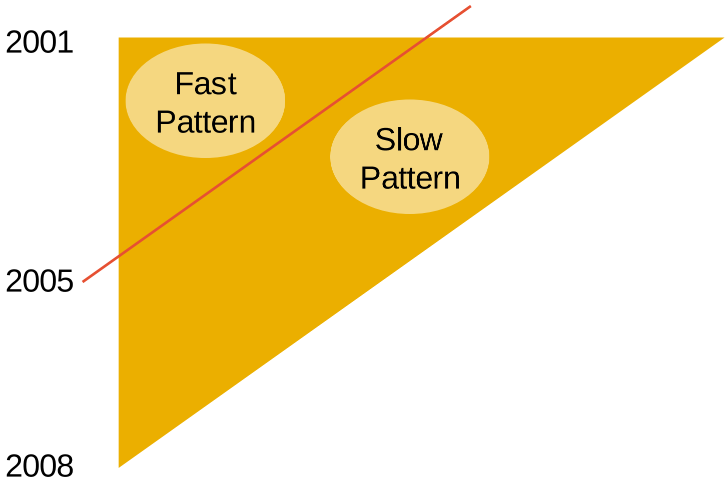
Selecting LDFs Changing Development Patterns

- As in the previous example, when creating our DFM for the purpose of Bootstrapping, we must be careful in our approach when:
  - selecting weighted averages, and/or
  - excluding LDFs
- This is especially true when considering triangles containing changes in development pattern



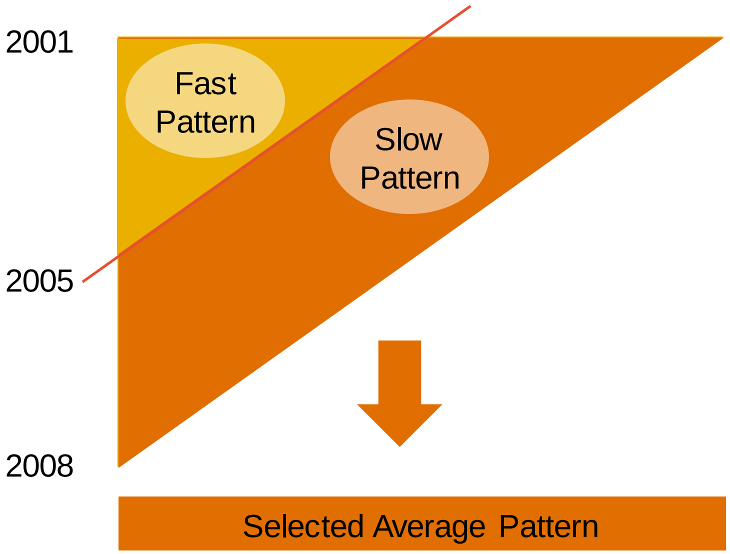
Selecting LDFs Changing Development Patterns

- For example, lets imagine we have a triangle of paid losses
- In 2005 a new claims process was put in place that reallocated shorter-tailed claims away from this aggregate class, leading to a slowing down in the payment pattern



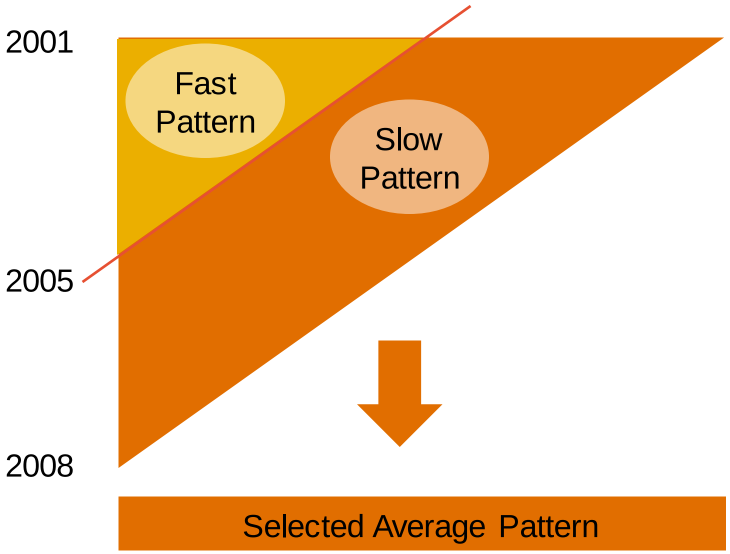
Selecting LDFs Changing Development Patterns

- We obviously do not want to include the 'old' pattern within our point-estimate projections
- To reflect this, we may choose to select an LDF equal to the volume-weighted average of the last 3-years



Selecting LDFs Changing Development Patterns

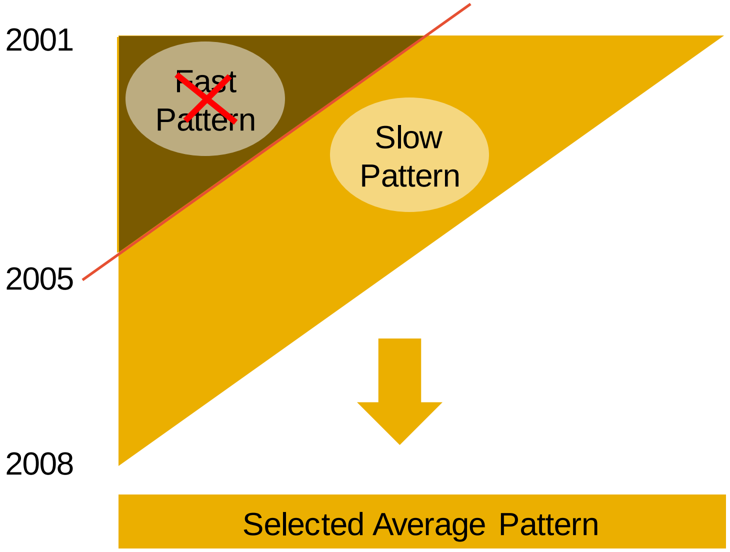
- We obviously do not want to include the 'old' pattern within our point-estimate projections
- To reflect this, we may choose to select an LDF equal to the volume-weighted average of the last 3-years
- Note, however, that while the top, 'fast' triangle of data is not influencing our point estimate projection, they will still be reflected in our variability model
- This is because:
  - they will still produce residuals that can be re-sampled ***anywhere*** within the triangle, and will therefore feed into the estimation variance
  - the residuals will also influence the alpha-values calculated at each lag, which will feed into the process variance (and also influence the scaling of the residuals across all periods)



Selecting LDFs Changing Development Patterns

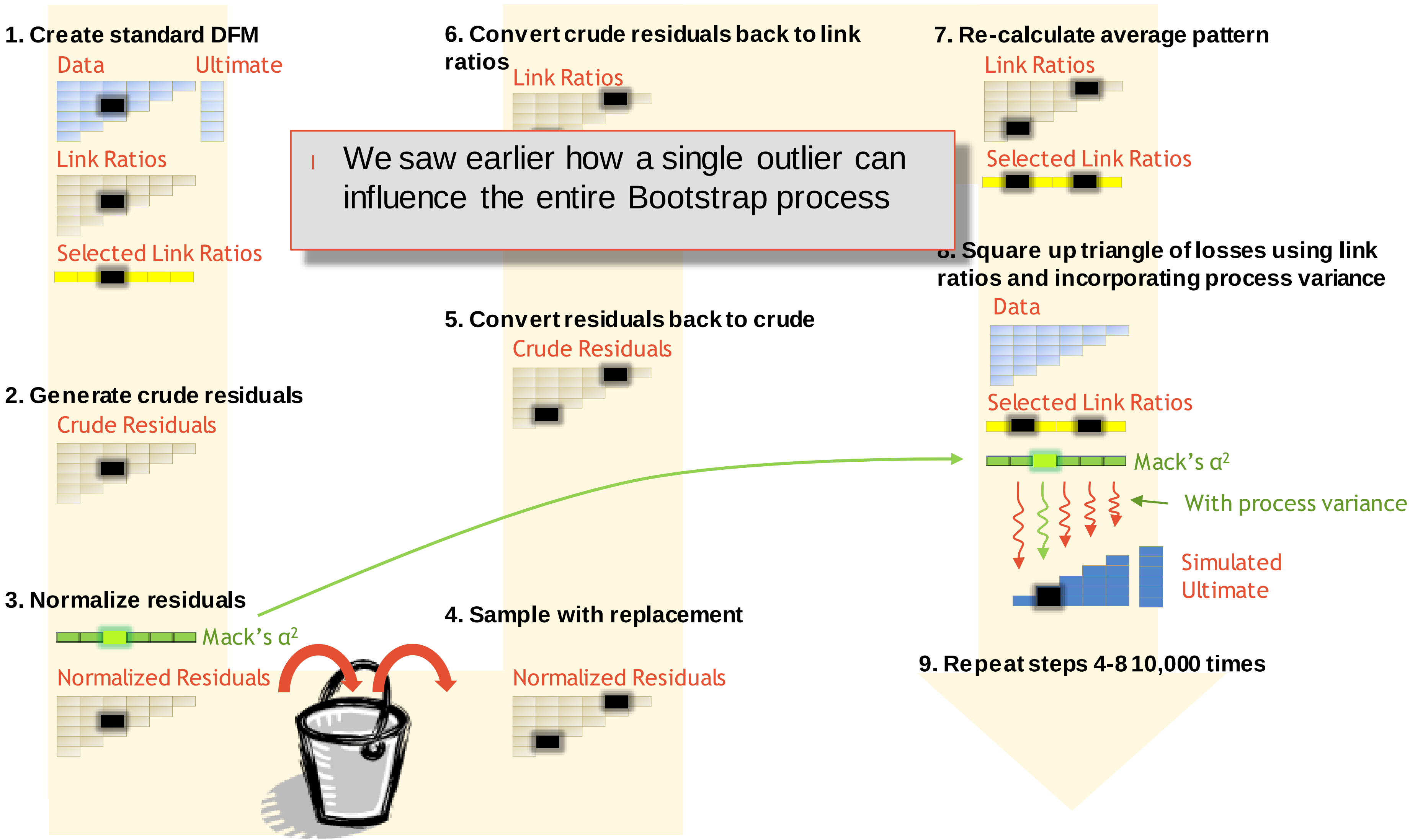
- Alternatively, we may wish to exclude these earlier LDFs from the underlying DFM
- Although this will produce the same point estimate result as the previous model in terms of selected pattern, ultimate and reserves, excluding the LDF's entirely will mean this earlier data is removed from both our point estimate model and also our variability model
- When it comes to DFM's for Bootstrapping…

Ignored ≠ Excluded



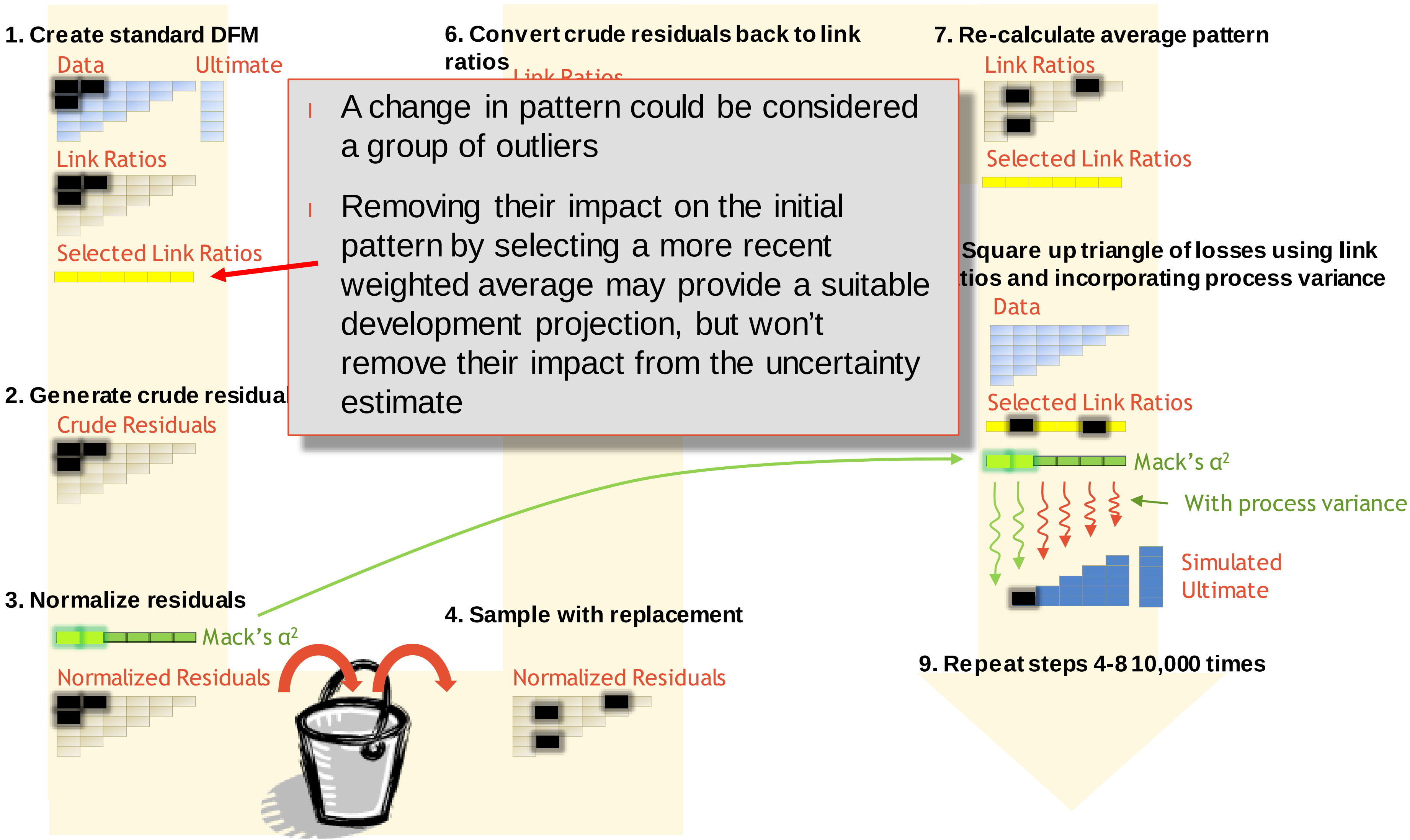
Selecting LDFs Changing Development Patterns

- We saw earlier how a single outlier can influence the entire Bootstrap process



Selecting LDFs Changing Development Patterns

- A change in pattern could be considered a group of outliers
- Removing their impact on the initial pattern by selecting a more recent weighted average may provide a suitable development projection, but won't remove their impact from the uncertainty estimate



Selecting LDFs Changing Development Patterns

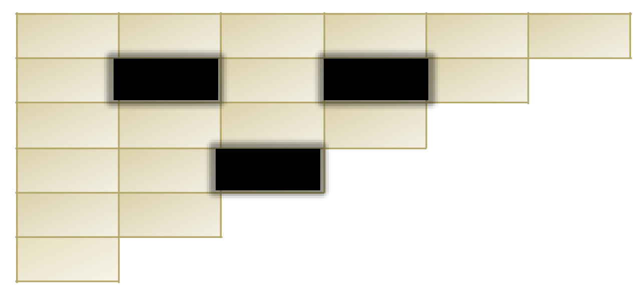
Case Study (3)

Selecting LDFs

Average Factor Selection

Selecting LDFs Average Factor Selection

- Selecting a volume-weighted average of all your historical development factors minimizes the variance in your model
- However, what happens if you have a number of outliers in your model and you select an average factor that, for example, excludes the highest and lowest values within each development period?



Selecting LDFs Average Factor Selection

- Imagine that we had a triangle with a number of 'large' outliers
- Using a volume-all excluding hi/lo, weighted average will produce a relatively smoother development pattern

However…

- This doesn't necessarily mean a Bootstrap around this model will produce a tighter, or less volatile distribution
- Because we did not 'exclude' these points from the model, they will still produce residuals used in the re-sampling stage
- Furthermore, the size of these residuals will increase along with the overall variance reflected in the alpha value that is used in both the estimation and process variance stages

However, once re-sampled and re-normalized, the hi/lo development ratios will be excluded from the simulated 'selected' average ratios

Also note that the 'smaller' residuals will reduce slightly in comparison to the original model, though, in most cases, this does not offset the overall impact on the variance

- Because of this, there is the potential that using an average that is not the volume-weighted average of all factors will actually **increase** your distribution of outcomes

Dealing with Sparse Data

Dealing With Sparse Data

- The Bootstrapping approach is entirely dependent on being able to produce a sample set of normalized residuals
- The standard Bootstrapping methodology samples only from those that you have available

  What if your sample size is simply too small?

- If your sample set of normalized residuals does not sufficiently reflect all points within the distribution, particularly with expected extremes, then re-sampling using a standard Bootstrap approach will not adequately reflect these possibilities
- This issue was raised at GIRO

Dealing With Sparse Data

Possible solutions include:

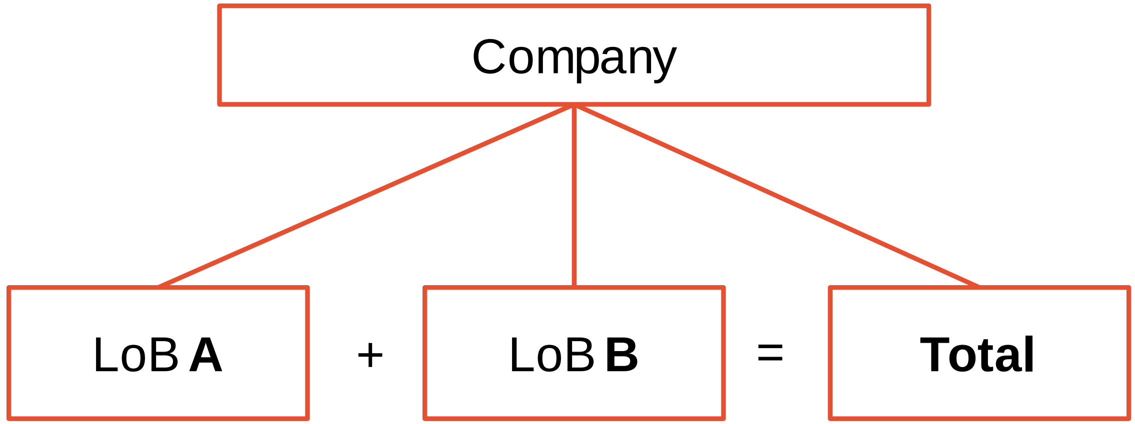
- Simulate an assumed reserve distribution (using Bootstrap Consolidation)
- Use a Parametric Bootstrap

Dealing with Sparse Data

Using Bootstrap Consolidation

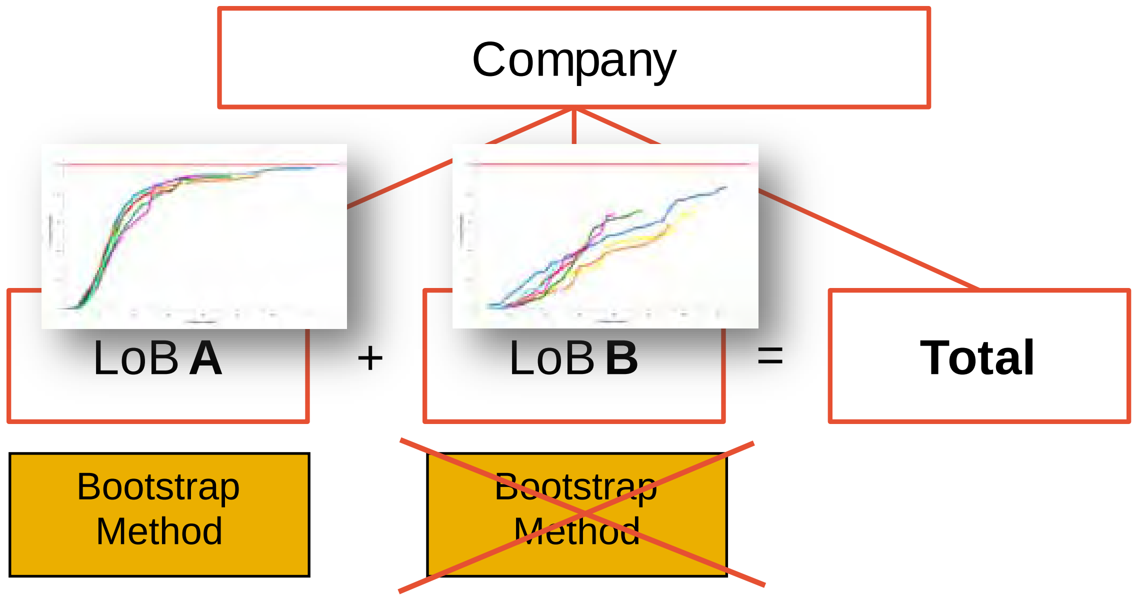
Dealing With Sparse Data Using Bootstrap Consolidation

- Imagine we have a company with 2 lines of business, A and B
- We wish to perform an reserve uncertainty analysis to produce a range around the entire business



Dealing With Sparse Data Using Bootstrap Consolidation

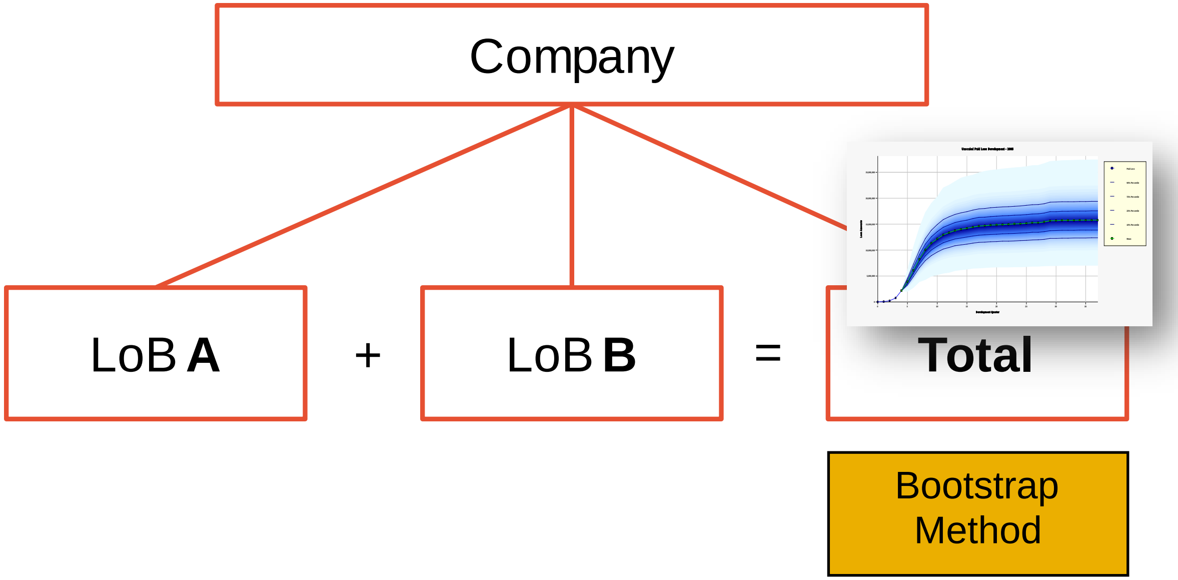
- LoB A is an established line with relatively stable development, good history and has been reserved using standard projection methods
- LoB B, however, consists of relatively immature business, with changing development patterns and has been reserved using simple expected loss ratio methods



- Given that we have one LoB that does not appear to adequately meet the assumptions underlying either a basic chain-ladder or a Bootstrap approach, how do we provide a predictive distribution around the business as a whole?

Dealing With Sparse Data Using Bootstrap Consolidation

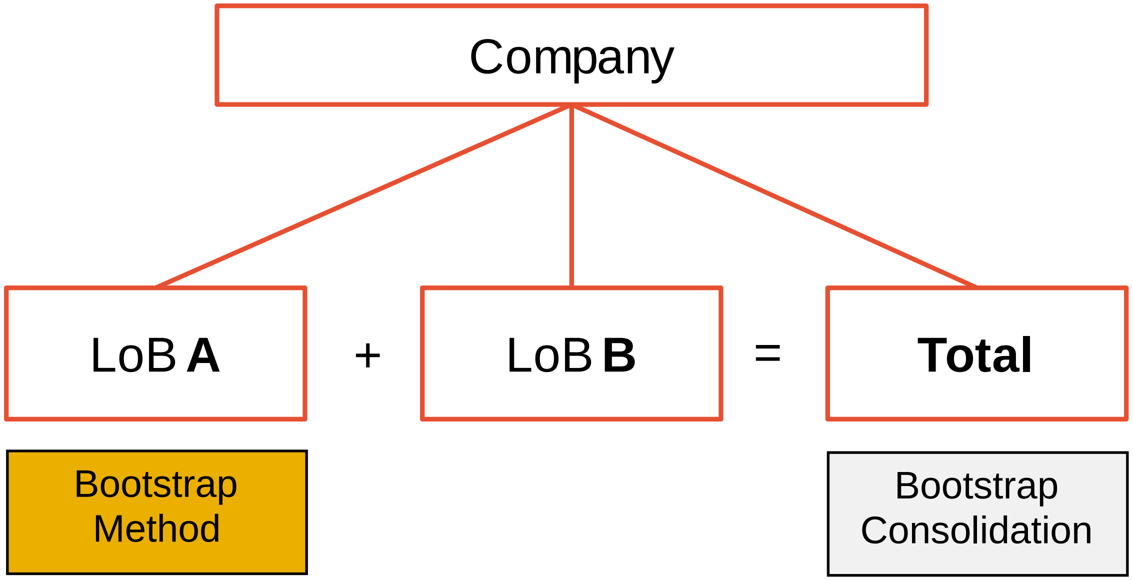
- We could look at combining the business and Bootstrapping at the Total level
- This would enable us to avoid the issue of finding a way of fitting a pattern to the more immature LoB
- It would also enables us to avoid having to deal with correlation assumptions between the two classes



- *However,*
  - Combining the business could threaten the validity of the chain-ladder and Bootstrap assumptions at the Total level
  - This could be due to the absolute volume of the volatile business and also how this changes over time, influencing the mix of business and therefore also the stability of the development patterns
  - We would also lose the ability to report on the uncertainty at the LoB level, which could provide useful information

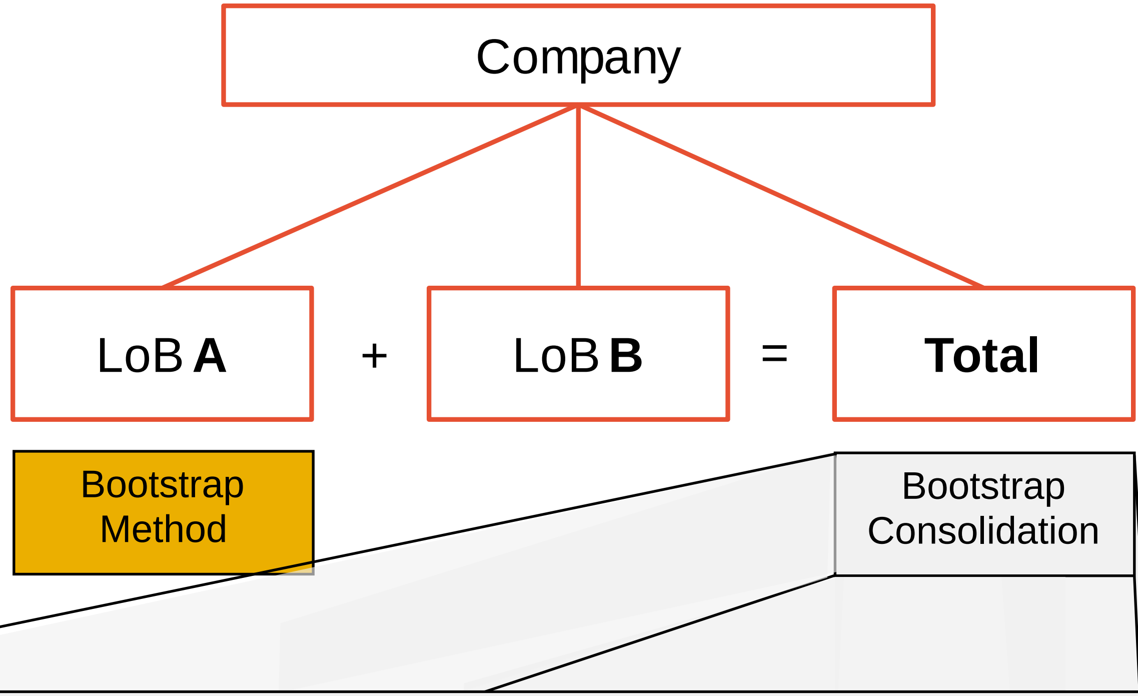
Dealing With Sparse Data Using Bootstrap Consolidation

- Alternatively, we could still apply a robust Bootstrap method to LoB A
- Using a Bootstrap Consolidation module, we could also produce a predictive distribution around the business as a whole
- For LoB B, for which no Bootstrap method is produced, we can simulate a distribution with a mean equal to our selected reserve and a specified, or benchmark CoV:



Dealing With Sparse Data Using Bootstrap Consolidation

- We can then combine these two distributions, using a specified correlation to produce a range around our combined results

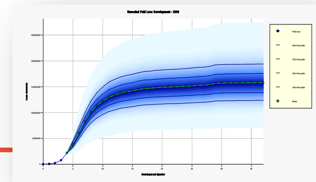


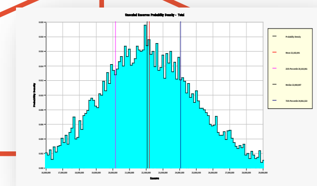
Bootstrap Consolidation Module			
Distribution	From Bootstrap method	LogNormal	Consolidated
Mean	From Bootstrap method	From specified vector / method	Consolidated
CoV	From Bootstrap method	User-defined	Consolidated

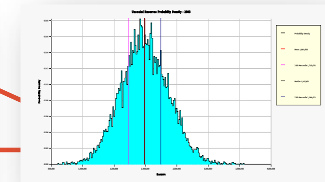
Dealing With Sparse Data Using Bootstrap Consolidation

- This approach allows us to maintain our robust Bootstrapping results, where possible, and report uncertainty either at a Total, or separate LoB level

Company







LoB **A** + LoB **B** = **Total**

Bootstrap Method

~~Bootstrap Method~~

Bootstrap Consolidation

Dealing With Sparse Data Using Bootstrap Consolidation

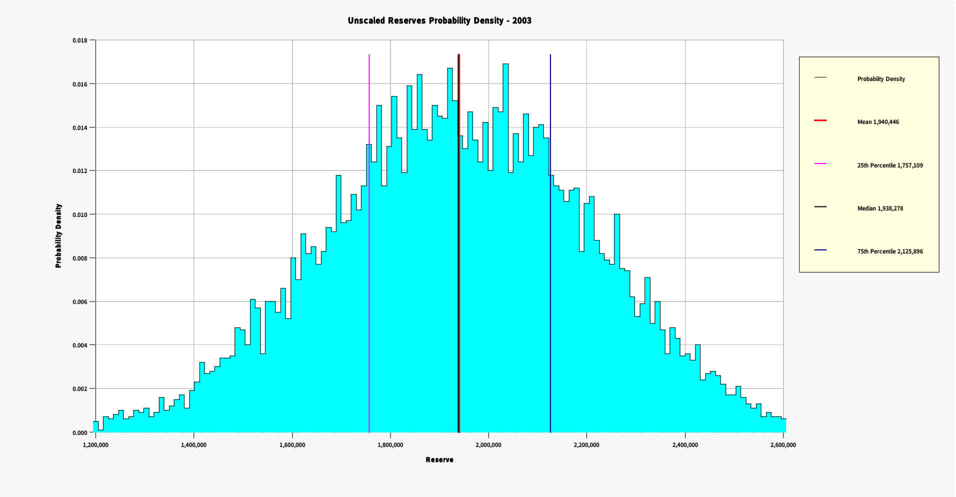
Case Study (4)

Dealing with Sparse Data

Using Parametric Bootstrap

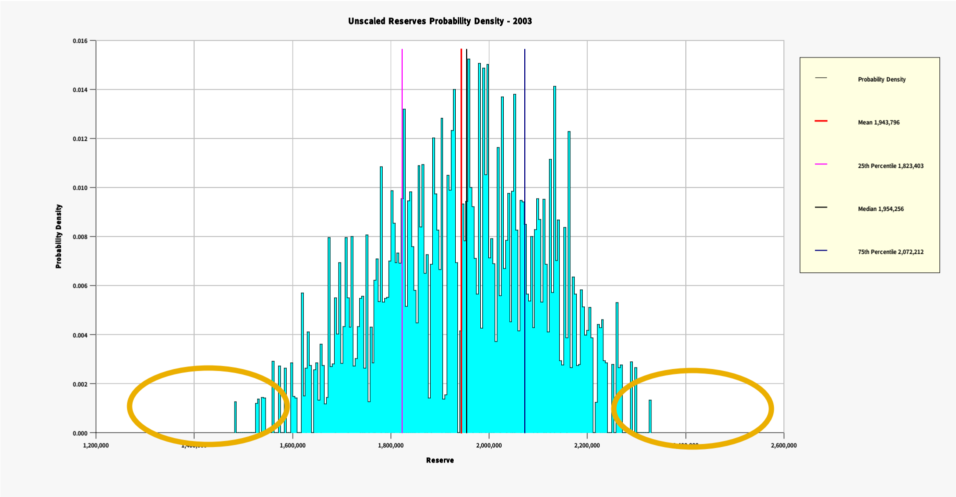
Dealing With Sparse Data Using Parametric Bootstrap

- The graph to the right shows the reserve distribution of an early accident year in a 5x5 triangle
- We ran 10,000 simulations using a 'standard' Bootstrap approach using Mack's model
- Initially, everything looks relatively sensible
- However, if we re-simulate the model, but exclude process variance, we get a different picture…



Dealing With Sparse Data Using Parametric Bootstrap

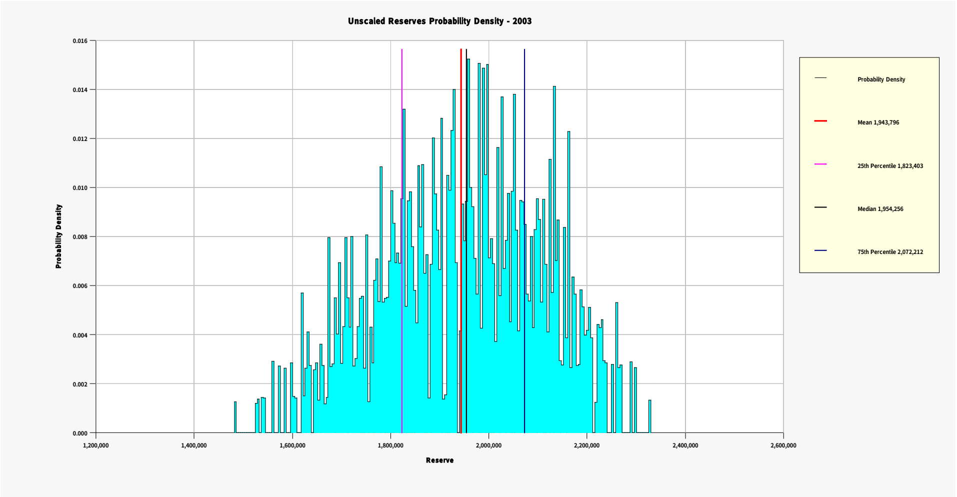
- The graph to the right shows the reserve distribution, but only incorporating estimation variance.
- I.e. the graph shows the results of simply using the re-sampled residuals produced by Mack's model and re-fitting the average development pattern
- Because of the small sample size there is a limit to the possible number of outcomes produced, giving the distribution a 'choppy' appearance

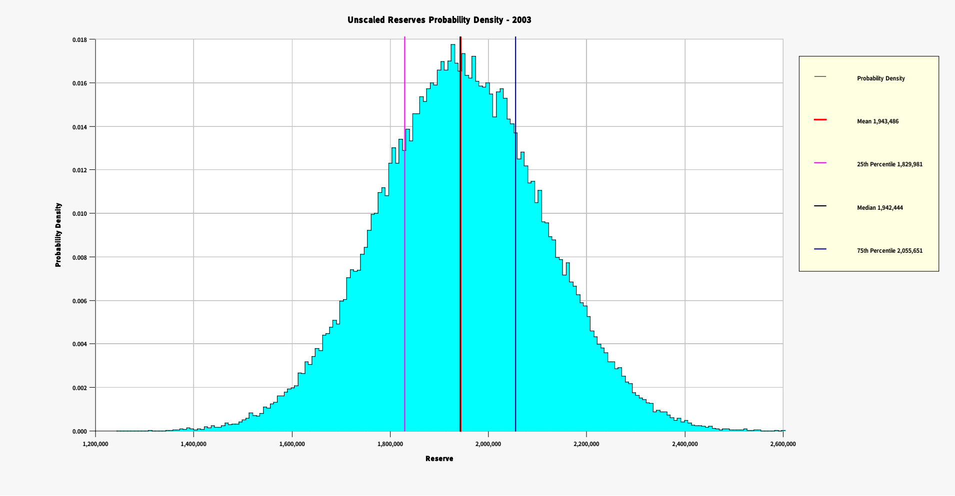


- Note also the lack of 'extreme' values produced, for the very same reason
- This is a common problem in that the Bootstrapping process in simplest form can only sample from the data points available
- Although adding in simulated process variance will help somewhat in producing a wider distribution of outcomes, the underlying problem will still exist if a model is run with a restrictive sample size

Dealing With Sparse Data Using Parametric Bootstrap

- On the bottom right, we can show the same graph from a similar Bootstrap model, also without any process variance included
- In this instance, we have performed a parametric Bootstrap and have simulated the estimation variance
- Note the smoother appearance of the graph and also the incidence of possible outcomes in the extremes
- Note also that the prediction error, CoV and even the 25th and 75th percentiles of the two models are very similar, highlighting the importance of not only viewing a summary of the uncertainty analysis, but also investigating the extreme simulations for reasonableness





Dealing With Sparse Data Using Parametric Bootstrap

Case Study (5)

Conclusion

- As in all actuarial approaches, the blind application of a method, without understanding the underlying mechanics of the calculations and assumptions, will lead to trouble
  - Bootstrapping is no different
- In order to produce a valid and meaningful model, we must understand the impact of both the data and also our decision-making process during the creation of our uncertainty analysis
- Important to understand the causes and symptoms of both estimation variance and process variance on the overall Bootstrap results
  - Characteristics of your data, such as outliers and the amount of history
  - Decisions made within the DFM
  - Decisions made within the Bootstrap model
- If a DFM is unreliable as a means for a point estimate, then Bootstrapping will also be unreliable

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