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Speculation in procurement auctions ☆

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Abstract

A speculator can take advantage of a procurement auction by acquiring items for sale before the auction. The accumulated market power can then be exercised in the auction and may lead to a large enough gain to cover the acquisition costs. I show that speculation always generates a positive expected profit in second-price auctions but could be unprofitable in first-price auctions. In the case where speculation is profitable in first-price auctions, it is more profitable in second-price auctions. This comparison in profitability is driven by different competition patterns in the two auction mechanisms: in first-price auctions, sellers who refuse to sell to the speculator bid more aggressively than in second-price auctions. In terms of welfare, speculation causes private value destruction and harms efficiency. Sellers benefit from the acquisition offer made by the speculator. Therefore, speculation comes at the expense of the auctioneer.

JEL classification: D44; D84; L41; C72

Keywords: Speculation; Procurement; Auction theory; Acquisition; Supply reduction; Supply withholding

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1. Introduction

1.1. Motivation

In anticipation of a procurement auction, speculators have the incentive to consolidate the market by acquiring items from different sellers. By doing so, speculators gain market power and can reduce competition in the procurement auction. Consequently, a speculator's expected gain from the procurement auction may be high and more than enough to cover the acquisition costs.

The speculative incentive offers a compelling explanation for the activities of some private equity firms around the time of the Federal Communications Commission's (FCC) Broadcast Incentive Auction.[1] LucasPoint Networks, NRJ TV, and OTA Broadcasting acquired 48 TV stations in the pre-auction stage (Doraszelski et al., 2017). To secure high transaction prices for the stations they actually sold in the auction, they withheld supply from the auction when the prices offered were still very high. Shortly after the auction, the firms resold many stations they refrained from selling in the auction. The resale price of a station is often much lower than the price at which the station is dropped out, indicating that the firms dropped out of bidding at high prices not because they value the stations more, but due to speculative reasons. For example, OTA Broadcasting sold five TV stations (WKHU-CD, WMVH-CD, WWKH-CD, WWLM-CD, and WJMB-CD) in Pittsburgh that it withheld from the incentive auction within one year after the auction had concluded.[2] The resale price was \$0.275 million, which is little more than 0.1% of the total price at which these stations were dropped out of the incentive auction, \$264 million.[3]

To study the profitability and welfare implications of such a speculation scheme, I incorporate a speculator and a pre-auction acquisition stage in an independent private value procurement auction model. To capture the economic insights within a parsimonious setting, I consider a single-object auction. In the model, the speculator is allowed to make a take-it-or-leave-it offer to every seller before the auction to buy the items they have for sale.[4]

I study speculation in both second-price auctions, which are the single-object analog to the deferred-acceptance auction design used by the FCC in the incentive auction,[5] and first-price auctions, which are adopted commonly in government procurement.

1.2. Overview of the results

The sellers weigh their prospects in the auction against the speculator's offer in deciding whether to sell to the speculator. As will be shown later, sellers who value their items less tend

[1] The FCC adopted a descending-clock auction to purchase broadcast rights from TV stations. At the beginning of the auction, each TV station was offered a very high price. Subsequently, the price was gradually reduced, and during the process, the bidders of the TV stations had the option to drop out of the auction. When the auction concluded, the remaining bidders became winners and received their last clock price. More details about the procurement auction design can be found in Milgrom and Segal (2020).

[2] See https://en.wikipedia.org/wiki/OTA_Broadcasting.

[3] Bidding data or the drop-out prices in the incentive auction can be found at https://auctiondata.fcc.gov/public/projects/1000/reports/reverse-bids.

[4] In the case of contract procurement, the obligation to fulfill the contract is interpreted as an item for sale.

[5] To be precise, (reverse) English auctions are the single-object analog to the deferred-acceptance clock auction used by the FCC in the incentive auction. But for the purpose of this paper, English auctions and second-price auctions are equivalent.

to accept the speculator's offer in equilibrium. This may appear counter-intuitive at first glance, since those sellers are more likely to win in a procurement auction than the sellers with high valuations for their items. But the comparison is misleading, as low-valuation sellers also have higher net gains than high-valuation sellers if they sell to the speculator. Notably, in the incentive auction example, Doraszelski et al. (2017) report that "[f]ew of the 48 TV stations [acquired by the private equity firms] are affiliated with major networks and many of them are failing or in financial distress."

After characterizing the equilibrium of the speculation game (Proposition 1 and Proposition 3),[6] I show that the profitability of the speculation scheme hinges on the auction format. In second-price auctions, the speculator earns positive expected profits by optimally choosing the acquisition price offered to the sellers (Proposition 2). In contrast, speculation could be unprofitable in first-price auctions (Proposition 4). Furthermore, the expected profit from speculation is always higher in second-price auctions than in first-price auctions (Proposition 5). To understand the contrast, first note that the speculator would be a strong bidder as compared with other sellers in the auction subgame, since he has no private value for the item(s). It has been recognized in the literature that a strong bidder would often be better off in second-price auctions vis-a-vis first-price auctions (see, for example, Maskin and Riley (2000)). This result, in turn, can be traced back to the different competition patterns under the two auction formats. In a second-price auction, the sellers do not respond to the speculator's entry in the auction, they still bid their true values as if no acquisition had happened. However, in a first-price auction, sellers respond to the speculator's entry in the auction by bidding more aggressively (Lemma 5). This undermines the speculator's effort in competition reduction and lessens the speculator's profit. Moreover, aggressive bidding in the subgame leads to a better prospect for sellers if they reject the acquisition offer and proceed to the auction. As a result, the speculator has to incur higher acquisition costs. A lower gain in competition reduction and higher acquisition costs account for the difference in speculation profitability across the two auction formats.

The presence of a speculator has non-trivial welfare implications. Since the speculator acquires multiple items solely for the purpose of selling one (at a high price) in the auction, the private values of all but one seller who sell to the speculator are wasted. This implies that speculation activities would induce efficiency losses. Sellers benefit from the presence of the speculator as the speculator overcompensates them for their loss of private value and for skipping the auction. This means that the speculator's profit comes at the auctioneer's expense. Therefore, in the case where speculation is unprofitable in first-price auctions, the auctioneer would prefer the first-price rule to the second-price rule. In this case, first-price auctions also outperform second-price auctions in terms of efficiency.

Two extensions to the second-price auction model are considered. The first one studies the profitability of the speculation scheme in a general setup where sellers' values are drawn from different distributions, and the speculator can only make acquisition offers to a subset of the sellers. Under quite general conditions, the expected profit from speculation is positive if and only if the speculator can access at least two of the sellers (Proposition $2'$). This is not surprising, since the speculator can reduce competition in the procurement auction only if he has the chance to possess more than one item. In the second extension, I consider an enhanced speculation scheme, in which the speculator auctions off the leftover items back to the sellers after the procurement

[6] One challenge in characterizing equilibrium in the first-price auction case is to deal with asymmetric auction subgames in which a speculator bids against some seller(s).

auction concludes. This fine-tuned approach, if feasible,[7] generates even more expected profit for the speculator (Corollary 3).

1.3. An illustrative example

To further illustrate the main insights of the paper, let us consider the following simple example. A buyer seeks to buy a TV station, and two sellers each have one station for sale. The buyer values a TV station for 1, and seller i's private value for his own station is v_i. The sellers' private values are independently and uniformly distributed on [0, 1]. While v_i is privately known to seller i, its distribution is common knowledge.

The buyer has scheduled a procurement auction with a reserve price of 1. Before the auction takes place, a speculator who owns no TV stations enters the market. Moreover, the speculator has no *private* value for any TV station.[8] The speculator makes take-it-or-leave-it offers to the sellers simultaneously to buy their TV stations, in the hope of selling one at a high price in the procurement auction. The sellers independently decide whether to accept the acquisition offer or reject it and participate in the procurement auction.

Suppose that the buyer uses a second-price sealed bid auction (SPA). The speculator's profit-maximizing acquisition offer is $p = 5/9$, which makes a seller with a private value of $v^* = 1/3$ indifferent between selling to the speculator or participating in the auction.[9] The premium of $p - v^* = 2/9$ reflects the marginal seller's expected gain from the auction. In equilibrium, sellers with lower realized values than $v^* = 1/3$ accept the speculator's offer, while sellers with higher values reject. So the probability of a seller accepting the acquisition offer is 1/3.

When both sellers sell to the speculator, which happens with probability 1/9, the speculator pays $2 \times p = 10/9$, sells one TV station at the reserve price of 1, and keeps the other. The speculator incurs an *ex post* loss of 1/9 in this case, because he was not aware that the competition between the sellers would have been intense (as they both have low realized values), and he overcompensated them.

When only one seller accepts the acquisition offer, which happens with a probability of 4/9, the speculator pays $p = 5/9$ and competes against the other seller in the second-price auction. Since truthful bidding is a weakly dominant strategy, the speculator bids 0, and the seller bids his true value. Therefore, the speculator always wins and gets paid the seller's true value, which is known to be above $v^* = 1/3$, since the seller rejected the acquisition offer. As a result, the speculator's expected revenue from the auction is $(1/3 + 1)/2 = 2/3$, and his expected profit is $2/3 - 5/9 = 1/9$ in this case.

Putting the two cases together, speculation generates an expected profit of $1/9 \times (-1/9) + 4/9 \times 1/9 = 1/27$ for the speculator, which is equivalent to 5.6% of the expected total surplus in the first-best outcome. The equilibrium outcomes of the SPA-speculation game are presented in

[7] For this approach to be feasible, either the items are homogeneous, or the timing allows the speculator to run the "return and refund" auction before delivering an item to the auctioneer. In the latter case, bidding in the procurement auction also needs to be for a generic item rather than a specific one. The feasibility of the approach is discussed in greater detail in Section 6.2.

[8] That the speculator has a value of 0 for a TV station is merely a normalization. It is straightforward to incorporate a scrap value. Specifically, one can assume that the speculator values the TV station for V, and the sellers' values are $V + v_i$.

[9] Formally, the indifference condition is $p = v^* + \int_{v^*}^{1} (1 - x)dx$, where the integral is the marginal seller's expected gain from an auction, as the integrand is a type-x seller's winning probability in the auction.

Table 1
Equilibrium Outcomes in the Illustrative Example.

	Expected Total Surplus	Expected Procurement Cost	Each Seller's Expected Payoff	Speculator's Expected Profit
FPA (No Speculation)	2/3	2/3	2/3	
SPA ($p = 5/9$, $v^* = 1/3$)	52/81	61/81	55/81	1/27
	Private Value Destruction	Supply Withholding Effect	Overcompensation from Speculator	
Difference	2/81	7/81	1/81	

the second row of Table 1. The expected total surplus is $52/81$, the expected procurement cost is $61/81$, and each seller's expected payoff is $55/81$.

In contrast, if a first-price sealed bid auction (FPA) is used, speculation would not be profitable for any acquisition price. As discussed in Section 1.2, in an FPA-speculation game, the acquisition costs are higher and the expected revenue is lower for the speculator, due to the sellers' aggressive bidding. For example, if the speculator would like to induce the acceptance/rejection cutoff $v^* = 1/3$, he must offer an acquisition price of $41/72$, exceeding $5/9$ in the SPA case. In the meantime, when only one seller accepts the acquisition offer, the speculator's expected revenue from competing against the other seller in the FPA is $3/8$, which is lower than $2/3$ in the SPA case.[10]

The equilibrium outcomes of the FPA-speculation game are presented in the first row of Table 1. Since speculation does not happen, the equilibrium outcomes are the same as those in an auction, be it SPA or FPA, without speculation. Incidentally, the expected total surplus, the expected procurement cost, and each seller's expected payoff are all $2/3$.

A comparison between the two auction formats is presented in the last row of Table 1. This comparison reveals the three components of the profit from speculation. First, because the speculator withholds one station from the auction when both sellers accept the acquisition offer, the expected procurement cost in the SPA is $7/81$ higher. This amount reflects the speculator's gain from supply withholding. Second, when the speculator buys two TV stations, the *private* value of one TV station is destroyed: Instead of being operated by the seller who values it the highest, it is held by the speculator. This accounts for the difference in expected total surplus with and without speculation. In this example, the loss from private value destruction amounts to $2/81$. This loss is suffered by a seller, but is passed through to the speculator. Third, the speculator overcompensates the sellers by paying more than their losses of private value and the opportunity of participating in the auction. The overcompensation makes each seller better off by $1/81$ as compared to the case without speculation. Overall, the speculator's profit is the gain from supply withholding minus the losses from private value destruction and overcompensation.

While this paper focuses on procurement auctions, its analysis can be readily extended to a mirror setting with forward auctions. This point can be illustrated by examining the counterpart to the SPA case in the illustrative example. Suppose there are $N = 2$ buyers who seek to purchase an item in an upcoming second-price auction. The buyers' values v_i are independently and uniformly distributed on $[0, 1]$. The reserve price in the auction is 0. In the meantime, an outside market sells this item at a price of 1. A speculator who owns no item offers to sell to the potential

[10] In the FPA, the speculator uses a mixed strategy and bids randomly on the interval $[3/8, 1/2]$. As a result, sellers with values below $1/2$ have the opportunity to win against the speculator and get a positive payoff.

buyers at a price of $p = 4/9$ before the auction starts. More precisely, the speculator offers each potential buyer a future contract. If a buyer accepts, the buyer pays p and the speculator promises to deliver the item *after the auction concludes*. The speculator's offer makes a buyer with value $v^* = 2/3$ indifferent between accepting (and skipping the auction) and rejecting (and participating in the auction).[11] Buyers with higher values accept the speculator's offer, and buyers with lower values reject it. The speculator's expected profit is again 1/27.[12]

1.4. Related literature

This paper is related to the literature on strategic demand/supply reduction in auctions (see, for example, Vickrey (1961), Ausubel et al. (2014), and Doraszelski et al. (2017)). In the procurement context, previous studies typically take the pre-auction ownership structure of goods as given.[13] This paper makes an effort to endogenize the ownership structure by considering a pre-auction acquisition stage. It is worth highlighting that, in this case, efficiency can no longer be restored by a VCG auction, because it does not eliminate the speculator's incentive to accumulate market power.

A growing literature is concerned with speculation in (forward) auctions with resale. In this literature, the speculator seeks to win the auction and resell to other bidders *after* the auction. In contrast, the current paper allows the speculator to transact with potential sellers *before* the auction. Put differently, while this literature focuses on speculation to gain market power in the resale stage and against other bidders, the current paper considers speculation to gain market power in the auction stage and against the auctioneer. Like the current paper, one strand of this literature considers speculation by special bidders who have no use value for the objects on sale. Garratt and Tröger (2006) show that in second-price auctions, there exist multiple equilibria in which the speculator wins the auction and makes positive profits, whereas speculation is unprofitable in first-price auctions. Pagnozzi (2010) studies speculation in complete information multi-object auctions with resale and whether the presence of speculators helps raise the auctioneer's revenue. Pagnozzi and Saral (2019), and Garratt and Georganas (2021) experimentally study the effects of speculation driven by post-auction resale.

In a forward auction setting, when there is a resale market, regular bidders can also engage in speculation by buying in the auction and selling afterward. Haile (2000, 2001, 2003) demonstrates how the strategies of regular bidders are affected by the existence of a resale market. The resale market emerges because bidders are uncertain about their own valuations, and therefore allocation is not efficient in the primary market. Even without such uncertainty, Georganas (2011) shows that resale (and speculation) can occur when there is some noise in bidders' strategies.

[11] Formally, the indifference condition is $v^* - p = \int_0^{v^*} x dx$, where the integral is the marginal buyer's expected gain from an auction, as the integrand is a type-x buyer's winning probability in the auction.

[12] Mirroring the procurement case, the speculator's expected profit can be calculated as follows. When both buyers buy from the speculator, which happens with probability 1/9, the speculator gets paid $2 \times p = 8/9$, buys one TV station at the reserve price of 0, and buys the other from the outside market at a price of 1. The speculator incurs an *ex post* loss of 1/9 in this case. When only one buyer takes the speculator's offer, which happens with a probability of 4/9, the speculator gets paid $p = 4/9$ and competes against the other buyer in the second-price auction. Since truthful bidding is a weakly dominant strategy, the speculator bids 1 and the buyer bids his true value. Therefore, the speculator always wins and pays the buyer's true value, which is known to be below $v^* = 2/3$, since the buyer rejected the speculator's offer. As a result, the speculator's expected payment in the auction is $(0 + 2/3)/2 = 1/3$, so his expected profit is $1/9$ in this case. Putting the two cases together, speculation generates an expected profit of 1/27 for the speculator.

[13] In the studies of multi-unit forward auctions, the demand structure of goods is typically taken as given.

Saral (2012) experimentally examines how the allocation of bargaining power in the resale stage affects bidders' behavior in the initial auction.

In a procurement setting, this paper concentrates on speculation by an exogenous speculator and assumes that regular sellers do not engage in speculation. This assumption is appropriate in some markets where acquisition in the pre-auction market requires a large budget, and therefore only some deep-pocketed third parties can afford to buy many items and accumulate market power. This is the case in the FCC's Broadcast Incentive Auction, as few TV stations can afford to purchase others, and private equity firms act as speculators. This assumption also highlights the effects of pure speculation by distinguishing it from *regular bidders'* endeavor to reduce prospective competition in the auction, which is the theme of the bidder collusion literature.

As the bidder collusion literature is also concerned with competition reduction in auctions, it is related to the current paper. One strand of the bidder collusion literature studies bidding rings' collusive arrangements while assuming that a non-strategic third-party coordinates the ring members' bidding and/or side payments (see, for example, Graham and Marshall (1987), Mailath and Zemsky (1991), and McAfee and McMillan (1992)). Another strand of this literature, forsaking the third-party approach, considers simple collusive plans proposed by a bidder. Eső and Schummer (2004), Rachmilevitch (2013, 2015), and Troyan (2017) allow a bidder to make a take-it-or-leave-it bribe to his opponent in exchange for the opponent's abstaining from (or bidding 0 in) the auction. Lu et al. (2021) allow two-option proposals, which include a bribe (for the opponent to abstain) and also a request (for the proposer to abstain). Technical challenges arise as a bidder's offer may leak his private information to the opponent. In response, the authors commonly restrict their attention to second-price auctions with two collusive bidders.[14,15]

Since the current paper also considers the case where a strategic player (the speculator) makes simple proposals in the hope of "gaming the system," it is more closely related to the latter strand of the bidder collusion literature. One distinctive feature of the current paper is that the speculation scheme does not rely on (potential) bidders' commitment power, whereas the collusive scheme depends crucially on bidders' ability to commit to abstaining. From a practical standpoint, explicit collusion is prohibited by law in many countries including the United States. However, a speculator may be able, legally, to buy up the items of several sellers and have substantially the same effect (of reducing competition and raising the price in the auction). In fact, as shown below in Corollary 4, in its extreme form, speculation can be as detrimental to the auctioneer as a strong cartel (in the sense of McAfee and McMillan (1992)): it eliminates any competition below the reserve price.

The remainder of the paper is organized as follows. Section 2 lays out the model of a dynamic speculation game. Section 3 characterizes and analyzes the equilibrium in the second-price auction case. Section 4 contains equilibrium characterization and analysis in the first-price auction case. Section 5 juxtaposes the results obtained under different auction formats. Section 6 includes two extensions to the baseline model. Section 7 concludes. Proofs omitted in the main text can be found in Appendix A.

2. Model

An auctioneer seeks to buy an item through a reverse auction, which can be a second-price sealed bid auction (SPA) or a first-price sealed bid auction (FPA). $N \geq 2$ risk-neutral sellers,

[14] One exception is Rachmilevitch (2013), in which two-bidder first-price auctions are considered.

[15] In this paper, I consider both first-price auctions and second-price auctions with $N \geq 2$ bidders.

indexed by the set $\mathcal{I} := \{1, 2, \cdots, N\}$, each has one such item for sale. Sellers' items need not be homogeneous as long as they are perfect substitutes for the auctioneer. Seller i's private value for the item is denoted by v_i, which is privately known to the seller.[16] The sellers' private values are independent and identically distributed on $[0, 1]$ according to the cumulative distribution function (CDF) $F(\cdot)$. The corresponding probability density function (PDF) is denoted by $f(\cdot)$. Throughout the paper, I assume that $F(0) = 0$ and that $F(\cdot)$ has full support on [0,1]. The reserve price of the procurement auction is $r \in (0, 1]$.

A risk-neutral speculator, who has no item for sale and no private value for any item, seeks to extract some surplus from the auction. To investigate the profitability and welfare implications of speculation, I consider the following two-stage game. At the beginning of the first stage, the speculator makes a take-it-or-leave-it offer to every seller to buy his item at a price of p.[17] Sellers choose simultaneously and individually between selling the item to the speculator, or rejecting the offer and then participating in the auction. The first stage ends after the sellers make their decisions.

In the second stage, the auction takes place. When the auction begins, the number of bidders and whether the speculator participates in the auction are announced publicly. Formally, the public history at the beginning of the auction is given by $h = (m, S) \in \{0, 1, \cdots, N\} \times \{0, 1\}$, where m is the number of sellers in the auction, and S describes the speculator's status, with $S = 1$ standing for "in the auction" and $S = 0$ for "not in the auction." If the speculator failed to buy any item in the first stage, he exits the game with zero profit. If at least one seller sold his item to the speculator, the speculator participates in the auction along with all the sellers who still possess an item. After the auction, if the speculator owns any surplus item(s), he gets a scrap value of 0 from each item.

I characterize and analyze the perfect Bayesian equilibrium (PBE) of the two-stage speculation game in the following sections.

3. Speculation in second-price procurement auctions

I first study the PBE of the SPA-speculation game, holding fixed an arbitrary price offer $p \in [0, r]$.[18] Once the equilibrium characterization is obtained, I proceed to investigate the speculator's choice of p.

3.1. Equilibrium characterization

To characterize the equilibrium, I use backward induction and first focus on the auction subgame. In an auction subgame, bidding truthfully is a weakly dominant strategy for the sellers, but not for the speculator. If the speculator enters the auction with multiple items, he has an incentive to strategically reduce the supply and drive up the price. In fact, it is weakly dominant for the speculator to bid 0 for one of his items and withhold the rest from the auction. I refer to this behavior as *strategic supply withholding*.[19] Notably, strategic supply withholding ensures that the

[16] Although the model is phrased for the procurement of goods and commodities, it can be applied in the case of contract procurement by interpreting the items as contractual obligations and the sellers' values as their costs.

[17] Since the sellers are symmetric, I assume the speculator offers the same price to all of them. In Section 6, I relax the seller symmetry assumption and consider individualized acquisition offers.

[18] Clearly, the speculator would never offer to pay more than r for an item.

[19] In the context of multi-unit auctions, the act of raising the bid for some units in order to increase the overall price is commonly referred to as "supply reduction." As this paper concentrates on single-unit auctions, the term "supply

speculator wins the auction for sure. The above analysis reveals the equilibrium in weakly dominant strategies of the second-price auction subgame. I focus on this equilibrium and maintain the following assumption throughout the paper.

Assumption 1 *(Dominant strategy equilibrium in the SPA).* In the SPA subgame, sellers bid truthfully, and the speculator engages in strategic supply withholding, meaning that he bids 0 for one of his items while withholding the rest from the auction.

Having established the equilibrium in the SPA subgame, I move backward to the acquisition stage and analyze the sellers' decisions regarding the speculator's offer. It may seem intuitive that sellers with low values for their items would reject the acquisition offer because they have a good chance of winning in the procurement auction. However, this intuition is incomplete, since it only takes into account a seller's loss of the opportunity to participate in the auction, and the seller's loss from giving up the item is ignored. It can be shown that a seller's overall cost of accepting the acquisition offer increases with his value. Formally, seller i's opportunity cost of accepting the acquisition offer, which is equivalent to the seller's expected payoff from participating in the auction, is given by

$$c_i(v_i) := \underbrace{v_i}_{\substack{\text{loss from giving} \\ \text{up the item}}} + \underbrace{\int_{v_i}^{r} \mathcal{WP}_i(x)dx}_{\text{loss from skipping the auction}}, \tag{1}$$

where $\mathcal{WP}_i(x)$ is seller i's interim expected winning probability in the procurement auction when his realized value is x.[20] It is clear that the loss from skipping the auction *decreases* with v_i while $c_i(v_i)$ *increases* with v_i. As a result, sellers with low values tend to accept the speculator's offer in equilibrium.

Based on the above analysis, Lemma 1 establishes that the sellers use a cutoff acceptance/rejection strategy. Moreover, Lemma 1 shows that the acceptance/rejection cutoff is the same for every seller.

Lemma 1. *Under Assumption 1 and holding fixed $p \in [0, r]$, in any PBE of the SPA-speculation game, there exists $v^* \in [0, r]$ such that seller $i \in \mathcal{I}$ accepts the speculator's offer if and only if $v_i < v^*$.*

In light of Lemma 1, the only task remaining for equilibrium characterization is to determine the cutoff v^*. This can be done by establishing the marginal seller's (with value v^*) indifference condition. By Lemma 1, a seller with value $x \in [v^*, r]$ wins in the procurement auction if and only if all the other sellers' values are above x.[21] So the interim expected winning probabil-

withholding" is utilized to differentiate between the two contexts, although the fundamental meaning of the two terms is the same.

[20] The payoff calculation in (1) follows from a standard payoff equivalence technique. Since a type-r seller's gain (in addition to his value) in the auction is 0, the type v_i seller's expected gain (in addition to his value) can be obtained by integrating his winning probability from v_i to r. This technique is used in many places throughout the paper.

[21] If some other seller's value is below x, that seller may sell to the speculator or participate in the auction. Either way, the type-x seller will lose in the auction.

ity is $[1-F(x)]^{N-1}$. Plugging this into (1) yields the marginal seller's expected payoff from participating in the auction:

$$c^*(v^*) := v^* + \int_{v^*}^{r} [1-F(x)]^{N-1} dx, \text{ with } v^* \in [0,r].$$

Therefore, if $r \geq p > c^*(0)$, the following indifference condition determines the cutoff v^*:

$$p = c^*(v^*). \tag{2}$$

If p is too low, i.e., $p \leq c^*(0)$, no seller would accept the acquisition offer and $v^* = 0$. Specifically, the acceptance/rejection cutoff v^* associated with a given price p is determined by

$$v^*(p) := \begin{cases} 0, & \text{if } p \leq c^*(0), \\ \text{the unique solution to (2)}, & \text{if } c^*(0) < p \leq r. \end{cases} \tag{3}$$

Proposition 1 presents the equilibrium characterization result.

Proposition 1. *Under Assumption 1 and holding fixed $p \in [0,r]$, there exists a unique PBE of the SPA-speculation game. In the equilibrium, seller $i \in \mathcal{I}$ accepts the speculator's offer if and only if $v_i < v^*(p)$.*

3.2. Analysis of the equilibrium

Proposition 1 paves the way for investigating the speculator's choice of $p \in [0,r]$. Since there is a one-to-one mapping between the price p and the cutoff v^* (except for the $v^* = 0$ case), it is more convenient to think of the speculator's problem as choosing a profit-maximizing cutoff level v^* from $[0,r]$.

For a fixed equilibrium cutoff $v^* \in (0,r]$, the corresponding acquisition price is $c^*(v^*)$. The speculator's expected profit is

$$\Pi^*(v^*) := \sum_{m=0}^{N-1} \binom{N}{m} [F(v^*)]^{N-m} [1-F(v^*)]^m y(m,v^*) - NF(v^*)c^*(v^*), \tag{4}$$

with the expected payment from an auction subgame against m sellers given by

$$y(m,v^*) := v^* + \int_{v^*}^{r} \left[\frac{1-F(x)}{1-F(v^*)}\right]^m dx.$$

The speculator's expected profit consists of three parts. First, the speculator gains from strategic supply withholding, meaning that when the speculator enters the auction with multiple items, he reduces the competition in the auction by bidding 0 for one item while withholding the rest from the auction. Second, the speculator loses from overcompensating the sellers: when the speculator purchases an item from a seller, he pays more than the expected payment that the seller would have received in an auction without speculation.[22] Third, the speculator loses from destroying private values: he has no private value for the items he owns after the auction, but to

[22] Consider a seller with realized value $v < v^*$. The expected payment the seller would have received in the absence of the speculator is $v + \int_v^r [1-F(x)]^{N-1} dx$, which is less than $c^*(v^*)$, the price paid by the speculator.

purchase them before the auction, he must compensate the sellers for their loss of private values. This decomposition is most clearly seen in the two-seller case. Fix $N = 2$, the speculator's expected profit is

$$\underbrace{[F(v^*)]^2 \left\{ r - \int_0^{v^*} x d\left[\frac{F(x)}{F(v^*)}\right]^2 \right\}}_{\text{gain from supply withholding}} - \underbrace{2\int_0^{v^*} [F(x)]^2 dx}_{\text{loss from overcompensating the sellers}} - \underbrace{\int_0^{v^*} x d[F(x)]^2}_{\text{loss from destroying private values}}.$$

To see that the first term represents the gain from supply withholding, note that $[F(v^*)]^2$ is the probability that two sellers both accept the acquisition offer, which is the premise of supply withholding; r is the payment received by the speculator with supply withholding, and; $\int_0^{v^*} x d[F(x)/F(v^*)]^2$ is the payment would have been made by the auctioneer in the absence of the speculator. The second term is the increment in the sellers' expected payoff in the speculator's presence, which comes from the speculator's overcompensation. The third term is the expected efficiency loss due to private value destruction. This loss is suffered by a seller, but is passed through to the speculator.

Notably, in the $N = 2$ case, as v^* approaches 0, the gain from supply withholding dominates the losses.[23] This implies that the speculator can always secure a positive expected profit by choosing v^* appropriately. Proposition 2 generalizes this result to the N-seller case.

Proposition 2. *With the optimally chosen acquisition price, the speculator earns a positive expected profit in the unique PBE of the SPA-speculation game.*

In fact, as v^* approaches 0, the major component of the speculator's profit is the gain from supply withholding when exactly two sellers accept the acquisition offer. Formally,

$$\lim_{v^* \to 0} \frac{\Pi^*(v^*)}{[F(v^*)]^2} = \binom{N}{2} \int_0^r [1 - F(x)]^{N-2} dx.$$

To see that the right-hand side is the gain from supply withholding when exactly two sellers accept the acquisition offer, note that $\int_0^r [1 - F(x)]^{N-2} dx$ is the limit of expected payment received by the speculator after supply withholding. The payment would have been made by the auctioneer in the absence of the speculator does not appear on the right-hand side because it is less than v^*, which approaches 0.

Proposition 2 implies that the speculator would always induce a positive cutoff in the speculation game. By inspecting the equilibrium outcome, the welfare implications of speculation are immediately obtained.

[23] To be precise, the gain is of the magnitude $[F(v^*)]^2 r$, whereas the losses are of the magnitude $[F(v^*)]^2 v^*$, which is a higher order infinitesimal.

Corollary 1. *Speculation results in an efficiency loss in the form of private value destruction. Sellers are better off in the presence of the speculator, while the auctioneer is worse off.*

It is worth highlighting that the inefficiency is not caused by strategic supply withholding alone. If, instead of a second-price auction, a VCG auction is used for the procurement, the speculator would bid truthfully for all of his items.[24] There would be no strategic supply withholding in the auction, yet the equilibrium outcome of the speculation game would not change. Although VCG auctions can restore efficiency by eliminating strategic supply/demand reduction in a setting where the ownership structure is fixed, they cannot eliminate the incentive to become a multi-unit owner in a setting where endogenous changes to the ownership structure can happen.

4. Speculation in first-price procurement auctions

In this section, I consider speculation in first-price auctions. For simplicity, I restrict attention to symmetric PBE in the FPA-speculation game. That is, sellers are assumed to use the same strategy in equilibrium. As in the previous section, I first characterize the equilibrium for any fixed $p \in [0, r]$, and then proceed to analyze the speculator's choice of p.

4.1. Equilibrium characterization

Characterizing the equilibrium of the FPA subgame is a non-trivial task due to potential asymmetries among bidders. When the speculator enters the auction, his value distribution differs from that of regular sellers since he has no private value for the item. The fact that the sellers' value distribution in the auction is conditional on their rejection of the acquisition offer further complicates things. Characterizing the equilibrium of the (asymmetric) FPA subgame for every possible conditional value distribution is a daunting task, so the usual backward induction approach is infeasible.

To address this problem, I establish a cutoff acceptance/rejection structure similar to the SPA case in Lemma 2. This helps determine the value distribution of the sellers who reject the acquisition offer and thereby simplifies the analysis of the FPA subgame.

Lemma 2. *Holding fixed $p \in [0, r]$, in any symmetric PBE of the FPA-speculation game, there exists $v^\star \in [0, r]$ such that seller $i \in \mathcal{I}$ accepts the speculator's offer if and only if $v_i < v^\star$.*[25]

In light of Lemma 2, it is useful to consider the subgame in which the speculator and $m \geq 1$ sellers compete with each other. The speculator has no value for his item,[26] while each seller's value is independently drawn from $[v^\star, 1]$, with $v^\star \in [0, r]$, according to the CDF $G(\cdot; v^\star) := [F(\cdot) - F(v^\star)]/[1 - F(v^\star)]$. The auction subgame is labeled as $\Gamma := \langle r, m, G(\cdot; v^\star)\rangle$.

I restrict attention to equilibrium in undominated strategies and maintain Assumption 2 throughout the rest of the paper.

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[24] For instance, if the speculator had acquired three items, he would bid $(0, 0, 0)$ in the VCG auction. In contrast, he effectively bids $(0, \infty, \infty)$ in the second-price auction.

[25] More precisely, there exists a F-measure 0 set $\mathcal{E}$ such that seller i accepts the speculator's offer if and only if $v_i \in [0, v^\star) \setminus \mathcal{E}$.

[26] The speculator may own many items, but he will withhold all but one from the auction.

Assumption 2 *(Undominated bidding in the FPA).* In the FPA subgame, bidders never bid below their values.

Lemma 3 establishes several equilibrium properties that serve as the basis for equilibrium characterization.

Lemma 3. *For any symmetric BNE in undominated strategies of the FPA subgame $\Gamma = \langle r, m, G(\cdot; v^\star)\rangle$, the following statements hold.*

(i) The speculator mixes with full support over an interval $[\underline{b}(m, v^\star), \bar{b}(m, v^\star)]$, where $v^\star \leq \underline{b}(m, v^\star) \leq \bar{b}(m, v^\star) \leq r$. If $\underline{b}(m, v^\star) = \bar{b}(m, v^\star)$, the mixed strategy degenerates to a pure strategy.

(ii) The sellers' strategy $\beta(v; m, v^\star)$ is strictly and continuously increasing in $v \in (v^\star, \bar{b}(m, v^\star))$, with $\beta(v^\star; m, v^\star) = \underline{b}(m, v^\star)$ and $\beta(\bar{b}(m, v^\star); m, v^\star) = \bar{b}(m, v^\star)$.

Using the properties presented in Lemma 3, the symmetric BNE of the FPA subgame can be obtained through three steps.

First, the speculator's bidding range $[\underline{b}(m, v^\star), \bar{b}(m, v^\star)]$ can be pinned down from the optimality of bidding $\underline{b}(m, v^\star)$ and $\bar{b}(m, v^\star)$. If the speculator bids $\underline{b}(m, v^\star)$, he wins for sure and gets a payoff of $\underline{b}(m, v^\star)$, since Lemma 3(ii) implies that no seller bids below $\underline{b}(m, v^\star)$.[27] If the speculator bids $\bar{b}(m, v^\star)$, by Lemma 3(ii) and the assumption of undominated bidding, the speculator wins if every seller's value is above $\bar{b}(m, v^\star)$, and therefore his expected payoff is $\bar{b}(m, v^\star)[1 - G(\bar{b}(m, v^\star); v^\star)]^m$. With an arbitrary bid $b \in [v^\star, r]$, the speculator's winning probability is at least $[1 - G(b; v^\star)]^m$, because the sellers never bid below their values. As a result, the speculator's expected payoff is at least $b[1 - G(b; v^\star)]^m$. The optimality of bidding $\underline{b}(m, v^\star)$ and $\bar{b}(m, v^\star)$ requires that

$$\underline{b}(m, v^\star) = \bar{b}(m, v^\star)[1 - G(\bar{b}(m, v^\star); v^\star)]^m \geq b[1 - G(b; v^\star)]^m \text{ for all } b \in [v^\star, r]. \tag{5}$$

This implies that $\bar{b}(m, v^\star)$ is a maximizer of $b[1 - G(b; v^\star)]^m$ on $[v^\star, r]$. Moreover, if multiple maximizers exist, $\bar{b}(m, v^\star)$ must be the minimum among the maximizers.[28] Consequently,

$$\underline{b}(m, v^\star) = \max_{r \geq b \geq v^\star} b[1 - G(b; v^\star)]^m \text{ and } \bar{b}(m, v^\star) = \min\left\{\arg\max_{r \geq b \geq v^\star} b[1 - G(b; v^\star)]^m\right\}. \tag{6}$$

When the two bounds coincide or, equivalently, when $b = v^\star$ maximizes $b[1 - G(b; v^\star)]^m$ over $b \in [v^\star, r]$, the speculator uses a pure strategy and bids $v^\star$ for sure.

Second, the sellers' bidding strategy can be derived from the speculator's indifference condition. By Lemma 3(i), the speculator must get the same expected payoff from bidding any

[27] In analyzing the FPA subgame, the term "payoff" pertains to the players' payoff within the FPA subgame, rather than their overall payoff within the speculation game.

[28] Otherwise, suppose $b^\dagger < \bar{b}(m, v^\star)$ also maximizes $b[1 - G(b; v^\star)]^m$ on $[v^\star, r]$. Then (5) holds at equality for $b^\dagger$, which implies that $\beta(b^\dagger; m, v^\star) = b^\dagger$, because otherwise if $\beta(b^\dagger; m, v^\star) > b^\dagger$, the speculator's payoff from bidding $b^\dagger$ would be strictly greater than $b^\dagger[1 - G(b^\dagger; v^\star)]^m = \underline{b}(m, v^\star)$, contradicting the optimality of bidding $\underline{b}(m, v^\star)$. Therefore, a type-$b^\dagger$ seller gets a payoff of 0. However, the type-$b^\dagger$ seller can deviate to bidding $b^\dagger + \epsilon < \bar{b}(m, v^\star)$ and get a positive payoff. A contradiction.

$b \in [\underline{b}(m, v^\star), \bar{b}(m, v^\star)]$. In particular, the speculator's expected payoff from bidding $b = \beta(v; m, v^\star) \in [\underline{b}(m, v^\star), \bar{b}(m, v^\star)]$ must equal that from bidding $\underline{b}(m, v^\star)$:

$$\beta(v; m, v^\star)[1 - G(v; v^\star)]^m = \underline{b}(m, v^\star) \text{ for } \beta(v; m, v^\star) \in [\underline{b}(m, v^\star), \bar{b}(m, v^\star)].$$

As a result, within the bidding range of $[\underline{b}(m, v^\star), \bar{b}(m, v^\star)]$, the sellers' strategy is given by $\beta(v; m, v^\star) = \underline{b}(m, v^\star)/[1 - G(v; v^\star)]^m$. By Lemma 3(ii), the bidding range $\beta(v; m, v^\star) \in [\underline{b}(m, v^\star), \bar{b}(m, v^\star)]$ corresponds to the value range $v \in [v^\star, \bar{b}(m, v^\star)]$. As for sellers with values higher than $\bar{b}(m, v^\star)$, since they never win in the auction, they may use any bidding strategy as long as that strategy does not make bidding above $\bar{b}(m, v^\star)$ a profitable deviation for the speculator. In particular, if $\beta(v; m, v^\star)$ is strictly increasing for $v \geq \bar{b}(m, v^\star)$, the speculator's no-deviation condition becomes

$$\beta(v; m, v^\star)[1 - G(v; v^\star)]^m \leq \underline{b}(m, v^\star) \text{ for } v \geq \bar{b}(m, v^\star).$$

This implies that sellers with $v \geq \bar{b}(m, v^\star)$ can bid according to any strictly increasing function $\beta(v; m, v^\star)$ that satisfies $v \leq \beta(v; m, v^\star) \leq \underline{b}(m, v^\star)/[1 - G(v; v^\star)]^m$. One such function is simply $\beta(v; m, v^\star) = v$.

Finally, the speculator's bidding strategy, which is described by the CDF $\Psi(b; m, v^\star)$, can be derived by invoking the standard payoff equivalence technique. Specifically, a type-v seller's interim expected payoff (net of his value) from the auction must equal the integral of winning probabilities:

$$\begin{aligned}&[\beta(v; m, v^\star) - v][1 - G(v; v^\star)]^{m-1} \left[1 - \Psi(\beta(v; m, v^\star); m, v^\star)\right] \\ &= \int_v^{\bar{b}(m, v^\star)} [1 - G(x; v^\star)]^{m-1} \left[1 - \Psi(\beta(x; m, v^\star); m, v^\star)\right] dx.\end{aligned}$$

One can solve for $\Psi(\beta(\cdot; m, v^\star); m, v^\star)$ from the above equation, and then obtain the CDF $\Psi(\cdot; m, v^\star)$ by substituting $\beta(\cdot; m, v^\star)$.

The above analysis yields the following equilibrium characterization result.

Lemma 4. *There exists an essentially unique symmetric BNE in undominated strategies of the FPA subgame $\Gamma = \langle r, m, G(\cdot; v^\star)\rangle$, which is described as follows.*[29]

(i) The sellers bid according to

$$\beta(v; m, v^\star) = \begin{cases} \underline{b}(m, v^\star)/[1 - G(v; v^\star)]^m, & \text{if } v^\star \leq v \leq \bar{b}(m, v^\star), \\ v, & \text{if } v > \bar{b}(m, v^\star), \end{cases}$$

where $\underline{b}(m, v^\star)$ and $\bar{b}(m, v^\star)$ are given by (6).

(ii) If $\underline{b}(m, v^\star) = \bar{b}(m, v^\star)$ or, equivalently, $v^\star \in \arg\max_{r \geq b \geq v^\star} b[1 - G(b; v^\star)]^m$, the speculator bids $v^\star$ for sure. Otherwise, the speculator mixes over $[\underline{b}(m, v^\star), \bar{b}(m, v^\star)]$ according to the CDF

[29] Changes in the sellers' strategy when $v > \bar{b}(m, v^\star)$ are the only potential cause of multiplicity, but they do not affect the equilibrium outcome, as those sellers never win the auction.

$$\Psi(b;m,v^\star) = 1 - \exp\left\{ - \int\limits_{v^\star}^{\beta^{-1}(b;m,v^\star)} \frac{[\beta(x;m,v^\star) + (m-1)x]}{[\beta(x;m,v^\star) - x][1 - G(x;v^\star)]} dG(x;v^\star) \right\}.$$

Lemma 4 paves the way for the analysis of the acquisition stage. Notably, in the FPA subgame, the speculator's expected payoff is $\underline{b}(m, v^\star)$ and a type-$v^\star$ seller's expected payoff is also $\underline{b}(m, v^\star)$. The payoff information can be used to set up the marginal seller's indifference condition in the acquisition stage. Specifically, the marginal seller's expected payoff from participating in the procurement auction is given by

$$c^\star(v^\star) := v^\star + \int\limits_{v^\star}^{r} [1 - F(x)]^{N-1} dx + \sum_{m=0}^{N-2} \binom{N-1}{m} [1 - F(v^\star)]^m [F(v^\star)]^{N-1-m} [\underline{b}(m+1, v^\star) - v^\star].$$

For the marginal seller to be indifferent between accepting the acquisition offer and participating in the procurement auction, the following must hold:

$$p = c^\star(v^\star). \tag{7}$$

It can be verified that $c^\star(v^\star)$ is strictly increasing in $v^\star \in (0, r)$.[30] As a result, a one-to-one mapping between p and $v^\star$ is defined implicitly by the indifference condition (7). For a given price p, the corresponding acceptance/rejection cutoff is given by

$$v^\star(p) := \begin{cases} 0, & \text{if } p \le c^\star(0) = c^*(0), \\ \text{the unique solution for } v^\star \text{ to (7)}, & \text{if } c^\star(0) < p \le r. \end{cases}$$

Again, if the acquisition price is below $c^\star(0) = c^*(0)$, no seller would sell to the speculator. Proposition 3 provides an equilibrium characterization for the speculation game.

Proposition 3. *Holding fixed $p \in [0, r]$, there exists an essentially unique symmetric PBE with undominated bidding of the FPA-speculation game, which is described as follows.*[31]

(i) Seller $i \in \mathcal{I}$ accepts the speculator's offer if and only if $v_i < v^\star(p)$.
(ii) In the procurement auction, the speculator and the sellers bid according to Lemma 4 (with $v^\star$ replaced by $v^\star(p)$).[32]

4.2. Analysis of the equilibrium

Again, the speculator's problem of choosing p can be reformulated as choosing $v^\star$. By Lemma 4 and Proposition 3, the speculator's expected profit with an arbitrary cutoff level $v^\star \in [0, r]$ is given by

30 See Appendix A for details.

31 The essential uniqueness is in the same sense as that in Lemma 4. See Footnote 29 for details.

32 If all the sellers sold their items to the speculator, the speculator bids r in the procurement auction; if all the sellers rejected the acquisition offer, they bid as in a standard FPA model with symmetric bidders.

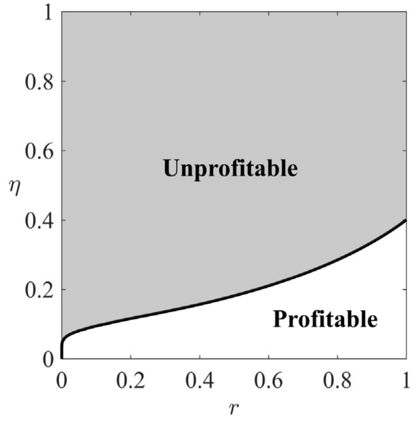

Fig. 1. Profitability of Speculation in FPAs with $N = 2$ and $F(v) = v^{\eta}$.

$$\Pi^{\star}(v^{\star}) := \sum_{m=1}^{N-1} \binom{N}{m} [1 - F(v^{\star})]^m [F(v^{\star})]^{N-m} \underline{b}(m, v^{\star}) + [F(v^{\star})]^N r - N[F(v^{\star})] c^{\star}(v^{\star}). \tag{8}$$

The following result on the profitability of speculation is readily available.

Proposition 4. *Speculation could be unprofitable in first-price auctions.*

Example 1 provides a proof for Proposition 4 and a comparative statics analysis on the profitability of speculation.

Example 1. Fix $N = 2$ and let $F(v) = v^{\eta}$, with $\eta > 0$. Since sellers are more likely to have high valuations (and be *less* competitive in the procurement auction) with a larger η, η can be thought of as a measure of sellers' competitiveness. Fig. 1 shows how the profitability of speculation in first-price auctions changes with the reserve price r, and the competitiveness parameter η.

For a fixed η, speculation is profitable if r is high enough and unprofitable otherwise. Since the speculator's profit comes from the auctioneer, it is not surprising that a higher willingness-to-pay of the auctioneer leaves more room for speculation.

For a fixed r, speculation is profitable if η is small enough and unprofitable otherwise. A small η implies that a seller is more likely to have a low valuation, so the seller would be willing to accept the acquisition offer at a low price. In the meantime, a small η means that other sellers are likely to have low valuations, so the competition in the procurement auction would be fierce. This further reduces the seller's willingness-to-accept for the acquisition offer. As a result, the speculator benefits from sellers' competitiveness.

5. A comparison of the two auction formats

In this section, I compare speculation under the two auction formats and identify the driving force underlying the difference.

5.1. Profitability ranking

In the previous sections, it is shown that speculation tends to be successful in second-price auctions but could be unprofitable in first-price auctions. Proposition 5 complements the result by quantitatively comparing the expected profit from speculation under the two auction formats.

Proposition 5. *Speculation is strictly more profitable in second-price auctions than in first-price auctions.*

The proof of Proposition 5 is kept here in the main text since it is informative about the underpinning of the result.

Proof. Consider the same equilibrium cutoff $v^\dagger \in (0, r)$ under the two auction formats. The speculator's expected profit is higher under the second-price rule than under the first-price rule—i.e., $\Pi^*(v^\dagger) > \Pi^\star(v^\dagger)$—for the following two reasons.

First, the corresponding acquisition price is higher in the first-price auction case, since sellers have better prospects in the first-price auction subgames. In particular, in an auction subgame with the speculator's participation, a seller never wins under the second-price rule but could win under the first-price rule. Formally,

$$c^\star(v^\dagger) - c^*(v^\dagger) = \sum_{m=0}^{N-2} \binom{N-1}{m} [1 - F(v^\dagger)]^m [F(v^\dagger)]^{N-1-m} [\underline{b}(m+1, v^\dagger) - v^\dagger],$$

where the right-hand side represents the extra expected payoff a type-$v^\dagger$ seller can get in first-price auction subgames.

Second, the speculator is better-paid in a second-price auction subgame than in a first-price auction subgame. If the speculator is competing against $1 \le m \le N-1$ sellers in an auction subgame summarized by $\Gamma = \langle r, m, G(\cdot; v^\dagger) \rangle$, the speculator's expected payoff is $y(m, v^\dagger)$ under the second-price rule and is $\underline{b}(m, v^\dagger)$ under the first-price rule. Lemma A.1 in Appendix A shows that $y(m, v^\dagger) > \underline{b}(m, v^\dagger)$ for $v^\dagger \in (0, r)$.

As a result, it is easy to see that $\Pi^*(v^\dagger) > \Pi^\star(v^\dagger)$ for all $v^\dagger \in (0, r)$ by comparing equations (4) and (8). An immediate implication is $\max_{v^\dagger \in [0,r]} \Pi^*(v^\dagger) \ge \max_{v^\dagger \in [0,r]} \Pi^\star(v^\dagger)$. If $\Pi^\star(v^\dagger)$ is not maximized by 0 or r, the inequality is strict. If $\Pi^\star(v^\dagger)$ is maximized by 0, the right-hand side of the inequality is 0. By Proposition 2, the left-hand side is positive, so the inequality is strict. Finally, $\Pi^\star(v^\dagger)$ cannot be maximized by r, since $\Pi^\star(r) = \left\{1 - [1 - F(r)]^N - NF(r)\right\} r < 0$.

In summary, $\max_{v^\dagger \in [0,r]} \Pi^*(v^\dagger) > \max_{v^\dagger \in [0,r]} \Pi^\star(v^\dagger)$. This concludes the proof. ■

The proof of Proposition 5 builds on the difference in the auction outcomes under the two formats, which can be traced back to the difference in competition patterns. Specifically, in second-price auctions, sellers do not respond to the presence of a strong competitor, namely, the speculator: they still bid truthfully in equilibrium. However, in first-price auctions, as will be shown below, sellers respond to the presence of a strong competitor by bidding more aggressively. This helps the sellers win the procurement auction sometimes, even if their values are surely above the speculator's. Therefore, sellers have better prospects in first-price auctions, whereas the speculator's expected payoff is lower.

To see how sellers respond to the speculator's presence in first-price auction subgames, first consider the following hypothetical benchmark. Suppose that in any first-price auction subgame

$\Gamma = \langle r, m, G(\cdot; v^\star)\rangle$ with $1 \le m \le N-1$, sellers bid as if the speculator was just another seller. That is, the speculator also drew his value from $[v^\star, 1]$ according to $G(\cdot; v^\star)$. In this case, sellers' bidding strategy is given by

$$\tilde{\beta}(v; m, v^\star) = \begin{cases} v + \int_v^r [1 - G(x; v^\star)]^m dx \big/ [1 - G(v; v^\star)]^m, & \text{if } v^\star \le v \le r, \\ v, & \text{if } v > r. \end{cases}$$

This is a suitable benchmark, since it hypothetically achieves revenue equivalence across the two auction formats. Specifically, the allocative outcome, the speculator's expected profit, the auctioneer's expected surplus and the sellers' expected payoffs would be the same under the two formats.[33]

In contrast with the benchmark, sellers bid against a stronger competitor in reality. They respond by bidding more aggressively as indicated by Lemma 5.

Lemma 5. *In an FPA subgame, sellers bid more aggressively when they compete against the speculator than when they compete against another seller. Formally, $\beta(v; m, v^\star) < \tilde{\beta}(v; m, v^\star)$ for all $v \in [v^\star, r)$.*

As is shown in the previous analysis, sellers' aggressive bidding undermines the profitability of speculation in first-price auctions. The difference in sellers' responses accounts for the result in Proposition 5 and also for the contrast between Proposition 2 and Proposition 4.

5.2. *Welfare comparison*

With speculation, the revenue equivalence between first-price auctions and second-price auctions breaks down. In this case, welfare ranking is generally ambiguous. In particular, although speculation is more profitable in second-price auctions, it does not necessarily lead to more efficiency loss. First-price auctions could be less efficient, as the profit-maximizing cutoff level could be higher therein. Even with the same cutoff level, a first-price auction is less efficient than a second-price auction, since it includes an additional source of inefficiency other than the destruction of private values: the allocative inefficiency in asymmetric auction subgames.

Since an explicit expression for the profit-maximizing cutoff level is unavailable under the two formats, a general welfare ranking result cannot be obtained. Nevertheless, when speculation is not profitable in first-price auctions, the welfare ranking is given by the following corollary.

Corollary 2. *If $\Pi^\star(v^\star) < 0$ for all $v^\star \in (0, r]$, the FPA generates lower expected procurement costs for the auctioneer and lower expected payoffs for the sellers, as compared to the SPA, while yielding greater efficiency.*

6. Extensions to the second-price auction model

Two extensions to the second-price auction model are presented in this section. First, I consider a more general setting and reexamine the profitability of speculation. Specifically, I relax the assumption that sellers draw their values from the same distribution. In the meantime, I consider

[33] Note that the speculator would bid $\tilde{\beta}(v^\star; m, v^\star)$ and always win the procurement auction in the hypothetical benchmark.

the case where the speculator can only make offers to a subset of the sellers. This limited access assumption is relevant when the speculator wishes to keep speculation discreet by operating on a limited scale, or when the speculator is subject to a budget constraint.

Formally, the baseline model is modified as follows for this section. For all $i \in \mathcal{I}$, seller i's value v_i is drawn from [0,1] according to the CDF $F_i(\cdot)$. The corresponding PDF is $f_i(\cdot)$. Again, I assume that $F_i(0) = 0$ and that $F_i(\cdot)$ has full support on [0,1]. The speculator can make (potentially different) acquisition offers only to a subset of the sellers, denoted by $\mathcal{A}$.

Second, I study an enhanced speculation approach which is fine-tuned to address the loss from private value destruction. While I focus mainly on the second-price auction case, a result regarding enhanced speculation in first-price auctions is included to facilitate comparison.

6.1. Speculation with limited access to asymmetric sellers

In this part, I reexamine the profitability of speculation in second-price auctions.

In light of the analysis of the baseline case, I consider price offers that will lead to a vector of acceptance/rejection cutoffs $\boldsymbol{v}^* := (v_j^*)_{j\in\mathcal{A}}$, with $v_j^* \in [0, r]$.[34] That is, seller $j \in \mathcal{A}$ accepts the speculator's offer if and only if $v_j < v_j^*$. Define[35]

$$c_j^*(\boldsymbol{v}^*) := v_j^* + \int_{v_j^*}^{r} \prod_{i\in\mathcal{I}\setminus\mathcal{A}} [1 - F_i(x)] \prod_{s\in\mathcal{A}\setminus\{j\}} \left[1 - F_s(\max\{x, v_s^*\})\right] dx. \tag{9}$$

Proposition 6 gives the equilibrium characterization in this case.

Proposition 6. *Suppose that the speculator offers $p_j = c_j^*(\boldsymbol{v}^*)$ to seller $j \in \mathcal{A}$, with $v_j^* \in [0, r]$. Then a PBE of the SPA-speculation game is described as follows. Seller $j \in \mathcal{A}$ accepts the speculator's offer if and only if $v_j < v_j^*$. In the procurement auction, sellers bid truthfully and the speculator engages in strategic supply withholding.*

Note that the speculator's profit originates from his ability to reduce potential competition in the auction. If the speculator cannot reduce the competition in a meaningful way, either because the speculator can make acquisition offer to only one seller, or because some sellers outside the speculator's reach are very competitive, speculation would not be profitable. Condition 1 addresses the two possibilities.

Condition 1. There exists $\{k, k'\} \subseteq \mathcal{A}$, such that for all $i \in \mathcal{I} \setminus \mathcal{A}$,

$$\lim_{v\to 0}[vF_i(v)/F_k(v)] = 0 \text{ and } \lim_{v\to 0}[vF_i(v)/F_{k'}(v)] = 0.$$

It is clear that Condition 1 rules out the case that $\mathcal{A}$ is a singleton. Condition 1 also requires the sellers outside the speculator's reach are not too competitive as compared with some sellers in the speculator's reach. Specifically, Condition 1 ensures the values of the sellers in $\mathcal{I} \setminus \mathcal{A}$ do not concentrate at the lower end much more than the values of some sellers in $\mathcal{A}$ do. This requirement does not appear to be too strong, and it is trivially met if sellers are symmetric.

[34] Clearly, $v_j^* > r$ (which implies that $p_j > r$) is suboptimal for the speculator.

[35] Throughout the paper, if $\mathcal{I} = \mathcal{A}$, $\prod_{i\in\mathcal{I}\setminus\mathcal{A}}[1 - F_i(x)]$ should be interpreted as 1.

Proposition 2 can be generalized to the current setting as follows.

Proposition 2′. *If Condition 1 is satisfied, speculation in the second-price procurement auction is profitable.*

6.2. Enhanced speculation

In this part, I consider an enhanced speculation scheme fined-tuned to address the issue of private value destruction. To that end, I assume that after the procurement auction, the speculator can auction off his surplus items back to the sellers who accepted the acquisition offer. The speculator's auction is referred to as the "return and refund" auction.

If sellers' items are heterogenous, they will not compete with each other in the return and refund auction, which undermines the speculator's incentive to carry out one. As the purpose of the analysis is to understand whether and by how much the enhanced speculation scheme can improve the speculator's profit, I assume in this part (Section 6.2) only that the items are homogeneous. Another way to have competition in the return and refund auction is that upon winning the procurement auction, the speculator can conduct the return and refund auction before delivering an item to the auctioneer. In this case, the bidding in the procurement auction must be for a generic item, rather than a specific one.[36]

The three-stage SPA-enhanced-speculation game is described as follows. The first two stages are the same as described in Section 2. In the third stage that happens after the procurement auction, the speculator conducts an auction to sell back his surplus items and get some refunds from the sellers who accepted the acquisition offer. I assume that the speculator's return and refund auction takes the form of a VCG auction. Specifically, if the speculator has $K \geq 1$ item(s) for sale after the procurement auction, the return and refund auction is a $(K+1)$-st price auction.[37]

The limited access and seller asymmetry specifications are maintained in this part. Again, I consider a set of prices that will induce a vector of acceptance/rejection cutoffs in equilibrium. For any vector $\boldsymbol{v}^* \equiv (v_j^*)_{j\in\mathcal{A}}$ with $v_j^* \in [0, r]$,[38] define

$$\begin{aligned}\bar{c}_j^*(\boldsymbol{v}^*) :=& \int_0^{v_j^*} \prod_{s\in\mathcal{A}\setminus\{j\}} \left[1 - F_s(\min\{x, v_s^*\})\right] dx + \\ & \int_{v_j^*}^{r} \prod_{i\in\mathcal{I}\setminus\mathcal{A}} [1 - F_i(x)] \prod_{s\in\mathcal{A}\setminus\{j\}} \left[1 - F_s(\max\{x, v_s^*\})\right] dx.\end{aligned} \tag{10}$$

Proposition 7 provides equilibrium characterization for the enhanced speculation game.

Proposition 7. *In the three-stage SPA-enhanced-speculation game, suppose that the speculator offers $p_j = \bar{c}_j^*(\boldsymbol{v}^*)$ to seller $j \in \mathcal{A}$, with $v_j^* \in [0, r]$. Then a PBE of the speculation game is*

[36] For example, if bidders bid for a road pavement contract by promising to finish the job without specifying the identity of the contractor they will use, the bidding is considered to be generic.

[37] It is clear that this enhanced speculation model has a forward auction counterpart, in which the speculator can promise to sell to more than one potential buyers before a forward auction. After the auction, the speculator conducts another auction to buy back some promises.

[38] It can be shown that $v_j^* > r$ is suboptimal for the speculator.

described as follows. Seller $j \in \mathcal{A}$ accepts the speculator's offer if and only if $v_j < v_j^*$. Sellers bid truthfully in the procurement auction or in the return and refund auction. The speculator engages in strategic supply withholding in the procurement auction.

From the speculator's perspective, the enhanced speculation scheme has two advantages over the simple speculation scheme. First, holding fixed a vector of acceptance/rejection cutoffs $\boldsymbol{v}^*$, the compensation to the sellers is lower under the enhanced speculation approach as the sellers have an opportunity to win their item back.[39] Second, the speculator can get some revenue from the return and refund auction. Corollary 3 ensues as a result.

Corollary 3. *Consider the equilibrium described in Proposition 7. The enhanced speculation approach with appropriately chosen acquisition offers generates strictly higher expected profits for the speculator than the simple speculation approach.*

Corollary 4 demonstrates the full potential of the enhanced speculation scheme in second-price auctions. If the speculator is capable of reaching every seller and organizing the return and refund auction, he is in a position to "take over" the procurement auction from the auctioneer.

Corollary 4. *Consider the equilibrium described in Proposition 7. If $\mathcal{A} = \mathcal{I}$, the speculator can "knock out" every seller by inducing $v_i^* = r$ for all $i \in \mathcal{I}$. Specifically, the speculator offers $p_j = r$ to each seller, and a seller accepts the acquisition offer if and only if his value is less than r. The equilibrium achieves the same outcome as if the speculator conducts a second-price auction with a reserve price of r to buy an item from the sellers, and then sells the item to the auctioneer at a price of r.*

Two remarks regarding the enhanced speculation scheme with full access (i.e., $\mathcal{A} = \mathcal{I}$) are in order. The first remark concerns the optimality of the return and refund auction. Conditional on that the procurement auction has concluded, the VCG auction is *not* a revenue-maximizing mechanism for the speculator to sell his leftover items. The speculator can use (possibly individualized) reserve prices to boost his revenue. However, the introduction of reserve prices will feed back into the acquisition stage and affect the sellers' decisions on whether to accept the acquisition offer. Because reserve prices reduce the sellers' chances of winning back their items in the return and refund auction, they will require more compensation for accepting the acquisition offer. In the case of symmetric sellers, it can be shown that the use of reserve prices will not increase the speculator's expected profit. Put differently, the complete knockout scenario is optimal for the speculator, even if he can use reserve prices in the return and refund auction. The formal analysis of enhanced speculation with the use of reserve prices in the return and refund auction is presented in Appendix B.

The second remark concerns the comparison between second-price auctions and first-price auctions in the enhanced speculation case. Fix $\mathcal{A} = \mathcal{I}$ and consider enhanced speculation in first-price auctions. Proposition 8 shows that complete knockout is generally not feasible. This suggests that enhanced speculation is also more profitable in second-price auctions.

[39] It is easy to see that $c_j^*(\boldsymbol{v}^*) - \bar{c}_j^*(\boldsymbol{v}^*) = \int_0^{v_j^*} \left\{1 - \prod_{s \in \mathcal{A} \setminus \{j\}} \left[1 - F_s(\min\{x, v_s^*\})\right]\right\} dx > 0$.

Proposition 8. *Suppose that $F(r) < 1$. Then in the FPA case, even if the speculator can conduct a VCG auction to return and refund the leftover items, there exists no PBE in which the complete knockout outcome (as described in Corollary 4) is achieved.*[40]

7. Conclusion

This paper studies speculation in procurement auctions by means of market power accumulation in the pre-auction stage and supply withholding in the auction stage. The PBE of a dynamic speculation game is characterized in both the case of second-price auctions and the case of first-price auctions. Speculation tends to be successful in second-price auctions but not so much in first-price auctions, because sellers respond differently in bidding to the speculator under the two formats. In particular, first-price auctions can prevent profitable speculation in some cases. Since speculation harms efficiency and inflates the procurement cost, in those cases, first-price auctions would outperform second-price auctions.

This paper represents a first step toward understanding the role of speculation in procurement auctions. There remains considerable room for extensions. For example, analyzing the profitability and welfare implications of speculation in a multi-unit auction setting deserves serious scholarly effort. In the context of the FCC's Broadcast Incentive Auction, it would be interesting to see how the deferred-acceptance auction design compares with alternative mechanisms, such as VCG auctions, in the presence of speculation. Exploring the design of profit-maximizing speculation schemes is another promising avenue for future research. While the current paper demonstrates that an enhanced-speculation approach can increase the speculator's profit in SPAs, it is still an open question as to what speculation approach would maximize the speculator's profit.

Declaration of competing interest

The author declares that he has no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Data availability

Data will be made available on request.

Appendix A. Proofs

Proof of Lemma 1. I prove the lemma in two steps. In step 1, I show that for each seller $i \in \mathcal{I}$, there exists $v_i^* \in [0, r]$ such that seller i takes the speculator's offer if and only if $v_i < v_i^*$. I then show in step 2 that v_i^*s are the same across $i \in \mathcal{I}$.

Step 1. Given that all the other sellers and the speculator play their equilibrium strategies, consider seller i's problem. Clearly, if seller i accepts the speculator's offer, his payoff is

[40] If $F(r) = 1$, the speculator can threaten to bid 0 if any seller rejects his offer. Then a complete knockout can be supported as a weak PBE. If $F(r) < 1$, because rejection happens with a positive probability on the equilibrium path, the threat would violate sequential rationality.

$\pi_i^A(v_i) = p.$

Suppose that all the other sellers reject the speculator's offer with probability 1. Then if seller i rejects the offer, he competes in a second-price auction against all the other sellers. It is straightforward to see that his payoff is

$$\pi_i^R(v_i) = v_i + \int_{v_i}^{r} [1 - F(x)]^{N-1} \mathbb{1}_{\{x \le r\}} dx.$$

Therefore,

$$\frac{\partial[\pi_i^A(v_i) - \pi_i^R(v_i)]}{\partial v_i} = \begin{cases} -\{1 - [1 - F(v_i)]^{N-1}\} < 0, & \text{if } 0 < v_i \le r, \\ -1 < 0, & \text{if } v_i > r. \end{cases}$$

This implies that $\pi_i^A(v_i) - \pi_i^R(v_i)$ strictly decreases with v_i, which further implies that there exists a desired cutoff v_i^*.

Next, suppose that the probability of at least one other seller taking the speculator's offer, denoted by $\mathcal{Q}$, is positive. If seller i rejects the offer, his payoff is 0 if at least one other seller accepts the offer, which happens with probability $\mathcal{Q}$. If no seller accepts the offer, or equivalently, after the public history $(N, 0)$, seller i's payoff is

$$\pi_i^R(v_i; N, 0) = v_i + \int_{v_i}^{r} \mathcal{WP}_i(x; N, 0) dx,$$

where $\mathcal{WP}_i(x; N, 0)$ stands for the winning probability of seller i with a realized value x. Then seller i's interim expected payoff is

$$\pi_i^R(v_i) = v_i + (1 - \mathcal{Q}) \int_{v_i}^{r} \mathcal{WP}_i(x; N, 0) dx,$$

which implies that

$$\frac{\partial[\pi_i^A(v_i) - \pi_i^R(v_i)]}{\partial v_i} = -[1 - (1 - \mathcal{Q})\mathcal{WP}_i(v_i; N, 0)] < 0.$$

Again, because $\pi_i^A(v_i) - \pi_i^R(v_i)$ strictly decreases with v_i, there exists a desired cutoff v_i^*. In particular, the following holds true for the cutoff v_i^*,

$$\pi_i^A(v_i^*) \le \pi_i^R(v_i^*), \text{ and } \pi_i^A(v_i^*) = \pi_i^R(v_i^*) \text{ if } v_i^* > 0. \tag{A.1}$$

Step 2. Without loss of generality, suppose that $v_1^* \le v_2^* \le \ldots \le v_N^*$. Assume by contradiction that $v_\ell^* < v_{\ell+1}^*$ for some $1 \le \ell < N$. Note that for each $i \in \mathcal{I}$, given $v_i \ge v_i^*$, seller i's winning probability is

$$\mathcal{WP}_i(v_i) = [1 - F(v_i)]^{s-1} \prod_{t=s+1}^{N} [1 - F(v_t^*)] \mathbb{1}_{\{v_i \le r\}}, \text{ if } v_s^* \le v_i < v_{s+1}^*.$$

Consider seller ℓ with value v_ℓ^*. If he rejects the acquisition offer, his expected payoff from the auction is

$$\pi_\ell^R(v_\ell^*) = v_\ell^* + \int_{v_\ell^*}^{r} \mathcal{WP}_\ell(x)dx.$$

It follows from (A.1) that

$$p \le \pi_\ell^R(v_\ell^*) = v_\ell^* + \int_{v_\ell^*}^{v_{\ell+1}^*} [1-F(x)]^{\ell-1} \prod_{t=\ell+1}^{N} [1-F(v_t^*)]\mathbb{1}_{\{v_\ell \le r\}}dx + \int_{v_{\ell+1}^*}^{r} \mathcal{WP}_\ell(x)dx. \tag{A.2}$$

Since

$$[1-F(x)]^{\ell-1} \prod_{t=\ell+1}^{N} [1-F(v_t^*)]\mathbb{1}_{\{v_\ell \le r\}} \le 1-F(v_{\ell+1}^*) < 1,$$

(A.2) implies that

$$p < v_\ell^* + v_{\ell+1}^* - v_\ell^* + \int_{v_{\ell+1}^*}^{r} \mathcal{WP}_\ell(x)dx. \tag{A.3}$$

Because $v_{\ell+1}^* > 0$, one can see that

$$p - v_{\ell+1}^* = \int_{v_{\ell+1}^*}^{r} \mathcal{WP}_{\ell+1}(x)dx = \int_{v_{\ell+1}^*}^{r} \mathcal{WP}_\ell(x)dx, \tag{A.4}$$

where the first equality follows from (A.1) and the second equality is due to the fact that $\mathcal{WP}_\ell(x) = \mathcal{WP}_{\ell+1}(x)$ for $x \ge v_{\ell+1}^*$. It is easy to see that a contradiction arises between (A.3) and (A.4). This completes the proof. ■

Proof of Proposition 1. By Lemma 1, it suffices to show that the equilibrium cutoff is unique and is given by $v^*(p)$.

If seller i rejects the speculator's offer, it is weakly dominant for him to bid truthfully in the procurement auction. Given that other sellers follow the equilibrium strategy, the winning probability of seller i is

$$\mathcal{WP}_i(v_i) = \begin{cases} [1-F(v^*)]^{N-1}\mathbb{1}_{\{v_i \le r\}}, & \text{if } v_i < v^*, \\ [1-F(v_i)]^{N-1}\mathbb{1}_{\{v_i \le r\}}, & \text{if } v_i \ge v^*. \end{cases}$$

Then (A.1) in the proof of Proposition 1 can be rewritten as follows,

$$p \le v^* + \int_{v^*}^{r} [1-F(x)]^{N-1}\mathbb{1}_{\{x \le r\}}dx, \text{ and } p = v^* + \int_{v^*}^{r} [1-F(x)]^{N-1}\mathbb{1}_{\{x \le r\}}dx \text{ if } v^* > 0.$$

This immediately implies that $v^* = v^*(p)$. ■

Proof of Proposition 2. Note that $\Pi^*(v^*)$ can be rewritten as follows.

$$\Pi^*(v^*) = \sum_{m=0}^{N-2} \binom{N}{m} [F(v^*)]^{N-m}[1-F(v^*)]^m \left\{ v^* + \int_{v^*}^{r} \left[\frac{1-F(x)}{1-F(v^*)}\right]^m dx \right\} - NF(v^*)\left\{1-[1-F(v^*)]^{N-1}\right\} v^*.$$

As $v^* \to 0$, the only non-zero term of $\lim_{v^*\to 0} \frac{\Pi^*(v^*)}{[F(v^*)]^2}$ is

$$\lim_{v^*\to 0} \binom{N}{N-2}[1-F(v^*)]^{N-2} \int_{v^*}^{r} \left[\frac{1-F(x)}{1-F(v^*)}\right]^{N-2} dx = \binom{N}{2}\int_0^r [1-F(x)]^{N-2}dx > 0.$$

Since $\frac{\Pi^*(v^*)}{[F(v^*)]^2}$ is continuous in $v^* \in [0,r]$, $\lim_{v^*\to 0} \frac{\Pi^*(v^*)}{[F(v^*)]^2} > 0$ implies that $\Pi^*(v^*) > 0$ for small enough $v^* > 0$. Consequently, the speculator's expected profit is $\max_{v^*\in[0,r]} \Pi^*(v^*) > 0$. This concludes the proof. ■

Proof of Lemma 2. Let $\alpha(v)$ denote the probability that a type-v seller accepts the speculator's offer in equilibrium. Define $\underline{v} := \inf\{v \in [0,1] : \exists \epsilon > 0$ such that $\alpha(v') < 1$ for all $v' \in \mathcal{U}_\epsilon(v) := (v, v+\epsilon)\}$. By construction, $\alpha(v) = 1$ for all $v < \underline{v}$ except for a set with F-measure 0. It is clear that $\underline{v} \le p \le r$.

It suffices to show that $\alpha(v) = 0$ for all $v > \underline{v}$. Consider seller i with $v_i > \underline{v}$. If seller i accepts the speculator's offer, his payoff is

$$\varpi^A(v_i) = p.$$

If seller i rejects the speculator's offer, the interim expected payoff to seller i can be established as follows by a standard payoff equivalence argument,

$$\begin{aligned}\varpi^R(v_i) - v_i = \varpi^R(\underline{v}) - \underline{v} &- \sum_{m=1}^{N-1} \Pr[h=(m,1)] \int_{\underline{v}}^{v_i} \mathcal{WP}^\star(x;m,1)dx \\ &- \Pr[h=(N,0)] \int_{\underline{v}}^{v_i} \mathcal{WP}^\star(x;N,0)dx,\end{aligned}$$

where $\mathcal{WP}^\star(x;h)$ is the winning probability of seller i with value x after a history h, given that all the other bidders in the auction use their equilibrium bidding strategies and seller i best-responds to the other bidders' equilibrium bidding. Note that

$$\frac{\partial[\varpi^A(v_i) - \varpi^R(v_i)]}{\partial v_i} = -1 + \sum_{m=1}^{N-1} \Pr[h=(m,1)]\mathcal{WP}^\star(v_i;m,1) + \Pr[h=(N,0)]\mathcal{WP}^\star(v_i;N,0) \le 0.$$

By the definition of $\underline{v}$, one can see that $\alpha(v_i) < 1$ for $v_i \in \mathcal{U}_\epsilon(\underline{v})$. Therefore, $\Pr[h=(N,0)] > 0$ and $\mathcal{WP}^\star(v_i;N,0) < 1$ for $v_i \in \mathcal{U}_\epsilon(\underline{v})$. This implies that $\partial[\varpi^A(v_i) - \varpi^R(v_i)]/\partial v_i < 0$ for $v_i \in \mathcal{U}_\epsilon(\underline{v})$. Further,

$$\varpi^A(v_i) - \varpi^R(v_i) = \varpi^A(\underline{v}) - \varpi^R(\underline{v}) + \int_{\underline{v}}^{v_i} \frac{\partial[\varpi^A(v_i) - \varpi^R(v_i)]}{\partial v_i} dx < \varpi^A(\underline{v}) - \varpi^R(\underline{v}). \tag{A.5}$$

Since the speculator's offer is rejected with a positive probability when a seller's value is in $\mathcal{U}_\epsilon(\underline{v})$, we know that

$$\varpi^A(\underline{v}) - \varpi^R(\underline{v}) \leq 0.$$

Plugging this into (A.5) yields that $\varpi^R(v_i) > \varpi^A(v_i)$, which implies that $\alpha(v_i) = 0$. This completes the proof. ■

Proof of Lemma 3. To start the proof, let $\underline{b}(m, v^\star)$ and $\bar{b}(m, v^\star)$ denote the speculator's lowest and the highest possible bid in equilibrium, respectively.[41] Clearly, $v^\star \leq \underline{b}(m, v^\star) \leq \bar{b}(m, v^\star) \leq r$.

It is convenient to prove Point (ii) first. Claim 1 through Claim 5 below collectively serve as a proof of Point (ii).

Claim 1. $\beta(v; m, v^\star) < \bar{b}(m, v^\star)$ *for all* $v < \bar{b}(m, v^\star)$.

Proof. Suppose to the contrary that $\beta(v'; m, v^\star) \geq \bar{b}(m, v^\star)$ for some $v' < \bar{b}(m, v^\star)$. Then a type-v' seller loses for sure. But he can profitably deviate to bidding $\bar{b}(m, v^\star) - \epsilon$ for some small $\epsilon > 0$. ■

Claim 2. $\beta(v; m, v^\star)$ *is strictly increasing in* $v \in (v^\star, \bar{b}(m, v^\star))$.

Proof. Suppose $v^\star < v' < v'' < \bar{b}(m, v^\star)$. The no-deviation condition of the two types are

$$[\beta(v'; m, v^\star) - v']\mathcal{WP}(\beta(v'; m, v^\star)) \geq [\beta(v''; m, v^\star) - v']\mathcal{WP}(\beta(v''; m, v^\star)),$$
$$[\beta(v''; m, v^\star) - v'']\mathcal{WP}(\beta(v''; m, v^\star)) \geq [\beta(v'; m, v^\star) - v'']\mathcal{WP}(\beta(v'; m, v^\star)),$$

where $\mathcal{WP}(b)$ stands for the winning probability of bidding b given that other players use their equilibrium bidding strategies. Adding up the two no-deviation conditions yields the following,

$$(v'' - v')[\mathcal{WP}(\beta(v'; m, v^\star)) - \mathcal{WP}(\beta(v''; m, v^\star))] \geq 0,$$

which implies that $\beta(v'; m, v^\star) \leq \beta(v''; m, v^\star)$. As a result, $\beta(v; m, v^\star)$ is weakly increasing in $v \in (v^\star, \bar{b}(m, v^\star))$.

It suffices to show that $\beta(v; m, v^\star)$ cannot remain constant on some interval $[v', v'']$. Suppose to the contrary that $\beta(v; m, v^\star) = b^\circ$ for $v \in [v', v'']$. By Claim 1, $b^\circ < \bar{b}(m, v^\star)$. Then a type-$v'$ seller can profitably deviate to bidding $b^\circ - \epsilon$ for a small enough $\epsilon > 0$. A contradiction. ■

Claim 3. $\beta(v; m, v^\star)$ *is continuous in* $v \in (v^\star, \bar{b}(m, v^\star))$.

[41] To be more precise, $\bar{b}(m, v^\star)$ should be interpreted as the maximum possible bid such that the left neighborhood $(\bar{b}(m, v^\star) - \epsilon, \bar{b}(m, v^\star))$ is assigned a positive probability. Similarly, $\underline{b}(m, v^\star)$ should be interpreted as the lowest possible bid such that the right neighborhood $(\underline{b}(m, v^\star), \underline{b}(m, v^\star) + \epsilon)$ is assigned a positive probability.

Proof. Suppose to the contrary that $\beta(v; m, v^\star)$ has a jump at $\hat{v} \in (v^\star, \bar{b}(m, v^\star))$. Formally, $\hat{b}^- := \lim_{v\to\hat{v}^-} \neq \hat{b}^+ := \lim_{v\to\hat{v}^+}$. By Claim 1 and Claim 2, $\hat{b}^- < \hat{b}^+ < \bar{b}(m, v^\star)$ and no seller bids between $\hat{b}^-$ and $\hat{b}^+$. This implies that bidding $\hat{b}^+$ is strictly more profitable than any bid between $\hat{b}^-$ and $\hat{b}^+$ for the speculator. As a result, the speculator does not bid between $\hat{b}^-$ and $\hat{b}^+$. Consequently, for a type-$\hat{v}^-$ seller, it is profitable to deviate to bidding $\hat{b}^+$. A contradiction. ■

Claim 4. $\beta(\bar{b}(m, v^\star); m, v^\star) = \bar{b}(m, v^\star)$.

Proof. By combining Claim 1 with the assumption of undominated bidding, it follows that $\beta(\bar{b}(m, v^\star); m, v^\star) = \bar{b}(m, v^\star)$, or more precisely, $\lim_{v\to\bar{b}(m,v^\star)^-} \beta(v; m, v^\star) = \bar{b}(m, v^\star)$. ■

Claim 5. $\beta(v^\star; m, v^\star) = \underline{b}(m, v^\star)$.

Proof. If $\beta(v^\star; m, v^\star) > \underline{b}(m, v^\star)$, the speculator can profitably deviate from bidding $\underline{b}(m, v^\star)$ to bidding $\beta(v^\star; m, v^\star)$, so $\beta(v^\star; m, v^\star) \le \underline{b}(m, v^\star)$. Suppose by contradiction that $\beta(v^\star; m, v^\star) < \underline{b}(m, v^\star)$. By Claim 2, Claim 3, and Claim 4, there exists $\hat{v} > v^\star$ such that $\beta(\hat{v}; m, v^\star) = \underline{b}(m, v^\star)$. Note that for $v \le \hat{v}$, the speculator's payoff from bidding $\beta(v; m, v^\star)$ is $[1 - G(v; v^\star)]^m \beta(v; m, v^\star)$. I show next that $[1 - G(v; v^\star)]^m \beta(v; m, v^\star)$ is decreasing in $v \in [v^\star, \hat{v}]$, which implies that the speculator can profitably deviate from bidding $\underline{b}(m, v^\star)$ to bidding $\beta(v^\star; m, v^\star)$.

By the standard payoff equivalence argument, the following holds for $v \in [v^\star, \hat{v}]$:

$$\varpi(\hat{v}) - \hat{v} + \int_v^{\hat{v}} [1 - G(x; v^\star)]^{m-1} dx = [\beta(v; m, v^\star) - v][1 - G(v; v^\star)]^{m-1},$$

where $\varpi(\hat{v})$ denotes the interim expected payoff of a type-$\hat{v}$ seller. Therefore,

$$\begin{aligned}\frac{d\left\{[1 - G(v; v^\star)]^{m-1}\beta(v; m, v^\star)\right\}}{dv} &= \frac{d\left\{\int_v^{\hat{v}}[1 - G(x; v^\star)]^{m-1} + v[1 - G(v; v^\star)]^{m-1}\right\}}{dv} \\ &= v\frac{d[1 - G(v; v^\star)]^{m-1}}{dv} < 0.\end{aligned}$$

This implies that $[1 - G(v; v^\star)]^{m-1}\beta(v; m, v^\star)$ decreases in $v \in [v^\star, \hat{v}]$. Consequently, $[1 - G(v; v^\star)]^m \beta(v; m, v^\star)$ decreases in $v \in [v^\star, \hat{v}]$. ■

I proceed to the proof of Point (i) next. It suffices to prove that the speculator does not skip bidding on any interval between $\underline{b}(m, v^\star)$ and $\bar{b}(m, v^\star)$. Suppose to the contrary that the speculator does not bid on $(\underline{b}^\dagger, \bar{b}^\dagger)$, with $\underline{b}(m, v^\star) \le \underline{b}^\dagger < \bar{b}^\dagger \le \bar{b}(m, v^\star)$. It is without loss to assume that the speculator *does* possibly bid $\bar{b}^\dagger$. By Point (ii), there exists $\hat{v} \in (v^\star, \bar{b}(m, v^\star))$ such that $\beta(\hat{v}; m, v^\star) = \bar{b}^\dagger$. By a similar argument as in the proof of Claim 5, I show in what follows that the speculator's expected payoff from bidding $\beta(v; m, v^\star)$, which is given by $[1 - G(v; v^\star)]^m \beta(v; m, v^\star)$, is decreasing in a small left neighborhood of $\hat{v}$, denoted by $[\hat{v} - \epsilon, \hat{v}]$. This leads to a contradiction as the speculator can profitably deviate from bidding $\bar{b}^\dagger$ to bidding $\beta(\hat{v} - \epsilon; m, v^\star)$.

By the standard payoff equivalence argument, the following holds for $v \in [\hat{v} - \epsilon, \hat{v}]$:

$$\varpi(\hat{v}) - \hat{v} + \int_{v}^{\hat{v}} [1 - G(x; v^{\star})]^{m-1} \widehat{\Psi} dx = [\beta(v; m, v^{\star}) - v][1 - G(v; v^{\star})]^{m-1} \widehat{\Psi},$$

where $\varpi(\hat{v})$ denotes the interim expected payoff of a type-$\hat{v}$ seller, and $\widehat{\Psi}$ denotes the probability that the speculator's bid is above $\bar{b}^{\dagger} = \beta(\hat{v}; m, v^{\star})$. Therefore,

$$\frac{d\left\{[1 - G(v; v^{\star})]^{m-1} \beta(v; m, v^{\star})\right\}}{dv} = v \frac{d[1 - G(v; v^{\star})]^{m-1}}{dv} < 0.$$

This implies that $[1 - G(v; v^{\star})]^{m-1} \beta(v; m, v^{\star})$ decreases in $v \in [v^{\star}, \hat{v}]$. Consequently, $[1 - G(v; v^{\star})]^{m} \beta(v; m, v^{\star})$ decreases in $v \in [v^{\star}, \hat{v}]$. ■

Proof of Lemma 4. In light of the analysis in the main text, it suffices to verify the strategies proposed in Lemma 4 constitute an equilibrium. By construction, the speculator has no profitable deviation.

Next, consider the problem of seller $i \in \mathcal{I}$, holding fixed all the other sellers' and the speculator's equilibrium bidding. If $v_i \geq \bar{b}(m, v^{\star})$, seller i can never win the auction with a positive payoff, bidding truthfully yields a zero payoff and thus is optimal for him.

Suppose $v_i \in [v^{\star}, \bar{b}(m, v^{\star}))$. Seller i never bids strictly less than $\underline{b}(m, v^{\star})$. Otherwise, he can deviate to a slightly higher bid and still win for sure. Bidding above $\bar{b}(m, v^{\star})$ is also not optimal for seller i since he loses for sure in that case. Therefore, seller i chooses a bid in $[\underline{b}(m, v^{\star}), \bar{b}(m, v^{\star})]$ to maximize his expected payoff. This is equivalent to choosing $\hat{v} \in [v^{\star}, \bar{b}(m, v^{\star})]$ and bidding $\beta(\hat{v}; m, v^{\star})$. Formally, seller i solves

$$\max_{\hat{v} \in [v^{\star}, \bar{b}(m, v^{\star})]} \varpi(\hat{v}; v_i) := [1 - G(\hat{v}; v^{\star})]^{m-1} [1 - \Psi(\beta(\hat{v}; m, v^{\star}))][\beta(\hat{v}; m, v^{\star}) - v_i] + v_i.$$

It can be verified that

$$\frac{\partial \varpi(\hat{v}; v_i)}{\partial \hat{v}} = \frac{m\beta(\hat{v}; m, v^{\star})}{\beta(\hat{v}; m, v^{\star}) - \hat{v}} [1 - G(\hat{v}; v^{\star})]^{m-2} G'(\hat{v}; v^{\star}) [1 - \Psi(\beta(\hat{v}; m, v^{\star}))](v_i - \hat{v}).$$

Therefore, $\varpi(\hat{v}; v_i)$ is maximized by $\hat{v} = v_i$ and it is optimal for seller i to bid $\beta(v_i; m, v^{\star})$. ■

Proof of Footnote 30. Consider the right derivative of $c^{\star}(v)$ for any $v \in (0, r)$. For notational ease, define

$$\mathcal{C}(v; m) := [1 - F(v)]^{m} [F(v)]^{N-1-m} [\underline{b}(m + 1, v) - v]. \tag{A.6}$$

I first prove that $\mathcal{C}'_{+}(v; m) := \lim_{\epsilon \to 0^{+}} \frac{\mathcal{C}(v+\epsilon; m) - \mathcal{C}(v; m)}{\epsilon} > -[1 - F(v)]^{m} [F(v)]^{N-1-m}$.

By definition,

$$\underline{b}(m + 1, v) = \frac{\max_{r \geq b \geq v} b[1 - F(b)]^{m+1}}{[1 - F(v)]^{m+1}}.$$

If there exists $b' \in (v, r]$ such that $b' \in \arg\max_{r \geq b \geq v} b[1 - F(b)]^{m+1}$, then increasing v marginally would not change the value of $\max_{r \geq b \geq v} b[1 - F(b)]^{m+1}$. It follows that

$$
\begin{aligned}
\mathcal{C}'_+(v;m) &= \frac{\partial\left\{[F(v)]^{N-1-m}b'[1-F(b')]^{m+1}/[1-F(v)]\right\}}{\partial v} \\
&\quad - \frac{\partial\left\{[1-F(v)]^m[F(v)]^{N-1-m}v\right\}}{\partial v} \\
&= b'[1-F(b')]^{m+1}\frac{(N-1-m)[F(v)]^{N-2-m}[1-F(v)]+[F(v)]^{N-1-m}}{[1-F(v)]^2}f(v) \\
&\quad - v\left\{(N-1-m)[F(v)]^{N-2-m}[1-F(v)]^m\right. \\
&\quad \left. - m[1-F(v)]^{m-1}[F(v)]^{N-1-m}\right\}f(v) \\
&\quad - [1-F(v)]^m[F(v)]^{N-1-m} \\
&> -[1-F(v)]^m[F(v)]^{N-1-m},
\end{aligned}
$$

where the inequality follows from the fact that $b'[1-F(b')]^{m+1} \geq v[1-F(v)]^{m+1}$.

If $v[1-F(v)]^{m+1} > b[1-F(b)]^{m+1}$ for all $b \in (v, r]$, there exists $\epsilon > 0$ such that $\underline{b}(m+1, v') = v'$ for all $v' \in [v, v+\epsilon]$. As a result, $\mathcal{C}(v';m) = 0$ for $v' \in [v, v+\epsilon]$. This implies that

$$\mathcal{C}'_+(v;m) = 0 > -[1-F(v)]^m[F(v)]^{N-1-m}.$$

To sum up, (A.6) holds and the following ensues,

$$
\begin{aligned}
\lim_{\epsilon\to 0^+}\frac{c^\star(v+\epsilon)-c^\star(v)}{\epsilon} &= 1-[1-F(v)]^{N-1}+\sum_{m=0}^{N-2}\binom{N-1}{m}\mathcal{C}'_+(v;m) \\
&> 1-[1-F(v)]^{N-1}-\sum_{m=0}^{N-2}\binom{N-1}{m}[1-F(v)]^m[F(v)]^{N-1-m} \\
&= 0.
\end{aligned}
$$

Consequently, $c^\star(v)$ is strictly increasing in $v \in (0, r)$. ■

Proof of Proposition 3. Point (i) follows immediately from Lemma 2 and the indifference condition (7).

To prove Point (ii), it suffices to verify the cutoff acceptance strategy is optimal for a representative seller $i \in \mathcal{I}$. If seller i, with a realized value of v_i, accepts the acquisition offer, his payoff is

$$\varpi_i^A(v_i) = p.$$

If seller i rejects the acquisition offer, he participates in the procurement auction. As is shown in the proof of Lemma 2, seller i's expected payoff $\varpi_i^R(v_i)$ satisfies

$$\frac{\partial \varpi_i^R(v_i)}{\partial v_i} \geq 0.$$

As a result, $\varpi_i^A(v_i) - \varpi_i^R(v_i)$ decreases with v_i. Since $\varpi_i^A(v^\star) = \varpi_i^R(v^\star)$, it is optimal for seller i to accept the acquisition offer if and only if $v_i < v^\star$. ■

Proof of Proposition 4. See Example 1 in the main text. ■

Lemma A.1. *For all* $v^\dagger \in [0, r)$,

$$y(m, v^\dagger) := v^\dagger + \int_{v^\dagger}^{r} \left[\frac{1 - F(x)}{1 - F(v^\dagger)}\right]^m dx > \underline{b}(m, v^\dagger) := \max_{r \geq b \geq v^\dagger} b[1 - G(b; v^\dagger)]^m.$$

Proof of Lemma A.1. Recall that $G(v; v^\dagger) := \frac{F(v) - F(v^\dagger)}{1 - F(v^\dagger)}$ for $v \in [v^\dagger, 1]$. So it suffices to prove that

$$v^\dagger + \int_{v^\dagger}^{r} \left[1 - G(x; v^\dagger)\right]^m dx > b[1 - G(b; v^\dagger)]^m \text{ for all } v^\dagger \in [0, r) \text{ and } b \in [v^\dagger, r].$$

In fact,

$$\begin{aligned} v^\dagger + \int_{v^\dagger}^{r} \left[1 - G(x; v^\dagger)\right]^m dx &\geq v^\dagger + \int_{v^\dagger}^{b} \left[1 - G(x; v^\dagger)\right]^m dx \\ &\geq v^\dagger + \int_{v^\dagger}^{b} \left[1 - G(b; v^\dagger)\right]^m dx \\ &= v^\dagger \left\{1 - \left[1 - G(b; v^\dagger)\right]^m\right\} + b[1 - G(b; v^\dagger)]^m \\ &\geq b[1 - G(b; v^\dagger)]^m. \end{aligned}$$

The equalities cannot hold at the same time, as that requires $b = r$ (from the first inequality), $G(x; v^\dagger) = G(b; v^\dagger)$ for almost every $x \in [v^\dagger, b]$ (from the second inequality), and $v^\dagger = 0$ (from the last inequality). ■

Proof of Lemma 5. It suffices to prove that $\tilde{\beta}(v; m, v^\star) > \beta(v; m, v^\star)$ for all $v \in [v^\star, \bar{b}(m, v^\star))$, which is equivalent to

$$\mathcal{Z}(v) := v[1 - G(v; v^\star)]^m + \int_{v}^{r} [1 - G(x; v^\star)]^m dx > \underline{b}(m, v^\star) \text{ for all } v \in [v^\star, \bar{b}(m, v^\star)).$$

It is easy to verify that $\mathcal{Z}(v)$ is strictly decreasing in $v \in [v^\star, \bar{b}(m, v^\star)]$. Then the proof is completed by the following fact,

$$\mathcal{Z}(\bar{b}(m, v^\star)) = \underline{b}(m, v^\star) + \int_{\bar{b}(m, v^\star)}^{r} [1 - G(x; v^\star)]^m dx \geq \underline{b}(m, v^\star). \quad ■$$

Proof of Proposition 6. It suffices to verify that the acceptance/rejection schedule is optimal for an arbitrary seller $j \in \mathcal{A}$, given that all the other sellers follow the equilibrium strategies described in Proposition 6.

If seller j, with a realized valuation $v_j \le r$,[42] accepts the offer, his payoff is $\pi_j^A(v_j) = p_j(\boldsymbol{v}^*)$. If he rejects the offer, he participates in the procurement auction. Seller j wins in the procurement auction if and only if all the other sellers' valuations are above v_j and the speculator failed to acquire any item. Formally, seller j's winning probability is

$$\mathcal{WP}_j(v_j) = \prod_{i \in \mathcal{I}\backslash\mathcal{A}} [1 - F_i(v_j)] \prod_{s \in \mathcal{A}\backslash\{j\}} \left[1 - F_s(\max\{v_j, v_s^*\})\right].$$

As a result, seller j's expected payoff in the auction is $\pi_j^R(v_j) = v_j + \int_{v_j}^r \mathcal{WP}_j(x)dx$.

The following two facts in combination verify the optimality of accepting the acquisition offer if and only if $v_j < v_j^*$:

$$\frac{\partial[\pi_j^A(v_j) - \pi_j^R(v_j)]}{\partial v_j} = -[1 - \mathcal{WP}_j(v_j)] \le 0, \text{ and } \pi_j^A(v_j^*) = \pi_j^R(v_j^*). \quad \blacksquare$$

Proof of Proposition 2′. I show in what follows that the speculator can obtain a positive expected profit by setting $v_j^* = v^*$ for all $j \in \mathcal{A}$ and appropriately choosing v^*.

Given that $v_j^* = v^*$ for all $j \in \mathcal{A}$, the speculator's profit from the events in which only one seller accepts the offer is

$$\Pi^*(v^*, 1) = \sum_{j \in \mathcal{A}} F_j(v^*) \prod_{j' \in \mathcal{A}\backslash\{j\}} [1 - F_{j'}(v^*)] \Bigg\{ \int_0^{v^*} \prod_{i \in \mathcal{I}\backslash\mathcal{A}} [1 - F_i(x)]dx$$
$$+ \int_{v^*}^r \frac{\prod_{i \in \mathcal{I}\backslash\{j\}}[1 - F_i(x)]}{\prod_{j' \in \mathcal{A}\backslash\{j\}}[1 - F_{j'}(v^*)]} dx - v^* - \int_{v^*}^r \prod_{i \in \mathcal{I}\backslash\{j\}} [1 - F_i(x)]dx \Bigg\}.$$

As v^* approaches 0, one can see that[43]

$$\begin{aligned}
\Pi^*(v^*, 1) = &\sum_{j \in \mathcal{A}} \sum_{j' \in \mathcal{A}\backslash\{j\}} F_j(v^*) F_{j'}(v^*) \int_{v^*}^r \prod_{i \in \mathcal{I}\backslash\{j\}} [1 - F_i(x)]dx \\
&+ \sum_{j \in \mathcal{A}, i \in \mathcal{I}\backslash\mathcal{A}} O\left(v^* F_j(v^*) F_i(v^*)\right) + \sum_{j \in \mathcal{A}, j' \in \mathcal{A}\backslash\{j\}} o\left(F_j(v^*) F_{j'}(v^*)\right) \\
= &\sum_{j \in \mathcal{A}, j' \in \mathcal{A}\backslash\{j\}} F_j(v^*) F_{j'}(v^*) \Bigg\{ \int_{v^*}^r \prod_{i \in \mathcal{I}\backslash\{j\}} [1 - F_i(x)]dx \\
&+ \int_{v^*}^r \prod_{i \in \mathcal{I}\backslash\{j'\}} [1 - F_i(x)]dx \Bigg\} \\
&+ \sum_{j \in \mathcal{A}, i \in \mathcal{I}\backslash\mathcal{A}} O\left(v^* F_j(v^*) F_i(v^*)\right) + \sum_{j \in \mathcal{A}, j' \in \mathcal{A}\backslash\{j\}} o\left(F_j(v^*) F_{j'}(v^*)\right).
\end{aligned}$$

[42] Note that $c_j^*(\boldsymbol{v}^*) \le r$, so seller j would reject the acquisition offer if $v_j > r$.

[43] I use $O(\cdot)$ to denote the same order infinitesimal and $o(\cdot)$ to denote higher order infinitesimal as $v^* \to 0$. That is, for an arbitrary function of v^* denoted by $T(v^*)$, $\lim_{v^* \to 0} O(T(v^*))/T(v^*)$ equals a non-zero constant, and $\lim_{v^* \to 0} o(T(v^*))/T(v^*) = 0$.

The speculator's expected profit from the events in which two sellers accept the offers is

$$\Pi^*(v^*,2) = \sum_{j\in\mathcal{A},\, j'\in\mathcal{A}\setminus\{j\}} F_j(v^*)F_{j'}(v^*) \times \prod_{j''\in\mathcal{A}\setminus\{j,j'\}}[1-F_{j''}(v^*)]\Bigg\{\int_0^{v^*}\prod_{i\in\mathcal{I}\setminus\mathcal{A}}[1-F_i(x)]dx + \int_{v^*}^{r}\frac{\prod_{i\in\mathcal{I}\setminus\{j,j'\}}[1-F_i(x)]}{\prod_{j''\in\mathcal{A}\setminus\{j,j'\}}[1-F_{j''}(v^*)]}dx - 2v^* - \int_{v^*}^{r}\prod_{i\in\mathcal{I}\setminus\{j,j'\}}[1-F_i(x)][2-F_j(x)-F_{j'}(x)]dx\Bigg\}.$$

As v^* approaches 0, one can see that

$$\Pi^*(v^*,2) = \sum_{j\in\mathcal{A},\, j'\in\mathcal{A}\setminus\{j\}} F_j(v^*)F_{j'}(v^*)\int_{v^*}^{r}\prod_{i\in\mathcal{I}\setminus\{j,j'\}}[1-F_i(x)][F_j(x)+F_{j'}(x)-1]dx + \sum_{j\in\mathcal{A},\, j'\in\mathcal{A}\setminus\{j\}} o\left(F_j(v^*)F_{j'}(v^*)\right).$$

Analogously, one can calculate $\Pi^*(v^*,m)$ for $m\geq 3$. For $m\geq 3$, it is clear that $\Pi^*(v^*,m)=\sum_{j\in\mathcal{A}}\sum_{j'\in\mathcal{A}\setminus\{j\}} o\left(F_j(v^*)F_{j'}(v^*)\right)$ as v^* approaches 0. Since the speculator's expected profit is $\Pi^*(v^*)=\sum_m \Pi^*(v^*,m)$, as v^* approaches 0,[44]

$$\Pi^*(v^*) = \sum_{j\in\mathcal{A}}\sum_{j'\in\mathcal{A}\setminus\{j\}} F_j(v^*)F_{j'}(v^*)\int_{v^*}^{r}\prod_{i\in\mathcal{I}\setminus\{j,j'\}}[1-F_i(x)]dx + \sum_{j\in\mathcal{A},\, i\in\mathcal{I}\setminus\mathcal{A}} O\left(v^*F_j(v^*)F_i(v^*)\right) + \sum_{j\in\mathcal{A},\, j'\in\mathcal{A}\setminus\{j\}} o\left(F_j(v^*)F_{j'}(v^*)\right).$$

One can find $j_1, j_2\in\mathcal{A}$ such that $\lim_{v\to 0}[F_j(v)/F_{j_1}(v)]<\infty$ for all $j\in\mathcal{A}$ and that $\lim_{v\to 0}[F_{j'}(v)/F_{j_2}(v)]<\infty$ for all $j'\in\mathcal{A}\setminus\{j_1\}$. Then j_1 and j_2 (as k and k') must satisfy Condition 1. From Condition 1 and the specification of j_1 and j_2, it follows that

$$\lim_{v\to 0}\frac{vF_j(v)F_i(v)}{F_{j_1}(v)F_{j_2}(v)}=0 \text{ and } \lim_{v\to 0}\frac{o\left(F_j(v^*)F_{j'}(v^*)\right)}{F_{j_1}(v)F_{j_2}(v)}=0, \forall j\in\mathcal{A},\, j'\in\mathcal{A}\setminus\{j\},$$

and $i\in\mathcal{I}\setminus\mathcal{A}$.

Therefore,

$$\lim_{v^*\to 0}\frac{\Pi^*(v^*)}{F_{j_1}(v^*)F_{j_2}(v^*)}\geq\int_0^r\prod_{i\in\mathcal{I}\setminus\{j_1,j_2\}}[1-F_i(x)]dx>0.$$

This completes the proof. ■

[44] Note that $|\mathcal{A}|>1$ is needed to make sure $\mathcal{A}\setminus\{j\}$ is not an empty set.

Proof of Proposition 7. First, it is useful to note that a seller can either participate in the procurement auction, or the return and refund auction, but not both. Since there are no continuation games for the sellers in the two auctions, it is easy to verify that the bidding strategies described in Proposition 7 are optimal for the sellers.

For the speculator, the return and refund auction happens after the procurement auction. Therefore, his bidding in the procurement auction may affect how many items he has for sale in the return and refund auction. Suppose that the speculator acquired $K \geq 1$ items from K sellers. The speculator's opportunity cost for selling one item in the procurement auction is that he has one less item to bring to the return and refund auction. Because bringing all K items to the return and refund auction yields a zero auction revenue, the speculator's opportunity cost for selling one item is 0. Therefore, strategic supply withholding—i.e., bidding 0 for one item and withholding the rest—is optimal for the speculator.

It only remains to verify that the acceptance/rejection schedule is optimal for an arbitrary seller $j \in \mathcal{A}$, given that all the other sellers follow their strategies described in Proposition 7. If seller $j \in \mathcal{A}$, with a realized valuation $v_j \in [0, 1]$, accepts the acquisition offer, he gets a payment of $\bar{c}_j^*(\boldsymbol{v}^*)$ and a chance to participate in the return and refund auction. In the return and refund auction, seller j wins if his valuation is not the lowest among the sellers who accepted the acquisition offer. The winning probability is

$$\overline{\mathcal{WP}}_j(v_j) = 1 - \prod_{s \in \mathcal{A} \setminus \{j\}} \left[1 - F_s(\min\{v_j, v_s^*\})\right].$$

Therefore, seller j's expected payoff in the return and refund auction is $\int_0^{v_j} \overline{\mathcal{WP}}_j(x)dx$. Further, seller j's expected payoff from accepting the acquisition offer is

$$\pi_j^A(v_j) = \bar{c}_j^*(\boldsymbol{v}^*) + \int_0^{v_j} \overline{\mathcal{WP}}_j(x)dx.$$

If seller j rejects the acquisition offer, he participates in the procurement auction. Seller j wins in the procurement auction if his valuation is lower than the reserve price and is the lowest among all the sellers, and no seller sold to the speculator. Formally, seller j's winning probability is

$$\mathcal{WP}_j(v_j) := \mathbb{1}_{\{v_j \leq r\}} \prod_{i \in \mathcal{I} \setminus \mathcal{A}} [1 - F_i(v_j)] \prod_{s \in \mathcal{A} \setminus \{j\}} \left[1 - F_s(\max\{v_j, v_s^*\})\right].$$

Therefore, seller j's expected payoff from the procurement auction is

$$\pi_j^R(v_j) = v_j + \int_{v_j}^{r} \mathcal{WP}_j(x)dx.$$

Note that $\pi_j^A(v_j^*) = \pi_j^R(v_j^*)$. Then it suffices to show that $\pi_j^A(v_j) - \pi_j^R(v_j)$ decreases with v_j. In fact,

$$\frac{\partial[\pi_j^A(v_j)-\pi_j^R(v_j)]}{\partial v_j} = -\Big\{ \prod_{s\in\mathcal{A}\setminus\{j\}} \big[1-F_s(\min\{v_j, v_s^*\})\big] - \mathbb{1}_{\{v_j\le r\}} \prod_{i\in\mathcal{I}\setminus\mathcal{A}} [1-F_i(v_j)] \prod_{s\in\mathcal{A}\setminus\{j\}} \big[1-F_s(\max\{v_j, v_s^*\})\big] \Big\} \le 0,$$

where the inequality follows from $\mathbb{1}_{\{v_j\le r\}} \prod_{i\in\mathcal{I}\setminus\mathcal{A}}[1-F_i(v_j)] \le 1$ and $1-F_s(\min\{v_j, v_s^*\}) \ge 1-F_s(\max\{v_j, v_s^*\})$. This completes the proof. ■

Proof of Proposition 8. Suppose that in an equilibrium, all the sellers with valuations lower than r are knocked out of the procurement auction. Because $F(r) < 1$, on the equilibrium path, rejection of the acquisition offer can still happen. Therefore, the speculator cannot tell whether a rejection is because of a deviation or because the seller has a valuation above r. So the speculator must bid r in the procurement auction, regardless of how many rejections occurred.

For a seller with value $v < r$, the equilibrium payoff in the complete knockout scenario is $v + \int_v^r [1-F(x)]^{N-1} dx$. However, if the seller deviates and bids $r-\epsilon$ in the procurement auction, where ϵ is an arbitrarily small number, he gets a higher payoff of $r-\epsilon$. This means complete knockout cannot happen in equilibrium. ■

Appendix B. Enhanced speculation with reserve prices in the return and refund auction

In this Appendix, I first extend the equilibrium characterization result in Proposition 7 to incorporate reserve prices in the return and refund auction, and then focus on the symmetric sellers case and show the optimality of not having a reserve price.

Suppose that the speculator holds a VCG auction with individualized reserve prices to return and refund any leftover items after the procurement auction has concluded. The reserve price for seller i, given that seller i had accepted the acquisition offer, is denoted by $\tilde{r}_i$.

Again, I consider a set of prices that will induce a vector of acceptance/rejection cutoffs in equilibrium. For any vector $\boldsymbol{v}^* \equiv (v_j^*)_{j\in\mathcal{A}}$ with $v_j^* \in [0, r]$ and $\tilde{\boldsymbol{r}} \equiv (\tilde{r}_j)_{j\in\mathcal{A}}$ with $\tilde{r}_j \le v_j^*$, define

$$\tilde{c}_j^*(\boldsymbol{v}^*) := \tilde{r}_j + \int_{\tilde{r}_j}^{v_j^*} \prod_{s\in\mathcal{A}\setminus\{j\}} \big[1-F_s(\min\{x, v_s^*\})\big] dx + \int_{v_j^*}^{r} \prod_{i\in\mathcal{I}\setminus\mathcal{A}} [1-F_i(x)] \prod_{s\in\mathcal{A}\setminus\{j\}} \big[1-F_s(\max\{x, v_s^*\})\big] dx. \tag{B.1}$$

Proposition 7′ generalizes Proposition 7 to incorporate reserve prices in the return and refund auction.

Proposition 7′. *In the three-stage SPA-enhanced-speculation game with reserve prices $\tilde{\boldsymbol{r}} \equiv (\tilde{r}_j)_{j\in\mathcal{A}}$ in the return and refund auction, suppose that the speculator offers $p_j = \tilde{c}_j^*(\boldsymbol{v}^*)$ to seller $j \in \mathcal{A}$, with $v_j^* \in [0, r]$. Then a PBE of the speculation game is described as follows. Seller $j \in \mathcal{A}$ accepts the speculator's offer if and only if $v_j < v_j^*$. Sellers bid truthfully in the procurement auction or in the return and refund auction. The speculator engages in strategic supply withholding in the procurement auction.*

Proof. The proof follows that of Proposition 7 closely and is therefore omitted. ∎

For the symmetric sellers case, Proposition B.1 shows the optimality of having a zero reserve price in the return and refund auction.

Proposition B.1. *Suppose that $\mathcal{A} = \mathcal{I}$, sellers are symmetric, and the speculator induces the same cutoff v^* and uses the same reserve price $\tilde{r}$ for all sellers. Consider the equilibrium described in Proposition 7′. Inducing $v^* = r$ while setting $\tilde{r} = 0$ maximizes the speculator's expected profit across all possible acceptance/rejection cutoffs v^* and reserve prices $\tilde{r}$.*

Proof. First, with $v^* = r$ and $\tilde{r} = 0$, the equilibrium outcome achieves allocative efficiency. Because the auctioneer's payoff is 0, the total expected payoff of the speculator and the sellers is maximized.

Second, it can be shown that each seller's expected payoff is minimized in the complete knockout scenario. Specifically, in this case, the probability that seller j eventually gives up his item is $[1 - F(v_j)]^{N-1}\mathbb{1}_{\{v_j \le r\}}$. This can be viewed as the seller's overall "winning probability" in a direct mechanism, if one applies the revelation principle to the enhanced-speculation game. As a result, seller j's expected gain from the enhanced-speculation game (net of his private value) is the integral of this winning probability from v_j to r. However, in other cases with non-zero reserve prices, the probability of seller j eventually giving up his item is higher than $[1 - F(v_j)]^{N-1}\mathbb{1}_{\{v_j \le r\}}$. Formally, if $v^* \ge \tilde{r} > 0$, seller j's probability of giving up his item is

$$\begin{cases} 1, & \text{if } v_j < \tilde{r}, \\ [1 - F(v_j) + F(\tilde{r})]^{N-1}, & \text{if } \tilde{r} \le v_j < v^*, \\ [1 - F(v_j)]^{N-1}, & \text{if } v^* \le v_j \le r, \\ 0, & \text{if } r < v_j. \end{cases}$$

Therefore, seller j's expected payoff is higher than that in the complete knockout scenario.

The two facts together imply that setting $v^* = r$ and $\tilde{r} = 0$ is optimal for the speculator. ∎

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