linNet

The Software for symbolic Analysis of linear Electronic Circuits
Version 1.0
User Guide

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November 2014

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Abbreviations

Abbreviation	Explanation
AC	Alternating current
CPU	Central processing unit
DC	Direct current
DLL	Dynamic link library
GCC	GNU compiler collection
GCD	Greatest common divisor
GNU	GNU is not UNIX
LCM	Least or lowest common multiple
LES	Linear equation system
LHS	Left-hand side (of an equation)
LTI	Linear time-invariant (system)
MIMO	Multiple input, multiple output (system)
NPN	N-type, p-type, n-type donated (transistor)
op-amp	Operational amplifier
R	Resistor
RHS	Right-hand side (of an equation)
SIMO	Single input, multiple output (system)
SISO	Single input, single output (system)
SW	Software
TBC	To be checked
TBD	To be defined
TODO	To be done

Table 1: Abbreviations

Chapter 1

Introduction

linNet is an application to compute the transfer function of linear, electronic circuits. The computation is done symbolically, not numerically, and the result is a formula rather than a number or a series of such. The found formula is the Laplace transform of the dependencies of the voltages and currents in the circuit on the input voltages and currents.

A linear electronic circuit is a combination of the supported basic devices as listed in table 1.1. The circuit is input to linNet. The representation of the circuit is a list of devices with connectivity information. The interconnections are expressed by references to nodes, where a node is a point of the circuit, which normally at least two devices are connected to. This leads to a simple formal syntax, the circuit network list. This list can be created and maintained with a text editor; there's no graphical interface for editing a circuit.

The computed formulas are printed to the console and to the application log file and can be used for further investigation or for publications or didactic purpose.

To make the application somewhat more attractive it exports the computed formulas as Octave or MATLAB script code, too. Numeric evaluation becomes a simple one-line command in Octave. The formulas are exported as LTI transfer function objects so that the complete set of analysis functions from the Octave control toolbox can be applied just like that. This reaches from simple transfer function plotting to stability analyses and system response computation on arbitrary system input.

Please refer to figure 1.1 as an example of how linNet works. This is a simple RLC element with a transfer function of second order. It can be represented by the following circuit netlist:

```
U Uin in  gnd
L L   in  K1
C C   K1  out
R R   out gnd
PLOT G U_out U_in
```

Given this was put into file rlc.cnl, then we can run linNet:

```
linNet -o rlc.cnl
```

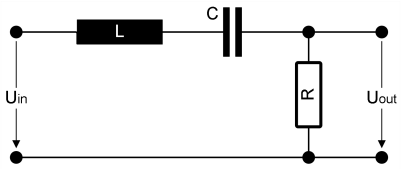


Figure 1.1: Simple example of a linear electronic circuit

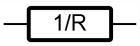
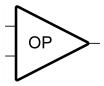
Device	ID	Symbol
Resistor	R	
Conductance	Y	
Capacitor	C	
Inductivity	L	
Ideal operational amplifier (op-amp)	OP	
Constant voltage source	U	
Voltage controlled voltage source	U(U)	
Current controlled voltage source	U(I)	
Constant current source	I	
Voltage controlled current source	I(U)	
Current controlled current source	I(I)	
Current probe (wire)	PI	

Table 1.1: Supported linear devices

and would yield the output:

```
User-defined result G (Bode plot):
The dependency of U_out on U_in:
  U_out(s) = N_U_out_U_in(s)/D_U_out_U_in(s) * U_in(s), with
    N_U_out_U_in(s) = R*C * s
    D_U_out_U_in(s) = L*C * s^2
                      +R*C * s
                      +1
```

Going to Octave and typing `G` to plot the transfer function (still using default device values) gives us figure 1.2. More plots or plots with altered device values are a matter of single commands in Octave.

linNet means "linear network". It founds on a symbolic solver for linear equation systems. There's no way to model any non linear effects like noise, voltage or current limits, non-linear distortions or switching operations. All of these effects play an important role in real electronic circuits and a good deal even uses these effects as their principle of operation – you won't find linNet helpful for an investigation of these kind of effects or circuits. There are many numeric circuit simulation tools, which are capable to do this, in the first place the popular open source tool SPICE with all its derivates. linNet is conceptually not a competitor of these tools, although it can behave a tiny bit alike when using Octave as numeric post-processor. linNet is not the worse SPICE, linNet is different.

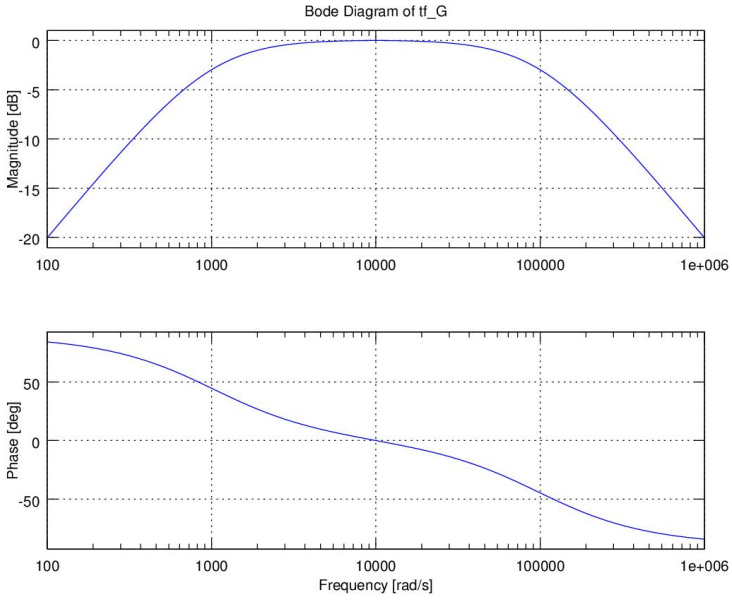


Figure 1.2: Octave plot of transfer function of the system in figure 1.1

Chapter 2

Installation

A valid installation looks like this: The linNet application is a simple executable file plus a folder with resources for the Octave code generator; this folder is called private and is located next to the linNet executable file, i.e. both are siblings in the same parent directory. The environment variable `LINNET_HOME` points to this parent directory.

Accordingly, this is what you have to do to install linNet:

Create an empty folder, the installation directory, e.g. /ProgramFiles/linNet. Copy the linNet executable file into this directory. Copy the folder private into this folder. Create a persistent environment variable `LINNET_HOME` and assign the path to the installation directory, `/ProgramFiles/linNet` in our example.

The usual way to use linNet is to run it from a shell window. You will probably organize your input circuits in a folder or a set of such and `cd` in your shell to this folder. To run linNet in this scenario in a convenient way it is useful to place a reference to the installation directory in the system environment variable `PATH`. In our example you'd append `:/ProgramFiles/linNet` (UNIX) and `;\ProgramFiles\linNet` (Windows) to the current value of this variable.

The definition of a shell command alias pointing to the linNet executable file can be a reasonable alternative to extending the environment variable `PATH`.

If the environment variable is not set then linNet makes an attempt to find its resources via the path to itself. However, it depends on the system, the shell in use and the way the command is typed, whether this information is completely passed to the application or not. If it can't find out where to find itself it'll refuse to generate Octave code.[1] Not setting the environment variable can be a safe option if you decide to run linNet solely from a shell window with an alias, which expands to the full, absolute path of the executable.

2.1 Linux and Mac OS

Unfortunately, Linux and Macintosh users will have to compile linNet from the sources prior to use. At the moment no pre-built executables are available for these systems. Please refer to section 6 on page 39.

[1] There are system specific functions which let an application find out, where its executable file is located, e.g. `GetModuleFileName` for Windows, but the use of such functions would make linNet system dependent. This price is much too high for an a bit simpler installation.

2.2 Windows

2.2.1 Pre-built executables

The download of linNet contains pre-build executable files for Windows. A build for 32 and 64 Bit is contained, each as DEBUG or PRODUCTION compilation. The executable files are already bundled with the resources folder private so that the installation reduces to copying the build folder of choice to any location, preferably c:\ProgramFiles, which doesn't contain blanks, and setting the environment variable `LINNET_HOME`; it would be set to c:\ProgramFiles\linNet if the suggested installation folder was used.

2.2.2 Required DLLs

The build of linNet has the executable file linNet.exe as only product. This executable file is however not self-contained; the application is linked against the compiler's C libraries, which in case of the GNU compiler collection build on some Windows DLLs. These DLLs need to be available in order to execute linNet. Normally, this should always be the case. In the rare case that such a library should be missing on your system then the only way out will be to install the GCC port of MinGW and to rebuild linNet. If you can run the simplest `helloWorld` application compiled with GCC then you should also be able to run linNet.

For a Windows 7 system and using the 32 Bit build the following list of DLLs appeared to be loaded by linNet. For different Windows releases or for the 64 Bit build the list may vary:

```
c:\Windows\SYSTEM32\ntdll.dll
c:\Windows\SYSTEM32\wow64.dll
c:\Windows\SYSTEM32\wow64cpu.dll
c:\Windows\SYSTEM32\wow64win.dll
c:\Windows\SysWOW64\ntdll.dll
c:\Windows\syswow64\KERNELBASE.dll
c:\Windows\syswow64\kernel32.dll
c:\Windows\syswow64\msvcrt.dll
```

2.2.3 Start from Windows Explorer (file name association)

If you want to start the computation of transfer functions directly from the Windows Explorer then you can add a file class definition to the Windows Registry. It'll extend the context menu entries of the Explorer for circuit netlist files. With right-click on the file you can load the netlist file into a text editor and by double-click you can run the computation with linNet. Please note, if starting linNet this way Windows will swallow all the program output and you will have to open the log file after computation to find out if the computation was successful.

Here's a template for a Windows registry script to make an appropriate file class definition:

```
REGEDIT4

[HKEY_CLASSES_ROOT\.cnl]
@="FileClass_cnl"

[HKEY_CLASSES_ROOT\.ckt]
@="FileClass_cnl"

[HKEY_CLASSES_ROOT\FileClass_cnl]
@="linNet Circuit Netlist"

[HKEY_CLASSES_ROOT\FileClass_cnl\shell]
@="Compute"
```

```
[HKEY_CLASSES_ROOT\FileClass_cnl\DefaultIcon]
@="c:\\ProgramFiles\\linNet\\linNet.exe,1"

[HKEY_CLASSES_ROOT\FileClass_cnl\shell\Edit]
@="&Edit"
[HKEY_CLASSES_ROOT\FileClass_cnl\shell\Edit\command]
@="c:\\ProgramFiles\\emacs-24.0.92\\bin\\emacsclient.exe -n \"%1\""

[HKEY_CLASSES_ROOT\FileClass_cnl\shell\Compute]
@="Run linNet"
[HKEY_CLASSES_ROOT\FileClass_cnl\shell\Compute\command]
@="c:\\ProgramFiles\\linNet\\linNet.exe -l -c -s -o \"%1\" %*"
```

Copy this code into a text file. Save it using a file name with extension .reg, e.g. fileClassCnl.reg.

Please note, that you will have to modify the proposed code as it uses absolute paths to the installed applications; replace these paths with your system's paths. Furthermore, the text editor for editing the circuit netlists is just a suggestion; you will probably replace the reference to Emacs with a reference to your favorite text editor.

After customization, double-click this file or run it from a shell window. If your system has no file name association for *.reg files then you need to import it explicitly using a command like `regedit.exe fileClassCnl.reg`. In either case you will need local administrator rights to complete.

Chapter 3

The Command Line Interface

linNet is a simple console application. It is fully controlled by the command line arguments and all its output is text based, either printed to the console or written into the output files.

All interaction with linNet is done from a shell window. From such a window type `linNet --help` to get a short reminder on the available command line options.

The command line may contain program options and the names of the input files, which are always circuit netlist files. The order of options and file names doesn't matter but usually all the file names are placed behind all the options. If a file name could be mixed up with an option then the double hyphen (`--`) can be placed on the command line: All remaining command line arguments are considered file names regardless of how they look like.

Circuit netlist files are named `*.cnl` by convention but they may follow any other name pattern. Only the file name extension `*.ckt` is reserved and must not be used.

All the information that impacts the conducted computations is placed in the circuit netlist files; the program options only affect the generated output. The verbositiy of the console output can be controlled, a log file can be created and the output for Octave is an option only. The command line arguments in detail:

- *-h, –help* linNet prints a short help text and terminates again. The return value to the shell is 0. Can be combined with `-r`

- *-r, –version* linNet prints the software revision and terminates again. The return value to the shell is 0. Can be combined with `-h`

- *-v LEVEL, –verbosity=LEVEL* The verbosity LEVEL of the application; LEVEL is an enumeration value. It doesn't matter whether it is written in upper or lower case characters. The blank between -v and LEVEL is optional. LEVEL applies in the same way to the console output and the application log file. In general, the same information is written to both of these streams. LEVEL is one out of:

  - `DEBUG`: A lot of internal information about the progress of the computation is printed to the console. Among more, the linear equation system is printed prior to and after running the solver. For complex circuits this can mean many thousand lines of output or much time to wait.

    A strong recommendation is to use this level of verbosity solely in conjunction with the other option `-s`, which suppresses all console output. (Writing to the log file only is times faster then writing to the console)

  - `INFO`: Some progress information is printed. Among more, the algebraic solution of the linear equation system is printed, which can still mean bulky output. Use this level of verbosity with care

  - `RESULT`: This is the default level of verbosity. All final results are printed, i.e. the requested variables in the frequency domain.

    Additionally, if a concise log format is chosen (see `-f`), which doesn't have time stamps, then `RESULT` will also print some timing information.
  - `WARN`: The warnings are printed. Computation results can be found only exported as Octave code (see `-o`)
  - `ERROR` The error messages are printed. Computation results can be found only exported as Octave code (see `-o`)
  - `FATAL`: Only the fatal error messages are printed. Fatal errors (as opposed to the "normal" errors) are those, which lead to immediate (but still controlled) program termination; the out-of-memory error is a prominent example. Computation results – if any – can be found only exported as Octave code (see `-o`)

  Default is `RESULT`. Any verbosity level includes all output of the less verbose levels.

- *-f FORMAT, –format-of-log-entry=FORMAT* Log entry format. FORMAT is an enumeration value. It doesn't matter whether it is written in upper or lower case characters. The blank between -f and FORMAT is optional. FORMAT is one out of:

  - `raw`: Only the pure message text is printed to the console and the log file
  - `short`: Each message is preceeded by a short line header, which contains the consumed CPU time and the severeness of the message
  - `long`: Each message is preceeded by a line header, which contains a time stamp, the consumed CPU time and the severeness of the message.

  The default format is `long`

- *-s, –silent* Silent operation, only a greeting is emitted. Do not write progress information to the console, only write into the log file. To avoid useless runs of the application it is not allowed to choose silent operation if no log file is in use (see `-l`)

- *-l[FILENAME], –log-file-name[=FILENAME]* This option enables the use of a log file and controls its name. No log file is opened if this option is not used.

  You may state the name of the log file but normally you won't: If the option is used without argument `FILENAME` then an appropriate file name is chosen by the application. It is derived from the name of the circuit netlist if a single input file is given and otherwise a generic name is chosen.

  There's a constraint: Either using a log file is specified with -l or `-s` (or `--silent`) is not given

- *-c, –clear-log-file* Clear the log file at the beginning of operation. Default is to append to a possibly already existing log file

- *-o[DIRNAME], –Octave-output-directory[=DIRNAME]* This option enables the generation of Octave script code and controls the location. No Octave code is generated if this option is not used.

  Octave code generated from a circuit netlist will be placed into a folder whose name is derived from the netlist file. As many output folders will be created as input files are given. All of these folders will be placed into a common parent directory and this directory is designated by `DIRNAME`. If `DIRNAME` is omitted then the current working directory is used as common parent directory

- *-i, –do-not-copy-common-Octave-code* The generated Octave code builds on some common scripts, which are normally copied into each of the netlist related output folders. This way the generated code will run just like that in Octave, without the need of manipulating Octave's search path. If you dislike the over and over copied, always identical files you can consider to install them once in your Octave environment and to let them no longer be produced by linNet.

If you give this option then copying the common files into each output folder is simply not done.

This switch is relevant only if `-o` is also given

If the command line parser detects a problem then it tends to print the help text for convenience. This is done into stdout. The concrete error report is however printed into stderr. It depends on your shell in which fashion these two text streams are separated (or interchanged). Carefully look for the actual error message; if you are lucky it's presented in a different color.

If linNet is integrated in a scripting environment then its return value may matter. Normally, linNet returns 0 to the calling shell. If the command line parser recognizes an error or if any input file can't be opened or if it causes a computational error then linNet returns -1. The error cause cannot be concluded from the return value, the log file needs to be inspected to find out.

Chapter 4

The Syntax of the Input File

Most of the functionality of linNet is controlled from the input file. The main part of the input file is the circuit netlist: Each line of code defines a single electronic device with the network nodes it is connected to. The number of nodes varies from two to four and depends on the kind of device. Nodes are defined implicitly the first time they are referenced by a device definition.

The remaining parts of the file are used to specify the wanted results. This part of the file may stay empty, in which case all unknown quantities are computed.

4.1 Basic syntax elements

A netlist file may of course be well-documented. You can use C/C++ style comments anywhere. Nested comments are not supported.

A semicolon is treated like an end of line character. Line concatenation can be achieved with a backslash as very last character of a line, as in the language C.

All elements of the formal syntax of the input file are treated case sensitive.

The names of devices, nodes, user-defined voltages and results are defined as identifiers in most computer languages. They begin with a letter or underscore and can continue with any number of the same or decimal digits. This particularly disables integer numbers to become node designations as it is common in SPICE netlists.

The names of knowns and unknowns are internally derived from the device and node names and are in turn "identifiers".

Suffix	Meaning	Value
`y`	yocto	10^{-24}
`z`	zepto	10^{-21}
`a`	atto	10^{-18}
`f`	femto	10^{-15}
`p`	pico	10^{-12}
`n`	nano	10^{-9}
`u`	micro	10^{-6}
`m`	milli	10^{-3}
`c`	centi	10^{-2}
`d`	deci	10^{-1}
`D`	deka	10^{1}
`h`	hecto	10^{2}

Table 4.1: Suffixes for exponents in floating point literals (continued on next page)

Suffix	Meaning	Value
k	kilo	10^3
M	mega	10^6
G	giga	10^9
T	tera	10^{12}
P	peta	10^{15}
X[1]	exa	10^{18}
Z	zetta	10^{21}
Y	yotta	10^{24}

Table 4.1: Suffixes for exponents in floating point literals

Numbers are either decimal integer literals as known from the language C or floating point numbers, depending on the context. Floating point numbers can be written as known from the language C. For convenience, the exponent may be abbreviated by a suffix character; just type this single character instead of the conventional exponent expression, write for example `5.6k` instead of `5.6e3`. Careful, the suffixes are case sensitive, too, `5.6K` would yield a syntax error. The supported suffixes are listed in table 4.1

More details on the syntax definition can be found in file parser-cnl-bnf.txt. The syntax is presented in commented Backus-Naur form. The file is printed in appendix A on page 61 of this document and it is part of the linNet sources.

4.2 The actual netlist

The main part of the input file is a list of device definitions. Each definition is placed on a new line or a semicolon is used to seperate two definitions on the same line. A device definition starts with the device type that indicates the kind of device. Table 1.1 on page 10 gives an overview on the available devices. The type identifier from table column ID is used as first token of a device definition in the netlist file.

The name of the device follows after some white space. All names need to be unique in order to avoid ambiguities in the result presentation. Then the names of the nodes the device is connected to follow up as a blank separated list. The kind of device determines the required number of nodes, see below.[2] There's no explicit definition of nodes; a node is defined implicitly when referencing it in a device definition the very first time. This doesn't hold for the voltage sensing inputs of a voltage controlled source (the third and forth node in the device definition). They are pure references to a pair of nodes, which need to be defined elsewhere. Normally the parser doesn't support forward references but here we have one of the few exceptions; the actual definition of the the source controlling nodes may be made with a later device definition:

```
/* This system has the input impedance Ri and an output voltage, which is the k-fold
   of the input voltage. */
U(U) k out gnd in gnd // Voltage controlled voltage source k is defined. Nodes out and
                     // gnd are implicitly defined. Nodes in and gnd are referenced;
                     // node in is not yet defined, we have a forward reference.
R Ri in gnd          // Resistor Ri is defined. Node in is implicitly defined and the
                     // above forward reference can be resolved.
U Ui in gnd          // The input voltage is modeled by a constant voltage source
```

[1] According to http://searchstorage.techtarget.com/definition/Kilo-mega-giga-tera-peta-and-all-that, as of Feb 9, 2014, exa should be abbreviated with `E`. This is ambiguous with the traditional form of an exponent. `1E+2` could be either $10^{18}+2$ or 100. We use `X` instead.

[2] The syntax format defines the nodes by position in the line only. If a node name is missing in the line the next token will be taken as this node and a missleading error message can result. Which might be particularly confusing if it is not the last node, which has been omitted.

An optional blank separated device relation may follow the node references. The device relation supports numeric post-processing and simplifications of the final result. The simplest form of a device relation is the assignment of a numeric value. Such an assignment has no impact on linNet's computation, it is solely passed on to Octave for numeric evaluation via the generated script code. An example would be

```
R R1 k0 k1 R1=5.6k
```

which defines the resistor `R1`, connects it to the two nodes `k0` and `k1` and assigns it a value of 5600 Ω. The generated Octave script code will initialize the variable `R1` with the numeric value 5600.0. All variables which no user-defined values are found for in the netlist file will get fixed standard values as initial values in the generated Octave script code. The standard values are defined such that the characteristic time constants of the circuit are probably in the audio frequency range. Changing these standard values to more appropriate values can then be done on the Octave command line. The hardcoded standard values of all affected devices are listed in table 7.1 on page 44.

More important than value assignments are relations between devices of same kind. The most simple case would be the identity of the values of two devices, e.g. `R1=R2`. Many circuits depend on such a relation; a voltage divider with two identical resistors is for example often used to pin the input voltage of an op-amp to half the operational voltage. If a pair of devices has a fixed value relation as a principle of operation of the circuit then this should be expressed in the netlist. The knowledge about value relations enables linNet to perform according simplifications of the final result, it would e.g. simplify a term like $R_1 R_2 C$ to ${R_2}^2 C$ in the previous example. An impressive example of such simplifications is found as `octagon.cnl` in the set of test cases in the source code distribution of linNet.

Device relations can be made recursively. The netlist[3]

```
U Uin in gnd
R R1 in k1
R R2 k1 k2  R2=R1
R R3 k2 k3  R3=R2
R R4 k3 gnd R4=R3
DEF Uout k3 gnd
RES VDiv Uout
```

defines a voltage divider into four identical voltages. Due to the given relations, the result formula will no longer contain any device symbol, but simply states $\frac{1}{4} U_{in}$ for the output voltage.

Device relations can use rational ratios. We could achieve the same result with the following netlist:

```
U Uin in gnd
R R1 in k1
R R2 k1 gnd R2=1/3*R1
DEF Uout k1 gnd
RES VDiv Uout
```

linNet claims to be a symbolic not a numeric program. Such a software is expected to produce exact, error free results only. Permitting rational ratios between two devices introduces some numeric calculations but these can still be carried out error free – with the exception of overflows. Making intensive use of such relations will easily produce such an overflow. linNet will recognize the overflow but not perform any approximative computations. You either get an accurate result or no result (but the error message). The danger of an overflow rises if you use ratios of large numbers that are prime to each other or if you use recursive device ratios as in the next example, which again has the same output voltage:

[3]The tokens `DEF` and `RES` are explained below in section 4.3.

```
U    Uin  in  gnd
R    Rb   in  b
PI   I_b  b   gnd
I(I) B    c   gnd I_b
R    Rc   c   gnd
DEF  Uout gnd c
PLOT G Uout U_in
```

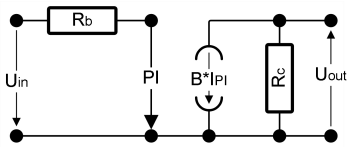


Figure 4.1: A simple bipolar transistor simulation

```
U Uin in gnd
R R1 in k1
R R2 k1 k2  R2=(7/8)*R1
R R3 k2 gnd R3=(5/7)*R2
DEF Uout k2 gnd
RES VDiv Uout
```

To reduce the danger of overflows due to careless use of the rational ratios the values of numerator and denominator of the ratios are limited to the range 1..999. Null and negative ratios are generally not permitted. Negative ratios can be useful with controlled sources; if so the appropriate sign has to be chosen by the polarity of either the source or the control voltage or current.

Device relations must not use forward references.

The definition of the device is closed with either a newline character or semicolon. Newline characters must not be used to spread a device definition over several lines but several device definitions can share the same line if the semicolon is used.

4.2.1 Which nodes to connect to

Passive device `R`, `Y`, `L`, `C`

The passive devices resistor `R`, conductance `Y`, inductivity `L` and capacitor `C` have two connectors and they are independent of the chosen polarity. Just connect them by referencing two nodes.

An example netlist can be found in section 1; please refer to figure 1.1, too.

```
U  Uin opIn1 GND
OP OP  opIn1 opIn2 opOut
R  R1  opOut opIn2
R  R2  opIn2 GND
PLOT G U_opOut Uin
```

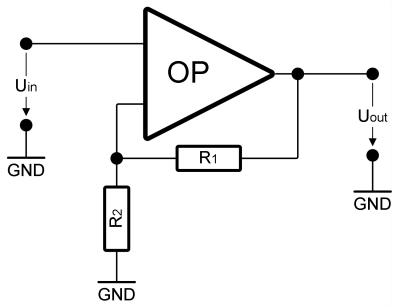


Figure 4.2: Non inverting op-amp

```
U    Uin  in    gnd
R    Rin  in    gnd
/* k has the meaning of R1/R2 in
   the inverting OP circuit */
U(U) k    out   gnd  gnd in
PLOT G    U_out U_in
```

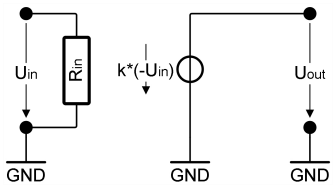


Figure 4.3: Model of the inverting op-amp

Constant voltage source `U`

The constant voltage source `U` has two connectors. The plus and minus pole of the source are the first and second referenced node, respectively. The polarity is defined such that the first and second referenced node designate the start and end of the arrow in the symbol of the source (or voltage).

The given voltage of the source is an independent variable to the computation but not considered a settable physical value; no value assignment or device relation can be made in the definition of a constant voltage source. The constant voltage appears as a variable in the results.

At least one constant source, either voltage or current, needs to be present in a computable netlist.

An example netlist is shown in figure 4.1.

Constant current source `I`

The constant current source `I` has two connectors. The polarity is defined such that the first and second referenced node designate the start and end of the arrow in the symbol of the source: The current is flowing into the network through the second referenced node and flowing back into the source from the first referenced node.

The given current of the source is an independent variable to the computation but not considered a settable physical value; no value assignment or device relation can be made in the definition of a constant current source. The constant current appears as a variable in the results.

At least one constant source, either voltage or current, needs to be present in a computable netlist.

An example netlist is shown in figure 4.4.

Current probe `PI`

The current probe `PI` has two connectors and can be connected in series with any other device to measure the current flowing through the device. The sensed current becomes a new unknown, which can be referenced in user-defined results.

The polarity of the measured current is defined such that the sensed current is flowing into the probe from the first referenced node and flowing back into the network through the second node.

```
I    Iin  GNDin N1
PI   in   N1    GNDin
U(I) k    N2    GNDout in
R    Rout N2    N3
RES  Uout U_N3
```

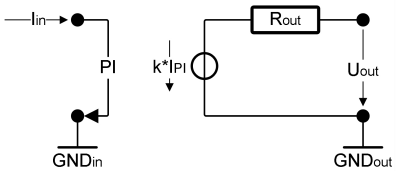


Figure 4.4: Current to voltage converter using a current controlled voltage source; $R_i = 0$, $R_o = R_{out}$

```
I     Iin   GNDin   N1
R     Rin   GNDin   N1
I(U)  k     GNDout  N2 N1 GNDin
R     Rout  N2      GNDout
RES   Uout  U_N2
```

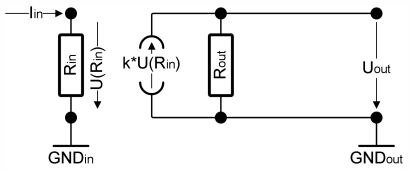


Figure 4.5: Current to voltage converter using a voltage controlled current source; $R_i = R_{in}$, $R_o = R_{out}$

A current probe is assumed an ideal wire. It has no physical value(s); no value assignment or device relation can be made in the definition of a current probe.

An example netlist is shown in figure 4.1.

Operational amplifier `OP`

An operational amplifier `OP` has three connectors; connect them by referencing three nodes.

The first two referenced nodes are the inputs. Different to real circuits no distinction needs to be made between the two inputs; there's no inverted or non inverted input. The computation needs to assume the linear operation – everytking else is out of scope of linNet – and in this sense the connection is always made correct.

The third referenced node is the output of the op-amp.

Please note that the ground node needs to be explicitly defined if an op-amp is used; see section 4.4 for details.

An op-amp is assumed ideal. It has no output voltage or frequency limits and hence no physical value(s); no value assignment or device relation can be made in the definition of an op-amp.

An example netlist is shown in figure 4.2.

Voltage controlled voltage source `U(U)`

The voltage controlled voltage source `U(U)` has four connectors. The actual source has two; these are the first two nodes referenced in the device definition. The first node designates the positive pole of the source, the second one the negative pole. First and second referenced node designate start and end of the voltage defining arrow in the schematic.

The control voltage is defined by the voltage potential difference between node three and four, in this order. The third and forth referenced node designate start and end of the voltage arrow in the schematic, respectively.

An example netlist is shown in figure 4.3. Please note, that the control voltage in this example is defined inverse to the input voltage in order to model the inverting op-amp.

Current controlled voltage source `U(I)`

The current controlled voltage source `U(I)` has two connectors. The first referenced node designates the positive pole of the source, the second one the negative pole. First and second referenced node designate start and end of the voltage defining arrow in the schematic.

The control current is specified by a reference to a current probe. The current probe is another, previously defined device (forward references are not supported), which behaves like an ideal wire. The current through this wire times the device constant of the controlled source is the voltage enforced between the two referenced nodes. The reference to the current probe is made by name; the name of the probe device follows the pair of referenced nodes.

An example netlist is shown in figure 4.4.

Voltage controlled current source `I(U)`

The voltage controlled current source `I(U)` has four connectors. The actual source has two; these are the first two nodes referenced in the device definition. The second node designates the pole of the source, which injects the current into the network. The same current flows back into the source from the first referenced node. This way the first and second referenced node designate start and end of the arrow in the symbol of the source, respectively.

The control voltage is defined by the voltage potential difference between node three and four, in this order. The third and forth referenced node designate start and end of the voltage arrow in the schematic, respectively.

An example netlist is shown in figure 4.5.

Current controlled current source `I(I)`

The current controlled current source `I(I)` has two connectors. The second node designates the pole of the source, which injects the current into the network. The same current flows back into the source from the first referenced node. This way the first and second referenced node designate start and end of the arrow in the symbol of the source, respectively.

The control current is specified by a reference to a current probe. The current probe is another, previously defined device (forward references are not supported), which behaves like an ideal wire. The current through this wire times the device constant of the controlled source is the injected current. The reference to the current probe is made by name; the name of the probe device follows the pair of referenced nodes.

An example netlist is shown in figure 4.1.

4.3 Result specification

The remaining parts of the file are used to specify the wanted results. This part of the file may stay empty; now, there is one implicitly defined result, a so called full result. It is named *allDependends* and contains the full set of dependencies of all unknown voltages and currents on all known voltages and currents. Full results can easily become very bulky (and practically useless) and should be avoided for all non trivial circuits.

Explicitly specified results consist of three elements, referred to by the keywords `DEF`, `RES` `PLOT`. The first token behind these keywords designates the name the specified object gets. This name must not clash with any other name already in use, regardless whether it are names of nodes or devices or if they have been introduced before by these keywords. Furthermore, the internally derived names of unknowns need to be avoided to. The current though a voltage source is e.g. derived from the name of the source by prefixing it with `I_`. Please refer to section 5 for a complete consideration.

4.3.1 User-defined voltages

The keyword `DEF` defines new voltages as difference of two nodes' voltage potentials. Such a so called user-defined voltage is a new explicitely defined unknown of the system. It doesn't matter if it is identical to an already existing unknown voltage, then the same voltage will be computed and reported twice with different names.

To give an example, we could extend our introductory example from section 1, figure 1.1, by adding the line

```
DEF U_c K1 out
```

to the netlist file. The first token behind keyword `DEF` is an identifier, which gives a name to the new voltage. The two referenced nodes follow. These nodes might be defined later; forward references are allowed here.

Our example would add a user-defined voltage U_c to the set of unknowns that models the voltage drop at the capacitor.

There's no keyword for user-defined currents: If a specific current (e.g. the current through the capacitor) should be put into the result then a special, virtual device is applied, the current probe; please see section 4.2.1.

4.3.2 Full results

If the set of user-defined voltages is complete then the keywords `RES` and `PLOT` are applied to specify the result set. The basic idea of both commands is to have dependent and independent quantities and to present how the former depend on the latter.

The first token behind keyword `RES` is an identifier, which gives a name to the new result. This name is used to title the found formulas in the program output, and the generated Octave script that (numerically) implements these formulas has the same name.

`RES` computes the full dependency of a set of unknown voltages or currents on all known voltages and currents. (Known voltage and currents are the device values of the constant voltage and current sources.) Since the independents are always *all* knowns of the system the command only has the set of desired unknowns as parameter, as a blank separated list of identifiers. Such an unknown can be a node's voltage potential, the current through a voltage source or current probe, the output current of an op-amp or a user-defined voltage. A complete list of knowns and unknowns can be found in table 5.1 on page 29.

In the Octave export this result is modeled as an LTI transfer function object of kind MIMO (multiple inputs, multiple outputs). The default operation of the generated script code is to plot a rectangular array of step responses, where each sub-plot presents the step reponse of one output if one but only one input undergoes a step and all others are constantly null.

Extending our example so far, we could add the line

```
RES U_c_r U_c U_out
```

to our file rlc.cnl. The specification of result `U_c_r` demands to compute the voltage drop at the capacitor and at the resistor. The result specification builds on the previously made user-defined voltage definition and exploits the fact that the voltage drop at the resistor is identical to the voltage potential of network node `out`; the other end of the resistor is namely connected to the ground node.

Two formulas can be expected. Both voltages will depend on the input voltage, which is the only independent quantity of the system. An LTI transfer function object of kind SIMO with one input and two outputs will be created by the generated Octave script rlc/U_c_r.m.

4.3.3 Bode plots and inverse transfer functions

The keyword `PLOT` specifies a result with exactely one independent and one dependent voltage or current. This is normally one of the partial transfer functions of a full result as specified with `RES`. The default operation of the generated Octave script is to open the Bode plot of this transfer function, which is the most common operation having such a transfer function.

The first token behind keyword `PLOT` is an identifier, which gives a name to the new result. This name is used to title the found formula in the program output, and the generated Octave script that (numerically) implements this formula has the same name.

`PLOT` is not just a sub-set of `RES`. Its added value is the larger freedom in chosing the dependent and independent quantity of the plot. The dependent quantity can be an actually given, known quantity, whereas the independent quantity can be an unknown to the internal computation. This permits to compute and plot inverse transfer functions. The most important use case is the computation of the input impedance of a circuit: The (known) input voltage is plotted in dependence on the (unknown) input current.

Even two unknowns can serve as a pair of independent and dependent quantity. Now, the transfer function shows how these two unknowns are related to each other if the input voltage is modulated. This mode is restricted to single input circuits, otherwise the relation wouldn't be unambiguous.

Refering to two known quantities is forbidden.

If we have the unknown u, which depends on the known k and if k is the only known of the system then the result specification `PLOT R u k` leads to the same computation as `RES R u` would. The only difference is the selected default operation in the Octave export: Using `PLOT` we get a Bode transfer function plot, using `RES` we get the step response. However, the generated LTI object is exactely the same; in either case you can get the other plot, too. It just needs a few more key strokes inside Octave.

In our introductory example we had used the line

```
PLOT G U_out U_in
```

to specify the result `G`, which means the SISO system's transfer function, the dependency of its single output on its single input. The result specification refers to the voltage potentials of the in- and output node, which is suitable as the other, negative end of the in- and output voltage definitions is the ground node.

A list of the knowns and unknowns, which can be referenced from a `PLOT` result specification can be found in table 5.1 on page 29.

4.3.4 Plot control

The generated Octave script code defaults to some initial plots. (Using the created LTI transfer function objects any other arbitrary plots and figures can be created in Octave.) These initial plots can be controlled in a rather limited way from the netlist file with some additional syntax elements.

A line starting with either `RES` or `PLOT`, which specifies a result, may be continued – after some separating white space – with a "plot-info" block. The plot-info begins with the keyword `LIN` for a plot with linear frequency axis or `LOG` for a logaritmic axis. An integer number follows up. It indicates the number of points computed for and displayed in the plot. For linear frequency axes this is the total number of points whereas for logarithmic axes it is the number of points per decade. The plot-info (and the result specification) is closed with a pair of positive floating point numbers, which designates the frequency range of the plot in Hz.[4]

If the default plot is a time domain plot (as for full results) then the plot-info still relates to the (actually not presented) Bode transfer function plot of the system. The according range of the time variable of the actual plot is derived from this information. Basically, the upper limit of the time range is the reciprocal of the upper limit of the specified frequency range.

If the optional plot-info block is not used then linNet won't pass any plot related information on to Octave. Most of the Ocatve functions from the control toolbox will safely find suitable ranges for plotting themselves so that this normally is the simplest and best way to do.

4.4 Ground nodes

A valid network of devices may consist of any number of unconnected sub-networks. (It's probably hard to find reasonable uses cases for unconnected sub-nets with maybe the exception of sub-nets coupled by controlled sources.) linNet assumes an unknown voltage potential for each but one of the sub-network nodes; the last voltage potential is linear dependent inside a sub-network. The excluded nodes are called the ground nodes and their voltage potential is null by definition. Normally linNet will select suitable ground nodes for you and the final results won't depend on this selection.

If you want to define the ground node of a sub-network yourself you can do this by naming. If there's one node whose name contains either "gnd" or "ground", either capitalized or in all lower or all upper

[4] Hz is a convenient unit to specify the desired frquency range but it can be confusing as Octave's frequency axis is scaled in rad/s; the displayed numeric range will differ by $2pi$ from your input.

case, then this node will become the ground node. If two or more nodes have such a name within the same sub-network then the computation is refused.

The explicit choice of a ground node is often advantageous to simplify the result specification. The voltage potentials of all nodes but the ground nodes can be referenced by name in the result specification. If you define your circuit such that system in- and output voltage are the voltage difference between an in- and output node and a common other node then you'd preferrably define this common node as ground node - system in- and output voltage are now directly accessible as node voltage potentials. If you leave the choice of the ground node to linNet or if you'd make another node the ground node then you had to define the system in- and output voltages explicitly as potential differences using the keyword `DEF`, please see section 4.3.1 for details.

The definition of the ground node is no longer a matter of convenience only but directly impacts the computation result if you use at least one operational amplifier in a sub-network. Op-amps are modeled as controlled voltage sources connected to the ground node, this is why. If an op-amp is used then linNet can no longer decide autonomously which node to take as ground node - it can't guess your intentions. You have to define the ground node explicitly, the computation is refused otherwise.

It's natural and considered good practice to always define the ground nodes of a circuit explicitly.

4.5 The deprecated netlist format `*.ckt`

The parser of linNet still supports an elder, deprecated input format. It had been used by an initial, much simpler implementation of the core algorithm, which had served as prove of concept. It had limited capabilities and its netlist syntax, which followed the SPICE netlist format could not easily be extended to express the features of the linNet release. Furthermore the similarity with the SPICE syntax turned out to be rather disadvantageous as it promised much more compatibility as could ever be kept due to the different concepts of the programs.

The *.ckt input format has been retained for regression testing only; the linNet release should yield the same results as the prove of concept. It is not intended to support this format in future releases and you're not encouraged to ever use it. This is why it is not explained in this manual. A formal syntax description in Backus-Naur form can however be found as parser-ckt-bnf.txt. The file is part of the linNet sources.

Chapter 5

Modelling of Devices

A linear electronic circuit is modeled by linNet as a system of linear equations. These equations basically describe the current balances at the nodes: since the nodes are conceptually free of the capabilty of storing charge all influent currents are in sum at any time equal to the sum of affluent currents. If the flow direction is expressed by a current's sign than this statement is simplified to the sum of currents is null at any time. This is applied to most of the nodes and gives us a number of equations.

If we have N nodes then we can make this statement mostly only for $N - 1$ of them; the current balance of the last node would become linear dependent, or with other words, the last equation would just be a weighted sum of the others.

The currents flowing in or out of the node in question are determined by the devices connected to this and neighboured nodes. The voltage potentials of the nodes become the unknowns of the equation system and the flowing currents can be expressed as functions of these potentials. Doing this consequently for all devices completes the equations or yields the full set of coefficients of the equation system in its matrix representation.

A netlist can describe a circuit, which are actually two, three or more unconnected sub-circuits. Here, unconnected means that there's no wired connection between the sub-circuits. The sub-circuits may but don't need to be logically independent; there may be logical interdependencies as introduced by controlled sources: the source is in one sub-circuit but the controlling voltage or current is found in another one. Reasonable use-cases are known, which result in such logically dependent but unconnected sub-circuits in a netlist.

If there are unconnected sub-circuits – regardless whether logically coupled or completely irrelated – then the number of equations expressing the current balances of nodes is further reduced; in general there will be one dependent equation per sub-circuit.

A voltage is always defined as the difference of two voltage potentials, in our application the voltage potential difference between two nodes only. The last, dependent node of any sub-circuit is called the ground node and gets the arbitrary voltage potential null at any time by definition. Which means that voltages were even defined between nodes of different, unconnected sub-circuits, although this is not described by the schematic and physically undefined. To avoid misleading results, which were correct only on condition of this definition, linNet refuses the definition of voltages between nodes in unconnected sub-circuits. This holds for user-defined voltages and control voltages of voltage controlled sources.

The terms of the current balance equation of a particular node N are principally dictated by the linear devices connected to this node as they conduct the in- and affluent currents but there are also some additional constraints, mainly introduced by the sources. How the equation system is build up from the list of devices (i.e. the netlist) is documented in the next sections. Table 5.1 gives an overview on the knowns and unknowns of the evolving equation system. The quantities listed in the table can be referenced when specifying a user-defined result in the netlist file.

Device	Known quantity	Unknown quantity	Meaning
Node	-	*U_<nameOfNode>*	Voltage potential of node. There's no such unknown for the ground node
Passive device	-	-	Passive devices, R, Y, C and L, don't introduce knowns or unknowns to the LES. They only appear as symbols in the computed results
Constant voltage source	*<nameOfDev>*	-	Given voltage of source; 1st connected node is plus pole
Constant current source	*<nameOfDev>*	-	Given current through source flowing from 1st to 2nd connected node
Op-amp	-	*I_<nameOfDev>*	Output current of op-amp. The current flows from ground node to output node, then into the network
Voltage controlled voltage source	-	*I_<nameOfDev>*	Current through source flowing from 2nd to 1st connected node
Current controlled voltage source	-	*I_<nameOfDev>*	Current through source flowing from 2nd to 1st connected node
Voltage controlled current source	-	*I_<nameOfDev>*	Current through source flowing from 1st to 2nd connected node
Current controlled current source	-	*I_<nameOfDev>*	Current through source flowing from 1st to 2nd connected node
Current probe	-	*I_<nameOfDev>* or *<nameOfDev>*	Measured current flowing through probe from 1st to 2nd connected node. The prefix *I_* in the name of the unknown current is omitted if the probe's name already begins with *I_* (case sensitive comparison)

Table 5.1: Knowns and unknowns of the LES. These quantities can be referenced in user-defined results

5.1 Passive devices

The passive devices are resistor, conductance (actually the same as a resistor but the device value is defined reciprocal), capacitor and impedance. All of these devices are characterized by the fact that the (complex) current is proportional to the (complex) voltage between the two ends of the device. All of these devices are handled exactely the same, only a different proportionality factor is considered.

However, the differences in the proportionality factors are considered only in the final result representation. linNet has a symbolic equation solver and during the complete elimination process the applied symbol has the meaning of complex conductance – whatever this is for the given device. For all of these devices holds $I_s = sU_s$, if s is the symbol representing the device. The effective voltage U_s in this equation, the voltage between the two ends of device s, is the difference of the voltage potentials of the nodes the device is connected to. The sign of a current is a matter of definition and linNet defines it to be positive if it is influent. We get: $I_s = s(U_{n_j} - U_{n_i})$, where n_i is the node under progress, i.e. the node the current balance is made for and n_j is the other node the device is connected to. If these terms are taken for all devices, which are connected to node n_i than the current balance of this node is widely done for this node; the other side of the equation simply is a null.

As usual, the equation system is represented by a matrix. This means only the coeffcients of the equations are actually represented, the product with the unknowns U_{n_j} and U_{n_i} is implicit. Which means that "take a term" just means to put the symbol s twice in the row of the matrix, which represents the

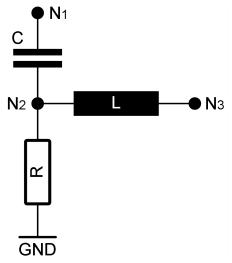


Figure 5.1: Fragment of circuit, handling of passive devices

Kind of device	Substitute of symbol	Meaning
R	$\frac{1}{R}$	Resistor
Y	Y	Conductance
L	$\frac{1}{Ls}$	Impedance
C	Cs	Capacitor

Table 5.2: Substitution of symbols in the final result

current balance of node n_i: in column j (assuming that this column represents unknown U_{n_j}) and as $-s$ in column i (assuming that this column represents unknown U_{n_i}).

The same is repeated for all independent nodes in the circuit. The same symbol s will appear again, when the process visits node n_j. This time the perspective is inverse and symbol s will appear with inverse signs in the row that represents the current balance of node n_j.

If one of the two nodes the device is connected to should be the ground node than there is no current balance for this node and one of the two rows of the matrix containing symbol s will disappear. Same for the columns, as the ground node's voltage potential doesn't belong to the set of unknowns. We end up with a single appearance of symbol s in the matrix.

Figure 5.1 shows a fragment of a circuit. If we assign node $n_i, i = 1 \ldots 3$ to row and column i of our matrix then this fragment would lead to the following matrix:

$$\begin{pmatrix} -C & C & 0 & \ldots \\ C & -C-L-R & L & \ldots \\ 0 & L & -L & \ldots \\ \vdots & \vdots & \vdots & \ddots \end{pmatrix}$$

5.1.1 Symbols, knowns and unknowns

The passive devices don't introduce knowns or additional unknowns to the equation system.

The name of a device is used as symbol in the internal computation and as variable in the final result representation. Although it is the same symbol it has different meanings in either case: In the internal computation it designates the complex conductance, whereas it has the common, physical meaning in the final result. For example, C would actually mean the frequency dependent complex conductivity Cs during the internal computation but designate the constant, real capacitance in Farad of a capacitor in the final result.

In the final result representation, the symbols of the internal computation are substituted by the physical expressions in the complex frequency variable s. This is done according to table 5.2.

The values for the R, Y, L and C in the final result, which are used as default in numeric post-processing with Octave can be found in table 7.1 on page 44.

5.2 Constant voltage source

The constant voltage source is a constraint on the LES. The source is connected to two nodes. These node's voltage potentials, which are both unknowns to the system are associated by the constraint that they differ by a constant, given, known voltage.

The constraint is expressed as a simple additional equation, which says $U_{n_i} - U_{n_j} - U_{ij} = 0$, where U_{ij} is the constant voltage of the source connected to nodes n_i and n_j.

The constant voltage source introduces a further known and thus a further column to the LES. The additional equation means a further row, too. The equation is just a triple of signed ones at the according columns. (If the source is connected to a ground node then only two ones remain.)

The additional equation comes along with an additional unknown. When setting up the current balance of the connected nodes we have to consider the additional circuit branch, the current flowing through the source. This current is unknown. It is defined to be the current flowing out of the source's plus pole; it's influent and thus positive at the node connected to the plus pole and affluent or negative at the minus pole's connetced node. We get a second additional column, which has a pair of ± 1 as only non null coefficients (or a single signed one if one of the nodes would be the ground node).

As an example, the source U_0 would be connected with its plus and minus pole between the nodes N_1 and N_3, respectively, of the circuit fragment in figure 5.1. The new unknown current *I_U0* is put to the right of the unknowns so far and the new known to the right of the knowns so far. The LES would be extended to:

$$\begin{pmatrix} -C & C & 0 & \dots & 1 & \dots & 0 \\ C & -C-L-R & L & \dots & 0 & \dots & \\ 0 & L & -L & \dots & -1 & & \vdots \\ \vdots & \vdots & \vdots & & & & 0 \\ 1 & 0 & -1 & 0 & \dots & 0 & -1 \end{pmatrix}$$

Only the constant sources, either voltage or current, introduce the knowns into the LES. At least one known, hence constant source, is required otherwise linNet refuses to compute the circuit. The system would remain unstimulated and all voltages and currents would anyway be null.

If more than one constant source is used then the system changes to a MIMO system and the result function for an unknown has several terms to describe the dependency of the unknown on each of the knowns (given a full results is requested).

5.2.1 Symbols, knowns and unknowns

The source is represented in the LES only as a number of coefficients ± 1 and its name does not appear as a symbol in the internal computation. It is a known and only apparent in the final result representation. Here, it has the meaning of the Laplace transform of the given source voltage in Volt.

The name of the additional unknown, the current through the source is derived from the device name by adding the prefix *I_* to the left of the user-defined device name. The name of the additional unknown does not appear as a symbol in the internal computation. It can be referenced in a result specification in order to get an according transfer function or full result.

5.3 Constant current source

A constant current source is modeled as a known current, which is influent (i.e. positive) at the node connected to the source's plus pole and affluent (i.e. negative) at its minus pole's connected node. This leads to a new column for the additional known, which has just a pair of ± 1 coefficients at the rows related to the connected nodes (and hence only a single signed one if one of these nodes is the ground node). All other coefficients in this column remain null.

```
/* LP with "normal" ground */
U    Uin1  in1    gnd1

OP   OP1   opInv1 gnd1   opOut1
R    Rin1  in1    opInv1
R    Rfb1  k1     opOut1
C    C1    opInv1 k1

DEF  Uout1 opOut1 gnd1
PLOT G1    Uout1  Uin1
PLOT Z1    Uin1   I_Uin1

/* Same LP with other ground */
U    Uin2  gnd2   null2

OP   OP2   opInv2 null2  opOut2
R    Rin2  gnd2   opInv2
R    Rfb2  k2     opOut2
C    C2    opInv2 k2

DEF  Uout2 opOut2 null2
PLOT G2    Uout2  Uin2
PLOT Z2    Uin2   I_Uin2
```

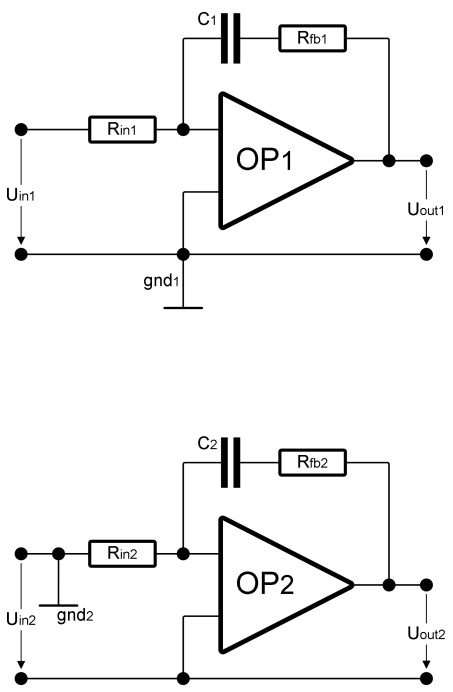


Figure 5.2: Low-pass filter with op-amp and two choices of ground node

As an example, the current source I_0 would be connected with its plus and minus pole between the nodes N_1 and N_3, respectively, of the circuit fragment in figure 5.1. The new known is put to the right of the knowns so far. The LES would be extended to:

$$\begin{pmatrix} -C & C & 0 & \dots & 1 \\ C & -C-L-R & L & \dots & 0 \\ 0 & L & -L & \dots & -1 \\ \vdots & \vdots & \vdots & & \end{pmatrix}$$

At least one constant current or voltage source is required otherwise linNet refuses to compute the circuit; see section 5.2 for more.

5.3.1 Symbols, knowns and unknowns

The source is represented in the LES only as a pair of coefficients ± 1 and its name does not appear as a symbol in the internal computation. It is a known and only apparent in the final result representation. Here, it has the meaning of the Laplace transform of the given source current in Ampere.

The constant current source doesn't add any unknowns to the LES.

5.4 The operational amplifier

Linear operational amplifier circuits are based on the principle, that the voltage between the two inputs is highly amplified but fed back to one of the inputs such that the input voltage is compensated till close to null. The remaining input voltage is in the magnitude of a few micro Volts only and the output voltage in the magnitude of a few volts. The high dynamic capabilities of the implementation of the op-amp ensure that this balance is kept at any time.

The output voltage of the op-amp leads to an output current. This current is controlled by a push-pull output stage, which lets the output current flow either from the positive supply voltage or into the negative supply voltage. (These supply voltages therefore limit the actual output voltage range.)

The real op-amp is modeled by linNet in an idealized way. The amplification is made infinite and the feedback by the outer circuitry thus leads to a voltage of exactely null between the inputs. The dynamic capabilities become infinitely fast; there's no reaction time, the equations hold at any time.

Controlling the output voltage by a push-pull output stage is a highly non linear behavior of the true implementation of an op-amp and can't be modeled as such by linNet. Instead, linNet approaches this behavior by a controlled voltage source, which is connected between ground and the op-amp's output. The voltage is controlled such that the input voltage becomes null.

Modelling an op-amp like this has several important implications. Most striking, this model doesn't require any distinction of the two inputs of the op-amp. The output condition simply is that the input voltage be null; only the real implementation of the circuitry requires careful selection of the right input so that the feedback leads to a voltage compensation rather than to a boost. Contrariwise, any kind of circuit, which founds on a voltage boost by toggling the inputs (like threshold detector, Schmitt trigger, etc.) can't simply be modeled with linNet.

Most obvious, there is no output voltage limitation. The output voltage will raise or drop to any voltage until the condition for the input voltage is met.

More in general, there is no modelling of the voltage supply at all. Using an op-amp in linNet only enables to compute the small-signal behavior of the true circuit; the DC behavior with all supply voltages and currents and the related power distribution can't be figured out. Furthermore, the decision to model the output by a voltage source, whose other end is connected to ground means that the full picture of the solution depends on the choice of the ground node. An example is given. A low-pass filter is implemented twice in a single netlist, where the only difference (besides indexing of the object names) is the different choice of the ground node. Please refer to figure 5.2.

linNet computes the same transfer function from input to output voltage for both variants of the filter. Please compare U_{out_1} with U_{out_2}:

```
RESULT - User-defined result G1 (Bode plot):
The dependency of Uout1 on Uin1:
  Uout1(s) = N_Uout1_Uin1(s)/D_Uout1_Uin1(s) * Uin1(s), with
    N_Uout1_Uin1(s) = -Rfb1*C1 * s
                      -1
    D_Uout1_Uin1(s) = Rin1*C1 * s
RESULT - User-defined result Z1 (Bode plot):
The dependency of Uin1 on I_Uin1:
  Uin1(s) = N_Uin1_I_Uin1(s)/D_Uin1_I_Uin1(s) * I_Uin1(s), with
    N_Uin1_I_Uin1(s) = Rin1
    D_Uin1_I_Uin1(s) = 1
RESULT - User-defined result G2 (Bode plot):
The dependency of Uout2 on Uin2:
  Uout2(s) = N_Uout2_Uin2(s)/D_Uout2_Uin2(s) * Uin2(s), with
    N_Uout2_Uin2(s) = -Rfb2*C2 * s
                      -1
    D_Uout2_Uin2(s) = Rin2*C2 * s
```

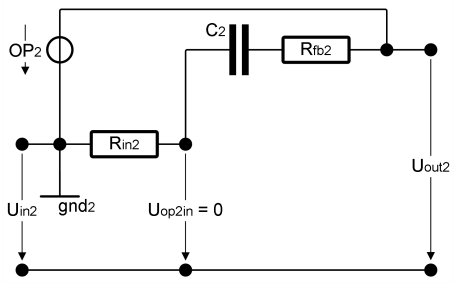


Figure 5.3: Internal, computed model of low-pass filter 2

```
RESULT - User-defined result Z2 (Bode plot):
The dependency of Uin2 on I_Uin2:
  Uin2(s) = N_Uin2_I_Uin2(s)/D_Uin2_I_Uin2(s) * I_Uin2(s), with
    N_Uin2_I_Uin2(s) = 1
    D_Uin2_I_Uin2(s) = 0
```

However, the input impedance (i.e. the transfer function Z from $I\,(U_{in})$ to U_{in}) differs significantly. Variant 1 of the filter yields R_{in}, which is what we expect but variant 2 has an input impedance of infinite (represented by $\frac{1}{0}$ in the result log). The infinite input impedance is explained by schematic 5.3 that shows how linNet models the op-amp in this case. The input connectors are open ended and no current will ever flow into the circuit; this is the meaning of an infinite input impedance.

It gets even more confusing if we define the node between resistor and capacitor in the feedback line to be the ground node, please refer to figure 5.4. The controlled voltage source in linNet's internal model of the op-amp now doesn't have any impact on the op-amp's input voltage; the current it can drive only flows through the resistor in the feedback line. The equation that the op-amp's input voltage needs to be null can't be fulfilled, the system determinant is null and the computation fails with error message.

Last but not least the chosen modelling of the op-amp implies that the output of the op-amp must not be connected to a ground node or to the output of another op-amp; the controlled voltage source would be short-circuited in the first case and the system would be underdetermined in the latter.

An op-amp in the netlist adds a new unknown to the linear equation system. It is the current flowing from the ground node through the device and out of its output. The name of this unknown is derived from the name of the op-amp, the prefix $I\_$ is added.

The new unknown current means a new column in the LES. It contains null coefficients anywhere but in the row that holds the current balance of the op-amp's output node. Here, the unknown current is influent and a one appears as coefficient.

The additional unknown comes along with the new equation $U_{in_1} - U_{in_2} = 0$. This equation means a row, which has just a pair of ± 1 coefficients at the columns, which are related to the two input node's voltage potentials (and hence only a single signed one if one of these nodes is the ground node).

As an example, an op-amp OP is connected with its two inputs and its output to the nodes N_2, GND and N_1, respectively, of the circuit fragment in figure 5.1. The new unknown current I_OP is put to the right of the unknowns so far. The LES would be extended to:

$$\begin{pmatrix} -C & C & 0 & \dots & 1 & \dots \\ C & -C-L-R & L & \dots & 0 & \dots \\ 0 & L & -L & \dots & 0 & \dots \\ \vdots & \vdots & \vdots & & \vdots & \ddots \\ 0 & 1 & 0 & \dots & 0 & 0 \end{pmatrix}$$

```
/* Same LP with 3rd choice of ground */
U    Uin3  in3    null3

OP   OP3   opInv3 null3  opOut3
R    Rin3  in3    opInv3
R    Rfb3  gnd3   opOut3
C    C3    opInv3 gnd3

DEF  Uout3 opOut3 null3
PLOT G3    Uout3  Uin3
PLOT Z3    Uin3   I_Uin3
```

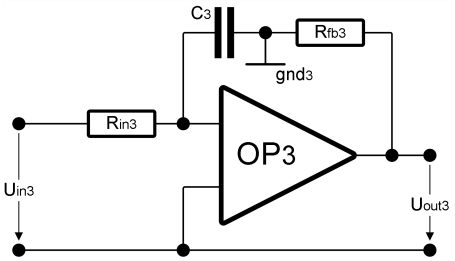


Figure 5.4: Same low-pass filter with invalid third choice of ground node

5.4.1 Symbols, knowns and unknowns

The op-amp is ideal and doesn't have a device constant or value. It neither adds a known to the LES nor a symbol to the computation.

The output current current of the op-amp is a new unknown of the linear equation system. The name of this unknown is derived from the name of the op-amp, the prefix $I\_$ is added. It can be referenced from a result definition.

5.5 Voltage controlled voltage source

The voltage controlled voltage source is modeled by an additional equation. The source is connected to two nodes. These node's voltage potentials, which are both unknowns to the system are associated by the constraint that they differ by a constant multiple k of a voltage, which is unknown but which can be expressed as difference of two nodes. The multiple k is the device constant and is a new symbol in the computation.

The additional equation is $U_{n_i} - U_{n_j} - kU_{n_u} + kU_{n_v} = 0$; the voltage source is connected to nodes n_i and n_j, k is its dimensionless device constant and the control voltage is the voltage potential difference between the nodes n_u and n_v.

The additional equation means a further row to the LES, which has all null coefficients but a pair of ± 1 and a pair of $\pm k$. (Both pairs can reduce to a single signed one or k if one of the references nodes is a ground node. If both nodes of the control voltage are the ground node then the $\pm k$ will even disappear entirely.)

The additional equation comes along with an additional unknown. When setting up the current balance of the nodes the source is connected to we have to consider the additional circuit branch, the current flowing through the source. This current is unknown. It is defined to be the current flowing out of the source's plus pole, which is the first connected node. This current is influent and thus positive at the node connected to the plus pole and affluent or negative at the minus pole's connected node. We get an additional column in the LES, which has a pair of ± 1 as only non null coefficients (or a single signed one if one of the connected nodes would be the ground node).

5.5.1 Symbols, knowns and unknowns

The voltage controlled voltage source doesn't introduce knowns to the equation system.

The name of the device appears as a symbol in the internal computation and in the final result. It has the meaning of the source's dimensionless voltage amplification k. The value for k used as default in

numeric post-processing with Octave can be found in table 7.1 on page 44.

The name of the additional unknown, the current through the source is derived from the device name by adding the prefix *I_* to the left of the user-defined device name. This name does not appear as a symbol in the internal computation. It can be referenced in a result specification in order to get an according transfer function or full result.

5.6 Current controlled voltage source

The current controlled voltage source is modeled by an additional equation. The source is connected to two nodes. These node's voltage potentials, which are both unknowns to the system are associated by the constraint that they differ by a constant multiple k of a current, which is an unknown of the LES. (It is the unknown current through a current probe; please refer to section 5.9 for details.) The multiple k is the device constant and it is a new symbol in the computation.

The additional equation is $U_{n_i} - U_{n_j} - kI_c = 0$; the voltage source is connected to nodes n_i and n_j, k is its device constant and I_c is the control current, an unknown of the LES.

The additional equation means a further row to the LES, which has all null coefficients but a pair of ± 1 (or a single signed one if one of the connected nodes would be the ground node) and a single signed k.

The additional equation comes along with an additional unknown. When setting up the current balance of the nodes the source is connected to we have to consider the additional circuit branch, the current flowing through the source. This current is unknown. It is defined to be the current flowing out of the source's plus pole, which is the first connected node. This current is influent and thus positive at the node connected to the plus pole and affluent or negative at the minus pole's connected node. We get an additional column in the LES, which has a pair of ± 1 as only non null coefficients (or a single signed one if one of the connected nodes would be the ground node).

5.6.1 Symbols, knowns and unknowns

The current controlled voltage source doesn't introduce knowns to the equation system.

The name of the device appears as a symbol in the internal computation and in the final result. It has the meaning of the source's current to voltage amplification k. The value for k used as default in numeric post-processing with Octave can be found in table 7.1 on page 44.

The name of the additional unknown, the current through the source is derived from the device name by adding the prefix *I_* to the left of the user-defined device name. This name does not appear as a symbol in the internal computation. It can be referenced in a result specification in order to get an according transfer function or full result.

5.7 Voltage controlled current source

The voltage controlled current source is modeled similar to a passive device. By definition, the current flowing through the voltage controlled current source is described by $I = k(U_{n_u} - U_{n_v})$, where k is the device constant, the amplification from voltage to current, and the control voltage is the voltage potential difference between the referenced nodes n_u and n_v.

This current is considered when setting up the current balance of the two nodes the source is connected to. It is defined to be the current flowing out of the second node in the netlist and it is influent and thus positive at this node. Vice versa, it is flowing back into the source at the first node in the netlist or affluent or negative at this node. In each of the two affected rows of the LES we get a pair of $\pm k$ as further addends of the coefficients of the unknown voltage potentials of the two referenced control nodes. (If one of the connected nodes is the ground node or if one of the nodes that define the control voltage is the ground node then two of the $\pm k$ disappear; if both holds then only a single signed k remains.) The appearance pattern of symbol k in the LES is nearly the same as for the passive devices; the only

difference is the possible appearance as a single pair $\pm k$. Neither an additional known, unknown nor equation is required.

5.7.1 Symbols, knowns and unknowns

The voltage controlled current source neither introduces knowns nor unknowns to the equation system. In particular, there's no unknown, which describes the current through the source. If this current is needed in a user-specified result then a current probe needs to be placed in series with the source.

The name of the device appears as a symbol in the internal computation and in the final result. It has the meaning of the source's voltage to current amplification k. The value for k used as default in numeric post-processing with Octave can be found in table 7.1 on page 44.

5.8 Current controlled current source

By definition, the current flowing through the current controlled current source is described by $I = kI_c$, where k is the device constant, the dimensionless amplification from control to output current, and I_c is the control current. I_c is the the current through a current probe device and such a current always is an unknown of the LES. (Please refer to section 5.9 for details.)

The current I is considered when setting up the current balance of the two nodes the source is connected to. It is defined to be the current flowing out of the second node in the netlist and it is influent and thus positive at this node. Vice versa, it is flowing back into the source at the first node in the netlist or affluent or negative at this node. A pair of $\pm k$ appears in the column of the LES that belongs to the unknown control current I_c (or a single signed k if one of the connected nodes is the ground node). Besides null coefficients this column can also contain the same pattern from other instances of such sources, if they are controlled by the same unknown current, and it will contain a pair of ± 1 from the referenced current probe itself. If the devices share their connected nodes then these ones and symbols share the same row, they become addends of the coefficient of the LES.

Neither an additional known, unknown nor equation is required.

5.8.1 Symbols, knowns and unknowns

The current controlled current source neither introduces knowns nor unknowns to the equation system. In particular, there's no unknown, which describes the current through the source. If this current is needed in a user-specified result then a current probe needs to be placed in series with the source.

The name of the device appears as a symbol in the internal computation and in the final result. It has the meaning of the source's dimensionless current to current amplification k. The value for k used as default in numeric post-processing with Octave can be found in table 7.1 on page 44.

5.9 The current probe

The current probe is an ideal wire, which connects two nodes. The voltage potentials of these two nodes become identical. As a kind of side-effect the current through this wire becomes a new unknown of the LES and can be queried in result specifications.

Actually, the current probe is a constant voltage source with voltage null. Please refer to section 5.2 to see how such a source is modeled. The current probe is different in two aspects: The way, the name of the new unknown is derived from the device name is different and no additional known (the constant source voltage) is introduced to the LES since the voltage is null by definition. All according terms of the result would have to be taken times null anyway.

The polarity of the unknown current through the current probe is defined as for the current sources: The unknown current is the one, which flows out of the second connected node into the network and

from the network node, which is mentioned first in the netlist back into the probe. This sounds counter-intuitive but isn't: This way, the first and second netlist node mean the start and end of the arrow in the symbol of the current probe in this order (see table 1.1 on page 10).

5.9.1 Symbols, knowns and unknowns

The current probe doesn't introduce a known to the equation system.

Neither in the internal computation and nor in the final result will the name of the device appear as a symbol.

The name of the additional unknown, the measured current, is basically derived from the device name by adding the prefix *I_* to the left of the user-defined device name – as usual for the sources. However, if the device name already starts with the character sequence *I_* then this name extension doesn't take place and the name of the unknown current is identical with the name of the device. This has been decided for convenience: It's natural to name a current probe like the measured current but this must not lead to names like *I_I_measured*. Sticking to the common naming pattern would mean to force the user to name the current probe device only *measured* in this example, which was a degradation of the readability of the netlist.

The name of the measured current does not appear as a symbol in the internal computation. It can be referenced in a result specification in order to get an according transfer function or full result.

Chapter 6

Building linNet

linNet is distributed as source code together with makefiles to compile and link the code. The distribution contains ready-to-use binaries for Windows, for both the win32 and win64 ABI and as debug or production compilation[1]. In all other environments a build of the sources needs to be done prior to the use of linNet.

To start the build you need to `cd` in a shell window to the root directory of the linNet distribution, where the file GNUmakefile is located. Here, you issue the command:

```
make build
```

The makefile has some more useful targets and options than just *build*. Tpye `make help` to find out.

6.1 Portability of makefiles

The makefile – actually it is a set of nested makefiles – is compatible with the GNU make processor of at least revision 3.81; 3.80 is not sufficient. Different derivates of make always tended to be quite incompatible to one another and other derivates of make will hence need heavy modifications of the makefiles.

The makefiles directly support the build of the software for Windows with GCC in the MinGW 32 and 64 Bit ports. The compilation and the compiled software have been tested with GCC 4.5.2 (32 Bit) and 4.8.1 (64 Bit) and under Windows 7 only. Other revisions may cause problems: Even between these two severe incompatibilities have been found, which had to be tackled by conditional code (see below).

The makefile needs to know, where GCC is installed. For Windows systems the environment variable *MINGW_HOME* has been introduced. It may be used to specify the root folder of a MinGW installation. If it is set, either persistently or – e.g. to be able to switch between 32 Bit and 64 Bit builds – on the command line of the make processor then the directory ${MINGW_HOME}/bin supersedes the normal system search path. If the variable is not set then all executables will be located via the system search path as specified in environment variable PATH. Please refer to file locateTools.mk for details.[2]

The makefiles are designed to support the compilation under Linux and Mac OS also. GNU make's `if/else/endif` statements have been applied to do so. This has however never been tested and will probably not work out of the box. Particularly, you should have a look at the tool localization in locateTools.mk and the compiler and linker command line options in compileAndLink.mk.

[1] Windows 7 has been used.

[2] Although designed for Windows, where typically several different competing ports of GCC can reside, the environment variable can be used for Linux and Mac OS, too. Don't bother with the name but let the variable point to the parent folder of GCC's bin folder.

6.1.1 Resource compiler

Under Windows the application icons are added to the executable binary file. This has no functional aspect; the image data is not accessed from the functional code. It's just a gimmick that allows to create illustrated associations between linNet in- and output files and the executable in the file system browser.

In the makefile the according build rules are placed in `if`/`endif` clauses; they should have no effect under either Linux or Mac OS. Similar mechanisms will probably exist for these operating systems as well. You may consider to extend the makefiles by adding appropriate `else` paths; an OS indicating variable already exists.

6.2 Portability of source code

The source code itself is system independent; only the abstract functions from the GNU C library are used.

The compilation with a different compiler tool chain (including another revision or port of GCC) will probably introduce some changes on the source files. The applied compiler needs to support the standard C99.

6.2.1 GNU extensions

In module *rat_rationalNumber* a GNU extension is used: Initialized structs are used as r-values. This has been done for convenience only, to achieve at tense and meaningful representation of the required functionality. It should however be easy and straightforward to eliminate these (few) constructs. All the rest is according to the standard C99.

6.2.2 Incompatibilities of linked libraries

Far the most necessary code modifications will result from incompatibilities of the linked libraries. Most obvious is the use of GCC's Basic Program/System Interface, the command line evaluation support. All of the command line evaluation related code will probably require a complete re-implementation with another compiler tool chain. Please refer to module *opt_getOpt* for details.

The two tested ports of GCC have different sets of standard functions. The MinGW 64 Bit port provides *stricmp* while MinGW 32 Bit doesn't. A simple implementation of this function has been added to the linNet sources. Depending on the availability of this function in your environment you will have to modify the according preprocessor switches to let it or don't let it compile the substitute. Please refer to file stricmp.h for details.

The standard function snprintf was found to be buggy in the used 64 Bit port of GCC. A work around has been implemented, which is compiled only for the 64 Bit MinGW ports. You will have to check if your revision of GCC has the same problems and maybe have to modify the preprocessor code that controls the conditional compilation of the substitute. Please refer to file snprintf.h for details.

6.3 Re-entrance of code

The implementation of linNet is widely but not completely reentrant. This can become an issue if the code should be integrated into a concurrent environment; for example a client-server structure is set up or multi-core support is implemented to speed up the computations. As long as the code is used as of today in a single-threaded self-contained application the following considerations are irrelevant.

Although the implementation of module *log_logger* is made in a reentrant style, the logging process is not reentrant by principle. All logger instances write to the same global stream *stdout* and concurrent use would lead to a mess of text at the console. Furthermore, all modules of linNet use one and the same global logger instance and they hence share the same file stream, too; here, we'd expect a similar mess.

For module *pci_parserCircuit*, the parser of the circuit netlist files, the decision to use a few global variables (e.g. for error reporting by side effect) has just been taken for convenience, to keep the code lean. An application for a fully reentrant implementation was simply not in the scope of the development. It's however straightforward and only little effort to make the parser reentrant. One just has to put the few global variables into a struct, which gets the meaning of a parser instance, add a constructor/destructor pair and pass the pointer to the parser instance to all the functions.

The situation differs for the other affected module *rat_rationalNumber*, the implementation of an exact arithmetics for rational numbers. Here, the decision to use global data and to loose re-entrance has been taken intentionally for performance reasons. A global variable is used to report an overflow. In addition, a global logger instance is used to visibly report problems. This logger isn't reentrant, neither. The client code can check the global variable after completing the operations. The flag is persistent and can thus be checked after a bunch of operations as an overall result. To avoid the flag we could return the overflow indication with every operation but this would lead to a manifold of computational effort. Or we could introduce rational number processor instances having each their own individual error handlers. The overhead of passing the instance pointer to all the functions would probably be less than in the first suggestion but still significant.

If the software is used in an environment, where re-entrance is a must then serialized access to the module could also be an option. A client is queued until the other one has completed its (bunch of) operations and has checked the global error information.

Chapter 7

Understanding the Octave Interface

7.1 Folders, files and functions

If the switch `-o` is put on the command line of linNet then the computed formulas are exported to Octave in form of a set of generated scripts (files *.m). Octave needs to be installed in order to make any use of these files. A further prerequisite is the availability of package control, which should normally be installed with Octave.[1] Please visit http://www.gnu.org/software/octave/ for information and download.

One dedicated script is generated for each of the results specified in the netlist file. The script has the name of the result in the netlist file. If no result is specified in the netlist file then linNet uses the implicit result definition *allDependents* and the generated Octave script has this name, too. All of the result related scripts are bundled in a common folder, which is also created and which has the name of the netlist file (except for the removed name extension). Several such folders will be created if the linNet command line specifies several netlist files. All of the created folders are placed in a common target directory, which can either be specified as argument of command line option `-o` or defaults to the current working directory. This directory needs to exist, linNet won't create it.

The generated, result related Octave scripts implement Octave functions. The functions in turn have the name of the result in the netlist file. They are self-contained in the sense that they can be called just like that if only the current working directory in the Octave session is the folder containing the script file, i.e. the folder that has the name of the processed netlist file. If you `cd` to the created folder and type the name of the result then you should get an Octave figure window with either a step response or a Bode plot – which one depends on the way the result was defined. The details of the result definition can be found at 4.3 on page 24.

The next step should be to type `help` followed by the name of the result.

The generated result related scripts use some (simple) sub-routines. These subroutines are implemented as a set of Octave files, which are bundled in folder private that is found inside the created netlist related result folder. The contents of folder private are invariant. As a matter of fact, the whole folder is just the copy of a template folder located somewhere in the installation of linNet. Therefore, getting this folder in every netlist related result folder is actually not essential and may even become annoying. (The design decision to do so has been taken for convenience, to make the result scripts self-contained and immediately usable.) To avoid these redundant files you can consider to make a copy of the files in the private folder into any folder holding Octave script files and to add this other folder to the Octave search path. From now on you would run linNet with command line option `-i` and folder private is no longer generated in the created result folders. The copied files on the Octave search path will be used instead of those in folder private.

[1] Type `pkg list` in the Octave command line to find out.

7.2 The two kinds of generated Octave functions

Depending on how a user-defined result is specified in the netlist file the generated Octave script will define a function of one out of two possible kinds. If the netlist command `PLOT` is used then the generated Octave function will show the Bode plot of a transfer function (or frequency response) of the system or one of its variables. If the netlist command `RES` is used then the generated Octave function will show the step response of the system or one of its variables.

The plots are presented if the function is called without assignment of the (optional) return values. The simplest way to do so is to just type the function name (i.e. result name) on the Octave command line.

7.3 Symmetries between the two kinds of generated Octave functions

Regardless of the fundamentally different plots the two kinds of generated Octave functions are quite similar.

Both kinds of functions have (nearly) the same signature. The difference is only their internal behavior, the internal decision to chose an appropriate kind of plot and this behavior is inhibited if a return value of the function is consumed by the calling code. If so, both kinds of functions behave identical: They generate and return the LTI object and maybe the parameter set and a default vector of either frequency values or time designations. (This last, optional return value is the only remaining difference in the behavior of the two kinds of functions.) As the first two return values are far the most relevant ones, both kinds of functions can mostly be used in an identical fashion. This means for example, that a Bode plot result can easily be used to plot a step response and vice versa:

Be `G.m` an Octave script generated from a `PLOT` command and `I_Uin.m` an Octave script generated from a `RES` command then will the direct call of

- `G` open a figure window with the Bode plot of (transfer function) G. This is the internal behavior of `PLOT` generated M functions,
- `I_Uin` open a figure window with the step response of (current) $I_{U_{in}}$. This is the internal behavior of `RES` generated M functions.

In contrast to this will a the first return value consuming call like

- `step(G)` open a figure window with the step response of the system described by (transfer function) G,
- `bode(I_Uin)` open a figure window with the Bode plot of the transfer function from system input U_{in} to current $I_{U_{in}}$.

Please note, that this is an example. In general, there are constraints. As an example, Octave refuses the Bode plot for systems other than SISO.

Using the generated M functions as argument of enclosing Octave expressions works well but is somewhat slow: Each time the expression is evaluated another (temporary) LTI object is created by the function. This causes a noticeable delay. In most situations it will be more appropriate to create the LTI object once in the workspace and to continue with the same object. The commands from the last example would change to:

```
tfG = G; step(tfG)
tfI_Uin = I_Uin; bode(tfI_Uin)
...
nyquist(tfG)
```

The last line is just meant to demonstrate how to benefit from the once and only once created objects.

Device	Symbol	Value	Remark
Resistor	R	100 Ω	RLC should yield audio frequencies
Conductance	Y	10 mS	
Capacitor	C	10 μF	
Impedance	L	1 mH	
Voltage controlled voltage source	U(U)	1	No particular use case aimed
Current controlled voltage source	U(I)	1 $\frac{V}{A}$	No particular use case aimed
Voltage controlled current source	I(U)	5 $\frac{mA}{V}$	Use case unipolar transistor
Current controlled current source	I(I)	250	Use case bipolar transistor

Table 7.1: Device standard values

7.4 Varying device values

A netlist file, which is the source of the Octave scripts that create the LTI objects, has the option to specify device values. This applies to the passive devices and the controlled sources. A resistor can get a value, which is understood as resistance in Ω, a conductance can get a value in $\frac{1}{\Omega}$, the capacitor is understood in F and the impedance in H. The device values of controlled sources are understood as the proportionality factor between control voltage/current and controlled voltage/current. Accordingly, they are dimensionless or understood as either $\frac{A}{V}$ or $\frac{V}{A}$.

If the netlist specifies a device value then this value is put into the generated Octave script as default value for all numeric computations. For linNet itself the value is meaningless, it is not used at all by any linNet computation.

All devices which no value is specified for in the netlist will get a hard coded standard value as default value in the generated Octave script. These standard values depend on the kind of device but do not consider values, which might have been specified in the netlist for other devices. A mixture of explicitly specified device values and standard values in one and the same netlist file is not recommended.

The hard coded standard values are chosen such that if no device values are specified in the netlist then the time constants of the system will probably be in the audio frequency range. This holds for the values of the four passive device kinds.

The standard values for the controlled sources are more difficult to choose. Such sources can be used to model totally different physical effects and a prototypic value is hard to find. If there is a typical use case like the bipolar transistor simulation with a current controlled current source then this use-case has let to the chosen standard value. Because of the difficult definition of these standard values it is recommended not to use controlled sources in numerical evaluations with Octave without the explicit specification of a realistic device value in the netlist.

All hard coded standard values of all affected devices are listed in table 7.1.

The values – either explicitly specified or standard values - are written into the generated Octave script and used if the scripts are run without more ado. In the Octave session they can be accessed and changed. This way the numeric system simulation can be repeated with altered values and the system behavior can be investigated in dependency of device value changes in an empiric way.

The second return value of the generated Octave functions is a `struct`. The members have the names of the devices. Each member holds the value of the related device. The struct implements the parameter set the returned LTI object has been created from.

The third return value of the generated Octave functions depends on the kind of result, Bode plot or full result. It is a vector of frequency values or time designations, respectively. Frequency values have the unit $\frac{rad}{s}$ as common for Octave functions. The retrieved vector is a suitable starting value for system simulations and can be used with the Octave functions *bode*, *step*, *impulse*, etc. It depends on the kind of command, whether a frequency or time vector is required.

Calling the generated function with consumption of at least one return value inhibits the internal behavior; no figure window is opened, no plot is made. Just the mentioned data is fetched if the return

values are assigned to workspace variables. These variables can then be altered and reused for subsequent computations.

The generated functions take a parameter set as only (optional) argument. In most cases, the parameter set will be the modified return value of a previous call of the function. Providing this argument makes the functions behave exactly as before but only using the altered parameters. You can either let them plot the standard figures (Bode plot or step response) or let them create a *new* LTI system object based on the altered parameters. Please note, it is not possible to change the parameters of a previously created, already in the workspace existing LTI object; instead, you will always have to replace or overwrite it.

Varying the device constants or parameters of a circuit is now completed with an example. Be G.m the generated Octave script:

```
G                   % Plot the transfer or step function as defined in the circuit file
[tf_G p] = G        % Get an LTI object (class tf) and the default set of device constants
p.R1 = 330;         % Modify device constant R1 in parameter set p
p.R2 = 2*p.R1       % Modify device constant R2 in parameter set p
G(p)                % Repeat the initial plot with the altered parameter set p
tf_G = G(p)         % Replace the LTI object with a new one using the altered device constants
step(tf_G)          % Plot the step response of the system with modified device constants
bode(tf_G)          % Plot the transfer function with modified device constants
nyquist(tf_G)       % Nyquist plot with modified device constants
```

7.5 Caveats of Octave

Although generally working well, has using Octave for numerical post-processing some weaknesses. The author has solely used Windows ports of Octave which might be worse than Linux releases with respect to functional failures and stability. The following is stated only for the Windows ports of release 3.4.3 and 3.6.4 running under Windows 7.

Octave 3.6.4 has general problems with the figure windows. The chosen graphics back-end was "fltk".

- The opened figure windows are initially black. The actual plots appear only after some clicking into the window pane or on the buttons to enforce a redraw
- If the figure window is enlarged by dragging the corners with the mouse than only the frame of the window is resized, the contents stay the same. This is particular annoying when MIMO systems produce grids of plots; these are initially tiny but can't be enlarged

These problems let to the decision to continue with the elder release 3.4.3. Window handling is okay in this release.

With Octave 3.4.3 we get a reproducible crash when running result G of netlist file bandStop.makesOctave3-4-3-win32Crash.cnl.[2] The linNet generated file G.m looks however unsuspiciously and resembles dozens of other generated files which are successfully processed. Octave 3.6.4 and MATLAB process file G.m successfully and plot the expected result.

[2]The file is contained in the source distribution of linNet.

Chapter 8

Typical Use Cases

8.1 Transfer function and full result

8.1.1 SISO system

The most common use case of linNet is the computation of the frequency response of a single in- and output system. This use case demonstrates how such a basic investigation can be done with linNet.

Please refer to figure 8.1. The circuit implements a typical op-amp based filter. It has two time constants; between the two edge frequencies the amplification rises from its initial DC value to the final value for high input frequencies. The device values are chosen such that all relevant frequencies are in the audio frequency range and the amplification increase is about 20 dB.

The netlist, which is printed on the left hand side of the figure be stored in the file opFilter.cnl. linNet is run with the command line

```
linnet -l -o opFilter.cnl
```

The most natural thing to do is to compute the transfer function of the filter, i.e. the dependency of the output signal on the input signal as a complex function. The line `PLOT G Uout Uin` specifies this result. linNet responds with:

```
RESULT - User-defined result G (Bode plot):
The dependency of Uout on Uin:
  Uout(s) = N_Uout_Uin(s)/D_Uout_Uin(s) * Uin(s), with
    N_Uout_Uin(s) = (R1*R2*C1 + R1*R2*C2) * s
                    +(R1 + R2)
    D_Uout_Uin(s) = R1*R2*C1 * s
                    +R2
```

The second requested result, *Gf*, uses the keyword `RES` in the specification. A full result is demanded. linNet responds with:

```
RESULT - User-defined result Gf:
The solution for unknown Uout:
  Uout(s) = N_Uout_Uin(s)/D_Uout_Uin(s) * Uin(s), with
    N_Uout_Uin(s) = (R1*R2*C1 + R1*R2*C2) * s
                    +(R1 + R2)
    D_Uout_Uin(s) = R1*R2*C1 * s
                    +R2
```

Actually, both results are just the same: The filter is a SISO system with a single input and the full dependency of the output on this input is the full dependency on *all* inputs at the same time.

```
// Definition of in- and output
U   Uin  Ip  gnd
DEF Uout out gnd

// The circuit
OP OP  Ip  Im  out
R  R1  out Im  R1=1k
C  C1  out Im  C1=1u
R  R2  Im  gnd R2=1k
C  C2  Im  gnd C2=22u

// Definition of results
PLOT G    Uout Uin
RES  Gf   Uout
RES  SIMO Uout U_Im I_OP
PLOT Iop  I_OP Uin
```

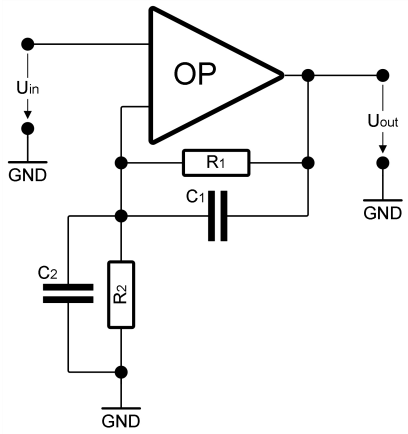


Figure 8.1: Op-amp filter with two time constants

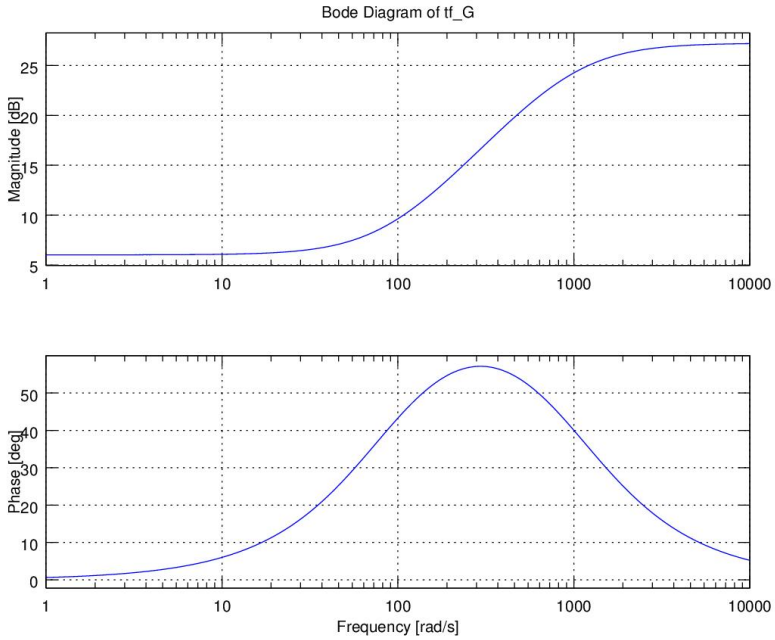


Figure 8.2: Default plot for linNet result of kind `PLOT`

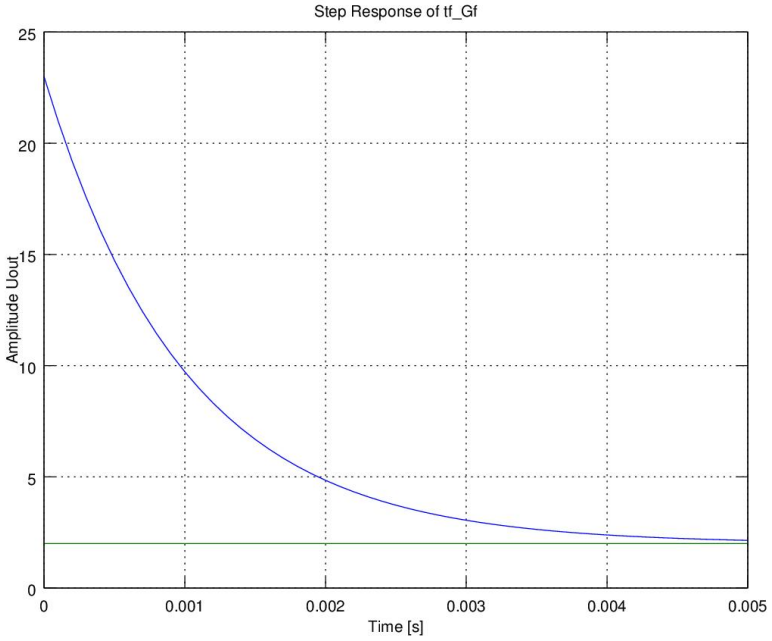


Figure 8.3: Default plot for linNet result of kind `RES`

Only when looking at the generated Octave code one will recognize a difference. While the generated LTI object is still identical differs the default operation performed on this object in both cases. The results have been implemented as Octave scripts G.m and Gf.m in the output folder opFilter. If called just like that the default operation is performed: The Octave command `G` opens a figure window with a Bode plot (see figure 8.2), command `Gf` presents the plot of the step response (see figure 8.3). Regardless, both Octave scripts can be applied to produce the same plots and numeric results if you make use of the optional function parameters and return values. Please, refer to section 7 for details.

The next result, which is specified in the netlist file is called *SIMO*. A full result is demanded for the three quantities U_{out}, U_{im} and I_{op}, where U_{im} is the voltage at the feedback input of the op-amp and I_{op} its output current. linNet responds with:

```
RESULT - User-defined result SIMO:
The solution for unknown Uout:
  Uout(s) = N_Uout_Uin(s)/D_Uout_Uin(s) * Uin(s), with
    N_Uout_Uin(s) = (R1*R2*C1 + R1*R2*C2) * s
                    +(R1 + R2)
    D_Uout_Uin(s) = R1*R2*C1 * s
                    +R2
The solution for unknown U_Im:
  U_Im(s) = N_U_Im_Uin(s)/D_U_Im_Uin(s) * Uin(s), with
    N_U_Im_Uin(s) = 1
    D_U_Im_Uin(s) = 1
The solution for unknown I_OP:
  I_OP(s) = N_I_OP_Uin(s)/D_I_OP_Uin(s) * Uin(s), with
    N_I_OP_Uin(s) = R1*R2*C1*C2 * s^2
                    +(R1*C1 + R2*C2) * s
                    +1
```

```
U    U1  in1 gnd
U    U2  in2 gnd

R    R1  in1 op1 R1=1k
R    R2  in2 op1 R2=1/2*R1
R    R3  op1 out R3=4*R1
R    R4  k1  out R4=1/10*R1
C    C1  op1 k1  C1=100n

OP   OP  op1 gnd out

RES  MIMO U_out I_OP
PLOT G1   U_out U1
```

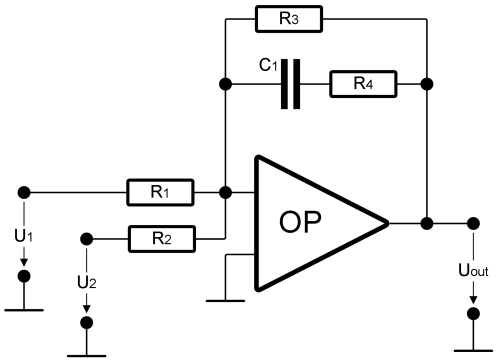


Figure 8.4: MIMO example: Analog adder

```
D_I_OP_Uin(s) = D_Uout_Uin(s)
```

Again, the found formula for U_{out} is exactly the same. However, this time it's only one formula out of three, which make up the complete result. For analytic considerations this is irrelevant. It doesn't matter if one specifies three single results with `RES` or one result with three dependent quantities. The only important difference can be found in the generated Octave script: A true SIMO LTI object is created. If typing `tf_SIMO=SIMO` on the Octave command line all three partial transfer functions of the SIMO system are printed.

The transfer function $I_{op}(U_{in})$ has a numerator degree in s of two but a denominator degree of only one, which means that the step response of the output current has a Dirac impulse; the two capacitors need to be loaded instantly. Octave can create such an LTI object, see before, but can't plot the unreal step response; it complains about an "improper" system.

The direct capacitive connection of the op-amp's output to ground further means an infinite output current for high frequencies. This is shown by the Bode plot Octave presents when typing `Iop`, which is the name of the last user-specified result. The Bode plot of an "improper" system is possible, as long as it is a SISO system. (Octave refuses to draw Bode plots of SIMO or MIMO systems.)

8.1.2 MIMO system

linNet is not restricted to SISO systems. If more than one independent sources are in the circuit then we have a multiple input system. linNet offers to compute all voltages and currents of a circuit, which leads to a multiple output system. The following use case presents a typical MIMO scenario.

Figure 8.4 shows an analog adder. The two input signals are added in a weighted sum in the ratio 1:2. The result specification `RES MIMO U_out I_OP` demands two dependents, this means a MIMO system with two in- and two output. linNet responds with:

```
RESULT - User-defined result MIMO:
The solution for unknown U_out:
  U_out(s) = N_U_out_U1(s)/D_U_out_U1(s) * U1(s)
             + N_U_out_U2(s)/D_U_out_U2(s) * U2(s), with
    N_U_out_U1(s) = -4*R1*C1 * s
                    -40
    D_U_out_U1(s) = 41*R1*C1 * s
                    +10
```

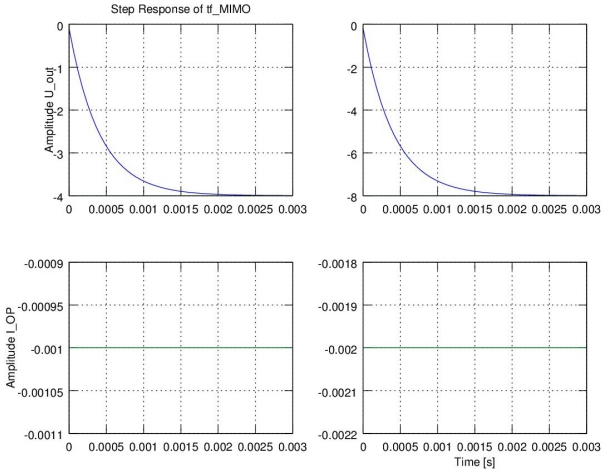


Figure 8.5: Step response of MIMO system created with `RES`

```
    N_U_out_U2(s) = -8*R1*C1 * s
                    -80
    D_U_out_U2(s) = D_U_out_U1(s)
The solution for unknown I_OP:
  I_OP(s) = N_I_OP_U1(s)/D_I_OP_U1(s) * U1(s)
            + N_I_OP_U2(s)/D_I_OP_U2(s) * U2(s), with
    N_I_OP_U1(s) = -1
    D_I_OP_U1(s) = R1
    N_I_OP_U2(s) = -2
    D_I_OP_U2(s) = R1
```

The composition of a single result from multiple formulas is already known from the previous use case, which introduced a SIMO system. Here, the representation of the multiple inputs is new. Each output is represented as a sum of two transfer functions, one for each input.

The generated Octave script MIMO.m will plot the step response of the system, see figure 8.5. The step response is presented in a rectangular grid for each in- and output. The meaning of a single figure is the response of the related output if the related input undergoes a step and all other inputs are constantly null.

The second user-specified result uses the keyword `PLOT`. The dependency of a single quantity on a single other one is presented as Bode plot. The result specification `PLOT G1 U_out U1` yields:

```
RESULT - User-defined result G1 (Bode plot):
The dependency of U_out on U1:
  U_out(s) = N_U_out_U1(s)/D_U_out_U1(s) * U1(s), with
    N_U_out_U1(s) = -4*R1*C1 * s
                    -40
    D_U_out_U1(s) = 41*R1*C1 * s
                    +10
```

This is not surprisingly one of the partial transfer functions of the MIMO system. In case of MIMO systems and if Bode plots are wanted the result kind `PLOT` needs to be used to pick out a single partial transfer function because the Octave command `bode` rejects any LTI object, which doesn't have SISO characteristics.

```
U Uin K1 gnd
DEF Uout K2 gnd

PLOT G   Uout Uin
PLOT Zin Uin I_Uin

I Iload gnd K2
PLOT Zout Uout Iload

C C  K1 K2
R Rc K1 K2
L L  K2 gnd
R Rl K2 gnd
```

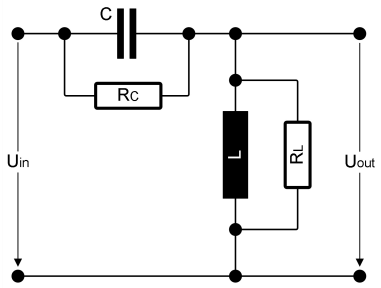


Figure 8.6: RLC circuit

8.2 In- and output impedance of a circuit

A typical issue in the discussion of an electronic circuit is the in- and output impedance as a function of the frequency. Since linNet is capable to figure out all voltages and currents in a linear circuit it is easily possible to request these functions as a user-defined result. Please refer to figure 8.6.

The focus of this use case is put on the definition of the results. The circuit itself is a simple damped LC resonator.

The user-defined result G is the conventional transfer function from stimulating input voltage U_{in} to resulting output voltage U_{out}. We have already seen this in a previous use case.

The definition of result *Zin* is new. Line `PLOT Zin Uin I_Uin` requests the transfer function that expresses the dependency of the input voltage on the input current. The ratio of these two is the input impedance and we will get it as a function of the signal frequency s. This function will contain the static DC input impedance as limit $s \to 0$. The new aspect of this result definition is that the dependent quantity, the input voltage U_{in}, actually is a known quantity to the internal computation; it's the device constant of the constant voltage source, which provides the stimulating input voltage. Vice versa, the independent quantity of the result, the input current $I_{U_{in}}$, is an unknown to the internal computation. With respect to the internal computation result *Zin* is the request for an inverse transfer function. linNet supports inverse transfer functions with the following limitations:

- Inverse transfer functions are limited to pairs of two quantities. There's no concept of an inverse full result
- In MIMO systems one and only one of the two quantities needs to be an independent quantity to the internal computation. Or with other words: one of the two quantities is a system input

The last constraint causes an inconvenience. Usually and like in figure 8.6 the input voltage is connected to a particular node N and the ground node. In which case the voltage potential of node N will be identical to the input voltage; but regardless of the triviality of this result it is a dependent quantity to the internal computation. If the impedance is defined as the dependency of node N's voltage potential on the input current then this result definition will be invalid for a MIMO system. As it is natural to name this particular node `in` and since the unknown node potential is addressed to by prefixing this name with `U_` it becomes a matter of `Uin` or `U_in` whether the result definition is valid or invalid, respectively.

The constraint disappears for SIMO systems: If the system only has a single input then the relationship between two dependent quantities is well defined; both follow a modulation of the only input in a certain, computable way and hence their relationship is known.

In principle, this behavior is natural and easy to understand but it might be hard to accept due to the in practice typically very similar names of a voltage source's device constant and the connected node's voltage potential.

For our example linNet returns:

```
RESULT - User-defined result Zin (Bode plot):
The dependency of Uin on I_Uin:
  Uin(s) = N_Uin_I_Uin(s)/D_Uin_I_Uin(s) * I_Uin(s), with
    N_Uin_I_Uin(s) = Rc*Rl*L*C * s^2
                     +(Rc*L + Rl*L) * s
                     +Rc*Rl
    D_Uin_I_Uin(s) = Rc*L*C * s^2
                     +(Rc*Rl*C + L) * s
                     +Rl
```

The input impedance is the transfer function `N_Uin_I_Uin(s)/D_Uin_I_Uin(s)`. The limit of this function for $s \to 0$ is R_C. For DC operation the capacitor is as not present and the impedance short-circuits the resistor R_L. The remaining resistance from the input side is R_C.

The limit $s \to \infty$ can also be interpreted. The formula yields R_L in this case. For very high frequencies the capacitor short-circuits the resistor R_C and L's impedance goes against ∞. The parallel resistor R_L is the remaining resistance from the input side.

Between these two ends the magnitude of the complex function has a local minimum.

If we run the generated Octave script `Zin.m` to plot the input impedance then the standard Bode plot settings are used. Which means that the y axis is scaled in decibel. For the purpose of an impedance value maybe not the most convenient convention. To get the values in Ohm you will have to inverse the dB definition, if you e.g. read a value of 80 dB in the plot then type `10^(80/20)` on the Ocatve command line to get the impedance in Ohm.

Alternatively you can evaluate the generated transfer function for a given frequency f in Hz. In Octave this is as simple as typing `abs(Zin()(2*pi*f))`.[1]

The computation of the output impedance is not that straightforward. It is defined as the ratio of the output voltage drop caused by an output current and this current. To put this in linNet manageable terms we need to extend the circuit by a load. If we use a current source then the output impedance is the dependency of the output voltage on this load current. The netlist in figure 8.6 already contains a load current source, see line `I Iload gnd K2`. (Insofar the schematic on the right hand side of the figure doesn't represent the circuit actually computed by linNet.)

We define the load current influent to the circuit. A negative voltage change (a voltage drop) will be caused by a negative current and the demanded ratio of both will become positive; this is the conventional sign definition for an output impedance.

Please note, that the extension of the circuit doesn't necessarily need to be done in a separate step. Other user-defined `PLOT` results, e.g. the transfer function G, do not suffer from this additional current source. If we demanded the full result for the output voltage then we'd get a changed formula, now with two addends instead of a single term: Each describing the impact of one of the sources on the output voltage. The partial transfer function G is just one of these two addends and doesn't depend on the load current.

The rest holds as for the input impedance. The definition of the output impedance Z_{out} as linNet result is made in line `PLOT Zout Uout Iload`. After running linNet you will discuss the found formula:

```
RESULT - User-defined result Zout (Bode plot):
The dependency of Uout on Iload:
  Uout(s) = N_Uout_Iload(s)/D_Uout_Iload(s) * Iload(s), with
    N_Uout_Iload(s) = Rc*Rl*L * s
```

[1] In MATLAB the LTI transfer function objects behave differently; the sample code won't work. You may instead use function *bode* with parameter $\omega = 2\pi f$ to get the impedance value at frequency f.

```
U    Ubatt   vss   gnd   // 12 V
U    Uin     in    gnd
C    Cin     in    b     Cin=100u
R    R1      vss   b     R1=10k
R    R2      b     gnd   R2=1.2k
R    Rc      vss   c     Rc=6.8k
R    Re      e     gnd   Re=680
R    Rec     e     rce   Rec=68
C    Ce      rce   gnd   Ce=4.7u
C    Cout    c     out   Cout=1.5u

/* The actual transistor model T1 */
PI   I_base  b     b1
U    Ube     b1    e     // 0.6 V
I(I) beta    c     e     I_base beta=250

RES  DC      U_c   U_b   I_Ubatt
PLOT G       U_out Uin   LOG 50  10 20k
PLOT Z       Uin   I_Uin LOG 50  10 20k
PLOT humSupp U_out Ubatt LOG 500 10 100
```

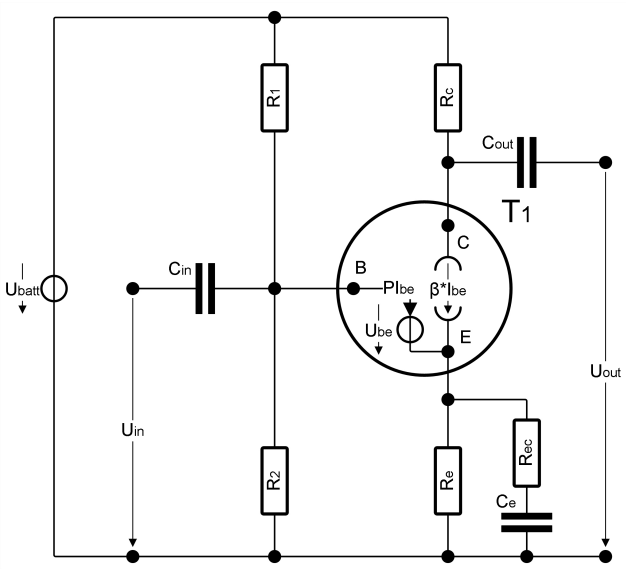


Figure 8.7: Bipolar transistor amplifier

```
D_Uout_Iload(s) = Rc*Rl*L*C * s^2
                  +(Rc*L + Rl*L) * s
                  +Rc*Rl
```

or run the generated Octave script Zout.m to plot the impedance or compute its values at particular frequencies. The Octave computations can be done for particular device values only; these are not configured in our example netlist and Octave will begin with initial standard values.

8.3 Transistor model

The final use case is the complexest one. Figure 8.7 shows a typical transistor amplifier circuit. The input voltage U_{in} is amplified to the output voltage U_{out}. Main characteristics of the circuit is the frequency response of the output in dependency of the input. This uses case demonstrates the usage of linNet in the analysis of such a prototypical circuit.

8.3.1 Circuit design

Initially, a rough calculation is made that leads to an initial set of numeric device values. The results of the later linNet computation can then be used to prove the made simplifications and to validate and perhaps refine the initial findings.

The design of the circuit is driven by some requirements and figured out based on some simplifications. Let's assume the following requirements:

- The amplification should be about 100 or 40 dB
- The output impedance should be in the magnitude of 10 kΩ

- The amplifier should be operational in a frequency range of 100 Hz ... 20 kHz
- The circuit should have a stable operating point

The starting point of the design is the yielded amplification. The nominal current amplification β of our transistor be 250. This leaves room for an emitter current based feedback: The rising voltage at the emitter resistor due to current amplification will be a counter-control of the amplification as it directly decreases the base-emitter voltage drop.[2]

The emitter voltage nearly follows the input voltage and the current through the emitter is nearly the current through the collector; consequently, the expected amplification (disregarding the inversion) is about the ratio of collector to emitter resistor, $|V| \simeq R_c/R_{e_{eff}}$.

A further simplification is that the collector current is relatively independent of the circuitry in the collector. An hypothetical output current will therefore always mean an additional voltage drop at R_c, which is proportional to this resistor and the hypothetical current. This means that the circuit's output impedance has nearly the same value as R_c. According to our requirement we choose a value of $R_c = 6.8\,\mathrm{k}$, which means $R_{e_{eff}} \simeq 68\,\Omega$.

The operating point of the collector (i.e. the voltage at input voltage null) should be somewhat above the middle of the supply voltage to get a symmetric limit for both polarities of the input voltage. Given $R_c = 6.8\,\mathrm{k}$ we want to have a collector current $I_c = 5.7\,\mathrm{V}/6.8\,\mathrm{k\Omega} = 0.84\,\mathrm{mA}$. It holds $I_c \simeq I_e$ and accordingly $U_b \simeq U_{be} + R_{e_{eff}} I_c$. The actual base voltage will be set with the ratio R_1/R_2. As U_{be} is considered constant the product $R_{e_{eff}} I_c$ will have to compensate for any deviation of U_b from the targeted value due to tolerances of R_1 or R_2. The larger $R_{e_{eff}}$ the smaller the deviation of I_c and thus of the targeted operating point. This consideration explains the emitter circuitry: For DC currents we want to have a large value to get a stable and targeted operating point but the design goal $V = 100$ already demanded a value of $68\,\Omega$ for the relevant input frequency range. The answer is: Place the $68\,\Omega$ behind a DC blocking capacitor and put a larger resistor for DC operation in parallel. We take the tenfold value for DC operation, $R_e = 680\,\Omega$; this won't affect the previous findings too much or with other words, $R_{e_{eff}}$ for the relevant input frequency range. For DC operation the effective emitter resistance is R_e and the wanted value for U_b becomes $U_b = 0.6\,\mathrm{V} + 680\,\Omega \cdot 0.84\,\mathrm{mA} = 1.17\,\mathrm{V}$.

The last step of the design is the voltage divider at the transistor's base. The simplification used here is that the base current can be neglected in comparison to the currents through the two resistors. This leads to the simple formula $U_b = R_2/(R_1 + R_2)U_{batt}$. Considering only resistor values from the E12 series (10% tolerance) we choose 10 k and 1.2 k to get the best fit. The current through the resistors is about one Milli Ampere, which is a hundredfold of the base current I_c/β so that the made simplification is applicable.

8.3.2 Modelling the transistor circuit

Normally a circuit like this is modeled as a so called small-signal circuit. The true circuit is reduced to those elements, which are relevant for the frequency response above $f = 0$. The resulting circuit would no longer be operational in a true construction and the DC operating point can't be computed any longer. In a second step one would then do the contrary to figure out the DC behavior. Using linNet we can decide to model all at once.

To linNet the circuit is a system with three independent inputs and a list of dependent quantities under which our output voltage U_{out}. For us, the two inputs U_{batt} and U_{be} are constant values, we will have to put the constant values 12 V and 0.6 V into the resulting formulas. However, seeing U_{batt} as a true input signal we can easily find e.g. the hum suppression of the circuit: Consider U_{batt} had a frequency component of the public power grid and ask for the transfer function from power supply to output voltage.

[2] In the true implementation of the circuit such a feedback reduces the impact of the non linearity of the real transistor device. This can't be illustrated, proven or quantified with linNet as linNet can't model any kind of nonlinearity.

The transistor model

The bipolar NPN transistor itself is modeled as an ideal current amplifier; the collector current is assumed to be β times the base current, where β is a constant. It is modeled without non-linearities and frequency dependencies. To get a full picture of the DC behaviour the voltage drop from base to emitter is modeled by a constant voltage source of 0.6 V.

The current amplification is implemented by a current controlled current source. The device constant is the factor between the source's current and the control current. Obviously, the former is the collector current and the latter the base current. linNet takes a device's name as device constant in the result representation. Therefore we name the device *beta*.

The current controlled current source references a current probe to specify the control current. The probe is placed in series to the base of the transistor and senses the base current. The name of the probe is *I_base* and linNet will use the same name for the sensed current if it should be referenced from a result definition. (If the probe's name wouldn't begin with `I_` then linNet would add this as a prefix in order to make transparent that the referenced quantity is a current.)

8.3.3 The voltage amplification

The principal characteristics of the transistor circuit is the resulting amplification, the ratio of magnitudes of out- and input voltage. Due to the decoupling capacitors we have amplification null for DC. We have to define the amplification as the voltage ratio at frequencies above the influence of the capacitors.

We look at the partial result for the output voltage, which only describes its dependency on the input voltage.

```
RESULT - User-defined result G (Bode plot):
The dependency of U_out on Uin:
  U_out(s) = N_U_out_Uin(s)/D_U_out_Uin(s) * Uin(s), with
    N_U_out_Uin(s) = -(beta*R1*R2*Rc*Re*Ce*Cin + beta*R1*R2*Rc*Rec*Ce*Cin) * s^2
                     -beta*R1*R2*Rc*Cin * s
    D_U_out_Uin(s) = (beta*R1*R2*Re*Rec*Ce*Cin + R1*R2*Re*Rec*Ce*Cin) * s^2
                     +(beta*R1*R2*Re*Cin + beta*R1*Re*Rec*Ce + beta*R2*Re*Rec*Ce
                       + R1*R2*Re*Ce + R1*R2*Re*Cin + R1*R2*Rec*Ce + R1*Re*Rec*Ce
                       + R2*Re*Rec*Ce
                      ) * s
                     +(beta*R1*Re + beta*R2*Re + R1*R2 + R1*Re + R2*Re)
```

Due to our initial considerations we take this result for high input frequencies, i.e. for high frequencies of U_{in}. The amplification V is got as limit $s \to \text{inf}$. We take the quotient of the numerator and denominator terms of highest power in s:

$$\begin{aligned} V &= \lim_{s\to\text{inf}} G(s) = -\frac{\beta R_1 R_2 R_c R_e C_e C_{in} + \beta R_1 R_2 R_c R_{ec} C_e C_{in}}{\beta R_1 R_2 R_e R_{ec} C_e C_{in} + R_1 R_2 R_e R_{ec} C_e C_{in}} = -\frac{\beta R_c R_e + \beta R_c R_{ec}}{\beta R_e R_{ec} + R_e R_{ec}} \\ &= -\frac{\beta}{\beta+1}\frac{R_c}{\frac{R_e R_{ec}}{R_e+R_{ec}}} \end{aligned} \tag{8.1}$$

The result (8.1) for the amplification can be interpreted. V is mainly determined by the second term, which can be read as the ratio of collector resistor to the parallel circuit of the two emitter resistors; the emitter capacitor behaves as if not present for high frequencies.

The earlier made approximation of V as the ratio of the collector resistor's value and the effective resistance at the emitter was based on the idea that the same current flew through both resistors and leads to voltage magnitudes proportional to their values, while due to the constant U_{be} the voltage drop at the emitter resistor follows the input voltage. Now we get the corrective first term in equation (8.1), which is

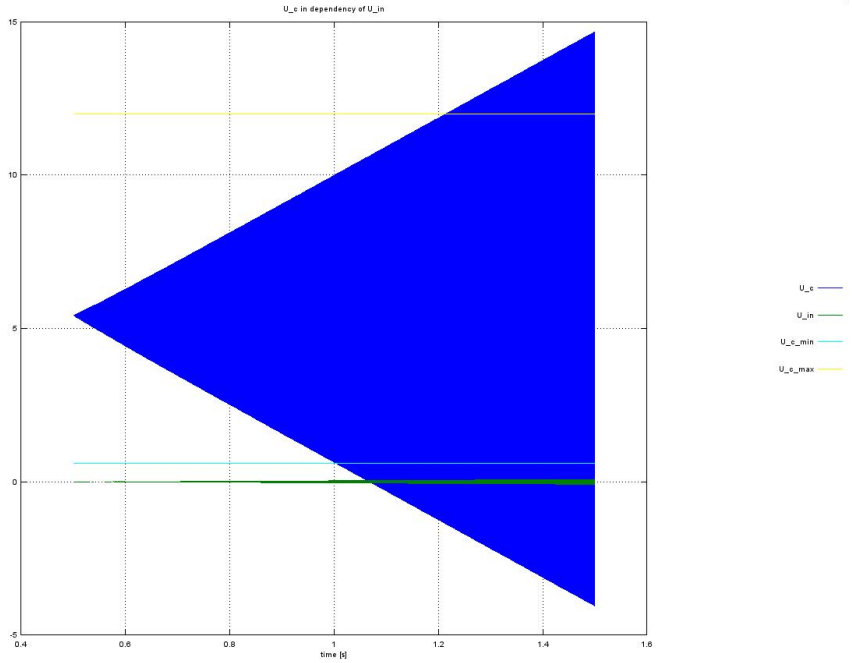


Figure 8.8: Simulation of bipolar amplifier circuit in figure 8.7 with modulated sinoid input

a number closed to one and reduces the approximation of the amplification a tiny bit. It expresses that the currents through emitter and collector are indeed similar but not identical. The voltage modulation at the collector has to be reduced by the portion of the base current.

8.3.4 Limits of operational range of transistor

The limitations of the true transistor can't be considered by linNet (as any non-linearity) and nothing about can be seen in any of the results.

As an idea to investigate the limits of the amplification we could request the result for the voltage U_c. The true ciruit reaches its limits if this voltage approaches either U_{batt} or U_e (U_e is nearly constant by design). In the numeric post-processing with Octave we could plot this quantity for a rising input signal and compare U_c with these limits. The point in time, when it first reaches one of the limits designates the maximum in- and output signal, which could be processed by a real implementation of the simulated circuit.

This is in one respect a new technique. So far, we focussed on transfer functions from one quantity to another one. For this investigation we need to simulate all the inputs since the DC operating point is an essential part of this computation. Neither the frequency response analysis nor the step response analysis Ocatve offers will suffice. We need to do a time domain simulation using all the inputs simultaneously.

An Octave script assembles the required input signal as a 3-column vector of samples and creates the LTI system object using the linNet generated result script. Then it runs this system with the input signal. The simulation result for U_c is shown in figure 8.8. At $t = 1\,\mathrm{s}$ the lower limit for U_c is reached. At this time the input signal has an amplitude of 50 mV. This is the maximum input value the true circuit could process without truncation. The figure shows that the operating point is not yet optimal; both limits should be reached at the same time to optimally exploit the available operating range.

The Octave script is printed as listing C.1 on page 86. Please refer to this listing to get more details on how to run a linNet generated LTI system with arbitrary input signals.

Using a former result, the amplification V, another approach is possible. If we achieved the design

goal that the operating point for the collector is in the center of the possible output voltage range then we can compute the maximum undistorted peak-peak input voltage by dividing this range by the found amplification. The found result should nearly be the same as with the time domain simulation and it would be formula based.

The design goal "operating point for collector voltage" can be validated by demanding the full result for U_c and evaluating the formula for $U_{in}(s) = 0$ and $s = 0$. linNet reports for U_c:

```
RESULT - User-defined result DC:
The solution for unknown U_c:
  U_c(s) = N_U_c_Ubatt(s)/D_U_c_Ubatt(s) * Ubatt(s)
           + N_U_c_Uin(s)/D_U_c_Uin(s) * Uin(s)
           + N_U_c_Ube(s)/D_U_c_Ube(s) * Ube(s), with
    N_U_c_Ubatt(s) = (beta*R1*R2*Re*Rec*Ce*Cin + R1*R2*Re*Rec*Ce*Cin) * s^2
                     +(beta*R1*R2*Re*Cin + beta*R1*Re*Rec*Ce - beta*R2*Rc*Re*Ce
                       - beta*R2*Rc*Rec*Ce + beta*R2*Re*Rec*Ce + R1*R2*Re*Ce
                       + R1*R2*Re*Cin + R1*R2*Rec*Ce + R1*Re*Rec*Ce + R2*Re*Rec*Ce
                      ) * s
                     +(beta*R1*Re - beta*R2*Rc + beta*R2*Re + R1*R2 + R1*Re
                       + R2*Re
                      )
    D_U_c_Ubatt(s) = (beta*R1*R2*Re*Rec*Ce*Cin + R1*R2*Re*Rec*Ce*Cin) * s^2
                     +(beta*R1*R2*Re*Cin + beta*R1*Re*Rec*Ce + beta*R2*Re*Rec*Ce
                       + R1*R2*Re*Ce + R1*R2*Re*Cin + R1*R2*Rec*Ce + R1*Re*Rec*Ce
                       + R2*Re*Rec*Ce
                      ) * s
                     +(beta*R1*Re + beta*R2*Re + R1*R2 + R1*Re + R2*Re)
    N_U_c_Uin(s) = -(beta*R1*R2*Rc*Re*Ce*Cin + beta*R1*R2*Rc*Rec*Ce*Cin) * s^2
                   -beta*R1*R2*Rc*Cin * s
    D_U_c_Uin(s) = D_U_c_Ubatt(s)
    N_U_c_Ube(s) = (beta*R1*R2*Rc*Re*Ce*Cin + beta*R1*R2*Rc*Rec*Ce*Cin) * s^2
                   +(beta*R1*R2*Rc*Cin + beta*R1*Rc*Re*Ce + beta*R1*Rc*Rec*Ce
                     + beta*R2*Rc*Re*Ce + beta*R2*Rc*Rec*Ce
                    ) * s
                   +(beta*R1*Rc + beta*R2*Rc)
    D_U_c_Ube(s) = D_U_c_Ubatt(s)
```

The operating point for U_c is found as:

$$\begin{aligned} U_{C0} &= \frac{(\beta R_1 R_e - \beta R_2 R_c + \beta R_2 R_e + R_1 R_2 + R_1 R_e + R_2 R_e) U_{batt} + (\beta R_1 R_c + \beta R_2 R_c) U_{be}}{\beta R_1 R_e + \beta R_2 R_e + R_1 R_2 + R_1 R_e + R_2 R_e} \\ &= U_{batt} - \frac{\beta R_c}{R_1 R_2 + (1+\beta)(R_1 + R_2) R_e} [R_2 U_{batt} - (R_1 + R_2) U_{be}] \end{aligned} \tag{8.2}$$

Copying the formula (8.2) into Octave and using $U_{be} = 0.6\,\mathrm{V}$ and $U_{batt} = 12\,\mathrm{V}$ we get $U_{c0} = 5.21\,\mathrm{V}$. This is not optimal; taking the constant emitter voltage of about 0.6 V into account the ideal value would be about 1 V higher; a refinement of the device values can be suggested.

Using Octave we can find out on the fly that the dependency of the operating point on the transistor amplification β is negligible; the variation $\beta = 150 \ldots 300$ yields a variation $U_{c0} = 5.26\,\mathrm{V} \ldots 5.20\,\mathrm{V}$.[3] This is important insofar as the transistor amplification has a high standard deviation for a given device type.

[3]The fasted way to reproduce these values is to directly use the linNet generated LTI object to evaluate the transfer function at s=0 and to compute the output as linear combination of the three input contributions: `[sys p]=DC; p.beta=150, sys=DC(p), F0=sys(0), U_c0=F0(1,:)*[12; 0; 0.6]` The first statement is only used to get parameter object p that can then be manipulated for the variation of β.

A more profound investigation could even state this as a formula, we can easily figure out the partial derivative of (8.2) with respect to β.

Another aspect: The partial derivatives of (8.2) with respect to β and the resistors would prove whether the trick with C_e and R_{ec} in parallel circuit to R_e really pays off: it has been made to reach the design goal for the amplification V without endangering a stable operating point. If the operating point would however still be tolerant against typical standard deviations of β and the resistors for a ten times smaller value of R_e then we could live without this trick.

8.3.5 Hum suppression

The hum suppression might be of interest for a real application of the circuit if it has a conventional power supply, which might contain some remains of the 50 Hz frequency of the public grid. Hum suppression describes to which extend such remains will be apparent at the circuit output.

The dependency of the collector voltage U_c on the supply voltage U_{batt} is part of the user-defined result *DC*, which we had discussed in the previous section. Even more convenient than evaluating result *DC* is to specify a result, which is more to the point. Please, refer to line

```
PLOT humSupp U_out Ubatt  LOG 500 10 100
```

in the circuit netlist: We demand the dedicated result *humSupp* and will get an Octave script of same name. The computed formula is exactly the extract of the full result *DC*, which we need. The default behavior of the Octave script humSupp.m is to present the result as a Bode plot. All we have to do is to run the script and to read the plot at the frequency of the public power grid. We get a value of only -0.3 dB, which means that the hum is propagated to the circuit output without noticeable damping.

The explanation: In the computation of a MIMO system and if we ask for the transfer function of one of its inputs to one of its outputs, then the answer holds under the assumption that the other inputs have given values, e.g. null all the time (however the actual input function is irrelevant). With other words its assumed that the inputs are driven with a source of output impedance null. For our circuit this has the consequence that the input capacitor C_{in} stabilizes the base voltage of the transistor - here we have a good hum suppression. Accordingly, the collector current is also stablized but unfortunately this means that the collector voltage exactely follows the battery voltage – and this is what we read from the Bode plot.

Please note that C_{out} doesn't appear in the transfer function U_{batt} to U_{out}. All computation is ideal and since there's no load the output capacitor is currentless all time long and hence has no impact on the output voltage. In any true application of the circuit it would introduce in conjunction with the input impedance of the following stage a further time constant in the overall system behavior.

8.3.6 Output impedance

As seen in a previous use case the output impedance of the circuit can be found by adding a defined load and asking for the dependency of the output voltage on this load. If we add the lines

```
I    Iload gnd   out
PLOT Zout  U_out Iload LOG 50 10 20k
```

to the input file then linNet finds a formula of third order in s for the output impedance. The limit of its value for $s \to 0$ is infinite. This is expected as the capacitor C_{out} decouples the DC component from the output signal. The limit for $s \to \infty$ is identical to R_c and in the Bode plot we see that this final value is reached at about 100 Hz, thus at the beginning of our defined frequency operation range. This meets our initial expectations.

The picture becomes even clearer if we modify the additional lines in the input file like this:

```
I    Iload gnd c
PLOT Zout  U_c Iload LOG 50 10 20k
```

Using these lines we actually remove the decoupling output capacitor from the circuit and define the output impedance at the collector of the transistor. Now linNet computes the resulting transfer function as:

```
RESULT - User-defined result Zout (Bode plot):
The dependency of U_c on Iload:
  U_c(s) = N_U_c_Iload(s)/D_U_c_Iload(s) * Iload(s), with
    N_U_c_Iload(s) = Rc
    D_U_c_Iload(s) = 1
```

The proportionality factor between current and voltage is identical to the value R_c of the collector resistor – this is what we had assumed in our initial design simplifications.

Appendices

Appendix A

Formal Syntax Description of Input File

The syntax of the the circuit defining input file (the netlist or *.cnl file) in Backus-Naur form:

```
<circuit>       = {[<deviceDef> | <voltageDef> | <resultDef> | <bodeResultDef>] (EOL|';')}
<deviceDef>     = <deviceType> <name> <node> <node> [<node> | <control>] [<relation>]
<deviceType>    = 'R'|'Y'|'C'|'L'|'PI'|'U'[<controlledBy>]|'I'[<controlledBy>]|'OP'
<controlledBy>  = '(' ('U'|'I') ')'
<control>       = <node> <node> | <name>
<relation>      = <name> '=' <quantityRef>
<quantityRef>   = <number> | <deviceRef>
<deviceRef>     = [<rationalNum> '*'] <deviceName>
<voltageDef>    = 'DEF' <name> <node> <node>
<resultDef>     = 'RES' <name> <unknown> {<unknown>} [<plotInfo>]
<bodeResultDef> = 'PLOT' <name> <unknown> <known> [<plotInfo>]
<node>          = <identifier>
<known>         = <name>
<unknown>       = <name>
<name>          = <identifier>
<plotInfo>      = ('LIN' | 'LOG') <integer> <number> <number>
<identifier>    = <character> {<character> | <digit>}
<character>     = 'a' | ... | 'z' | 'A' | ... | 'Z' | '_'
<number>        = <integer> ['.' [<integer>]] [<exponent>]
<rationalNum>   = ['('] <integer> '/' <integer> [')']
<integer>       = <digit> {<digit>}
<exponent>      = ('y' | 'z' | 'a' | 'f' | 'p' | 'n' | 'u' | 'm' | 'c' | 'd'
                   | 'D' | 'h' | 'k' | 'M' | 'G' | 'T' | 'P' | 'X' | 'Z' | 'Y'
                  )
                  | ('E' ['+'|'-'] <integer>)
<digit>         = '0' | ... | '9'
```

Remarks:

Whitespace separates syntax elements and has not been integrated into the syntax graph. Blanks and tabs are permitted.

Comments are defined as in C/C++ and are not shown in the syntax graph.

Forward references are generally not supported. This is why result definitions should appear at the end of the file; they typically refer to nodes, which need to be already defined. An exception are node references made in the control inputs of voltage controlled sources; the referenced nodes may be defined later by being connected to actual devices.

`<deviceDef>`: The number of nodes depends:

- Most devices have two connectors. They specify two nodes (2 times `<node>`), which they are connected to
- The case having 3 times `<node>` is for operational amplifiers only
- The case having 4 times `<node>` is for the voltage controlled sources. The first two nodes indicate where the source is connected and the second pair (`<control>`) defines the controlling voltage potential difference
- The case having 2 times `<node>` followed by a device name (`<control>`) is for the current controlled sources. The device name references an already defined current probe element (type `PI`). The current through this element is the control current of the source

`<deviceRef>`: `<rationalNum>` is a quotient of two positive integer numbers in the range [1..999]. For clarity, the quotient may be enclosed in parenthesis.

Devices of type `U`, `I`, `OP` and `PI` can't have a value and thus don't have a `<relation>`.

Explanation of the specification of the desired computation results:

`<voltageDef>`: Definition of any voltage in the network as potential difference between two nodes. The voltage gets a name.

`<resultDef>`: Definition of a full result. Such a result means the formula how a number of unknowns is composed from all knowns. Named voltages according to `<voltageDef>` and/or any unknown introduced and named by the LES creator can be referenced. The generated Octave script code will contain a LTI MIMO transfer function object, which describes the full set of dependencies between knowns and unknowns.

`<bodeResultDef>`: Definition of a frequency response plot. This plot is limited to the dependency of one quantity on one other one. The result is implemented as an LTI SISO transfer function object in the generated Octave script.

`<plotInfo>`: The first number is the number of plotted points, in total for linear frequency axes or per decade for logarithmic axes. The following pair of numbers is the frequency range.

Appendix B

Math

B.1 The elimination method

The basic principle of the linear equation system solver is the Gaussian elimination method. This algorithm replaces a row of coefficients of the LES with a linear combination of this row and a reference row such that the leading coefficient of the row becomes a null. Doing this with two nested loops in an systematic way leads to a triangular matrix of coefficients, that still represents an equivalent LES and the solution of the last unknown is already figured out. (The others need additional, similar operations.) Normally, this algorithm is applied to numerals, which form an algebraic field. If so, the linear combination of the two rows is the sum of the reference row times a numeric factor and the row under progress. To compute this factor the division needs to be defined on the coefficients and this requires field characteristics.

The symbolic solver operates on a simpler algebraic structure, a ring, which only knows sum and product but not the division; a prominent example of this structure are integer numbers.

If we want to apply the Gaussian method to coefficients of a ring than we have to use another linear combination: Both combined rows are multiplied crosswise with the leading coefficient of the other row and these two products are then subtracted. Replacing the row under operation with this linear combination still fulfills the requirement of shaping null values in a controlled way and we indeed end up with a triangular matrix and a solution for the last unknown (and could proceed to figure out the others as in the standard case).

The problem we face is of purely practical nature: The absolute value of the coefficients increases in strong exponential order; for our class of coefficients this would mean an exploding increase of both memory and computation time. The proposed way out is a known divisor d; given a is the coefficient resulting from the previously explained linear combination then it holds $a = d\tilde{a}$, where d and $\tilde{a}$ are existing instances of the class of coefficients. Since d is identical for all coefficients of a row we can modify the elimination rule: We don't replace the row under progress with the results a of the linear combination so far but with the $\tilde{a}$.

The known divisor d is easily computed: it can be found as the last recently computed coefficient on the diagonal of the matrix and it is one in the very first iteration.

Some proves are given in this appendix: The main prove confirms the existance of the factorization $a = d\tilde{a}$ with the proposed d, which is followed by the proves of particular characteristics of the coefficients in the given (physical) context, characteristics that strongly determine the chosen implementation. Finally the correctness and finiteness of the algorithm that implements the factorization is proven – either is not apparent from the source code.

Prior to the proves we give an example of the proposed elimination method.

B.1.1 Example: Elimination with integer numbers

Be the following LES given:

$$\left(\begin{array}{ccccc|c} \text{-7} & 3 & 5 & 7 & \text{-6} & 1 \\ 1 & 0 & \text{-2} & \text{-5} & 3 & 0 \\ 0 & 2 & 0 & \text{-1} & 13 & 0 \\ \text{-3} & 3 & \text{-7} & \text{-11} & 8 & \text{-2} \\ \text{-1} & \text{-2} & 5 & \text{-3} & 7 & 0 \end{array}\right) \tag{B.1}$$

1st Elimination step, the reference row is the first one, it is combined with rows $2 \ldots 5$ and the known divisor is $d = 1$. The reference row itself is not touched. This yields:

$$\left(\begin{array}{ccccc|c} \text{-7} & 3 & 5 & 7 & \text{-6} & 1 \\ 0 & \text{-3} & 9 & 28 & \text{-15} & \text{-1} \\ 0 & 0 & 126 & 371 & 63 & \text{-14} \\ 0 & \text{-12} & 64 & 98 & \text{-74} & 17 \\ 0 & 17 & \text{-30} & 28 & \text{-55} & 1 \end{array}\right)$$

2nd elimination step, the reference row is the second one, it is combined with rows $3 \ldots 5$. The known divisor is the diagonal coefficient of the 1st elimination step, $d = -7$. This yields:

$$\left(\begin{array}{ccccc|c} \text{-7} & 3 & 5 & 7 & \text{-6} & 1 \\ 0 & \text{-3} & 9 & 28 & \text{-15} & \text{-1} \\ 0 & 0 & \text{126/-7} & \text{371/-7} & \text{63/-7} & \text{-14/-7} \\ 0 & 0 & \text{-84/-7} & \text{42/-7} & \text{42/-7} & \text{-63/-7} \\ 0 & 0 & \text{-63/-7} & \text{-560/-7} & \text{420/-7} & \text{14/-7} \end{array}\right) =$$

$$\left(\begin{array}{ccccc|c} \text{-7} & 3 & 5 & 7 & \text{-6} & 1 \\ 0 & \text{-3} & 9 & 28 & \text{-15} & \text{-1} \\ 0 & 0 & \text{-18} & \text{-53} & \text{-9} & 2 \\ 0 & 0 & 12 & \text{-6} & \text{-6} & 9 \\ 0 & 0 & 9 & 80 & \text{-60} & \text{-2} \end{array}\right)$$

3rd elimination step, the reference row is the third one, it is combined with rows 4 and 5. The known divisor is the diagonal coefficient of the second elimination step, $d = -3$. This yields:

$$\left(\begin{array}{ccccc|c} \text{-7} & 3 & 5 & 7 & \text{-6} & 1 \\ 0 & \text{-3} & 9 & 28 & \text{-15} & \text{-1} \\ 0 & 0 & \text{-18} & \text{-53} & \text{-9} & 2 \\ 0 & 0 & 0 & \text{744/-3} & \text{216/-3} & \text{-186/-3} \\ 0 & 0 & 0 & \text{-963/-3} & \text{1161/-3} & \text{18/-3} \end{array}\right) =$$

$$\left(\begin{array}{ccccc|c} \text{-7} & 3 & 5 & 7 & \text{-6} & 1 \\ 0 & \text{-3} & 9 & 28 & \text{-15} & \text{-1} \\ 0 & 0 & \text{-18} & \text{-53} & \text{-9} & 2 \\ 0 & 0 & 0 & \text{-248} & \text{-72} & 62 \\ 0 & 0 & 0 & 321 & \text{-387} & \text{-6} \end{array}\right)$$

Last elimination step, the reference row is the fourth one, it is combined with row 5. The known divisor is the diagonal coefficient of the third elimination step, $d = -18$. This yields:

$$\left(\begin{array}{ccccc|c} \text{-7} & 3 & 5 & 7 & \text{-6} & 1 \\ 0 & \text{-3} & 9 & 28 & \text{-15} & \text{-1} \\ 0 & 0 & \text{-18} & \text{-53} & \text{-9} & 2 \\ 0 & 0 & 0 & \text{-248} & \text{-72} & 62 \\ 0 & 0 & 0 & 0 & \text{119088/-18=-6616} & \text{-18414/-18=1023} \end{array}\right)$$

The solution for the last unknown is $1023/-6616$. The coefficients of the last row of the last elimination step designate the determinants of the LES (i.e. the main determinant and the one related to the unknown according to Cramer's rule):

$$\begin{vmatrix} -7 & 3 & 5 & 7 & -6 \\ 1 & 0 & -2 & -5 & 3 \\ 0 & 2 & 0 & -1 & 13 \\ -3 & 3 & -7 & -11 & 8 \\ -1 & -2 & 5 & -3 & 7 \end{vmatrix} = -6616$$

$$\begin{vmatrix} -7 & 3 & 5 & 7 & 1 \\ 1 & 0 & -2 & -5 & 0 \\ 0 & 2 & 0 & -1 & 0 \\ -3 & 3 & -7 & -11 & -2 \\ -1 & -2 & 5 & -3 & 0 \end{vmatrix} = 1023$$

B.1.2 Prove of statement about known divisor

The prove of the statement that we can find a common divisor in each elimination step as the diagonal coefficient of the previous elimination step is based on some algebraic transformations, which will show that after the elimination step each coefficient has the meaning of the determinant of a sub-matrix of the LES. Due to the Laplace expansion of a determinant, which computes the determinant only with products and sums, this must be an existing element of the class of coefficients.

Definition: Be $M_{mn}^{[k]}$ the sub-matrix of M, which has the dimension $k \times k$ and the coefficients a_{ij} of M at $i = 1..k-1, m$ and $j = 1..k-1, n$ with $m, n \geq k$. (B.2)

$M_{mn}^{[k]}$ is a square matrix. It inherits the upper-left square of M and has one additional row that has the coefficients of row m of M and one additional column that has the coefficients of row n of M.

Examples: If M is the matrix (B.1) from the example in section B.1.1 then we have $M_{14}^{[1]} = 7$,

$$M_{23}^{[2]} = \begin{pmatrix} -7 & 5 \\ 1 & -2 \end{pmatrix}$$

or

$$M_{46}^{[4]} = \begin{pmatrix} -7 & 3 & 5 & 1 \\ 1 & 0 & -2 & 0 \\ 0 & 2 & 0 & 0 \\ -3 & 3 & -7 & -2 \end{pmatrix}$$

Lemma: Be M the representation of a LES of dimension $p \times q$ with $q \geq p$ and $M^{(k)}$ with $k = 1 \ldots p-1$ the result of elimination step k of the proposed elimination method and $a_{mn}^{(k)}$ the coefficient of $M^{(k)}$ at position mn. It holds: (B.3)

$$a_{mn}^{(k)} = \left| M_{mn}^{[k+1]} \right| \qquad \text{with } m, n > k$$

Lemma (B.3) is what we still have to prove: If we know that the resulting coefficients are determinants of sub-matrices of the original, the LES describing matrix M then it must be existing valid instances from the class of coefficients and the factorization was possible.

The prove of lemma (B.3) is given for a particular coefficient a_{ij} of the LES. We have to consider the position of the coefficient in the matrix and and the elimination step. The elimination step is denoted by the top index in parenthesis. The original LES is represented by matrix $M^{(0)}$. (Elimination step 0 means prior to the first elimination step. $M^{(0)}$ is also simply written as M – the explicit indication of elimination

step 0 is mostly omitted.) The lemma relates the coefficient $a_{ij}^{(k)}$ that is the result of elimination step k with the determinant of a particular, square sub-matrix of $M^{(0)}$. The idea of the prove is to look at the coefficients of $M^{(0\dots k)}$ at the positions that belong to this sub-matrix and to keep track what the operations of the eliminations steps mean to the value of the determinant of the sub-matrix. This leads to a chained equation, which finally proves the lemma. (Opposed to the relation of the $M^{(0\dots k)}$, which is an equivalence of LESs.) This looking at the coefficients of the sub-matrix means to reduce the prove for any elimination step of any rectangular matrix to the final elimination result of a square matrix.

B.1.3 Reducing the problem to square matrices

It is essential that all the sub-matrices $M_{ij}^{[1\dots n](0)}$ in question are square and that they share the upper-left square area of M. All the factors of the linear combinations of rows to be computed are derived only from the coefficients found in this shared area and therefore the linear combinations made on the rows of the LES are always the same as one would have to use to perform the elimination of only the sub-matrix or of its determinant. This is now illustrated with an example:

Be M the matrix, which represents a LES with 5 unknowns and 2 knowns:

$$M = \left(\begin{array}{ccccc:cc} a_{11} & a_{12} & a_{13} & a_{14} & a_{15} & a_{16} & a_{17} \\ a_{21} & a_{22} & a_{23} & a_{24} & a_{25} & a_{26} & a_{27} \\ a_{31} & a_{32} & a_{33} & a_{34} & a_{35} & a_{36} & a_{37} \\ a_{41} & a_{42} & a_{43} & a_{44} & a_{45} & a_{46} & a_{47} \\ a_{51} & a_{52} & a_{53} & a_{54} & a_{55} & a_{56} & a_{57} \end{array}\right) \tag{B.4}$$

Let's take the coefficient at row 5, column 4: The lemma says

$$a_{54}^{(1)} = \left|M_{54}^{[2]}\right| = \begin{vmatrix} a_{11} & a_{14} \\ a_{51} & a_{54} \end{vmatrix}$$

When the first iteration of the elimination of the matrix M reaches the last row it replaces:

$$R_5^{(1)} = a_{11}R_5^{(0)} - a_{51}R_1^{(0)},$$

which is – for the affected four coefficients shared between M and $M_{54}^{[2]}$ – exactly the same operation as an elimination of $M_{54}^{[2]}$ would be:

$$M_{54}^{[2](1)} = \begin{pmatrix} a_{11} & a_{14} \\ 0 & a_{11}a_{54} - a_{51}a_{14} \end{pmatrix}$$

All other operations of the first iteration of the elimination on M affect rows and columns outside of $M_{54}^{[2]}$ and have no impact on any of the coefficients of $M_{54}^{[2](1)}$.

Or let's take the coefficient at row 4, column 6: The lemma says

$$a_{46}^{(2)} = \left|M_{46}^{[3]}\right| = \begin{vmatrix} a_{11} & a_{12} & a_{16} \\ a_{21} & a_{22} & a_{26} \\ a_{41} & a_{42} & a_{46} \end{vmatrix}$$

(Please note, that this example refers to a coefficient, which has not the final value of the complete elimination, there will also be the value $a_{46}^{(3)}$ during further processing.)

In the first elimination step and when the process reaches rows 2 and 4 then shared coefficients are touched. The factors of the linear combination of rows ((a_{11}, a_{21}) and (a_{11}, a_{41}) for rows 2 and 4, respectively) are located at according positions in M and $M_{46}^{[3]}$ (i.e. in the first column) and thus the elimination of M is exactly the same operation as seen in its sub-matrix $M_{46}^{[3]}$. For both matrices the identical row replacements

$$\begin{aligned} R_2^{(1)} &= a_{11}R_2^{(0)} - a_{21}R_1^{(0)} \\ R_4^{(1)} &= a_{11}R_4^{(0)} - a_{21}R_4^{(0)} \end{aligned}$$

take place (using the same global position indices in the matrix representations).

If we look at the second elimination step it's just the same: the operation performed on the rows of matrix $M^{(1)}$ is

$$R_i^{(2)} = a_{22}^{(1)} R_i^{(1)} - a_{i2}^{(1)} R_2^{(1)} \qquad \text{with } i = 3 \dots 5$$

Coefficients, which are shared with the sub-matrix are affected only for $i = 4$ and then we see the same operation on the coefficients of the sub-matrix that would be used to run the second elimination step directly on the isolated sub-matrix. The required factors $(a_{22}^{(1)}, a_{42}^{(1)})$ of the linear combination are found at the appropriate position in the sub-matrix.

This idea holds in general for all sub-matrices in question because they all share their $k-1$ top-most rows and left-most columns with the original matrix M and the factors of all the linear combinations are taken from these rows and columns - be it the original coefficients or the products of the $k-2$ involved, first elimination steps.

This is why we can now reduce the prove to the elimination of the square sub-matrices. We have to show that the final value of the bottom-right coefficient in the square area is identical to the determinant of the original sub-matrix. This result for a particular but arbitrary coefficients can then be applied to any coefficient computed in any elimination step.

But before we can start with the prove we need to introduce a minor lemma, which is applied in the following.

B.1.4 Linear combination of rows of determinant

For the following steps we need to understand how the value of the determinant of a square matrix is changed if a row of the matrix is replaced by the linear combination of itself and another row.

Lemma: Be M a square matrix and $\hat{M}$ the same matrix, where only row R_j has been replaced with the linear combination $c_j R_j + c_i R_i$ of this row and the other row R_i. It holds: $|\hat{M}| = c_j |M|$. (B.5)

To prove lemma (B.5) we just need to expand $|\hat{M}|$ along its row j:

$$\begin{aligned} \left|\hat{M}\right| &= \begin{vmatrix} a_{11} & \dots & a_{1n} \\ \vdots & & \vdots \\ a_{i1} & \dots & a_{in} \\ \vdots & & \vdots \\ c_j a_{j1} + c_i a_{i1} & \dots & c_j a_{jn} + c_i a_{in} \\ \vdots & & \vdots \\ a_{n1} & \dots & a_{nn} \end{vmatrix} \\ &= \sum_{c=1}^{n} (-1)^{i+c} (c_j a_{jc} + c_i a_{ic}) D_{jc} \qquad \text{with } D_{ij} \text{ being the sub-determinant at position } ij \\ &= c_j \sum_{c=1}^{n} (-1)^{i+c} a_{jc} D_{jc} + c_i \sum_{c=1}^{n} (-1)^{i+c} a_{ic} D_{jc} \qquad \text{(B.6)} \\ &= c_j \, |M| + c_i \cdot 0 \end{aligned}$$

Both determinants in lemma (B.5), $|M|$ and $|\hat{M}|$, share all rows but row j, hence all sub-determinants $D_{jc}, c = 1 \dots n$ are identical for both. Therefore the sum in the first addend in (B.6) is the Laplace expansion of the determinant of M. The sum in the second addend in (B.6) is null; it is the Laplace expansion of a similar determinant: all rows but j are identical but row j is replaced by row i. The determinant of a matrix with two identical thus linear dependent rows is null. Equation (B.6) reduces to lemma (B.5).

B.1.5 Prove for square sub-matrix

From now on we take the view at the sub-matrix only. To simplify the explanations we change the position indices of the coefficients. We are going to use consecutive one based indices as usual. The positional relationship with the matrix of the LES might be obscured but is out of scope in this section anyway.

We investigate the elimination of an arbitrary square sub-matrix S according to definition (B.2) of M, the matrix describing the original LES:

$$\begin{aligned} S^{(0)} &= M_{ij}^{[n](0)} \qquad \text{with } n > 1, i \geq n, j \geq n \\ &= \begin{pmatrix} a_{11} & \dots & a_{1n} \\ \vdots & \ddots & \vdots \\ a_{n1} & \dots & a_{nn} \end{pmatrix} \qquad \text{(here using the renamed position indices)} \end{aligned}$$

and define D to be its determinant:

$$\begin{aligned} D &:= \left| S^{(0)} \right| \\ &= \begin{vmatrix} a_{11} & \dots & a_{1n} \\ \vdots & \ddots & \vdots \\ a_{n1} & \dots & a_{nn} \end{vmatrix} \end{aligned}$$

Please note that due to the redefinition of the position indices a_{ij} no longer is the coefficient of M in row i and column j if either $i = n$ or $j = n$.

After the first elimination step of the LES – and according to section B.1.3 the first elimination step of S – the matrix and its determinant change as follows:

$$\begin{aligned} S^{(1)} &= \begin{pmatrix} a_{11} & \dots & & a_{1n} \\ 0 & a_{22}^{(1)} & \dots & a_{2n}^{(1)} \\ \vdots & \vdots & \ddots & \vdots \\ 0 & a_{n2}^{(1)} & \dots & a_{nn}^{(1)} \end{pmatrix} \\ D^{(1)} &:= \left| S^{(1)} \right| \\ &= \frac{(a_{11})^{(n-1)}}{1^{(n-1)}} \left| S(0) \right| \\ &= \frac{(a_{11})^{(n-1)}}{1^{(n-1)}} D \end{aligned} \tag{B.7}$$

The factor in front of D in equation (B.7) is explained by the $n - 1$ row operations that form the elimination step: The rows $2 \dots n$ are all multiplied by a_{11} as part of the linear combination with the first row, which explains according to lemma (B.5) the numerator of the factor. Then the rows are all

divided by the common divisor, which is one in the first elimination step. This explains the denominator of the factor.

If we now present the second elimination step, too, then a general pattern becomes apparent, which gives us the wanted result for any size of matrix and its determinant:

$$S^{(2)} = \begin{pmatrix} a_{11} & \dots & & & a_{1n} \\ 0 & a_{22}^{(1)} & \dots & & a_{2n}^{(1)} \\ 0 & 0 & a_{33}^{(2)} & \dots & a_{3n}^{(2)} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & a_{n3}^{(2)} & \dots & a_{nn}^{(2)} \end{pmatrix} \quad \text{with } a_{ij}^{(2)} = \frac{a_{22}^{(1)} a_{ij}^{(1)} - a_{i2}^{(1)} a_{2j}^{(1)}}{a_{11}} \tag{B.8}$$

$$\begin{aligned} D^{(2)} &= \frac{\left(a_{22}^{(1)}\right)^{(n-2)}}{(a_{11})^{(n-2)}} D^{(1)} \\ &= \frac{\left(a_{22}^{(1)}\right)^{(n-2)}}{(a_{11})^{(n-2)}} \frac{(a_{11})^{(n-1)}}{1^{(n-1)}} D \\ &= (a_{22}^{(1)})^{(n-2)} a_{11} D \end{aligned} \tag{B.9}$$

The pattern: In elimination step k we multiply $n-k$ rows with the value $v = a_{kk}^{(k-1)}$, which temporarily and according to lemma (B.5) increases the determinant of the modified matrix by v^{n-k} but in the next elimination step $k+1$, when $n-k-1$ rows are processed the same value v is used as common divisor, which decreases the determinant again by v^{n-k-1} – and a simple v to the power of one remains as additional factor in the equation between $D^{(k)}$ and D. This leads to the final elimination result, the result of elimination step $n-1$:

$$\begin{aligned} S^{(n-1)} &= \begin{pmatrix} a_{11} & \dots & & & a_{1n} \\ 0 & a_{22}^{(1)} & \dots & & a_{2n}^{(1)} \\ 0 & 0 & a_{33}^{(2)} & \dots & a_{3n}^{(2)} \\ \vdots & \vdots & & \ddots & \vdots \\ 0 & 0 & \dots & & a_{nn}^{(n-1)} \end{pmatrix} \\ D^{(n-1)} &= \prod_{i=1}^{n-1} a_{ii}^{(i-1)} D \end{aligned} \tag{B.10}$$

$S^{(n-1)}$ is a triangular matrix. Its determinant is the product of all the diagonal elements. This product can be equated with (B.10). Please note, that the expressions in parenthesis denote the elimination step not a power:

$$\left|S^{(n-1)}\right| = \prod_{i=1}^{n} a_{ii}^{(i-1)} = D^{(n-1)} = \prod_{i=1}^{n-1} a_{ii}^{(i-1)} D$$

or

$$a_{nn}^{(n-1)} = D \tag{B.11}$$

This is what we wanted to prove; the last diagonal element after the last elimination step has the value D of the determinant of the original matrix S.

The prove holds for all $n > 1$ and any coefficient.[1] All coefficients at any position and after any elimination step of the matrix M, which represents the LES have the meaning of the determinant of the one or other sub-matrix of the *original* matrix M.

All the appearing determinants and thus all ever appearing coefficients can be represented as a sum of products of original coefficients. This defines the arithmetic class of the coefficients and their implementation with respect to storage and needed operations. To perform the elimination step we need to conduct the linear combination of a pair of coefficients (the linear combination of rows as used above is broken down to coefficients via an iteration along the rows), followed by the factorization with respect to the known divisor. Both together is called the "elementary step" in the implementation. The elementary step can be further broken down into the operations product, sum and factorization. A real division is not required.

In the particular use case of linNet the original coefficients are derived from physical facts that bear specific properties, which further refines the definition of their arithmetic class; this is explained in section B.2 and the following.

B.1.6 Pivoting

The linear combination used in an elimination step must never multiply the destination row with null; all the information of the related equation would be discarded and the final solution could not be figured out. Therefore, an implementation of the Gauss elimination method uses pivoting and so does linNet's extended method. If it holds $a_{kk}^{(k-1)} = 0$ for the diagonal coefficient at the beginning of elimination step k then the algorithm looks for a coefficient $a_{ck}^{(k-1)} \neq 0$ with $c > k$. If there is such a coefficient then the related row is swapped with row k. If there is no such coefficient then the elimination has completed with the result that there is no unambiguous solution for the unknowns of the LES. Because of the immediate abortion of the elimination the latter case has no impact on our proves. But even the former case is irrelevant because the instant row swapping "in case" is identical to the same (hypothetic) swapping of rows prior to the complete elimination process. "Identical" means that all chosen linear combinations, all common divisors and all resulting coefficients in all elimination steps would be just the same. And since the proves hold for any LES and matrix it also holds for the (hypothetic) LES and matrix with rows swapped prior to the elimination.

B.1.7 Computational effort

The order of complexity of the computation has not been investigated in depth. Obviously, as we use a modified Gauss elimination, it is $O(n^3)$ in terms of elementary steps to bring the matrix in triangular shape. However, different to the Gauss elimination method there's no inverse elimination to bring it into diagonal form. The proposed method requires n-fold elimination for the n unknowns, which means a complexity of $O(n^4)$ in terms of elementary steps for a full solution of the LES.

In the linNet use case the latter is irrelevant as the number of unknowns of interest, i.e. the really figured out unknowns, does typically not depend on the size of the circuit or the total number of unknowns; for this use case with a rather constant number of computed unknowns $O(n^3)$ still holds.

The bigger problem is the elementary step itself. It is the atomic operation, that combines a four-tuple of coefficients to a new coefficient. This operation is far from constant; the more the algorithm advances the longer the coefficients become and the longer this operation takes. Finally this operation yields the wanted result. In theory an upper boundary of this operation would be $O(n!)$; in a fully interconnected network, where each node is connected to each other node we will need to have $n!$ independent terms in the resulting expressions and these will need an according amount of time to be figured out. For practical relevant use cases full interconnectivity won't ever occur, sparse matrices are typical. Which makes it better but the order will still be far from good.

A possible approach is the simplification that a network has a (more or less) constant number c of interconnections (i.e. devices) between its nodes. If we furthermore disregard terms, which eliminate

[1] And for $n = 1$ by definition, too, if we consider the original matrix the result of the "0-th" elimination step.

each other in the computed products of coefficients then the lengths of the coefficients were taken to the power of two in each elimination step and when using the unmodified Gauss elimination with linear combinations of rows. The length at the beginning is c. Repeatedly increasing this length leads to the complexity of $O(c^{(2^{n-1})})$.

Due to the applied common divisor the proposed elimination method significantly reduces the growth in length of the coefficients. Across the eliminations steps $k = 1 \ldots (n-1)$ the length will now grow like $c^2, c^3 = \frac{(c^2)^2}{c}, c^4 = \frac{(c^3)^2}{c^2}, \ldots$ This leads to a complexity of still $O(c^n)$.

Combining these considerations with the complexity of the elimination pattern, i.e. considering the quadraticly decreasing number of affected coefficients, could be tried to refine the result but the complexity of the elementary step will anyway be dominant.

The improvement of the proposed factorization is illustrated by a numeric example. If we put $c = 3$ and $n = 8$ then $O(c^{(2^{n-1})})$ means about 10^{61} units of effort and $O(c^n)$ only about 6500 units, units however, which are significantly more expensive in terms of computational effort due to the factorization.

The lengths of the coefficients and the computational effort for the elementary steps are closely related as linear lists are applied. The order of effort in terms of memory usage will be the same as for computational effort. The main impact on the computational effort is the growing length of the coefficients and the length of the coefficients is proportional to the memory consumption.

Since the growth in length of the coefficients seems to have the overwhelming impact on memory and computational effort a possible optimization idea is an extended pivoting. Rather than looking for the first non null coefficient we could look for the row with the shortest coefficients. (If the longer rows are processed later than far less coefficients are affected.) However this kind of pivoting would in turn bring a significant additional complexity as it in turn depends on all (remaining) coefficients and their lengths. The next problem would be the definition of "row with the shortest coefficients": which norm to be applied; the average length is not necessarily the right choice.

One has to see that the complexity of the duration of the computation is in line with the complexity of the eventually yielded result: Given you were very patient and would wait a number of years for the result then you'd probably be frustrated by this result. Kilometers of paper to print the formula and no practical use at all. As a matter of experience we can state that the analysis of circuits that have meaningful and still manageable transfer functions is uncritical with respect to computational effort.

In this sense the theoretically poor order of the algorithm is harmless with respect to the imaginable practical use cases of the software.

B.1.8 Common divisors in final solution

The proposed elimination does not guarantee that the found solution (i.e. the found numerator and denominator for the right-most unknown) is free of common divisors. It can only guarantee that it won't introduce additional common divisors as the unmodified Gaussian elimination without the factorization would. If the method is e.g. applied to a matrix of integers, where all coefficients of a row or column have a common divisor then this common divisor will be propagated into the final result.

In our application of the method the initial coefficients of the matrix describe independent and unrelated physical devices. The elimination method itself doesn't introduce common divisors, which means that the final result should always be fully canceled.

For explanatory purpose it has to be mentioned that the relations between the values of devices of same kind, which can be stated in the netlist file are completely irrelevant to the elimination process of the LES. These relations are simply not known by the solver. They are applied only in the final rendering of the found solution and respecting the wanted absence of common divisors in the rendered result becomes a simple matter of traditional computation of the greatest common divisor of all affected numeric factors.

B.2 Prove of non quadratic coefficients

The elimination method as pointed out in section B.1 will work with any kind of objects of kind ring. The implementation of the solver will however strongly depend on the specific kind of object. The basic operations store, add and multiply and the factorization $a = d\tilde{a}$ need to be implemented. Our kind of objects, i.e. our kind of matrix coefficients, has some particular characteristics, which are strongly exploited in the implementation.

Section 5 on page 28 explains how the LES is set up. The modeling of the electronic devices determines how the coefficients of the LES look like (prior to the elimination). Summarizing this section we can state: At the beginning, all coefficients are sums of symbols and/or the number one. The addends of these sums may have both signs. No products of symbols appear and in particular no symbol is taken to a power other than one. No two addends of a coefficient are identical, so there's no possibility to agglomerate symbols leading to a product of a symbol and a number other than ± 1.

During the elimination steps the initial coefficients are multiplied and added and factorized. The resulting coefficients will still be sums. The addends will now either be products of different symbols and ± 1 or the number ± 1. The further characteristics of the initial coefficients (symbols taken to the power of one, no identical addends leading to products of symbols and a number other than ± 1) are retained. The coefficients can thus be represented by a list of signed products of symbols. Each known symbol is either present in the product (power of one) or not (power of null). The product can be represented by a bit vector, where each bit is related to one known symbol. The sign of an addend could be represented by a Boolean, but actually isn't: The numeric factor in an addend is ± 1 for a coefficient prior to and after each elimination step but can take (small) integer values in intermediate computation results. To avoid different representations of expressions during the computation and in the computation result we use integer numbers in general to represent the numeric factor.

While the characteristics of the initial coefficients result from the modelling of the devices in the setup phase of the LES is the preservation of these characteristics during the elimination far from self-explanatory. This one and the next sections give the required proves. In the first step it is proven that the symbols in the products in the addends will never appear with a power other than one or null.

Lemma: The value of the determinant of any of the sub-matrices of the LES according to definition (B.2) can be brought into the form of a sum of products of a numeric factor f and all the device symbols taken to a power of either null or one. In this form no two addends of the sum have the same set of symbol powers. (B.12)

Since all in the course of the elimination ever appearing coefficients have the value of one of these subdeterminants the same statement holds for all coefficients of the matrix after all the elimination steps.

The device symbols are the given R, L, C, etc. They appear in the addends of the coefficients (power = 1) or they don't appear (power = 0) but – and this is the gist of the lemma – they will never appear with a negative power or with a power of two or higher. To put it in an example: $R_1R_2C + R_1L$ could be a coefficient but $R_1R_1C + R_1L$ or $\frac{R_1}{R_2}C + R_1L$ are expressions, which will definitely never appear as a coefficient of the matrix after an elimination step.

The importance of the lemma is evident: The addends of the coefficients can be implemented as bit vectors and their products become cheap bit-wise integer operations.

The prove of lemma (B.12) exploits that there are symmetries of the symbols in the LES. Any symbol s always appears in a characteristic pattern:

$$\begin{pmatrix} \ddots & & & & \\ & \pm s & \dots & \mp s & \\ & \vdots & & \vdots & \\ & \mp s & \dots & \pm s & \\ & & & & \ddots \end{pmatrix} \tag{B.13}$$

The explanation of the pattern is the meaning of symbol s: It designates the current flow through device s from one node of the network to another one[2]; and the current is the same at both ends. The sign of a current is defined from the node's perspective: Effluent is a negative current influent is positive by definition. A row of the LES is the current balance of a node, the columns are related to the unknown node's voltage potentials. An increase of the own potential will lead to an affluent current (thus negative sign), whereas an increase of the potential of the far end's node will lead to an influent current with positive sign. This leads to the pair $(+s, -s)$ in a row of the LES. The other row containing the inverse pair results accordingly from the current balance of the far end's node. Here the pair appears inverse as we take the perspective of the other node.

Two exceptions have to be considered. We always have a ground node, which no current balance is made for, and s might be connected to this node. In which case one of the two rows would disappear. Furthermore, the ground node has null potential by definition, there's no related unknown or column. One of the two columns disappears. s appears only once in the LES with negative sign.

The controlled sources introduce coefficients s which also appear in pairs, but here we have the additional freedom that the source is connected two one or two non ground nodes and the control input is connected to one or two non ground nodes. Consequently, the disappearing row and the disappearing column can happen independently resulting in either two pairs, or a vertical pair only or a horizontal pair only or a single appearance of s (the latter here with positive sign). Summarizing we can observe only the following patterns of appearance of s in the LES (simplified representation, disregarding other symbols and coefficients):

$$(\pm s) \quad \begin{pmatrix} \pm s & \mp s \end{pmatrix} \quad \begin{pmatrix} \pm s \\ \mp s \end{pmatrix} \quad \begin{pmatrix} \pm s & \mp s \\ \mp s & \pm s \end{pmatrix} \tag{B.14}$$

It has been proven that all appearing coefficients have the meaning of the determinant of a sub-matrice of the LES, see lemma (B.3). All such sub-matrices retain the same structural characteristics of the appearance of s; by extracting a subdeterminant from the matrix a row or a column with s might disappear or s might completely disappear, but we will never see a new pattern for the remaining symbols in the subdeterminant.

According to lemma (B.5) the value of a determinant is not changed if we replace a row or column by the sum of this and another row or column, respectively. Applying this operation first to the two columns that contain the symbol and then to the two rows that contain the symbol, we can always reduce all possible patterns to the simple pattern of having the symbol just once in the determinant. (Obviously, it doesn't matter if we skip one or both of theses steps in case s appears in one of the simpler patterns.)

The coefficients of the LES are sums of differing symbols. A coefficient can thus be written as the symbol of interest, s, plus a remainder, an arbitrary sum of *other* symbols: $a_{ij} = \pm s + R_{ij}$. We can modify the determinant of the sub-matrix as follows:

$$\begin{vmatrix} \ddots & & & \\ & \pm s + R_{iu} & \dots & \mp s + R_{iv} & \\ & \vdots & & \vdots & \\ & \mp s + R_{ju} & \dots & \pm s + R_{jv} & \\ & & & & \ddots \end{vmatrix} = \dots$$

[2]In the internal representation of the network all devices have the meaning of conductance, i.e. the complex proportionality factor between current and voltage.

$$\begin{vmatrix} \ddots & & & \\ & \pm s + R_{iu} & \dots & R_{iu} + R_{iv} \\ & \vdots & & \vdots \\ & \mp s + R_{ju} & \dots & R_{ju} + R_{jv} \\ & & & \ddots \end{vmatrix} = \dots$$

$$\begin{vmatrix} \ddots & & & \\ & \pm s + R_{iu} & \dots & R_{iu} + R_{iv} \\ & \vdots & & \vdots \\ & R_{iu} + R_{ju} & \dots & R_{iu} + R_{iv} + R_{ju} + R_{jv} \\ & & & \ddots \end{vmatrix} \tag{B.15}$$

The modifications of the determinant are shown for the case if s appears in the most complex appearance pattern. For the simpler patterns less modification steps are required but the final form of Equation (B.15) with a single s will be the same.

Only one statement about the R_{ij} is important in this context: They do not contain any appearance of symbol s.

To compute the value of the determinant, we perform the Laplace expansion across row i, which still contains the one and only appearance of s. If we are in column $n \neq u$ the coefficient visited in row i doesn't contain s and nor do the subdeterminants we have to multiply with. If we are in column u of row i then we have to take the coefficient $s + R_{iu}$ times a subdeterminant, which doesn't contain s. R_{iu} also has no s, thus the only term of the product containing s is s times the s-free subdeterminant, which can only lead to a term of s to the power of 1. The further expansion means to sum up all such terms; this operation doesn't create any higher power of s.

It is proven, that s can't appear in the determinant of any sub-matrix with a power higher than 1; it might not appear at all, which means power of 0. Since s was an arbitrary chosen symbol out of the set of all symbols the prove holds for all the symbols.

B.3 Prove of numeric factor one in coefficients' addends

According to lemmas (B.3) and (B.12) all coefficients possibly appearing as result of an elimination step can be represented as a sum of products of a numeric factor and a subset of the device symbols. Here, we will give the prove that this factor can only have the values plus and minus one. We show, that the expansion of the determinant of any of the sub-matrices of the LES only has addends, which differ (i.e. differ in the tuple of multiplied symbols). Since all addends (whose sum is the value of the determinant) are different in this sense, no pair of two of those can be gathered together, which would yield a numeric factor other than absolute one.

Lemma: The numeric factor f mentioned in lemma (B.12) has a value of either one or minus one. (B.16)

In the first step, this is shown for determinants of sub-matrices solely containing null values and sums of symbols in the possible appearance patterns according to equation (B.14).

The prove is recursive, we start with a determinant of dimension two. An example of such a subdeterminant is given as illustration. It contains five symbols, using all possible appearance patterns at least once:

$$\begin{vmatrix} s + a - b - d & -s + b + d \\ -s - a + d & s + c - d \end{vmatrix}$$

We will show for the particular symbol s that there will be no two identical terms in the expansion of the determinant, which contain s. First we replace the other symbols from the example with a general form, the remainders R_{ij}, the sums of all other symbols, and bring the determinant in the equivalent form with identic value but a single appearance of s only; the same has been done in detail in section B.2:

$$\begin{vmatrix} \pm s + R_{11} & R_{11} + R_{i2} \\ R_{11} + R_{21} & R_{11} + R_{12} + R_{21} + R_{22} \end{vmatrix}$$

If we expand this determinant it is evident that the only term that contains s is the product $\pm s(R_{11} + R_{12} + R_{21} + R_{22})$. The sum of the four remainders will not contain identical addends: It are all four initial remainders, which means that for any symbol holds that all its occurrences according to the patterns of equation (B.14) are summed up. Depending on the appearance pattern, the symbol is either eliminated or it appears just once. In the previous example we would see $s(a - b - d + b + d + (-a + d) + c - d) = sc$ but nothing like $s(a + a) = 2sa$. This is what we wanted to prove.

It's not obvious but trivial to show that the same holds if s has one of the other possible appearance patterns in the initial form of the determinant. We always end up with s times a sum of the R_{ij} as only addend containing s. The sum of the R_{ij} either is a single R_{ij} or the sum of a pair of two neighboured R_{ij}. In both cases the sum can't contain multiple occurances of any symbol since symbols appearing in both R_{ij} will eliminate each other.

Since s was an arbitrarily chosen symbol the same holds true for all other symbols as well.

The recursion from dimension $N - 1$ to N is done with the Laplace expansion of the determinant. Again, we look at the particular symbol s of the determinant of an arbitrary sub-matrix of dimension N of the LES. In the first step we do the same transformation as shown in (B.15) and get a determinant of identical value and with a single occurrence of s.

The transformation doesn't destroy the appearance pattern of any symbol $t \neq s$ in a relevant way. An example, using two symbols s and t:

$$\begin{vmatrix} \ddots & & & & & \\ & \pm s \pm t & \dots & \mp t & \dots & \mp s \\ & \vdots & & \vdots & & \vdots \\ & \mp s & \dots & & \dots & \pm s \\ & \vdots & & \vdots & & \vdots \\ & \mp t & \dots & \pm t & \dots & \\ & & & & & \ddots \end{vmatrix} = \dots$$

$$\begin{vmatrix} \ddots & & & & & \\ & \pm s \pm t & \dots & \mp t & \dots & \pm t \\ & \vdots & & \vdots & & \vdots \\ & \mp s & \dots & & \dots & \\ & \vdots & & \vdots & & \vdots \\ & \mp t & \dots & \pm t & \dots & \mp t \\ & & & & & \ddots \end{vmatrix} = \dots$$

$$\begin{vmatrix} \ddots & & & & & & \\ & \pm s \pm \pm t & \dots & \mp t & \dots & \pm t & \\ & \vdots & & \vdots & & \vdots & \\ & \pm t & \dots & \mp t & \dots & \pm t & \\ & \vdots & & \vdots & & \vdots & \\ & \mp t & \dots & \pm t & \dots & \mp t & \\ & & & & & & \ddots \end{vmatrix} \tag{B.17}$$

Although we see a new appearance pattern for t in the transformed determinant this is not relevant. In the next step we expand the determinant along the row, which still contains s. All subdeterminants can't contain s. The only terms in the result, which might contain symbol s are got, when we reach the column with s and now the subdeterminant (whose value has to be taken times s) again has the required appearance pattern for t: The additional occurrences of t do not belong to this subdeterminant. The subdeterminant exposes the assumed appearance pattern for all its symbols and symbol s doesn't belong to these symbols.

What has been shown for our example t holds in general: The two operations copy the symbols, which are found in row and column of the finally remaining s to another row and column. The symbols at the origin of this copying will never be part of the subdeterminant, so there are at maximum two new pairs of the copied symbol in the relevant subdeterminant. If the other symbol t has a simpler appearance pattern or if it shares more positions with s as in the example then there will be only one pair or even only a single appearance of t in the subdeterminant due to elimination.

The only addends in the final expansion that contain s result from the product of $\pm s$ with the value of this subdeterminant; if lemma (B.16) holds for this determinant, than its value doesn't contain any term with a numeric factor other than ± 1 and nor will the product with $\pm s$ do. In the final expansion result there are no two addends containing symbol s times a product of the same sub-set of other symbols or with other words there are no addends containing symbol s and a numeric factor other than ± 1.

As before, s is an arbitrary chosen symbol, hence these considerations hold true for all symbols.

The subdeterminant is a determinant of dimension $N - 1$ and it fulfills the assumption of the prove, the appearance pattern: The recursion is done, the same consideration can be done in turn on this subdeterminant, and so on, till we reach our anchor with dimension $N = 2$.

In the next step we need to extend the prove to some more patterns: With respect to the *symbols* everything has been said but the coefficients of the LES also have addends ± 1 or with other words addends with all symbols' power equal to null. Arbitrarily placed ones in the LES (and hence in the determinant of its sub-matrices) would immediately break our prove but fortunately the ones also have specific appearance pattern and similar considerations as for the symbols prove that lemma (B.16) still holds.[3]

B.3.1 Extension of prove to other devices

The first part of the prove has been done for matrices with symbols only. These matrices exhibit the appearance pattern (B.14) for all the symbols and their coefficients are sums of such symbols and nothing else. We get this kind of matrices from networks with passive devices only. The prove is now extended to the matrices, which are seen if the other supported devices are part of the network.

[3]The prove could end here if the implementation of the software would undergo a minor change: The ones in the LES result from the conditions of specific devices, mostly the sources. Each of these devices produces ones in pairs, that appear in the patterns, which are the prerequisite of the prove so far. If the implementation would handle the ones as device specific constants, thus make a difference between *this* device's ones and *that* device's ones then we had already reached our goal. It would indeed mean a minor implementation change only and no significant perform loss to do so. However, the actual implementation aggregates all ones in a single numeric factor and this requires the extension of the prove.

This part of the prove begins with a single constant voltage source; the other devices will be similar. The determinants consisting only of sums of symbols result from passive networks without any sources. If we add a constant voltage source then the LES is extended by a new row and two new columns. (One of the new columns is a new right-hand side of the equation system.) The new unknown is the current through the source and it flows into one node and returns to the source from another node; this leads to a column with a vertical pair of ± 1. The new unknown comes along with the additional equation that says that the difference between the source's voltage and the potential difference between the connected nodes becomes null; this leads to a horizontal pair of ± 1 and the one and only one in the new RHS column. This last mentioned one actually has a negative sign as the internal representation of the equations is defined such that summing up all columns (including the "RHS") is equal to null. We get the following structure of the LES. The S_{ij} are the sums of the symbols:

$$\left(\begin{array}{ccccccccc|c} S_{11} & & S_{1i} & & S_{1j} & & S_{1z} & 0 & 0 \\ & \ddots & & \ddots & & \ddots & & \vdots & \vdots \\ S_{m1} & & S_{mi} & & S_{mj} & & S_{mz} & \pm 1 & \\ & \ddots & & \ddots & & \ddots & & \vdots & \\ S_{n1} & & S_{ni} & & S_{nj} & & S_{nz} & \mp 1 & \\ & \ddots & & \ddots & & \ddots & & \vdots & \vdots \\ S_{z1} & & S_{zi} & & S_{zj} & & S_{zz} & 0 & 0 \\ 0 & \dots & \pm 1 & \dots & \mp 1 & \dots & 0 & 0 & -1 \end{array}\right) \tag{B.18}$$

The upper left area of the matrix consists of coefficients S, which are sums of symbols as seen before, particularly with the typical appearance patterns for all the symbols. In the implementation the matrix is not necessarily arranged as shown in (B.18), the additional row and columns may appear at different positions but this has no impact on the prove since a rearrangement of rows and columns of a determinant is always possible without another effect then a change of sign. Which is important is that the pairs ± 1 both appear next to the symbols, either in vertical or horizontal direction.

All possible sub-matrices, which we have to investigate, will share the appearance with the matrix. These sub-matrices have the upper-left area of the matrix besides their last row and column; the last row and column are taken from the LES somewhere downwards and somewhere to the right, respectively. Which leads to a determinant with

- a symbol area in the upper left corner,
- one right-most column with the pattern of one of the two right-most columns of the LES and
- a bottom row with the pattern of the last row of the matrix.

Depending on which sub-matrix is taken any of the three elements might be not present. Furthermore the pairs ± 1 might be reduced to a single ± 1 and the right-most column might even contains all nulls. (actually, even the original matrix pattern may only contain single ones instead of pairs if one of the two related nodes is the ground node.)

Most of the possible patterns of the sub-matrix do not require a further consideration. If a column holds only null values it's trivial since the determinant is null, too. If the new ± 1-rows or columns are not contained it's the same situation as in the first step of the prove. If a sub-matrix has inherited the column with the single one then its determinant can be expanded along this column and – disregarding the sign – it can be replaced by the according only subdeterminant, which doesn't have such a column.

If a subdeterminant has inherited at least one out of the new row or columns then we can modify it similar to the first step of the prove. If a pair ± 1 is present in the last row (column) then one of the columns (rows) is replaced by the sum of both, which reduces the pair to a single one. We will always end up with a determinant with modified symbol area and a row and a column (both optional) with all nulls and a single one. The modification of such a determinant is show for LES (B.18); be the number of

rows N, then equation (B.19) presents $\left|M_{NN}^{[N]}\right|$ as an example (with no loss of generality the sign of the ± 1 has been chosen and set):

$$\begin{vmatrix} S_{11} & & S_{1i} & & S_{1i}+S_{1j} & & S_{1z} & 0 \\ & \ddots & & \ddots & & \ddots & & \vdots \\ S_{m1} & & S_{mi} & & S_{mi}+S_{mj} & & S_{mz} & 1 \\ & \ddots & & \ddots & & \ddots & & \vdots \\ S_{m1}+S_{n1} & & S_{mi}+S_{ni} & & S_{mi}+S_{mj}+S_{ni}+S_{nj} & & S_{mz}+S_{nz} & 0 \\ & \ddots & & \ddots & & \ddots & & \vdots \\ S_{z1} & & S_{zi} & & S_{zi}+S_{zj} & & S_{zz} & 0 \\ 0 & \dots & 1 & \dots & 0 & \dots & 0 & 0 \end{vmatrix} \tag{B.19}$$

The suggested modifications will reduce the subdeterminant always to a row and column (both optional) with all nulls and a single one and a symbol area. The expansion is done along the column with the single one and leads to a single sub-subdeterminant. It has inherited the special row and can be expanded along this row; this in turn leads to a single sub-sub-subdeterminant. The latter only contains coefficients from the (modified) symbol area. We disregard the sign; the absolute value of the determinant (B.19) is equal to:

$$\begin{vmatrix} S_{11} & & S_{1i}+S_{1j} & & S_{1z} \\ & \ddots & & \ddots & \\ S_{m1}+S_{n1} & & S_{mi}+S_{mj}+S_{ni}+S_{nj} & & S_{mz}+S_{nz} \\ & \ddots & & \ddots & \\ S_{z1} & & S_{zi}+S_{zj} & & S_{zz} \end{vmatrix} \tag{B.20}$$

S_{ij} denotes the original matrix coefficients in the symbol area. Each S_{ij} is a sum of any symbols, where the appearance pattern of each symbol across the S_{ij} in the original matrix is again described by (B.14). To end this step of the prove we still need to show that in (B.20) the appearance pattern is unharmed for all symbols. If we e.g. look at the middle column: For all rows but the middle it is the sum of the original column j with original column i. The original row i has been scratched out during the expansion of the determinant, so the sum of symbols means that a symbol s from original column i has either been moved to the middle column or it has been eliminated if in the original matrix both columns contained this symbol and due to the inverse signs in the appearance patter. Both possibilities retain the appearance pattern for s. This consideration holds for all symbols in the S and it holds likewise for the row operation. Its the same situation as explained above for symbol t in determinant (B.17): The appearance pattern of the symbols is retained in the relevant sub-determinant.

Please note: These considerations include the situation, where the symbol area is empty (meaning the area is null not the coefficients). In which case the expansion of the subdeterminant will always yield the absolute value one.

In the first step lemma (B.16) has already been proven for symbol determinants, which exhibit the appearance pattern (B.14). In this step we showed that all the subdeterminants $\left|M_{ij}^{(k)}\right|$ of matrix or LES (B.18) can be rearranged to such a symbol determinant and consequently at this point the prove is extended to all the LES with a single constant voltage source.

The considerations of this step can be applied recursively to handle all the LES with any number of constant voltage sources. Additional sources add additional rows and pairs of columns to the LES and each have the same structure with respect to the ones and null values. All of these rows and columns can be reduced to all nulls and a single one in the same way and the appearance pattern in the symbol area is still not harmed: If the required quality is retained in the first iteration than the output of this

iteration is legal input to the same prove or consideration again, which proves the second iteration, and so on.

The repeated application of the proposed manipulation of the subdeterminants just needs one additional consideration: The first iteration will not only affect the symbol area but also the other additional rows and columns introduced by the other constant voltage sources. With two sources the LES (B.18) could become:

$$\left(\begin{array}{ccccccccccc|cc}
S_{11} & & S_{1i} & & S_{1j} & & S_{1k} & & S_{1z} & 0 & 0 & 0 & 0 \\
& \ddots & & \ddots & & \ddots & & \ddots & & \vdots & \vdots & \vdots & \vdots \\
S_{m1} & & S_{mi} & & S_{mj} & & S_{mk} & & S_{mz} & \pm 1 & & & \\
& \ddots & & \ddots & & \ddots & & \ddots & & \vdots & & & \\
S_{n1} & & S_{ni} & & S_{nj} & & S_{nk} & & S_{nz} & \mp 1 & & & \\
& \ddots & & \ddots & & \ddots & & \ddots & & \vdots & & & \\
S_{o1} & & S_{oi} & & S_{oj} & & S_{ok} & & S_{oz} & 0 & \pm 1 & & \\
& \ddots & & \ddots & & \ddots & & \ddots & & \vdots & \vdots & & \\
S_{p1} & & S_{pi} & & S_{pj} & & S_{pk} & & S_{pz} & & \mp 1 & & \\
& \ddots & & \ddots & & \ddots & & \ddots & & \vdots & \vdots & \vdots & \vdots \\
S_{z1} & & S_{zi} & & S_{zj} & & S_{zk} & & S_{zz} & 0 & 0 & 0 & 0 \\
0 & \dots & \pm 1 & \dots & \mp 1 & \dots & 0 & \dots & 0 & 0 & 0 & -1 & 0 \\
0 & \dots & & \dots & \mp 1 & \dots & \pm 1 & \dots & 0 & 0 & 0 & 0 & -1
\end{array}\right) \tag{B.21}$$

Please note, in the example the ± 1 pairs in the two bottom rows share a column index. This might happen if the sources are connected to the same node but is neither essential for nor contradictory to our argumentation. It has been done only to reduce the size of the representation.

Now it is important that the pairs ± 1 in the additional rows and columns of the LHS of the LES are all in the index range of the symbol area. Furthermore, the appearance pattern of the ones is one of the patterns we'd considered for the symbol area. Consequently, the first iteration will retain this pattern in all the other additional rows (columns) for the same reasons as explained before for the symbols in the symbol area. This is why the iteration can then in turn be done on the next additional row (column).

Because this recursion can be done lemma (B.16) is now proven for all the LES created from networks consisting of passive devices (leading to the symbol area) plus any number of constant voltage sources.

linNet supports more kinds of sources and the ideal operational amplifier. All of these devices are handled by introducing additional unknown currents and thus adding a LHS column to the LES and an additional equation. The constant current source also adds a RHS column. The character of the additional rows and columns is always the same; the symbols (for controlled sources only) appear in pairs and the only numbers are pairs of ± 1 or a single one in a column of nulls otherwise. The idea of reducing the additional rows and columns to a single value without destroying the appearance pattern of the symbols in the symbol area is the same as shown in detail for the constant voltage sources. The determinant is then expanded along the rows and columns with single ones until the only remaining subdeterminant only consists of a symbol area which the statement was proven for in section B.3. In general, we will never see a numeric factor other than ± 1 in the coefficients of the LES.

B.3.2 Intermediate expressions

An important note for the implementation is that the prove only holds for coefficients that are the result of a completed iteration of the elimination. There's no such prove for intermediate results during the elimination step. Actually, the elementary step is implemented by first adding two products of two coefficients each and then doing the factorization. The intermediate sum can indeed have factors other than ± 1 due to identical addends in both products. This is one of the reasons why the implementation still uses integers instead of Booleans as numeric factors.

Nonetheless, the prove is even useful for the intermediate results: An implication of the prove is that the numeric factor appearing in the intermediate result can only grow incrementally, i.e. one by one with the result's addends being figured out in a loop. (In far the most cases it still stays absolute one.) The implementation foregos a range check for the required integer operations: It's proven that an overrun can occur only after a pseudo-infinite computation time and an error report which would be printed in an unreachable future is useless and doesn't add any value to the software or its behavior.

B.4 User-defined voltages

It can be proven that the properties of the coefficients – no symbol powers other than null or one and all numeric factors absolute one – hold for the user-defined voltages, too. User-defined voltages are differences of two node potentials, where the two nodes are arbitrarily chosen. If one of the nodes is the ground node then the difference is trivial, either an existing node potential or the negated value. Obviously the properties are retained in these trivial results. It's less evident for the general situation of two nodes, whose potentials are both unknowns of the solved LES. The symbol powers are still simple, as we compute a difference there's no way to get new powers in the products of symbols. However there could be addends with same symbol products and same sign of the numeric factor, which would lead to a numeric factor of two but no longer absolute one. This can be excluded by looking at the LES and how it is transformed by using derived unknowns.

Be M the matrix, which represents a LES with m unknowns and $n - m$ knowns:

$$M = \left(\begin{array}{ccc:ccc} a_{11} & \dots & a_{1m} & a_{1,m+1} & \dots & a_{1n} \\ \vdots & & \vdots & \vdots & & \vdots \\ a_{m1} & \dots & a_{mm} & a_{m,m+1} & \dots & a_{mn} \end{array}\right)$$

Be the user-defined voltage $\hat{U}$ the difference of the unknowns in column u and v, i.e. $\hat{U} = U_u - U_v$. This equation could be put into the LES in order to directly figure out the user wanted $\hat{U}$. We substitute e.g. U_u by $U_u = \hat{U} + U_v$. The LES is transformed into an equivalent LES using a new vector of unknowns, where $\hat{U}$ replaces U_u:

$$\hat{M} = \left(\begin{array}{ccccccc:ccc} a_{11} & \dots & a_{1u} & \dots & a_{1v} + a_{1u} & \dots & a_{1m} & a_{1,m+1} & \dots & a_{1n} \\ \vdots & & \vdots & & \vdots & & \vdots & \vdots & & \vdots \\ a_{m1} & \dots & a_{mu} & \dots & a_{mv} + a_{mu} & \dots & a_{mm} & a_{m,m+1} & \dots & a_{mn} \end{array}\right)$$

The new unknown $\hat{U}$ inherits the coefficients of the substituted unknown U_u and the coefficients of unknown U_v become the sum of two columns of the original matrix M. To apply the elimination method according to section (B.1) for $\hat{U}$ we need to swap two columns so that $\hat{U}$ becomes the last unknown in the again new vector of unknowns:

$$\tilde{M} = \left(\begin{array}{ccccccc:ccc} a_{11} & \dots & a_{1m} & \dots & a_{1v} + a_{1u} & \dots & a_{1u} & a_{1,m+1} & \dots & a_{1n} \\ \vdots & & \vdots & & \vdots & & \vdots & \vdots & & \vdots \\ a_{m1} & \dots & a_{mm} & \dots & a_{mv} + a_{mu} & \dots & a_{mu} & a_{m,m+1} & \dots & a_{mn} \end{array}\right)$$

The solution for $\hat{U}$ is an expression consisting of the $n - m + 1$ right-most coefficients of the last row of $\tilde{M}$ after the last elimination step. Using the position indexes of $\tilde{M}$ these are $a_{mm}^{(m-1)}$ till $a_{m,n}^{(m-1)}$. According to lemma (B.3) these coefficients are the determinants of $\tilde{M}_{mm}^{[m]}$ till $\tilde{M}_{mn}^{[m]}$, respectively.

Disregarding the sign change due to the swapping of columns, $|M_{mm}^{[m]}|$ is the unchanged system determinant: If we replace column v with the difference of itself and column m, which doesn't alter the determinant's value according to lemma (B.5), then we have the same determinant as for the original LES M.

The $n-m$ other determinants share the left-most $m-1$ columns of $\tilde{M}$ and have columns $m+1$ till n of $\tilde{M}$ as right-most column. These determinants contain the column v with the sum of two original coefficients but don't contain the column where the added coefficients origin (i.e. column m of $\tilde{M}$ or column u of the original matrix M). With respect to the appearance patterns of symbols according to (B.14) this means a move of the symbols from the original column u to column v, which doesn't harm the patterns. Due to elimination particular symbols might show a different pattern but all appearing patterns are still as listed in (B.14). Since the appearance patterns are the prerequisite for the proves of the properties of the coefficients of the elimination steps these proves still hold for the $n-m$ coefficients which form the numerators of the solution for $\hat{U}$.

All coefficients that form the solution of the user-defined voltage $\hat{U}$ have the same properties as shown for the coefficients of the actually solved LES with the original unknowns. Also in the user-defined voltages we will never see a numeric factor other than absolute one and there won't be any symbol with a power of higher than one.

B.5 Implementation of elementary step

This section explains the implementation of the elementary step of the elimination method (see section B.1). This operation combines the four matrix coefficients under progress and the common divisor to a new coefficient, the resulting coefficient of that elimination step. The implementation is not done very explicit but uses simplifications and omissions, which are justified in the following. The source code is not self-explaining and will be hard to understand without the explanations given here.

The elementary step is defined as

$$a_{ij}^{(k)} = \frac{a_{kk}^{(k-1)} a_{ij}^{(k-1)} - a_{ik}^{(k-1)} a_{kj}^{(k-1)}}{a_{(k-1)(k-1)}^{(k-1)}} \tag{B.22}$$

in elimination step k. For all affected $a_{ij}^{(k)}$ – including the resulting $a^{(k)}$ at the LHS of equation (B.22) – it holds due to former proves:

- a is a sum of addends. Each addend is a product of symbols to the power of either null or one and a numeric factor of either plus or minus one
- No two addends of a have the same product of symbols, i.e. the vectors of powers are different between all pairs of addends

It follows that the result of the division can be carried out without a remainder. With other words, the numerator of equation (B.22) step can be brought into the shape of a product of a coefficient with the above properties and the denominator of equation (B.22).

The implementation basically follows the definition of the elementary step. It carries out the first product of the numerator by multiplying and summing up all pairs of addends between first and second factor. This is followed by the second product; here the products of the pairs of addends are immediately subtracted from the sum so far. After this step a polynomial division is carried out.

The mentioned omissions relate to such (intermediate) terms we can predict for that they won't appear in the final result because of the known properties of this result. These terms are simply discarded but neither summed up nor stored.

A hypothetic implementation of the data representation is introduced, which can represent all coefficients and all intermediate and the final result (referred to as "variable" in the following). This implementation is actually not used but below we will justify why it can be safely reduced to the actually implemented variable representation.

The product of the addends of the coefficients in the numerator can produce symbol powers of 0, 1 or 2. The hypothetic implementation of a variable will use a 2 Bit integer value to store a symbol's power. It'll furthermore order these two Bit values in a certain order of symbols, which is defined arbitrarily but

fixed for all variables. (Alphabetic order of symbols would e.g. suffice.) Reading this sequence of bits as a whole as an unsigned binary number gives us a one-by-one relation between any appearing combination of symbols and an integer number. We call this number the "value" of the symbols or simply symbol value of an addend of the variable. In this sense products of symbols become an ordered set.

The addends of a variable will always be ordered in decreasing order of their symbol value.

For the input variables we know that the numeric factor of each addend is ± 1 but this is not proven (and indeed not the truth) for the numerator in equation (B.22); its two addends can contribute with same symbol combinations. Therefore the implementation needs to use an integer type here.

For the further we assume that all input variables, all intermediate results and the final result are represented in the outlined form.

The order of addends according to their symbol value is important for the further and therefore we now have a look at the required operations product and quotient of addends with respect to the symbol value:

The product of two addends is figured out by multiplying their numeric factors. The result is an integer as the operands are. The products of symbols are multiplied by adding the powers. All the 2 Bit fields have to be added separately, one pair of 2 Bit values for each symbol. However, since we only add two values each either null or one, we will never see an overrun in any of the 2 Bit sums and therefore the result of all the 2 Bit sums is identical to the arithmetic sum of the symbol values of the addends. This implies for example that the product of a variable with an addend will not change the order of addends – the symbol value of all addends rises by the same value, the symbol value of the multiplied addend.

The quotient is analog. Now, the symbol powers are subtracted. Instead of subtracting all the 2 Bit values of the distinct symbols separately we can directly subtract the entire symbol values of the affected addends. We will only perform divisions which are possible; this is why there won't ever be an underrun in any of the 2 Bit differences and this is why the 2 Bit operations again can be computed as a single arithmetic difference of symbol values.

Based on what has been said so far and adding some straightforward algorithmics the numerator of equation (B.22) can be figured out and stored in the hypothetic data representation. The products of coefficients are reduced to the sum of all crosswise products of addends and the sum of addends is a matter of list sorting according to the symbol value of the addends and adding the numeric factors.

In the hypothetic implementation the polynomial division starts when all addends of the numerator in equation (B.22) are figured out. A loop iterates along the numerator variable in decreasing order of the symbol value, i.e. from the left to the right. The computation finishes when the last addend of the numerator has been processed. The first operation is to divide the addend under progress by the first, left-most addend of the denominator variable. Three possibilities exist:

a) The division is possible with respect to the symbols (i.e. the numerator addend contains a superset of symbols of the denominator addend) and results in symbol powers of either null or one
b) Like a) but at least one symbol power of two results
c) The division is not possible (i.e. the numerator addend lacks at least one symbol from the denominator addend's set)

Please note, that the division of the numeric factors of the addends will always succeed without a remainder: The denominator variable is an unmodified input coefficient, which is proven to have numeric factors of ± 1 only.

The hypothetic implementation continues in all three cases:

a) The result of the division is put into the result variable as its next addend
b) Like a) but here we can predict that a later contribution to the result will eliminate this addend again; we know that the final result won't contain addends with symbol powers greater than one
c) The numerator addend is left as is in the numerator variable. For now this is a remainder of the polynomial division. The operation is completed and the loop of the polynomial division will handle the next numerator addend

In cases a) and b) the back-multiplication is carried out as a principle of the polynomial division. The new result addend is multiplied with all addends of the denominator and each product of addends is subtracted from the numerator variable, which gets the meaning of the still undivided remainder of the

operation. The number of products is finite and defined by the given length of the denominator variable. Obviously, the first subtracted product eliminates the addend under progress of the numerator variable, the following are sorted into this variable; they might eliminate existing terms, constitute new ones or change the numeric factor of existing ones.

All variables are ordered in falling symbol value of the addends. This means that the first subtracted product, which is got by back-multiplication with the first addend of the denominator has a higher symbol value than all other addends of the back-multiplication. This first addend eliminates the addend under progress in the numerator; all further might remain in the numerator. Due to the order of addends of the numerator the addend under progress is the one with the currently still highest symbol value. Consequently and although the back-multiplication can lengthen the numerator variable it can be said that the highest remaining symbol power in the numerator is lower than before (remainders according to case c) are disregarded here) and that all possibly new addends in the numerator are to the right of the addend under progress and will be processed later in the main loop.

B.5.1 Finiteness of algorithm

The last statement already proves the finiteness of the algorithm. Although the numerator can become longer due to the back-multiplication the highest symbol value, i.e. the symbol value of the addend under progress, is lowered in every cycle of the main loop. At latest when this symbol value is lower than the symbol value of the left-most addend of the denominator the algorithm would terminate: If the symbol value of the divisor's addend is greater than the symbol value of the numerator's addend then at least one symbol is missing in the numerator's symbol product to make the division possible. The addend under progress and all further addends (which have even lower symbol values) would belong to case c) and hence be part of the remainder of the division.

B.5.2 Omission of addends

The real implementation of the elementary step discards some addends during computation of the numerator of (B.22) and during back-multiplication in the polynomial division. When reading the source code this looks arbitrarily if not faulty. The prove why it can be safely done follows.

Each addend is tested whether the division with the left-most addend of the denominator is possible. As the numeric factor of all the denominator addends is absolute one this is just a matter of "fitting" symbols: All 2 Bit power values of the tested addend need to be greater or equal to the according 2 Bit values of the denominator addend.

Let's have a closer look at case c) of the hypothetic implementation: The addend under progress can't be divided by the denominator's addend and it is preliminarily left as an addend of the remainder of the quotient. From our prerequisites we know that the division can be carried out without a remainder so at the end of the operation the left addend must have disappeared. On the first glance this seems to be possible by elimination of the unwanted addend by subtracted addends from the back-multiplication with later computed result terms. However due to the sort order according to the symbol values we can exclude this: All subsequently handled numerator terms have lower symbol values and consequently no subsequently subtracted addend from the back-multiplication will ever have a symbol value greater or equal to the symbol value of the unwanted addend and it would remain till the end of the complete operation. This is contradictory to the prove of a remainder-less quotient. Which means that it can't ever happen that the main loop encounters a numerator addend under progress, which can't be divided by the left-most denominator addend. Case c) was necessary for the explanation but doesn't ever occur and needs not to be considered by the actual implementation.

The last statement does not mean that there can't be any addends in the numerator, which can't be divided by the left-most denominator addend – it's rather a matter of *when* they may appear: They can appear during and at the end of the computation of the sum of products in the numerator of (B.22) and they can even remain in the numerator variable during the polynomial division until the main loop processes their preceding addend – at latest now they will be eliminated by addends from the back-

multiplication. Earlier elimination is also possible: During computation of the numerator expression such addends might first appear as a result of the first product and later be eliminated by addends from the second product. Anyhow, it's for sure that they will be eliminated from the numerator variable prior to the attempt to divide them by the left-most addend of the denominator. Instead of keeping track of addends that will be anyway eliminated somewhere in the future the real implementation decides to discard them immediately. If one of the products yields an indivisible addend it is thrown away without any ado. Most likely, counterparts of these addends will appear during the back-multiplication; now the implementation needs to throw them away, too – elimination in the numerator by subtraction can't succeed due to the former decision.

Similar considerations hold for case b). The products in the numerator of (B.22) can yield symbols to the power of two. If such an addend is divisible by the left-most addend of the denominator we can get a result term with symbol powers of two. From our prerequisites it's yet for sure that this addend won't appear in the final result. So there must be a later contribution to the result, which eliminates the unwanted result term again. However, due to the sort order of the numerator addends, all later result addends will have symbol values that are lower than the one of the unwanted addend and the elimination of an already found result term is simply impossible. This is a contradiction with the prove of symbol powers only being null or one. Which means that it can't ever happen that the main loop encounters a numerator addend under progress, which yields a power of two for at least one symbol when being divided by the left-most denominator addend.

The rest is just the same: Such addends may temporarily occur in the numerator variable but they are surely eliminated before the main loop reached them, before they become the addend under progress. Elimination can occur due to negated addends during the computation of the second numerator product or due to subtraction of back-multiplication addends in earlier cycles of the main loop of the polynomial division. Anyhow, the main loop will never encounter and process such an addend and this is why the real implementation immediately discards such addends during the computation of the numerator of (B.22) and why it then has to throw according counterpart-addends away during back-multiplication.

A further optimization becomes possible in the real implementation: Any numerator addend which yields a power of two for at least one symbol after division is immediately thrown away, thus not stored in the numerator variable. All stored addends yield symbol powers of either null or one after division by the left-most addend of the denominator. The real implementation doesn't safe the addends of the numerator of (B.22) but the addends of (B.22) *after* division by the left-most addend of the denominator. This is possible because any addend which can't be divided for one of the two criteria is discarded, as said. Consequently, the real implementation can use 1 Bit fields for the symbols.

The hypothetic implementation reduces the product and quotient of symbol powers to the arithmetic sum and difference, respectively. The real implementation uses bit operations. The operations are equivalent since the test "division is possible" ensures that there is neither an over- nor an underrun of symbol powers from one field into another one. The same holds for the two tests themselves: Since the power of three doesn't occur anywhere the hypothetic arithmetic 2 Bit operations can be carried out by a combination of bit operations only. All of these optimizations lead to an efficient real implementation, which is equivalent to the hypothetic implementation used in the considerations here.

A last remark is required. The real implementation doesn't store the (relevant) addends of the numerator of (B.22) but the same, already divided by the left-most addend of the denominator. This does not change the argumentation of the hypothetic implementation: The division means that the symbol value of the left-most addend of the denominator is subtracted from all addends in the numerator variable. The order of addends is not effected and nor are the considerations affected, which base on the statement that a symbol value as large as the symbol value of the addend under progress will never be seen again in the back-multiplication of this cycle or in any addend of any later cycle of the main loop of the polynomial division. All appearing symbol values are lowered by a constant value but their mutual relations are not changed.

B.5.3 Numeric factor one

In section B.3 the prove was given that the numeric factor in the addends of all appearing coefficients will only be ± 1. The prove given here refers to this statement only at one point: The division of the numeric factor of the addend under progress by the factor of the first addend of the denominator would always succeed as the latter is absolute one. Actually, this just shortens the explanations above but is irrelevant for the correct operation of the actual implementation. With one minor exception the implementation of the elementary step (and the elimination method according to section B.1 anyway) would continue to work with LESs, where the addends can have arbitrary numeric integer factors (but still symbols, which are restricted to a power of null or one).

The critical point is the decision a), b) or c). With arbitrary integer factors we could expect addends, which are indivisible with regard to a remainder-less result of the quotient of the numeric factors. But actually, this can never happen for the same reason why case c) couldn't occur above: A remainder in the quotient of the numeric factors would mean to leave the addend under progress with reduced numeric factor in the numerator variable. Due to the consideration with strictly monotonically decreasing symbol values this addend could not be eliminated any more - a contradiction to the prove of a remainder-less final result.

A difference to the same consideration for the quotient of products of symbols is that the addends of the numerator computation, which show indivisible numeric factors must not be discarded. Here, combination with later addends (i.e. either in the further computation of the numerator of equation (B.22) or in a later back-multiplication of the polynomial division) can change them to hold well divisible numeric factors but won't surely eliminate them as shown for addends with indivisible symbol products.

The mentioned minor exception from the irrelevance of the factor-one statement is the integer overflow handling. If the implementation would be applied to a system with arbitrary integer factors overflow recognition becomes a must.[4] The current use case may however safely ignore integer overflows due to the restriction of the factors to ± 1.

[4] Actually, there's one more change required on the implementation: The ± 1 statement is double-checked by assertions. These assertions needed to be replaced by an assertion that proved the remainder-less division of the numeric factors.

Appendix C

Listing of Octave LTI Simulation

Here is the listing of an Octave script that can be used to run the LTI system generated from the bipolar transistor model. In section 8.3.4 this script has been used to find the limits of the linear simulation; with other words the operating range of the linear solution linNet had figured out. Outside this range the found solution will no longer show the behavior of the true circuit.

To run the script you need to copy it into the same folder, where linNet generated the user-defined result *DC*, i.e. where the script DC.m is located.

```
%    BipolarTModel_opRangeByLsim() - This script runs a simulation of the
%    found LTI object in order to find the operating range of the amplifier
%    circuit.
%
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% We operate in the audio frequency range and will use a input signal of
% 1 kHz. This allows to do the needed amplitude modulation within one
% second. A sampling period of 50us is sufficient, we see 20 samples per
% period.
tiEndInit = 0.5;
tiEnd = tiEndInit + 1;
tiSample = 50e-6;
tiSim = [0:tiSample:tiEnd].';
noSamples = length(tiSim);

% The battery voltage and the base-emitter voltage drop are modelled as
% constants.
```

```
U_batt = ones(noSamples, 1) * 12;
U_be = ones(noSamples, 1) * 0.6;

% The LF input signal is modelled as a sinoid of 1 kHz with a rising
% amplitude. The first 500ms the input is null: This way we await all
% transients from switching U_batt and U_be from initially null to their
% constant values. (The better approach would be to specify the initial
% conditions for the internal states of the LTI object but this is much
% more difficult.)
noInitSamples = ceil(tiEndInit/tiSample);
noSineSamples = noSamples - noInitSamples;
U_in_init = zeros(noInitSamples, 1);
U_in_f = 1000;
U_in_sine = sin(2*pi*U_in_f*tiSim(1:noSineSamples));

% The modulation function of the input should rise from null to the final
% value. The final value should be somewhat above the expected maximum
% input amplitude. This maximum amplitude can be approximated: We have an
% amplification of about 100 and the maximum output amplitude is about
% half the battery voltage.
F_mod = (1:noSineSamples).'/noSineSamples * 1.5 * (12/2)/100;
U_in_sine = U_in_sine .* F_mod;

% Compose the LF input signal from the segments.
U_in = [U_in_init; U_in_sine];
assert(all(size(U_in) == [noSamples 1]));

% Compose the system input from the three input signals.
U = [U_batt U_in U_be];

% Create the LTI system object. Use the linNet generated function DC to do
% so.
[tfDC par] = DC

% The LTI system DC is: U_batt, U_in, U_be -> U_c, U_base, I_batt
%   Run the simulation.
Y = lsim(tfDC, U, tiSim);

% Extract samples of interest.
%   Side note: The initialization phase is not only out of scope. It is
% furthermore wrong if we expect to see the behavior of the true circuit.
% The true circuit starts up with strong non linearities while the linNet
% solution is linear all the time. The voltage at the transistor's base
% rises slowly from null due to input voltage null and the input capacitor
% C_in. In the true system the base current is also null and hence the
% collector current. In the linNet solution the base current has a
% negative start value (since U_be already has its positive, constant
% value) and hence (strictly linear behavior!) the collector current is
% strongly negative. Consequently the collector voltage shows a large
% positive start value, significantly above the battery voltage.
tiPlot = tiSim(noInitSamples+1:end);
U_c = Y(noInitSamples+1:end,1);
```

```
U_base = Y(noInitSamples+1:end,2);
I_batt = Y(noInitSamples+1:end,3);

% Plot the result
figure
hold on;
plot(tiPlot, [U_c U_in(noInitSamples+1:end)]);
plot([tiPlot(1) tiPlot(end)].', [0.6; 0.6].', 'c');
plot([tiPlot(1) tiPlot(end)].', [12; 12].', 'y');
title('U_c in dependency of U_in');
xlabel('time [s]');
ylabel('');
legend('U_c', 'U_in', 'U_c_min', 'U_c_max', 'location', 'eastoutside');
grid on
zoom on
```

Listing C.1: Time domain simulation of bipolar transistor model

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Version 1.3, 3 November 2008

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