2010

1. 考慮 phase leading 控制器之通式如下：

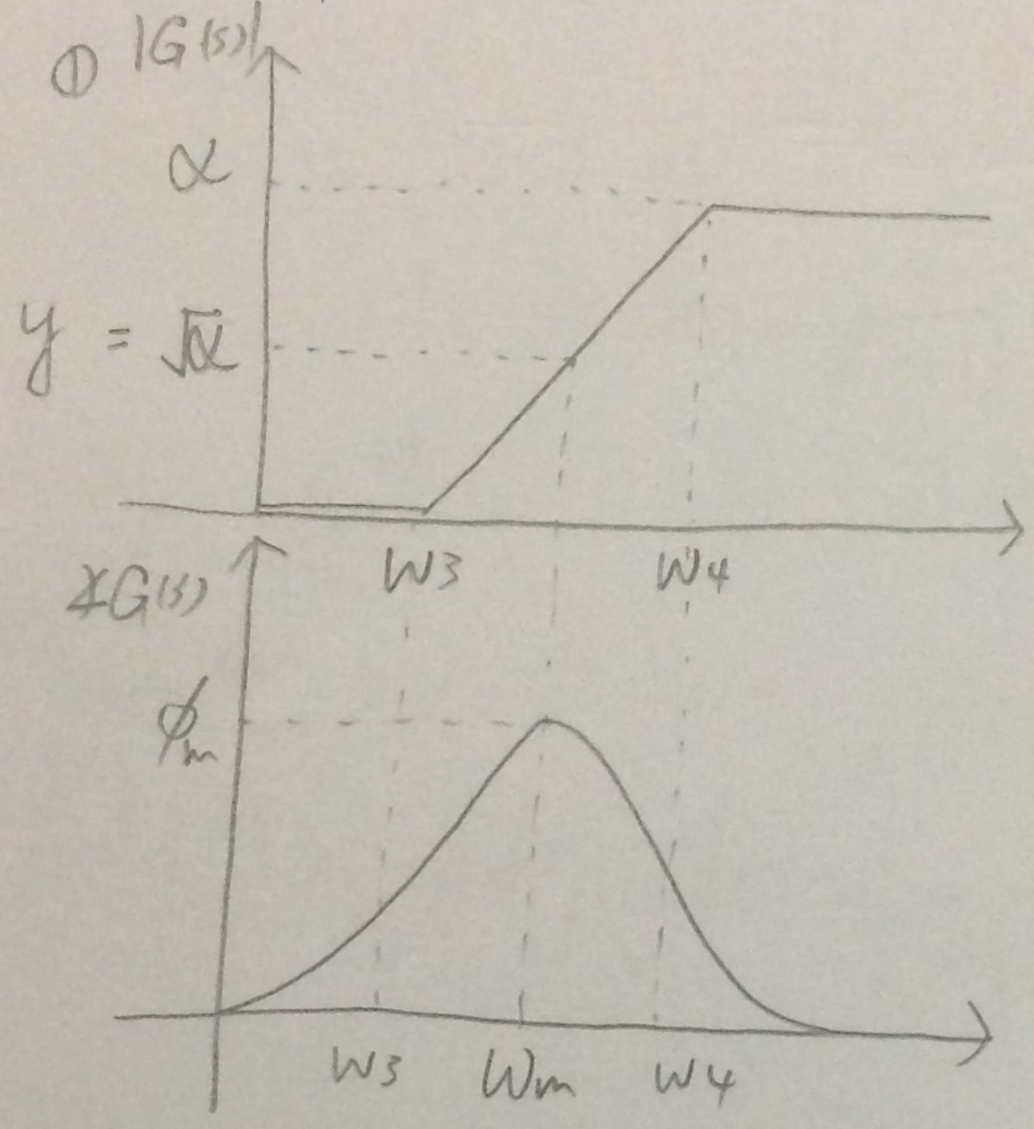
$$G(s) = \alpha \cdot \frac{s+w_3}{s+w_4}, \quad 其中\ \alpha = \frac{w_4}{w_3}$$

① 試繪出 $G(s)$ 之完整 Bode plot

② 試證明其可補償之最大相位 $\phi_m = \sin^{-1}\frac{\alpha-1}{\alpha+1}$

1

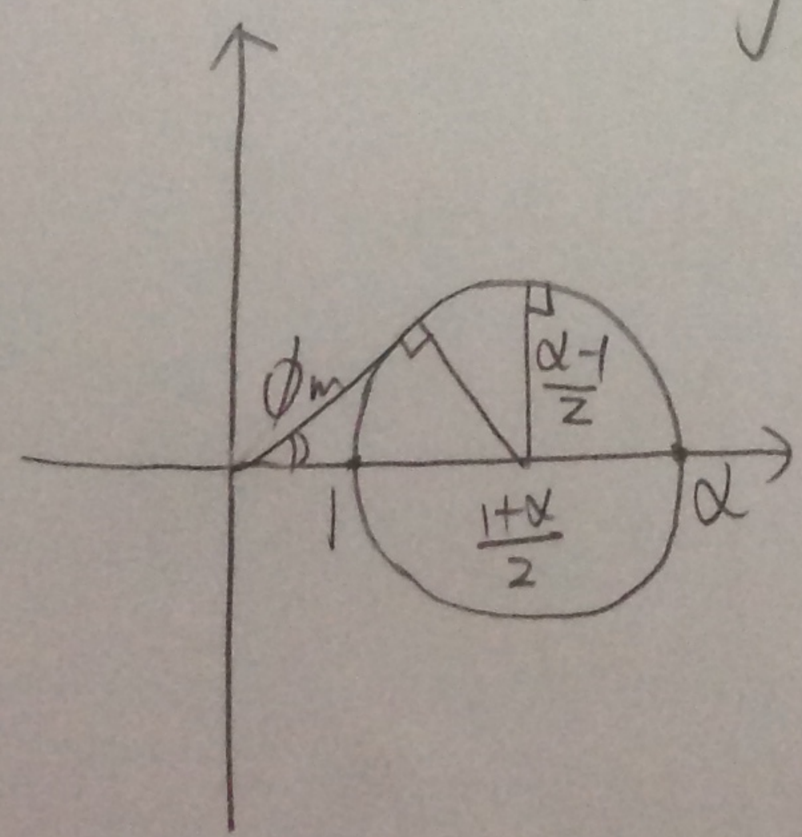
①



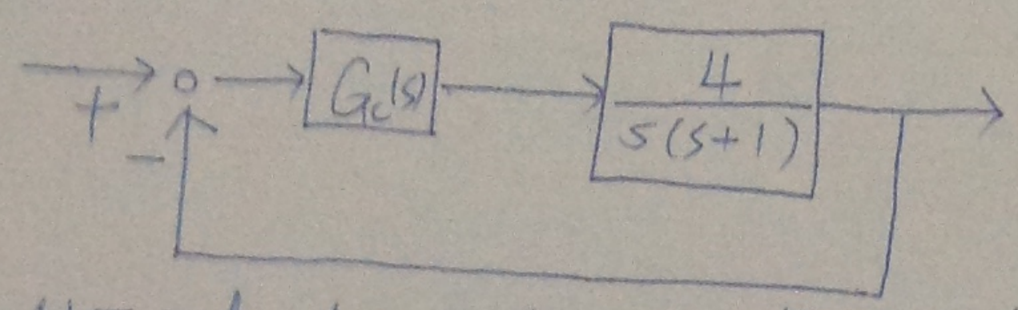
② 由圖①，$\frac{y}{1} = \frac{w_m}{w_3} = \frac{\sqrt{w_3 w_4}}{w_3} = \sqrt{\frac{w_4}{w_3}} = \sqrt{\alpha}$

$$\phi_m = \angle G(jw_m) = \tan^{-1}\frac{w_m}{w_3} - \tan^{-1}\frac{w_m}{w_4}$$

$$= \sin^{-1}\frac{\alpha-1}{\alpha+1}, \quad \alpha = \frac{w_4}{w_3}$$



2. 考慮下列回授控制系統



利用Bode plot 設計方法設計 phase leading controller $G_c(s)$，
使得開迴路系統滿足下列規格：

① $e_{ss|ramp} \le 5\%$

② $PM \ge 45°$

if $G_c(s)=1 \rightarrow G_c(s)=5$

$\frac{1}{\sqrt{\alpha}} = |olt\,f(j\omega_m)| \Rightarrow$ 得 ω_m

step 1: if $G_c(s)=1$, $olt\,f(s)=\frac{4}{s(s+1)}$, $k_v=\lim_{s\to 0} s\cdot\frac{4}{s(s+1)}=4$

$\Rightarrow$ ∴ 選擇 $G_c(s)=5$，可得 $k_v=20 \rightarrow e_{ss|ramp}=\frac{1}{k_v}=5\%$

step 2: $G_c(s)=5$ $olt\,f(s)=\frac{20}{s(s+1)}$,

求PM: $|olt\,f(j\omega_{gc})|=1=\left|\frac{20}{j\omega_{gc}(j\omega_{gc}+1)}\right|=\left|\frac{20}{-\omega_{gc}^2+j\omega_{gc}}\right|$

$=\frac{20}{\sqrt{\omega_{gc}^4+\omega_{gc}^2}} \Rightarrow \omega_{gc}=4.47$

$\measuredangle olt\,f(j\omega_{gc})=-\measuredangle j\omega_{gc}-\measuredangle(j\omega_{gc}+1)$

$=-90°-77.25°=-167.25°$

$PM=180+(-167.25°)=12.75°$

∴ PM 尚缺 $45°-12.75°=32.25°$，補 $45°$ ϕ_m

$\phi_m=\sin^{-1}\left(\frac{\alpha-1}{\alpha+1}\right)=45°$

$\Rightarrow \alpha=3+2\sqrt{2}=5.83$

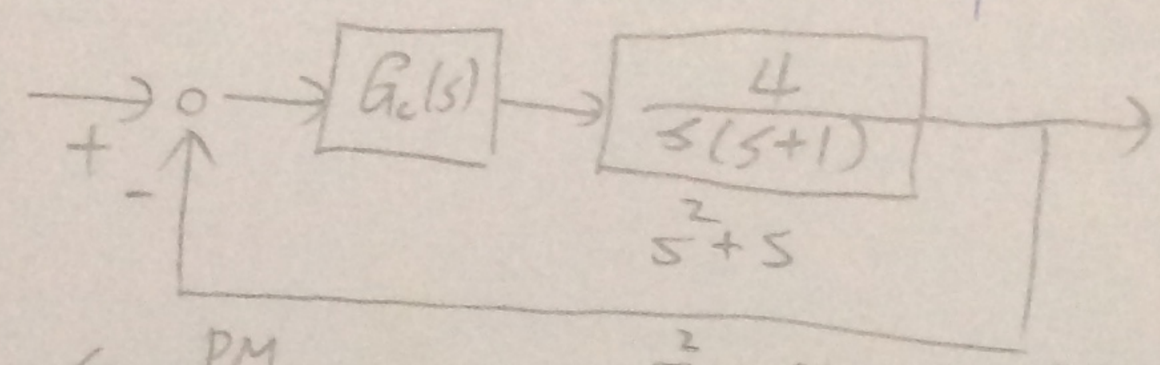
$\frac{1}{\sqrt{\alpha}}=\left|\frac{20}{j\omega_m(j\omega_m+1)}\right| \Rightarrow \omega_m=6.9128$ rad/s

$\omega_4=\sqrt{\alpha}\,\omega_m=16.69$

$\omega_3=\frac{\omega_m}{\sqrt{\alpha}}=2.8634$

$\rightarrow G_c(s)=5\times\frac{1+\frac{s}{2.8634}}{1+\frac{s}{16.69}}$

3. 同 2.，利用 root locus 設計方法設計 phase leading controller $G_c(s)$.



$\xi = \frac{PM}{100} = 0.45$ $\qquad s^2+2\xi\omega_n s+\omega_n^2$

s^2+15

$\cos\theta = \xi = 0.45 \Rightarrow \theta = 63.25$

$\tan 63.25° = \frac{x}{0.5} \Rightarrow x = 1$

主極点: $-4 \pm 7.94j$

$s = -\xi\omega_n \pm j\sqrt{1-\xi^2}\,\omega_n$

$t_s = 1 = \frac{4}{\xi\omega_n}$ $\quad \omega_n = 8.89$

root locus 上的一点須滿足 $olt\,f(s) = k\cdot\frac{4}{s(s+1)} = -1$

$\Rightarrow k\cdot\frac{4}{s(s+1)}\Big|_{s=-\frac{1}{2}+j} = -1 \Rightarrow k = \frac{4}{5}$ (for PM)

$e_{ss}|_{ramp} = k_v = \lim_{s\to 0} s\cdot olt\,f(s) = \frac{16}{5} \rightarrow e_{ss}|_{ramp} = \frac{1}{k_v} = 31\% >> 5\%$

$\Rightarrow \boxed{G(s) = k\cdot\frac{s+z}{s+p}}$ (加上 pole & zero) $\qquad c\frac{dV_c}{dt}$

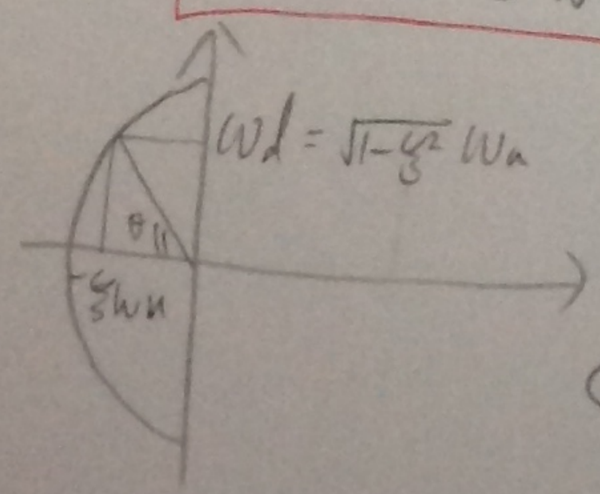
$\hookrightarrow \frac{di_L}{dt}$

Not!

$olt\,f(s) = k\cdot\frac{s+z}{s+p}\left(\frac{4}{s(s+1)}\right)$

$s = -\xi\omega_n \pm j\sqrt{1-\xi^2}\,\omega_n$

$\boxed{t_s = 1 = \frac{4}{\xi\omega_n}} \Rightarrow \xi\omega_n = 4$, $\xi = 0.45 \Rightarrow \omega_n = 8.89$, $\omega_d = 7.94$



① 決定 z

$-z < -4$

主極点 $= -4 \pm 7.94j$

$\therefore$ 選 $z = 4$

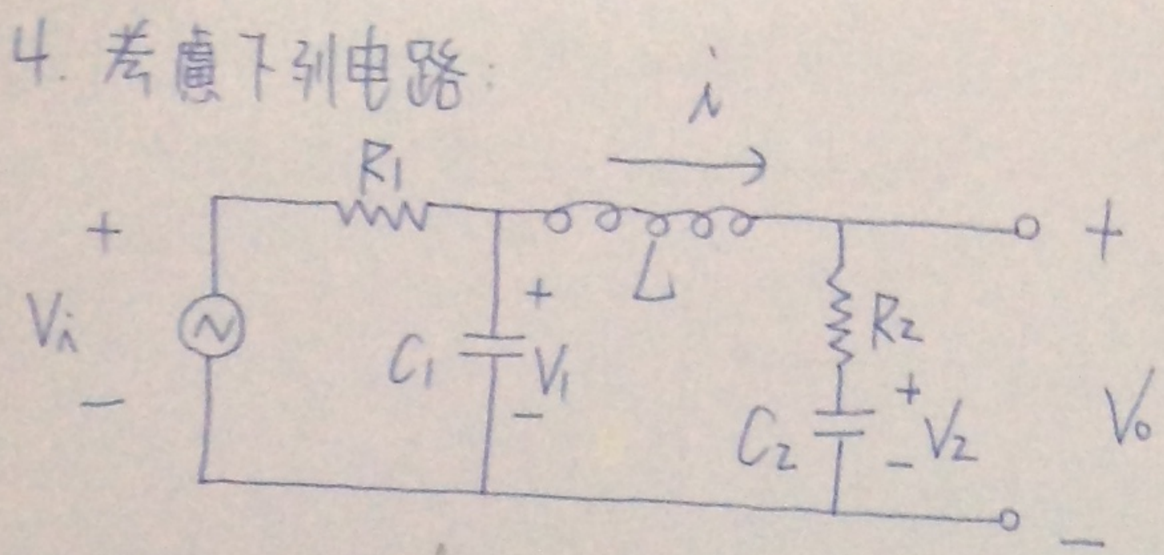
② $olt\,f(s) = k\underbrace{\frac{s+4}{s+p}}_{\text{phase leading}}\frac{4}{s(s+1)}$, $olt\,f(s)|_{s=-4+7.94j} = -1$

$= -180°$

$\measuredangle olt\,f(s) = \measuredangle k + \measuredangle(s+4) - \measuredangle(s+p) - \measuredangle(s) - \measuredangle(s+1)$

$= 0 + 90° - \tan^{-1}\frac{7.94}{p-4} + 63.26° + 69.312° = -180°$

4. 考慮下列电路：



以 V_1、V_2、i 為狀態變數，
V_i 為輸入電壓，V_o 為輸出電壓，
試寫出此系統之狀態空間表示式。

$$
\begin{cases}
L_1 \dfrac{di}{dt} = V_1 - (V_2 + iR_2) \\[2ex]
C_1 \dfrac{dV_1}{dt} = \left(\dfrac{V_i - V_1}{R_1} - i \right) \\[2ex]
C_2 \dfrac{dV_2}{dt} = i
\end{cases}
$$

5. ① 試述可控性(controllability)之定義

② 試述可觀性(observability)之定義

③ $\dot{x} = \overset{A}{\begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 4 \\ 1 & 3 & 1 \end{bmatrix}} x + \overset{B}{\begin{bmatrix} 1 \\ 1 \\ 2 \end{bmatrix}} u$

$y = \underset{C}{[1 \quad 0 \quad 2]} x$

試檢視此系統之可控性、可觀性、及穩定性

在一有限時段 $t_f \geq t_0$ 中，
對任一初始時間 $t = t_0$，
若存在一片斷連續 input $u(t)$，
能驅使 $x(t_0) \to x(t_f)$，則系統可控

在一有限時段 $t_f \geq t_0$ 中，
由已知 $u(t)$ 和 $y(t)$ 就可
決定 $t = t_0$ 時的任意
狀態 $x(t_0)$，則系統可觀

① $x(t_0)$ 為可控，

if 存在一個 piecewise continuous input $u(t)$，

可推測 $x(t_0) \longrightarrow x(t_f)$ for a finite time $t_f \geq t_0$

$\dot{x} = Ax + Bu$

$x(t_f)$ final state

$\text{rank}\,[B \quad AB \quad A^2B \cdots]$

$x(t_0)$ initial state

② $x(t_0)$ 為可觀，

if 給任一 input $u(t)$，存在 a finite time $t_f \geq t_0$，

$\begin{cases} u(t) \\ y(t) \end{cases}$ for $t_0 \leq t \leq t_f$ 可唯一決定 $x(t_0)$

$\text{rank} \begin{bmatrix} C \\ CA \\ CA^2 \\ \vdots \end{bmatrix}$

6. $G(s) = \dfrac{s+6}{s^4-3s^2+4s+1}$ C

① 將其用 controllable canonical form 表示之

② 將其用 observable canonical form 表示之

③ 決定狀態回授增益，將閉迴路之極点置放於 $-1\pm 3j$，$-2\pm j$

① controllable canonical form:

$$\dot{x} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -1 & -4 & 3 & 0 \end{bmatrix} x + \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} u$$

$$y = \begin{bmatrix} 6 & 1 & 0 & 0 \end{bmatrix} x$$

② CCF

$$\begin{cases} \dot{x} = Ax + Bu \\ y = Cx + Du \end{cases} \qquad \dot{x} = \begin{bmatrix} 0 & 0 & 0 & -1 \\ 1 & 0 & 0 & -4 \\ 0 & 1 & 0 & 3 \\ 0 & 0 & 1 & 0 \end{bmatrix} + \begin{bmatrix} 6 \\ 1 \\ 0 \\ 0 \end{bmatrix} u$$

OCF

$$\begin{cases} \dot{x} = A^T x + C^T u \\ y = B^T x + D^T u \end{cases} \qquad y = \begin{bmatrix} 0 & 0 & 0 & 1 \end{bmatrix} x$$

③ $$[sI-(A-Bk)] = \begin{bmatrix} s & -1 & 0 & 0 \\ 0 & s & -1 & 0 \\ 0 & 0 & s & -1 \\ 1+k_1 & 4+k_2 & -3+k_3 & s+k_4 \end{bmatrix} \qquad A-Bk = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -1-k_1 & -4-k_2 & 3-k_3 & -k_4 \end{bmatrix}$$

$k_1 = 49$

$k_2 = 46$

$k_3 = 26$

$k_4 = 6$

$$\begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -1 & -4 & 3 & 0 \end{bmatrix} - \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} \begin{bmatrix} k_1 & k_2 & k_3 & k_4 \end{bmatrix}$$