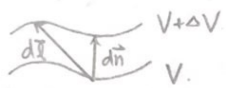
（裝 訂 線）

國立成功大學 學年度第 學期考試試卷 National Cheng Kung University Examination Sheet for Academic Year: Semester:			107 年 11 月 6 日 Year Month Day
姓名 Name		科目名稱 Subject Name	電磁學
學號 Student No.			教師簽章 Signature of Instructor
院系 College	工 學院 College 工科 系 Department 三 年 Year 班 Class	評閱成績 Score: 95	

Part A.

1、gradient (梯度)：在純量場中，純量隨空間的最大變化率及方向。 △詳述細節.

+13 +5



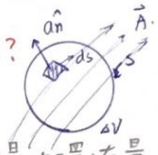
$$\left|\frac{\Delta V}{\Delta l}\right| < \left|\frac{\Delta V}{\Delta n}\right| \qquad \nabla V = \hat{a}_n \frac{dV}{dn}$$

2、divergence (散度)：從微小體積上，其外圍封閉曲面上的通量總和。

+4

$$\nabla\cdot\vec{A} = \lim_{\Delta V\to 0}\frac{\oint_S \vec{A}\cdot d\vec{s}}{\Delta V}$$

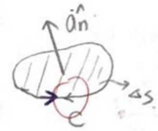
illustration?



3、curl (旋度)：向量場中，從微小面積上，其外圍contour的最大環流量及其相對應的方向

+4

$$\nabla\times\vec{A} = \lim_{\Delta s\to 0}\frac{[\hat{a}_n \oint \vec{A}\cdot d\vec{l}\,]_{max}}{\Delta s}$$



2、fundamental postulates of electrostatics in free space.

+18

[Differential form] [Integral form]

+3 ① $\nabla\cdot\vec{E} = \frac{\rho_V}{\varepsilon_0}$? ?

② $\nabla\times\vec{E} = 0$ +5

ε_0：真空介電係數

ρ_V：體電荷密度

+5 ① $\int_V \nabla\cdot\vec{E}\,dV = \int_V \frac{\rho_V}{\varepsilon_0}dV = \frac{Q}{\varepsilon_0}$

Divergence Theorem ↓ $\oint_S \vec{E}\cdot d\vec{s} = \frac{Q}{\varepsilon_0}$ (Gauss's Law)

在一個封閉曲面上電量的通量會等於封閉曲面內所有電量總和除以ε_0? 體電荷密度

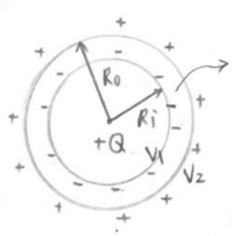
② $\int_S (\nabla\times\vec{E})\cdot d\vec{s} = 0$

Stoke's Theorem ↓ $\oint_C \vec{E}\cdot d\vec{l} = 0$ +5

在電場中沿任意封閉路徑移動電荷，作功量為0.

3、No. $\vec{E} = -\nabla V$ $\vec{E}=0$ 不代表V為0，是代表$\nabla V = 0$.

+10 Ex. 在靜電場中的金屬球殼



球殼中的電荷會被+Q吸引及排斥到導體表面，導致其內部沒有自由電荷存在 ⇒ $\rho_V = 0$ 由fundamental postulate $\nabla\cdot\vec{E} = \frac{\rho_V}{\varepsilon_0} = 0 \Rightarrow \vec{E} = 0$

∵ 在導體內部 $\vec{E}=0$ ∴ 移動電荷並不會使其電位能增加

$$V_1 = V_2 = \frac{Q}{4\pi\varepsilon_0 R_i}$$

由此即可證明，即使$\vec{E}=0$，並不代表V亦為0.

4、良好的金屬導體中沒有自由電荷

(續寫轉背頁)

+5 是因為其內部電荷會被吸引及排斥到導體表面，如上題所繪之圖形.

Part B. 1、

① $0 \le R \le b$. $\rho_v = -\rho$

$$\oint_s \vec{E}\cdot d\vec{s} = \frac{Q}{\varepsilon_0} = \frac{-\rho\left(\frac{4}{3}\pi R^3\right)}{\varepsilon_0}$$

+20

$$\vec{E} = E_R\,\hat{a}_R \qquad d\vec{s} = ds\,\hat{a}_R$$

$$\oint_s \vec{E}\cdot d\vec{s} = E_R(4\pi R^2) = \frac{-\rho\left(\frac{4}{3}\pi R^3\right)}{\varepsilon_0} \Rightarrow E_R = \frac{-\rho R}{3\varepsilon_0}$$

△ why 徑向 $\vec{E} = E_R\hat{a}_R$

$$\underline{\vec{E} = \frac{-\rho R}{3\varepsilon_0}\hat{a}_R} \ \divideontimes$$

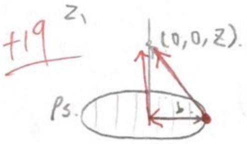
② $R > b$. $\rho_v = 0$.

$$\oint_s \vec{E}\cdot d\vec{s} = \frac{Q}{\varepsilon_0} = \frac{-\rho\left(\frac{4}{3}\pi b^3\right)}{\varepsilon_0}$$

$$\vec{E} = E_R\hat{a}_R \quad d\vec{s} = ds\,\hat{a}_R \quad \oint_s \vec{E}\cdot d\vec{s} = E_R(4\pi R^2) = \frac{-\rho\left(\frac{4}{3}\pi b^3\right)}{\varepsilon_0}$$

$$\Rightarrow E_R = \frac{-\rho b^3}{3\varepsilon_0 R^2} \qquad \underline{\vec{E} = \frac{-\rho b^3}{3\varepsilon_0 R^2}\hat{a}_R} \ \divideontimes$$

+19



$$\vec{E} = \frac{q}{4\pi\varepsilon_0 R^2}\hat{a}_R \qquad d\vec{E} = \frac{\rho_s\,ds'\,\vec{R}}{4\pi\varepsilon_0 R^3} \qquad \vec{R} = -r\hat{a}_r + z\hat{a}_z$$

$$R = \sqrt{r^2+z^2}$$

$$\vec{E} = \frac{\rho_s}{4\pi\varepsilon_0}\int_s \frac{-r\hat{a}_r + z\hat{a}_z}{\left(\sqrt{r^2+z^2}\right)^3}\,ds'$$

$$= \underbrace{\frac{\rho_s}{4\pi\varepsilon_0}\int_0^{2\pi}\int_0^b \frac{-r}{(r^2+z^2)^{3/2}}\,r\,dr\,d\phi\,\hat{a}_r}_{\text{會有對稱項互消}} + \frac{\rho_s}{4\pi\varepsilon_0}\int_0^{2\pi}\int_0^b \frac{z}{(r^2+z^2)^{3/2}}\,r\,dr\,d\phi\,\hat{a}_z$$

+15

$$= \frac{\rho_s}{4\pi\varepsilon_0}\int_0^{2\pi}\int_0^b \frac{zr}{(r^2+z^2)^{3/2}}\,dr\,d\phi\,\hat{a}_z \qquad 令\ r^2+z^2 = u$$

$$du = 2r\,dr \Rightarrow r\,dr = \frac{1}{2}du$$

$$= \frac{\rho_s}{4\pi\varepsilon_0}\int_0^{2\pi}\int_{z^2}^{z^2+b^2} z u^{-\frac{3}{2}} \times \frac{1}{2}\,du\,d\phi\,\hat{a}_z$$

$$= \frac{\rho_s}{4\pi\varepsilon_0} \times 2\pi \times \frac{1}{2} \times z \times \left(-2u^{-\frac{1}{2}}\right)\Big|_{z^2}^{b^2+z^2}\ \hat{a}_z$$

$$= \underline{-\frac{\rho_s z}{2\varepsilon_0}\left(\frac{1}{\sqrt{b^2+z^2}} - \frac{1}{\sqrt{z^2}}\right)\hat{a}_z} \ \divideontimes$$

if b is large.

$$\underline{\vec{E} \simeq -\frac{\rho_s z}{2\varepsilon_0}\left(\frac{1}{b} - \frac{1}{|z|}\right)\hat{a}_z} \ \divideontimes$$

0

+4

3. (a)

+10

$$\rho_{ps} = \vec{P}\cdot\hat{a}_n \qquad \hat{a}_n = \hat{a}_R$$

$$= P_0\hat{a}_x\cdot\hat{a}_R = \underline{P_0\sin\theta\cos\phi} \ \divideontimes$$

$$\rho_{pv} = -\nabla\cdot\vec{P} = -\frac{\partial}{\partial x}P_0 = \underline{0} \ \divideontimes$$

(b).

$$\oint_s \rho_{ps}\,ds = \int_0^{2\pi}\int_0^{\pi} P_0\sin\theta\cos\phi\ R_0^2\sin\theta\,d\theta\,d\phi$$

$$= P_0R_0^2\int_0^{2\pi}\cos\phi\,d\phi\int_0^{\pi}\sin^2\theta\,d\theta = \underline{0} \ \divideontimes$$

$$\int_v \rho_{pv}\,dV = \underline{0} \ \divideontimes$$