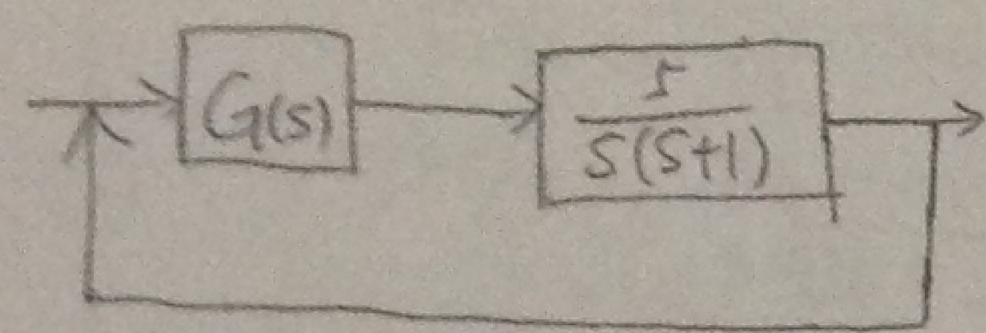
2009.

1. root locus, phase leading



$oltf(s) = k\frac{s+z}{s+p}\frac{5}{s(s+1)}$

令 $ts = 1s$ $\quad ts = \frac{4}{\xi w_n}$ $\quad \xi w_n = 4$

$w_d = \sqrt{1-\xi^2}\, w_n$

$\xi = \frac{PM}{100}$ $\quad PM = 45°$ $\quad \xi = 0.45$ $\quad w_n = 8.89$. $\cancel{-12.646} + 0 + -1 = -8 + x$

$w_d = \sqrt{1-0.45^2} \times 8.89 = 7.94$

$z = 4$

有 pole $= -4 \pm j7.94$

$\angle oltf(s)|_{s=-4+j7.94} = -180°$

$\angle k \cdot \frac{(-4+j7.94+4)}{-4+j7.94+p} \times \frac{5}{(-4+j7.94)(-3+j7.94)} = -180°$

① $\angle k = 0$

$\angle(-4+j7.94+4) = 90°$

$\angle(-4+j7.94+p) = ?$

$\angle(-4+j7.94) = 116.74$

$\angle(-3+j7.94) = 110.7$

$0 + 90° - ? - 116.74 - 110.7 = -180°$

$? = 42.56°$

0.918>6

$\tan^{-1}\frac{7.94}{P-4} = 42.56$

$P = 12.646$

$oltf(s) = -1$

~~$|oltf(s)|_{s=-4+j7.94}| = 1$~~

$-0.01776 - 885.83$

$\boxed{k \times \left|\frac{j7.94 \times 5}{(8.646+j7.94)(-4+j7.94)(-3+j7.94)}\right| = 1}$

$39.7k = 885.83$ $\quad k = 22.31$

Grant's law

$\Sigma cltpole = \Sigma$ open loop pole

驗證.

$\frac{PM}{100} = 0.45 = \xi$ ok!

$ess = \frac{1}{kv} = \frac{1}{\lim_{s\to 0} s \cdot oltf(s)}$

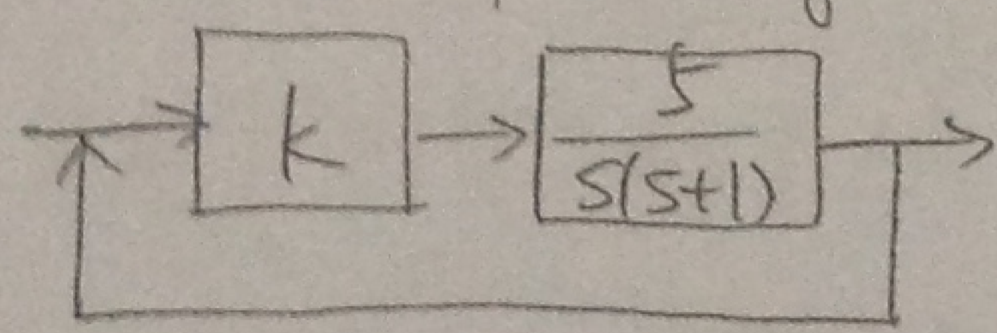
$= \frac{1}{22.31 \times \frac{4}{12.646} \times \frac{5}{1}} = 2.834\%$

$< 5\%$

可以

2.

Bode plot, phase lag



$e_{ss}|_{ramp} = 5\%$

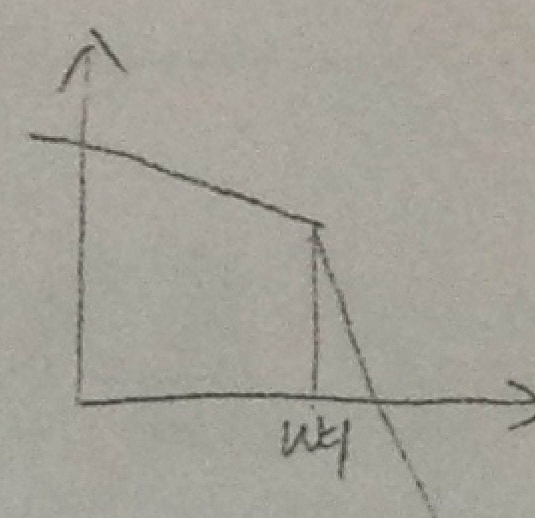
$k_v = 20$

$k_v = \lim_{s\to 0} s \cdot k \cdot \frac{5}{s(s+1)} = 5k \quad k=4$

$\left|\frac{20}{j\omega(j\omega+1)}\right| = \left|\frac{20}{-\omega^2+j\omega}\right| = 1$

$\omega^4+\omega^2 = 400$ $\quad 5\times 4 = 20$

$\omega = 4.41658, -4.41658$



$\angle \frac{20}{-4.41658^2+j4.41658} = -(180 - \tan^{-1}\frac{1}{4.41658})$

$= -179.99$

$PM = 180 + \cancel{179.99} = \cancel{0.01}$ 不夠

求讓 $\angle oltf(j\omega) = -135°$

$-(180 - \tan^{-1}\frac{1}{\omega}) = -135° \quad \omega_c = 1$

$|oltf(j\omega)|_{\omega=1} = \frac{20}{\sqrt{2}} = 10\sqrt{2} = 14.14$

$\frac{1}{14.14} \times \frac{\cancel{40}}{s(s+2)} \quad \frac{20}{s(s+1)}$

$oltf(s) = \frac{1}{14.14} \times \frac{s+\omega_2}{s+\omega_1} \times \frac{40}{s(s+2)}$

$\omega_2 = \frac{1}{10} = 0.1$

$\frac{\omega_2}{\omega_1} = 14.14 \quad \omega_1 = 7.072\times 10^{-3}$

$controller = \frac{1}{14.14} \times \frac{s+0.1}{s+7.072\times10^{-3}}$

驗証：① $k_v = \lim_{s\to 0} s \cdot \frac{1}{14.14} \times \frac{0.1}{7.072\times10^{-3}} \times 20 = 20$ ok.

② $PM = \angle \frac{20}{s(s+1)} = -135°$

$\angle \frac{1}{14.14} \times \frac{s+0.1}{s+7.072\times10^{-3}}\Big|_{s=j1} = \angle j+0.1 - \angle j+7.072\times10^{-3} = -5.67$

$PM = 45° - \cancel{0.974°}$ 不足 取 $\omega_2 = 0.05 \quad \omega_1 = 3.536\times10^{-3}$. $PM = 45° - 2.6598$

① phase leading locus

(i) $oltf(s) = k \frac{s+z}{s+p} \boxed{}$

(ii) $t_s = 1 \Rightarrow \omega_n \Rightarrow \omega_d$

(iii) $\angle oltf(s) = -180°$, $|oltf(s)| = 1$.

$\omega_d = \sqrt{1-\xi^2}\,\omega_n$

② phase lag, Bode

(i) k, $|oltf| = 1$, PM

(ii) $\phi = -135° \Rightarrow \omega \Rightarrow \alpha$

(iii) $\omega_2 = 0.1 \quad \frac{\omega_2}{\omega_1} = \alpha$

③ phase leading Bode

(i) $k \quad |oltf| = 1 \Rightarrow \omega \Rightarrow PM$

(ii) $\phi_m \Rightarrow \alpha$, $|oltf| = \frac{1}{\sqrt{\alpha}} \Rightarrow \omega_3, \omega_4$

④ phase lag locus

(i) $k \boxed{}\Big|_{s=-1+2j}^{\xi=0.47} = -1 \Rightarrow k$

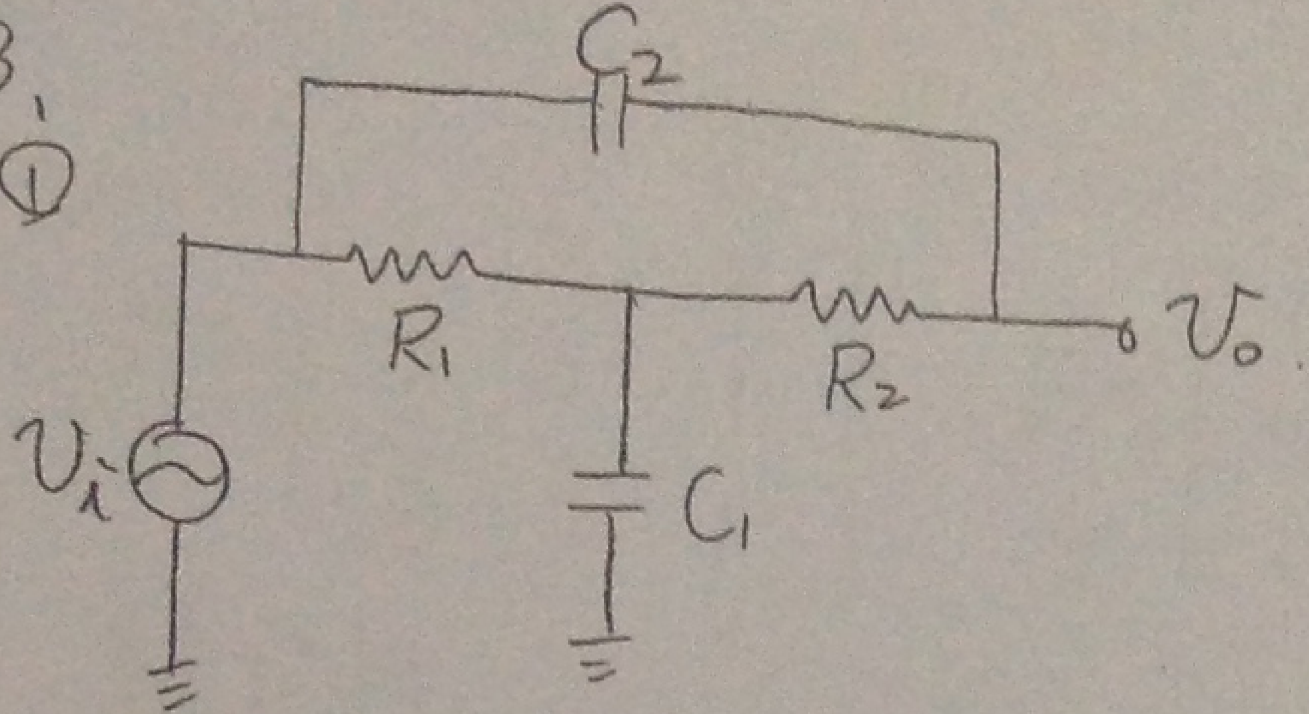
(ii) $\frac{s+z}{s+p} k \boxed{} \Rightarrow \frac{s+z}{s+p}$

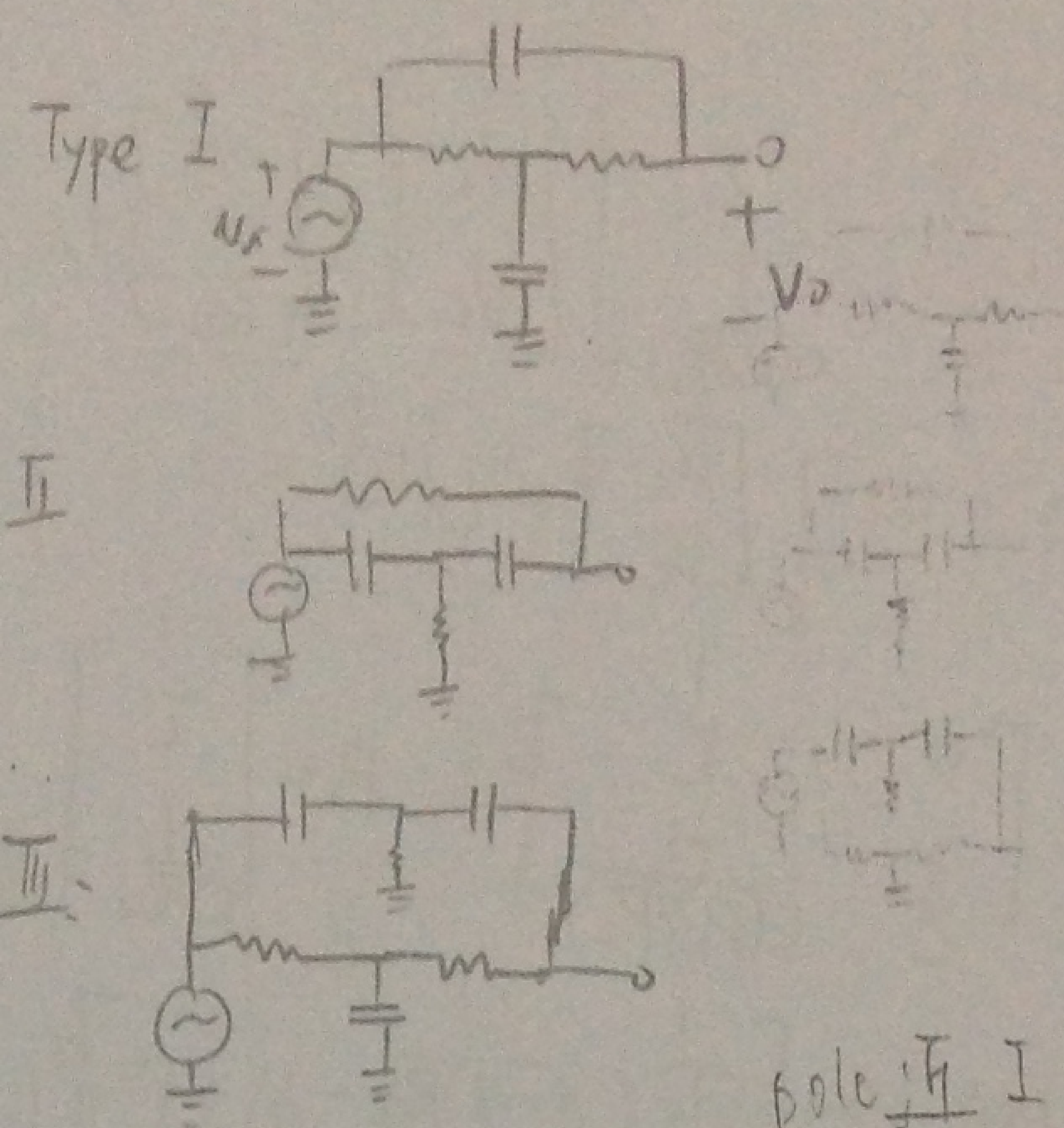
$\xi\omega_n = 4$

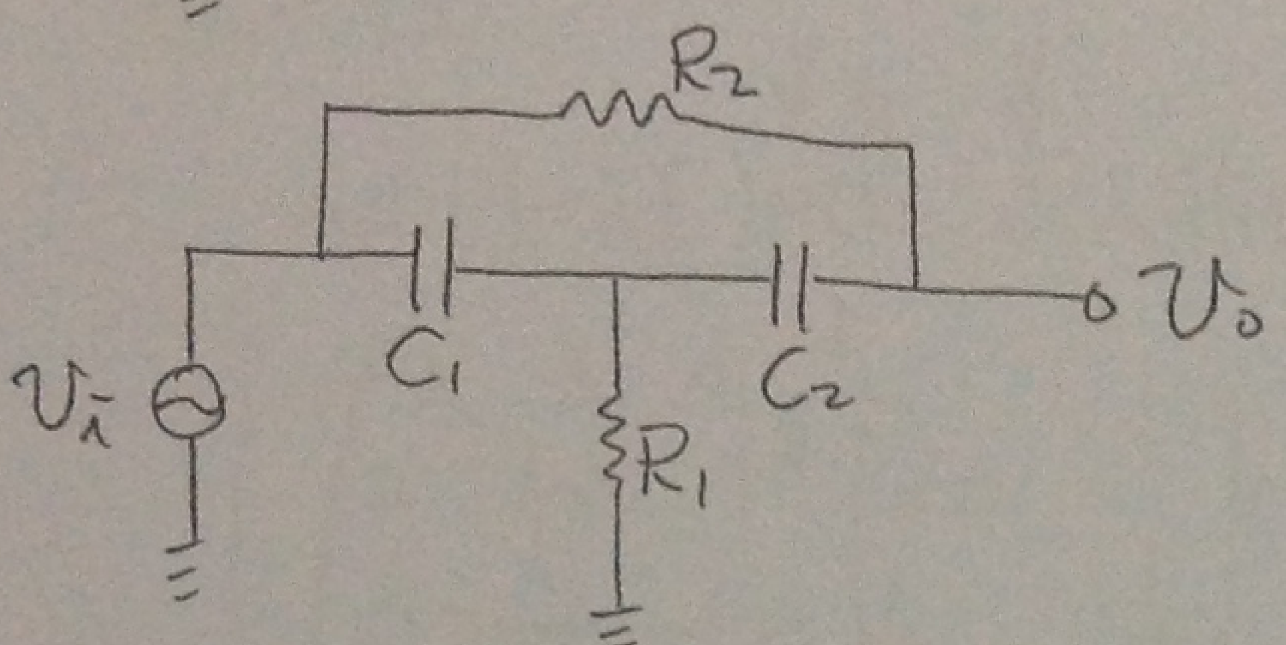
$t_s = 1 = \frac{4}{\xi\omega_n}$

2009.

3. ①

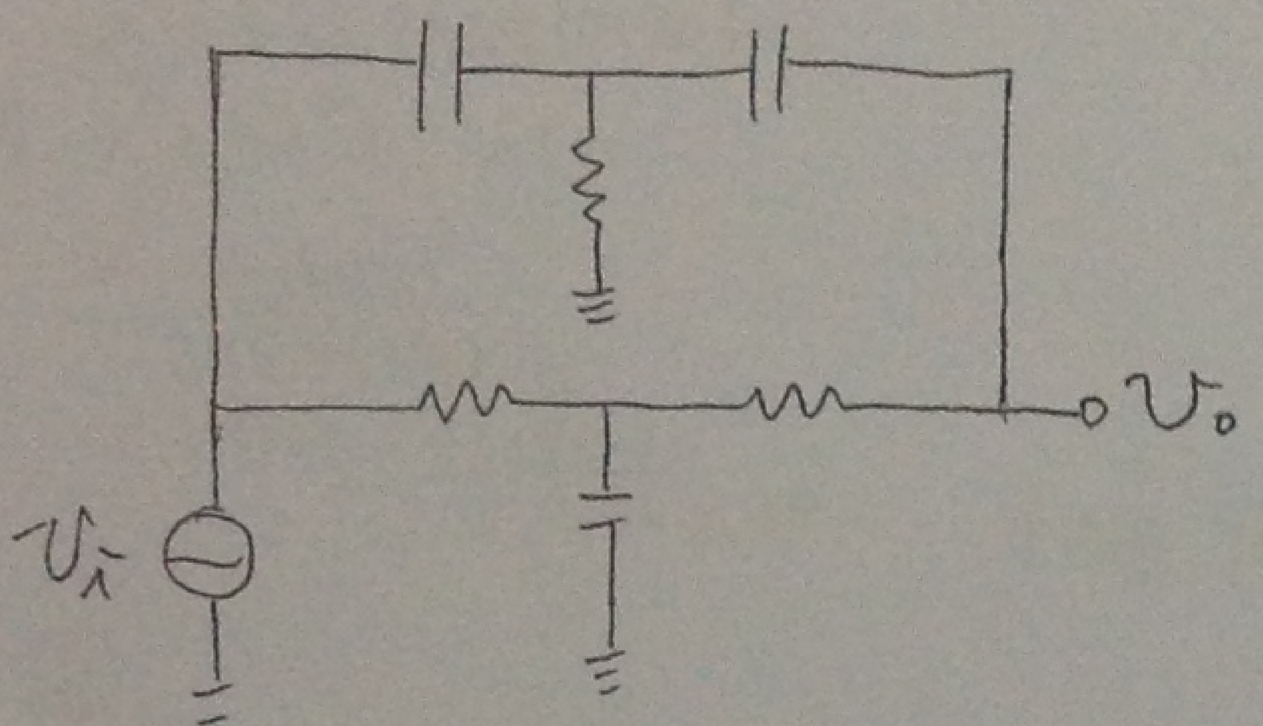






pole近I軸易震

→ Bridged-T 消去

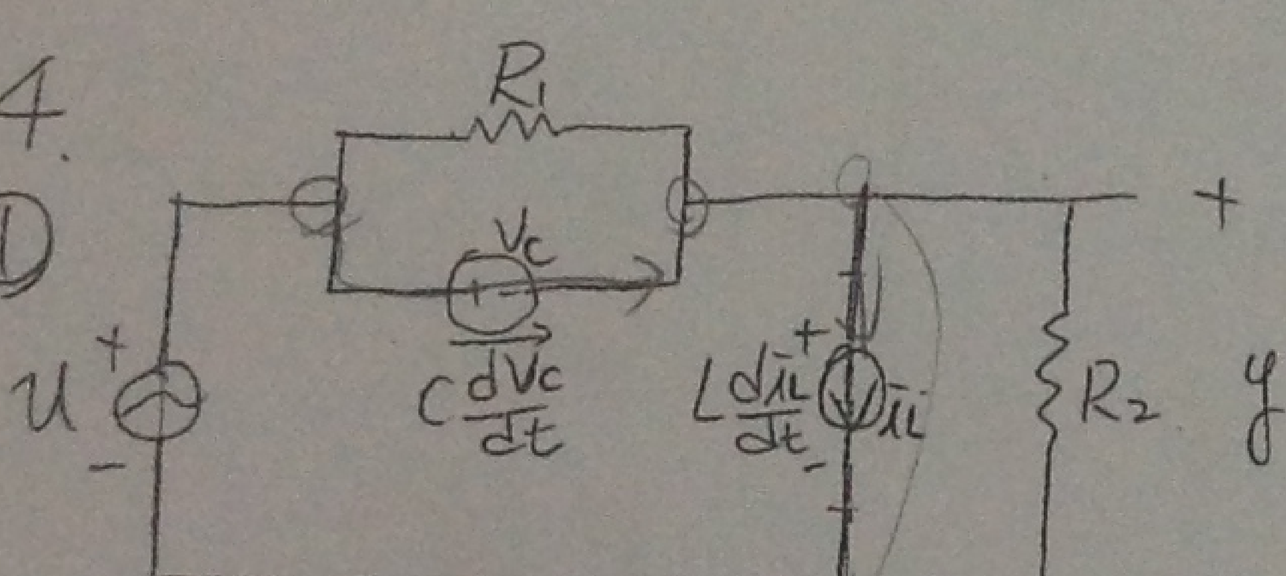


② 當系統有pole很靠近jω軸時，因為會易振，所以必需用Bridge-T 去消去pole pole-zero cancellation.

共軛zero

4.

①



$u - V_c = L\frac{di_L}{dt}$

$\frac{V_c}{R_1} + \frac{u-V_c}{R_2}$ $\quad \frac{V_c}{R_1} - \frac{V_c}{R_2}$ $\quad \frac{R_2-R_1}{R_1R_2}$

$$L\frac{di_L}{dt} = (0)i_L + (-1)V_c + (1)u \Rightarrow \frac{di_L}{dt} = 0\,i_L + \left(\frac{-1}{L}\right)V_c + \frac{1}{L}u$$

$$C\frac{dV_c}{dt} = (1)i_L + \left(-\frac{1}{R_1//R_2}\right)V_c + \left(\frac{1}{R_2}\right)u \Rightarrow \frac{dV_c}{dt} = \frac{1}{C}i_L + \frac{-1}{C(R_1//R_2)}V_c + \frac{1}{CR_2}u$$

$$y = L\frac{di_L}{dt} = 0\,i_L + (-1)V_c + u$$

$$\begin{bmatrix} \frac{di_L}{dt} \\ \frac{dV_c}{dt} \end{bmatrix} = \begin{bmatrix} 0 & -\frac{1}{L} \\ \frac{1}{C} & -\frac{1}{C(R_1//R_2)} \end{bmatrix} \begin{bmatrix} i_L \\ V_c \end{bmatrix} + \begin{bmatrix} \frac{1}{L} \\ \frac{1}{CR_2} \end{bmatrix} u$$

$$y = \begin{bmatrix} 0 & -1 \end{bmatrix} \begin{bmatrix} i_L \\ V_c \end{bmatrix} + u.$$

② $\begin{bmatrix} \frac{di_L}{dt} \\ \frac{dV_c}{dt} \end{bmatrix} = \begin{bmatrix} 0 & -\frac{1}{L} \\ \frac{1}{C} & \frac{-1}{C(R_1 /\!/ R_2)} \end{bmatrix} \begin{bmatrix} i_L \\ V_c \end{bmatrix} + \begin{bmatrix} \frac{1}{L} \\ \frac{1}{CR_2} \end{bmatrix} u$

$y = \begin{bmatrix} 0 & -1 \end{bmatrix} \begin{bmatrix} i_L \\ V_c \end{bmatrix} + u$

可控性:

$A = \begin{bmatrix} 0 & -\frac{1}{L} \\ \frac{1}{C} & \frac{-1}{C(R_1 /\!/ R_2)} \end{bmatrix} \quad B = \begin{bmatrix} \frac{1}{L} \\ \frac{1}{CR_2} \end{bmatrix}$ B AB

$\text{Rank}[B \vdots AB] = \begin{bmatrix} \frac{1}{L} & -\frac{1}{LR_2} \\ \frac{1}{CR_2} & \frac{1}{LC} - \frac{1}{R_2 C^2 (R_1 /\!/ R_2)} \end{bmatrix}$

$\begin{bmatrix} \frac{1}{L} & -\frac{1}{LCR_2} \\ \frac{1}{CR_2} & \frac{1}{CL} - \frac{1}{R_2 C^2 (R_1 /\!/ R_2)} \end{bmatrix}$

$\frac{-\frac{1}{LR_2}}{\frac{1}{L}} = \frac{\frac{1}{LC} - \frac{1}{R_2 C^2 (R_1 /\!/ R_2)}}{\frac{1}{R_2}} = -\frac{1}{R_2}$

只要 $\frac{1}{LC} - \frac{1}{R_2 C^2 (R_1 /\!/ R_2)} \neq -\frac{1}{R_2^2}$ 就可控.

可觀性:

$C = \begin{bmatrix} 0 & -1 \end{bmatrix}$

$\text{rank} \begin{bmatrix} C \\ CA \end{bmatrix} = \begin{bmatrix} 0 & -1 \\ -\frac{1}{C} & \frac{1}{C(R_1 /\!/ R_2)} \end{bmatrix} = 2$ 可觀 $C \neq 0$

5.

①

$$A=\begin{bmatrix}1&3&4\\2&1&1\\3&2&1\end{bmatrix}\quad B=\begin{bmatrix}1\\0\\1\end{bmatrix}\quad C=\begin{bmatrix}1&4&0\end{bmatrix}$$

穩定性

$A-\lambda I$

$\det(A-\lambda I)=0$ 求λ的位置

$$\begin{bmatrix}1-\lambda&3&4\\2&1-\lambda&1\\3&2&1-\lambda\end{bmatrix}=0$$

$(1-\lambda)^3+9+16-12(1-\lambda)-2(1-\lambda)-6(1-\lambda)=0$

$1-3\lambda+3\lambda^2-\lambda^3+25-12+12\lambda-2+2\lambda-6+6\lambda=0$

$-\lambda^3+3\lambda^2+17\lambda+6=0$

$\lambda=-0.38196$ or $\dfrac{-3.38196\pm\sqrt{24.271}}{-2}$

$\dfrac{3.38196\pm\sqrt{24.271}}{2}$

$-\lambda^2+3.38196\lambda+15.70823$

$\lambda+0.38196\,\big)\,-\lambda^3+3\lambda^2+17\lambda+6$

$-\lambda^3-0.38196\lambda^2$

$3.38196\lambda^2+17\lambda$

$3.38196\lambda^2+1.29177\lambda$

$15.70823\lambda+6$

不穩定

②

$$\text{rank}\,[B\,\vdots\,AB\,\vdots\,A^2B]=\text{rank}\begin{bmatrix}1&5&30\\0&3&17\\1&4&25\end{bmatrix}=3$$

可控性

③

$$\text{rank}\begin{bmatrix}C\\CA\\CA^2\end{bmatrix}=\begin{bmatrix}1&4&0\\9&7&8\\47&50&51\end{bmatrix}=3$$

可觀性

$\det[SI-A]\Rightarrow s^n\cdots$

$S=[B\,\vdots\,AB\;A^2B\cdots$

M

$P=SM$

$C'=CP$

④

$$\det[SI-A]=\begin{bmatrix}s-1&-3&-4\\-2&s-1&-1\\-3&-2&s-1\end{bmatrix}=(s-1)^3-9-16-12(s-1)-2(s-1)-6(s-1)$$

$=s^3-3s^2+3s-1-25-12s+12-2s+2-6s+6$

$=s^3-3s^2-17s-6$

$$S=[B\,\vdots\,AB\,\vdots\,A^2B]=\begin{bmatrix}1&5&30\\0&3&17\\1&4&25\end{bmatrix}$$

$$M=\begin{bmatrix}-17&-3&1\\-3&1&0\\1&0&0\end{bmatrix}\qquad P=SM=\begin{bmatrix}-2&2&1\\8&3&0\\-4&1&1\end{bmatrix}$$

$$newC = CP = \begin{bmatrix} 1 & 4 & 0 \end{bmatrix} \begin{bmatrix} -2 & 2 & 1 \\ 8 & 3 & 0 \\ -4 & 1 & 1 \end{bmatrix} = \begin{bmatrix} 30 & 14 & 1 \end{bmatrix}.$$

$$\dot{x} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 6 & 17 & 3 \end{bmatrix} x + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} u$$

$$y = \begin{bmatrix} 30 & 14 & 1 \end{bmatrix} x.$$