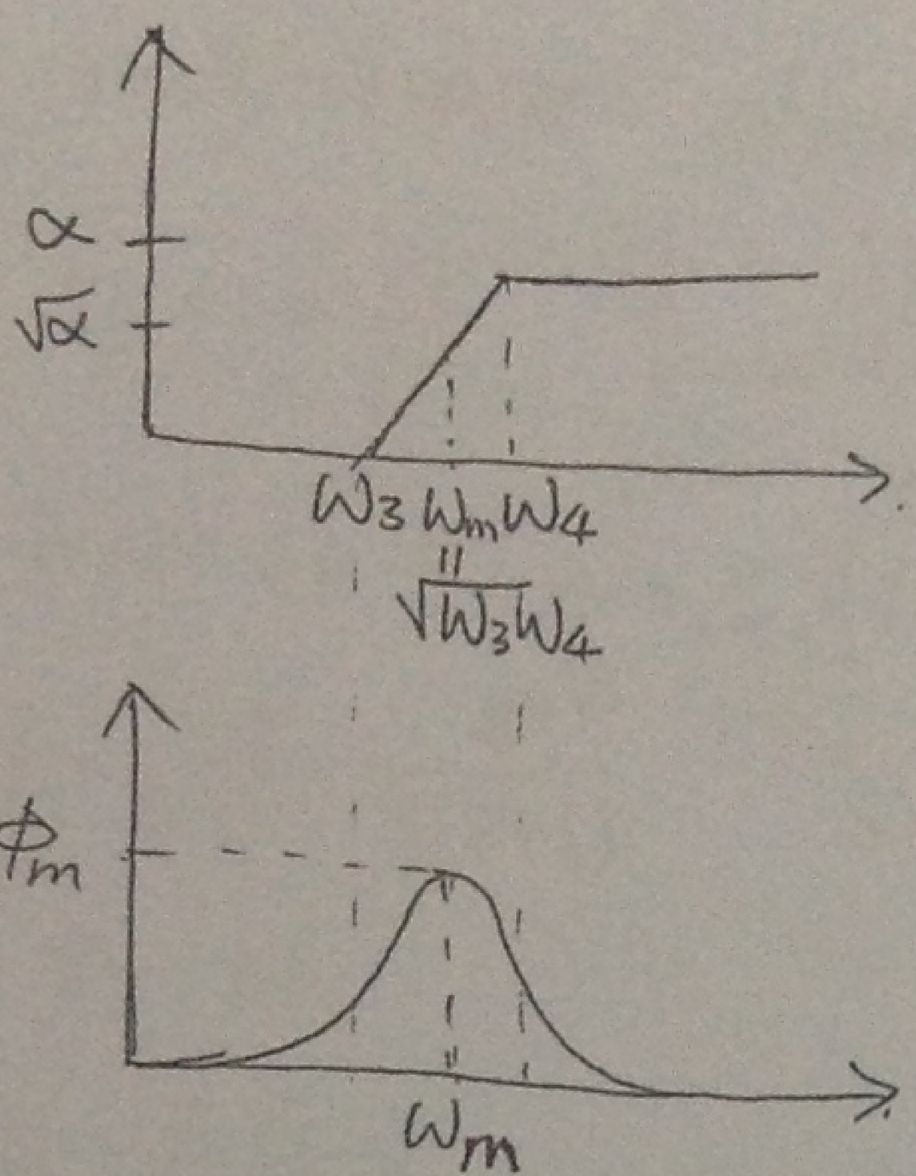
…期末考
…則不予計分・線…

2010

1、

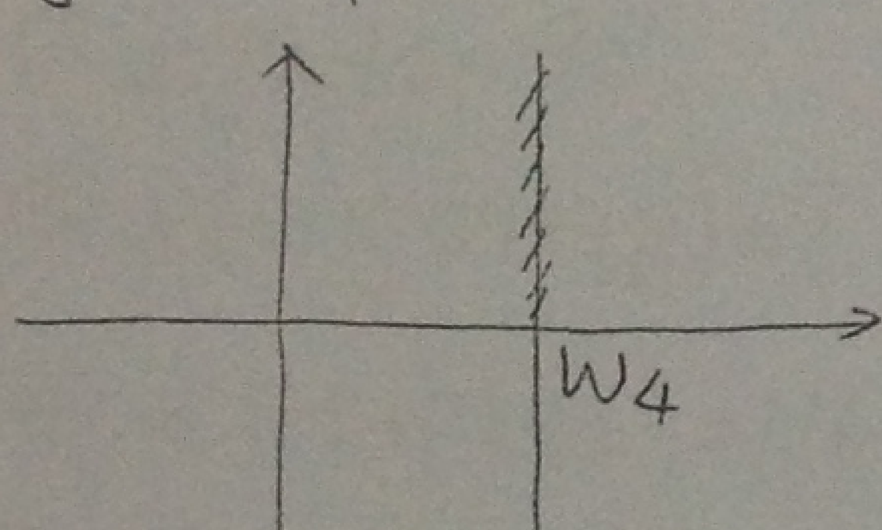
①



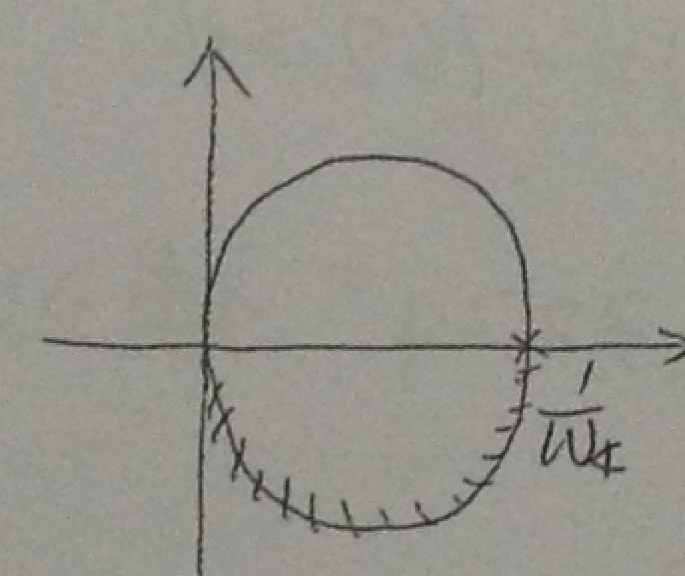
②

$$G(s) = \alpha \frac{s+w_3}{s+w_4} \qquad G(jw) = \alpha \frac{jw+w_3}{jw+w_4} = \alpha\left(1 - \frac{w_4 - w_3}{jw+w_4}\right)$$

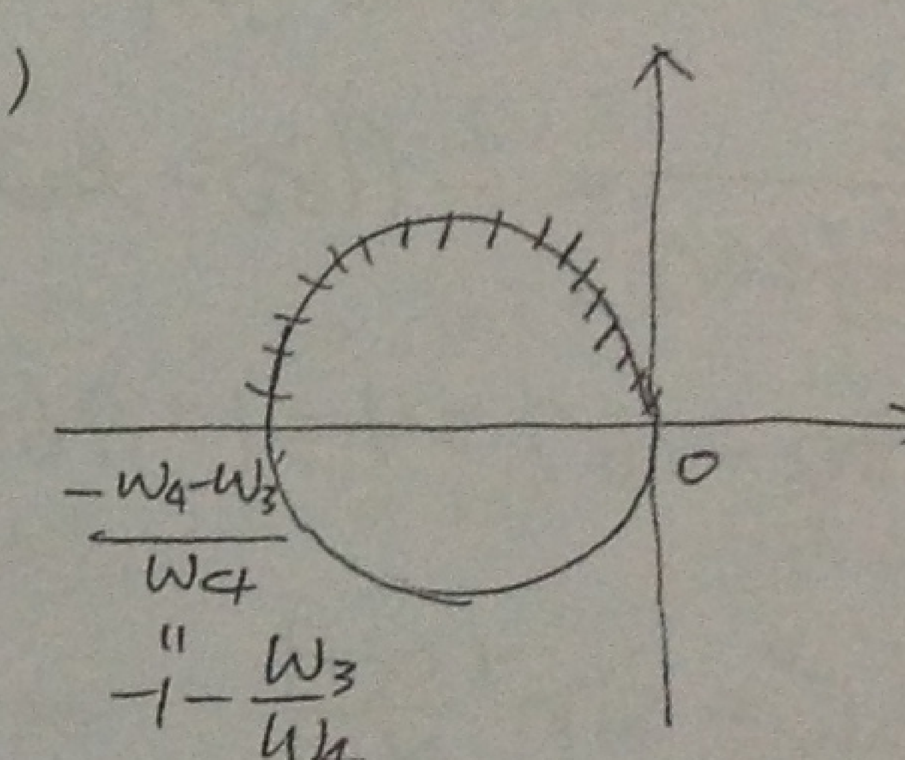
(1) $jw + w_4$



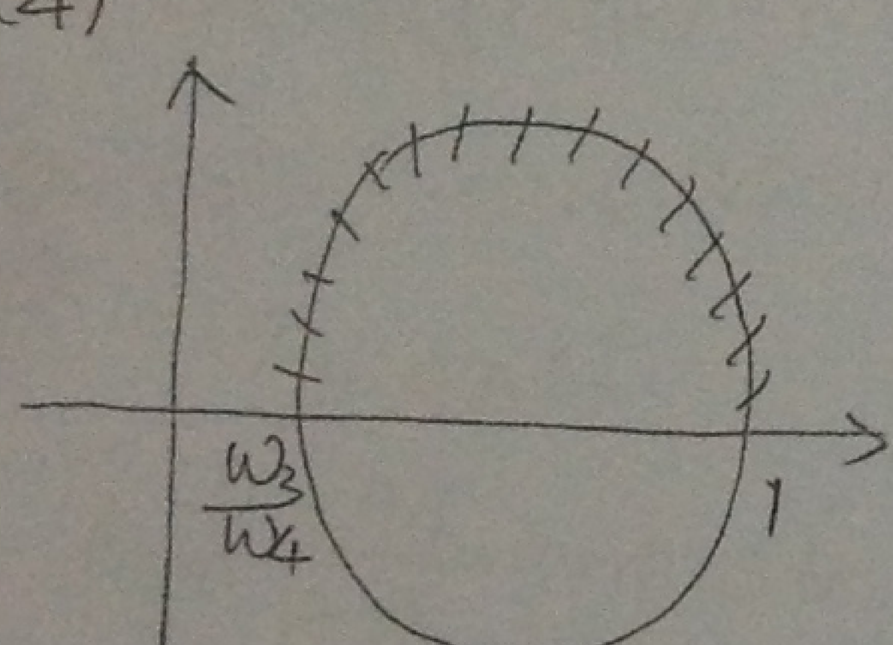
(2)



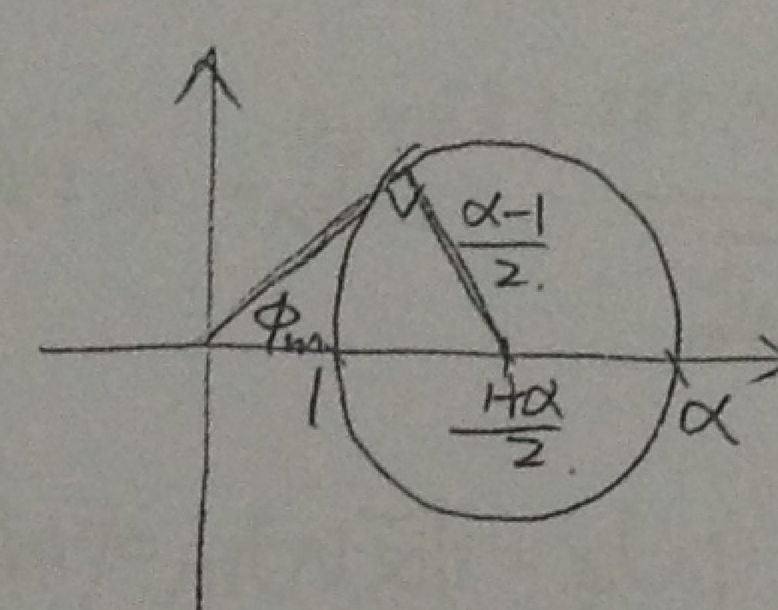
(3)



(4)



(5)



在垂直時 ϕ_m 有最大值. $\sin\phi_m = \frac{\alpha-1}{\alpha+1}$

$$\phi_m = \sin^{-1}\frac{\alpha-1}{\alpha+1}$$

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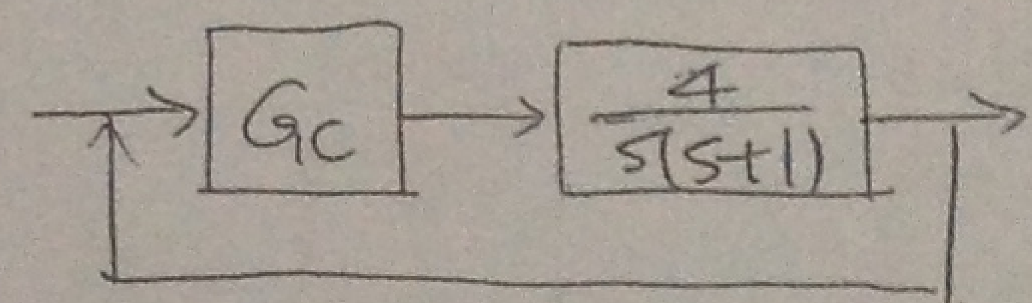
2. Bode plot phase leading



① $e_{ss}|_{ramp} = 5\%$

$k_v = 20$

$\lim_{s\to 0} sk\frac{4}{s(s+1)} = 20 \quad k=5$

② $\frac{20}{s(s+1)}$ 求PM.

$\left|\frac{20}{s(s+1)}\right| = 1 = \frac{|20|}{|-w^2+jw|} \quad \sqrt{w^4+w^2}=20 \quad w^4+w^2-400=0. \quad w=4.417.$

$\angle \frac{20}{-4.417^2+j4.417} = -(180^\circ - \tan^{-1}\frac{1}{4.417}) = -167.24$

$PM = 180-167.24 = 12.76$ 不足32.24° 補35°

$\phi_m = 35^\circ = \sin^{-1}\frac{\alpha-1}{\alpha+1} \quad \alpha = 3.69$

$\frac{1}{\sqrt{3.69}} = \left|\frac{20}{jw(jw+1)}\right| = \frac{20}{\sqrt{w_m^4+w_m^2}} \quad w_m = 6.158.$

$w_3 = \frac{w_m}{\sqrt{\alpha}} = \frac{6.158}{\sqrt{3.69}} = 3.21 \qquad w_4 = \sqrt{\alpha}\, w_m = 11.829$

$controller = 5\times 3.69 \times \frac{s+3.21}{s+11.829} = 18.45\times\frac{s+3.21}{s+11.829}$

驗證:

① $k_v = \lim_{s\to 0} s \cdot 18.45\times\frac{s+3.21}{s+11.829}\times\frac{4}{(s+1)s} = \underline{20}$ ok.

② $PM = \angle\, 18.45\times\frac{j\times 6.158+3.21}{j6.158+11.829}\times\frac{4}{-6.158^2+j6.158}$

$= \tan^{-1}\frac{6.158}{3.21} - \tan^{-1}\frac{6.158}{11.829} - (180^\circ - \tan^{-1}\frac{1}{6.158}) = -135.808$

$PM = 44.19$ 不足 -0.8° 補40°再做一次

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3. root locus phase leading

$t_s = 1 = \frac{4}{\xi\omega_n}$ $\quad \xi\omega_n = 4$ $\quad \xi = \frac{PM}{100} = 0.45$ $\quad \omega_n = 8.89$

$\omega_d = \sqrt{1-\xi^2}\,\omega_n = \sqrt{1-0.45^2}\times 8.89 = 7.94$

$k\times\frac{s+z}{s+p}\,\frac{4}{s(s+1)}$ $\quad s = -4 \pm j7.94$

$z = +4$

$\angle oltf(s)|_{s=-4+j7.94} = -180^\circ$

$k\times\frac{j7.94}{(-4+j7.94+p)}\times\frac{4}{(-4+j7.94)(-3+j7.94)} = -180^\circ$

① $\angle k = 0^\circ$

② $\angle j7.94 = 90^\circ$

③ $\angle(-4+j7.94+p) = ?$

④ $\angle(-4+j7.94) = 116.74$

⑤ $\angle(-3+j7.94) = 110.7.$

$0.068+885.886$

$0+90^\circ - ? - 116.74 - 110.7 = -180^\circ$

$? = 42.56$

$\tan^{-1}\left(\frac{7.94}{p-4}\right) = 42.56 \quad p = 12.647$

$\left| k\times\frac{s+4}{s+12.647}\times\frac{4}{s(s+1)}\Big|_{s=-4+j7.94}\right| = 1$

$k\times\frac{|j7.94\times 4|}{|(8.647+j7.94)(-4+j7.94)(-3+j7.94)|} = 1$

$31.76k = 885.886$

$k = 27.89$

$\text{controller} = 27.89\times\frac{s+4}{s+12.647}\times\frac{4}{s(s+1)}$

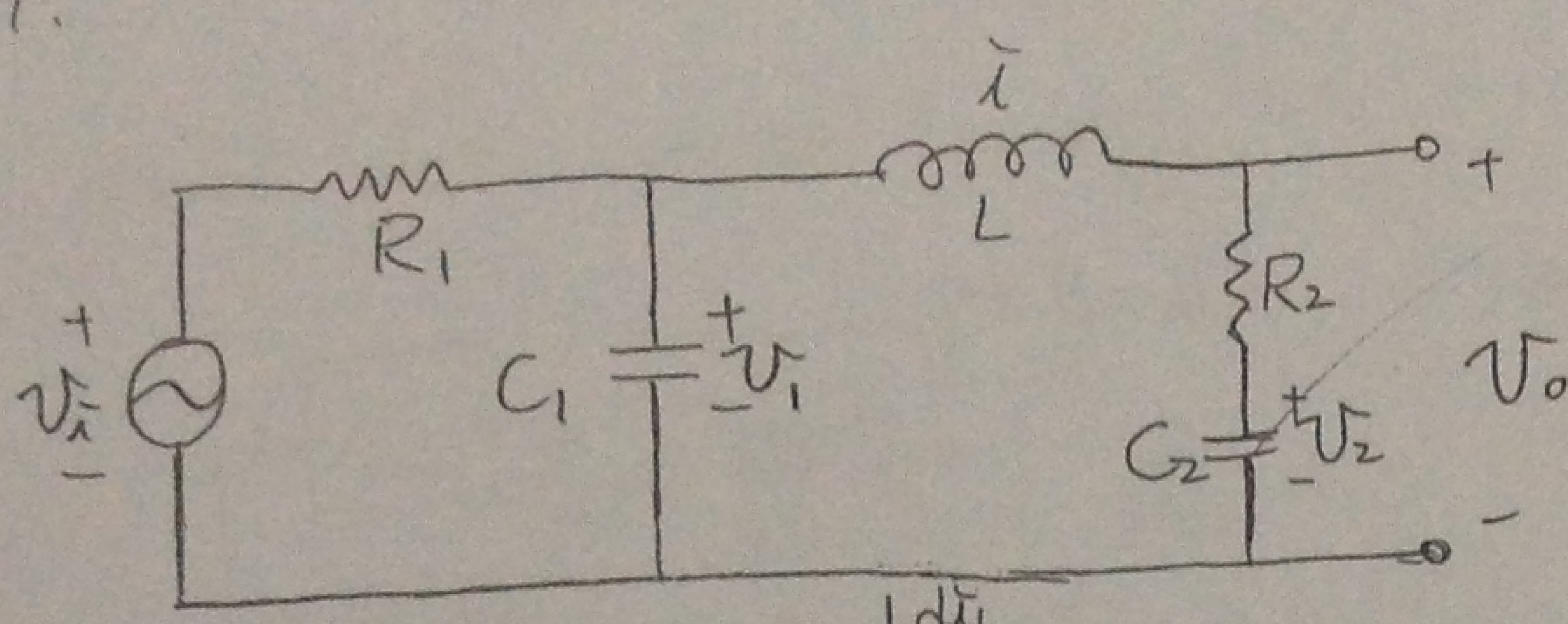
驗證:

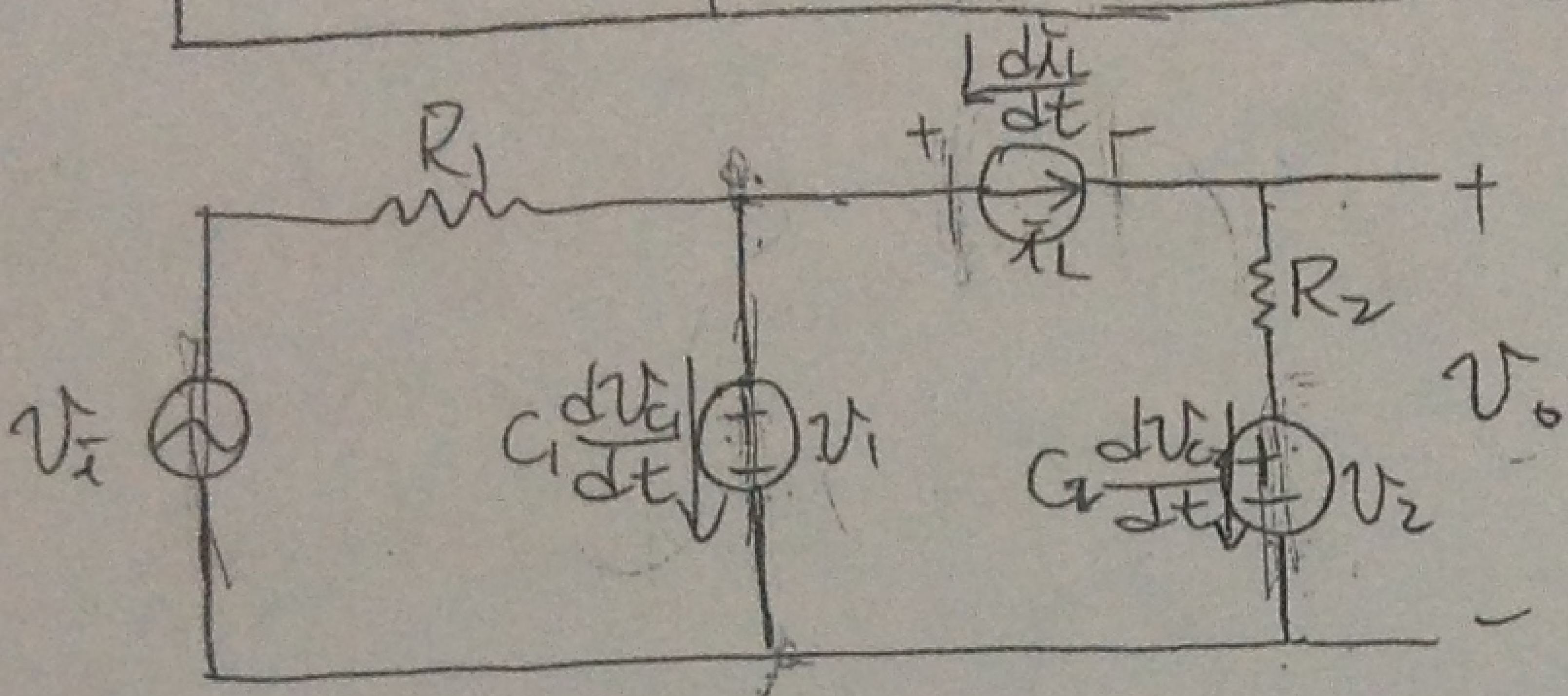
① $PM = \xi\times 100 = 45^\circ$ ok

② $k_v = \lim_{s\to 0} s\cdot 27.89\times\frac{s+4}{s+12.647}\times\frac{4}{s(s+1)} = 35.284.$

$e_{ss} = \frac{1}{k_v} = \frac{1}{35.284} = 2.834\% < 5\%$

4.





$$L\frac{di_L}{dt} = -R_2\, i_L + 1\, v_1 + -1\, v_2 + 0\, v_i$$

$$C_1\frac{dv_1}{dt} = -1\, i_L + -\frac{1}{R_1}\, v_1 + 0\, v_2 + \frac{1}{R_1}\, v_i$$

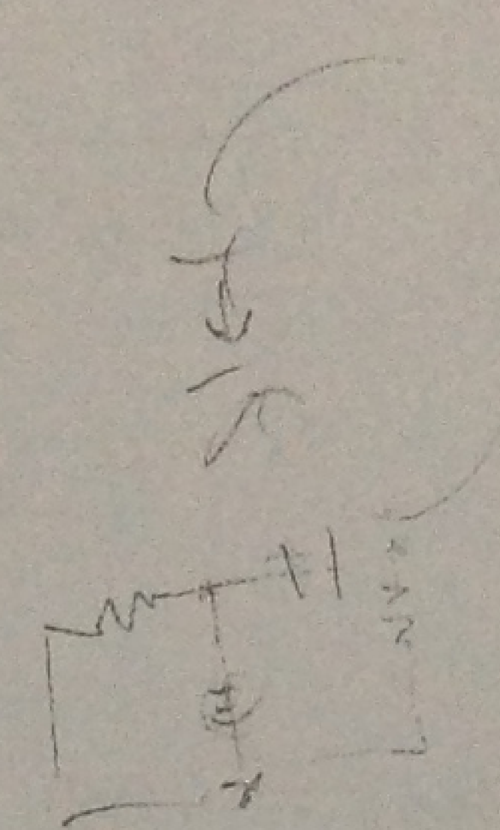
$$C_2\frac{dv_2}{dt} = 1\, i_L + 0\, v_1 + 0\, v_2 + 0\, v_i$$

$$v_0 = v_1 - L\frac{di_L}{dt} = R_2 i_L + v_2$$

$$\begin{bmatrix} \frac{di_L}{dt} \\ \frac{dv_1}{dt} \\ \frac{dv_2}{dt} \end{bmatrix} = \begin{bmatrix} -\frac{R_2}{L} & \frac{1}{L} & \frac{-1}{L} \\ \frac{-1}{C_1} & \frac{-1}{R_1C_1} & 0 \\ \frac{1}{C_2} & 0 & 0 \end{bmatrix} \begin{bmatrix} i_L \\ v_1 \\ v_2 \end{bmatrix} + \begin{bmatrix} 0 \\ \frac{1}{C_1R_1} \\ 0 \end{bmatrix} v_i$$

$$v_0 = \begin{bmatrix} R_2 & 0 & 1 \end{bmatrix} \begin{bmatrix} i_L \\ v_1 \\ v_2 \end{bmatrix}$$

lag L



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5.

① $x(t)$ is controllable at $t=t_0$ if there exists a piecewise continuous input $u(t)$, that will drive the state $x(t_0)\to x(t_f)$ a finite time $t_f \geq t_0$. 輸入片段連續 $u(t)$ 可使 $x(t_0)$ 在有限時間到 $x(t_f)$ 即為可控

② $x(t_0)$ is observable if given any input $u(t)$, there exists a finite time $t_f \geq t_0$ such that knowledge of $\begin{cases}u(t)\\y(t)\end{cases}$ for $t_0 \leq t \leq t_f$ are sufficient to determine $x(t_0)$

輸入任何 $u(t)$ 可使 $u(t)$, $y(t)$ 決定 $x(t_0)$, 即為可觀 $t_f \geq t \geq t_0$

③ $A=\begin{bmatrix}1&2&3\\0&1&4\\1&3&1\end{bmatrix}$ $B=\begin{bmatrix}1\\1\\2\end{bmatrix}$ $C=\begin{bmatrix}1&0&2\end{bmatrix}$

可控性:

$$\text{rank}\left[B \vdots AB \vdots A^2B\right]=\begin{bmatrix}1&9&45\\1&9&33\\2&6&42\end{bmatrix}=3$$

可控

可觀性:

$$\text{rank}\begin{bmatrix}C\\CA\\CA^2\end{bmatrix}=\begin{bmatrix}1&0&2\\3&8&5\\8&29&46\end{bmatrix}=3$$

可觀。

穩定性:

$$\det\begin{bmatrix}1-\lambda&2&3\\0&1-\lambda&4\\1&3&1-\lambda\end{bmatrix}=0$$

$$(1-\lambda)^3+8-3(1-\lambda)-12(1-\lambda)=0$$

$$1-3\lambda+3\lambda^2-\lambda^3+8-3+3\lambda-12+12\lambda=0$$

$$-\lambda^3+3\lambda^2+12\lambda-6=0$$

$$\lambda=0.4559,\ \frac{-2.5441\pm\sqrt{60.868}}{-2}$$

$$\begin{array}{r}
-\lambda^2+2.5441\lambda+13.599\\
\lambda-0.4559\ \overline{)\,-\lambda^3+3\lambda^2+12\lambda-6}\\
-\lambda^3+0.4559\lambda^2\\
\hline
2.5441\lambda^2+12\lambda\\
2.5441\lambda^2-1.599\lambda\\
\hline
13.599\lambda-6
\end{array}$$

不穩定

6.

① $$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \\ \dot{x}_4 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -1 & -4 & 3 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} u.$$

$$y = \begin{bmatrix} 6 & 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix}$$

② $$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \\ \dot{x}_4 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 & -1 \\ 1 & 0 & 0 & -4 \\ 0 & 1 & 0 & 3 \\ 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} + \begin{bmatrix} 6 \\ 1 \\ 0 \\ 0 \end{bmatrix} u$$

$$y = \begin{bmatrix} 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix}$$