EXAM I
HEAT TRANSFER

May 10, 2013

I. The one-dimensional Fourier law can be written as

$$q'' = -k\frac{\partial T}{\partial z}$$

(1) What are q'', k and $\partial T/\partial z$? (6%)

(2) What is the meaning of the negative sign in the equation? (3%)

(3) If the Fourier law is re-written as

$$q'' = k\frac{\partial T}{\partial z},$$

is it correct? Why or Why not? (3%)

II. Answer the following questions: (33%)

(1) From the viewpoint of heat transfer, how does heattech clothing (發熱衣) work?

(2) In what conditions can the heat conduction equation be written as

$$k\frac{\partial T}{\partial x} = \text{constant ?}$$

(3) Why does a refrigerator need defrosting?

(4) Why do tanks（戰車）use air cooling instead of water cooling?

(5) When a fin is attached to a high-temperature body, can it be regarded as a thermal resistance to the body? Why or Why not?

(6) Write down the finite difference expression of $\partial T/\partial t$ for the point (i, j) using the forward difference.

(7) What is the special heat-conduction feature of graphite?

(8) In solving a 2-D steady heat conduction problem, the variable separation is used. It is assumed $T = X(x)Y(y)$. Put this relation into the energy equation and the equation can rearranged as

$$-\frac{1}{X}\frac{d^2X}{dx^2} = \frac{1}{Y}\frac{d^2Y}{dy^2} = \text{constant}$$

Why are they equal to constant?

(9) What are the two kinds of error by using the finite difference to simulate the heat transfer problems?

(10) Why is it said that the flowers and trees facing the south can easily feel spring coming（向南花木易爲春）?

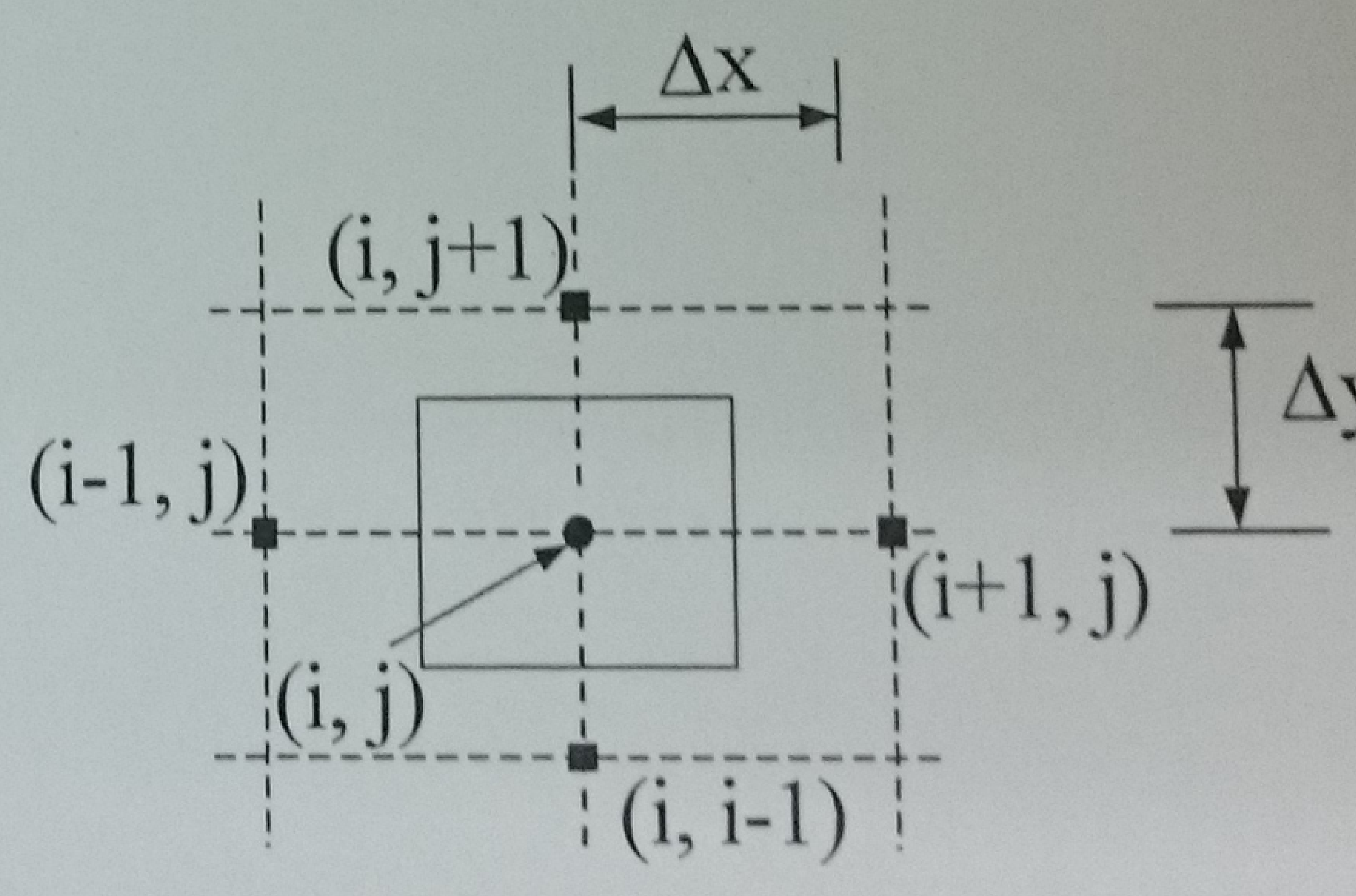
(11) A heat-insulated layer is put between two solids with different temperatures. Why can the layer not completely block the heat transfer between the two solids?

III. A plane wall of thickness 2L, thermal conductivity k, and diffusivity α (shown in the following figure) has an initial temperature of T_i and the surrounding temperature of T_∞.The fluid convective environment is such that h >> k/L. The governing heat conduction equation is

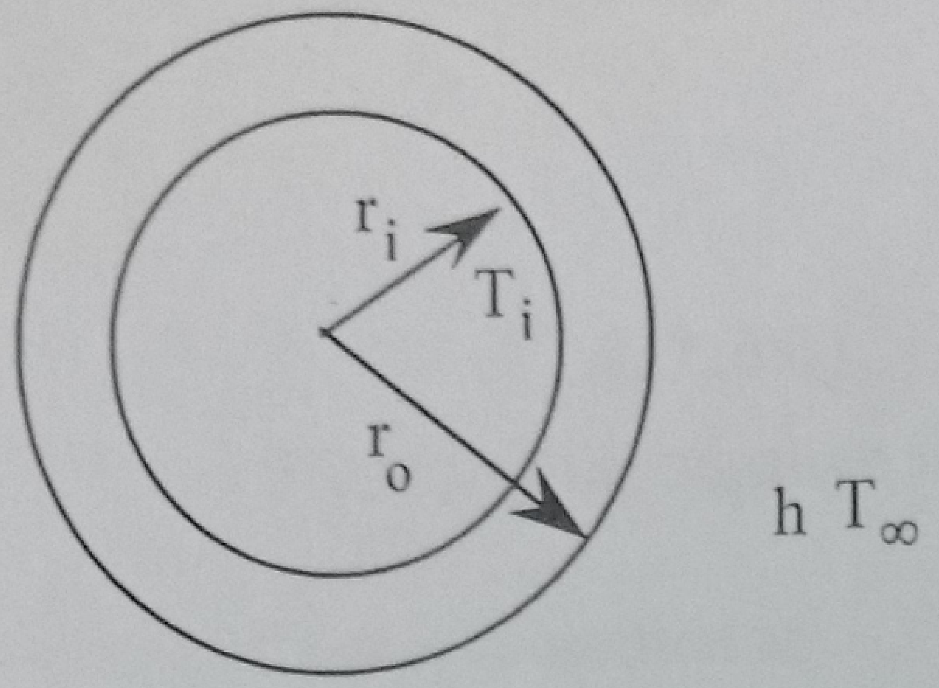
$$\frac{\partial^2 T}{\partial x^2} = \frac{1}{\alpha}\frac{\partial T}{\partial t}$$

By using superposition, separation of variables, and appropriate boundary and initial conditions, obtain an expression for the transient temperature profile. (12%)

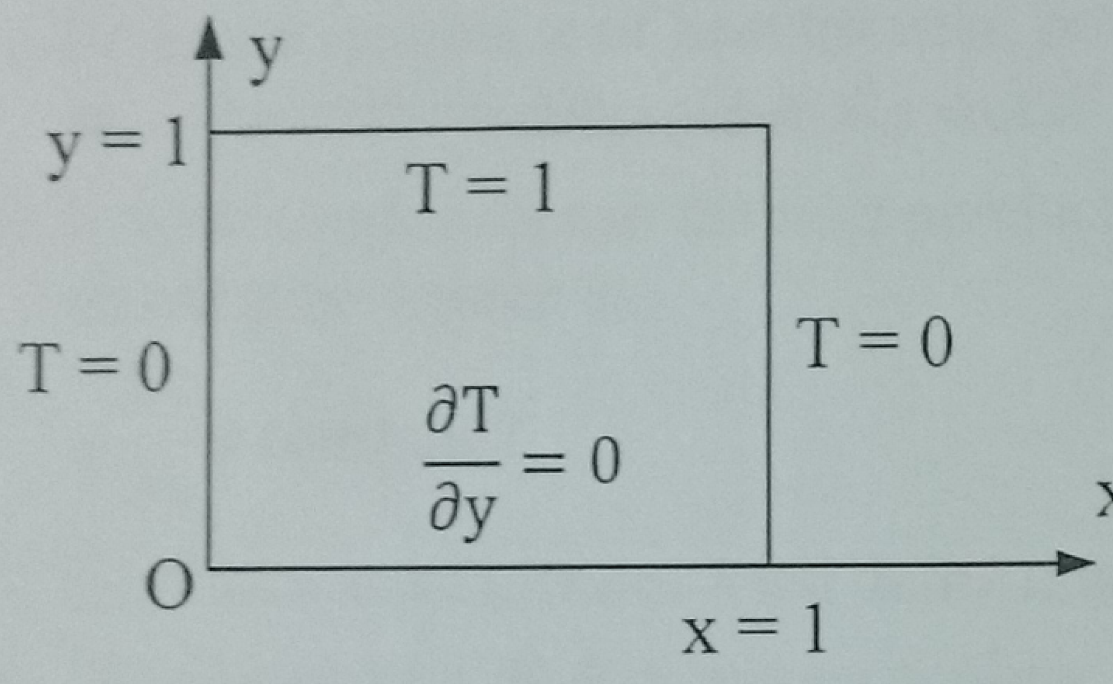
IV. For a 2-D steady heat conduction problem without heat source, please derive the finite difference equation for point (i, j) shown in the following figure by considering the energy conservation in the control volume of that point (the shadow area). (8%)



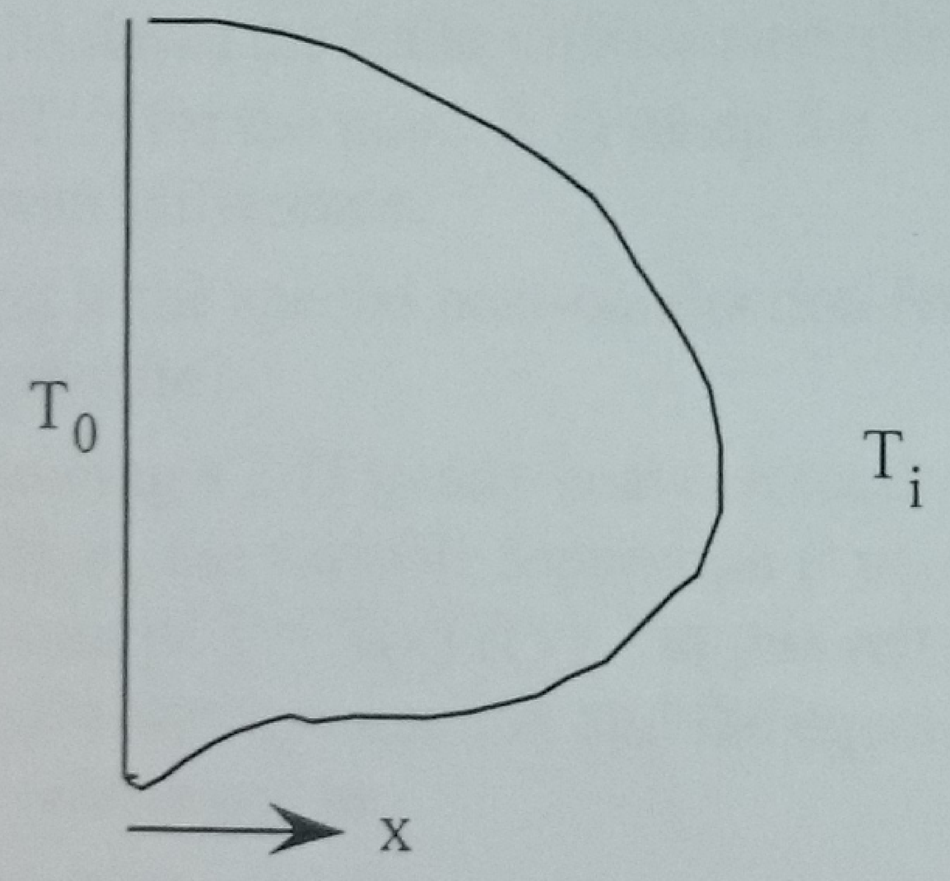
V. Consider a layer of insulation which is installed around a circular pipe (see the following figure). The outer radius of the pipe is r_i and the outer radius of the insulation is r_o. Prove that the critical radius of insulation is $r_o = k/h$. (8%)



VI. Consider a steady-state heat conduction problem in a rectangular plate. Its boundary conditions are shown in the following figure. Find the temperature solution of the plate. (12%)

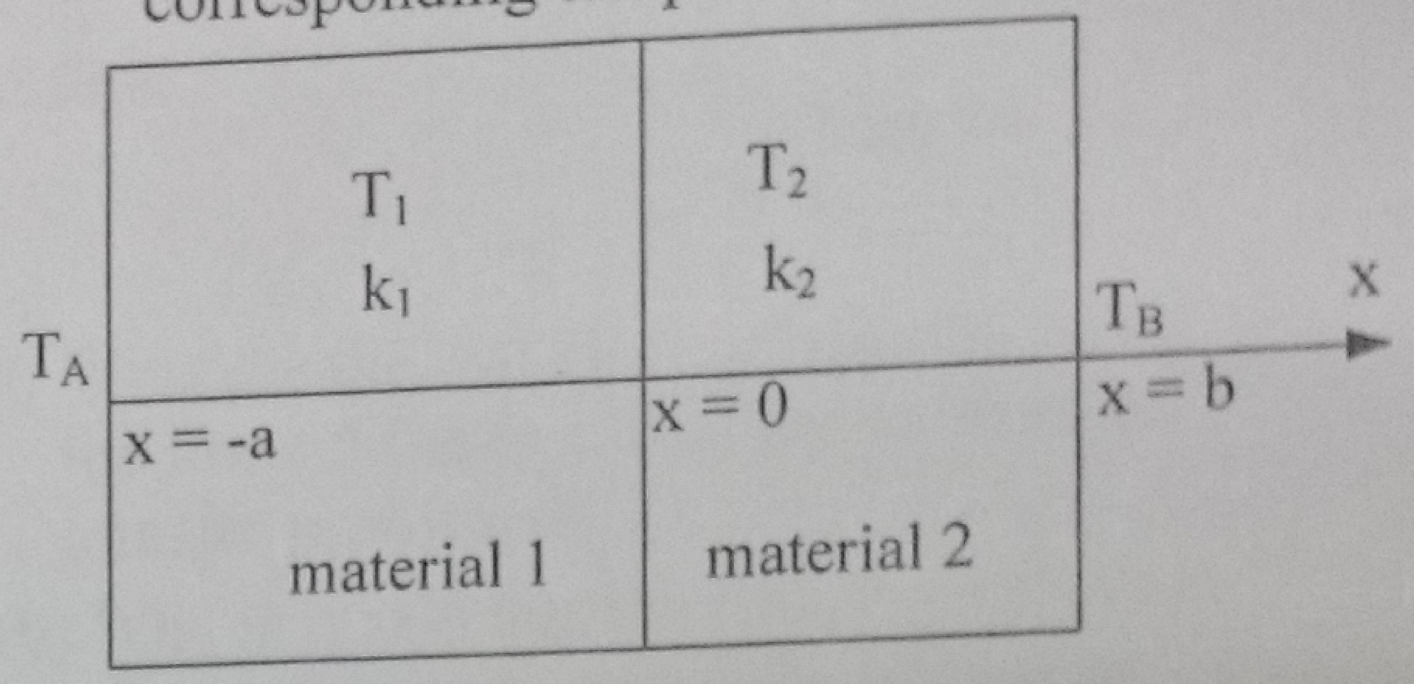


VII. Consider a semi-infinite solid shown in the following figure maintained at some initial temperature T_i. The surface temperature is suddenly lowered and maintain at a temperature T_0.



Solve the temperature distribution for this transient problem. (10%)

VIII. In the following figure, two materials with k_1 and k_2 thermal conductivities have perfect contact and their corresponding temperatures are T_1 and T_2.



(i) Write down the boundary conditions at x=0, x=a and x=b. (6%)

(ii) Solve the T_1 and T_2 under the assumptions of 1-D steady state with no heat source and constant k_1 and k_2. (8%)

IX. One side of a plane wall is maintained at 100°C, while the other side is exposed to a convection environment having T = 10°C and h = 12 W/(m^2·°C). The wall has k = 1.2 W/(m·°C) and is 40 cm thick. Calculate the heat flux through the wall. (10%)

X. A one-dimensional steady heat transfer problem of a cylinder with a uniform heat source $\dot{q}$ is shown in the following figure. What is the heat flux on the cylinder surface (i.e., at r = R)? (8%)