Numerical method
Midterm Exam (2020/04/20 9:10~12:00)

1. Convert the following decimal numbers to a 32-bit binary number. (6%)
   (a) -3125.3125
   (b) 10^{-100}

2. Derive the following finite difference formula. (10%)

$$\frac{\partial f_i}{\partial x} = \frac{2f_{i+3} - 9f_{i+2} + 18f_{i+1} - 11f_i}{6\Delta x} + O(\Delta x^3)$$

3. Given the data table.

x	1.0	1.3	1.6	1.9
f(x)	0.76	0.62	0.46	0.28

   (a) Use the Newton's divided-difference formula. Find f(1.5). (8%)
   (b) Use the third-degree Lagrange polynomial. Find f(5.5). (7%)

4. The Pade' scheme are derived by writing approximations of the form:

$$\beta f'_{i-2} + \alpha f'_{i-1} + f'_i + \alpha f'_{i+1} + \beta f'_{i+2} = c\frac{f_{i+3} - f_{i-3}}{6h} + b\frac{f_{i+2} - f_{i-2}}{4h} + a\frac{f_{i+1} - f_{i-1}}{2h}$$

   list the Taylor table and find α, β, a, b, and c.(10%)

5. A general Padé type boundary scheme for the first derivative can be written as

$$f'_0 + \alpha f'_1 = \frac{1}{h}(af_0 + bf_1 + cf_2 + df_3)$$

   show that requiring this scheme to be at least third-order accurate would constrain the coefficients to

$$a = -\frac{11 + 2\alpha}{6}, \quad b = \frac{6 - \alpha}{2}, \quad c = \frac{2\alpha - 3}{2}, \quad d = \frac{2 - \alpha}{6}$$

   Which value of α would you choose and why? (10%)

6. Let $f(x) = (x+2)(x+1)x(x-1)^3(x-2)$.
   (a) Which root of $f(x)$ can be found by the Bisection method for the interval [-3.0, 2.5], why? (10%)
   (b) Comments on the disadvantage of the Bisection method. (5%)

(背面尚有試題，請翻面作答)