Discrete Mathematics
HW5

11-1 2.
(a)
Trail: can have repeated vertices but no repeat edges, open.
b-d not a trail: {b,e} {e,f} {e,g} {g,e} {e,b} {b,c} {c,d}
(b)
Path: no repeated vertices and edges, open.
b-d trail and not a path: {b,e} {e,f} {f,g} {g,e} {e,d}
(c)
b-d path: {b,e} {e,d}
(d)
Closed walk: can have repeated vertices and edges, closed.
Circuit: can have repeated vertices but no repeat edges, closed.
b-b closed but not a circuit: {b,e} {e,f} {f,g} {g,e} {e,b}
(e)
Cycle: no repeated edges and circuits, closed.
b-b circuit but not cycle: {b,c} {c,d} {d,e} {e,g} {g,f} {f,e} {e,b}
(f)
b-b cycle: {b,a} {a,c} {c,b}

11-1 5.
(1) path from a to h needs to go through {b,g},
so the answer = # of path a-b * # of path g-h.

a-b path:	g-h path:
{a,b}	{g,h}
{a,c} {c,b}	{g,f} {f,h}
{a,c} {c,d} {d,b}	{g,e} {e,f} {f,h}

a-h path: 3 *3 = 9
(2) length 5: length a-b + length g-h + {b,g} = 5
Length a-b + length g-h = 4 = 1 + 3 = 2 + 2 = 3 + 1

上面三種情況都各只有一種搭配可以符合，所以共三種

a-h and length 5 path: 3 kinds.

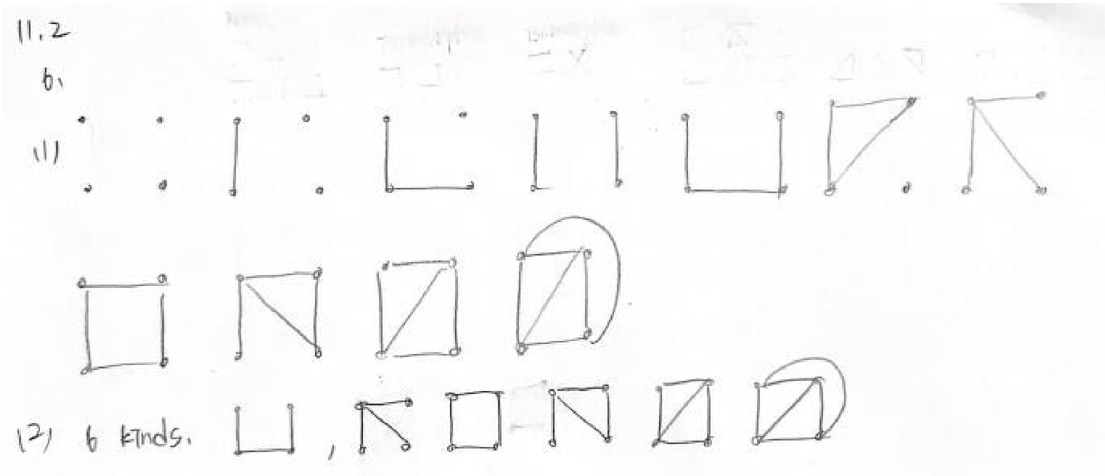
11-2 4.
Spanning graph: $V_1 = V$, no restrict with E
Induced graph: if $U \subseteq V$, induced subgraph’s edges contain all edges (from G) $\{x, y\}$ if both $x, y \in U$

因為 Spanning graph 的點包含了原先 G 的所有點，如果他要是 induced subgraph 的話，就需要包含所有點在 G 裡面構成的邊，也就只有 G 本身自己可以滿足這個條件。

A: 1 種，G 本身

11-2 6.

(1) 11 種，如下圖



11-3 2.

希望 |V| 越大，則 deg(v)應該越小越好，所以可以的話盡量讓 deg(v)=3
And by theorem, we know $\Sigma_{v \in V} deg(v) = 2|E|$

Then, $\Sigma_{v \in V} deg(v) = 34 \geq 3|V|$, maximum |V|=11 (3*11+1，有 10 個 vertex deg(v)=3，一個 deg(v)=4)

11-3 9.
(a)
# of edges of a cube of dimension n is $n * 2^{n-1}$.
So, $n * 2^{n-1} = 524288$
$n = 16$
(b)
# of vertices of n dimension hypercube is 2^n
$n * 2^{n-1} = 4980736$
$n = 19$
# of vertices= $2^{19} = 524288$

11-4 8.
(a)
$K_{1,4}$: edge in G {a,b}, $a \in V_1, b \in V_2$ 且 V_1每個點都有對應到 V_2

所以在這個 case 裡面最多可以有 4 個 edge，從 V_1 到 V_2的四個點

而最長 path 可為 2，從 V_2出發到 V_1 再回到 V_2

(b)
In this case, 因為比較少點的那一邊有 3 個點，最好情況是 V_1 每個

點(假設有 a, b, c)都與 V_2 每個點(1,2,3…,7)相連

最長 path 就可以從V_2的 1出發，到 V_1 的 a，接著連回 V_2 的 2，V_1

的 b，V_2 的 3，V_1 的 c，V_2 的 4，共 7 個點 6 條邊

其中 a,b,c 和 1,2,3,4......並非指特定點，只是代表不同集合的不同點

(c)
同理，最長的 path 應該是從有 12 個 vertices 的 V_2 出發，然後拜訪

V_1 全部共 7 個，來回總共 15 個點 14 條邊（=2*7=14）

(d)
Since m<n, the longest path in $K_{m,n} = 2 * m = 2m$

11-5 7.

(a)

我們可以視為有 n 個點要排成列，共有 n!種排法，但因為要排成圓型，1->2->3->1 和 2->3->1->2 會變成同一種，需除 n，且因為沒有方向概念，1->2->3->1 和 1->3->2->1 視為同一種，要再除 2，所以共 $\frac{(n-1)!}{2}$ 種。

(b)

For a K_n , we have $\binom{n}{2} = \frac{n(n-1)}{2}$ edges. And each Hamilton cycles has n edges, so we will have $\frac{n(n-1)}{2} * \frac{1}{n} = \frac{n-1}{2}$ kinds of edge-disjoint Hamilton cycles.

To have a better answer, n should be odd.

When n=21, $\frac{21-1}{2} = 10$

(c)

Since they need to hold hands form a cycle, we can think it as edge-disjoint Hamilton cycle, so just like we need to find total number of edge-disjoint Hamilton cycle on a K_{19} . when n=19, $\frac{19-1}{2} = 9$, we have 9 edge-disjoint Hamilton cycle in K_{19} .

所以 9 天內所有學生旁邊的人都不是重複的。

11-6 2.

Each committees is a vertex. If someone attends two committees, for example c_i, c_j. Then draw the edge joining the vertices for c_i, c_j, and we will get graph G. Then the least number of meeting times is the chromatic number of this graph (in hours) .