12.5 Evaluate and compare with the exact solution δ, C_{fx}, and C_{fL} for the laminar boundary layer over a flat plate, using the velocity profile

$$v_x = \alpha \sin by.$$

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$$\begin{cases} V_x \dfrac{\partial V_x}{\partial x} + V_y \dfrac{\partial V_x}{\partial y} = \nu \dfrac{\partial^2 V_x}{\partial y^2} \\ \dfrac{\partial V_x}{\partial x} + \dfrac{\partial V_y}{\partial y} = 0 \rightarrow \text{stream function } \Psi(x, y) \Rightarrow \dfrac{\partial \Psi}{\partial x} = -V_y, \quad \dfrac{\partial \Psi}{\partial y} = V_x \end{cases}$$

B.C. ① $V_x(0) = 0$

② $V_x(\delta) = V_\infty = \alpha \sin(b\delta)$

③ $\dfrac{dV_x(\delta)}{dy} = 0 = \alpha b \cos(b\delta)$

$\cos b\delta = \cos\frac{\pi}{2}$

$b = \frac{\pi}{2\delta}$ 代入② 得 $\alpha = V_\infty$

$\therefore V_x = V_\infty \sin\left(\frac{\pi y}{2\delta}\right)$

von Kármán integral eq.

$$\frac{\tau_0}{\rho} = \cancel{\left(\frac{d}{dx}V_\infty\right)\int_0^\delta (V_\infty - V_x)\,dy}^{\,0} + \frac{d}{dx}\int_0^\delta V_x(V_\infty - V_x)\,dy$$

$$\Rightarrow \frac{\tau_0}{\rho} = \frac{d}{dx}\int_0^\delta V_x(V_\infty - V_x)\,dy$$

$$= \frac{d}{dx}\int_0^\delta \left[V_\infty^2 \sin\left(\frac{\pi y}{2\delta}\right) - V_\infty^2 \sin^2\left(\frac{\pi y}{2\delta}\right)\right] dy$$

$$= \frac{d}{dx} V_\infty^2 \int_0^\delta \left[\sin\frac{\pi y}{2\delta} - \left(\frac{1-\cos\left(\frac{\pi y}{\delta}\right)}{2}\right)\right] dy$$

$$= \frac{d}{dx} V_\infty^2 \left[-\frac{2\delta}{\pi}\cos\frac{\pi y}{2\delta} - \frac{1}{2}y + \frac{\delta}{2\pi}\sin\frac{\pi y}{\delta}\right]_0^\delta$$

$$= \frac{d}{dx} V_\infty^2 \left[\left(-\frac{2\delta}{\pi}\left(\cancel{\cos\frac{\pi}{2}}^{\,0} - 1\right)\right) + \left(-\frac{1}{2}(\delta)\right) + \left(\frac{\delta}{2\pi}\right)\left(\cancel{\sin\pi}^{\,0} - 0\right)\right]$$

$$= \frac{d}{dx} V_\infty^2 \left(\frac{2\delta}{\pi} - \frac{\delta}{2}\right) = \frac{d\delta}{dx} V_\infty^2 \left(\frac{2}{\pi} - \frac{1}{2}\right)$$

equal

also $\dfrac{\tau_0}{\rho} = \dfrac{\mu}{\rho}\left.\dfrac{dV_x}{dy}\right|_{y=0} = \dfrac{\mu}{\rho}\alpha b(\cos by) = \dfrac{\mu}{\rho} V_\infty \dfrac{\pi}{2\delta}$

$$\frac{d\delta}{dx} V_\infty^{\cancel{2}}\left(\frac{2}{\pi} - \frac{1}{2}\right) = \frac{\mu}{\rho}\cancel{V_\infty}\frac{\pi}{2\delta}$$

$$\int \delta\, d\delta = \int \frac{\mu}{V_\infty \rho}\frac{2\pi^2}{(4-\pi)\cancel{2}}\,dx$$

$$\frac{1}{2}\delta^2 = \frac{\mu\pi^2 x}{V_\infty\rho(4-\pi)}$$

$Re_x = \dfrac{\rho V x}{\mu}$

$$\delta = \sqrt{\frac{2\mu\pi^2 x}{V_\infty\rho(4-\pi)}} = \sqrt{\frac{2\mu\pi^2 x^2}{V_\infty\rho x(4-\pi)}} = \sqrt{\frac{2\pi^2}{4-\pi}}\frac{x}{\sqrt{Re_x}} = 4.788\frac{x}{\sqrt{Re_x}} \quad \#$$

$$\therefore \tau_0 = \mu V_\infty \frac{\pi}{2}\sqrt{\frac{V_\infty\rho(4-\pi)}{2\mu\pi^2 x}}$$

for $C_{fx} = \dfrac{2\tau}{\rho V_\infty^2} = \dfrac{\cancel{2}\cdot\mu\cdot\cancel{V_\infty}\frac{\pi}{\cancel{2}}}{\rho V_\infty^{\cancel{2}}}\sqrt{\dfrac{V_\infty\rho(4-\pi)}{2\mu\pi^2 x}} = \sqrt{\dfrac{\mu^{\cancel{2}}\cdot\cancel{\pi^2}\cdot\cancel{V_\infty}\cdot\cancel{\rho}\cdot(4-\pi)}{\rho^{\cancel{2}}\cdot V_\infty^{\cancel{2}}\cdot 2\cancel{\mu}\cancel{\pi^2}\cdot x}} = \sqrt{\dfrac{\mu}{\rho V_\infty x}}\cdot\sqrt{\dfrac{4-\pi}{2}} = \dfrac{0.655}{\sqrt{Re_x}} \quad \#$

for $C_{fL} = \dfrac{1}{L}\displaystyle\int_0^L C_{fx}\,dx = \dfrac{1}{L}\int_0^L \sqrt{\dfrac{\mu}{\rho V_\infty x}}\sqrt{\dfrac{4-\pi}{2}}\,dx = \dfrac{1}{L}\sqrt{\dfrac{\mu}{\rho V_\infty}}\sqrt{\dfrac{4-\pi}{2}}\int_0^L x^{-\frac{1}{2}}\,dx = \dfrac{2}{L}\sqrt{\dfrac{\mu}{\rho V_\infty}}\sqrt{\dfrac{4-\pi}{2}}\sqrt{L}$

$$= \sqrt{\frac{\mu}{\rho V_\infty L}}\sqrt{\frac{4(4-\pi)}{2}} = \frac{1.311}{\sqrt{Re_x}} \quad \#$$

12.17 A baseball (sphere) has a circumference of 9¼ in. and a weight of 5¼ ounces. At 95 mph determine

a. the Reynolds number

b. the drag force

c. the type of flow (see the illustration for Problem 12.7)

$V = 95\ mph = 139.3\ ft/s.$

$C = 9.25\ in = 0.77\ ft$

$D = \frac{C}{\pi} = \frac{0.77}{\pi} = 0.245\ ft$

diameter

查表 $T = 80°F$, $\nu = \frac{\mu}{\rho} = 0.169 \times 10^{-3}\ ft^2/s$

$\rho = 0.0735\ lbm/ft^3$

(a) $Re = \frac{\rho V D}{\mu} = \frac{V \cdot D}{\nu} = \frac{(139.3) \cdot (0.245)}{0.169 \times 10^{-3}} = 201943.787 \doteq 2 \times 10^5$ ※

(b)

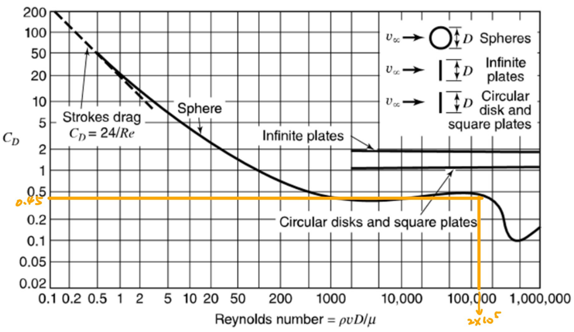

Figure 12.4 Drag coefficient versus Reynolds number for various objects.

$\frac{F}{A_p} = C_D \frac{\rho V_\infty^2}{2}$

$F = A_p C_D \frac{\rho V_\infty^2}{2}$

$= \left(\frac{\pi}{4} D^2\right) C_D \frac{1}{2} \rho V_\infty^2 = \frac{\pi}{4} \cdot (0.245^2) \cdot (0.45) \cdot \left(\frac{1}{2}\right) (0.0735) (139.3)^2$

$= 15.1\ \frac{lbm \cdot ft}{s^2}$

$= 0.47\ lbf.$ ※

(c)

(a) $Re_x < 2 \times 10^5$ the boundary layer is laminar
(b) $2 \times 10^5 < Re_x < 3 \times 10^6$ the boundary layer may be either laminar or turbulent
(c) $3 \times 10^6 < Re_x$ the boundary layer is turbulent

flow is near transition.

12.26 For a thin a plate 6 in. wide and 3 ft long, estimate the friction force in air at a velocity of 40 fps, assuming

a. turbulent flow

b. laminar flow

The flow is parallel to the 6-in. dimension.

$w = 6\,in = 0.5\,ft$

$L = 3\,ft$

$v = 40\,fps.$

代入 $T = 80°F$

$\nu = \frac{\mu}{\rho} = 0.169 \times 10^{-3}\ \frac{ft^2}{s}$

$\rho = 0.0735\ \frac{lbm}{ft^3}$

$Re_x = \frac{\rho V w}{\mu} = \frac{40 \cdot (0.5)}{0.169 \times 10^{-3}} = 118343 \Rightarrow Re < 10^7$

(a) for turbulent flow

by $\frac{1}{7}$ power law for turbulent boundary layer

$\frac{V_x}{V_{x\,max}} = \left(\frac{y}{\delta}\right)^{\frac{1}{7}} \Rightarrow V_x = V_\infty \left(\frac{y}{\delta}\right)^{\frac{1}{7}}$

$\tau_0 = 0.0225 \rho V_{x\,max}^2 \left(\frac{\nu}{V_{x\,max}\,\delta}\right)^{\frac{1}{4}}$

代入 von Kármán integral eq $\frac{\tau_0}{\rho} = \cancel{\left(\frac{d}{dx} V_\infty\right) \int_0^\delta (V_\infty - V_x)\, dy} + \frac{d}{dx}\int_0^\delta V_x (V_\infty - V_x)\, dy$

flat plat V_∞ is const.

$0.0225\, \cancel{V_\infty^2} \left(\frac{\nu}{V_\infty \delta}\right)^{\frac{1}{4}} = \frac{d}{dx}\int_0^\delta \cancel{V_\infty^2} \left[\left(\frac{y}{\delta}\right)^{\frac{1}{7}} - \left(\frac{y}{\delta}\right)^{\frac{2}{7}}\right] dy$

$\Rightarrow 0.0225 \left(\frac{\nu}{V_\infty \delta}\right)^{\frac{1}{4}} = \frac{d}{dx}\int_0^\delta \left(\frac{y}{\delta}\right)^{\frac{1}{7}} - \left(\frac{y}{\delta}\right)^{\frac{2}{7}} \Big] dy = \frac{d}{dx}\left[\frac{7}{8}\left(\frac{1}{\delta}\right)^{\frac{1}{7}} y^{\frac{8}{7}} - \frac{7}{9}\left(\frac{1}{\delta}\right)^{\frac{2}{7}} y^{\frac{9}{7}}\right]_0^\delta = \frac{d}{dx}\left(\frac{7}{8}\delta - \frac{7}{9}\delta\right) = \frac{d\delta}{dx}\frac{7}{72}$

$\Rightarrow 0.0225 \left(\frac{\nu}{V_\infty \delta}\right)^{\frac{1}{4}} = \frac{7}{72}\frac{d\delta}{dx}$

$\int \left(\frac{\nu}{V_\infty}\right)^{\frac{1}{4}} dx = \frac{7}{72 \cdot (0.0225)} \int (\delta)^{\frac{1}{4}} d\delta \Rightarrow \left(\frac{\nu}{V_\infty}\right)^{\frac{1}{4}} x = 4.32\ \frac{4}{5}\ \delta^{\frac{5}{4}} = 3.456\, \delta^{\frac{5}{4}} + C$ 代入 B.C. $x = 0,\ \delta = 0\ \therefore C = 0$

$\left(\frac{\nu}{V_\infty x}\right)^{\frac{1}{4}} = 3.45 \left(\frac{\delta}{x}\right)^{\frac{5}{4}} \Rightarrow 0.376 (Re_x)^{-\frac{1}{5}} = \frac{\delta}{x}$

$C_{fx} = \frac{2\tau_0}{\rho V_\infty^2} = 0.045 \left(\frac{\nu Re_x^{\frac{1}{5}}}{V_\infty (0.376 x)}\right)^{\frac{1}{4}} = \frac{0.074}{Re_x^{\frac{1}{5}}}$

$C_{fT} = \frac{0.074}{Re^{0.2}} = \frac{0.074}{118343^{\frac{1}{5}}} = 0.00715$

$F_{fT} = A \cdot C_{fT} \frac{\rho V_\infty^2}{2} = (0.5) \cdot (3) \cdot (0.00715) \frac{1}{2} (0.0735)(40)^2 = 0.63\ \frac{lbm \cdot ft}{s^2} = 0.02\ lbf. \ ※$

(b) for larminan flow

$\because$ thin plate 代入 The Blasius's solution for flat plate for laminar

$C_{fx} = \frac{0.664}{\sqrt{Re_x}}$

$C_{fL} = \frac{1}{L}\int C_{fx}\, dx = \frac{1}{L}\int_0^L 0.664 \sqrt{\frac{\mu}{\rho V_\infty x}}\, dx = \frac{1.328}{\sqrt{Re_x}} = 1.328 \sqrt{\frac{\mu}{\rho V_\infty L}}$

$C_{fL} = \frac{1.328}{\sqrt{Re}} = \frac{1.328}{\sqrt{118343}} = 0.00386$

$F_{fL} = A\, C_{fL} \frac{\rho V_\infty^2}{2} = (0.5) \cdot (3) \cdot (0.00386) \cdot \frac{1}{2} (0.0735)(40)^2$

$= 0.34\ \frac{lbm\, ft}{s^2}$

$= 0.01\ lbf \ ※$