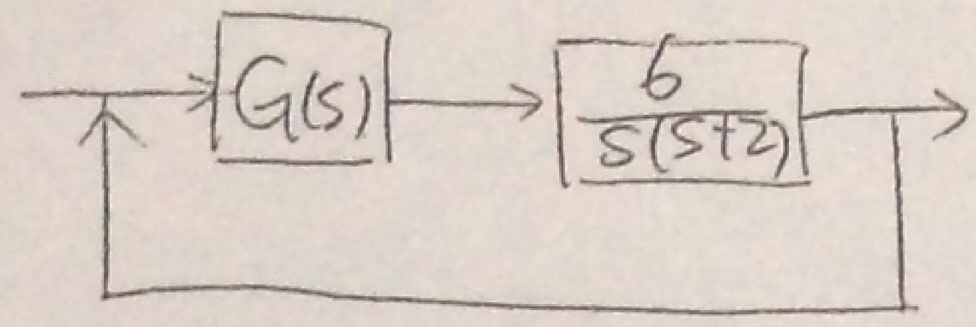
2011

1.



① $t_s = 1s = \frac{4}{\xi\omega_n}$ $\quad \xi\omega_n = 4$ $\quad \xi = \frac{PM}{100} = 0.45$ $\quad \omega_n = 8.89$

$\omega_d = \sqrt{1-\xi^2}\,\omega_n = 7.94.$

$oltf(s) = k\,\frac{s+z}{s+p} \times \frac{6}{s(s+2)}$

$z = 4$

$\angle oltf(s)|_{s=-4+j7.94} = -180^\circ$

① $\angle k = 0$

② $\angle s+z = 90^\circ$

③ $\angle s+p = ?$

④ $\angle s = 180^\circ - \tan^{-1}\frac{7.94}{4} = 116.74$

⑤ $\angle s+2 = 180^\circ - \tan^{-1}\frac{7.94}{2} = 104.14$

$0 + 90^\circ - ? - 116.74 - 104.14 = -180$

$? = 49.12.$

$\tan^{-1}\frac{7.94}{p-4} = 49.12 \quad p = 10.873$

362.248
1437.705

$$\left| k \times \frac{s+4}{s+10.873} \times \frac{6}{s(s+2)} \Big|_{s=-4+j7.94} \right| = 1$$

$$k \times \frac{|j7.94 \times 6|}{|(6.873+j7.94)\times(-4+j7.94)(-2+j7.94)|} = 1$$

$47.64k = 1482.64 \quad k = 31.12.$

驗証.

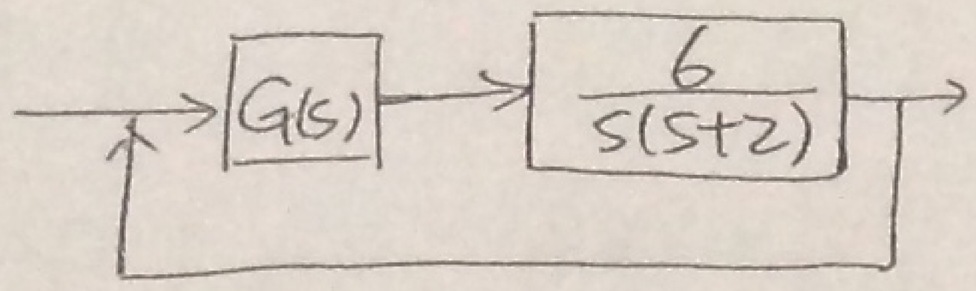
① $PM = \xi \times 100 = 45^\circ$ ok

② $k_v = \lim_{s\to 0} s \cdot 31.12 \times \frac{s+4}{s+10.873} \times \frac{6}{s(s+2)} = 34.345$

$e_{ss} = \frac{1}{34.345} = 2.911\% < 3\%$ ok.

2011

2\. Bode plot phase lag



$\frac{1}{kv} = \frac{30}{100}$

$e_{ss} = 3\%$ $kv = \frac{10}{3}$

$kv = \lim_{s \to 0} s \cdot k \cdot \frac{6}{s(s+2)} = 3k = \frac{10}{3}$ $k = \frac{10}{9}$

$oltf(s) = \frac{20}{3s(s+2)}$

$\left|\frac{20}{3j\omega(j\omega+2)}\right| = 1$ $\frac{20}{\sqrt{9\omega^4+36\omega^2}} = 1$ $\omega = 2.227$

$\measuredangle oltf(j\omega) = \measuredangle \frac{20}{-3\times 2.227^2 + j\times 6\times 2.227} = -\left(180^\circ - \tan^{-1}\frac{6}{3\times 2.227}\right) = -138.07$

PM = 41.926 不夠

$\measuredangle oltf(j\omega) = \frac{20}{-3\omega^2 + j6\omega} = -\left(180^\circ - \tan^{-1}\frac{2}{\omega}\right) = -135^\circ$ $\omega = 2.$

$\left|oltf(s)\right|_{s=2j} = 1.179$

$oltf(s) = \frac{1}{1.179} \times \frac{s+\omega_2}{s+\omega_1} \times \frac{6}{s(s+2)} \times \frac{10}{9}$

$\omega_2 = \frac{2}{10} = 0.2$

$\frac{\omega_2}{\omega_1} = 1.179$ $\omega_1 = 0.17.$

$controller = \frac{1}{1.179} \times \frac{s+0.2}{s+0.17} \times \frac{10}{9}$

驗証：

① $kv = \lim_{s \to 0} s \times \frac{1}{1.179} \times \frac{s+0.2}{s+0.17} \times \frac{6}{s(s+2)} \times \frac{10}{9} = \frac{10}{3}$ ok.

② PM = $\measuredangle \frac{6}{s(s+2)} = -135^\circ$

$\measuredangle \frac{1}{1.179} \times \frac{s+0.2}{s+0.17}\Big|_{s=2j} = \tan^{-1}\frac{2}{0.2} - \tan^{-1}\frac{2}{0.17} = -0.8521$

PM = 45 − 0.851 = 44.15 不夠

取 $\omega_2 = \frac{2}{20} = 0.1$ $\omega_1 = 0.014$ 再做一次

$\dot{x} = Ax + Bu$

$y = Cx + Du$

$z = p^{-1}x$

$pz = x$ $\quad z = p^{-1}x$ $\quad \dot{z} = APz + Bu$ $\quad \dot{z} = p^{-1}APz + p^{-1}Bu$

$y = pcz + Du$

pz $\quad \dot{z} =$

$A - \lambda I$

$\det(A - \lambda I) = 0$

$\det(P^{-1}AP - \lambda I) = \det[P^{-1}(A - \lambda I)P]$

$= \det P^{-1} \det(A - \lambda I) = \det P$

$\det(A - \lambda I) = 0.$

$= \det P^{-1}P \det(A - \lambda I)$

$= \det(A - \lambda I)$

2011

4. system I: $\dot{x} = Ax + Bu$

$y = Cx + Du$

$z = p^{-1}x \quad x = pz$

system II: $\dot{z} = p^{-1}APz + p^{-1}Bu$

$y = CPz + Du$

① stability:

system I:

$\det(A - \lambda I) = 0$

system II:

$$\det(p^{-1}AP - \lambda I) = \det[P^{-1}(A - \lambda I)P]$$

$$= \det(p^{-1})\det(A - \lambda I)\det(p)$$

$$= \det(p^{-1})\det(P)\det(A - \lambda I)$$

$$= \det(p^{-1}p)\det(A - \lambda I)$$

$$= \det(A - \lambda I)$$

請依題序作答,否則不予計分・總分110分・

② controllability

system I: $\operatorname{rank}[B \,\vdots\, AB \,\vdots\, \cdots\; A^{n-1}B]$

system II:

$$\operatorname{rank}[\,P^{-1}B \,\vdots\, (P^{-1}AP)(P^{-1}B) \,\vdots\, (P^{-1}AP)(P^{-1}AP)(P^{-1}B)\cdots\,]$$

$$=\operatorname{rank}\{P^{-1}[B \,\vdots\, AB \,\vdots\, A^2B \,\vdots\, \cdots\; A^{n-1}B]\}$$

$$=\operatorname{rank}[B \,\vdots\, AB \,\vdots\, A^2B \,\vdots\, \cdots\; A^{n-1}B]$$

③ observability

system I:

$$\operatorname{rank}\begin{bmatrix} C \\ CA \\ CA^2 \\ \vdots \\ CA^{n-1} \end{bmatrix}$$

system II:

$$\operatorname{rank}\begin{bmatrix} CP \\ CP(P^{-1}AP) \\ CP(P^{-1}AP)(P^{-1}AP) \\ \vdots \end{bmatrix} = \operatorname{rank}\begin{bmatrix} CP \\ CAP \\ CA^2P \\ \vdots \\ CA^{n-1}P \end{bmatrix} = \operatorname{rank}\left\{\begin{bmatrix} C \\ CA \\ CA^2 \\ \vdots \\ CA^{n-1} \end{bmatrix} P\right\}$$

$$=\operatorname{rank}\begin{bmatrix} C \\ CA \\ CA^2 \\ \vdots \\ CA^{n-1} \end{bmatrix}$$

2011
5.

①
$$X_1 = \frac{1}{s+1}(U+X_3)$$
$$\dot{x}_1 = -x_1 + x_3 + u$$
$$X_2 = (5X_2 + X_1)\frac{1}{s+3}$$
$$sX_2 + 3X_2 = 5X_2 + X_1$$
$$sX_2 = -2X_2 + X_1$$
$$\dot{x}_2 = -2x_2 + x_1$$
$$X_3 = \frac{1}{s+2}X_2$$
$$\dot{x}_3 = -2x_3 + x_2$$

$$\dot{x}_1 = -x_1 + x_3 + u$$
$$\dot{x}_2 = -2x_2 + x_1$$
$$\dot{x}_3 = -2x_3 + x_2$$
$$y = x_2$$

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \begin{bmatrix} -1 & 0 & 1 \\ 1 & -2 & 0 \\ 0 & 1 & -2 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} u$$

$$y = \begin{bmatrix} 0 & 1 & 0 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

②
$$A = \begin{bmatrix} -1 & 0 & 1 \\ 1 & -2 & 0 \\ 0 & 1 & -2 \end{bmatrix} \quad B = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} \quad C = \begin{bmatrix} 0 & 1 & 0 \end{bmatrix}$$

$$\text{rank}\left[B \,\vdots\, AB \,\vdots\, A^2B\right] = \begin{bmatrix} 1 & -1 & 1 \\ 0 & 1 & -3 \\ 0 & 0 & 1 \end{bmatrix} = 3$$

可控

③
$$\text{rank}\begin{bmatrix} C \\ CA \\ CA^2 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 1 & -2 & 0 \\ -3 & 4 & 1 \end{bmatrix} = 3$$

可觀

2011

6.

$$G(s)=\frac{s^2+3s+1}{s^5-6s^4+8s^3+4s^2+2s+1}$$

$$\begin{bmatrix}\dot{x}_1\\ \dot{x}_2\\ \dot{x}_3\\ \dot{x}_4\\ \dot{x}_5\end{bmatrix}=\begin{bmatrix}0&1&0&0&0\\0&0&1&0&0\\0&0&0&1&0\\0&0&0&0&1\\-1&-2&-4&-8&6\end{bmatrix}\begin{bmatrix}x_1\\x_2\\x_3\\x_4\\x_5\end{bmatrix}+\begin{bmatrix}0\\0\\0\\0\\1\end{bmatrix}u.$$

$$y=\begin{bmatrix}1&3&1&0&0\end{bmatrix}\begin{bmatrix}x_1\\x_2\\x_3\\x_4\\x_5\end{bmatrix}$$