電子電工學
Lecture 12

Recap: Phasors for AC

$$e^{j\theta} = \cos\theta + j\sin\theta$$

$$\sqrt{-1} = j = e^{j\frac{\pi}{2}}$$

Time-variant Signal $v(t)$, $i(t)$ → Complex V, I → Phasor Diagram

Differential Eq.

$$V(t) = \frac{1}{j\omega C} I(t) \qquad V(t) = j\omega L I(t) \qquad V(t) = R I(t)$$

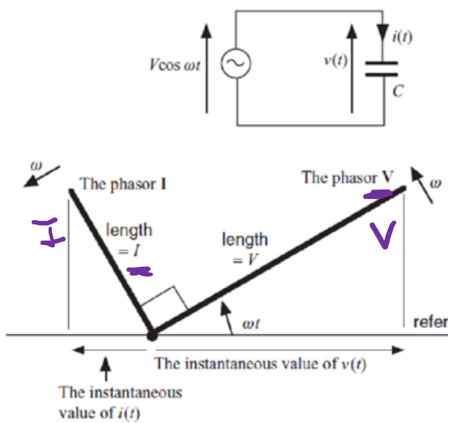


Figure 9.12 A phasor diagram showing phasors representing capacito
Note that the phasors rotate at an angular frequency ω and it is th
reference axis that determines the actual scalar instantaneous values o

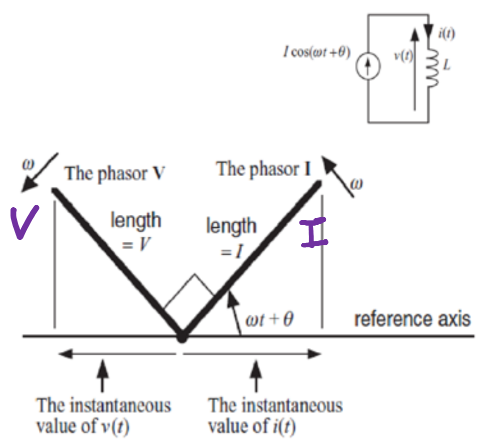


Figure 9.13 A Phasor diagram showing phasors representing inductor voltage and current

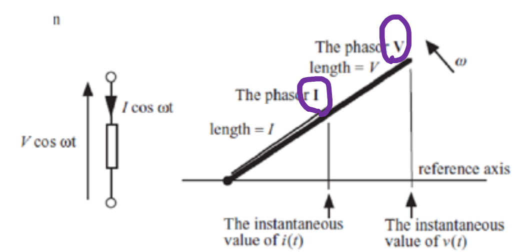


Figure 9.14 The phasors representing the sinusoidal voltage and current of a resistor ar
in phase. The current phasor has been offset slightly for clarity

KCL, KVL

Recap: Phasor addition

$$v_1(t) = V_1 \cos(\omega t + \theta_1)$$

$$+)\quad v_2(t) = V_2 \cos(\omega t + \theta_2)$$

$$v(t) = v_1(t) + v_2(t) \neq \underline{(V_1 + V_2)} \cos(\omega t + \theta_?)$$

$\rightarrow$

$$\mathbf{V}_1 = V_1 e^{j(\omega t + \theta_1)}$$

$$\mathbf{V}_2 = V_2 e^{j(\omega t + \theta_2)}$$

$$\mathbf{V} = \mathbf{V}_1 + \mathbf{V}_2$$

phasor Diagram

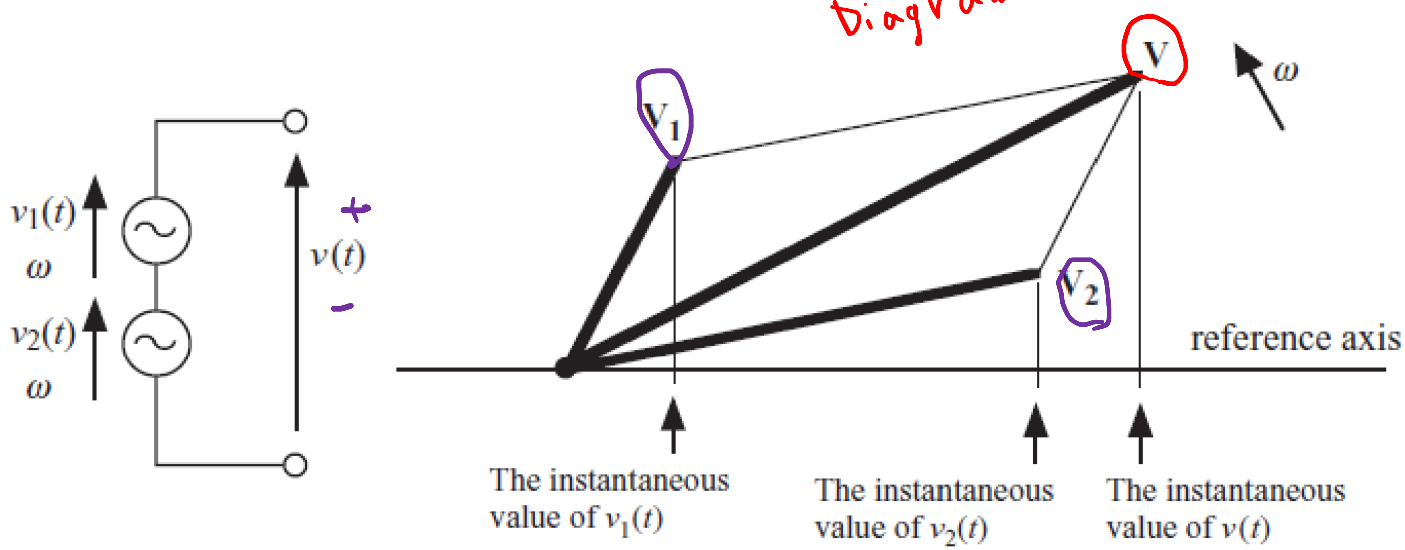


Figure 9.15 A Phasor diagram showing that the phasor addition of voltages obeys KVL. Hairlines are used to indicate construction

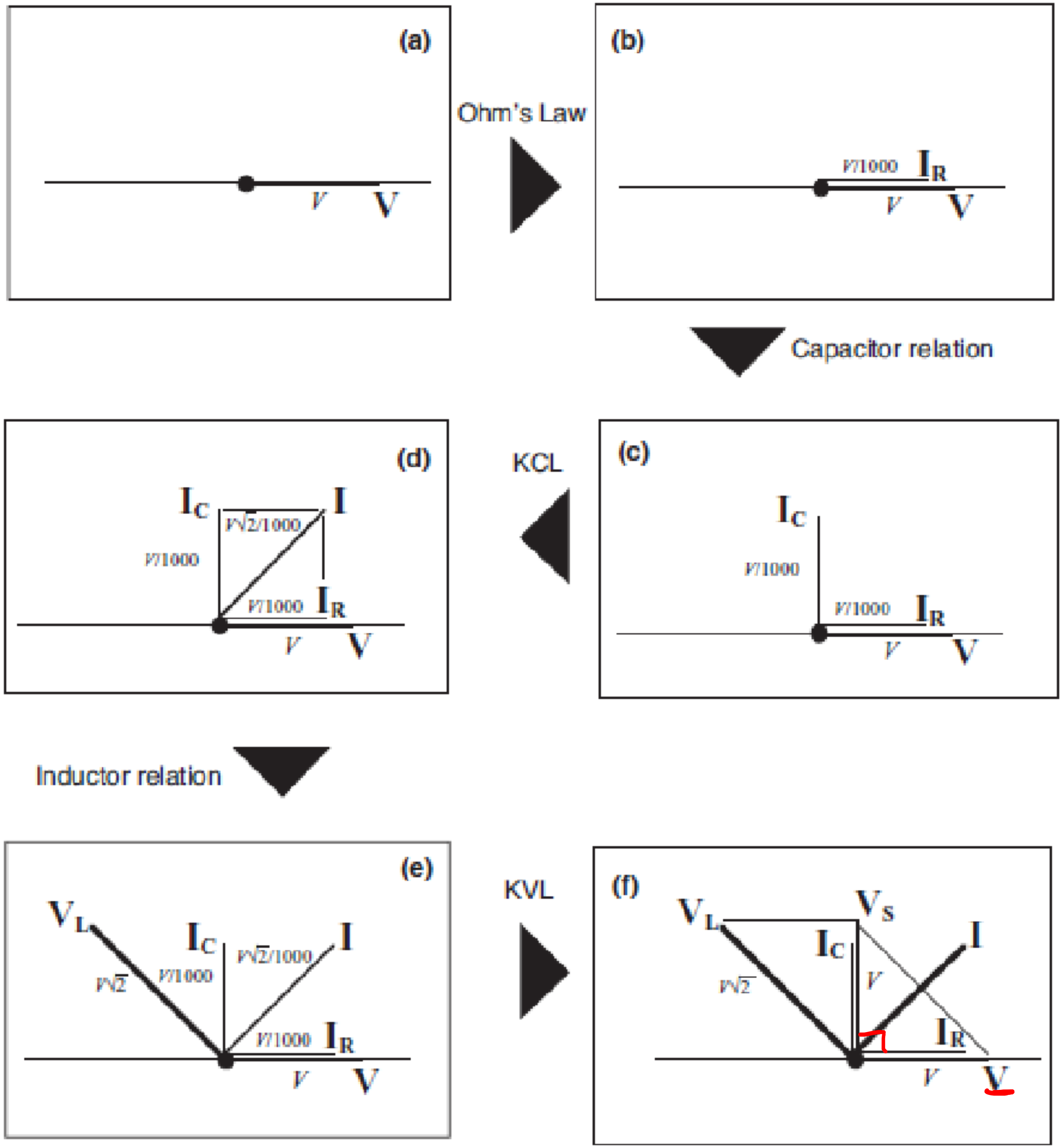
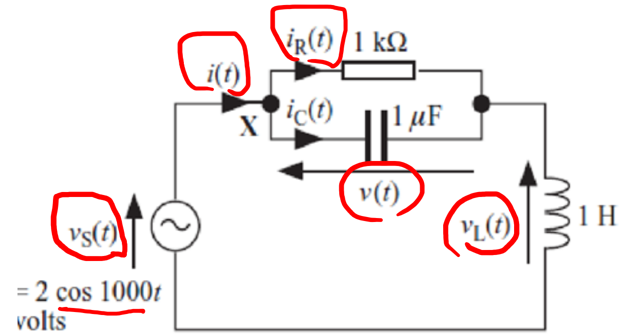


Figure 9.17 Steps in the construction of a phasor diagram for the circuit of Figure 9.16. Where possible phasor lengths have been indicated beside the phasor. Some overlapping phasors have been offset slightly for clarity. Hairlines are included to indicate constructions



6 The circuit whose phasor diagram is to be constructed

$V_s \rightarrow 2\cos(1000t)$

$V \rightarrow 2\cos(1000t - 90°)$

$V_L \rightarrow v_L(t)$

$I \rightarrow i(t)$

$I_c \rightarrow i_c(t)$

$I_L \rightarrow i_L(t)$

Constructing a phasor diagram

Resonance

R-L-C Parallel

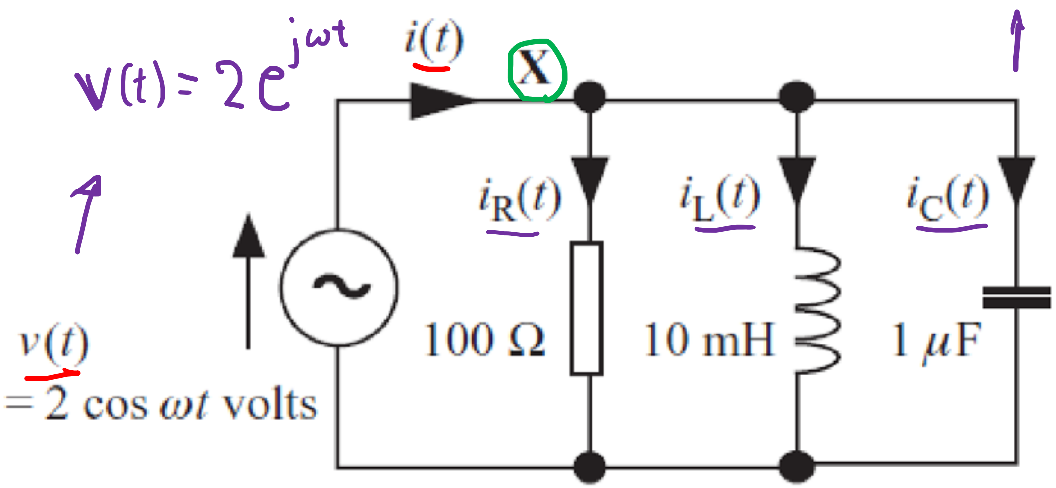
Time domain $v(t) \longrightarrow i(t)$

Involve 2^{nd} order ODE

$I_R(t) = \dfrac{V}{R}$ $\quad I_L(t) = \dfrac{V}{j\omega L}$

$I_C(t) = (j\omega C)V$

$V(t) = 2e^{j\omega t}$



KCL@

$I(t) = I_R(t) + I_L(t) + I_C(t)$

$= V(t) \cdot \left[\dfrac{1}{R} + \dfrac{1}{j\omega L} + j\omega C\right]$

Figure 9.18 A resonant circuit

Resonance

Cartesian Coord. $(a, b) \longrightarrow (r, \theta)$ Polar Coor.

For ω= 10000√2≈14142 rad/s

$$I(t) = V(t)\cdot\left[\frac{1}{R} + \frac{1}{j\omega L} + j\omega C\right]$$

$$= V(t)\cdot\left[\frac{1}{R} + j\left[-\frac{1}{\omega L} + \omega C\right]\right]$$

$$= V(t)\cdot\left[\frac{1}{100} + j\left[-\frac{1}{100\sqrt{2}} + \frac{\sqrt{2}}{100}\right]\right]$$

$$a+bj = re^{j\theta}$$

$$\approx 12.25\, e^{j35^\circ}$$

$$\approx 24.5\, e^{j(\omega t + 35^\circ)} \text{ (mA)}$$

$v(t) = 2e^{j\omega t}$

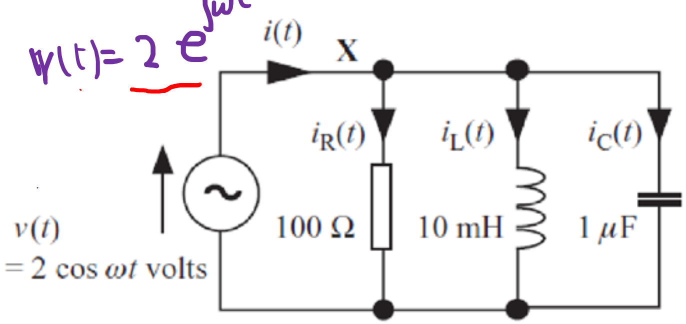


Figure 9.18 A resonant circuit

Table 9.1 Relevant to the analysis of the circuit of Figure 9.18

ω radians/sec	ωC siemens	1/ωC ohms	ωL ohms
10^4	10^{-2}	100	100
$10^4\sqrt{2}$	$\sqrt{2}/100$	$100/\sqrt{2}$	$100\sqrt{2}$

The values of V/I for the inductor and capacitor of Figure 9.18 for two values of ω, the radian frequency of the current source

Resonance

For ω= 10000 rad/s

$$I(t) = V(t)\left[\frac{1}{R} + j\Big(\underbrace{-\frac{1}{\omega L}}_{-0.01} + \underbrace{\omega C}_{0.01}\Big)\right]$$

$$= V(t)\cdot\frac{1}{R}$$

$$= 20\, e^{j\omega t} \quad (mA)$$

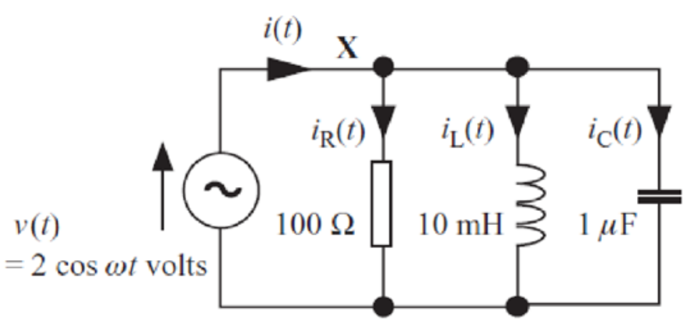


Figure 9.18 A resonant circuit

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The values of V/I for the inductor and capacitor of Figure 9.18 for two values of ω, the radian frequency of the current source

$\omega = 10000\sqrt{2}$

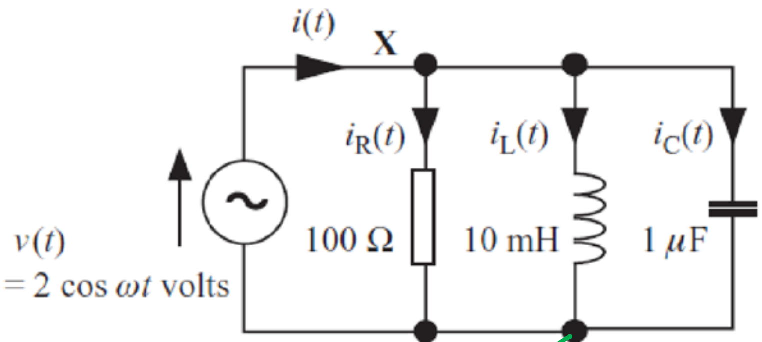


Figure 9.18 A resonant circuit

$I_L = \frac{V}{j\omega L}$

$I_C = (j\omega C)V$

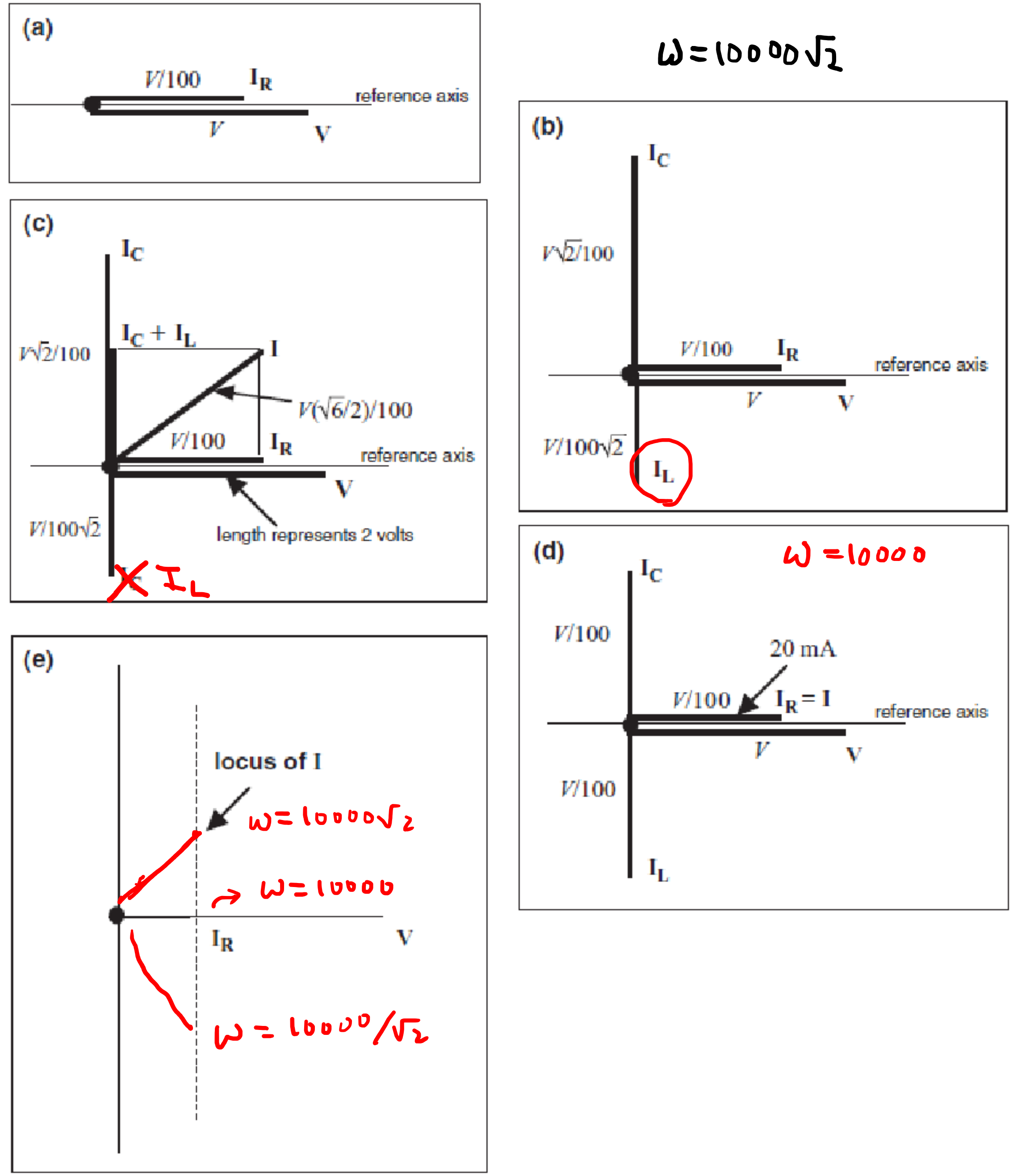
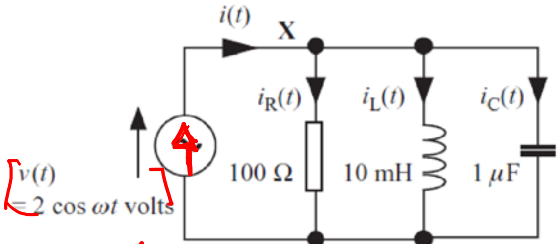


Figure 9.19 Construction of phasor diagrams for the circuit of Figure 9.18. Some overlapping phasors have been offset for clarity



$i(t) = 1\cos(\omega t)$

$+$ $v(t)$ $-$

Figure 9.18 A resonant circuit

Resonance

@ $\omega C - \frac{1}{\omega L} = 0$

$\Rightarrow \omega^2 = 1/LC$

$\Rightarrow \omega = \frac{1}{\sqrt{LC}} = 10^4$ (rad/s)

Replace source with 1 mA current source
(ω adjustable)

$$\mathbb{I}(t) = 1e^{j\omega t}$$

$$\mathbb{V}(t) = \mathbb{I}(t)\cdot\left[100\Omega \,//\, j\omega L \,//\, \frac{1}{j\omega C}\right]$$

$$= \mathbb{I}(t)\cdot\left[\frac{1}{\frac{1}{100} + j\left(\omega C - \frac{1}{\omega L}\right)}\right]$$

Amplitude

$$|\mathbb{V}(t)| = |\mathbb{I}(t)|\left|\frac{1}{\frac{1}{100} + j\left(\omega C - \frac{1}{\omega L}\right)}\right|$$

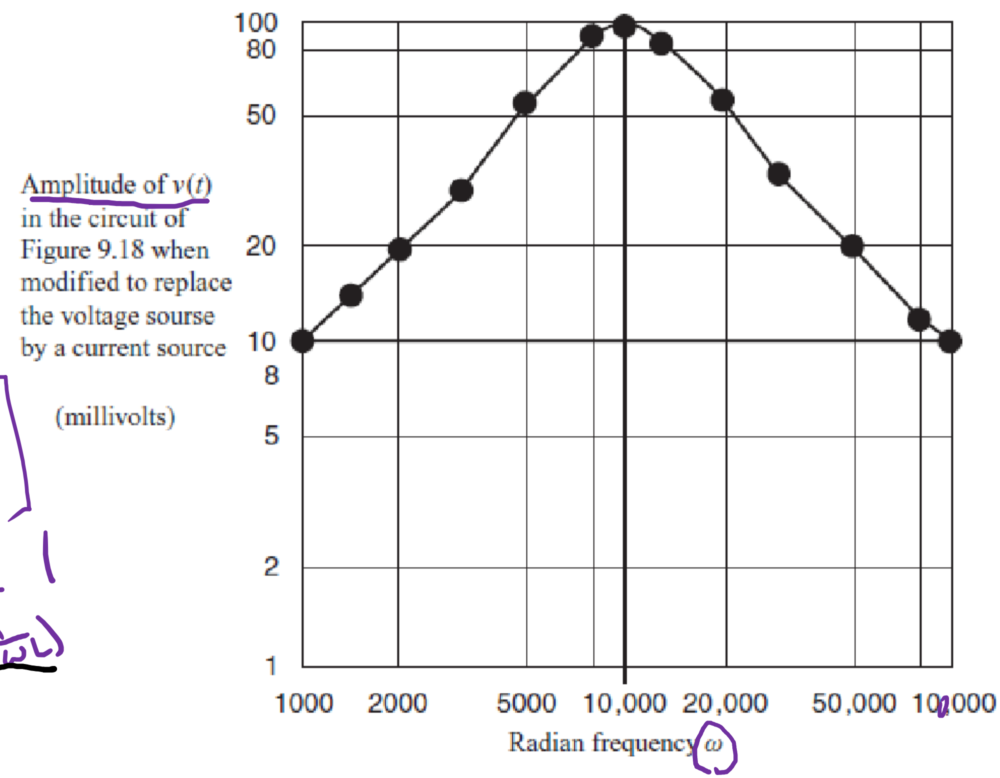


Figure 9.20 Pertinent to the circuit of Figure 9.18 in which the sinusoidal voltage source is replaced by a sinusoidal current source of 1 mA amplitude. The sketch shows the variation of the ampitude of the voltage $v(t)$ as the source frequency ω varies

Chapter 10

AC circuit example

Sinusoidal wave source

$v_s(t) = V \cos(\omega t + \theta)$

Amplitude, Freq., Phase

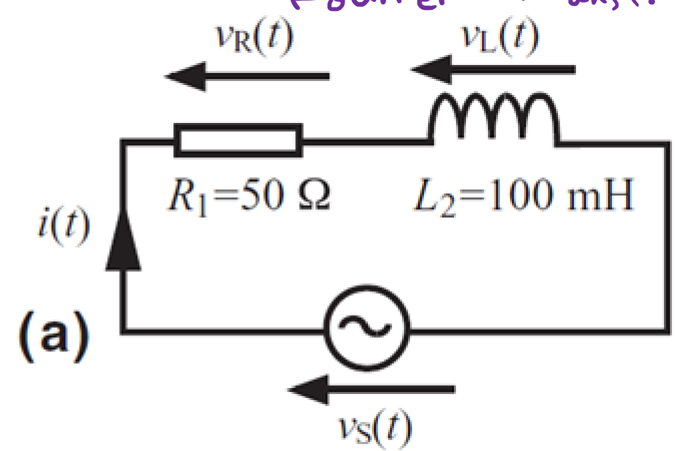
$v_s(t) = v_R(t) + v_L(t)$

ODE: 1^{st}, 2^{nd}

Trigonometric Identity

Simplify $\longrightarrow$

- Phasor diagram
- Complex voltage, current
- Laplace transf.
- Fourier transf.



(40 V peak-to-peak amplitude, 500 rad/s)

$20 \cos(500t)$

Euler's Theorem

i current

$$e^{j\theta} = \cos\theta + j\sin\theta$$

$j = \sqrt{-1}$

$$|e^{j\theta}| = \sqrt{\cos^2\theta + \sin^2\theta}$$

$$= 1$$

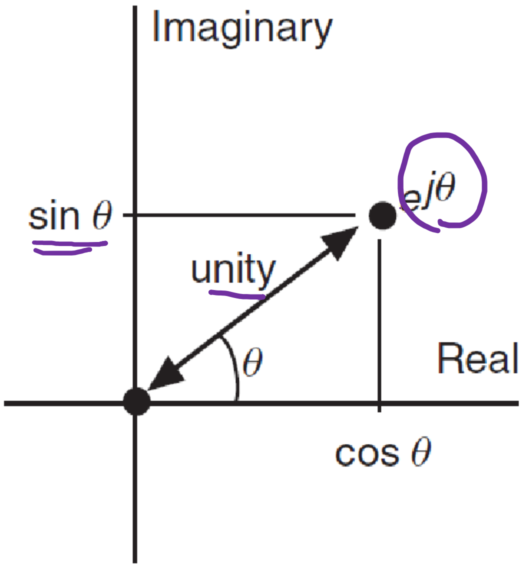


Figure 10.1 Representation of $e^{j\theta}$ in the complex rane

Recall: Complex numbers

- Conjugate

共軛

$$\overline{a+jb} = a-jb$$

$$\overline{e^{j\theta}} = \cos\theta - j\sin\theta$$

$$= e^{-j\theta}$$

- Absolute value

$$|a+jb| = \sqrt{a^2+b^2} = \sqrt{(a+jb)(a-jb)}$$

$$e^{j\theta} = \sqrt{e^{j\theta}e^{-j\theta}} = 1$$

$$e^{j\theta} = \cos\theta + j\sin\theta$$

$$\theta = \omega t$$

$$\rightarrow e^{j\omega t} = \cos\omega t + j\sin\omega t$$

$$|e^{j\omega t}| = 1 \quad \forall t$$

$$r e^{j\omega t}$$

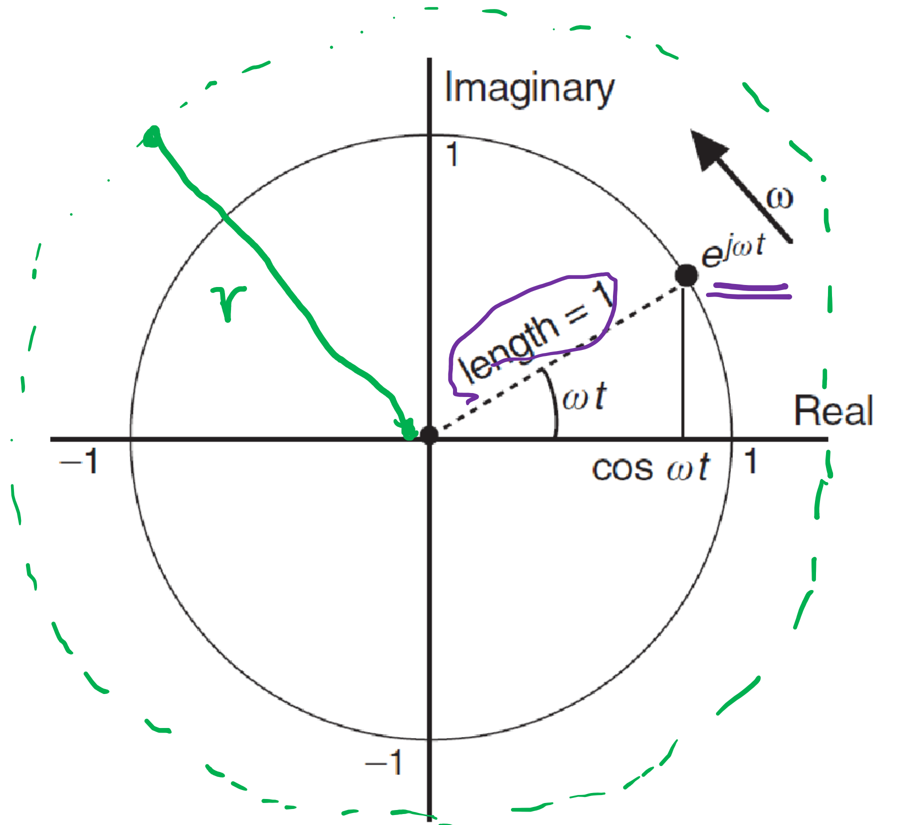


Figure 10.2 Properties of $e^{j\omega t}$ in the complex plane

$$a + jb \overset{?}{=} r e^{j\theta}$$

$$= r\cos\theta + jr\sin\theta$$

$$\begin{cases} a = r\cos\theta \\ b = r\sin\theta \end{cases}$$

$$\rightarrow \begin{cases} r = \sqrt{a^2+b^2} \\ \theta = \arctan\left(\frac{b}{a}\right) \end{cases}$$

Polar coordinate

$$(a, b) \rightarrow (r, \theta)$$

x $a+jb$

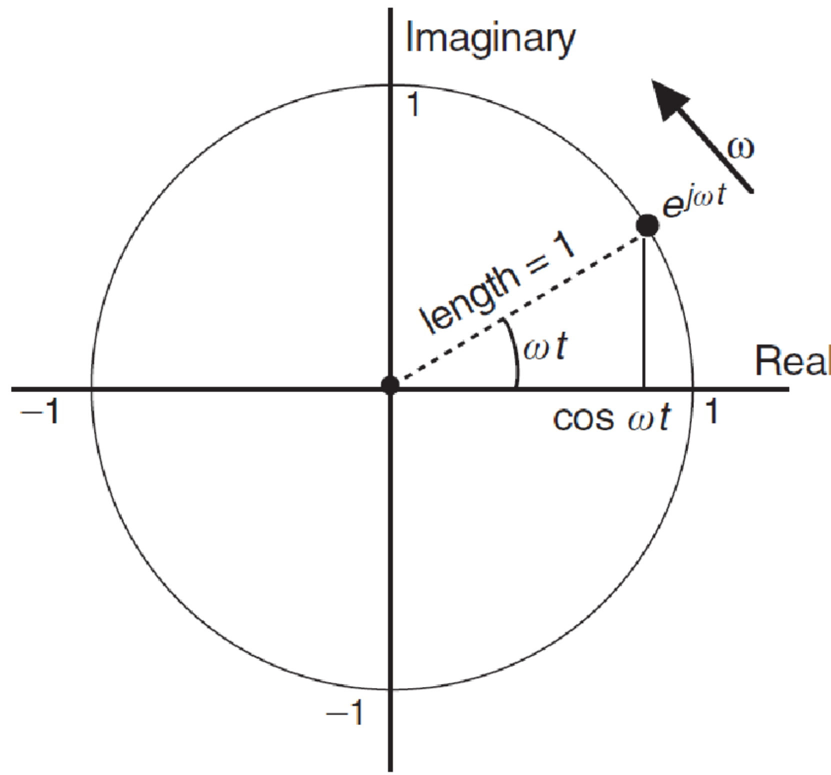


Figure 10.2 Properties of $e^{j\omega t}$ in the complex plane

Complex Voltages and Currents

Real-domain Signal
(Actual)

$$v(t) = V\cos(\omega t + \theta)$$

$$= \mathrm{Re}\left[V e^{j(\omega t + \theta)} \right]$$

complex-domain

$$= \mathrm{Re}\left[\underline{\underline{V e^{j\theta}}} \, e^{j\omega t} \right]$$

$$= \mathrm{Re}\left[\underline{\underline{V}} \, e^{j\omega t} \right]$$

(Time-indep.) Complex-Voltage

(phasor)

Real-domain current

Complex current

$$i(t) = \mathrm{Re}\left[\mathbb{I} \, e^{j\omega t} \right]$$

AC circuit example

$$0.1\,\frac{di(t)}{dt} + 50\,i(t) = 20\cos(500t)$$

complex domain

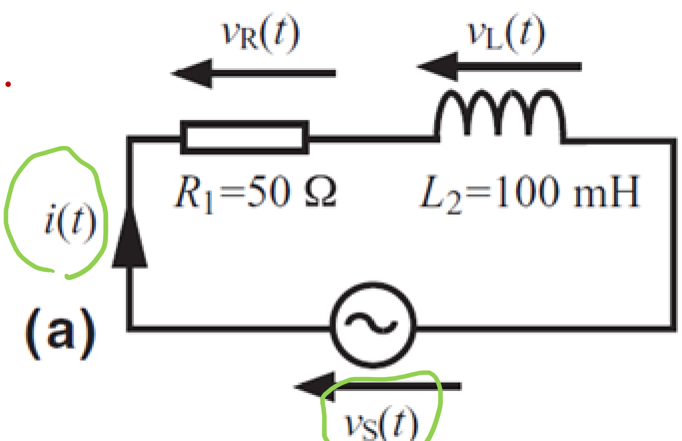
$$0.1\,\frac{d}{dt}\left[I e^{j\omega t}\right] + 50\,I e^{j\omega t} = 20\,e^{j500t} \qquad (\omega = 500)$$

$$\Rightarrow \quad 0.1\,I \cdot j\omega \cdot \left[e^{j\omega t}\right] + 50\,I\left[e^{j\omega t}\right] = 20\left[e^{j\omega t}\right]$$

$$\Rightarrow \quad (0.1 \cdot j\omega + 50)\,I = 20$$

Time-independent Eq.

$$\Rightarrow \quad I = \frac{20}{50 + j \cdot 0.1 \cdot 500} = \frac{20}{50 + j50}$$



(40 V peak-to-peak amplitude, 500 rad/s)

AC circuit example

$$\mathbb{I} = \frac{20}{50 + j50}$$

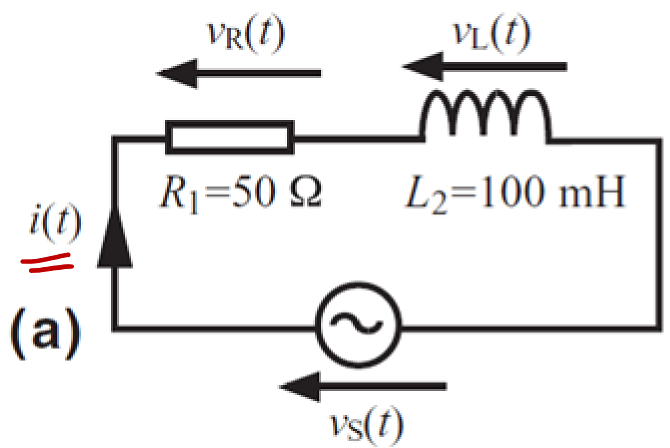
$$= \frac{20}{50} \frac{1}{1+j} \frac{1-j}{1-j}$$

$$= \frac{20}{50} \frac{1}{2} (1-j)$$

$$= 0.2(1-j)$$

$$\longrightarrow \quad i(t) = \mathrm{Re}\left[\mathbb{I}\, e^{j\omega t}\right]$$

$$= \mathrm{Re}\left[0.2(1-j)\, e^{j\omega t}\right]$$



(40 V peak-to-peak amplitude, 500 rad/s)

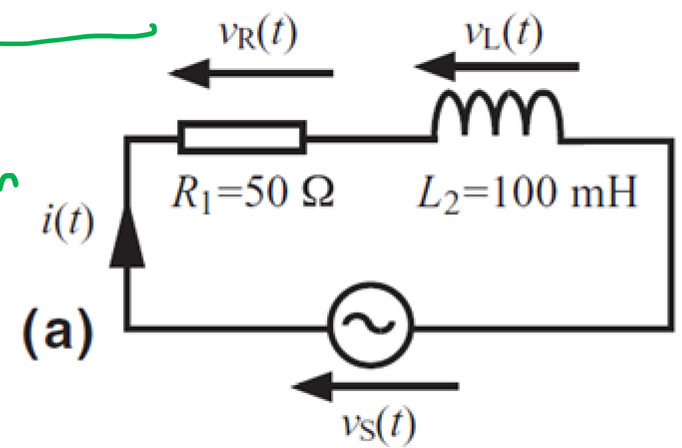
AC circuit example

$$i(t) = \mathrm{Re}\left[\, 0.2\,(1-j)\, e^{j\omega t} \right]$$

$$= \mathrm{Re}\left[\, 0.2\,(1-j)\,(\cos\omega t + j\sin\omega t) \right]$$

$$= 0.2 \times \left[\cos\omega t + \sin\omega t\right]$$

$$= \underbrace{0.2\left[\cos 500t + \sin 500t\right]}_{\text{Steady-state sol'n}}$$



(40 V peak-to-peak amplitude, 500 rad/s)

Component relations (capacitor)

$$i(t) = C\,\frac{d\,v(t)}{dt}$$

Complex Domain

$$\mathbb{I}\left[e^{j\omega t}\right] = C\,\frac{d}{dt}\left[\mathbb{V}\,e^{j\omega t}\right]$$

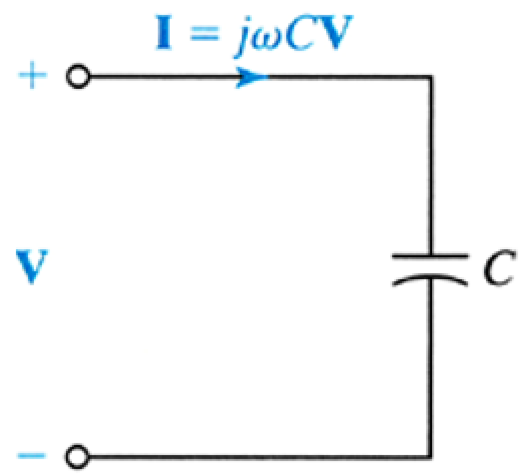
$$= C\,\mathbb{V}\,j\omega\left[e^{j\omega t}\right]$$

$$\Rightarrow \quad \mathbb{I} = (j\omega C)\,\mathbb{V}$$

$$\mathbb{V} = \left[\frac{1}{j\omega C}\right]\mathbb{I}$$

Generalized Ohm's Law

Impedance $\boxed{\mathbb{Z}_c = \frac{1}{j\omega C}}$



Component relations (inductor)

$$v(t) = L\,\frac{di(t)}{dt}$$

Complex domain:

$$\mathbb{V} e^{j\omega t} = L\,\frac{d}{dt}\left[\mathbb{I}\, e^{j\omega t}\right]$$

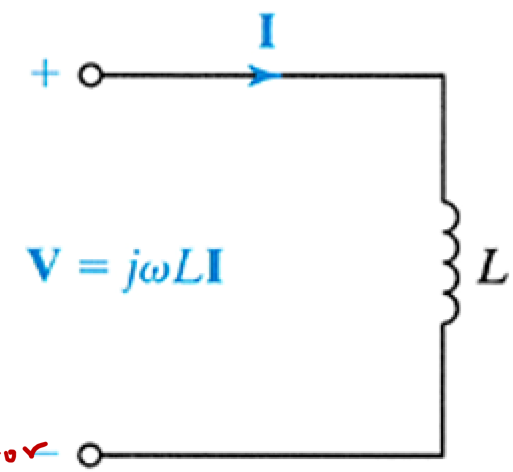
....

$$\mathbb{V} = (j\omega L)\,\mathbb{I}$$

$$\downarrow$$

Impedance of Inductor

$$Z_L = j\omega L$$



Component relations (resistor)

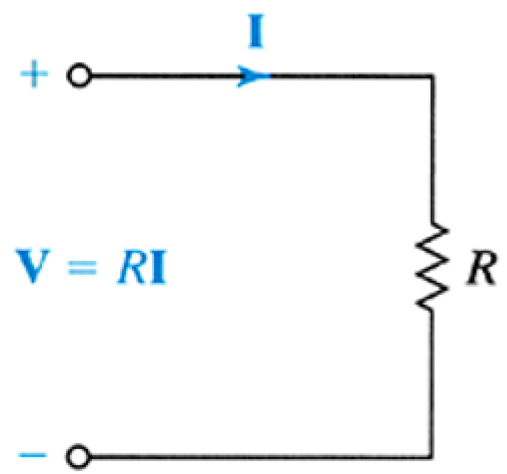
$$v(t) = R \cdot i(t)$$

Complex domain

$$\mathbf{V} = R\,\mathbf{I}$$

Impedance of Resistor

$$Z_R = R$$



Interconnection KCL

@ Node A: $i_1(t) + i_2(t) + i_3(t) = 0$

Complex $\mathrm{Re}\left[\mathbb{I}_1 e^{j\omega t}\right] + \mathrm{Re}\left[\mathbb{I}_2 e^{j\omega t}\right] + \mathrm{Re}\left[\mathbb{I}_3 e^{j\omega t}\right] = 0$

$$\mathrm{Re}\left[(\mathbb{I}_1 + \mathbb{I}_2 + \mathbb{I}_3)\, e^{j\omega t}\right] = 0$$

$$\Rightarrow \quad \mathbb{I}_1 + \mathbb{I}_2 + \mathbb{I}_3 = 0$$

KCL holds in complex domain.

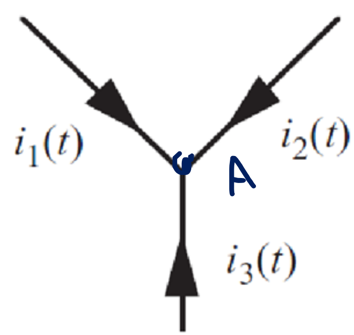
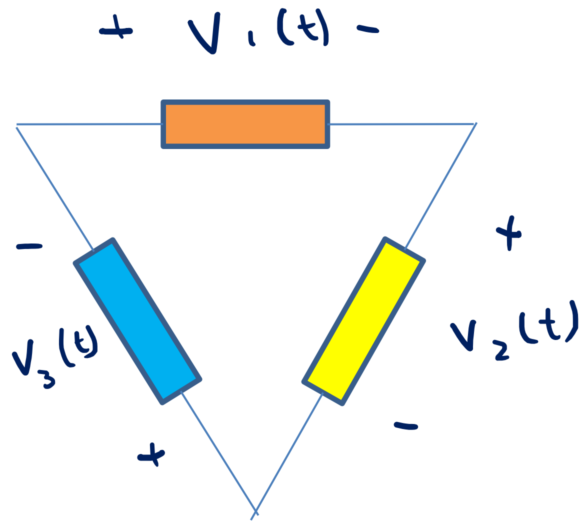


Figure 10.3 Relevant to the discussion of whether KCL is valid for complex currents

Complex KVL

Loop: $v_1(t) + v_2(t) + v_3(t) = 0$

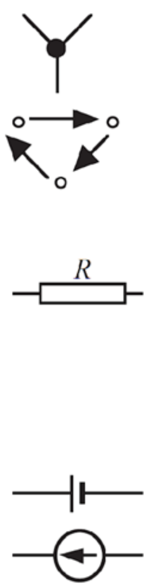
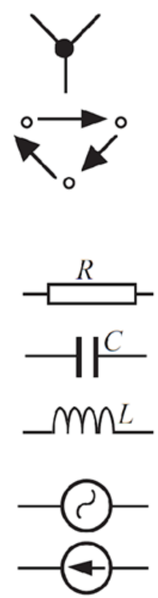
Complex domain $\hookrightarrow$ $\mathbb{V}_1 + \mathbb{V}_2 + \mathbb{V}_3 = 0$



AC Circuit Analysis

NODAL ANALYSIS
SUPERPOSITION
THEVENIN'S EQUIV CKT
NORTON EQUIV CKT

Table 10.1 Similarities between the component relations and connection constraints in DC circuits and AC circuits

DC Currents and voltages			AC Currents and voltages	
representation	*relation*		*relation*	*representation*
		**Connections**		
	$\sum_{node} I = 0$	*at node*	$\sum_{node} \mathbf{I} = 0$	
	$\sum_{loop} V = 0$	*around loop*	$\sum_{loop} \mathbf{V} = 0$	
		Components		
	$V = R\,I$	*resistor*	$\mathbf{V} = R\,\mathbf{I}$	
		capacitor	$\mathbf{I} = j\omega C\,\mathbf{V}$	
		inductor	$\mathbf{V} = j\omega L\,\mathbf{I}$	
	V = constant	*voltage source*	$\mathbf{V}$ = constant	
	I = constant	*current source*	$\mathbf{I}$ = constant	

KCL

KVL

Generalized Ohm's Law (Impedance)

fixed ω

Example 10.1

Given $v_s(t)$

Find $i(t)$

ODE → Solution

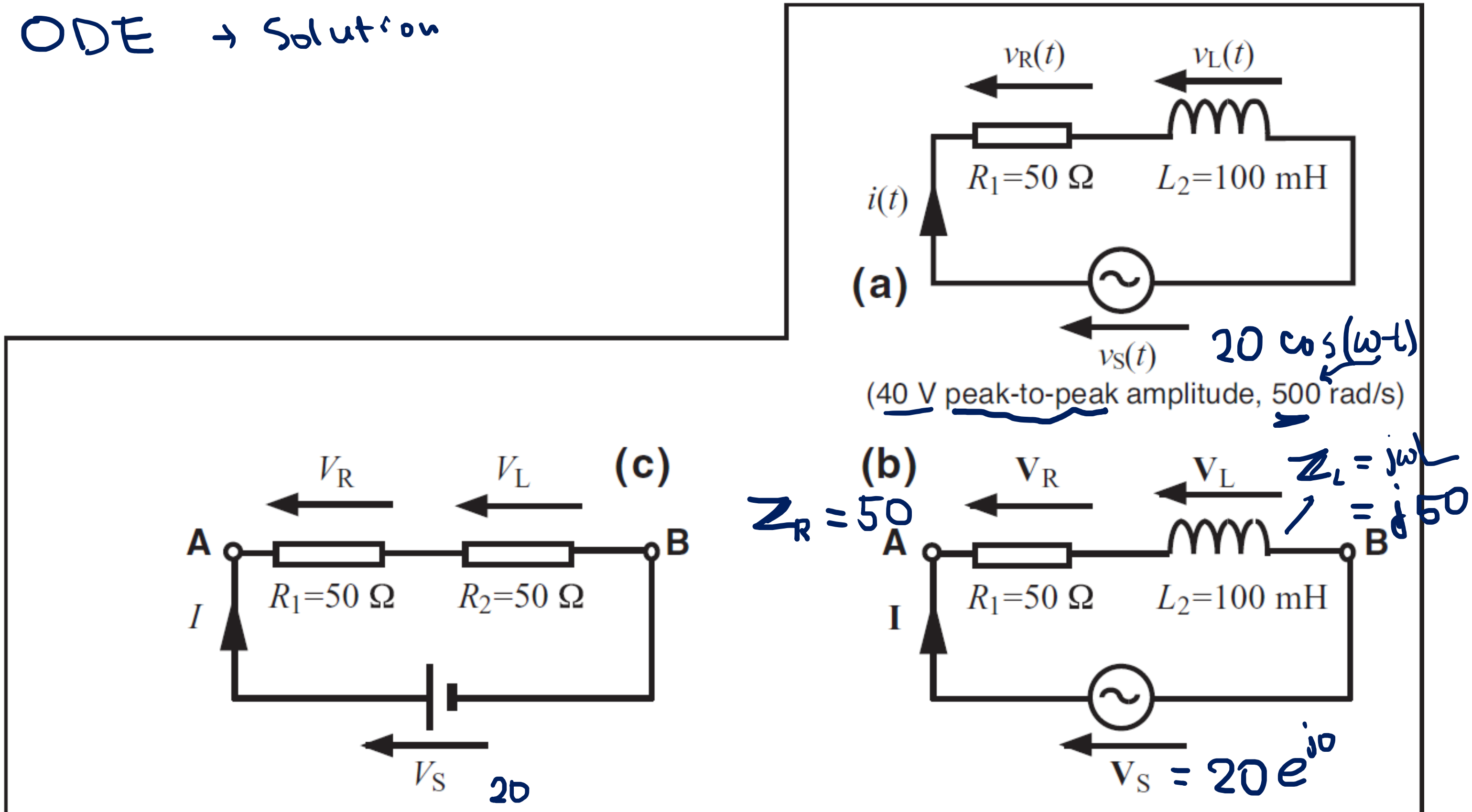
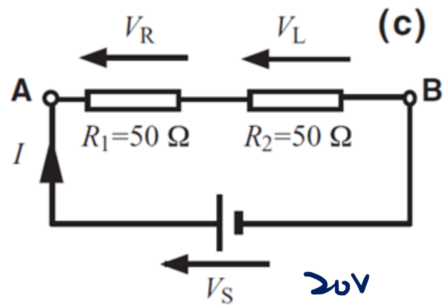


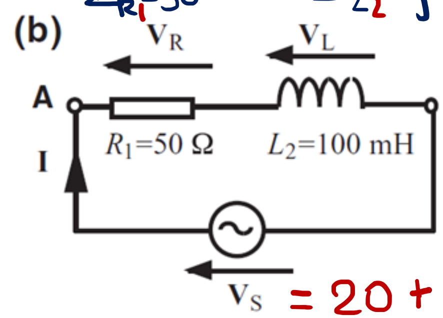
Figure 10.4 Relevant to a demonstration of the similarities between DC and AC analysis

Example 10.1



20V

$Z_{R1}=50$ $\quad Z_{L2}=j50$



$= 20 + j0$

$$R_{AB} = R_1 + R_2 = 100\Omega$$

$$I = V_s/R_{AB} = \frac{20}{100} = 0.2\ A$$

$$V_R = R_1 \cdot I = 10V$$

$$V_L = R_2 \cdot I = 10\ V$$

$\boxed{DC}$

$$Z_{AB} = Z_{R1} + Z_{L2}$$
$$= 50 + j50$$

$$I = V_s/Z_{AB} = \frac{20}{50+j50}$$
$$= 0.2 - j0.2$$

$$V_R = R_1 \cdot I = 10 - j10$$

$$V_L = Z_L \cdot I = (j50)(0.2 - j0.2)$$
$$= 10 + j10$$

$V_s = V_R + V_L$

check !

$\boxed{AC}$

Example 10.1

$\mathrm{I} = 0.2 - j\,0.2 \longrightarrow i(t) = \mathrm{Re}[\,\mathrm{I}\, e^{j\omega t}\,]$

$r = \sqrt{a^2+b^2}$

$\theta = \arctan(b/a)$

$\downarrow$ polar coordinate

$= 0.2\sqrt{2}\; e^{-j\frac{\pi}{4}} = 0.2\sqrt{2}\angle -45^0$

$i(t) = \mathrm{Re}\left[0.2\sqrt{2}\, e^{-j\frac{\pi}{4}} \cdot e^{j\omega t}\right]$

$= \underbrace{0.2\sqrt{2}}_{\text{Amplitude}} \cdot \cos\left(\omega t - \underbrace{\frac{\pi}{4}}_{\text{Phase}}\right)$

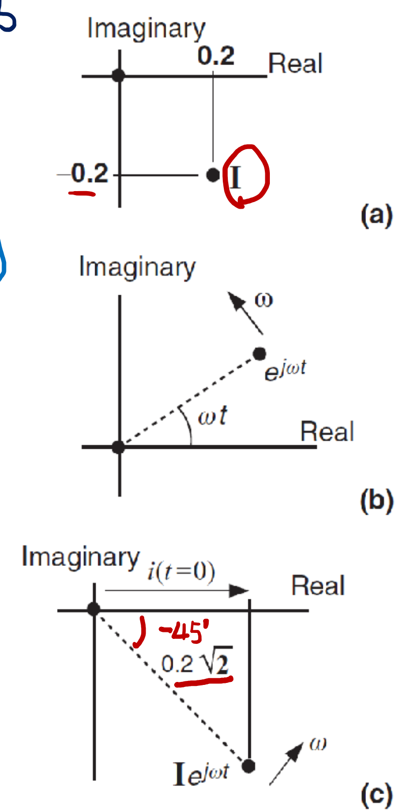


Figure 10.5 Steps in the transformation from a complex current to the actual value at a particular time

Example 10.1

$$\mathbb{V}_R = 10 - j10 = 10\sqrt{2}\, e^{-j\frac{\pi}{4}} = 10\sqrt{2} \angle -45^\circ$$

time domain

$$v_R(t) = 10\sqrt{2} \cos\left(\omega t - \frac{\pi}{4}\right)$$

$$\mathbb{V}_L = 10 + j10 = 10\sqrt{2}\, e^{+j\frac{\pi}{4}} = 10\sqrt{2} \angle 45^\circ$$

time domain

$$v_L(t) = 10\sqrt{2} \cos\left(\omega t + \frac{\pi}{4}\right)$$

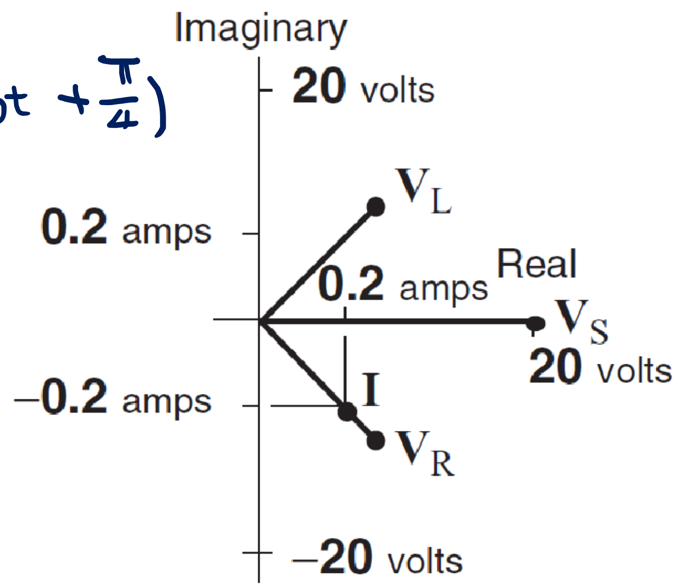


Figure 10.6 Complex representation of voltages in the circuit of Figure 10.4a

Simplified phasor notations

$$1\, e^{j\theta} = 1 \angle \theta$$

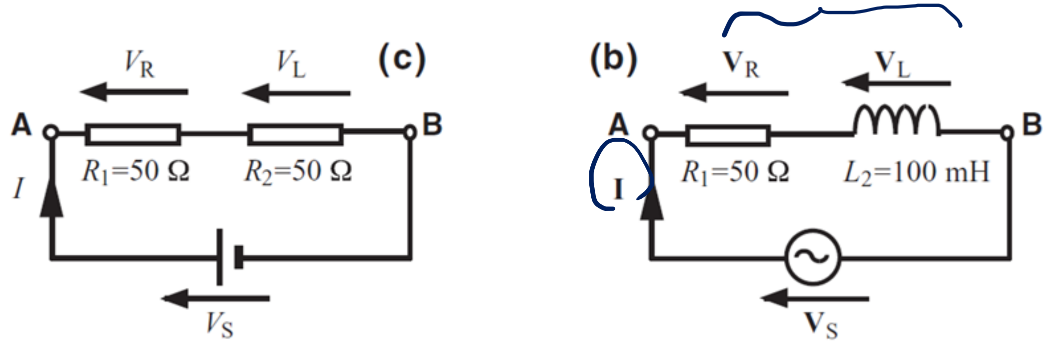
$$(A \angle \theta)(B \angle \phi) = AB \angle (\theta + \phi)$$

$$\frac{A \angle \theta}{B \angle \phi} = \frac{A}{B} \angle (\theta - \phi)$$

e.g. $(2 \angle 45^\circ)(3 \angle 20^\circ) = 6 \angle 65^\circ$

$$\frac{2 \angle 45^\circ}{3 \angle 20^\circ} = \frac{2}{3} \angle 25^\circ$$

Example 10.1



Using simplified phasor notations

$$\mathbf{V}_s = 20\angle 0^\circ$$

$$\mathbf{Z}_{AB} = 50 + j\omega L_2 = 50 + j50 = 50\sqrt{2}\angle 45^\circ$$

$$\mathbf{I} = \frac{20\angle 0^\circ}{50\sqrt{2}\angle 45^\circ} = 0.2\sqrt{2}\angle -45^\circ$$

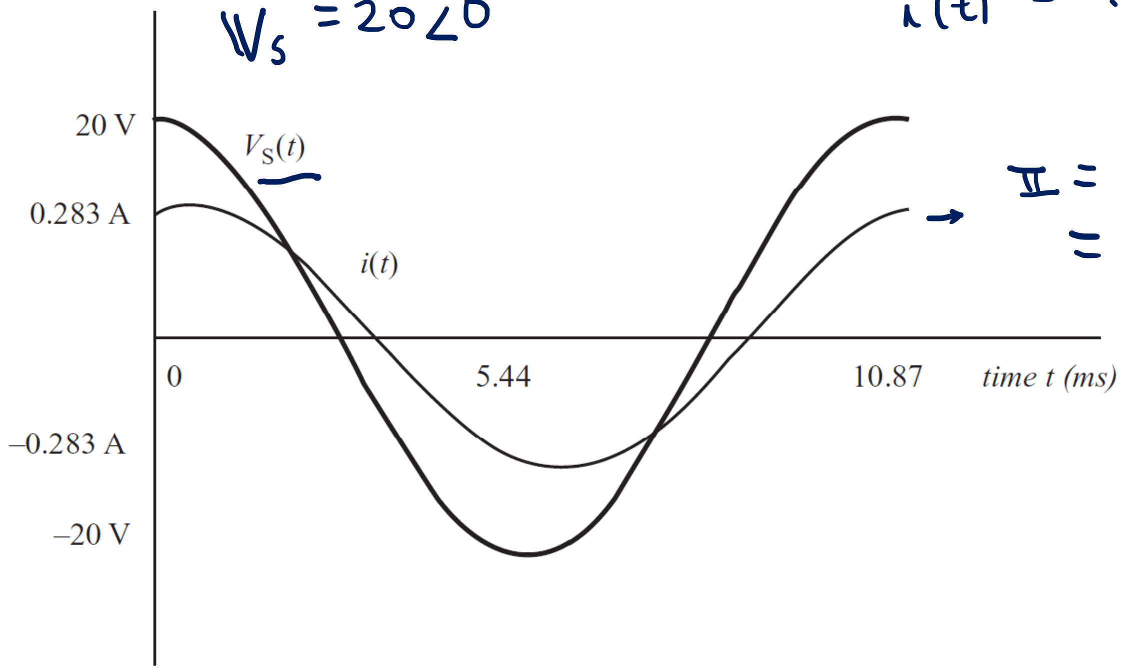
$$\Rightarrow i(t) = 0.2\sqrt{2}\cos(500t - 45^\circ)$$

Example 10.1

$v_s(t) = 20\cos(\omega t)$

$\mathbb{V}_s = 20\angle 0^\circ$

$i(t) = 0.283\cos(\omega t - 45^\circ)$



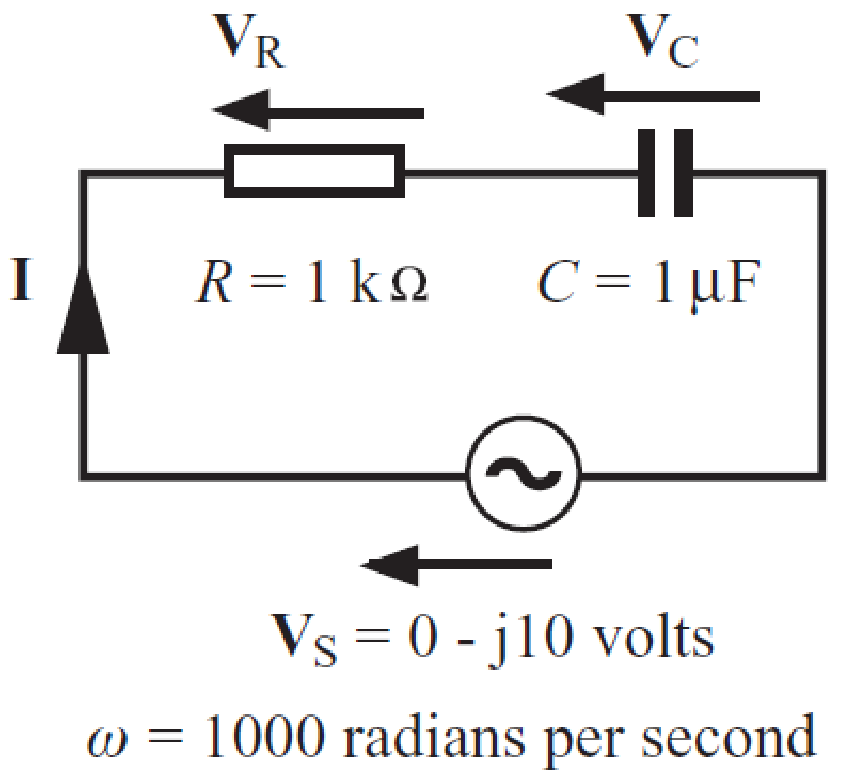
$\rightarrow \mathbb{I} = 0.2\sqrt{2}\angle -45^\circ$

$= 0.283\angle -45^\circ$

Figure 10.7 One cycle of the waveforms of $v_S(t)$ and $i(t)$

Quiz

Find the values of $\mathbf{V}_R$, $\mathbf{V}_C$ and $\mathbf{I}$ in the circuit of the following figure. Express these quantities in polar (simplified) form.



Quiz review

$$\mathbb{V}_s = 10\angle -90^\circ \quad (V)$$

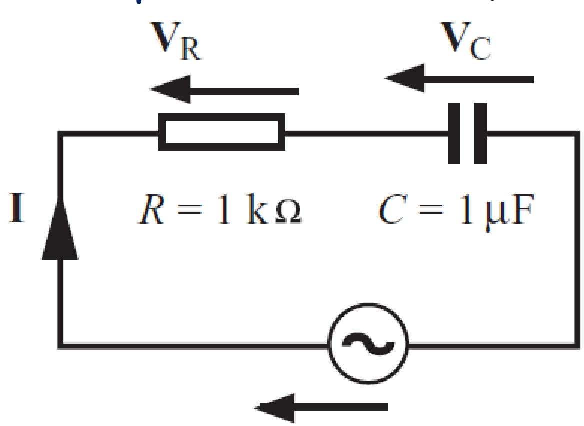
$$\mathbb{Z} = R + \frac{1}{j\omega C} = 1000 + \frac{1}{j\cdot 1000\cdot 10^{-6}} = 1000 - j1000$$

$$= 1000\sqrt{2}\angle -45^\circ \quad (\Omega)$$

$$\mathbb{I} = \frac{\mathbb{V}_s}{\mathbb{Z}} = \frac{10\angle -90^\circ}{1000\sqrt{2}\angle -45^\circ} = 5\sqrt{2}\angle -45^\circ \quad (mA)$$

$$\mathbb{V}_R = R\cdot \mathbb{I} = 5\sqrt{2}\angle -45^\circ \quad (V)$$

$$\mathbb{V}_c = \frac{1}{j\omega C}\mathbb{I} \qquad \mathbb{Z} \qquad 5\sqrt{2}\angle -135^\circ$$



$\mathbf{V}_S = 0 - j10$ volts

$\omega = 1000$ radians per second

Simplified AC analysis

$$Z = R + jX$$

Impedance Resistance Reactance

C : $\frac{1}{j\omega C}$ 容抗

L : $j\omega L$ 感抗

$Y = \frac{1}{Z}$

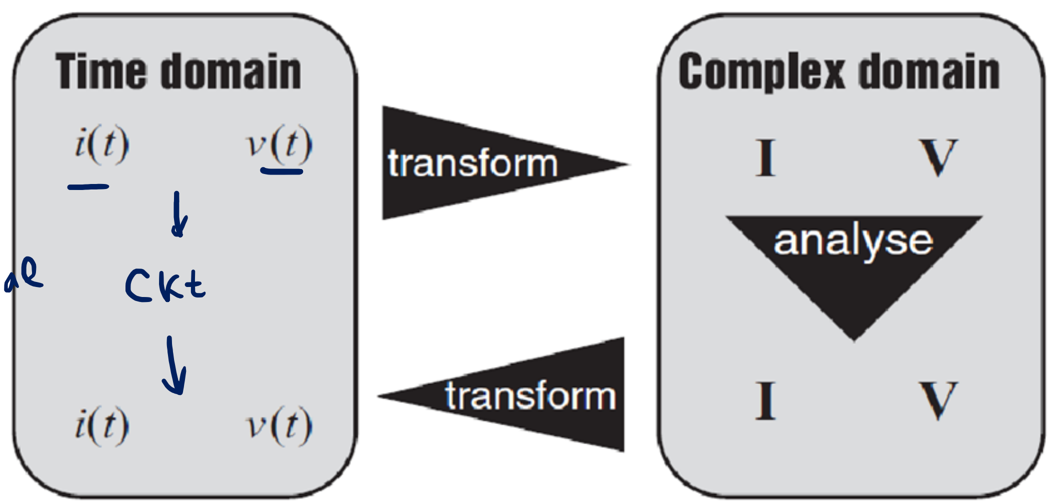
$$Y = G + jB$$

Admittance 導納 Conductance Succeptance

C 容納

L 感納

Impedance



(phasor)

Differential Equation

Algebra

Figure 10.8 Illustrating the approach to AC analysis based on the use of complex currents and voltages