equivalent circuit the current through the 1.6 kΩ resistor is easily found by Ohm's law as 4/(2.4 + 1.6) = 1 mA.

Just as the voltage divider principle can be applied to a circuit of the form of Figure A4.12(c), a 'current divider' principle can be applied to analyse the circuit of Figure A4.12(d). The conductance of the 1.6 kΩ resistor is 0.625 mS, and that of the 2.4 kΩ resistor is 0.4167 mS. By the current divider principle the current through the 1.6 kΩ resistor is (1.667).[0.625/(0.625 + 0.4167)] which is 1 mA.

Answer 4.13

The open-circuit voltage between terminals A and B is, by KVL, $-8 + (5\,\text{k}\Omega \times 4\,\text{mA}) = 12\,\text{V}$ (there is no voltage drop across the right-hand 5 kΩ resistor because no current flows in or out of terminal A).

To find the Thevenin resistance R_O we set the values of the voltage source and current source to zero, thereby resulting in two 5 kΩ resistors connected in series between A and B: the equivalent resistance, R_O, is 10 kΩ.

The connection of a 12 V source in the polarity described would result in no current flow (since the external 12 V source directly opposes the internal 12 V source in the Thevenin model).

Answer 5.2

We select a voltage reference node indicated by the earth symbol in Figure A5.2 and label the nodes (A, B) for which the nodal voltage is unknown.

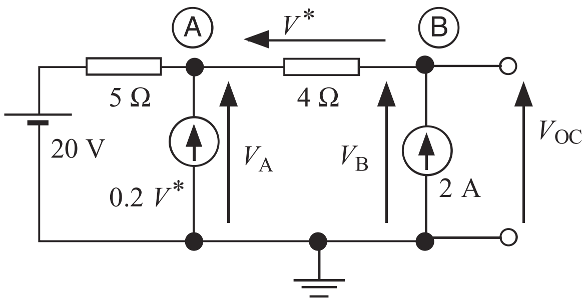

Figure A5.2

By application of KCL at node A we obtain:

$$\frac{(20 - V_A)}{5} + \frac{(V_B - V_A)}{4} + 0.2(V_A - V_B) = 0$$

which can be rearranged as

$$-0.25\,V_A + 0.05\,V_B = -4 \tag{5.1}$$

The application of KCL at node B leads to

$$2 + \frac{(V_A - V_B)}{4} = 0$$

which can be rearranged as

$$0.25\,V_A - 0.25\,V_B = -2 \tag{5.2}$$

Addition of these two nodal Equations (5.1) and (5.2) yields $V_B = 30\,\text{V}$.
Since V_B and V_{OC} are identical, $V_{OC} = 30\,\text{V}$.

Answer 5.4

To find the resistance between the external terminals of the grey box we connect (Figure A5.4) a current of 1 A between them and calculate the voltage V that appears: the ratio $V/(1\text{ A})$ is then the required resistance.

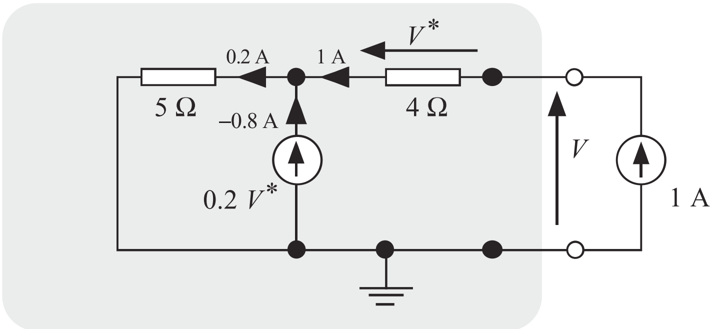

Figure A5.4

From the circuit we see, from Ohm's law, that $V^* = -4\,\text{V}$. The value of the controlled source is therefore $0.2\,(-4) = -0.8\,\text{A}$. By KCL it follows that the current through the 5Ω resistor is 0.2 A (right to left). By Ohm's law the voltage across the 5Ω resistor is 1 V. Using KVL we can write

$$V = 1 + 4 = 5\,\text{V}.$$

A two-terminal box whose current is 1 A and whose voltage is 5 V is equivalent to a resistance R_O of 5 Ω.

Answer 5.5

We first create the circuit of Figure A5.5(a) to calculate the current I due to the voltage source acting alone. With the reference node for voltage as shown we apply KCL at node A to obtain

$$-\frac{V}{1} + 0.2\,V + \frac{(3 - V)}{1} = 0$$

which gives $V = 1.666\,\text{V}$. By KVL the voltage across the top $1\,\text{k}\Omega$ resistor is 1.333 V and the current $I = -1.333\,\text{mA}$ in the reference direction shown.

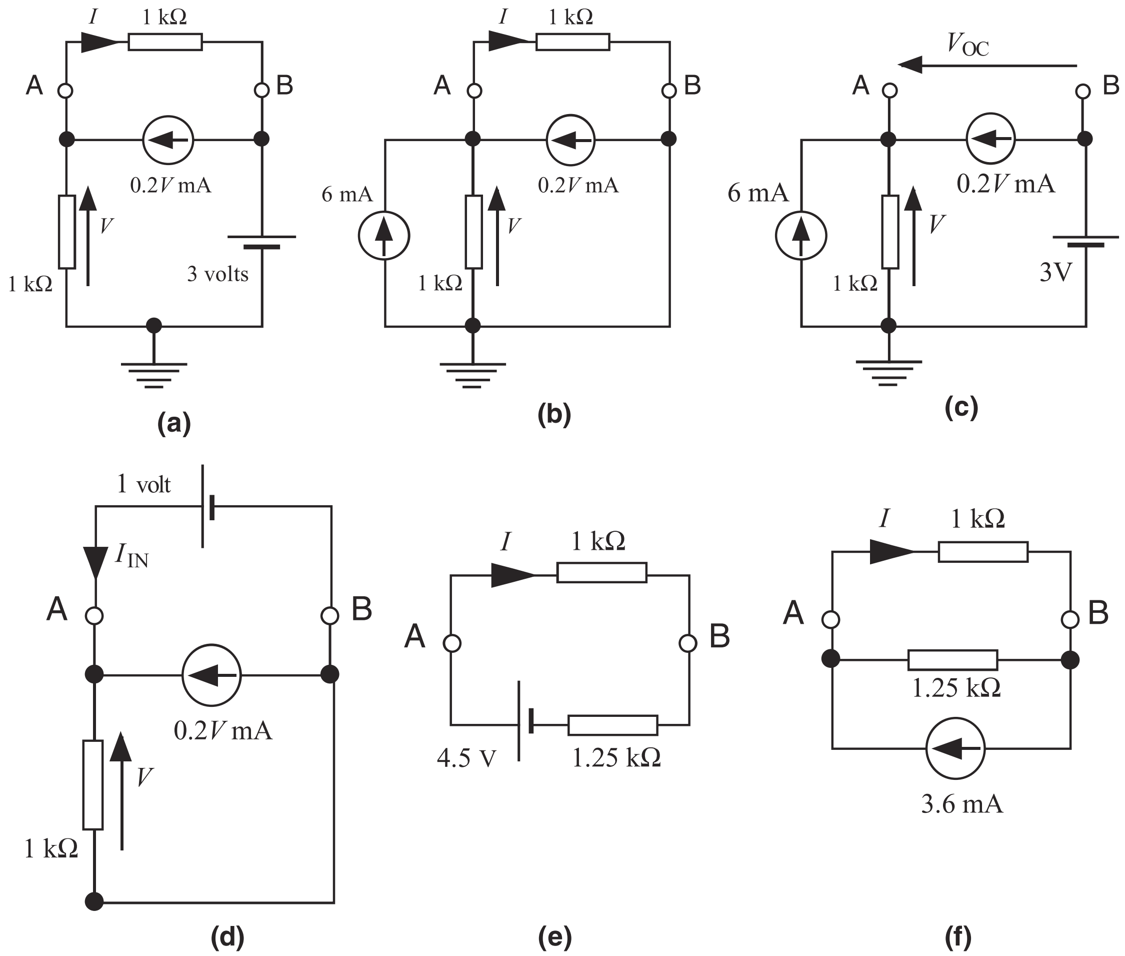

Figure A5.5

We now create the circuit of Figure A5.5(b) to calculate the current I due to the independent current source acting alone. With the reference node as shown, application of KCL at node A gives

$$6 - \frac{V}{1} + 0.2\,V - \frac{V}{1} = 0$$

which gives $V = 3.333\,\text{V}$ and, by Ohm's law, $I = 3.333\,\text{mA}$ in the reference direction shown.

Application of the superposition principle allows us to say that the actual value of I in the original circuit in the reference direction shown is $3.333 - 1.333 = 2\,\text{mA}$,

To obtain the requested Thevenin equivalent circuit we first find the open-circuit voltage V_{OC} between terminals A and B (see Figure A5.5c). With the chosen voltage reference node, application of KCL at node A leads to

$$6 - \frac{V}{1} + 0.2\,V = 0$$

such that $V = 7.5\,\text{V}$ and, by KVL, $V_{OC} = V - 3 = 4.5\,\text{V}$.

To find the Thevenin resistance R_O we set the independent sources to zero, *leaving the VCCS in place*, to obtain the circuit of Figure A5.5(d). Here we are applying a voltage of 1 V between terminals A and B and calculating the resulting input current I_{IN}, because $(1\,\text{V})/I_{IN}$ will then be the resistance between terminals A and B. Noting that $V = 1\,\text{V}$ we apply KCL to obtain

$$I_{IN} + 0.2 = 1/1 \text{ giving } I_{IN} = 0.8A.$$

The resistance R_O is therefore $1/0.8 = 1.25\,\text{k}\Omega$.

We now connect the 1 kΩ resistor to the terminals A and B of the Thevenin equivalent circuit (Figure A5.5e) and calculate the current I to be 2 mA, which is in agreement with the value obtained by application of the superposition principle.

The Norton equivalent is easily derived from the Thevenin model: the current source is the Thevenin voltage source V_{OC} divided by R_O and the resistance is identical with R_O (Figure A5.5f). By applying the current divider principle we can write that

$$I = 3.6[1/1 + 0.8)] = 2\text{ mA}.$$

By reference to Figure A5.5(e) we see that if the external 1 kΩ resistor were to be replaced by a 4.5 V source with its positive terminal connected to A, the current I would be zero.

Answer 5.6

To find the Thevenin equivalent circuit we first analyse the circuit with nothing attached to the external terminals (Figure A5.6a). Application of KCL at node A leads to

$$\frac{(20 - V_{OC})}{1} - (V_{OC} - 20) - \frac{V_{OC}}{1} = 0 \text{ which gives}$$

$V_{OC} = 13.33\,\text{V}$.

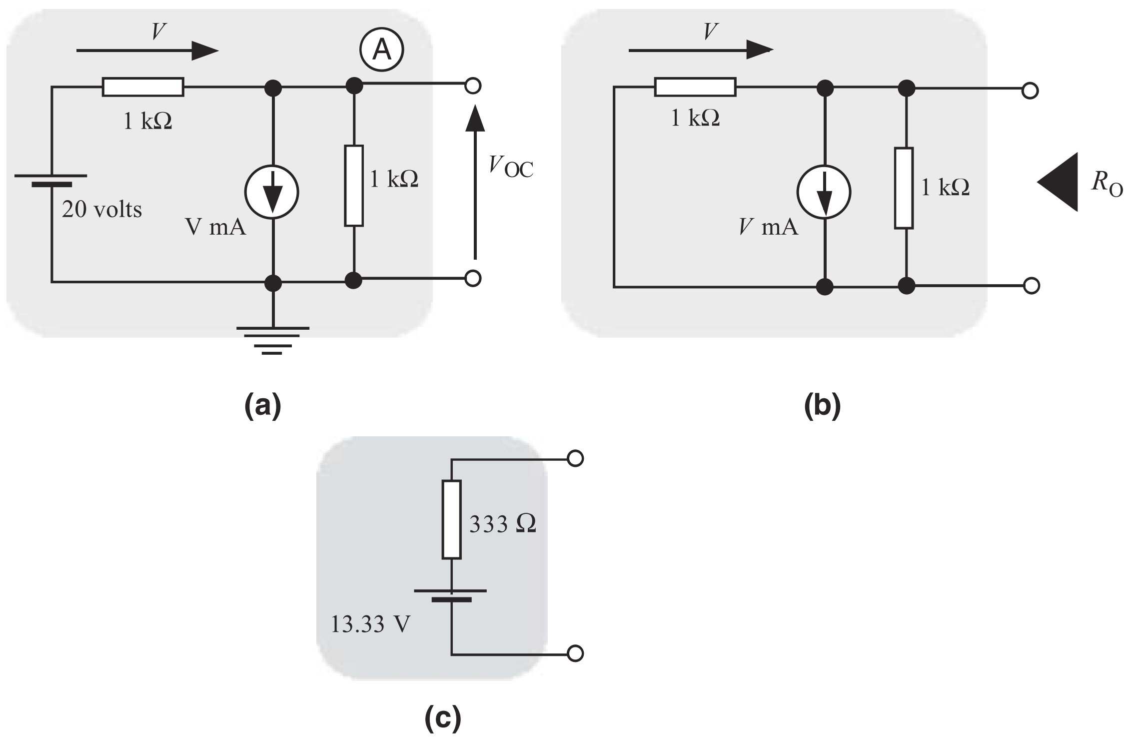

Figure A5.6

To find the Thevenin resistance R_O we set the independent voltage source equal to zero and remember not to set the dependent source to zero (Figure A5.6b). The calculation of the resistance R_O between the external terminals is considerably simplified by observing that the voltage controlling the current source appears directly across the current source, which is therefore equivalent to a resistance, in this case of value 1 kΩ (A voltage V across a two-terminal component creating a current of V mA through it is characteristic of a resistance of value 1 kΩ). Thus, by reference to the figure, we have three 1 kΩ resistors in parallel, equivalent to 333Ω. Thus, $R_O = 330\,\Omega$. The complete Thevenin equivalent circuit is shown in Figure A5.6(c).

Answer 5.9

In Figure A5.9 the added points correspond to a constant product (300 mW) of V and I, and the boundary sketched in indicates the permissible region of operation. The plotted load-line is associated with the series connection of the 8 V source and the resistor R for the case in which the intersection of the load-line and the Zener diode characteristic is at the minimum, for R, consistent with the 300 mW limit of dissipated power. If the intersection of the load-line with the diode current axis is estimated to be 160 mA, the corresponding value of R is (8 V)/(160 mA) = 50 Ω.

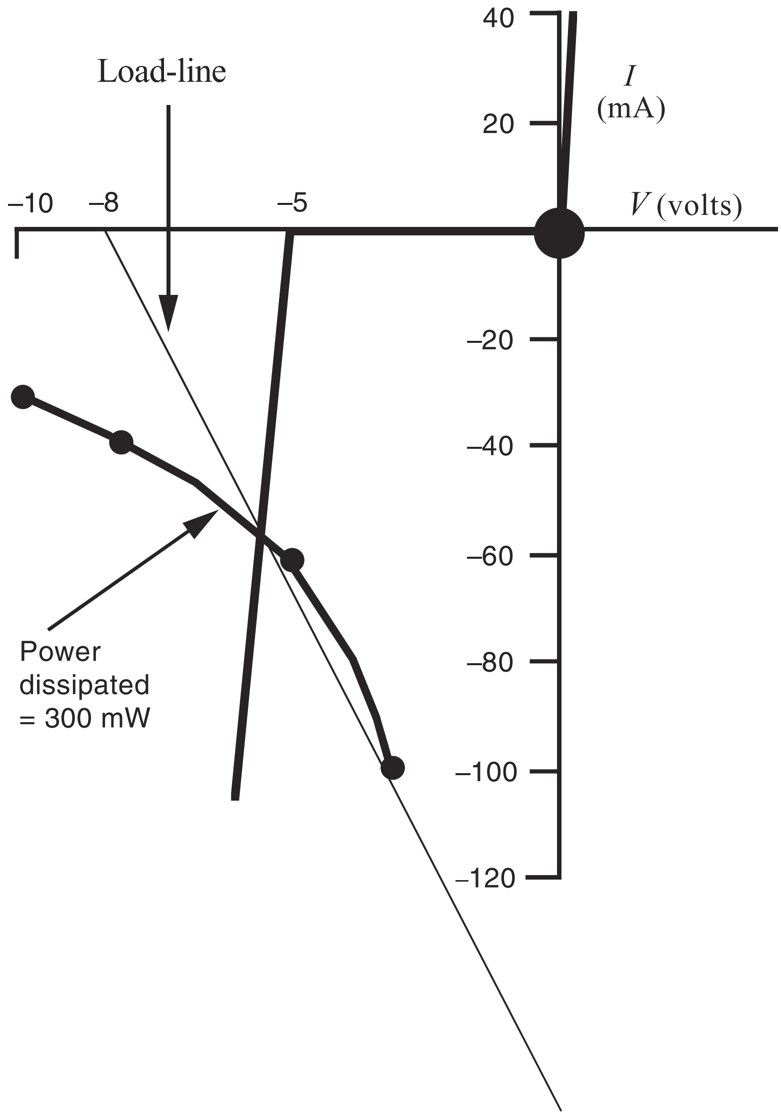

Figure A5.9

The load-line intersects the Zener characteristic at about 5.5 V, so the voltage across the resistor R would be $8 - 5.5 = 2.5$ V. The intersection indicates a current of approximately 50 mA, so the power dissipated by the resistor is $(2.5\,\text{V}) \times (50\,\text{mA}) = 125$ mW.

Answer 6.1

(a) $V^+ > V^-$ therefore $V_I > 0$. Hence V_O is also positive and equal to $+10$ V.

(b) $V^+ = 0$, so $V_I = V^+ - V^- = 0 - 4 = -4$ V. Since V_I is negative V_O must be negative and equal to -10 V.

(c) $V_I = (-5) - (-4) = -1$ V. Since V_I is negative V_O must be equal to -10 V.

(d) $V_I = 4 - 5 = -1$ V. Therefore $V_O = -10$ V.

Answer 6.3

The circuit is a cascade of two triggers.

We first examine the trigger involving opamp X in order to determine the threshold values at which the voltage at A will cause a change of state. When the voltage at C is 15 V, then $V^+ = 4$ V. Therefore the voltage at C changes from +15 to −15 V when the voltage at C exceeds 4 V. Similarly, the voltage at C changes from −15 to +15 V when the voltage at A falls below −4 V.

We now examine the trigger involving opamp Y. To establish the threshold voltage levels pertinent to the voltage at C we examine the circuit of Figure A6.3(a). The opamp Y changes state (voltage at B changes from +15 to −15 V) when the voltage at X just starts to go negative. With $V_X = 0$ the current through the 10 kΩ resistor is 1.5 mA. This current flows through the 5 kΩ resistor, creating a voltage of 7.5 V so that the voltage at C is −7.5 V. Thus the output of opamp Y changes from +15 to −15 V when the voltage at C decreases below −7.5 V, and it changes back from −15 to +15 V when the voltage at C increases above 7.5 V.

At $t = 2\,T$ the voltage at A increases above 4 V, so the voltage at C drops from +15 to −15 V. As a result the voltage at B also drops to −15 V.

At $t = 4\,T$ there is no effect because the threshold for the voltage at A is now −4 V.

At $t = 4.5\,T$ the voltage at A decreases below −4 V so the voltage at C changes from −15 to +15 V. As a result the voltage at B also changes from −15 to +15 V.

Waveforms of the voltages at B and C are shown in Figure A6.3(b)

The current I has two components. The current through the series connection of the 11 and 4 kΩ resistors (equivalent to 15 kΩ since no current is drawn by the positive input terminal of opamp X) is 1 mA when the voltage at C is +15 V and −1 mA when it is −15 V. The other component of the current I is the current through the 5 kΩ resistor. But since the voltages at C and B are always identical this current has a value of zero. Thus, the waveform of the current I is as shown in Figure A6.3(b).

Refer to the circuit of Figure A6.3c. If the voltage at B stays at 15 V when the voltage at C falls to −15 V the voltage V^+ at node X is −5 V because the current through the 10 kΩ resistor is (30 V)/15 kΩ = 2 mA. To maintain the input voltage V_I of opamp Y positive (to keep B at +15 V) the voltage V^- at the negative input terminal of opamp Y must therefore be less than −5 V. The connection of a 6 V source (note the polarity shown in Figure A6.3 c) would ensure that the value of V_I for opamp Y is positive so that the voltage at B remains unchanged at +15 V.

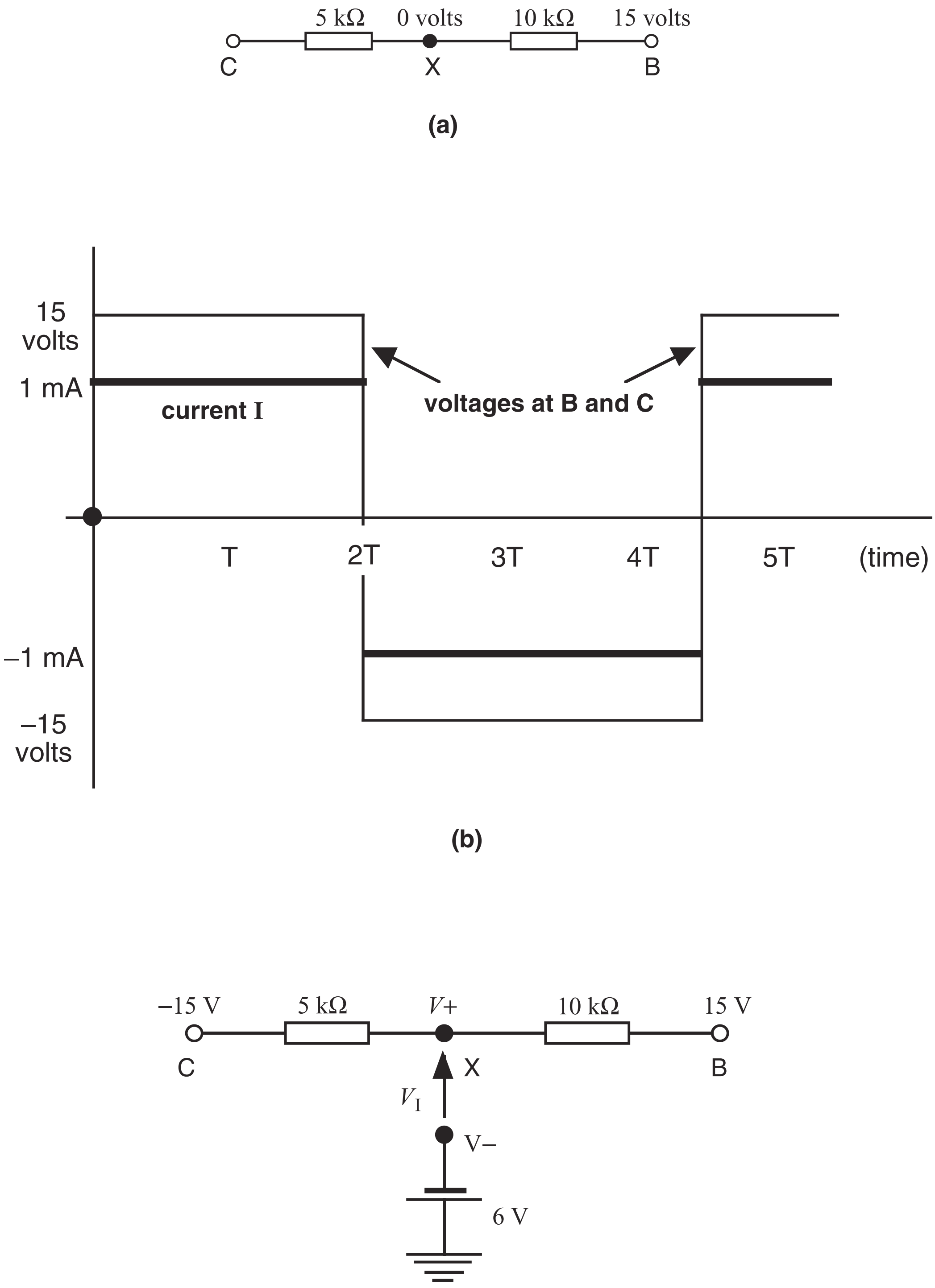
5 kΩ
0 volts
10 kΩ
15 volts
C
X
B
(a)
15 volts
1 mA
current I
voltages at B and C
T
2T
3T
4T
5T
(time)
–1 mA
–15 volts
(b)
–15 V
5 kΩ
V+
10 kΩ
15 V
C
X
B
V_I
V–
6 V
(c)

Figure A6.3

Answer 6.4

The same voltage V is applied to two comparators (but note the polarity of the opamp input terminals). For any value of V, therefore, it is possible to determine the output voltage of each opamp. Only when these two voltages are the same will no current flow in the 10 kΩ resistor.

Corresponding values of V and the outputs V_L and V_R of the left-hand and right-hand opamps respectively are (in volts):

V	-1	0	0.99	1.1	1.9	2.1	3
V_L	-10	-10	-10	-10	-10	$+10$	$+10$
V_R	$+10$	$+10$	$+10$	-10	-10	-10	-10

In fact, the critical transitions occur when $V = 1.0$ and 2.0 V. Within that range the output voltages of the two opamps are identical and no current will flow through the 10 kΩ resistor.

Answer 7.3

Since no current enters the negative input terminal of the opamp we can apply the voltage divider principle to calculate the voltage at the junction of the 4 and 1 kΩ resistors: it is $(4/5)\times V = 0.8V$. Again, because there is no voltage drop across the 11 kΩ resistor, the voltage at the negative input terminal is also $0.8V$.

With negative feedback we assume a virtual short circuit between the input terminals of the opamp, so the voltage at the positive input terminal is also $0.8V$. The voltage at this point is also equal to 4 V because there is no current through, and therefore no voltage across, the 17 kΩ resistor. Thus,

$$0.8\,V = 4 \text{ giving } V = 5\,\text{V}.$$

Note the redundant nature of the 17 and 11 kΩ resistors.

Answer 7.5

Since the voltage at the negative input terminal of the opamp is essentially the same as at the positive input terminal (i.e., there is a virtual short-circuit between the two terminals) and hence at earth voltage, the direct voltage provided by the voltage source appears directly across the photodiode and provides it with the reverse bias necessary to its successful operation. The only factor that controls the current through the photodiode is the incident radiation.

Any current I_D passed by the diode must flow through the resistor R, setting up a voltage $I_D R$ across it. Thus, the voltage measured by the voltmeter is, by KVL, equal to $-I_D R$ since the negative input terminal is essentially at zero voltage. For each microwatt of radiation the diode generates 0.5μA and sets up a measured voltage having a magnitude of $(0.5\mu\text{A})\times R$. But we are told that the scale factor of the voltmeter is to be 2.5 μW/mV, so the measured voltage corresponding to a microwatt of incident radiation is 0.4 mV. Thus,

$$(0.5\ \mu\text{A}) \times R \times (2.5) = 1\ \text{mV, giving } R = 800\ \Omega.$$

Answer 7.7

Because there is a virtual short-circuit between the input terminals of each opamp, the voltage V_2 appears at the top of the 10 kΩ resistor and the voltage V_1 at the bottom. The voltage across the 10 kΩ resistor is therefore $(V_1 - V_2)$ and the current through it is $(V_1 - V_2)/10\,\text{k}\Omega$ upwards. All this current must flow in the 50 kΩ resistors because no current can flow into the negative terminals of the opamps. Thus, by Ohm's law,

$$V_Y = V_2 - 5(V_1 - V_2) = -5\,V_1 + 6\,V_2$$
$$V_X = V_1 + 5(V_1 - V_2) = 6\,V_1 - 5\,V_2$$

In calculating V_{OUT} we can regard V_X and V_Y as fixed at the values given above and we can otherwise ignore the circuit to the left of the 5 kΩ resistors.

We now employ superposition to calculate the effects of V_X and V_Y. If we set $V_X = 0$ the voltage at the positive input terminal of the right-hand opamp is zero. Recalling the expression for the voltage gain of an inverter we can write:

$$V_{OUT} \text{ due to } V_Y = -(50\,\text{k}\Omega/5\,\text{k}\Omega) \times V_Y = -10\,V_Y$$

If we now set $V_Y = 0$ the voltage at the negative input terminal of the right-hand opamp is, by voltage divider action, $V_{OUT}[5/(5 + 50)] = V_{OUT}/11$. This must also be the voltage at the positive input terminal in view of the virtual short-circuit between the input terminals. But, again by voltage divider action, this voltage must be $V_X[50/(50 + 5)]$. Thus,

$$V_X(50/55) = V_{OUT}/11 \text{ giving } V_{OUT} \text{ (due to } V_X) = 10\,V_1$$

Adding the contributions of V_X and V_Y we find that

$$V_{OUT} = 10(V_X - V_Y) = 110(V_1 - V_2).$$

Answer 7.9

First, we calculate the open-circuit voltage between terminals A and B.

With negative feedback applied to both opamps, a virtual short-circuit will occur between their input terminals (see Figure A7.9a). Because $V_I = 0$ for each opamp, the voltage of 6 V appears across the 6 kΩ resistor, giving rise to a current of 1 mA flowing in the direction X to Y. That current must flow through the 10 and 1 kΩ resistors, setting up voltages of 10 and 1 V, respectively. So, the voltage between P and Q is, by Ohm's law, $1\ \text{mA} \times (10\ \text{k}\Omega + 6\ \text{k}\Omega + 1\ \text{k}\Omega) = 17\ \text{V}$. There is no current through the 5 kΩ resistor and therefore no voltage across it, so the open-circuit voltage is 17 V.

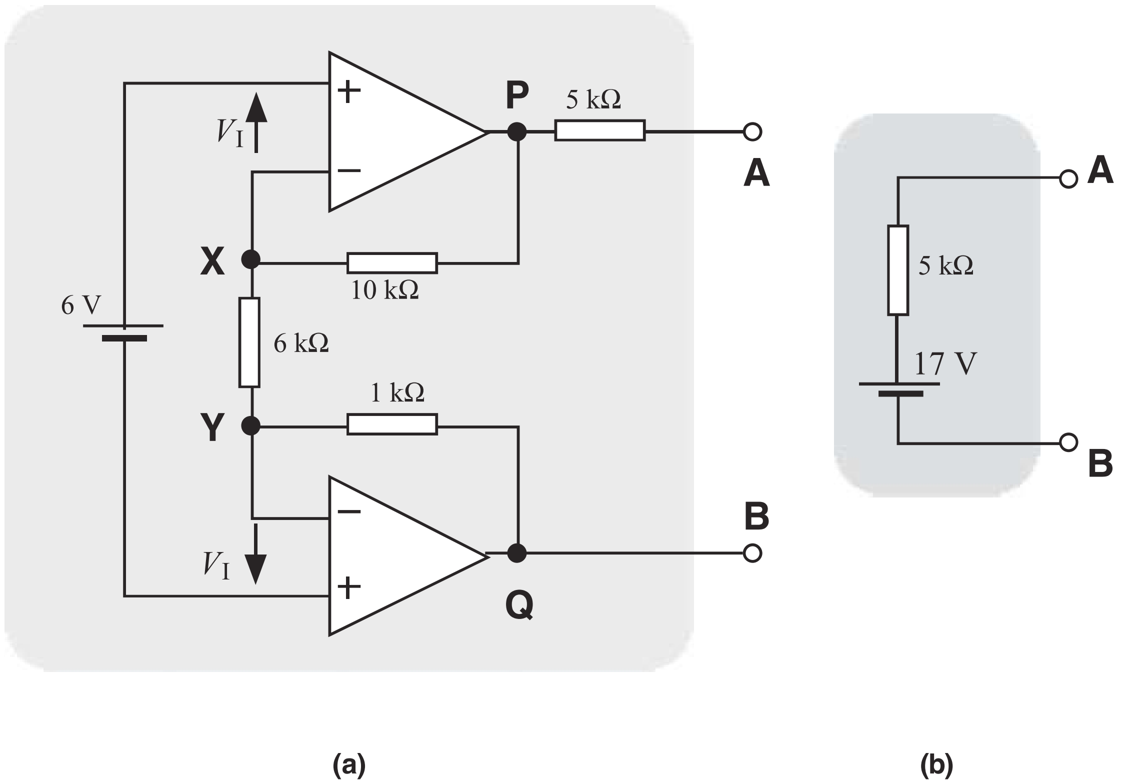

Figure A7.9

To find the Thevenin resistance R_O we set the independent sources to zero: in this case we replace the 6 V source by a short-circuit. There is now no voltage across the 6 kΩ resistor and hence no current through it. There is therefore no current through the 10 and 1 kΩ resistors. By Ohm's law there is therefore no voltage between terminals P and Q. The resistance between terminals A and B is therefore 5 kΩ.

The Thevenin equivalent circuit of the shaded box is as shown in Figure A7.9(b).

Answer 7.12

No current flows into the positive input terminal of the opamp so we can apply the voltage divider principle to calculate the voltage at that terminal to be zero.

With a virtual short-circuit between the opamp's input terminals the voltage at the negative input terminal must also be zero. The current through the left-hand 4 kΩ resistor is therefore, by Ohm's law, 1 mA. Again by Ohm's law, the current through the 8 kΩ resistor is zero. Applying KCL at the negative input terminal shows that 1 mA flows through the left-hand 2 kΩ resistor, creating a voltage of 2 V across it. Since the voltage at the negative input terminal is zero, $V = -2$ V.

Answer 8.1

The relevant equation is $i = C\mathrm{d}v/\mathrm{d}t$ where i and v are the capacitor current and voltage.

From $t = 0$ to $t = 5$ ms, $i = 1$ mA. Substituting in the equation we have $10^{-3} = 10^{-6}\,\mathrm{d}v/\mathrm{d}t$, so $\mathrm{d}v/\mathrm{d}t = 1000$ V/s. The voltage v therefore increases linearly by $5 \times 10^{-3} \times 1000 = 5$ V.

From $t = 5$ to $t = 10$ ms, $i = 0$, so v does not change.

From $t = 10$ to 15 ms, $i = 2$ mA, so $\mathrm{d}v/\mathrm{d}t = 2000$ V/s and v changes by 10 V.

From $t = 15$ to 20 ms, $i = -1$ mA, so $\mathrm{d}v/\mathrm{d}t = -1000$ V/s and v decreases by 5 V.

From $t = 20$ to 25 ms, $i = 0$, so v does not change.

From $t = 25$ to 30 ms, $i = -2$ mA so v decreases by 10 V, its final value being zero.

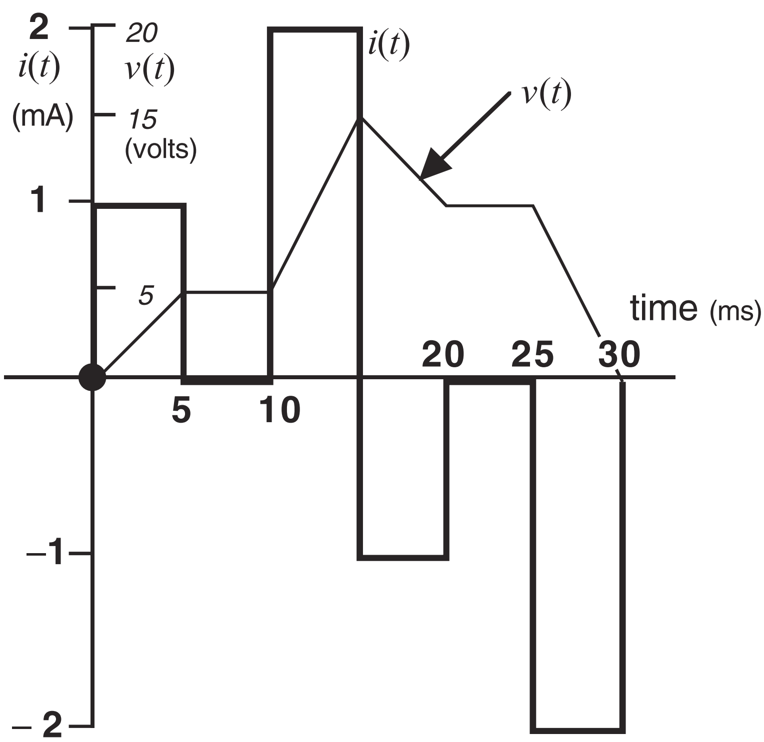

Figure A8.1