Lab00-Proof

CS214-Algorithm and Complexity, Xiaofeng Gao, Spring 2020.

∗ If there is any problem, please contact TA Yiming Liu.
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1. Prove that for any integer $n > 2$, there is a prime p satisfying $n < p < n!$. (Hint: consider a prime factor p of $n! - 1$ and prove by contradiction)

2. Use the minimal counterexample principle to prove that for any integer $n > 17$, there exist integers $i_n \geq 0$ and $j_n \geq 0$, such that $n = i_n \times 4 + j_n \times 7$.

3. Let $P = \{p_1, p_2, \cdots\}$ the set of all primes. Suppose that $\{p_i\}$ is monotonically increasing, i.e., $p_1 = 2$, $p_2 = 3$, $p_3 = 5$, $\cdots$. Please prove: $p_n < 2^{2^n}$. (Hint: $p_i \nmid (1 + \prod_{j=1}^{n} p_j), i = 1, 2, \cdots, n$.)

4. Prove that a plane divided by n lines can be colored with only 2 colors, and the adjacent regions have different colors.

Remark: You need to include your .pdf and .tex files in your uploaded .rar or .zip file.