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# **Sage Reference Manual: Cryptography**

***Release 6.3***

**The Sage Development Team**

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# CRYPTOSYSTEMS

This module contains base classes for various cryptosystems, including symmetric key and public-key cryptosystems. The classes defined in this module should not be called directly. It is the responsibility of child classes to implement specific cryptosystems. Take for example the Hill or matrix cryptosystem as implemented in `HillCryptosystem`. It is a symmetric key cipher so `HillCryptosystem` is a child class of `SymmetricKeyCryptosystem`, which in turn is a child class of `Cryptosystem`. The following diagram shows the inheritance relationship of particular cryptosystems:

```
Cryptosystem
+ SymmetricKeyCryptosystem
| + HillCryptosystem
| + LFSRCryptosystem
| + ShiftCryptosystem
| + ShrinkingGeneratorCryptosystem
| + SubstitutionCryptosystem
| + TranspositionCryptosystem
| + VigenereCryptosystem
+ PublicKeyCryptosystem
```

```
class sage.crypto.cryptosystem.Cryptosystem(plaintext_space, ciphertext_space, key_space,
                                             block_length=1, period=None)
```

```
Bases: sage.structure.parent_old.Parent, sage.structure.parent.Set_generic
```

A base cryptosystem class. This is meant to be extended by other specialized child classes that implement specific cryptosystems. A cryptosystem is a pair of maps

$$E : \mathcal{K} \rightarrow \text{Hom}(\mathcal{M}, \mathcal{C})$$

$$D : \mathcal{K} \rightarrow \text{Hom}(\mathcal{C}, \mathcal{M})$$

where  $\mathcal{K}$  is the key space,  $\mathcal{M}$  is the plaintext or message space, and  $\mathcal{C}$  is the ciphertext space. In many instances  $\mathcal{M} = \mathcal{C}$  and the images will lie in  $\text{Aut}(\mathcal{M})$ . An element of the image of  $E$  is called a cipher.

We may assume that  $E$  and  $D$  are injective, hence identify a key  $K$  in  $\mathcal{K}$  with its image  $E_K := E(K)$  in  $\text{Hom}(\mathcal{M}, \mathcal{C})$ .

The cryptosystem has the property that for every encryption key  $K_1$  there is a decryption key  $K_2$  such that  $D_{K_2} \circ E_{K_1}$ . A cryptosystem with the property that  $K := K_2 = K_1$ , is called a symmetric cryptosystem. Otherwise, if the key  $K_2 \neq K_1$ , nor is  $K_2$  easily derived from  $K_1$ , we call the cryptosystem asymmetric or public key. In that case,  $K_1$  is called the public key and  $K_2$  is called the private key.

INPUT:

- `plaintext_space` – the plaintext alphabet.

- `ciphertext_space` – the ciphertext alphabet.
- `key_space` – the key alphabet.
- `block_length` – (default: 1) the block length.
- `period` – (default: None) the period.

**EXAMPLES:**

Various classical cryptosystems:

```
sage: ShiftCryptosystem(AlphabeticStrings())
Shift cryptosystem on Free alphabetic string monoid on A-Z
sage: SubstitutionCryptosystem(HexadecimalStrings())
Substitution cryptosystem on Free hexadecimal string monoid
sage: HillCryptosystem(BinaryStrings(), 3)
Hill cryptosystem on Free binary string monoid of block length 3
sage: TranspositionCryptosystem(OctalStrings(), 5)
Transposition cryptosystem on Free octal string monoid of block length 5
sage: VigenereCryptosystem(Radix64Strings(), 7)
Vigenere cryptosystem on Free radix 64 string monoid of period 7
```

**block\_length()**

Return the block length of this cryptosystem. For some cryptosystems this is not relevant, in which case the block length defaults to 1.

**EXAMPLES:**

The block lengths of various classical cryptosystems:

```
sage: ShiftCryptosystem(AlphabeticStrings()).block_length()
1
sage: SubstitutionCryptosystem(HexadecimalStrings()).block_length()
1
sage: HillCryptosystem(BinaryStrings(), 3).block_length()
3
sage: TranspositionCryptosystem(OctalStrings(), 5).block_length()
5
sage: VigenereCryptosystem(Radix64Strings(), 7).block_length()
1
```

**cipher\_codomain()**

Return the alphabet used by this cryptosystem for encoding ciphertexts. This is the same as the ciphertext space.

**EXAMPLES:**

The cipher codomains, or ciphertext spaces, of various classical cryptosystems:

```
sage: ShiftCryptosystem(AlphabeticStrings()).cipher_codomain()
Free alphabetic string monoid on A-Z
sage: SubstitutionCryptosystem(HexadecimalStrings()).cipher_codomain()
Free hexadecimal string monoid
sage: HillCryptosystem(BinaryStrings(), 3).cipher_codomain()
Free binary string monoid
sage: TranspositionCryptosystem(OctalStrings(), 5).cipher_codomain()
Free octal string monoid
sage: VigenereCryptosystem(Radix64Strings(), 7).cipher_codomain()
Free radix 64 string monoid
```

**cipher\_domain()**

Return the alphabet used by this cryptosystem for encoding plaintexts. This is the same as the plaintext

space.

#### EXAMPLES:

The cipher domains, or plaintext spaces, of various classical cryptosystems:

```
sage: ShiftCryptosystem(AlphabeticStrings()).cipher_domain()
Free alphabetic string monoid on A-Z
sage: SubstitutionCryptosystem(HexadecimalStrings()).cipher_domain()
Free hexadecimal string monoid
sage: HillCryptosystem(BinaryStrings(), 3).cipher_domain()
Free binary string monoid
sage: TranspositionCryptosystem(OctalStrings(), 5).cipher_domain()
Free octal string monoid
sage: VigenereCryptosystem(Radix64Strings(), 7).cipher_domain()
Free radix 64 string monoid
```

#### **ciphertext\_space()**

Return the ciphertext alphabet of this cryptosystem.

#### EXAMPLES:

The ciphertext spaces of various classical cryptosystems:

```
sage: ShiftCryptosystem(AlphabeticStrings()).ciphertext_space()
Free alphabetic string monoid on A-Z
sage: SubstitutionCryptosystem(HexadecimalStrings()).ciphertext_space()
Free hexadecimal string monoid
sage: HillCryptosystem(BinaryStrings(), 3).ciphertext_space()
Free binary string monoid
sage: TranspositionCryptosystem(OctalStrings(), 5).ciphertext_space()
Free octal string monoid
sage: VigenereCryptosystem(Radix64Strings(), 7).ciphertext_space()
Free radix 64 string monoid
```

#### **key\_space()**

Return the alphabet used by this cryptosystem for encoding keys.

#### EXAMPLES:

The key spaces of various classical cryptosystems:

```
sage: ShiftCryptosystem(AlphabeticStrings()).key_space()
Ring of integers modulo 26
sage: SubstitutionCryptosystem(HexadecimalStrings()).key_space()
Free hexadecimal string monoid
sage: HillCryptosystem(BinaryStrings(), 3).key_space()
Full MatrixSpace of 3 by 3 dense matrices over Ring of integers modulo 2
sage: TranspositionCryptosystem(OctalStrings(), 5).key_space()
Symmetric group of order 5! as a permutation group
sage: VigenereCryptosystem(Radix64Strings(), 7).key_space()
Free radix 64 string monoid
```

#### **period()**

`x.__init__(...)` initializes `x`; see `help(type(x))` for signature

#### **plaintext\_space()**

Return the plaintext alphabet of this cryptosystem.

#### EXAMPLES:

The plaintext spaces of various classical cryptosystems:

```
sage: ShiftCryptosystem(AlphabeticStrings()).plaintext_space()
Free alphabetic string monoid on A-Z
sage: SubstitutionCryptosystem(HexadecimalStrings()).plaintext_space()
Free hexadecimal string monoid
sage: HillCryptosystem(BinaryStrings(), 3).plaintext_space()
Free binary string monoid
sage: TranspositionCryptosystem(OctalStrings(), 5).plaintext_space()
Free octal string monoid
sage: VigenereCryptosystem(Radix64Strings(), 7).plaintext_space()
Free radix 64 string monoid
```

```
class sage.crypto.cryptosystem.PublicKeyCryptosystem(plaintext_space, ciphertext_space,
                                                    key_space, block_length=1, pe-
                                                    riod=None)
```

Bases: `sage.crypto.cryptosystem.Cryptosystem`

The base class for asymmetric or public-key cryptosystems.

```
class sage.crypto.cryptosystem.SymmetricKeyCryptosystem(plaintext_space, ciphertext_space, key_space,
                                                         block_length=1, period=None)
```

Bases: `sage.crypto.cryptosystem.Cryptosystem`

The base class for symmetric key, or secret key, cryptosystems.

**alphabet\_size()**

Return the number of elements in the alphabet of this cryptosystem. This only applies to any cryptosystem whose plaintext and ciphertext spaces are the same alphabet.

EXAMPLES:

```
sage: ShiftCryptosystem(AlphabeticStrings()).alphabet_size()
26
sage: ShiftCryptosystem(BinaryStrings()).alphabet_size()
2
sage: ShiftCryptosystem(HexadecimalStrings()).alphabet_size()
16
sage: SubstitutionCryptosystem(OctalStrings()).alphabet_size()
8
sage: SubstitutionCryptosystem(Radix64Strings()).alphabet_size()
64
```



# CIPHERS

```
class sage.crypto.cipher.Cipher (parent, key)  
    Bases: sage.structure.element.Element  
  
    Cipher class  
  
    codomain ()  
        x.__init__(...) initializes x; see help(type(x)) for signature  
  
    domain ()  
        x.__init__(...) initializes x; see help(type(x)) for signature  
  
    key ()  
        x.__init__(...) initializes x; see help(type(x)) for signature  
  
class sage.crypto.cipher.PublicKeyCipher (parent, key, public=True)  
    Bases: sage.crypto.cipher.Cipher  
  
    Public key cipher class  
  
class sage.crypto.cipher.SymmetricKeyCipher (parent, key)  
    Bases: sage.crypto.cipher.Cipher  
  
    Symmetric key cipher class
```



# CLASSICAL CRYPTOSYSTEMS

A convenient user interface to various classical ciphers. These include:

- affine cipher; see `AffineCryptosystem`
- Hill or matrix cipher; see `HillCryptosystem`
- shift cipher; see `ShiftCryptosystem`
- substitution cipher; see `SubstitutionCryptosystem`
- transposition cipher; see `TranspositionCryptosystem`
- Vigenere cipher; see `VigenereCryptosystem`

These classical cryptosystems support alphabets such as:

- the capital letters of the English alphabet; see `AlphabeticStrings()`
- the hexadecimal number system; see `HexadecimalStrings()`
- the binary number system; see `BinaryStrings()`
- the octal number system; see `OctalStrings()`
- the radix-64 number system; see `Radix64Strings()`

AUTHORS:

- David Kohel (2007): initial version with the Hill, substitution, transposition, and Vigenere cryptosystems.
- Minh Van Nguyen (2009-08): shift cipher, affine cipher

**class** `sage.crypto.classical.AffineCryptosystem(A)`

Bases: `sage.crypto.cryptosystem.SymmetricKeyCryptosystem`

Create an affine cryptosystem.

Let  $A = \{a_0, a_1, a_2, \dots, a_{n-1}\}$  be a non-empty alphabet consisting of  $n$  unique elements. Define a mapping  $f : A \rightarrow \mathbf{Z}/n\mathbf{Z}$  from the alphabet  $A$  to the set  $\mathbf{Z}/n\mathbf{Z}$  of integers modulo  $n$ , given by  $f(a_i) = i$ . Thus we can identify each element of the alphabet  $A$  with a unique integer  $0 \leq i < n$ . A key of the affine cipher is an ordered pair of integers  $(a, b) \in \mathbf{Z}/n\mathbf{Z} \times \mathbf{Z}/n\mathbf{Z}$  such that  $\gcd(a, n) = 1$ . Therefore the key space is  $\mathbf{Z}/n\mathbf{Z} \times \mathbf{Z}/n\mathbf{Z}$ . Since we assume that  $A$  does not have repeated elements, the mapping  $f : A \rightarrow \mathbf{Z}/n\mathbf{Z}$  is bijective. Encryption and decryption functions are both affine functions. Let  $(a, b)$  be a secret key, i.e. an element of the key space, and let  $p$  be a plaintext character and consequently  $p \in \mathbf{Z}/n\mathbf{Z}$ . Then the ciphertext character  $c$  corresponding to  $p$  is given by

$$c \equiv ap + b \pmod{n}$$

Similarly, given a ciphertext character  $c \in \mathbf{Z}/n\mathbf{Z}$  and a secret key  $(a, b)$ , we can recover the corresponding plaintext character as follows:

$$p \equiv a^{-1}(c - b) \pmod{n}$$

where  $a^{-1}$  is the inverse of  $a$  modulo  $n$ . Use the bijection  $f : A \longrightarrow \mathbf{Z}/n\mathbf{Z}$  to convert  $c$  and  $p$  back to elements of the alphabet  $A$ . Currently, only the following alphabet is supported for the affine cipher:

- capital letters of the English alphabet as implemented in `AlphabeticStrings()`

#### EXAMPLES:

Encryption and decryption over the capital letters of the English alphabet:

```
sage: A = AffineCryptosystem(AlphabeticStrings()); A
Affine cryptosystem on Free alphabetic string monoid on A-Z
sage: P = A.encoding("The affine cryptosystem generalizes the shift cipher.")
sage: P
THEAFFINECRYPTOSYSTEMGENERALIZESTHESHIFTCIPHER
sage: a, b = (9, 13)
sage: C = A.enciphering(a, b, P); C
CYXNGGHAXFKVSCJTVTCXRPXAXKNIHEXTCYXTYHGCFHSYXK
sage: A.deciphering(a, b, C)
THEAFFINECRYPTOSYSTEMGENERALIZESTHESHIFTCIPHER
sage: A.deciphering(a, b, C) == P
True
```

We can also use functional notation to work through the previous example:

```
sage: A = AffineCryptosystem(AlphabeticStrings()); A
Affine cryptosystem on Free alphabetic string monoid on A-Z
sage: P = A.encoding("The affine cryptosystem generalizes the shift cipher.")
sage: P
THEAFFINECRYPTOSYSTEMGENERALIZESTHESHIFTCIPHER
sage: a, b = (9, 13)
sage: E = A(a, b); E
Affine cipher on Free alphabetic string monoid on A-Z
sage: C = E(P); C
CYXNGGHAXFKVSCJTVTCXRPXAXKNIHEXTCYXTYHGCFHSYXK
sage: aInv, bInv = A.inverse_key(a, b)
sage: D = A(aInv, bInv); D
Affine cipher on Free alphabetic string monoid on A-Z
sage: D(C)
THEAFFINECRYPTOSYSTEMGENERALIZESTHESHIFTCIPHER
sage: D(C) == P
True
sage: D(C) == P == D(E(P))
True
```

Encrypting the ciphertext with the inverse key also produces the plaintext:

```
sage: A = AffineCryptosystem(AlphabeticStrings())
sage: P = A.encoding("Encrypt with inverse key.")
sage: a, b = (11, 8)
sage: C = A.enciphering(a, b, P)
sage: P; C
ENCRYPTWITHINVERSEKEY
AVENMRJQSJHSVFANYAOAM
sage: aInv, bInv = A.inverse_key(a, b)
sage: A.enciphering(aInv, bInv, C)
ENCRYPTWITHINVERSEKEY
```

```
sage: A.enciphering(aInv, bInv, C) == P
True
```

For a secret key  $(a, b) \in \mathbf{Z}/n\mathbf{Z} \times \mathbf{Z}/n\mathbf{Z}$ , if  $a = 1$  then any affine cryptosystem with key  $(1, b)$  for any  $b \in \mathbf{Z}/n\mathbf{Z}$  is a shift cryptosystem. Here is how we can create a Caesar cipher using an affine cipher:

```
sage: caesar = AffineCryptosystem(AlphabeticStrings())
sage: a, b = (1, 3)
sage: P = caesar.encoding("abcdef"); P
ABCDEF
sage: C = caesar.enciphering(a, b, P); C
DEFGHI
sage: caesar.deciphering(a, b, C) == P
True
```

Any affine cipher with keys of the form  $(a, 0) \in \mathbf{Z}/n\mathbf{Z} \times \mathbf{Z}/n\mathbf{Z}$  is called a decimation cipher on the Roman alphabet, or decimation cipher for short:

```
sage: A = AffineCryptosystem(AlphabeticStrings())
sage: P = A.encoding("A decimation cipher is a specialized affine cipher.")
sage: a, b = (17, 0)
sage: C = A.enciphering(a, b, P)
sage: P; C
ADECIMATIONCIPHERISASPECIALIZEDAFFINECIPHER
AZQIGWALGENIGVPQDUAUVQIGAFGJQZAHGNGQIGVPQD
sage: A.deciphering(a, b, C) == P
True
```

Generate a random key for encryption and decryption:

```
sage: A = AffineCryptosystem(AlphabeticStrings())
sage: P = A.encoding("An affine cipher with a random key.")
sage: a, b = A.random_key()
sage: C = A.enciphering(a, b, P)
sage: A.deciphering(a, b, C) == P
True
```

## TESTS:

The binary number system is currently not a supported alphabet of this affine cryptosystem:

```
sage: AffineCryptosystem(BinaryStrings())
Traceback (most recent call last):
...
TypeError: A (= Free binary string monoid) is not supported as a cipher domain of this affine cr
```

Nor are the octal, hexadecimal, and radix-64 number systems supported:

```
sage: AffineCryptosystem(OctalStrings())
Traceback (most recent call last):
...
TypeError: A (= Free octal string monoid) is not supported as a cipher domain of this affine cry
sage: AffineCryptosystem(HexadecimalStrings())
Traceback (most recent call last):
...
TypeError: A (= Free hexadecimal string monoid) is not supported as a cipher domain of this affi
sage: AffineCryptosystem(Radix64Strings())
Traceback (most recent call last):
...
TypeError: A (= Free radix 64 string monoid) is not supported as a cipher domain of this affine
```

A secret key  $(a, b)$  must be an element of  $\mathbf{Z}/n\mathbf{Z} \times \mathbf{Z}/n\mathbf{Z}$  with  $\gcd(a, n) = 1$ . This rules out the case  $a = 0$  irrespective of the value of  $b$ . For the upper-case letters of the English alphabet, where the alphabet size is  $n = 26$ ,  $a$  cannot take on any even value:

```
sage: A = AffineCryptosystem(AlphabeticStrings())
sage: A(0, 1)
Traceback (most recent call last):
...
ValueError: (a, b) = (0, 1) is outside the range of acceptable values for a key of this affine c
sage: A(2, 1)
Traceback (most recent call last):
...
ValueError: (a, b) = (2, 1) is outside the range of acceptable values for a key of this affine c
```

#### REFERENCES:

**brute\_force** (*C*, *ranking*='none')

Attempt a brute force cryptanalysis of the ciphertext *C*.

INPUT:

- *C* – A ciphertext over one of the supported alphabets of this affine cryptosystem. See the class [AffineCryptosystem](#) for documentation on the supported alphabets.
- *ranking* – (default "none") the method to use for ranking all possible keys. If *ranking*="none", then do not use any ranking function. The following ranking functions are supported:
  - "chi\_square" – the chi-square ranking function as implemented in the method [rank\\_by\\_chi\\_square\(\)](#).
  - "squared\_differences" – the squared differences ranking function as implemented in the method [rank\\_by\\_squared\\_differences\(\)](#).

OUTPUT:

- All the possible plaintext sequences corresponding to the ciphertext *C*. This method effectively uses all the possible keys in this affine cryptosystem to decrypt *C*. The method is also referred to as exhaustive key search. The output is a dictionary of key, candidate decipherment pairs.

#### EXAMPLES:

Cryptanalyze using all possible keys with the option *ranking*="none":

```
sage: A = AffineCryptosystem(AlphabeticStrings())
sage: a, b = (3, 7)
sage: P = A.encoding("Linear"); P
LINEAR
sage: C = A.enciphering(a, b, P)
sage: L = A.brute_force(C)
sage: sorted(L.items())[:26] # display 26 candidate decipherments

[(1, 0), OFUTHG),
(1, 1), NETSGF),
(1, 2), MDSRFE),
(1, 3), LCRQED),
(1, 4), KBQPDC),
(1, 5), JAPOCB),
(1, 6), IZONBA),
(1, 7), HYNMAZ),
(1, 8), GXMLZY),
(1, 9), FWLKYX),
```

```

((1, 10), EVKJXW),
((1, 11), DUJIWV),
((1, 12), CTIHVU),
((1, 13), BSHGUT),
((1, 14), ARGFTS),
((1, 15), ZQFESR),
((1, 16), YPEDRQ),
((1, 17), XODCQP),
((1, 18), WNCBPO),
((1, 19), VMBAON),
((1, 20), ULAZNM),
((1, 21), TKZYML),
((1, 22), SJYXLK),
((1, 23), RIXWKJ),
((1, 24), QHWVJI),
((1, 25), PGVUIH)]

```

Use the chi-square ranking function, i.e. `ranking="chisquare"`:

```

sage: A = AffineCryptosystem(AlphabeticStrings())
sage: a, b = (3, 7)
sage: P = A.encoding("Linear functions for encrypting and decrypting."); P
LINEARFUNCTIONSFORENCRYPTINGANDDECRYPTING
sage: C = A.enciphering(a, b, P)
sage: Rank = A.brute_force(C, ranking="chisquare")
sage: Rank[:10] # display only the top 10 candidate keys

```

```

[ ((3, 7), LINEARFUNCTIONSFORENCRYPTINGANDDECRYPTING),
  ((23, 25), VYTCGPBMTENYSTOBSPCTEPIRNYTAGTDDCEPIRNYTA),
  ((1, 12), CTIHVUKDIBATLIXKLUHIBUPOATINVIEEHBUPOATIN),
  ((11, 15), HSRYELDAROVSWRQDWLYROLUBVVSRIERTTYOLUBVSRI),
  ((25, 1), NWHIUVMHOPWEHSFEVIHOVABPWHCUHLLIOVABPWHC),
  ((25, 7), TCNOABLSNUVCKNYLKBONUBGHVCNIANRROUBGHVCNI),
  ((15, 4), SHIBVOWZILEHDIJWDOBILOFYEHIRVIGGBLOFYEHIR),
  ((15, 23), PEFYSLTWFI BEAFGTALYFILCVBEFOSFDDYILCVBEFO),
  ((7, 10), IDUFHSYXUTEDNULYNSFUTSVGEDURHUMMFTSVGEDUR),
  ((19, 22), QVETRGABEFUVLENALGTEFGDSUVEHREMMTFGDSUVEH) ]

```

Use the squared differences ranking function, i.e. `ranking="squared_differences"`:

```

sage: Rank = A.brute_force(C, ranking="squared_differences")
sage: Rank[:10] # display only the top 10 candidate keys

```

```

[ ((3, 7), LINEARFUNCTIONSFORENCRYPTINGANDDECRYPTING),
  ((23, 6), GJENRAMXEPYJDEZMDANEPATCYJELREONPATCYJEL),
  ((23, 25), VYTCGPBMTENYSTOBSPCTEPIRNYTAGTDDCEPIRNYTA),
  ((19, 22), QVETRGABEFUVLENALGTEFGDSUVEHREMMTFGDSUVEH),
  ((19, 9), DIRGETNORSHIYRANYTGRSTQFHIRUERZZGSTQFHIRU),
  ((23, 18), KNIRVEQBITCNHIDQHERITEXGCNIPVISSRTEXGCNIP),
  ((17, 16), GHORBEIDOMHFOVIFEROJETWMHOZBOAARJETWMHOZ),
  ((21, 14), AHEZRMOFEVQHTEBOTMZEVMNIQHEDREKKZVMNIQHED),
  ((1, 12), CTIHVUKDIBATLIXKLUHIBUPOATINVIEEHBUPOATIN),
  ((7, 18), SNEPRCIHEDONXEVIXCPEDCFQONEBREWWDPCFQONEB) ]

```

#### TESTS:

Currently, the binary number system is not supported as an alphabet of this affine cryptosystem:

```
sage: A = AffineCryptosystem(AlphabeticStrings())
sage: BinStr = BinaryStrings()
sage: C = BinStr.encoding("abc")
sage: A.brute_force(C)
Traceback (most recent call last):
...
TypeError: Ciphertext must be encoded using one of the supported cipher domains of this affine cipher
```

Nor are the octal, hexadecimal, and radix-64 number systems supported:

```
sage: OctStr = OctalStrings()
sage: C = OctStr([1, 2, 3])
sage: A.brute_force(C)
Traceback (most recent call last):
...
TypeError: Ciphertext must be encoded using one of the supported cipher domains of this affine cipher
sage: HexStr = HexadecimalStrings()
sage: C = HexStr.encoding("abc")
sage: A.brute_force(C)
Traceback (most recent call last):
...
TypeError: Ciphertext must be encoded using one of the supported cipher domains of this affine cipher
sage: RadStr = Radix64Strings()
sage: C = RadStr([1, 2, 3])
sage: A.brute_force(C)
Traceback (most recent call last):
...
TypeError: Ciphertext must be encoded using one of the supported cipher domains of this affine cipher
```

Only the chi-square and squared-differences ranking functions are currently supported. The keyword ranking must take on either of the values "none", "chisquare" or "squared\_differences":

```
sage: A = AffineCryptosystem(AlphabeticStrings())
sage: a, b = (3, 7)
sage: P = A.encoding("Linear")
sage: C = A.encrypting(a, b, P)
sage: A.brute_force(C, ranking="chi")
Traceback (most recent call last):
...
ValueError: Keyword 'ranking' must be either 'none', 'chisquare', or 'squared_differences'.
sage: A.brute_force(C, ranking="")
Traceback (most recent call last):
...
ValueError: Keyword 'ranking' must be either 'none', 'chisquare', or 'squared_differences'.
```

### **deciphering**(*a*, *b*, *C*)

Decrypt the ciphertext *C* with the key (*a*, *b*) using affine cipher decryption.

INPUT:

- *a*, *b* – a secret key belonging to the key space of this affine cipher. This key must be an element of  $\mathbb{Z}/n\mathbb{Z} \times \mathbb{Z}/n\mathbb{Z}$  such that  $\gcd(a, n) = 1$  with *n* being the size of the ciphertext and plaintext spaces.
- *C* – a string of ciphertext; possibly an empty string. Characters in this string must be encoded using one of the supported alphabets. See the method `encoding()` for more information.

OUTPUT:

- The plaintext corresponding to the ciphertext *C*.



## EXAMPLES:

Decryption over the capital letters of the English alphabet:

```
sage: A = AffineCryptosystem(AlphabeticStrings())
sage: a, b = (5, 2)
sage: P = A.encoding("Affine functions are linear functions.")
sage: C = A.enciphering(a, b, P); C
CBBQFWBYPMQTUPOCJWFQFWCJBYPMTQUPO
sage: P == A.deciphering(a, b, C)
True
```

The previous example can also be worked through using functional notation:

```
sage: A = AffineCryptosystem(AlphabeticStrings())
sage: a, b = (5, 2)
sage: P = A.encoding("Affine functions are linear functions.")
sage: E = A(a, b); E
Affine cipher on Free alphabetic string monoid on A-Z
sage: C = E(P); C
CBBQFWBYPMQTUPOCJWFQFWCJBYPMTQUPO
sage: aInv, bInv = A.inverse_key(a, b)
sage: D = A(aInv, bInv); D
Affine cipher on Free alphabetic string monoid on A-Z
sage: D(C) == P
True
```

If the ciphertext is an empty string, then the plaintext is also an empty string regardless of the value of the secret key:

```
sage: a, b = A.random_key()
sage: A.deciphering(a, b, A.encoding(""))
sage: A.deciphering(a, b, A.encoding(" "))
```

## TESTS:

The key must be an ordered pair  $(a, b) \in \mathbf{Z}/n\mathbf{Z} \times \mathbf{Z}/n\mathbf{Z}$  with  $n$  being the size of the plaintext and ciphertext spaces. Furthermore,  $a$  must be relatively prime to  $n$ , i.e.  $\gcd(a, n) = 1$ :

```
sage: A.deciphering(2, 6, P)
Traceback (most recent call last):
...
ValueError: (a, b) = (2, 6) is outside the range of acceptable values for a key of this affi
```

**enciphering** ( $a, b, P$ )

Encrypt the plaintext  $P$  with the key  $(a, b)$  using affine cipher encryption.

## INPUT:

- $a, b$  – a secret key belonging to the key space of this affine cipher. This key must be an element of  $\mathbf{Z}/n\mathbf{Z} \times \mathbf{Z}/n\mathbf{Z}$  such that  $\gcd(a, n) = 1$  with  $n$  being the size of the ciphertext and plaintext spaces.
- $P$  – a string of plaintext; possibly an empty string. Characters in this string must be encoded using one of the supported alphabets. See the method `encoding()` for more information.

## OUTPUT:

- The ciphertext corresponding to the plaintext  $P$ .

## EXAMPLES:

Encryption over the capital letters of the English alphabet:

```
sage: A = AffineCryptosystem(AlphabeticStrings())
sage: a, b = (3, 6)
sage: P = A.encoding("Affine ciphers work with linear functions.")
sage: A.enciphering(a, b, P)
GVVETSMEZBSFIUWFKUELBNETSGFVOTMLEWTI
```

Now work through the previous example using functional notation:

```
sage: A = AffineCryptosystem(AlphabeticStrings())
sage: a, b = (3, 6)
sage: P = A.encoding("Affine ciphers work with linear functions.")
sage: E = A(a, b); E
Affine cipher on Free alphabetic string monoid on A-Z
sage: E(P)
GVVETSMEZBSFIUWFKUELBNETSGFVOTMLEWTI
```

If the plaintext is an empty string, then the ciphertext is also an empty string regardless of the value of the secret key:

```
sage: a, b = A.random_key()
sage: A.enciphering(a, b, A.encoding(""))

sage: A.enciphering(a, b, A.encoding(" "))
```

TESTS:

The key must be an ordered pair  $(a, b) \in \mathbf{Z}/n\mathbf{Z} \times \mathbf{Z}/n\mathbf{Z}$  with  $n$  being the size of the plaintext and ciphertext spaces. Furthermore,  $a$  must be relatively prime to  $n$ , i.e.  $\gcd(a, n) = 1$ :

```
sage: A.enciphering(2, 6, P)
Traceback (most recent call last):
...
ValueError: (a, b) = (2, 6) is outside the range of acceptable values for a key of this affi
```

### **encoding**( $S$ )

The encoding of the string  $S$  over the string monoid of this affine cipher. For example, if the string monoid of this cryptosystem is `AlphabeticStringMonoid`, then the encoding of  $S$  would be its upper-case equivalent stripped of all non-alphabetic characters. Only the following alphabet is supported for the affine cipher:

- capital letters of the English alphabet as implemented in `AlphabeticStrings()`

INPUT:

- $S$  – a string, possibly empty.

OUTPUT:

- The encoding of  $S$  over the string monoid of this cryptosystem. If  $S$  is an empty string, return an empty string.

EXAMPLES:

Encoding over the upper-case letters of the English alphabet:

```
sage: A = AffineCryptosystem(AlphabeticStrings())
sage: A.encoding("Affine cipher over capital letters of the English alphabet.")
AFFINECIPHEROVERCAPITALLETTERSOFTHEENGLISHALPHABET
```

The argument  $S$  can be an empty string, in which case an empty string is returned:

```
sage: AffineCryptosystem(AlphabeticStrings()).encoding("")
```

### **inverse\_key**( $a, b$ )

The inverse key corresponding to the secret key  $(a, b)$ . If  $p$  is a plaintext character so that  $p \in \mathbf{Z}/n\mathbf{Z}$  and  $n$  is the alphabet size, then the ciphertext  $c$  corresponding to  $p$  is

$$c \equiv ap + b \pmod{n}$$

As  $(a, b)$  is a key, then the multiplicative inverse  $a^{-1}$  exists and the original plaintext can be recovered as follows

$$p \equiv a^{-1}(c - b) \pmod{n} \equiv a^{-1}c + a^{-1}(-b) \pmod{n}$$

Therefore the ordered pair  $(a^{-1}, -ba^{-1})$  is the inverse key corresponding to  $(a, b)$ .

INPUT:

- $a, b$  – a secret key for this affine cipher. The ordered pair  $(a, b)$  must be an element of  $\mathbf{Z}/n\mathbf{Z} \times \mathbf{Z}/n\mathbf{Z}$  such that  $\gcd(a, n) = 1$ .

OUTPUT:

- The inverse key  $(a^{-1}, -ba^{-1})$  corresponding to  $(a, b)$ .

EXAMPLES:

```
sage: A = AffineCryptosystem(AlphabeticStrings())
sage: a, b = (1, 2)
sage: A.inverse_key(a, b)
(1, 24)
sage: A.inverse_key(3, 2)
(9, 8)
```

Suppose that the plaintext and ciphertext spaces are the capital letters of the English alphabet so that  $n = 26$ . If  $\varphi(n)$  is the Euler phi function of  $n$ , then there are  $\varphi(n)$  integers  $0 \leq a < n$  that are relatively prime to  $n$ . For the capital letters of the English alphabet, there are 12 such integers relatively prime to  $n$ :

```
sage: euler_phi(A.alphabet_size())
12
```

And here is a list of those integers:

```
sage: n = A.alphabet_size()
sage: L = [i for i in xrange(n) if gcd(i, n) == 1]; L
[1, 3, 5, 7, 9, 11, 15, 17, 19, 21, 23, 25]
```

Then a secret key  $(a, b)$  of this shift cryptosystem is such that  $a$  is an element of the list  $L$  in the last example. Any inverse key  $(A, B)$  corresponding to  $(a, b)$  is such that  $A$  is also in the list  $L$  above:

```
sage: a, b = (3, 9)
sage: a in L
True
sage: aInv, bInv = A.inverse_key(a, b)
sage: aInv, bInv
(9, 23)
sage: aInv in L
True
```

TESTS:

Any ordered pair of the form  $(0, b)$  for any integer  $b$  cannot be a secret key of this affine cipher. Hence  $(0, b)$  does not have a corresponding inverse key:

```

sage: A = AffineCryptosystem(AlphabeticStrings())
sage: A.inverse_key(0, 1)
Traceback (most recent call last):
...
ValueError: (a, b) = (0, 1) is outside the range of acceptable values for a key of this affi

```

**random\_key()**

Generate a random key within the key space of this affine cipher. The generated secret key is an ordered pair  $(a, b) \in \mathbf{Z}/n\mathbf{Z} \times \mathbf{Z}/n\mathbf{Z}$  with  $n$  being the size of the cipher domain and  $\gcd(a, n) = 1$ . Let  $\varphi(n)$  denote the Euler phi function of  $n$ . Then the affine cipher has  $n \cdot \varphi(n)$  possible keys (see page 10 of [Sti06]).

OUTPUT:

- A random key within the key space of this affine cryptosystem. The output key is an ordered pair  $(a, b)$ .

EXAMPLES:

```

sage: A = AffineCryptosystem(AlphabeticStrings())
sage: A.random_key() # random
(17, 25)

```

If  $(a, b)$  is a secret key and  $n$  is the size of the plaintext and ciphertext alphabets, then  $\gcd(a, n) = 1$ :

```

sage: a, b = A.random_key()
sage: n = A.alphabet_size()
sage: gcd(a, n)
1

```

**rank\_by\_chi\_square(C, pdict)**

Use the chi-square statistic to rank all possible keys. Currently, this method only applies to the capital letters of the English alphabet.

ALGORITHM:

Consider a non-empty alphabet  $A$  consisting of  $n$  elements, and let  $C$  be a ciphertext encoded using elements of  $A$ . The plaintext  $P$  corresponding to  $C$  is also encoded using elements of  $A$ . Let  $M$  be a candidate decipherment of  $C$ , i.e.  $M$  is the result of attempting to decrypt  $C$  using a key  $(a, b)$  which is not necessarily the same key used to encrypt  $P$ . Suppose  $F_A(e)$  is the characteristic frequency probability of  $e \in A$  and let  $F_M(e)$  be the message frequency probability with respect to  $M$ . The characteristic frequency probability distribution of an alphabet is the expected frequency probability distribution for that alphabet. The message frequency probability distribution of  $M$  provides a distribution of the ratio of character occurrences over message length. One can interpret the characteristic frequency probability  $F_A(e)$  as the expected probability, while the message frequency probability  $F_M(e)$  is the observed probability. If  $M$  is of length  $L$ , then the observed frequency of  $e \in A$  is

$$O_M(e) = F_M(e) \cdot L$$

and the expected frequency of  $e \in A$  is

$$E_A(e) = F_A(e) \cdot L$$

The chi-square rank  $R_{\chi^2}(M)$  of  $M$  corresponding to a key  $(a, b) \in \mathbf{Z}/n\mathbf{Z} \times \mathbf{Z}/n\mathbf{Z}$  is given by

$$R_{\chi^2}(M) = \sum_{e \in A} \frac{(O_M(e) - E_A(e))^2}{E_A(e)}$$

Cryptanalysis by exhaustive key search produces a candidate decipherment  $M_{a,b}$  for each possible key  $(a, b)$ . For a set  $D = \{M_{a_1, b_1}, M_{a_2, b_2}, \dots, M_{a_k, b_k}\}$  of all candidate decipherments corresponding to a

ciphertext  $C$ , the smaller is the rank  $R_{\chi^2}(M_{a_i, b_i})$  the more likely that  $(a_i, b_i)$  is the secret key. This key ranking method is based on the Pearson chi-square test [PearsonTest09].

INPUT:

- $C$  – The ciphertext, a non-empty string. The ciphertext must be encoded using the upper-case letters of the English alphabet.
- $pdict$  – A dictionary of key, possible plaintext pairs. This should be the output of `brute_force()` with `ranking="none"`.

OUTPUT:

- A list ranking the most likely keys first. Each element of the list is a tuple of key, possible plaintext pairs.

EXAMPLES:

Use the chi-square statistic to rank all possible keys and their corresponding decipherment:

```
sage: A = AffineCryptosystem(AlphabeticStrings())
sage: a, b = (3, 7)
sage: P = A.encoding("Line.")
sage: C = A.enciphering(a, b, P)
sage: Plist = A.brute_force(C)
sage: Rank = A.rank_by_chi_square(C, Plist)
sage: Rank[:10] # display only the top 10 candidate keys
```

```
[((1, 1), NETS),
((3, 7), LINE),
((17, 20), STAD),
((5, 2), SLOT),
((5, 5), HADI),
((9, 25), TSLI),
((17, 15), DELO),
((15, 6), ETUN),
((21, 8), ELID),
((7, 17), HCTE)]
```

As more ciphertext is available, the reliability of the chi-square ranking function increases:

```
sage: A = AffineCryptosystem(AlphabeticStrings())
sage: a, b = (11, 24)
sage: P = A.encoding("Longer message is more information for cryptanalysis.")
sage: C = A.enciphering(a, b, P)
sage: Plist = A.brute_force(C)
sage: Rank = A.rank_by_chi_square(C, Plist)
sage: Rank[:10] # display only the top 10 candidate keys
```

```
[((11, 24), LONGERMESAGEISMOREINFORMATIONFORCRYPTANALYSIS),
((17, 9), INURFSBFLLRFDLBNSFDUYNBHDNUYNSTSVGEHUHIVLDL),
((9, 18), RMFIUHYUOOSIUWOYMHUWFBMHYSVWMFBMHGHETVSFSREOWO),
((15, 12), VSTACPUCOGACYOUSPCYTBSPUGNYSTBSPEPIRNGTGVIYO),
((3, 22), PAFOYLKYGGSOYEGKALYEFTALKSBEAFTALILCVBSFSPCGEG),
((25, 3), OHSRNADNPPFRNVPDHDANVSCHADFEVHSCHAJABWEFSFOBPVP),
((7, 25), GHYNVIPVRRNLNVRPHIVFYEHIPLAFHYEHIDITQALYLGTRFR),
((5, 2), NEHCIVKISSUCIWSKEVIWHFEVKUPWEHFEVOVABPUHUNASWS),
((15, 25), IFGNPCHPBNTNPLBHFCLGOFCHTALFGOFCRCVEATGTIVBLB),
((9, 6), BWPSERIEYYCSEGYIWREGPLWRICFGWPLWRQRODFCPCBOYGY)]
```

TESTS:

The ciphertext cannot be an empty string:

```
sage: A.rank_by_chi_square("", Plist)
Traceback (most recent call last):
...
AttributeError: 'str' object has no attribute 'parent'
sage: A.rank_by_chi_square(A.encoding(""), Plist)
Traceback (most recent call last):
...
ValueError: The ciphertext must be a non-empty string.
sage: A.rank_by_chi_square(A.encoding(" "), Plist)
Traceback (most recent call last):
...
ValueError: The ciphertext must be a non-empty string.
```

The ciphertext must be encoded using the capital letters of the English alphabet as implemented in `AlphabeticStrings()`:

```
sage: H = HexadecimalStrings()
sage: A.rank_by_chi_square(H.encoding("shift"), Plist)
Traceback (most recent call last):
...
TypeError: The ciphertext must be capital letters of the English alphabet.
sage: B = BinaryStrings()
sage: A.rank_by_chi_square(B.encoding("shift"), Plist)
Traceback (most recent call last):
...
TypeError: The ciphertext must be capital letters of the English alphabet.
```

The dictionary `pdict` cannot be empty:

```
sage: A.rank_by_chi_square(C, {})
Traceback (most recent call last):
...
KeyError: (1, 0)
```

#### **`rank_by_squared_differences(C, pdict)`**

Use the squared-differences measure to rank all possible keys. Currently, this method only applies to the capital letters of the English alphabet.

ALGORITHM:

Consider a non-empty alphabet  $A$  consisting of  $n$  elements, and let  $C$  be a ciphertext encoded using elements of  $A$ . The plaintext  $P$  corresponding to  $C$  is also encoded using elements of  $A$ . Let  $M$  be a candidate decipherment of  $C$ , i.e.  $M$  is the result of attempting to decrypt  $C$  using a key  $(a, b)$  which is not necessarily the same key used to encrypt  $P$ . Suppose  $F_A(e)$  is the characteristic frequency probability of  $e \in A$  and let  $F_M(e)$  be the message frequency probability with respect to  $M$ . The characteristic frequency probability distribution of an alphabet is the expected frequency probability distribution for that alphabet. The message frequency probability distribution of  $M$  provides a distribution of the ratio of character occurrences over message length. One can interpret the characteristic frequency probability  $F_A(e)$  as the expected probability, while the message frequency probability  $F_M(e)$  is the observed probability. If  $M$  is of length  $L$ , then the observed frequency of  $e \in A$  is

$$O_M(e) = F_M(e) \cdot L$$

and the expected frequency of  $e \in A$  is

$$E_A(e) = F_A(e) \cdot L$$

The squared-differences, or residual sum of squares, rank  $R_{RSS}(M)$  of  $M$  corresponding to a key  $(a, b) \in \mathbf{Z}/n\mathbf{Z} \times \mathbf{Z}/n\mathbf{Z}$  is given by

$$R_{RSS}(M) = \sum_{e \in A} (O_M(e) - E_A(e))^2$$

Cryptanalysis by exhaustive key search produces a candidate decipherment  $M_{a,b}$  for each possible key  $(a, b)$ . For a set  $D = \{M_{a_1,b_1}, M_{a_2,b_2}, \dots, M_{a_k,b_k}\}$  of all candidate decipherments corresponding to a ciphertext  $C$ , the smaller is the rank  $R_{RSS}(M_{a_i,b_i})$  the more likely that  $(a_i, b_i)$  is the secret key. This key ranking method is based on the residual sum of squares measure [RSS09].

INPUT:

- **C** – The ciphertext, a non-empty string. The ciphertext must be encoded using the upper-case letters of the English alphabet.
- **pdict** – A dictionary of key, possible plaintext pairs. This should be the output of `brute_force()` with `ranking="none"`.

OUTPUT:

- A list ranking the most likely keys first. Each element of the list is a tuple of key, possible plaintext pairs.

EXAMPLES:

Use the method of squared differences to rank all possible keys and their corresponding decipherment:

```
sage: A = AffineCryptosystem(AlphabeticStrings())
sage: a, b = (3, 7)
sage: P = A.encoding("Line.")
sage: C = A.enciphering(a, b, P)
sage: Plist = A.brute_force(C)
sage: Rank = A.rank_by_squared_differences(C, Plist)
sage: Rank[:10] # display only the top 10 candidate keys
```

```
[((1, 1), NETS),
 ((15, 6), ETUN),
 ((7, 17), HCTE),
 ((3, 7), LINE),
 ((17, 15), DELO),
 ((9, 4), EDWT),
 ((9, 9), POHE),
 ((21, 8), ELID),
 ((17, 20), STAD),
 ((7, 18), SNEP)]
```

As more ciphertext is available, the reliability of the squared-differences ranking function increases:

```
sage: A = AffineCryptosystem(AlphabeticStrings())
sage: a, b = (11, 24)
sage: P = A.encoding("Longer message is more information for cryptanalysis.")
sage: C = A.enciphering(a, b, P)
sage: Plist = A.brute_force(C)
sage: Rank = A.rank_by_squared_differences(C, Plist)
sage: Rank[:10] # display only the top 10 candidate keys
```

```
[((11, 24), LONGERMESAGEISMOREINFORMATIONFORCRYPTANALYSIS),
 ((9, 14), DYRUGTKGAAEUGIAKYTGIRNYTKEHIYRNYTSTQFHEREDQAIA),
 ((23, 24), DSNEUHIUMMAEUOMISHUONZSHIAROSNZSHKHQXRANADQMOM),
 ((23, 1), ETOFVIJVNNBFVFNJTIVPOATIJBSPTOATILIRYSBOBERNPN),
 ((21, 16), VEBGANYAQOGAMQYENAMBDENYOTMEBDENUNIHTOBOVIQMQ),
```

```
((7, 12), TULAIVCIEEYAISECUVISLRUVVCYNSULRUVQVGDNYLYTGESE),
((5, 20), ZQTOUHWUEEGOUIEWQHUITRQHWGBIQTRQHAHMNBGTGZMEIE),
((21, 8), JSPUOBMOEECUOAEMSBOAPRSEBCHASPRSBIBWVHCPCJWEAE),
((25, 7), SLWVREHRTTJVRZTHLERZWGLEHJIZLWGLENEFAIJWSFTZT),
((25, 15), ATEDZMPZBBRDZHBPTMZHEOTMPRQHTTEOTMVMNIQREERANBHB)]
```

**TESTS:**

The ciphertext cannot be an empty string:

```
sage: A.rank_by_squared_differences("", Plist)
Traceback (most recent call last):
...
AttributeError: 'str' object has no attribute 'parent'
sage: A.rank_by_squared_differences(A.encoding(""), Plist)
Traceback (most recent call last):
...
ValueError: The ciphertext must be a non-empty string.
sage: A.rank_by_squared_differences(A.encoding(" "), Plist)
Traceback (most recent call last):
...
ValueError: The ciphertext must be a non-empty string.
```

The ciphertext must be encoded using the capital letters of the English alphabet as implemented in `AlphabeticStrings()`:

```
sage: H = HexadecimalStrings()
sage: A.rank_by_squared_differences(H.encoding("line"), Plist)
Traceback (most recent call last):
...
TypeError: The ciphertext must be capital letters of the English alphabet.
sage: B = BinaryStrings()
sage: A.rank_by_squared_differences(B.encoding("line"), Plist)
Traceback (most recent call last):
...
TypeError: The ciphertext must be capital letters of the English alphabet.
```

The dictionary `pdict` cannot be empty:

```
sage: A.rank_by_squared_differences(C, {})
Traceback (most recent call last):
...
KeyError: (1, 0)
```

**class** `sage.crypto.classical.HillCryptosystem`(*S*, *m*)

Bases: `sage.crypto.cryptosystem.SymmetricKeyCryptosystem`

Create a Hill cryptosystem defined by the  $m \times m$  matrix space over  $\mathbf{Z}/N\mathbf{Z}$ , where  $N$  is the alphabet size of the string monoid  $S$ .

**INPUT:**

- $S$  - a string monoid over some alphabet
- $m$  - integer  $> 0$ ; the block length of matrices that specify block permutations

**OUTPUT:**

- A Hill cryptosystem of block length  $m$  over the alphabet  $S$ .

**EXAMPLES:**



```

sage: S = AlphabeticStrings()
sage: E = HillCryptosystem(S, 3)
sage: E
Hill cryptosystem on Free alphabetic string monoid on A-Z of block length 3
sage: R = IntegerModRing(26)
sage: M = MatrixSpace(R, 3, 3)
sage: A = M([[1, 0, 1], [0, 1, 1], [2, 2, 3]])
sage: A
[1 0 1]
[0 1 1]
[2 2 3]
sage: e = E(A)
sage: e
Hill cipher on Free alphabetic string monoid on A-Z of block length 3
sage: e(S("LAMAISONBLANCHE"))
JYVKSQPELAYKPV

```

**TESTS:**

```

sage: S = AlphabeticStrings()
sage: E = HillCryptosystem(S, 3)
sage: E == loads(dumps(E))
True

```

**block\_length()**

The row or column dimension of a matrix specifying a block permutation. Encryption and decryption keys of a Hill cipher are square matrices, i.e. the row and column dimensions of an encryption or decryption key are the same. This row/column dimension is referred to as the *block length*.

**OUTPUT:**

- The block length of an encryption/decryption key.

**EXAMPLES:**

```

sage: A = AlphabeticStrings()
sage: n = randint(1, A.ngens() - 1)
sage: H = HillCryptosystem(A, n)
sage: H.block_length() == n
True

```

**deciphering(A, C)**

Decrypt the ciphertext C using the key A.

**INPUT:**

- A - a key within the key space of this Hill cipher
- C - a string (possibly empty) over the string monoid of this Hill cipher

**OUTPUT:**

- The plaintext corresponding to the ciphertext C.

**EXAMPLES:**

```

sage: H = HillCryptosystem(AlphabeticStrings(), 3)
sage: K = H.random_key()
sage: M = H.encoding("Good day, mate! How ya going?")
sage: H.deciphering(K, H.encrypting(K, M)) == M
True

```

**enciphering** ( $A, M$ )

Encrypt the plaintext  $M$  using the key  $A$ .

INPUT:

- $A$  - a key within the key space of this Hill cipher
- $M$  - a string (possibly empty) over the string monoid of this Hill cipher.

OUTPUT:

- The ciphertext corresponding to the plaintext  $M$ .

EXAMPLES:

```
sage: H = HillCryptosystem(AlphabeticStrings(), 3)
sage: K = H.random_key()
sage: M = H.encoding("Good day, mate! How ya going?")
sage: H.deciphering(K, H.enciphering(K, M)) == M
True
```

**encoding** ( $M$ )

The encoding of the string  $M$  over the string monoid of this Hill cipher. For example, if the string monoid of this Hill cipher is `AlphabeticStringMonoid`, then the encoding of  $M$  would be its upper-case equivalent stripped of all non-alphabetic characters.

INPUT:

- $M$  - a string, possibly empty

OUTPUT:

- The encoding of  $M$  over the string monoid of this Hill cipher.

EXAMPLES:

```
sage: M = "The matrix cipher by Lester S. Hill."
sage: A = AlphabeticStrings()
sage: H = HillCryptosystem(A, 7)
sage: H.encoding(M) == A.encoding(M)
True
```

**inverse\_key** ( $A$ )

The inverse key corresponding to the key  $A$ .

INPUT:

- $A$  - an invertible matrix of the key space of this Hill cipher

OUTPUT:

- The inverse matrix of  $A$ .

EXAMPLES:

```
sage: S = AlphabeticStrings()
sage: E = HillCryptosystem(S, 3)
sage: A = E.random_key()
sage: B = E.inverse_key(A)
sage: M = S("LAMAISONBLANCHE")
sage: e = E(A)
sage: c = E(B)
sage: c(e(M))
LAMAISONBLANCHE
```

**random\_key()**

A random key within the key space of this Hill cipher. That is, generate a random  $m \times m$  matrix to be used as a block permutation, where  $m$  is the block length of this Hill cipher. If  $n$  is the size of the cryptosystem alphabet, then there are  $n^{m^2}$  possible keys. However the number of valid keys, i.e. invertible  $m \times m$  square matrices, is smaller than  $n^{m^2}$ .

OUTPUT:

- A random key within the key space of this Hill cipher.

**EXAMPLES:**

```
sage: A = AlphabeticStrings()
sage: n = 3
sage: H = HillCryptosystem(A, n)
sage: K = H.random_key()
sage: Ki = H.inverse_key(K)
sage: M = "LAMAISONBLANCHE"
sage: e = H(K)
sage: d = H(Ki)
sage: d(e(A(M))) == A(M)
True
```

**class sage.crypto.classical.ShiftCryptosystem(A)**

Bases: `sage.crypto.cryptosystem.SymmetricKeyCryptosystem`

Create a shift cryptosystem.

Let  $A = \{a_0, a_1, a_2, \dots, a_{n-1}\}$  be a non-empty alphabet consisting of  $n$  unique elements. Define a mapping  $f : A \rightarrow \mathbf{Z}/n\mathbf{Z}$  from the alphabet  $A$  to the set  $\mathbf{Z}/n\mathbf{Z}$  of integers modulo  $n$ , given by  $f(a_i) = i$ . Thus we can identify each element of the alphabet  $A$  with a unique integer  $0 \leq i < n$ . A key of the shift cipher is an integer  $0 \leq k < n$ . Therefore the key space is  $\mathbf{Z}/n\mathbf{Z}$ . Since we assume that  $A$  does not have repeated elements, the mapping  $f : A \rightarrow \mathbf{Z}/n\mathbf{Z}$  is bijective. Encryption works by moving along the alphabet by  $k$  positions, with wrap around. Decryption reverses the process by moving backwards by  $k$  positions, with wrap around. More generally, let  $k$  be a secret key, i.e. an element of the key space, and let  $p$  be a plaintext character and consequently  $p \in \mathbf{Z}/n\mathbf{Z}$ . Then the ciphertext character  $c$  corresponding to  $p$  is given by

$$c \equiv p + k \pmod{n}$$

Similarly, given a ciphertext character  $c \in \mathbf{Z}/n\mathbf{Z}$  and a secret key  $k$ , we can recover the corresponding plaintext character as follows:

$$p \equiv c - k \pmod{n}$$

Use the bijection  $f : A \rightarrow \mathbf{Z}/n\mathbf{Z}$  to convert  $c$  and  $p$  back to elements of the alphabet  $A$ . Currently, the following alphabets are supported for the shift cipher:

- capital letters of the English alphabet as implemented in `AlphabeticStrings()`
- the alphabet consisting of the hexadecimal number system as implemented in `HexadecimalStrings()`
- the alphabet consisting of the binary number system as implemented in `BinaryStrings()`

**EXAMPLES:**

Some examples illustrating encryption and decryption over various alphabets. Here is an example over the upper-case letters of the English alphabet:

```
sage: S = ShiftCryptosystem(AlphabeticStrings()); S
Shift cryptosystem on Free alphabetic string monoid on A-Z
sage: P = S.encoding("The shift cryptosystem generalizes the Caesar cipher.")
```

```
sage: P
THESHIFTCRYPTOSYSTEMGENERALIZESTHECAESARCIPHER
sage: K = 7
sage: C = S.enciphering(K, P); C
AOLZOPMAJYFWAVZFZALTNLULYHSPGLZAOLJHLZHYJPWOLY
sage: S.deciphering(K, C)
THESHIFTCRYPTOSYSTEMGENERALIZESTHECAESARCIPHER
sage: S.deciphering(K, C) == P
True
```

The previous example can also be done as follows:

```
sage: S = ShiftCryptosystem(AlphabeticStrings())
sage: P = S.encoding("The shift cryptosystem generalizes the Caesar cipher.")
sage: K = 7
sage: E = S(K); E
Shift cipher on Free alphabetic string monoid on A-Z
sage: C = E(P); C
AOLZOPMAJYFWAVZFZALTNLULYHSPGLZAOLJHLZHYJPWOLY
sage: D = S(S.inverse_key(K)); D
Shift cipher on Free alphabetic string monoid on A-Z
sage: D(C) == P
True
sage: D(C) == P == D(E(P))
True
```

Over the hexadecimal number system:

```
sage: S = ShiftCryptosystem(HexadecimalStrings()); S
Shift cryptosystem on Free hexadecimal string monoid
sage: P = S.encoding("Encryption & decryption shifts along the alphabet."); P
456e6372797074696f6e20262064656372797074696f6e2073686966747320616c6f6e672074686520616c7068616265
sage: K = 5
sage: C = S.enciphering(K, P); C
9ab3b8c7cec5c9beb4b3757b75b9bab8c7cec5c9beb4b375c8bdbbebbc9c875b6b1b4b3bc75c9bdba75b6b1c5bdb6b7ba
sage: S.deciphering(K, C)
456e6372797074696f6e20262064656372797074696f6e2073686966747320616c6f6e672074686520616c7068616265
sage: S.deciphering(K, C) == P
True
```

And over the binary number system:

```
sage: S = ShiftCryptosystem(BinaryStrings()); S
Shift cryptosystem on Free binary string monoid
sage: P = S.encoding("The binary alphabet is very insecure."); P
010101000110100001100101001000000110001001101001011011100110000101110010011110010010000001100001
sage: K = 1
sage: C = S.enciphering(K, P); C
10101011100101111001101011011111100111011001011010010001100111101000110110000110110111110011110
sage: S.deciphering(K, C)
010101000110100001100101001000000110001001101001011011100110000101110010011110010010000001100001
sage: S.deciphering(K, C) == P
True
```

A shift cryptosystem with key  $k = 3$  is commonly referred to as the Caesar cipher. Create a Caesar cipher over the upper-case letters of the English alphabet:

```
sage: caesar = ShiftCryptosystem(AlphabeticStrings())
sage: K = 3
sage: P = caesar.encoding("abcdef"); P
```

```

ABCDEF
sage: C = caesar.encrypting(K, P); C
DEFGHI
sage: caesar.decrypting(K, C) == P
True

```

Generate a random key for encryption and decryption:

```

sage: S = ShiftCryptosystem(AlphabeticStrings())
sage: P = S.encoding("Shift cipher with a random key.")
sage: K = S.random_key()
sage: C = S.encrypting(K, P)
sage: S.decrypting(K, C) == P
True

```

Decrypting with the key K is equivalent to encrypting with its corresponding inverse key:

```

sage: S.encrypting(S.inverse_key(K), C) == P
True

```

#### TESTS:

Currently, the octal number system is not supported as an alphabet for this shift cryptosystem:

```

sage: ShiftCryptosystem(OctalStrings())
Traceback (most recent call last):
...
TypeError: A (= Free octal string monoid) is not supported as a cipher domain of this shift cryptosystem

```

Nor is the radix-64 number system supported:

```

sage: ShiftCryptosystem(Radix64Strings())
Traceback (most recent call last):
...
TypeError: A (= Free radix 64 string monoid) is not supported as a cipher domain of this shift cryptosystem

```

Testing of dumping and loading objects:

```

sage: SA = ShiftCryptosystem(AlphabeticStrings())
sage: SA == loads(dumps(SA))
True
sage: SH = ShiftCryptosystem(HexadecimalStrings())
sage: SH == loads(dumps(SH))
True
sage: SB = ShiftCryptosystem(BinaryStrings())
sage: SB == loads(dumps(SB))
True

```

The key  $K$  must satisfy the inequality  $0 \leq K < n$  with  $n$  being the size of the plaintext, ciphertext, and key spaces. For the shift cryptosystem, all these spaces are the same alphabet. This inequality must be satisfied for each of the supported alphabets. The capital letters of the English alphabet:

```

sage: S = ShiftCryptosystem(AlphabeticStrings())
sage: S(2 + S.alphabet_size())
Traceback (most recent call last):
...
ValueError: K (=28) is outside the range of acceptable values for a key of this shift cryptosystem
sage: S(-2)
Traceback (most recent call last):
...
ValueError: K (=-2) is outside the range of acceptable values for a key of this shift cryptosystem

```

The hexadecimal number system:

```
sage: S = ShiftCryptosystem(HexadecimalStrings())
sage: S(1 + S.alphabet_size())
Traceback (most recent call last):
...
ValueError: K (=17) is outside the range of acceptable values for a key of this shift cryptosystem
sage: S(-1)
Traceback (most recent call last):
...
ValueError: K (=-1) is outside the range of acceptable values for a key of this shift cryptosystem
```

The binary number system:

```
sage: S = ShiftCryptosystem(BinaryStrings())
sage: S(1 + S.alphabet_size())
Traceback (most recent call last):
...
ValueError: K (=3) is outside the range of acceptable values for a key of this shift cryptosystem
sage: S(-2)
Traceback (most recent call last):
...
ValueError: K (=-2) is outside the range of acceptable values for a key of this shift cryptosystem
```

**brute\_force**(*C*, *ranking*='none')

Attempt a brute force cryptanalysis of the ciphertext *C*.

INPUT:

- *C* – A ciphertext over one of the supported alphabets of this shift cryptosystem. See the class [ShiftCryptosystem](#) for documentation on the supported alphabets.
- *ranking* – (default "none") the method to use for ranking all possible keys. If *ranking*="none", then do not use any ranking function. The following ranking functions are supported:
  - "chisquare" – the chi-square ranking function as implemented in the method [rank\\_by\\_chi\\_square\(\)](#).
  - "squared\_differences" – the squared differences ranking function as implemented in the method [rank\\_by\\_squared\\_differences\(\)](#).

OUTPUT:

- All the possible plaintext sequences corresponding to the ciphertext *C*. This method effectively uses all the possible keys in this shift cryptosystem to decrypt *C*. The method is also referred to as exhaustive key search. The output is a dictionary of key, plaintext pairs.

EXAMPLES:

Cryptanalyze using all possible keys for various alphabets. Over the upper-case letters of the English alphabet:

```
sage: S = ShiftCryptosystem(AlphabeticStrings())
sage: P = S.encoding("The shift cryptosystem generalizes the Caesar cipher.")
sage: K = 7
sage: C = S.enciphering(K, P)
sage: Dict = S.brute_force(C)
sage: for k in xrange(len(Dict)):
...     if Dict[k] == P:
...         print "key =", k
...
key = 7
```

Over the hexadecimal number system:

```
sage: S = ShiftCryptosystem(HexadecimalStrings())
sage: P = S.encoding("Encryption & decryption shifts along the alphabet.")
sage: K = 5
sage: C = S.enciphering(K, P)
sage: Dict = S.brute_force(C)
sage: for k in xrange(len(Dict)):
...     if Dict[k] == P:
...         print "key =", k
...
key = 5
```

And over the binary number system:

```
sage: S = ShiftCryptosystem(BinaryStrings())
sage: P = S.encoding("The binary alphabet is very insecure.")
sage: K = 1
sage: C = S.enciphering(K, P)
sage: Dict = S.brute_force(C)
sage: for k in xrange(len(Dict)):
...     if Dict[k] == P:
...         print "key =", k
...
key = 1
```

Don't use any ranking functions, i.e. ranking="none":

```
sage: S = ShiftCryptosystem(AlphabeticStrings())
sage: P = S.encoding("Shifting using modular arithmetic.")
sage: K = 8
sage: C = S.enciphering(K, P)
sage: pdict = S.brute_force(C)
sage: sorted(pdict.items())
```

```
[(0, APQNBQVOCAQVOUWLCTIZIZQBUMBQK),
(1, ZOPMAPUNBZPUNTVKBSHYHYPAOTLAPJ),
(2, YNOLZOTMAYOTMSUJARGXGXOZNSKZOI),
(3, XMNKYNSLZXNSLRTIZQFWFWMYMRJYNH),
(4, WLMJXMRKYWMRKQSHYPEVEVMXLQIXMG),
(5, VKLIWLQJXVLQJPRGXODUDULWKPHWLF),
(6, UJKHVKPIWUKPIOQFWNCTCTKVJOGVKE),
(7, TIJGUJOHVTJOHNPEVMSBSJUINFUJD),
(8, SHIFTINGUSINGMODULARARITHMETIC),
(9, RGHESHMFTRHMFNLCTKZQZQHSGLDSHB),
(10, QFGDRGLESQGLEKMBSJYPYPGRFKCRGA),
(11, PEFCQFKDRPFKDJLARIXOXOFQEBQFZ),
(12, ODEBPEJCQOEJCIKZQHWNWNEPDIAPEY),
(13, NCDAODIBPNDIBHJYPGVMVMDOCHZODX),
(14, MBCZNCHAOMCHAGIXOFULULCNBGYNCW),
(15, LABYMBGZNLBGZFHWNETKTKBMAFXMBV),
(16, KZAXLAFYMKAFYEGVMDJSJSJALZEWLAU),
(17, JYZWKZEXLJZEXDFULCRIRIZKYDVKZT),
(18, IXYVJYDWKIYDWCETKBQHQHYJXCUIYS),
(19, HWXUIXCVJHXCVBDSJAPGPGXIWTIXR),
(20, GVWTHWBUIGWBUACRIZOFOFWHVAHWQ),
(21, FUVSGVATHFVATZBQHYNENEVGUZRGVP),
(22, ETURFUZSGEUZSYAPGXMDMDUFTYQFUO),
(23, DSTQETYRFDTYRXZOFWLCLCTESXPETN),
(24, CRSPDSXQECXQWYNEVKBKBSDRWODSM),
```

```
(25, BQROCRWPDBRWPVXMDUJAJARCQVNCRL) ]
```

Use the chi-square ranking function, i.e. `ranking="chisquare"`:

```
sage: S.brute_force(C, ranking="chisquare")
```

```
[ (8, SHIFTINGUSINGMODULARARITHMETIC),
  (14, MBCZNCHAOMCHAGIXOFULULCNBGYNCW),
  (20, GVWTHWBUIGWBUACRIZOFOFWHVASHWQ),
  (13, NCDAODIBPNDIBHJYPGVMVMDOCHZODX),
  (1, ZOPMAPUNBZPUNTVKBSHYHYPAOTLAPJ),
  (23, DSTQETYRFDTYRXZOFWLCLCTESXPETN),
  (10, QFGDRGLESQGLEKMBSJYPYPGRFKCRGA),
  (6, UJKHVKPIWUKPIOQFWNCTCTKVJOGVKE),
  (22, ETURFUZSGEUZSYAPGXMDMDUFTYQFUO),
  (15, LABYMBGZNLBGZFWHNETKTKBMAFXMBV),
  (12, ODEBPEJCQOEJCIKZQHWNWNEPDIAPEY),
  (21, FUVSGVATHFVATZBQHYNENEVGUZRGVP),
  (16, KZAXLAFYMKAFYEGVMDSJSJALZEWL AU),
  (25, BQROCRWPDBRWPVXMDUJAJARCQVNCRL),
  (9, RGHESHMFTRHMFNLCTKZQZQHSGLDSHB),
  (24, CRSPDSXQEC SXQWYNEVKBKBSDRWODSM),
  (3, XMNKYNSLZXNSLRTIZQFWFWMYMRJYNH),
  (5, VKLIWLQJXVLQJPRGXODUDULWKPWWLF),
  (7, TIJGUJOHVTJOHNPEVMSBSJUINFUJD),
  (2, YNOLZOTMAYOTMSUJARGXGXOZNSKZOI),
  (18, IXYVJYDWKIYDWCE TKBQH QHYJXCUIJS),
  (4, WLMJXMRKYWMRKQSHYPEVEVMXLQIXMG),
  (11, PEFCQFKDRPFKDJLARIXOXOFQEBQFZ),
  (19, HWXUIXCVJHXCVBDSJAPGPGXIWTIXR),
  (0, APQNBQVOCAQVOUWLCTIZIZQB PUMBQK),
  (17, JYZWKZEXLJZEXDFULCRIRIZKYDVKZT) ]
```

Use the squared differences ranking function, i.e. `ranking="squared_differences"`:

```
sage: S.brute_force(C, ranking="squared_differences")
```

```
[ (8, SHIFTINGUSINGMODULARARITHMETIC),
  (23, DSTQETYRFDTYRXZOFWLCLCTESXPETN),
  (12, ODEBPEJCQOEJCIKZQHWNWNEPDIAPEY),
  (2, YNOLZOTMAYOTMSUJARGXGXOZNSKZOI),
  (9, RGHESHMFTRHMFNLCTKZQZQHSGLDSHB),
  (7, TIJGUJOHVTJOHNPEVMSBSJUINFUJD),
  (21, FUVSGVATHFVATZBQHYNENEVGUZRGVP),
  (22, ETURFUZSGEUZSYAPGXMDMDUFTYQFUO),
  (1, ZOPMAPUNBZPUNTVKBSHYHYPAOTLAPJ),
  (16, KZAXLAFYMKAFYEGVMDSJSJALZEWL AU),
  (20, GVWTHWBUIGWBUACRIZOFOFWHVASHWQ),
  (24, CRSPDSXQEC SXQWYNEVKBKBSDRWODSM),
  (14, MBCZNCHAOMCHAGIXOFULULCNBGYNCW),
  (13, NCDAODIBPNDIBHJYPGVMVMDOCHZODX),
  (3, XMNKYNSLZXNSLRTIZQFWFWMYMRJYNH),
  (10, QFGDRGLESQGLEKMBSJYPYPGRFKCRGA),
  (15, LABYMBGZNLBGZFWHNETKTKBMAFXMBV),
  (6, UJKHVKPIWUKPIOQFWNCTCTKVJOGVKE),
  (11, PEFCQFKDRPFKDJLARIXOXOFQEBQFZ),
  (25, BQROCRWPDBRWPVXMDUJAJARCQVNCRL),
  (17, JYZWKZEXLJZEXDFULCRIRIZKYDVKZT),
  (19, HWXUIXCVJHXCVBDSJAPGPGXIWTIXR),
```



```
(4, WLMJXMRKYWMRKQSHYPEVEVMXLQIXMG),
(0, APQNBQVOCAQVOUWLCTIZIZQBPUMBQK),
(18, IXYVJYDWKIYDWCETKBQHGHYJXCUIYS),
(5, VKLIWLQJXVLQJPRGXODUDULWKPHWLF)]
```

**TESTS:**

Currently, the octal number system is not supported as an alphabet for this shift cryptosystem:

```
sage: SA = ShiftCryptosystem(AlphabeticStrings())
sage: OctStr = OctalStrings()
sage: C = OctStr([1, 2, 3])
sage: SA.brute_force(C)
Traceback (most recent call last):
...
TypeError: ciphertext must be encoded using one of the supported cipher domains of this shift
```

Nor is the radix-64 alphabet supported:

```
sage: Rad64 = Radix64Strings()
sage: C = Rad64([1, 2, 3])
sage: SA.brute_force(C)
Traceback (most recent call last):
...
TypeError: ciphertext must be encoded using one of the supported cipher domains of this shift
```

**deciphering** (*K*, *C*)

Decrypt the ciphertext *C* with the key *K* using shift cipher decryption.

**INPUT:**

- *K* – a secret key; a key belonging to the key space of this shift cipher. This key is an integer  $k$  satisfying the inequality  $0 \leq k < n$ , where  $n$  is the size of the cipher domain.
- *C* – a string of ciphertext; possibly an empty string. Characters in this string must be encoded using one of the supported alphabets. See the method `encoding()` for more information.

**OUTPUT:**

- The plaintext corresponding to the ciphertext *C*.

**EXAMPLES:**

Let's perform decryption over the supported alphabets. Here is decryption over the capital letters of the English alphabet:

```
sage: S = ShiftCryptosystem(AlphabeticStrings())
sage: P = S.encoding("Stop shifting me."); P
STOPSHIFTINGME
sage: K = 13
sage: C = S.enciphering(K, P); C
FGBCFUVSGVATZR
sage: S.deciphering(K, C) == P
True
```

Decryption over the hexadecimal number system:

```
sage: S = ShiftCryptosystem(HexadecimalStrings())
sage: P = S.encoding("Shift me now."); P
5368696674206d65206e6f772e
sage: K = 7
sage: C = S.enciphering(K, P); C
```

```
cadfd0ddeb97d4dc97d5d6ee95
sage: S.deciphering(K, C) == P
True
```

Decryption over the binary number system:

```
sage: S = ShiftCryptosystem(BinaryStrings())
sage: P = S.encoding("OK, enough shifting."); P
0100111101001011001011000010000001100101011011100110111101110101011001110110100001000000111
sage: K = 1
sage: C = S.enciphering(K, P); C
1011000010110100110100111101111110011010100100011001000010001010100110001001011111011111000
sage: S.deciphering(K, C) == P
True
```

### **enciphering**(*K*, *P*)

Encrypt the plaintext *P* with the key *K* using shift cipher encryption.

INPUT:

- *K* – a key belonging to the key space of this shift cipher. This key is an integer  $k$  satisfying the inequality  $0 \leq k < n$ , where  $n$  is the size of the cipher domain.
- *P* – a string of plaintext; possibly an empty string. Characters in this string must be encoded using one of the supported alphabets. See the method `encoding()` for more information.

OUTPUT:

- The ciphertext corresponding to the plaintext *P*.

EXAMPLES:

Let's perform encryption over the supported alphabets. Here is encryption over the capital letters of the English alphabet:

```
sage: S = ShiftCryptosystem(AlphabeticStrings())
sage: P = S.encoding("Shift your gear."); P
SHIFTYOURGEAR
sage: K = 3
sage: S.enciphering(K, P)
VKLIWBRXUJH DU
```

Encryption over the hexadecimal number system:

```
sage: S = ShiftCryptosystem(HexadecimalStrings())
sage: P = S.encoding("Capitalize with the shift key."); P
4361706974616c697a65207769746820746865207368696674206b65792e
sage: K = 5
sage: S.enciphering(K, P)
98b6c5bec9b6b1becfba75ccbec9bd75c9bdba75c8bdbbebbc975b0bace73
```

Encryption over the binary number system:

```
sage: S = ShiftCryptosystem(BinaryStrings())
sage: P = S.encoding("Don't shift."); P
01000100011011110110111000100111011101000010000001110011011010000110100101100110011101000010
sage: K = 1
sage: S.enciphering(K, P)
1011101110010000100100011101100010001011110111110001100100101111001011010011001100010111101
```

### **encoding**(*S*)

The encoding of the string *S* over the string monoid of this shift cipher. For example, if the string monoid

of this cryptosystem is `AlphabeticStringMonoid`, then the encoding of  $S$  would be its upper-case equivalent stripped of all non-alphabetic characters. The following alphabets are supported for the shift cipher:

- capital letters of the English alphabet as implemented in `AlphabeticStrings()`
- the alphabet consisting of the hexadecimal number system as implemented in `HexadecimalStrings()`
- the alphabet consisting of the binary number system as implemented in `BinaryStrings()`

INPUT:

- $S$  – a string, possibly empty.

OUTPUT:

- The encoding of  $S$  over the string monoid of this cryptosystem. If  $S$  is an empty string, return an empty string.

EXAMPLES:

Encoding over the upper-case letters of the English alphabet:

```
sage: S = ShiftCryptosystem(AlphabeticStrings())
sage: S.encoding("Shift cipher on capital letters of the English alphabet.")
SHIFTCIPHERONCAPITALLETTERSOFTHEENGLISHALPHABET
```

Encoding over the binary system:

```
sage: S = ShiftCryptosystem(BinaryStrings())
sage: S.encoding("Binary")
010000100110100101101110011000010111001001111001
```

Encoding over the hexadecimal system:

```
sage: S = ShiftCryptosystem(HexadecimalStrings())
sage: S.encoding("Over hexadecimal system.")
4f7665722068657861646563696d616c2073797374656d2e
```

The argument  $S$  can be an empty string, in which case an empty string is returned:

```
sage: ShiftCryptosystem(AlphabeticStrings()).encoding("")
sage: ShiftCryptosystem(HexadecimalStrings()).encoding("")
sage: ShiftCryptosystem(BinaryStrings()).encoding("")
```

### **inverse\_key( $K$ )**

The inverse key corresponding to the key  $K$ . For the shift cipher, the inverse key corresponding to  $K$  is  $-K \bmod n$ , where  $n > 0$  is the size of the cipher domain, i.e. the plaintext/ciphertext space. A key  $k$  of the shift cipher is an integer  $0 \leq k < n$ . The key  $k = 0$  has no effect on either the plaintext or the ciphertext.

INPUT:

- $K$  – a key for this shift cipher. This must be an integer  $k$  such that  $0 \leq k < n$ , where  $n$  is the size of the cipher domain.

OUTPUT:

- The inverse key corresponding to  $K$ .

EXAMPLES:

Some random keys and their respective inverse keys:

```
sage: S = ShiftCryptosystem(AlphabeticStrings())
sage: key = S.random_key(); key # random
2
sage: S.inverse_key(key) # random
24
sage: S = ShiftCryptosystem(HexadecimalStrings())
sage: key = S.random_key(); key # random
12
sage: S.inverse_key(key) # random
4
sage: S = ShiftCryptosystem(BinaryStrings())
sage: key = S.random_key(); key # random
1
sage: S.inverse_key(key) # random
1
sage: key = S.random_key(); key # random
0
sage: S.inverse_key(key) # random
0
```

Regardless of the value of a key, the addition of the key and its inverse must be equal to the alphabet size. This relationship holds exactly when the value of the key is non-zero:

```
sage: S = ShiftCryptosystem(AlphabeticStrings())
sage: K = S.random_key()
sage: while K == 0:
...     K = S.random_key()
...
sage: invK = S.inverse_key(K)
sage: K + invK == S.alphabet_size()
True
sage: invK + K == S.alphabet_size()
True
sage: K = S.random_key()
sage: while K != 0:
...     K = S.random_key()
...
sage: invK = S.inverse_key(K)
sage: K + invK != S.alphabet_size()
True
sage: K; invK
0
0
```

TESTS:

The key  $K$  must satisfy the inequality  $0 \leq K < n$  with  $n$  being the size of the plaintext, ciphertext, and key spaces. For the shift cryptosystem, all these spaces are the same alphabet. This inequality must be satisfied for each of the supported alphabets. The capital letters of the English alphabet:

```
sage: S = ShiftCryptosystem(AlphabeticStrings())
sage: S.inverse_key(S.alphabet_size())
Traceback (most recent call last):
...
ValueError: K (=26) is outside the range of acceptable values for a key of this shift cryptosystem
sage: S.inverse_key(-1)
Traceback (most recent call last):
...
```

```
ValueError: K (-1) is outside the range of acceptable values for a key of this shift cryptosystem
```

The hexadecimal number system:

```
sage: S = ShiftCryptosystem(HexadecimalStrings())
sage: S.inverse_key(S.alphabet_size())
Traceback (most recent call last):
...
ValueError: K (=16) is outside the range of acceptable values for a key of this shift cryptosystem
sage: S.inverse_key(-1)
Traceback (most recent call last):
...
ValueError: K (-1) is outside the range of acceptable values for a key of this shift cryptosystem
```

The binary number system:

```
sage: S = ShiftCryptosystem(BinaryStrings())
sage: S.inverse_key(S.alphabet_size())
Traceback (most recent call last):
...
ValueError: K (=2) is outside the range of acceptable values for a key of this shift cryptosystem
sage: S.inverse_key(-1)
Traceback (most recent call last):
...
ValueError: K (-1) is outside the range of acceptable values for a key of this shift cryptosystem
```

#### **random\_key()**

Generate a random key within the key space of this shift cipher. The generated key is an integer  $0 \leq k < n$  with  $n$  being the size of the cipher domain. Thus there are  $n$  possible keys in the key space, which is the set  $\mathbf{Z}/n\mathbf{Z}$ . The key  $k = 0$  has no effect on either the plaintext or the ciphertext.

OUTPUT:

- A random key within the key space of this shift cryptosystem.

EXAMPLES:

```
sage: S = ShiftCryptosystem(AlphabeticStrings())
sage: S.random_key() # random
18
sage: S = ShiftCryptosystem(BinaryStrings())
sage: S.random_key() # random
0
sage: S = ShiftCryptosystem(HexadecimalStrings())
sage: S.random_key() # random
5
```

Regardless of the value of a key, the addition of the key and its inverse must be equal to the alphabet size. This relationship holds exactly when the value of the key is non-zero:

```
sage: S = ShiftCryptosystem(AlphabeticStrings())
sage: K = S.random_key()
sage: while K == 0:
...     K = S.random_key()
...
sage: invK = S.inverse_key(K)
sage: K + invK == S.alphabet_size()
True
sage: invK + K == S.alphabet_size()
True
```

```

sage: K = S.random_key()
sage: while K != 0:
...     K = S.random_key()
...
sage: invK = S.inverse_key(K)
sage: K + invK != S.alphabet_size()
True
sage: K; invK
0
0

```

### **rank\_by\_chi\_square** (*C*, *pdict*)

Use the chi-square statistic to rank all possible keys. Currently, this method only applies to the capital letters of the English alphabet.

ALGORITHM:

Consider a non-empty alphabet  $A$  consisting of  $n$  elements, and let  $C$  be a ciphertext encoded using elements of  $A$ . The plaintext  $P$  corresponding to  $C$  is also encoded using elements of  $A$ . Let  $M$  be a candidate decipherment of  $C$ , i.e.  $M$  is the result of attempting to decrypt  $C$  using a key  $k \in \mathbf{Z}/n\mathbf{Z}$  which is not necessarily the same key used to encrypt  $P$ . Suppose  $F_A(e)$  is the characteristic frequency probability of  $e \in A$  and let  $F_M(e)$  be the message frequency probability with respect to  $M$ . The characteristic frequency probability distribution of an alphabet is the expected frequency probability distribution for that alphabet. The message frequency probability distribution of  $M$  provides a distribution of the ratio of character occurrences over message length. One can interpret the characteristic frequency probability  $F_A(e)$  as the expected probability, while the message frequency probability  $F_M(e)$  is the observed probability. If  $M$  is of length  $L$ , then the observed frequency of  $e \in A$  is

$$O_M(e) = F_M(e) \cdot L$$

and the expected frequency of  $e \in A$  is

$$E_A(e) = F_A(e) \cdot L$$

The chi-square rank  $R_{\chi^2}(M)$  of  $M$  corresponding to a key  $k \in \mathbf{Z}/n\mathbf{Z}$  is given by

$$R_{\chi^2}(M) = \sum_{e \in A} \frac{(O_M(e) - E_A(e))^2}{E_A(e)}$$

Cryptanalysis by exhaustive key search produces a candidate decipherment  $M_k$  for each possible key  $k \in \mathbf{Z}/n\mathbf{Z}$ . For a set  $D = \{M_{k_1}, M_{k_2}, \dots, M_{k_r}\}$  of all candidate decipherments corresponding to a ciphertext  $C$ , the smaller is the rank  $R_{\chi^2}(M_{k_i})$  the more likely that  $k_i$  is the secret key. This key ranking method is based on the Pearson chi-square test [PearsonTest09].

INPUT:

- *C* – The ciphertext, a non-empty string. The ciphertext must be encoded using the upper-case letters of the English alphabet.
- *pdict* – A dictionary of key, possible plaintext pairs. This should be the output of `brute_force()` with `ranking="none"`.

OUTPUT:

- A list ranking the most likely keys first. Each element of the list is a tuple of key, possible plaintext pairs.

EXAMPLES:

Use the chi-square statistic to rank all possible keys and their corresponding decipherment:

```

sage: S = ShiftCryptosystem(AlphabeticStrings())
sage: P = S.encoding("Shi."); P
SHI
sage: K = 5
sage: C = S.enciphering(K, P)
sage: Pdict = S.brute_force(C)
sage: S.rank_by_chi_square(C, Pdict)

[(9, ODE),
 (5, SHI),
 (20, DST),
 (19, ETU),
 (21, CRS),
 (10, NCD),
 (25, YNO),
 (6, RGH),
 (12, LAB),
 (8, PEF),
 (1, WLM),
 (11, MBC),
 (18, FUV),
 (17, GVW),
 (2, VKL),
 (4, TIJ),
 (3, UJK),
 (0, XMN),
 (16, HWX),
 (15, IXY),
 (23, APQ),
 (24, ZOP),
 (22, BQR),
 (7, QFG),
 (13, KZA),
 (14, JYZ)]

```

As more ciphertext is available, the reliability of the chi-square ranking function increases:

```

sage: P = S.encoding("Shift cipher."); P
SHIFTCIPHER
sage: C = S.enciphering(K, P)
sage: Pdict = S.brute_force(C)
sage: S.rank_by_chi_square(C, Pdict)

[(5, SHIFTCIPHER),
 (9, ODEBPYELDAN),
 (18, FUVSGPVCURE),
 (2, VKLIWFLSKHU),
 (20, DSTQENTASPC),
 (19, ETURFOUBTQD),
 (21, CRSPDMSZROB),
 (6, RGHESBHOGDQ),
 (7, QFGDRAGNFCP),
 (12, LABYMBVBIAXK),
 (17, GVWTHQWDVSF),
 (24, ZOPMAJPWOLY),
 (1, WLMJXGMTLIV),
 (0, XMNKYHNUMJW),
 (11, MBCZNWCJBYL),
 (8, PEFCQZFMEBO),

```

```
(25, YNOLZIOVNKX),
(10, NCDAOXDKCZM),
(3, UJKHVEKRJGT),
(4, TIJGUDJQIFS),
(22, BQROCLRYQNA),
(16, HWXUIRXEWGT),
(15, IXYVJSYFXUH),
(14, JYZWKTZGYVI),
(13, KZAXLUAHZWJ),
(23, APQNBKQXPMZ)]
```

#### TESTS:

The ciphertext cannot be an empty string:

```
sage: S.rank_by_chi_square("", Pdict)
Traceback (most recent call last):
...
AttributeError: 'str' object has no attribute 'parent'
sage: S.rank_by_chi_square(S.encoding(""), Pdict)
Traceback (most recent call last):
...
ValueError: The ciphertext must be a non-empty string.
sage: S.rank_by_chi_square(S.encoding(" "), Pdict)
Traceback (most recent call last):
...
ValueError: The ciphertext must be a non-empty string.
```

The ciphertext must be encoded using the capital letters of the English alphabet as implemented in `AlphabeticStrings()`:

```
sage: H = HexadecimalStrings()
sage: S.rank_by_chi_square(H.encoding("shift"), Pdict)
Traceback (most recent call last):
...
TypeError: The ciphertext must be capital letters of the English alphabet.
sage: B = BinaryStrings()
sage: S.rank_by_chi_square(B.encoding("shift"), Pdict)
Traceback (most recent call last):
...
TypeError: The ciphertext must be capital letters of the English alphabet.
```

The dictionary `pdict` cannot be empty:

```
sage: S.rank_by_chi_square(C, {})
Traceback (most recent call last):
...
KeyError: 0
```

#### REFERENCES:

##### **`rank_by_squared_differences`** (*C*, *pdict*)

Use the squared-differences measure to rank all possible keys. Currently, this method only applies to the capital letters of the English alphabet.

#### ALGORITHM:

Consider a non-empty alphabet  $A$  consisting of  $n$  elements, and let  $C$  be a ciphertext encoded using elements of  $A$ . The plaintext  $P$  corresponding to  $C$  is also encoded using elements of  $A$ . Let  $M$  be a candidate decipherment of  $C$ , i.e.  $M$  is the result of attempting to decrypt  $C$  using a key  $k \in \mathbb{Z}/n\mathbb{Z}$  which



is not necessarily the same key used to encrypt  $P$ . Suppose  $F_A(e)$  is the characteristic frequency probability of  $e \in A$  and let  $F_M(e)$  be the message frequency probability with respect to  $M$ . The characteristic frequency probability distribution of an alphabet is the expected frequency probability distribution for that alphabet. The message frequency probability distribution of  $M$  provides a distribution of the ratio of character occurrences over message length. One can interpret the characteristic frequency probability  $F_A(e)$  as the expected probability, while the message frequency probability  $F_M(e)$  is the observed probability. If  $M$  is of length  $L$ , then the observed frequency of  $e \in A$  is

$$O_M(e) = F_M(e) \cdot L$$

and the expected frequency of  $e \in A$  is

$$E_A(e) = F_A(e) \cdot L$$

The squared-differences, or residual sum of squares, rank  $R_{RSS}(M)$  of  $M$  corresponding to a key  $k \in \mathbf{Z}/n\mathbf{Z}$  is given by

$$R_{RSS}(M) = \sum_{e \in A} (O_M(e) - E_A(e))^2$$

Cryptanalysis by exhaustive key search produces a candidate decipherment  $M_k$  for each possible key  $k \in \mathbf{Z}/n\mathbf{Z}$ . For a set  $D = \{M_{k_1}, M_{k_2}, \dots, M_{k_r}\}$  of all candidate decipherments corresponding to a ciphertext  $C$ , the smaller is the rank  $R_{RSS}(M_{k_i})$  the more likely that  $k_i$  is the secret key. This key ranking method is based on the residual sum of squares measure [RSS09].

INPUT:

- `C` – The ciphertext, a non-empty string. The ciphertext must be encoded using the upper-case letters of the English alphabet.
- `pdict` – A dictionary of key, possible plaintext pairs. This should be the output of `brute_force()` with `ranking="none"`.

OUTPUT:

- A list ranking the most likely keys first. Each element of the list is a tuple of key, possible plaintext pairs.

EXAMPLES:

Use the method of squared differences to rank all possible keys and their corresponding decipherment:

```
sage: S = ShiftCryptosystem(AlphabeticStrings())
sage: P = S.encoding("Shi."); P
SHI
sage: K = 5
sage: C = S.enciphering(K, P)
sage: Pdict = S.brute_force(C)
sage: S.rank_by_squared_differences(C, Pdict)
```

```
[(19, ETU),
(9, ODE),
(20, DST),
(5, SHI),
(8, PEF),
(4, TIJ),
(25, YNO),
(21, CRS),
(6, RGH),
(10, NCD),
(12, LAB),
```

```
(23, APQ),
(24, ZOP),
(0, XMN),
(13, KZA),
(15, IXY),
(1, WLM),
(16, HWX),
(22, BQR),
(11, MBC),
(18, FUV),
(2, VKL),
(17, GVW),
(7, QFG),
(3, UJK),
(14, JYZ)]
```

As more ciphertext is available, the reliability of the squared differences ranking function increases:

```
sage: P = S.encoding("Shift cipher."); P
SHIFTCIPHER
sage: C = S.enciphering(K, P)
sage: Pdict = S.brute_force(C)
sage: S.rank_by_squared_differences(C, Pdict)
```

```
[(20, DSTQENTASPC),
(5, SHIFTCIPHER),
(9, ODEBPYELDAN),
(19, ETURFOUBTQD),
(6, RGHESBHOGDQ),
(16, HWXUIRSEWTG),
(8, PEFQZFMBO),
(21, CRSPDMSZROB),
(22, BQROCLRYQNA),
(25, YNOLZIOVNKX),
(3, UJKHVEKRJGT),
(18, FUVSGPVCURE),
(4, TIJGUDJQIFS),
(10, NCDAXDKCZM),
(7, QFGDRAGNFCP),
(24, ZOPMAJPWOLY),
(2, VKLIWFLSKHU),
(12, LABYMBVBIAXK),
(17, GVWTHQWDVSF),
(1, WLMJXGMTLIV),
(13, KZAXLUAHZWJ),
(0, XMNKYHNUMJW),
(15, IXYVJSYFXUH),
(14, JYZWKTZGYVI),
(11, MBCZNWCJBYL),
(23, APQNBKQXPMZ)]
```

TESTS:

The ciphertext cannot be an empty string:

```
sage: S.rank_by_squared_differences("", Pdict)
Traceback (most recent call last):
...
AttributeError: 'str' object has no attribute 'parent'
```

```

sage: S.rank_by_squared_differences(S.encoding(""), Pdict)
Traceback (most recent call last):
...
ValueError: The ciphertext must be a non-empty string.
sage: S.rank_by_squared_differences(S.encoding(" "), Pdict)
Traceback (most recent call last):
...
ValueError: The ciphertext must be a non-empty string.

```

The ciphertext must be encoded using the capital letters of the English alphabet as implemented in `AlphabeticStrings()`:

```

sage: H = HexadecimalStrings()
sage: S.rank_by_squared_differences(H.encoding("shift"), Pdict)
Traceback (most recent call last):
...
TypeError: The ciphertext must be capital letters of the English alphabet.
sage: B = BinaryStrings()
sage: S.rank_by_squared_differences(B.encoding("shift"), Pdict)
Traceback (most recent call last):
...
TypeError: The ciphertext must be capital letters of the English alphabet.

```

The dictionary `pdict` cannot be empty:

```

sage: S.rank_by_squared_differences(C, {})
Traceback (most recent call last):
...
KeyError: 0

```

#### REFERENCES:

```

class sage.crypto.classical.SubstitutionCryptosystem(S)
Bases: sage.crypto.cryptosystem.SymmetricKeyCryptosystem

```

Create a substitution cryptosystem.

INPUT:

- `S` - a string monoid over some alphabet

OUTPUT:

- A substitution cryptosystem over the alphabet `S`.

#### EXAMPLES:

```

sage: M = AlphabeticStrings()
sage: E = SubstitutionCryptosystem(M)
sage: E
Substitution cryptosystem on Free alphabetic string monoid on A-Z
sage: K = M([ 25-i for i in range(26) ])
sage: K
ZYXWVUTSRQPONMLKJIHGFEDCBA
sage: e = E(K)
sage: m = M("THECATINTHEHAT")
sage: e(m)
GSVXZGRMGSVSZG

```

TESTS:

```
sage: M = AlphabeticStrings()
sage: E = SubstitutionCryptosystem(M)
sage: E == loads(dumps(E))
True
```

**deciphering** ( $K, C$ )

Decrypt the ciphertext  $C$  using the key  $K$ .

INPUT:

- $K$  - a key belonging to the key space of this substitution cipher
- $C$  - a string (possibly empty) over the string monoid of this cryptosystem.

OUTPUT:

- The plaintext corresponding to the ciphertext  $C$ .

EXAMPLES:

```
sage: S = SubstitutionCryptosystem(AlphabeticStrings())
sage: K = S.random_key()
sage: M = S.encoding("Don't substitute me!")
sage: S.deciphering(K, S.enciphering(K, M)) == M
True
```

**enciphering** ( $K, M$ )

Encrypt the plaintext  $M$  using the key  $K$ .

INPUT:

- $K$  - a key belonging to the key space of this substitution cipher
- $M$  - a string (possibly empty) over the string monoid of this cryptosystem.

OUTPUT:

- The ciphertext corresponding to the plaintext  $M$ .

EXAMPLES:

```
sage: S = SubstitutionCryptosystem(AlphabeticStrings())
sage: K = S.random_key()
sage: M = S.encoding("Don't substitute me.")
sage: S.deciphering(K, S.enciphering(K, M)) == M
True
```

**encoding** ( $M$ )

The encoding of the string  $M$  over the string monoid of this substitution cipher. For example, if the string monoid of this cryptosystem is `AlphabeticStringMonoid`, then the encoding of  $M$  would be its upper-case equivalent stripped of all non-alphabetic characters.

INPUT:

- $M$  - a string, possibly empty

OUTPUT:

- The encoding of  $M$  over the string monoid of this cryptosystem.

EXAMPLES:

```
sage: M = "Peter Pan(ning) for gold."
sage: A = AlphabeticStrings()
sage: S = SubstitutionCryptosystem(A)
```

```
sage: S.encoding(M) == A.encoding(M)
True
```

**inverse\_key(K)**

The inverse key corresponding to the key  $K$ . The specified key is a permutation of the cryptosystem alphabet.

INPUT:

- $K$  - a key belonging to the key space of this cryptosystem

OUTPUT:

- The inverse key of  $K$ .

EXAMPLES:

```
sage: S = AlphabeticStrings()
sage: E = SubstitutionCryptosystem(S)
sage: K = E.random_key()
sage: L = E.inverse_key(K)
sage: M = S("THECATINTHEHAT")
sage: e = E(K)
sage: c = E(L)
sage: c(e(M))
THECATINTHEHAT
```

**random\_key()**

Generate a random key within the key space of this substitution cipher. The generated key is a permutation of the cryptosystem alphabet. Let  $n$  be the length of the alphabet. Then there are  $n!$  possible keys in the key space.

OUTPUT:

- A random key within the key space of this cryptosystem.

EXAMPLES:

```
sage: A = AlphabeticStrings()
sage: S = SubstitutionCryptosystem(A)
sage: K = S.random_key()
sage: Ki = S.inverse_key(K)
sage: M = "THECATINTHEHAT"
sage: e = S(K)
sage: d = S(Ki)
sage: d(e(A(M))) == A(M)
True
```

**class** sage.crypto.classical.**TranspositionCryptosystem**( $S, n$ )

Bases: sage.crypto.cryptosystem.SymmetricKeyCryptosystem

Create a transposition cryptosystem of block length  $n$ .

INPUT:

- $S$  - a string monoid over some alphabet
- $n$  - integer  $> 0$ ; a block length of a block permutation

OUTPUT:

- A transposition cryptosystem of block length  $n$  over the alphabet  $S$ .

EXAMPLES:

```
sage: S = AlphabeticStrings()
sage: E = TranspositionCryptosystem(S,14)
sage: E
Transposition cryptosystem on Free alphabetic string monoid on A-Z of block length 14
sage: K = [ 14-i for i in range(14) ]
sage: K
[14, 13, 12, 11, 10, 9, 8, 7, 6, 5, 4, 3, 2, 1]
sage: e = E(K)
sage: e(S("THECATINTHEHAT"))
TAHEHTNITACEHT
```

**TESTS:**

```
sage: S = AlphabeticStrings()
sage: E = TranspositionCryptosystem(S,14)
sage: E == loads(dumps(E))
True
```

**deciphering** (*K*, *C*)

Decrypt the ciphertext *C* using the key *K*.

**INPUT:**

- *K* - a key belonging to the key space of this transposition cipher
- *C* - a string (possibly empty) over the string monoid of this cryptosystem.

**OUTPUT:**

- The plaintext corresponding to the ciphertext *C*.

**EXAMPLES:**

```
sage: T = TranspositionCryptosystem(AlphabeticStrings(), 14)
sage: K = T.random_key()
sage: M = T.encoding("The cat in the hat.")
sage: T.deciphering(K, T.enciphering(K, M)) == M
True
```

**enciphering** (*K*, *M*)

Encrypt the plaintext *M* using the key *K*.

**INPUT:**

- *K* - a key belonging to the key space of this transposition cipher
- *M* - a string (possibly empty) over the string monoid of this cryptosystem

**OUTPUT:**

- The ciphertext corresponding to the plaintext *M*.

**EXAMPLES:**

```
sage: T = TranspositionCryptosystem(AlphabeticStrings(), 14)
sage: K = T.random_key()
sage: M = T.encoding("The cat in the hat.")
sage: T.deciphering(K, T.enciphering(K, M)) == M
True
```

**encoding** (*M*)

The encoding of the string *M* over the string monoid of this transposition cipher. For example, if the

string monoid of this cryptosystem is `AlphabeticStringMonoid`, then the encoding of `M` would be its upper-case equivalent stripped of all non-alphabetic characters.

INPUT:

- `M` - a string, possibly empty

OUTPUT:

- The encoding of `M` over the string monoid of this cryptosystem.

EXAMPLES:

```
sage: M = "Transposition cipher is not about matrix transpose."
sage: A = AlphabeticStrings()
sage: T = TranspositionCryptosystem(A, 11)
sage: T.encoding(M) == A.encoding(M)
True
```

**`inverse_key(K, check=True)`**

The inverse key corresponding to the key `K`.

INPUT:

- `K` - a key belonging to the key space of this transposition cipher
- `check` - bool (default: `True`); check that `K` belongs to the key space of this cryptosystem.

OUTPUT:

- The inverse key corresponding to `K`.

EXAMPLES:

```
sage: S = AlphabeticStrings()
sage: E = TranspositionCryptosystem(S, 14)
sage: K = E.random_key()
sage: Ki = E.inverse_key(K)
sage: e = E(K)
sage: d = E(Ki)
sage: M = "THECATINTHEHAT"
sage: C = e(S(M))
sage: d(S(C)) == S(M)
True
```

**`random_key()`**

Generate a random key within the key space of this transposition cryptosystem. Let  $n > 0$  be the block length of this cryptosystem. Then there are  $n!$  possible keys.

OUTPUT:

- A random key within the key space of this cryptosystem.

EXAMPLES:

```
sage: S = AlphabeticStrings()
sage: E = TranspositionCryptosystem(S, 14)
sage: K = E.random_key()
sage: Ki = E.inverse_key(K)
sage: e = E(K)
sage: d = E(Ki)
sage: M = "THECATINTHEHAT"
sage: C = e(S(M))
sage: d(S(C)) == S(M)
True
```

**class** `sage.crypto.classical.VigenereCryptosystem(S, n)`  
Bases: `sage.crypto.cryptosystem.SymmetricKeyCryptosystem`

Create a Vigenere cryptosystem of block length  $n$ .

INPUT:

- $S$  – a string monoid over some alphabet
- $n$  – integer  $> 0$ ; block length of an encryption/decryption key

OUTPUT:

- A Vigenere cryptosystem of block length  $n$  over the alphabet  $S$ .

EXAMPLES:

```
sage: S = AlphabeticStrings()
sage: E = VigenereCryptosystem(S, 14)
sage: E
Vigenere cryptosystem on Free alphabetic string monoid on A-Z of period 14
sage: K = S('ABCDEFGHIJKLMN')
sage: K
ABCDEFGHIJKLMN
sage: e = E(K)
sage: e
Cipher on Free alphabetic string monoid on A-Z
sage: e(S('THECATINTHEHAT'))
TIGFEYOUBQOSMG
```

TESTS:

```
sage: S = AlphabeticStrings()
sage: E = VigenereCryptosystem(S, 14)
sage: E == loads(dumps(E))
True
```

**deciphering** ( $K, C$ )

Decrypt the ciphertext  $C$  using the key  $K$ .

INPUT:

- $K$  – a key belonging to the key space of this Vigenere cipher
- $C$  – a string (possibly empty) over the string monoid of this cryptosystem

OUTPUT:

- The plaintext corresponding to the ciphertext  $C$ .

EXAMPLES:

```
sage: V = VigenereCryptosystem(AlphabeticStrings(), 24)
sage: K = V.random_key()
sage: M = V.encoding("Jack and Jill went up the hill.")
sage: V.deciphering(K, V.enciphering(K, M)) == M
True
```

**enciphering** ( $K, M$ )

Encrypt the plaintext  $M$  using the key  $K$ .

INPUT:

- $K$  – a key belonging to the key space of this Vigenere cipher



- M - a string (possibly empty) over the string monoid of this cryptosystem

OUTPUT:

- The ciphertext corresponding to the plaintext M.

EXAMPLES:

```
sage: V = VigenereCryptosystem(AlphabeticStrings(), 24)
sage: K = V.random_key()
sage: M = V.encoding("Jack and Jill went up the hill.")
sage: V.deciphering(K, V.enciphering(K, M)) == M
True
```

### **encoding** (M)

The encoding of the string M over the string monoid of this Vigenere cipher. For example, if the string monoid of this cryptosystem is AlphabeticStringMonoid, then the encoding of M would be its upper-case equivalent stripped of all non-alphabetic characters.

INPUT:

- M - a string, possibly empty

OUTPUT:

- The encoding of M over the string monoid of this cryptosystem.

EXAMPLES:

```
sage: A = AlphabeticStrings()
sage: V = VigenereCryptosystem(A, 24)
sage: M = "Jack and Jill went up the hill."
sage: V.encoding(M) == A.encoding(M)
True
```

### **inverse\_key** (K)

The inverse key corresponding to the key K.

INPUT:

- K - a key within the key space of this Vigenere cryptosystem

OUTPUT:

- The inverse key corresponding to K.

EXAMPLES:

```
sage: S = AlphabeticStrings()
sage: E = VigenereCryptosystem(S, 14)
sage: K = E.random_key()
sage: L = E.inverse_key(K)
sage: M = S("THECATINTHEHAT")
sage: e = E(K)
sage: c = E(L)
sage: c(e(M))
THECATINTHEHAT
```

### **random\_key** ()

Generate a random key within the key space of this Vigenere cryptosystem. Let  $n > 0$  be the length of the cryptosystem alphabet and let  $m > 0$  be the block length of this cryptosystem. Then there are  $n^m$  possible keys.

OUTPUT:

- A random key within the key space of this cryptosystem.

**EXAMPLES:**

```
sage: A = AlphabeticStrings()
sage: V = VigenereCryptosystem(A, 14)
sage: M = "THECATINTHEHAT"
sage: K = V.random_key()
sage: Ki = V.inverse_key(K)
sage: e = V(K)
sage: d = V(Ki)
sage: d(e(A(M))) == A(M)
True
```

# CLASSICAL CIPHERS

**class** `sage.crypto.classical_cipher.AffineCipher` (*parent, key*)  
Bases: `sage.crypto.cipher.SymmetricKeyCipher`

Affine cipher class. This is the class that does the actual work of encryption and decryption. Users should not directly instantiate or create objects of this class. Instead, functionalities of this class should be accessed via `AffineCryptosystem` as the latter provides a convenient user interface.

**class** `sage.crypto.classical_cipher.HillCipher` (*parent, key*)  
Bases: `sage.crypto.cipher.SymmetricKeyCipher`

Hill cipher class

**inverse** ()  
    `x.__init__(...)` initializes x; see `help(type(x))` for signature

**class** `sage.crypto.classical_cipher.ShiftCipher` (*parent, key*)  
Bases: `sage.crypto.cipher.SymmetricKeyCipher`

Shift cipher class. This is the class that does the actual work of encryption and decryption. Users should not directly instantiate or create objects of this class. Instead, functionalities of this class should be accessed via `ShiftCryptosystem` as the latter provides a convenient user interface.

**class** `sage.crypto.classical_cipher.SubstitutionCipher` (*parent, key*)  
Bases: `sage.crypto.cipher.SymmetricKeyCipher`

Substitution cipher class

**inverse** ()  
    `x.__init__(...)` initializes x; see `help(type(x))` for signature

**class** `sage.crypto.classical_cipher.TranspositionCipher` (*parent, key*)  
Bases: `sage.crypto.cipher.SymmetricKeyCipher`

Transition cipher class

**inverse** ()  
    `x.__init__(...)` initializes x; see `help(type(x))` for signature

**class** `sage.crypto.classical_cipher.VigenereCipher` (*parent, key*)  
Bases: `sage.crypto.cipher.SymmetricKeyCipher`

Vigenere cipher class

**inverse** ()  
    `x.__init__(...)` initializes x; see `help(type(x))` for signature



# SIMPLIFIED DES

A simplified variant of the Data Encryption Standard (DES). Note that Simplified DES or S-DES is for educational purposes only. It is a small-scale version of the DES designed to help beginners understand the basic structure of DES.

AUTHORS:

- Minh Van Nguyen (2009-06): initial version

```
class sage.crypto.block_cipher.sdes.SimplifiedDES
    Bases: sage.structure.sage_object.SageObject
```

This class implements the Simplified Data Encryption Standard (S-DES) described in [Sch96]. Schaefer's S-DES is for educational purposes only and is not secure for practical purposes. S-DES is a version of the DES with all parameters significantly reduced, but at the same time preserving the structure of DES. The goal of S-DES is to allow a beginner to understand the structure of DES, thus laying a foundation for a thorough study of DES. Its goal is as a teaching tool in the same spirit as Phan's *Mini-AES* [Pha02].

EXAMPLES:

Encrypt a random block of 8-bit plaintext using a random key, decrypt the ciphertext, and compare the result with the original plaintext:

```
sage: from sage.crypto.block_cipher.sdes import SimplifiedDES
sage: sdes = SimplifiedDES(); sdes
Simplified DES block cipher with 10-bit keys
sage: bin = BinaryStrings()
sage: P = [bin(str(randint(0, 1))) for i in xrange(8)]
sage: K = sdes.random_key()
sage: C = sdes.encrypt(P, K)
sage: plaintext = sdes.decrypt(C, K)
sage: plaintext == P
True
```

We can also encrypt binary strings that are larger than 8 bits in length. However, the number of bits in that binary string must be positive and a multiple of 8:

```
sage: from sage.crypto.block_cipher.sdes import SimplifiedDES
sage: sdes = SimplifiedDES()
sage: bin = BinaryStrings()
sage: P = bin.encoding("Encrypt this using S-DES!")
sage: Mod(len(P), 8) == 0
True
sage: K = sdes.list_to_string(sdes.random_key())
sage: C = sdes(P, K, algorithm="encrypt")
sage: plaintext = sdes(C, K, algorithm="decrypt")
sage: plaintext == P
True
```

## REFERENCES:

**block\_length()**

Return the block length of Schaefer's S-DES block cipher. A key in Schaefer's S-DES is a block of 10 bits.

## OUTPUT:

- The block (or key) length in number of bits.

## EXAMPLES:

```
sage: from sage.crypto.block_cipher.sdes import SimplifiedDES
sage: sdes = SimplifiedDES()
sage: sdes.block_length()
10
```

**decrypt(C, K)**

Return an 8-bit plaintext corresponding to the ciphertext  $C$ , using S-DES decryption with key  $K$ . The decryption process of S-DES is as follows. Let  $P$  be the initial permutation function,  $P^{-1}$  the corresponding inverse permutation,  $\Pi_F$  the permutation/substitution function, and  $\sigma$  the switch function. The ciphertext block  $C$  first goes through  $P$ , the output of which goes through  $\Pi_F$  using the second subkey. Then we apply the switch function to the output of the last function, and the result is then fed into  $\Pi_F$  using the first subkey. Finally, run the output through  $P^{-1}$  to get the plaintext.

## INPUT:

- $C$  – an 8-bit ciphertext; a block of 8 bits
- $K$  – a 10-bit key; a block of 10 bits

## OUTPUT:

The 8-bit plaintext corresponding to  $C$ , obtained using the key  $K$ .

## EXAMPLES:

Decrypt an 8-bit ciphertext block:

```
sage: from sage.crypto.block_cipher.sdes import SimplifiedDES
sage: sdes = SimplifiedDES()
sage: C = [0, 1, 0, 1, 0, 1, 0, 1]
sage: K = [1, 0, 1, 0, 0, 0, 0, 0, 1, 0]
sage: sdes.decrypt(C, K)
[0, 0, 0, 1, 0, 1, 0, 1]
```

We can also work with strings of bits:

```
sage: C = "01010101"
sage: K = "1010000010"
sage: sdes.decrypt(sdes.string_to_list(C), sdes.string_to_list(K))
[0, 0, 0, 1, 0, 1, 0, 1]
```

## TESTS:

The ciphertext must be a block of 8 bits:

```
sage: from sage.crypto.block_cipher.sdes import SimplifiedDES
sage: sdes = SimplifiedDES()
sage: sdes.decrypt("C", "K")
Traceback (most recent call last):
...
TypeError: ciphertext must be a list of 8 bits
sage: sdes.decrypt([], "K")
```

```
Traceback (most recent call last):
...
ValueError: ciphertext must be a list of 8 bits
sage: sdes.decrypt([1, 2, 3, 4], "K")
Traceback (most recent call last):
...
ValueError: ciphertext must be a list of 8 bits
```

The key must be a block of 10 bits:

```
sage: sdes.decrypt([1, 0, 1, 0, 1, 1, 0, 1], "K")
Traceback (most recent call last):
...
TypeError: the key must be a list of 10 bits
sage: sdes.decrypt([1, 0, 1, 0, 1, 1, 0, 1], [])
Traceback (most recent call last):
...
TypeError: the key must be a list of 10 bits
sage: sdes.decrypt([1, 0, 1, 0, 1, 1, 0, 1], [1, 2, 3, 4, 5])
Traceback (most recent call last):
...
TypeError: the key must be a list of 10 bits
```

The value of each element of C or K must be either 0 or 1:

```
sage: C = [1, 2, 3, 4, 5, 6, 7, 8]
sage: K = [11, 12, 13, 14, 15, 16, 17, 18, 19, 20]
sage: sdes.decrypt(C, K)
Traceback (most recent call last):
...
TypeError: Argument x (= 2) is not a valid string.
sage: C = [0, 1, 0, 0, 1, 1, 1, 0]
sage: K = [11, 12, 13, 14, 15, 16, 17, 18, 19, 20]
sage: sdes.decrypt(C, K)
Traceback (most recent call last):
...
TypeError: Argument x (= 13) is not a valid string.
```

### **encrypt** ( $P, K$ )

Return an 8-bit ciphertext corresponding to the plaintext  $P$ , using S-DES encryption with key  $K$ . The encryption process of S-DES is as follows. Let  $P$  be the initial permutation function,  $P^{-1}$  the corresponding inverse permutation,  $\Pi_F$  the permutation/substitution function, and  $\sigma$  the switch function. The plaintext block  $P$  first goes through  $P$ , the output of which goes through  $\Pi_F$  using the first subkey. Then we apply the switch function to the output of the last function, and the result is then fed into  $\Pi_F$  using the second subkey. Finally, run the output through  $P^{-1}$  to get the ciphertext.

INPUT:

- $P$  – an 8-bit plaintext; a block of 8 bits
- $K$  – a 10-bit key; a block of 10 bits

OUTPUT:

The 8-bit ciphertext corresponding to  $P$ , obtained using the key  $K$ .

EXAMPLES:

Encrypt an 8-bit plaintext block:

```
sage: from sage.crypto.block_cipher.sdes import SimplifiedDES
sage: sdes = SimplifiedDES()
sage: P = [0, 1, 0, 1, 0, 1, 0, 1]
sage: K = [1, 0, 1, 0, 0, 0, 0, 0, 1, 0]
sage: sdes.encrypt(P, K)
[1, 1, 0, 0, 0, 0, 0, 1]
```

We can also work with strings of bits:

```
sage: P = "01010101"
sage: K = "1010000010"
sage: sdes.encrypt(sdes.string_to_list(P), sdes.string_to_list(K))
[1, 1, 0, 0, 0, 0, 0, 1]
```

#### TESTS:

The plaintext must be a block of 8 bits:

```
sage: from sage.crypto.block_cipher.sdes import SimplifiedDES
sage: sdes = SimplifiedDES()
sage: sdes.encrypt("P", "K")
Traceback (most recent call last):
...
TypeError: plaintext must be a list of 8 bits
sage: sdes.encrypt([], "K")
Traceback (most recent call last):
...
ValueError: plaintext must be a list of 8 bits
sage: sdes.encrypt([1, 2, 3, 4], "K")
Traceback (most recent call last):
...
ValueError: plaintext must be a list of 8 bits
```

The key must be a block of 10 bits:

```
sage: sdes.encrypt([1, 0, 1, 0, 1, 1, 0, 1], "K")
Traceback (most recent call last):
...
TypeError: the key must be a list of 10 bits
sage: sdes.encrypt([1, 0, 1, 0, 1, 1, 0, 1], [])
Traceback (most recent call last):
...
TypeError: the key must be a list of 10 bits
sage: sdes.encrypt([1, 0, 1, 0, 1, 1, 0, 1], [1, 2, 3, 4, 5])
Traceback (most recent call last):
...
TypeError: the key must be a list of 10 bits
```

The value of each element of P or K must be either 0 or 1:

```
sage: P = [1, 2, 3, 4, 5, 6, 7, 8]
sage: K = [11, 12, 13, 14, 15, 16, 17, 18, 19, 20]
sage: sdes.encrypt(P, K)
Traceback (most recent call last):
...
TypeError: Argument x (= 2) is not a valid string.
sage: P = [0, 1, 0, 0, 1, 1, 1, 0]
sage: K = [11, 12, 13, 14, 15, 16, 17, 18, 19, 20]
sage: sdes.encrypt(P, K)
Traceback (most recent call last):
```



```
...
TypeError: Argument x (= 13) is not a valid string.
```

**initial\_permutation** (*B*, *inverse=False*)

Return the initial permutation of *B*. Denote the initial permutation function by  $P$  and let  $(b_0, b_1, b_2, \dots, b_7)$  be a vector of 8 bits, where each  $b_i \in \{0, 1\}$ . Then

$$P(b_0, b_1, b_2, b_3, b_4, b_5, b_6, b_7) = (b_1, b_5, b_2, b_0, b_3, b_7, b_4, b_6)$$

The inverse permutation is  $P^{-1}$ :

$$P^{-1}(b_0, b_1, b_2, b_3, b_4, b_5, b_6, b_7) = (b_3, b_0, b_2, b_4, b_6, b_1, b_7, b_5)$$

INPUT:

- *B* – list; a block of 8 bits
- *inverse* – (default: *False*) if *True* then use the inverse permutation  $P^{-1}$ ; if *False* then use the initial permutation  $P$

OUTPUT:

The initial permutation of *B* if *inverse=False*, or the inverse permutation of *B* if *inverse=True*.

EXAMPLES:

The initial permutation of a list of 8 bits:

```
sage: from sage.crypto.block_cipher.sdes import SimplifiedDES
sage: sdes = SimplifiedDES()
sage: B = [1, 0, 1, 1, 0, 1, 0, 0]
sage: P = sdes.initial_permutation(B); P
[0, 1, 1, 1, 0, 0, 0]
```

Recovering the original list of 8 bits from the permutation:

```
sage: Pinv = sdes.initial_permutation(P, inverse=True)
sage: Pinv; B
[1, 0, 1, 1, 0, 1, 0, 0]
[1, 0, 1, 1, 0, 1, 0, 0]
```

We can also work with a string of bits:

```
sage: S = "10110100"
sage: L = sdes.string_to_list(S)
sage: P = sdes.initial_permutation(L); P
[0, 1, 1, 1, 0, 0, 0]
sage: sdes.initial_permutation(sdes.string_to_list("01111000"), inverse=True)
[1, 0, 1, 1, 0, 1, 0, 0]
```

TESTS:

The input block must be a list:

```
sage: from sage.crypto.block_cipher.sdes import SimplifiedDES
sage: sdes = SimplifiedDES()
sage: sdes.initial_permutation("B")
Traceback (most recent call last):
...
TypeError: input block must be a list of 8 bits
sage: sdes.initial_permutation(())
Traceback (most recent call last):
```

```
...
TypeError: input block must be a list of 8 bits
```

The input block must be a list of 8 bits:

```
sage: sdes.initial_permutation([])
Traceback (most recent call last):
...
ValueError: input block must be a list of 8 bits
sage: sdes.initial_permutation([1, 2, 3, 4, 5, 6, 7, 8, 9])
Traceback (most recent call last):
...
ValueError: input block must be a list of 8 bits
```

The value of each element of the list must be either 0 or 1:

```
sage: sdes.initial_permutation([1, 2, 3, 4, 5, 6, 7, 8])
Traceback (most recent call last):
...
TypeError: Argument x (= 2) is not a valid string.
```

#### **left\_shift** ( $B, n=1$ )

Return a circular left shift of  $B$  by  $n$  positions. Let  $B = (b_0, b_1, b_2, b_3, b_4, b_5, b_6, b_7, b_8, b_9)$  be a vector of 10 bits. Then the left shift operation  $L_n$  is performed on the first 5 bits and the last 5 bits of  $B$  separately. That is, if the number of shift positions is  $n=1$ , then  $L_1$  is defined as

$$L_1(b_0, b_1, b_2, b_3, b_4, b_5, b_6, b_7, b_8, b_9) = (b_1, b_2, b_3, b_4, b_0, b_6, b_7, b_8, b_9, b_5)$$

If the number of shift positions is  $n=2$ , then  $L_2$  is given by

$$L_2(b_0, b_1, b_2, b_3, b_4, b_5, b_6, b_7, b_8, b_9) = (b_2, b_3, b_4, b_0, b_1, b_7, b_8, b_9, b_5, b_6)$$

INPUT:

- $B$  – a list of 10 bits
- $n$  – (default: 1) if  $n=1$  then perform left shift by 1 position; if  $n=2$  then perform left shift by 2 positions. The valid values for  $n$  are 1 and 2, since only up to 2 positions are defined for this circular left shift operation.

OUTPUT:

The circular left shift of each half of  $B$ .

EXAMPLES:

Circular left shift by 1 position of a 10-bit string:

```
sage: from sage.crypto.block_cipher.sdes import SimplifiedDES
sage: sdes = SimplifiedDES()
sage: B = [1, 0, 0, 0, 0, 0, 1, 1, 0, 0]
sage: sdes.left_shift(B)
[0, 0, 0, 0, 1, 1, 1, 0, 0, 0]
sage: sdes.left_shift([1, 0, 1, 0, 0, 0, 0, 0, 1, 0])
[0, 1, 0, 0, 1, 0, 0, 1, 0, 0]
```

Circular left shift by 2 positions of a 10-bit string:

```
sage: B = [0, 0, 0, 0, 1, 1, 1, 0, 0, 0]
sage: sdes.left_shift(B, n=2)
[0, 0, 1, 0, 0, 0, 0, 0, 1, 1]
```

Here we work with a string of bits:

```
sage: S = "1000001100"
sage: L = sdes.string_to_list(S)
sage: sdes.left_shift(L)
[0, 0, 0, 0, 1, 1, 1, 0, 0, 0]
sage: sdes.left_shift(sdes.string_to_list("1010000010"), n=2)
[1, 0, 0, 1, 0, 0, 1, 0, 0, 0]
```

TESTS:

The input block must be a list:

```
sage: from sage.crypto.block_cipher.sdes import SimplifiedDES
sage: sdes = SimplifiedDES()
sage: sdes.left_shift("B")
Traceback (most recent call last):
...
TypeError: input block must be a list of 10 bits
sage: sdes.left_shift(())
Traceback (most recent call last):
...
TypeError: input block must be a list of 10 bits
```

The input block must be a list of 10 bits:

```
sage: sdes.left_shift([])
Traceback (most recent call last):
...
ValueError: input block must be a list of 10 bits
sage: sdes.left_shift([1, 2, 3, 4, 5])
Traceback (most recent call last):
...
ValueError: input block must be a list of 10 bits
```

The value of each element of the list must be either 0 or 1:

```
sage: sdes.left_shift([1, 2, 3, 4, 5, 6, 7, 8, 9, 10])
Traceback (most recent call last):
...
TypeError: Argument x (= 2) is not a valid string.
```

The number of shift positions must be either 1 or 2:

```
sage: B = [0, 0, 0, 0, 1, 1, 1, 0, 0, 0]
sage: sdes.left_shift(B, n=-1)
Traceback (most recent call last):
...
ValueError: input n must be either 1 or 2
sage: sdes.left_shift(B, n=3)
Traceback (most recent call last):
...
ValueError: input n must be either 1 or 2
```

### `list_to_string(B)`

Return a binary string representation of the list B.

INPUT:

- B – a non-empty list of bits

OUTPUT:

The binary string representation of B.

EXAMPLES:

A binary string representation of a list of bits:

```
sage: from sage.crypto.block_cipher.sdes import SimplifiedDES
sage: sdes = SimplifiedDES()
sage: L = [0, 0, 0, 0, 1, 1, 0, 1, 0, 0]
sage: sdes.list_to_string(L)
0000110100
```

TESTS:

Input B must be a non-empty list:

```
sage: from sage.crypto.block_cipher.sdes import SimplifiedDES
sage: sdes = SimplifiedDES()
sage: sdes.list_to_string("L")
Traceback (most recent call last):
...
TypeError: input B must be a non-empty list of bits
sage: sdes.list_to_string([])
Traceback (most recent call last):
...
ValueError: input B must be a non-empty list of bits
```

Input must be a non-empty list of bits:

```
sage: sdes.list_to_string([0, 1, 2])
Traceback (most recent call last):
...
IndexError: tuple index out of range
```

#### **permutation10(B)**

Return a permutation of a 10-bit string. This permutation is called  $P_{10}$  and is specified as follows. Let  $(b_0, b_1, b_2, b_3, b_4, b_5, b_6, b_7, b_8, b_9)$  be a vector of 10 bits where each  $b_i \in \{0, 1\}$ . Then  $P_{10}$  is given by

$$P_{10}(b_0, b_1, b_2, b_3, b_4, b_5, b_6, b_7, b_8, b_9) = (b_2, b_4, b_1, b_6, b_3, b_9, b_0, b_8, b_7, b_5)$$

INPUT:

- B – a block of 10-bit string

OUTPUT:

A permutation of B.

EXAMPLES:

Permute a 10-bit string:

```
sage: from sage.crypto.block_cipher.sdes import SimplifiedDES
sage: sdes = SimplifiedDES()
sage: B = [1, 1, 0, 0, 1, 0, 0, 1, 0, 1]
sage: sdes.permutation10(B)
[0, 1, 1, 0, 0, 1, 1, 0, 1, 0]
sage: sdes.permutation10([0, 1, 1, 0, 1, 0, 0, 1, 0, 1])
[1, 1, 1, 0, 0, 1, 0, 0, 1, 0]
sage: sdes.permutation10([1, 0, 1, 0, 0, 0, 0, 0, 1, 0])
[1, 0, 0, 0, 0, 0, 1, 1, 0, 0]
```

Here we work with a string of bits:

```

sage: S = "1100100101"
sage: L = sdes.string_to_list(S)
sage: sdes.permutation10(L)
[0, 1, 1, 0, 0, 1, 1, 0, 1, 0]
sage: sdes.permutation10(sdes.string_to_list("0110100101"))
[1, 1, 1, 0, 0, 1, 0, 0, 1, 0]

```

**TESTS:**

The input block must be a list:

```

sage: from sage.crypto.block_cipher.sdes import SimplifiedDES
sage: sdes = SimplifiedDES()
sage: sdes.permutation10("B")
Traceback (most recent call last):
...
TypeError: input block must be a list of 10 bits
sage: sdes.permutation10(())
Traceback (most recent call last):
...
TypeError: input block must be a list of 10 bits

```

The input block must be a list of 10 bits:

```

sage: sdes.permutation10([])
Traceback (most recent call last):
...
ValueError: input block must be a list of 10 bits
sage: sdes.permutation10([1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11])
Traceback (most recent call last):
...
ValueError: input block must be a list of 10 bits

```

The value of each element of the list must be either 0 or 1:

```

sage: sdes.permutation10([1, 2, 3, 4, 5, 6, 7, 8, 9, 10])
Traceback (most recent call last):
...
TypeError: Argument x (= 3) is not a valid string.

```

**permutation4(B)**

Return a permutation of a 4-bit string. This permutation is called  $P_4$  and is specified as follows. Let  $(b_0, b_1, b_2, b_3)$  be a vector of 4 bits where each  $b_i \in \{0, 1\}$ . Then  $P_4$  is defined by

$$P_4(b_0, b_1, b_2, b_3) = (b_1, b_3, b_2, b_0)$$

**INPUT:**

- B – a block of 4-bit string

**OUTPUT:**

A permutation of B.

**EXAMPLES:**

Permute a 4-bit string:

```

sage: from sage.crypto.block_cipher.sdes import SimplifiedDES
sage: sdes = SimplifiedDES()
sage: B = [1, 1, 0, 0]
sage: sdes.permutation4(B)

```

```
[1, 0, 0, 1]
sage: sdes.permutation4([0, 1, 0, 1])
[1, 1, 0, 0]
```

We can also work with a string of bits:

```
sage: S = "1100"
sage: L = sdes.string_to_list(S)
sage: sdes.permutation4(L)
[1, 0, 0, 1]
sage: sdes.permutation4(sdes.string_to_list("0101"))
[1, 1, 0, 0]
```

#### TESTS:

The input block must be a list:

```
sage: from sage.crypto.block_cipher.sdes import SimplifiedDES
sage: sdes = SimplifiedDES()
sage: sdes.permutation4("B")
Traceback (most recent call last):
...
TypeError: input block must be a list of 4 bits
sage: sdes.permutation4(())
Traceback (most recent call last):
...
TypeError: input block must be a list of 4 bits
```

The input block must be a list of 4 bits:

```
sage: sdes.permutation4([])
Traceback (most recent call last):
...
ValueError: input block must be a list of 4 bits
sage: sdes.permutation4([1, 2, 3, 4, 5])
Traceback (most recent call last):
...
ValueError: input block must be a list of 4 bits
```

The value of each element of the list must be either 0 or 1:

```
sage: sdes.permutation4([1, 2, 3, 4])
Traceback (most recent call last):
...
TypeError: Argument x (= 2) is not a valid string.
```

#### **permutation8**(*B*)

Return a permutation of an 8-bit string. This permutation is called  $P_8$  and is specified as follows. Let  $(b_0, b_1, b_2, b_3, b_4, b_5, b_6, b_7, b_8, b_9)$  be a vector of 10 bits where each  $b_i \in \{0, 1\}$ . Then  $P_8$  picks out 8 of those 10 bits and permutes those 8 bits:

$$P_8(b_0, b_1, b_2, b_3, b_4, b_5, b_6, b_7, b_8, b_9) = (b_5, b_2, b_6, b_3, b_7, b_4, b_9, b_8)$$

INPUT:

- *B* – a block of 10-bit string

OUTPUT:

Pick out 8 of the 10 bits of *B* and permute those 8 bits.

## EXAMPLES:

Permute a 10-bit string:

```
sage: from sage.crypto.block_cipher.sdes import SimplifiedDES
sage: sdes = SimplifiedDES()
sage: B = [1, 1, 0, 0, 1, 0, 0, 1, 0, 1]
sage: sdes.permutation8(B)
[0, 0, 0, 0, 1, 1, 1, 0]
sage: sdes.permutation8([0, 1, 1, 0, 1, 0, 0, 1, 0, 1])
[0, 1, 0, 0, 1, 1, 1, 0]
sage: sdes.permutation8([0, 0, 0, 0, 1, 1, 1, 0, 0, 0])
[1, 0, 1, 0, 0, 1, 0, 0]
```

We can also work with a string of bits:

```
sage: S = "1100100101"
sage: L = sdes.string_to_list(S)
sage: sdes.permutation8(L)
[0, 0, 0, 0, 1, 1, 1, 0]
sage: sdes.permutation8(sdes.string_to_list("0110100101"))
[0, 1, 0, 0, 1, 1, 1, 0]
```

## TESTS:

The input block must be a list:

```
sage: from sage.crypto.block_cipher.sdes import SimplifiedDES
sage: sdes = SimplifiedDES()
sage: sdes.permutation8("B")
Traceback (most recent call last):
...
TypeError: input block must be a list of 10 bits
sage: sdes.permutation8(())
Traceback (most recent call last):
...
TypeError: input block must be a list of 10 bits
```

The input block must be a list of 10 bits:

```
sage: sdes.permutation8([])
Traceback (most recent call last):
...
ValueError: input block must be a list of 10 bits
sage: sdes.permutation8([1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11])
Traceback (most recent call last):
...
ValueError: input block must be a list of 10 bits
```

The value of each element of the list must be either 0 or 1:

```
sage: sdes.permutation8([1, 2, 3, 4, 5, 6, 7, 8, 9, 10])
Traceback (most recent call last):
...
TypeError: Argument x (= 6) is not a valid string.
```

**permute\_substitute** (*B*, *key*)

Apply the function  $\Pi_F$  on the block *B* using subkey *key*. Let  $(b_0, b_1, b_2, b_3, b_4, b_5, b_6, b_7)$  be a vector of 8 bits where each  $b_i \in \{0, 1\}$ , let *L* and *R* be the leftmost 4 bits and rightmost 4 bits of *B* respectively, and

let  $F$  be a function mapping 4-bit strings to 4-bit strings. Then

$$\Pi_F(L, R) = (L \oplus F(R, S), R)$$

where  $S$  is a subkey and  $\oplus$  denotes the bit-wise exclusive-OR function.

The function  $F$  can be described as follows. Its 4-bit input block  $(n_0, n_1, n_2, n_3)$  is first expanded into an 8-bit block to become  $(n_3, n_0, n_1, n_2, n_1, n_2, n_3, n_0)$ . This is usually represented as follows

$$\begin{array}{c|cc|c} n_3 & n_0 & n_1 & n_2 \\ n_1 & n_2 & n_3 & n_0 \end{array}$$

Let  $K = (k_0, k_1, k_2, k_3, k_4, k_5, k_6, k_7)$  be an 8-bit subkey. Then  $K$  is added to the above expanded input block using exclusive-OR to produce

$$\begin{array}{c|cc|c} n_3 + k_0 & n_0 + k_1 & n_1 + k_2 & n_2 + k_3 \\ n_1 + k_4 & n_2 + k_5 & n_3 + k_6 & n_0 + k_7 \end{array} = \begin{array}{c|cc|c} p_{0,0} & p_{0,1} & p_{0,2} & p_{0,3} \\ p_{1,0} & p_{1,1} & p_{1,2} & p_{1,3} \end{array}$$

Now read the first row as the 4-bit string  $p_{0,0}p_{0,3}p_{0,1}p_{0,2}$  and input this 4-bit string through S-box  $S_0$  to get a 2-bit output.

$S_0 =$	Input	Output	Input	Output
	0000	01	1000	00
	0001	00	1001	10
	0010	11	1010	01
	0011	10	1011	11
	0100	11	1100	11
	0101	10	1101	01
	0110	01	1110	11
	0111	00	1111	10

Next read the second row as the 4-bit string  $p_{1,0}p_{1,3}p_{1,1}p_{1,2}$  and input this 4-bit string through S-box  $S_1$  to get another 2-bit output.

$S_1 =$	Input	Output	Input	Output
	0000	00	1000	11
	0001	01	1001	00
	0010	10	1010	01
	0011	11	1011	00
	0100	10	1100	10
	0101	00	1101	01
	0110	01	1110	00
	0111	11	1111	11

Denote the 4 bits produced by  $S_0$  and  $S_1$  as  $b_0b_1b_2b_3$ . This 4-bit string undergoes another permutation called  $P_4$  as follows:

$$P_4(b_0, b_1, b_2, b_3) = (b_1, b_3, b_2, b_0)$$

The output of  $P_4$  is the output of the function  $F$ .

INPUT:

- B – a list of 8 bits
- key – an 8-bit subkey

OUTPUT:

The result of applying the function  $\Pi_F$  to B.

EXAMPLES:

Applying the function  $\Pi_F$  to an 8-bit block and an 8-bit subkey:



```

sage: from sage.crypto.block_cipher.sdes import SimplifiedDES
sage: sdes = SimplifiedDES()
sage: B = [1, 0, 1, 1, 1, 1, 0, 1]
sage: K = [1, 1, 0, 1, 0, 1, 0, 1]
sage: sdes.permute_substitute(B, K)
[1, 0, 1, 0, 1, 1, 0, 1]

```

We can also work with strings of bits:

```

sage: B = "10111101"
sage: K = "11010101"
sage: B = sdes.string_to_list(B); K = sdes.string_to_list(K)
sage: sdes.permute_substitute(B, K)
[1, 0, 1, 0, 1, 1, 0, 1]

```

#### TESTS:

The input B must be a block of 8 bits:

```

sage: from sage.crypto.block_cipher.sdes import SimplifiedDES
sage: sdes = SimplifiedDES()
sage: sdes.permute_substitute("B", "K")
Traceback (most recent call last):
...
TypeError: input B must be an 8-bit string
sage: sdes.permute_substitute([], "K")
Traceback (most recent call last):
...
ValueError: input B must be an 8-bit string

```

The input key must be an 8-bit subkey:

```

sage: sdes.permute_substitute([0, 1, 0, 0, 1, 1, 1, 0], "K")
Traceback (most recent call last):
...
TypeError: input key must be an 8-bit subkey
sage: sdes.permute_substitute([0, 1, 0, 0, 1, 1, 1, 0], [])
Traceback (most recent call last):
...
ValueError: input key must be an 8-bit subkey

```

The value of each element of B or key must be either 0 or 1:

```

sage: B = [1, 2, 3, 4, 5, 6, 7, 8]
sage: K = [0, 1, 2, 3, 4, 5, 6, 7]
sage: sdes.permute_substitute(B, K)
Traceback (most recent call last):
...
TypeError: Argument x (= 2) is not a valid string.
sage: B = [0, 1, 0, 0, 1, 1, 1, 0]
sage: K = [1, 2, 3, 4, 5, 6, 7, 8]
sage: sdes.permute_substitute(B, K)
Traceback (most recent call last):
...
TypeError: Argument x (= 2) is not a valid string.

```

**random\_key()**

Return a random 10-bit key.

EXAMPLES:

The size of each key is the same as the block size:

```
sage: from sage.crypto.block_cipher.sdes import SimplifiedDES
sage: sdes = SimplifiedDES()
sage: key = sdes.random_key()
sage: len(key) == sdes.block_length()
True
```

**sbox()**

Return the S-boxes of simplified DES.

EXAMPLES:

```
sage: from sage.crypto.block_cipher.sdes import SimplifiedDES
sage: sdes = SimplifiedDES()
sage: sbox = sdes.sbox()
sage: sbox[0]; sbox[1]
(1, 0, 3, 2, 3, 2, 1, 0, 0, 2, 1, 3, 3, 1, 3, 2)
(0, 1, 2, 3, 2, 0, 1, 3, 3, 0, 1, 0, 2, 1, 0, 3)
```

**string\_to\_list(S)**

Return a list representation of the binary string S.

INPUT:

- S – a string of bits

OUTPUT:

A list representation of the string S.

EXAMPLES:

A list representation of a string of bits:

```
sage: from sage.crypto.block_cipher.sdes import SimplifiedDES
sage: sdes = SimplifiedDES()
sage: S = "0101010110"
sage: sdes.string_to_list(S)
[0, 1, 0, 1, 0, 1, 0, 1, 1, 0]
```

TESTS:

Input must be a non-empty string:

```
sage: from sage.crypto.block_cipher.sdes import SimplifiedDES
sage: sdes = SimplifiedDES()
sage: sdes.string_to_list("")
Traceback (most recent call last):
...
ValueError: input S must be a non-empty string of bits
sage: sdes.string_to_list(1)
Traceback (most recent call last):
...
TypeError: input S must be a non-empty string of bits
```

Input must be a non-empty string of bits:

```
sage: sdes.string_to_list("0123")
Traceback (most recent call last):
...
TypeError: Argument x (= 2) is not a valid string.
```

**subkey** ( $K, n=1$ )

Return the  $n$ -th subkey based on the key  $K$ .

INPUT:

- $K$  – a 10-bit secret key of this Simplified DES
- $n$  – (default: 1) if  $n=1$  then return the first subkey based on  $K$ ; if  $n=2$  then return the second subkey. The valid values for  $n$  are 1 and 2, since only two subkeys are defined for each secret key in Schaefer's S-DES.

OUTPUT:

The  $n$ -th subkey based on the secret key  $K$ .

EXAMPLES:

Obtain the first subkey from a secret key:

```
sage: from sage.crypto.block_cipher.sdes import SimplifiedDES
sage: sdes = SimplifiedDES()
sage: key = [1, 0, 1, 0, 0, 0, 0, 0, 1, 0]
sage: sdes.subkey(key, n=1)
[1, 0, 1, 0, 0, 1, 0, 0]
```

Obtain the second subkey from a secret key:

```
sage: key = [1, 0, 1, 0, 0, 0, 0, 0, 1, 0]
sage: sdes.subkey(key, n=2)
[0, 1, 0, 0, 0, 0, 1, 1]
```

We can also work with strings of bits:

```
sage: K = "1010010010"
sage: L = sdes.string_to_list(K)
sage: sdes.subkey(L, n=1)
[1, 0, 1, 0, 0, 1, 0, 1]
sage: sdes.subkey(sdes.string_to_list("0010010011"), n=2)
[0, 1, 1, 0, 1, 0, 1, 0]
```

TESTS:

Input  $K$  must be a 10-bit key:

```
sage: from sage.crypto.block_cipher.sdes import SimplifiedDES
sage: sdes = SimplifiedDES()
sage: sdes.subkey("K")
Traceback (most recent call last):
...
TypeError: input K must be a 10-bit key
sage: sdes.subkey([])
Traceback (most recent call last):
...
ValueError: input K must be a 10-bit key
```

There are only two subkeys:

```
sage: key = [1, 0, 1, 0, 0, 0, 0, 0, 1, 0]
sage: sdes.subkey(key, n=0)
Traceback (most recent call last):
...
ValueError: input n must be either 1 or 2
sage: sdes.subkey(key, n=3)
```

```
Traceback (most recent call last):
...
ValueError: input n must be either 1 or 2
```

**switch(B)**

Interchange the first 4 bits with the last 4 bits in the list B of 8 bits. Let  $(b_0, b_1, b_2, b_3, b_4, b_5, b_6, b_7)$  be a vector of 8 bits, where each  $b_i \in \{0, 1\}$ . Then the switch function  $\sigma$  is given by

$$\sigma(b_0, b_1, b_2, b_3, b_4, b_5, b_6, b_7) = (b_4, b_5, b_6, b_7, b_0, b_1, b_2, b_3)$$

INPUT:

- B – list; a block of 8 bits

OUTPUT:

A block of the same dimension, but in which the first 4 bits from B has been switched for the last 4 bits in B.

EXAMPLES:

Interchange the first 4 bits with the last 4 bits:

```
sage: from sage.crypto.block_cipher.sdes import SimplifiedDES
sage: sdes = SimplifiedDES()
sage: B = [1, 1, 1, 0, 1, 0, 0, 0]
sage: sdes.switch(B)
[1, 0, 0, 0, 1, 1, 1, 0]
sage: sdes.switch([1, 1, 1, 1, 0, 0, 0, 0])
[0, 0, 0, 0, 1, 1, 1, 1]
```

We can also work with a string of bits:

```
sage: S = "11101000"
sage: L = sdes.string_to_list(S)
sage: sdes.switch(L)
[1, 0, 0, 0, 1, 1, 1, 0]
sage: sdes.switch(sdes.string_to_list("11110000"))
[0, 0, 0, 0, 1, 1, 1, 1]
```

TESTS:

The input block must be a list:

```
sage: from sage.crypto.block_cipher.sdes import SimplifiedDES
sage: sdes = SimplifiedDES()
sage: sdes.switch("B")
Traceback (most recent call last):
...
TypeError: input block must be a list of 8 bits
sage: sdes.switch(())
Traceback (most recent call last):
...
TypeError: input block must be a list of 8 bits
```

The input block must be a list of 8 bits:

```
sage: sdes.switch([])
Traceback (most recent call last):
...
ValueError: input block must be a list of 8 bits
sage: sdes.switch([1, 2, 3, 4, 5, 6, 7, 8, 9])
```

```
Traceback (most recent call last):  
...  
ValueError: input block must be a list of 8 bits
```

The value of each element of the list must be either 0 or 1:

```
sage: sdes.switch([1, 2, 3, 4, 5, 6, 7, 8])  
Traceback (most recent call last):  
...  
TypeError: Argument x (= 5) is not a valid string.
```



# MINI-AES

A simplified variant of the Advanced Encryption Standard (AES). Note that Mini-AES is for educational purposes only. It is a small-scale version of the AES designed to help beginners understand the basic structure of AES.

AUTHORS:

- Minh Van Nguyen (2009-05): initial version

**class** `sage.crypto.block_cipher.miniaes.MiniaES`  
 Bases: `sage.structure.sage_object.SageObject`

This class implements the Mini Advanced Encryption Standard (Mini-AES) described in [P02]. Note that Phan's Mini-AES is for educational purposes only and is not secure for practical purposes. Mini-AES is a version of the AES with all parameters significantly reduced, but at the same time preserving the structure of AES. The goal of Mini-AES is to allow a beginner to understand the structure of AES, thus laying a foundation for a thorough study of AES. Its goal is as a teaching tool and is different from the SR small scale variants of the AES. SR defines a family of parameterizable variants of the AES suitable as a framework for comparing different cryptanalytic techniques that can be brought to bear on the AES.

EXAMPLES:

Encrypt a plaintext:

```
sage: from sage.crypto.block_cipher.miniaes import MiniAES
sage: maes = MiniAES()
sage: K = FiniteField(16, "x")
sage: MS = MatrixSpace(K, 2, 2)
sage: P = MS([K("x^3 + x"), K("x^2 + 1"), K("x^2 + x"), K("x^3 + x^2")]); P

[  x^3 + x    x^2 + 1]
[  x^2 + x  x^3 + x^2]
sage: key = MS([K("x^3 + x^2"), K("x^3 + x"), K("x^3 + x^2 + x"), K("x^2 + x + 1")]); key

[      x^3 + x^2      x^3 + x]
[x^3 + x^2 + x    x^2 + x + 1]
sage: C = maes.encrypt(P, key); C

[      x      x^2 + x]
[x^3 + x^2 + x    x^3 + x]
```

Decrypt the result:

```
sage: plaintext = maes.decrypt(C, key)
sage: plaintext; P

[  x^3 + x    x^2 + 1]
[  x^2 + x  x^3 + x^2]
```

```
[ x^3 + x  x^2 + 1]
[ x^2 + x x^3 + x^2]
sage: plaintext == P
True
```

We can also work directly with binary strings:

```
sage: from sage.crypto.block_cipher.miniaes import MiniAES
sage: maes = MiniAES()
sage: bin = BinaryStrings()
sage: key = bin.encoding("KE"); key
0100101101000101
sage: P = bin.encoding("Encrypt this secret message!"); P
010001010110111001100011011100100111100101110000011101000010000001110100011010000110100101110011
sage: C = maes(P, key, algorithm="encrypt"); C
1000100010100110111100000111100001001100111011010100011101101101010010111011111010110011100111
sage: plaintext = maes(C, key, algorithm="decrypt")
sage: plaintext == P
True
```

Now we work with integers  $n$  such that  $0 \leq n \leq 15$ :

```
sage: from sage.crypto.block_cipher.miniaes import MiniAES
sage: maes = MiniAES()
sage: P = [n for n in xrange(16)]; P
[0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15]
sage: key = [2, 3, 11, 0]; key
[2, 3, 11, 0]
sage: P = maes.integer_to_binary(P); P
0000000100100011010001010110011110001001101010111100110111101111
sage: key = maes.integer_to_binary(key); key
00100011101110000
sage: C = maes(P, key, algorithm="encrypt"); C
1100100000100011111001010101010101011011100111110001000011100001
sage: plaintext = maes(C, key, algorithm="decrypt")
sage: plaintext == P
True
```

Generate some random plaintext and a random secret key. Encrypt the plaintext using that secret key and decrypt the result. Then compare the decrypted plaintext with the original plaintext:

```
sage: from sage.crypto.block_cipher.miniaes import MiniAES
sage: maes = MiniAES()
sage: MS = MatrixSpace(FiniteField(16, "x"), 2, 2)
sage: P = MS.random_element()
sage: key = maes.random_key()
sage: C = maes.encrypt(P, key)
sage: plaintext = maes.decrypt(C, key)
sage: plaintext == P
True
```

## REFERENCES:

### `GF_to_binary(G)`

Return the binary representation of  $G$ . If  $G$  is an element of the finite field  $\mathbb{F}_{2^4}$ , then obtain the binary representation of  $G$ . If  $G$  is a list of elements belonging to  $\mathbb{F}_{2^4}$ , obtain the 4-bit representation of each element of the list, then concatenate the resulting 4-bit strings into a binary string. If  $G$  is a matrix with entries over  $\mathbb{F}_{2^4}$ , convert each matrix entry to its 4-bit representation, then concatenate the 4-bit strings. The concatenation is performed starting from the top-left corner of the matrix, working across left to right,



top to bottom. Each element of  $\mathbf{F}_{2^4}$  can be associated with a unique 4-bit string according to the following table:

4-bit string	$\mathbf{F}_{2^4}$	4-bit string	$\mathbf{F}_{2^4}$
0000	0	1000	$x^3$
0001	1	1001	$x^3 + 1$
0010	$x$	1010	$x^3 + x$
0011	$x + 1$	1011	$x^3 + x + 1$
0100	$x^2$	1100	$x^3 + x^2$
0101	$x^2 + 1$	1101	$x^3 + x^2 + 1$
0110	$x^2 + x$	1110	$x^3 + x^2 + x$
0111	$x^2 + x + 1$	1111	$x^3 + x^2 + x + 1$

INPUT:

- $G$  – an element of  $\mathbf{F}_{2^4}$ , a list of elements of  $\mathbf{F}_{2^4}$ , or a matrix over  $\mathbf{F}_{2^4}$

OUTPUT:

- A binary string representation of  $G$ .

EXAMPLES:

Obtain the binary representation of all elements of  $\mathbf{F}_{2^4}$ :

```
sage: from sage.crypto.block_cipher.miniaes import MiniAES
sage: maes = MiniAES()
sage: K = FiniteField(16, "x")
sage: S = Set(K); len(S) # GF(2^4) has this many elements
16
sage: [maes.GF_to_binary(S[i]) for i in xrange(len(S))]
```

```
[0000,
0001,
0010,
0011,
0100,
0101,
0110,
0111,
1000,
1001,
1010,
1011,
1100,
1101,
1110,
1111]
```

The binary representation of a list of elements belonging to  $\mathbf{F}_{2^4}$ :

```
sage: from sage.crypto.block_cipher.miniaes import MiniAES
sage: maes = MiniAES()
sage: K = FiniteField(16, "x")
sage: G = [K("x^2 + x + 1"), K("x^3 + x^2"), K("x"), K("x^3 + x + 1"), K("x^3 + x^2 + x + 1")]
sage: maes.GF_to_binary(G)
01111100001010111111011000010111
```

The binary representation of a matrix over  $\mathbf{F}_{2^4}$ :

```

sage: from sage.crypto.block_cipher.miniaes import MiniAES
sage: maes = MiniAES()
sage: K = FiniteField(16, "x")
sage: MS = MatrixSpace(K, 2, 2)
sage: G = MS([K("x^3 + x^2"), K("x + 1"), K("x^2 + x + 1"), K("x^3 + x^2 + x")]); G

[      x^3 + x^2      x + 1]
[ x^2 + x + 1 x^3 + x^2 + x]
sage: maes.GF_to_binary(G)
1100001101111110
sage: MS = MatrixSpace(K, 2, 4)
sage: G = MS([K("x^2 + x + 1"), K("x^3 + x^2"), K("x"), K("x^3 + x + 1"), K("x^3 + x^2 + x + 1")]); G

[      x^2 + x + 1      x^3 + x^2      x      x^3 + x + 1]
[x^3 + x^2 + x + 1      x^2 + x      1      x^2 + x + 1]
sage: maes.GF_to_binary(G)
0111110000101011111011000010111

```

**TESTS:**

Input must be an element of  $\mathbf{F}_{2^4}$ :

```

sage: from sage.crypto.block_cipher.miniaes import MiniAES
sage: maes = MiniAES()
sage: K = FiniteField(8, "x")
sage: G = K.random_element()
sage: maes.GF_to_binary(G)
Traceback (most recent call last):
...
TypeError: input G must be an element of GF(16), a list of elements of GF(16), or a matrix over GF(16)

```

A list of elements belonging to  $\mathbf{F}_{2^4}$ :

```

sage: maes.GF_to_binary([])
Traceback (most recent call last):
...
ValueError: input G must be an element of GF(16), a list of elements of GF(16), or a matrix over GF(16)
sage: G = [K.random_element() for i in xrange(5)]
sage: maes.GF_to_binary(G)
Traceback (most recent call last):
...
KeyError:...

```

A matrix over  $\mathbf{F}_{2^4}$ :

```

sage: MS = MatrixSpace(FiniteField(7, "x"), 4, 5)
sage: maes.GF_to_binary(MS.random_element())
Traceback (most recent call last):
...
TypeError: input G must be an element of GF(16), a list of elements of GF(16), or a matrix over GF(16)

```

**`GF_to_integer(G)`**

Return the integer representation of the finite field element  $G$ . If  $G$  is an element of the finite field  $\mathbf{F}_{2^4}$ , then obtain the integer representation of  $G$ . If  $G$  is a list of elements belonging to  $\mathbf{F}_{2^4}$ , obtain the integer representation of each element of the list, and return the result as a list of integers. If  $G$  is a matrix with entries over  $\mathbf{F}_{2^4}$ , convert each matrix entry to its integer representation, and return the result as a list of integers. The resulting list is obtained by starting from the top-left corner of the matrix, working across left to right, top to bottom. Each element of  $\mathbf{F}_{2^4}$  can be associated with a unique integer according to the

following table:

integer	$\mathbf{F}_{2^4}$	integer	$\mathbf{F}_{2^4}$
0	0	8	$x^3$
1	1	9	$x^3 + 1$
2	$x$	10	$x^3 + x$
3	$x + 1$	11	$x^3 + x + 1$
4	$x^2$	12	$x^3 + x^2$
5	$x^2 + 1$	13	$x^3 + x^2 + 1$
6	$x^2 + x$	14	$x^3 + x^2 + x$
7	$x^2 + x + 1$	15	$x^3 + x^2 + x + 1$

INPUT:

- $G$  – an element of  $\mathbf{F}_{2^4}$ , a list of elements belonging to  $\mathbf{F}_{2^4}$ , or a matrix over  $\mathbf{F}_{2^4}$

OUTPUT:

- The integer representation of  $G$ .

EXAMPLES:

Obtain the integer representation of all elements of  $\mathbf{F}_{2^4}$ :

```
sage: from sage.crypto.block_cipher.miniaes import MiniAES
sage: maes = MiniAES()
sage: K = FiniteField(16, "x")
sage: S = Set(K); len(S) # GF(2^4) has this many elements
16
sage: [maes.GF_to_integer(S[i]) for i in xrange(len(S))]
[0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15]
```

The integer representation of a list of elements belonging to  $\mathbf{F}_{2^4}$ :

```
sage: from sage.crypto.block_cipher.miniaes import MiniAES
sage: maes = MiniAES()
sage: K = FiniteField(16, "x")
sage: G = [K("x^2 + x + 1"), K("x^3 + x^2"), K("x"), K("x^3 + x + 1"), K("x^3 + x^2 + x + 1")]
sage: maes.GF_to_integer(G)
[7, 12, 2, 11, 15, 6, 1, 7]
```

The integer representation of a matrix over  $\mathbf{F}_{2^4}$ :

```
sage: from sage.crypto.block_cipher.miniaes import MiniAES
sage: maes = MiniAES()
sage: K = FiniteField(16, "x")
sage: MS = MatrixSpace(K, 2, 2)
sage: G = MS([K("x^3 + x^2"), K("x + 1"), K("x^2 + x + 1"), K("x^3 + x^2 + x")]); G

[      x^3 + x^2      x + 1]
[ x^2 + x + 1 x^3 + x^2 + x]
sage: maes.GF_to_integer(G)
[12, 3, 7, 14]
sage: MS = MatrixSpace(K, 2, 4)
sage: G = MS([K("x^2 + x + 1"), K("x^3 + x^2"), K("x"), K("x^3 + x + 1"), K("x^3 + x^2 + x + 1")]); G

[      x^2 + x + 1      x^3 + x^2      x      x^3 + x + 1]
[x^3 + x^2 + x + 1      x^2 + x      1      x^2 + x + 1]
sage: maes.GF_to_integer(G)
[7, 12, 2, 11, 15, 6, 1, 7]
```

TESTS:

Input must be an element of  $\mathbf{F}_{2^4}$ :

```
sage: from sage.crypto.block_cipher.miniaes import MiniAES
sage: maes = MiniAES()
sage: K = FiniteField(7, "x")
sage: G = K.random_element()
sage: maes.GF_to_integer(G)
Traceback (most recent call last):
...
TypeError: input G must be an element of GF(16), a list of elements of GF(16), or a matrix over GF(16)
```

A list of elements belonging to  $\mathbf{F}_{2^4}$ :

```
sage: maes.GF_to_integer([])
Traceback (most recent call last):
...
ValueError: input G must be an element of GF(16), a list of elements of GF(16), or a matrix over GF(16)
sage: G = [K.random_element() for i in xrange(5)]
sage: maes.GF_to_integer(G)
Traceback (most recent call last):
...
KeyError:...
```

A matrix over  $\mathbf{F}_{2^4}$ :

```
sage: MS = MatrixSpace(FiniteField(7, "x"), 4, 5)
sage: maes.GF_to_integer(MS.random_element())
Traceback (most recent call last):
...
TypeError: input G must be an element of GF(16), a list of elements of GF(16), or a matrix over GF(16)
```

**add\_key** (*block*, *rkey*)

Return the matrix addition of *block* and *rkey*. Both *block* and *rkey* are  $2 \times 2$  matrices over the finite field  $\mathbf{F}_{2^4}$ . This method just return the matrix addition of these two matrices.

INPUT:

- *block* – a  $2 \times 2$  matrix with entries over  $\mathbf{F}_{2^4}$
- *rkey* – a round key; a  $2 \times 2$  matrix with entries over  $\mathbf{F}_{2^4}$

OUTPUT:

- The matrix addition of *block* and *rkey*.

EXAMPLES:

We can work with elements of  $\mathbf{F}_{2^4}$ :

```
sage: from sage.crypto.block_cipher.miniaes import MiniAES
sage: maes = MiniAES()
sage: K = FiniteField(16, "x")
sage: MS = MatrixSpace(K, 2, 2)
sage: D = MS([ [K("x^3 + x^2 + x + 1"), K("x^3 + x")], [K("0"), K("x^3 + x^2")] ]); D

[x^3 + x^2 + x + 1      x^3 + x]
[      0      x^3 + x^2]
sage: k = MS([ [K("x^2 + 1"), K("x^3 + x^2 + x + 1")], [K("x + 1"), K("0")] ]); k

[      x^2 + 1  x^3 + x^2 + x + 1]
[      x + 1      0]
```

```
sage: maes.add_key(D, k)
```

```
[ x^3 + x    x^2 + 1]
[   x + 1 x^3 + x^2]
```

Or work with binary strings:

```
sage: bin = BinaryStrings()
```

```
sage: B = bin.encoding("We"); B
```

```
0101011101100101
```

```
sage: B = MS(maes.binary_to_GF(B)); B
```

```
[   x^2 + 1 x^2 + x + 1]
```

```
[   x^2 + x    x^2 + 1]
```

```
sage: key = bin.encoding("KY"); key
```

```
0100101101011001
```

```
sage: key = MS(maes.binary_to_GF(key)); key
```

```
[   x^2 x^3 + x + 1]
```

```
[ x^2 + 1    x^3 + 1]
```

```
sage: maes.add_key(B, key)
```

```
[   1 x^3 + x^2]
```

```
[ x + 1 x^3 + x^2]
```

We can also work with integers  $n$  such that  $0 \leq n \leq 15$ :

```
sage: N = [2, 3, 5, 7]; N
```

```
[2, 3, 5, 7]
```

```
sage: key = [9, 11, 13, 15]; key
```

```
[9, 11, 13, 15]
```

```
sage: N = MS(maes.integer_to_GF(N)); N
```

```
[   x    x + 1]
```

```
[ x^2 + 1 x^2 + x + 1]
```

```
sage: key = MS(maes.integer_to_GF(key)); key
```

```
[   x^3 + 1    x^3 + x + 1]
```

```
[ x^3 + x^2 + 1 x^3 + x^2 + x + 1]
```

```
sage: maes.add_key(N, key)
```

```
[x^3 + x + 1    x^3]
```

```
[   x^3    x^3]
```

TESTS:

The input block and key must each be a matrix:

```
sage: from sage.crypto.block_cipher.miniaes import MiniAES
```

```
sage: maes = MiniAES()
```

```
sage: K = FiniteField(16, "x")
```

```
sage: MSB = MatrixSpace(K, 2, 2)
```

```
sage: B = MSB([ [K("x^3 + 1"), K("x^2 + x")], [K("x^3 + x^2"), K("x + 1")] ])
```

```
sage: maes.add_key(B, "key")
```

```
Traceback (most recent call last):
```

```
...
```

```
TypeError: round key must be a 2 x 2 matrix over GF(16)
```

```
sage: maes.add_key("block", "key")
```

```
Traceback (most recent call last):
```

```
...
```

```
TypeError: input block must be a 2 x 2 matrix over GF(16)
```

In addition, the dimensions of the input matrices must each be  $2 \times 2$ :

```
sage: MSB = MatrixSpace(K, 1, 2)
sage: B = MSB([ [K("x^3 + 1"), K("x^2 + x")] ]])
sage: maes.add_key(B, "key")
Traceback (most recent call last):
...
TypeError: input block must be a 2 x 2 matrix over GF(16)
sage: MSB = MatrixSpace(K, 2, 2)
sage: B = MSB([ [K("x^3 + 1"), K("x^2 + x")], [K("x^3 + x^2"), K("x + 1")] ]])
sage: MSK = MatrixSpace(K, 1, 2)
sage: key = MSK([ [K("x^3 + x^2"), K("x^3 + x^2 + x + 1")] ]])
sage: maes.add_key(B, key)
Traceback (most recent call last):
...
TypeError: round key must be a 2 x 2 matrix over GF(16)
```

### **binary\_to\_GF**(B)

Return a list of elements of  $\mathbf{F}_{2^4}$  that represents the binary string B. The number of bits in B must be greater than zero and a multiple of 4. Each nibble (or 4-bit string) is uniquely associated with an element of  $\mathbf{F}_{2^4}$  as specified by the following table:

4-bit string	$\mathbf{F}_{2^4}$	4-bit string	$\mathbf{F}_{2^4}$
0000	0	1000	$x^3$
0001	1	1001	$x^3 + 1$
0010	$x$	1010	$x^3 + x$
0011	$x + 1$	1011	$x^3 + x + 1$
0100	$x^2$	1100	$x^3 + x^2$
0101	$x^2 + 1$	1101	$x^3 + x^2 + 1$
0110	$x^2 + x$	1110	$x^3 + x^2 + x$
0111	$x^2 + x + 1$	1111	$x^3 + x^2 + x + 1$

INPUT:

- B – a binary string, where the number of bits is positive and a multiple of 4

OUTPUT:

- A list of elements of the finite field  $\mathbf{F}_{2^4}$  that represent the binary string B.

EXAMPLES:

Obtain all the elements of the finite field  $\mathbf{F}_{2^4}$ :

```
sage: from sage.crypto.block_cipher.miniaes import MiniAES
sage: maes = MiniAES()
sage: bin = BinaryStrings()
sage: B = bin("0000000100100011010001010110011110001001101010111100110111101111")
sage: maes.binary_to_GF(B)
```

```
[0,
1,
x,
x + 1,
x^2,
x^2 + 1,
x^2 + x,
x^2 + x + 1,
```

```

x^3,
x^3 + 1,
x^3 + x,
x^3 + x + 1,
x^3 + x^2,
x^3 + x^2 + 1,
x^3 + x^2 + x,
x^3 + x^2 + x + 1]

```

**TESTS:**

The input B must be a non-empty binary string, where the number of bits is a multiple of 4:

```

sage: from sage.crypto.block_cipher.miniaes import MiniAES
sage: maes = MiniAES()
sage: maes.binary_to_GF("")
Traceback (most recent call last):
...
ValueError: the number of bits in the binary string B must be positive and a multiple of 4
sage: maes.binary_to_GF("101")
Traceback (most recent call last):
...
ValueError: the number of bits in the binary string B must be positive and a multiple of 4

```

**binary\_to\_integer(B)**

Return a list of integers representing the binary string B. The number of bits in B must be greater than zero and a multiple of 4. Each nibble (or 4-bit string) is uniquely associated with an integer as specified by the following table:

4-bit string	integer	4-bit string	integer
0000	0	1000	8
0001	1	1001	9
0010	2	1010	10
0011	3	1011	11
0100	4	1100	12
0101	5	1101	13
0110	6	1110	14
0111	7	1111	15

**INPUT:**

- B – a binary string, where the number of bits is positive and a multiple of 4

**OUTPUT:**

- A list of integers that represent the binary string B.

**EXAMPLES:**

Obtain the integer representation of every 4-bit string:

```

sage: from sage.crypto.block_cipher.miniaes import MiniAES
sage: maes = MiniAES()
sage: bin = BinaryStrings()
sage: B = bin("0000000100100011010001010110011110001001101010111100110111101111")
sage: maes.binary_to_integer(B)
[0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15]

```

**TESTS:**

The input B must be a non-empty binary string, where the number of bits is a multiple of 4:

```

sage: from sage.crypto.block_cipher.miniaes import MiniAES
sage: maes = MiniAES()
sage: maes.binary_to_integer("")
Traceback (most recent call last):
...
ValueError: the number of bits in the binary string B must be positive and a multiple of 4
sage: maes.binary_to_integer("101")
Traceback (most recent call last):
...
ValueError: the number of bits in the binary string B must be positive and a multiple of 4

```

**block\_length()**

Return the block length of Phan's Mini-AES block cipher. A key in Phan's Mini-AES is a block of 16 bits. Each nibble of a key can be considered as an element of the finite field  $\mathbf{F}_{2^4}$ . Therefore the key consists of four elements from  $\mathbf{F}_{2^4}$ .

OUTPUT:

- The block (or key) length in number of bits.

EXAMPLES:

```

sage: from sage.crypto.block_cipher.miniaes import MiniAES
sage: maes = MiniAES()
sage: maes.block_length()
16

```

**decrypt(C, key)**

Use Phan's Mini-AES to decrypt the ciphertext  $C$  with the secret key  $key$ . Both  $C$  and  $key$  must be  $2 \times 2$  matrices over the finite field  $\mathbf{F}_{2^4}$ . Let  $\gamma$  denote the operation of nibble-sub,  $\pi$  denote shift-row,  $\theta$  denote mix-column, and  $\sigma_{K_i}$  denote add-key with the round key  $K_i$ . Then decryption  $D$  using Phan's Mini-AES is the function composition

$$D = \sigma_{K_0} \circ \gamma^{-1} \circ \pi \circ \theta \circ \sigma_{K_1} \circ \gamma^{-1} \circ \pi \circ \sigma_{K_2}$$

where  $\gamma^{-1}$  is the nibble-sub operation that uses the S-box for decryption, and the order of execution is from right to left.

INPUT:

- $C$  – a ciphertext block; must be a  $2 \times 2$  matrix over the finite field  $\mathbf{F}_{2^4}$
- $key$  – a secret key for this Mini-AES block cipher; must be a  $2 \times 2$  matrix over the finite field  $\mathbf{F}_{2^4}$

OUTPUT:

- The plaintext corresponding to  $C$ .

EXAMPLES:

We encrypt a plaintext, decrypt the ciphertext, then compare the decrypted plaintext with the original plaintext. Here we work with elements of  $\mathbf{F}_{2^4}$ :

```

sage: from sage.crypto.block_cipher.miniaes import MiniAES
sage: maes = MiniAES()
sage: K = FiniteField(16, "x")
sage: MS = MatrixSpace(K, 2, 2)
sage: P = MS([ [K("x^3 + 1"), K("x^2 + x")], [K("x^3 + x^2"), K("x + 1")] ]); P

[ x^3 + 1  x^2 + x]
[x^3 + x^2  x + 1]
sage: key = MS([ [K("x^3 + x^2"), K("x^3 + x^2 + x + 1")], [K("x + 1"), K("0")] ]); key

```



```

[      x^3 + x^2 x^3 + x^2 + x + 1]
[      x + 1      0]
sage: C = maes.encrypt(P, key); C

[x^2 + x + 1      x^3 + x^2]
[      x      x^2 + x]
sage: plaintext = maes.decrypt(C, key)
sage: plaintext; P

[ x^3 + 1      x^2 + x]
[x^3 + x^2      x + 1]

[ x^3 + 1      x^2 + x]
[x^3 + x^2      x + 1]
sage: plaintext == P
True

```

But we can also work with binary strings:

```

sage: bin = BinaryStrings()
sage: P = bin.encoding("de"); P
0110010001100101
sage: P = MS(maes.binary_to_GF(P)); P

[x^2 + x      x^2]
[x^2 + x x^2 + 1]
sage: key = bin.encoding("ke"); key
0110101101100101
sage: key = MS(maes.binary_to_GF(key)); key

[      x^2 + x x^3 + x + 1]
[      x^2 + x      x^2 + 1]
sage: C = maes.encrypt(P, key)
sage: plaintext = maes.decrypt(C, key)
sage: plaintext == P
True

```

Here we work with integers  $n$  such that  $0 \leq n \leq 15$ :

```

sage: P = [3, 5, 7, 14]; P
[3, 5, 7, 14]
sage: key = [2, 6, 7, 8]; key
[2, 6, 7, 8]
sage: P = MS(maes.integer_to_GF(P)); P

[      x + 1      x^2 + 1]
[ x^2 + x + 1 x^3 + x^2 + x]
sage: key = MS(maes.integer_to_GF(key)); key

[      x      x^2 + x]
[x^2 + x + 1      x^3]
sage: C = maes.encrypt(P, key)
sage: plaintext = maes.decrypt(C, key)
sage: plaintext == P
True

```

TESTS:

The input block must be a matrix:

```

sage: from sage.crypto.block_cipher.miniaes import MiniAES
sage: maes = MiniAES()
sage: K = FiniteField(16, "x")
sage: MS = MatrixSpace(K, 2, 2)
sage: key = MS([ [K("x^3 + x^2"), K("x^3 + x^2 + x + 1")], [K("x + 1"), K("0")] ])
sage: maes.decrypt("C", key)
Traceback (most recent call last):
...
TypeError: ciphertext block must be a 2 x 2 matrix over GF(16)
sage: C = MS([ [K("x^3 + 1"), K("x^2 + x")], [K("x^3 + x^2"), K("x + 1")] ])
sage: maes.decrypt(C, "key")
Traceback (most recent call last):
...
TypeError: secret key must be a 2 x 2 matrix over GF(16)

```

In addition, the dimensions of the input matrices must be  $2 \times 2$ :

```

sage: MS = MatrixSpace(K, 1, 2)
sage: C = MS([ [K("x^3 + 1"), K("x^2 + x")] ])
sage: maes.decrypt(C, "key")
Traceback (most recent call last):
...
TypeError: ciphertext block must be a 2 x 2 matrix over GF(16)
sage: MSC = MatrixSpace(K, 2, 2)
sage: C = MSC([ [K("x^3 + 1"), K("x^2 + x")], [K("x^3 + x^2"), K("x + 1")] ])
sage: MSK = MatrixSpace(K, 1, 2)
sage: key = MSK([ [K("x^3 + x^2"), K("x^3 + x^2 + x + 1")] ])
sage: maes.decrypt(C, key)
Traceback (most recent call last):
...
TypeError: secret key must be a 2 x 2 matrix over GF(16)

```

#### **encrypt** ( $P, key$ )

Use Phan's Mini-AES to encrypt the plaintext  $P$  with the secret key  $key$ . Both  $P$  and  $key$  must be  $2 \times 2$  matrices over the finite field  $\mathbb{F}_{2^4}$ . Let  $\gamma$  denote the operation of nibble-sub,  $\pi$  denote shift-row,  $\theta$  denote mix-column, and  $\sigma_{K_i}$  denote add-key with the round key  $K_i$ . Then encryption  $E$  using Phan's Mini-AES is the function composition

$$E = \sigma_{K_2} \circ \pi \circ \gamma \circ \sigma_{K_1} \circ \theta \circ \pi \circ \gamma \circ \sigma_{K_0}$$

where the order of execution is from right to left. Note that  $\gamma$  is the nibble-sub operation that uses the S-box for encryption.

INPUT:

- $P$  – a plaintext block; must be a  $2 \times 2$  matrix over the finite field  $\mathbb{F}_{2^4}$
- $key$  – a secret key for this Mini-AES block cipher; must be a  $2 \times 2$  matrix over the finite field  $\mathbb{F}_{2^4}$

OUTPUT:

- The ciphertext corresponding to  $P$ .

EXAMPLES:

Here we work with elements of  $\mathbb{F}_{2^4}$ :

```

sage: from sage.crypto.block_cipher.miniaes import MiniAES
sage: maes = MiniAES()
sage: K = FiniteField(16, "x")
sage: MS = MatrixSpace(K, 2, 2)

```

```

sage: P = MS([ [K("x^3 + 1"), K("x^2 + x")], [K("x^3 + x^2"), K("x + 1")] ]); P

[  x^3 + 1   x^2 + x]
[x^3 + x^2   x + 1]
sage: key = MS([ [K("x^3 + x^2"), K("x^3 + x^2 + x + 1")], [K("x + 1"), K("0")] ]); key

[      x^3 + x^2 x^3 + x^2 + x + 1]
[      x + 1      0]
sage: maes.encrypt(P, key)

[x^2 + x + 1   x^3 + x^2]
[      x      x^2 + x]

```

But we can also work with binary strings:

```

sage: bin = BinaryStrings()
sage: P = bin.encoding("de"); P
0110010001100101
sage: P = MS(maes.binary_to_GF(P)); P

[x^2 + x   x^2]
[x^2 + x x^2 + 1]
sage: key = bin.encoding("ke"); key
0110101101100101
sage: key = MS(maes.binary_to_GF(key)); key

[  x^2 + x x^3 + x + 1]
[  x^2 + x   x^2 + 1]
sage: C = maes.encrypt(P, key)
sage: plaintext = maes.decrypt(C, key)
sage: plaintext == P
True

```

Now we work with integers  $n$  such that  $0 \leq n \leq 15$ :

```

sage: P = [1, 5, 8, 12]; P
[1, 5, 8, 12]
sage: key = [5, 9, 15, 0]; key
[5, 9, 15, 0]
sage: P = MS(maes.integer_to_GF(P)); P

[      1   x^2 + 1]
[      x^3 x^3 + x^2]
sage: key = MS(maes.integer_to_GF(key)); key

[      x^2 + 1      x^3 + 1]
[x^3 + x^2 + x + 1      0]
sage: C = maes.encrypt(P, key)
sage: plaintext = maes.decrypt(C, key)
sage: plaintext == P
True

```

TESTS:

The input block must be a matrix:

```

sage: from sage.crypto.block_cipher.miniaes import MiniAES
sage: maes = MiniAES()
sage: K = FiniteField(16, "x")
sage: MS = MatrixSpace(K, 2, 2)

```

```

sage: key = MS([ [K("x^3 + x^2"), K("x^3 + x^2 + x + 1")], [K("x + 1"), K("0")] ])
sage: maes.encrypt("P", key)
Traceback (most recent call last):
...
TypeError: plaintext block must be a 2 x 2 matrix over GF(16)
sage: P = MS([ [K("x^3 + 1"), K("x^2 + x")], [K("x^3 + x^2"), K("x + 1")] ])
sage: maes.encrypt(P, "key")
Traceback (most recent call last):
...
TypeError: secret key must be a 2 x 2 matrix over GF(16)

```

In addition, the dimensions of the input matrices must be  $2 \times 2$ :

```

sage: MS = MatrixSpace(K, 1, 2)
sage: P = MS([ [K("x^3 + 1"), K("x^2 + x")]])
sage: maes.encrypt(P, "key")
Traceback (most recent call last):
...
TypeError: plaintext block must be a 2 x 2 matrix over GF(16)
sage: MSP = MatrixSpace(K, 2, 2)
sage: P = MSP([ [K("x^3 + 1"), K("x^2 + x")], [K("x^3 + x^2"), K("x + 1")] ])
sage: MSK = MatrixSpace(K, 1, 2)
sage: key = MSK([ [K("x^3 + x^2"), K("x^3 + x^2 + x + 1")] ])
sage: maes.encrypt(P, key)
Traceback (most recent call last):
...
TypeError: secret key must be a 2 x 2 matrix over GF(16)

```

#### `integer_to_GF(N)`

Return the finite field representation of  $N$ . If  $N$  is an integer such that  $0 \leq N \leq 15$ , return the element of  $\mathbf{F}_{2^4}$  that represents  $N$ . If  $N$  is a list of integers each of which is  $\geq 0$  and  $\leq 15$ , then obtain the element of  $\mathbf{F}_{2^4}$  that represents each such integer, and return a list of such finite field representations. Each integer between 0 and 15, inclusive, can be associated with a unique element of  $\mathbf{F}_{2^4}$  according to the following table:

integer	$\mathbf{F}_{2^4}$	integer	$\mathbf{F}_{2^4}$
0	0	8	$x^3$
1	1	9	$x^3 + 1$
2	$x$	10	$x^3 + x$
3	$x + 1$	11	$x^3 + x + 1$
4	$x^2$	12	$x^3 + x^2$
5	$x^2 + 1$	13	$x^3 + x^2 + 1$
6	$x^2 + x$	14	$x^3 + x^2 + x$
7	$x^2 + x + 1$	15	$x^3 + x^2 + x + 1$

INPUT:

- $N$  – a non-negative integer less than or equal to 15, or a list of such integers

OUTPUT:

- Elements of the finite field  $\mathbf{F}_{2^4}$ .

EXAMPLES:

Obtain the element of  $\mathbf{F}_{2^4}$  representing an integer  $n$ , where  $0 \leq n \leq 15$ :

```

sage: from sage.crypto.block_cipher.miniaes import MiniAES
sage: maes = MiniAES()
sage: maes.integer_to_GF(0)

```

```

0
sage: maes.integer_to_GF(2)
x
sage: maes.integer_to_GF(7)
x^2 + x + 1

```

Obtain the finite field elements corresponding to all non-negative integers less than or equal to 15:

```

sage: from sage.crypto.block_cipher.miniaes import MiniAES
sage: maes = MiniAES()
sage: lst = [n for n in xrange(16)]; lst
[0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15]
sage: maes.integer_to_GF(lst)

```

```

[0,
1,
x,
x + 1,
x^2,
x^2 + 1,
x^2 + x,
x^2 + x + 1,
x^3,
x^3 + 1,
x^3 + x,
x^3 + x + 1,
x^3 + x^2,
x^3 + x^2 + 1,
x^3 + x^2 + x,
x^3 + x^2 + x + 1]

```

#### TESTS:

The input  $N$  can be an integer, but it must be such that  $0 \leq N \leq 15$ :

```

sage: from sage.crypto.block_cipher.miniaes import MiniAES
sage: maes = MiniAES()
sage: maes.integer_to_GF(-1)
Traceback (most recent call last):
...
KeyError:...
sage: maes.integer_to_GF(16)
Traceback (most recent call last):
...
KeyError:...
sage: maes.integer_to_GF("2")
Traceback (most recent call last):
...
TypeError: N must be an integer 0 <= N <= 15 or a list of such integers

```

The input  $N$  can be a list of integers, but each integer  $n$  in the list must be bounded such that  $0 \leq n \leq 15$ :

```

sage: maes.integer_to_GF([])
Traceback (most recent call last):
...
ValueError: N must be an integer 0 <= N <= 15 or a list of such integers
sage: maes.integer_to_GF([""])
Traceback (most recent call last):
...
KeyError:...

```

```
sage: maes.integer_to_GF([0, 2, 3, "4"])
Traceback (most recent call last):
...
KeyError:...
sage: maes.integer_to_GF([0, 2, 3, 16])
Traceback (most recent call last):
...
KeyError:...
```

**integer\_to\_binary** (*N*)

Return the binary representation of *N*. If *N* is an integer such that  $0 \leq N \leq 15$ , return the binary representation of *N*. If *N* is a list of integers each of which is  $\geq 0$  and  $\leq 15$ , then obtain the binary representation of each integer, and concatenate the individual binary representations into a single binary string. Each integer between 0 and 15, inclusive, can be associated with a unique 4-bit string according to the following table:

4-bit string	integer	4-bit string	integer
0000	0	1000	8
0001	1	1001	9
0010	2	1010	10
0011	3	1011	11
0100	4	1100	12
0101	5	1101	13
0110	6	1110	14
0111	7	1111	15

INPUT:

- *N* – a non-negative integer less than or equal to 15, or a list of such integers

OUTPUT:

- A binary string representing *N*.

EXAMPLES:

The binary representations of all integers between 0 and 15, inclusive:

```
sage: from sage.crypto.block_cipher.miniaes import MiniAES
sage: maes = MiniAES()
sage: lst = [n for n in xrange(16)]; lst
[0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15]
sage: maes.integer_to_binary(lst)
0000000100100011010001010110011110001001101010111100110111101111
```

The binary representation of an integer between 0 and 15, inclusive:

```
sage: from sage.crypto.block_cipher.miniaes import MiniAES
sage: maes = MiniAES()
sage: maes.integer_to_binary(3)
0011
sage: maes.integer_to_binary(5)
0101
sage: maes.integer_to_binary(7)
0111
```

TESTS:

The input *N* can be an integer, but must be bounded such that  $0 \leq N \leq 15$ :

```

sage: from sage.crypto.block_cipher.miniaes import MiniAES
sage: maes = MiniAES()
sage: maes.integer_to_binary(-1)
Traceback (most recent call last):
...
KeyError:...
sage: maes.integer_to_binary("1")
Traceback (most recent call last):
...
TypeError: N must be an integer 0 <= N <= 15 or a list of such integers
sage: maes.integer_to_binary("")
Traceback (most recent call last):
...
TypeError: N must be an integer 0 <= N <= 15 or a list of such integers

```

The input  $N$  can be a list of integers, but each integer  $n$  of the list must be  $0 \leq n \leq 15$ :

```

sage: from sage.crypto.block_cipher.miniaes import MiniAES
sage: maes = MiniAES()
sage: maes.integer_to_binary([])
Traceback (most recent call last):
...
ValueError: N must be an integer 0 <= N <= 15 or a list of such integers
sage: maes.integer_to_binary([""])
Traceback (most recent call last):
...
KeyError:...
sage: maes.integer_to_binary([0, 1, 2, 16])
Traceback (most recent call last):
...
KeyError:...

```

#### **mix\_column**(*block*)

Return the matrix multiplication of *block* with a constant matrix. The constant matrix is

$$\begin{bmatrix} x+1 & x \\ x & x+1 \end{bmatrix}$$

If the input block is

$$\begin{bmatrix} c_0 & c_2 \\ c_1 & c_3 \end{bmatrix}$$

then the output block is

$$\begin{bmatrix} d_0 & d_2 \\ d_1 & d_3 \end{bmatrix} = \begin{bmatrix} x+1 & x \\ x & x+1 \end{bmatrix} \begin{bmatrix} c_0 & c_2 \\ c_1 & c_3 \end{bmatrix}$$

INPUT:

- *block* – a  $2 \times 2$  matrix with entries over  $\mathbf{F}_{2^4}$

OUTPUT:

- A  $2 \times 2$  matrix resulting from multiplying the above constant matrix with the input matrix *block*.

EXAMPLES:

Here we work with elements of  $\mathbf{F}_{2^4}$ :

```
sage: from sage.crypto.block_cipher.miniaes import MiniAES
sage: maes = MiniAES()
sage: K = FiniteField(16, "x")
sage: MS = MatrixSpace(K, 2, 2)
sage: mat = MS([ [K("x^2 + x + 1"), K("x^3 + x^2 + 1")], [K("x^3"), K("x")] ])
sage: maes.mix_column(mat)

[      x^3 + x      0]
[      x^2 + 1 x^3 + x^2 + x + 1]
```

Multiplying by the identity matrix should leave the constant matrix unchanged:

```
sage: eye = MS([ [K("1"), K("0")], [K("0"), K("1")] ])
sage: maes.mix_column(eye)

[x + 1      x]
[      x x + 1]
```

We can also work with binary strings:

```
sage: bin = BinaryStrings()
sage: B = bin.encoding("rT"); B
0111001001010100
sage: B = MS(maes.binary_to_GF(B)); B

[x^2 + x + 1      x]
[      x^2 + 1      x^2]
sage: maes.mix_column(B)

[      x + 1 x^3 + x^2 + x]
[      1      x^3]
```

We can also work with integers  $n$  such that  $0 \leq n \leq 15$ :

```
sage: P = [10, 5, 2, 7]; P
[10, 5, 2, 7]
sage: P = MS(maes.integer_to_GF(P)); P

[      x^3 + x      x^2 + 1]
[      x x^2 + x + 1]
sage: maes.mix_column(P)

[x^3 + 1      1]
[      1      x + 1]
```

TESTS:

The input block must be a matrix:

```
sage: from sage.crypto.block_cipher.miniaes import MiniAES
sage: maes = MiniAES()
sage: maes.mix_column("mat")
Traceback (most recent call last):
...
TypeError: input block must be a 2 x 2 matrix over GF(16)
```

In addition, the dimensions of the input matrix must be  $2 \times 2$ :

```
sage: K = FiniteField(16, "x")
sage: MS = MatrixSpace(K, 1, 2)
```



```

sage: mat = MS([[K("x^3 + x^2 + x + 1"), K("0")]])
sage: maes.mix_column(mat)
Traceback (most recent call last):
...
TypeError: input block must be a 2 x 2 matrix over GF(16)

```

**nibble\_sub** (*block*, *algorithm*='encrypt')

Substitute a nibble (or a block of 4 bits) using the following S-box:

Input	Output	Input	Output
0000	1110	1000	0011
0001	0100	1001	1010
0010	1101	1010	0110
0011	0001	1011	1100
0100	0010	1100	0101
0101	1111	1101	1001
0110	1011	1110	0000
0111	1000	1111	0111

The values in the above S-box are taken from the first row of the first S-box of the Data Encryption Standard (DES). Each nibble can be thought of as an element of the finite field  $\mathbb{F}_{2^4}$  of 16 elements. Thus in terms of  $\mathbb{F}_{2^4}$ , the S-box can also be specified as:

Input	Output
0	$x^3 + x^2 + x$
1	$x^2$
$x$	$x^3 + x^2 + 1$
$x + 1$	1
$x^2$	$x$
$x^2 + 1$	$x^3 + x^2 + x + 1$
$x^2 + x$	$x^3 + x + 1$
$x^2 + x + 1$	$x^3$
$x^3$	$x + 1$
$x^3 + 1$	$x^3 + x$
$x^3 + x$	$x^2 + x$
$x^3 + x + 1$	$x^3 + x^2$
$x^3 + x^2$	$x^2 + 1$
$x^3 + x^2 + 1$	$x^3 + 1$
$x^3 + x^2 + x$	0
$x^3 + x^2 + x + 1$	$x^2 + x + 1$

Note that the above S-box is used for encryption. The S-box for decryption is obtained from the above S-box by reversing the role of the Input and Output columns. Thus the previous Input column for encryption now becomes the Output column for decryption, and the previous Output column for encryption is now

the Input column for decryption. The S-box used for decryption can be specified as:

Input	Output
0	$x^3 + x^2 + x$
1	$x + 1$
$x$	$x^2$
$x + 1$	$x^3$
$x^2$	1
$x^2 + 1$	$x^3 + x^2$
$x^2 + x$	$x^3 + x$
$x^2 + x + 1$	$x^3 + x^2 + x + 1$
$x^3$	$x^2 + x + 1$
$x^3 + 1$	$x^3 + x^2 + 1$
$x^3 + x$	$x^3 + 1$
$x^3 + x + 1$	$x^2 + x$
$x^3 + x^2$	$x^3 + x + 1$
$x^3 + x^2 + 1$	$x$
$x^3 + x^2 + x$	0
$x^3 + x^2 + x + 1$	$x^2 + 1$

INPUT:

- `block` – a  $2 \times 2$  matrix with entries over  $\mathbf{F}_{2^4}$
- `algorithm` – (default: "encrypt") a string; a flag to signify whether this nibble-sub operation is used for encryption or decryption. The encryption flag is "encrypt" and the decryption flag is "decrypt".

OUTPUT:

- A  $2 \times 2$  matrix resulting from applying an S-box on entries of the  $2 \times 2$  matrix `block`.

EXAMPLES:

Here we work with elements of the finite field  $\mathbf{F}_{2^4}$ :

```
sage: from sage.crypto.block_cipher.miniaes import MiniAES
sage: maes = MiniAES()
sage: K = FiniteField(16, "x")
sage: MS = MatrixSpace(K, 2, 2)
sage: mat = MS([[K("x^3 + x^2 + x + 1"), K("0")], [K("x^2 + x + 1"), K("x^3 + x")]])
sage: maes.nibble_sub(mat, algorithm="encrypt")
```

```
[ x^2 + x + 1 x^3 + x^2 + x]
[          x^3          x^2 + x]
```

But we can also work with binary strings:

```
sage: bin = BinaryStrings()
sage: B = bin.encoding("bi"); B
0110001001101001
sage: B = MS(maes.binary_to_GF(B)); B

[x^2 + x      x]
[x^2 + x x^3 + 1]
sage: maes.nibble_sub(B, algorithm="encrypt")

[ x^3 + x + 1 x^3 + x^2 + 1]
[ x^3 + x + 1      x^3 + x]
sage: maes.nibble_sub(B, algorithm="decrypt")
```

```
[      x^3 + x      x^2]
[      x^3 + x x^3 + x^2 + 1]
```

Here we work with integers  $n$  such that  $0 \leq n \leq 15$ :

```
sage: P = [2, 6, 8, 14]; P
[2, 6, 8, 14]
sage: P = MS(maes.integer_to_GF(P)); P

[      x      x^2 + x]
[      x^3 x^3 + x^2 + x]
sage: maes.nibble_sub(P, algorithm="encrypt")

[x^3 + x^2 + 1  x^3 + x + 1]
[      x + 1      0]
sage: maes.nibble_sub(P, algorithm="decrypt")

[      x^2      x^3 + x]
[x^2 + x + 1      0]
```

#### TESTS:

The input block must be a matrix:

```
sage: from sage.crypto.block_cipher.miniaes import MiniAES
sage: maes = MiniAES()
sage: maes.nibble_sub("mat")
Traceback (most recent call last):
...
TypeError: input block must be a 2 x 2 matrix over GF(16)
```

In addition, the dimensions of the input matrix must be  $2 \times 2$ :

```
sage: K = FiniteField(16, "x")
sage: MS = MatrixSpace(K, 1, 2)
sage: mat = MS([[K("x^3 + x^2 + x + 1"), K("0")]])
sage: maes.nibble_sub(mat)
Traceback (most recent call last):
...
TypeError: input block must be a 2 x 2 matrix over GF(16)
```

The value for the option algorithm must be either the string "encrypt" or "decrypt":

```
sage: K = FiniteField(16, "x")
sage: MS = MatrixSpace(K, 2, 2)
sage: mat = MS([[K("x^3 + x^2 + x + 1"), K("0")], [K("x^2 + x + 1"), K("x^3 + x")]])
sage: maes.nibble_sub(mat, algorithm="abc")
Traceback (most recent call last):
...
ValueError: the algorithm for nibble-sub must be either 'encrypt' or 'decrypt'
sage: maes.nibble_sub(mat, algorithm="e")
Traceback (most recent call last):
...
ValueError: the algorithm for nibble-sub must be either 'encrypt' or 'decrypt'
sage: maes.nibble_sub(mat, algorithm="d")
Traceback (most recent call last):
...
ValueError: the algorithm for nibble-sub must be either 'encrypt' or 'decrypt'
```

`random_key()`

A random key within the key space of this Mini-AES block cipher. Like the AES, Phan's Mini-AES is a symmetric-key block cipher. A Mini-AES key is a block of 16 bits, or a  $2 \times 2$  matrix with entries over the finite field  $\mathbb{F}_{2^4}$ . Thus the number of possible keys is  $2^{16} = 16^4$ .

OUTPUT:

- A  $2 \times 2$  matrix over the finite field  $\mathbb{F}_{2^4}$ , used as a secret key for this Mini-AES block cipher.

EXAMPLES:

Each nibble of a key is an element of the finite field  $\mathbb{F}_{2^4}$ :

```
sage: K = FiniteField(16, "x")
sage: from sage.crypto.block_cipher.miniaes import MiniAES
sage: maes = MiniAES()
sage: key = maes.random_key()
sage: [key[i][j] in K for i in xrange(key.nrows()) for j in xrange(key.ncols())]
[True, True, True, True]
```

Generate a random key, then perform encryption and decryption using that key:

```
sage: from sage.crypto.block_cipher.miniaes import MiniAES
sage: maes = MiniAES()
sage: K = FiniteField(16, "x")
sage: MS = MatrixSpace(K, 2, 2)
sage: key = maes.random_key()
sage: P = MS.random_element()
sage: C = maes.encrypt(P, key)
sage: plaintext = maes.decrypt(C, key)
sage: plaintext == P
True
```

**round\_key** (*key*, *n*)

Return the round key for round *n*. Phan's Mini-AES is defined to have two rounds. The round key  $K_0$  is generated and used prior to the first round, with round keys  $K_1$  and  $K_2$  being used in rounds 1 and 2 respectively. In total, there are three round keys, each generated from the secret key *key*.

INPUT:

- *key* – the secret key
- *n* – non-negative integer; the round number

OUTPUT:

- The *n*-th round key.

EXAMPLES:

Obtaining the round keys from the secret key:

```
sage: from sage.crypto.block_cipher.miniaes import MiniAES
sage: maes = MiniAES()
sage: K = FiniteField(16, "x")
sage: MS = MatrixSpace(K, 2, 2)
sage: key = MS([ [K("x^3 + x^2"), K("x^3 + x^2 + x + 1")], [K("x + 1"), K("0")] ])
sage: maes.round_key(key, 0)

[      x^3 + x^2  x^3 + x^2 + x + 1]
[      x + 1      0]
sage: key

[      x^3 + x^2  x^3 + x^2 + x + 1]
[      x + 1      0]
```

```

sage: maes.round_key(key, 1)

[
      x + 1 x^3 + x^2 + x + 1]
[
      0 x^3 + x^2 + x + 1]
sage: maes.round_key(key, 2)

[x^2 + x x^3 + 1]
[x^2 + x x^2 + x]

```

#### TESTS:

Only two rounds are defined for this AES variant:

```

sage: from sage.crypto.block_cipher.miniaes import MiniAES
sage: maes = MiniAES()
sage: K = FiniteField(16, "x")
sage: MS = MatrixSpace(K, 2, 2)
sage: key = MS([ [K("x^3 + x^2"), K("x^3 + x^2 + x + 1")], [K("x + 1"), K("0")] ])
sage: maes.round_key(key, -1)
Traceback (most recent call last):
...
ValueError: Mini-AES only defines two rounds
sage: maes.round_key(key, 3)
Traceback (most recent call last):
...
ValueError: Mini-AES only defines two rounds

```

The input key must be a matrix:

```

sage: maes.round_key("key", 0)
Traceback (most recent call last):
...
TypeError: secret key must be a 2 x 2 matrix over GF(16)

```

In addition, the dimensions of the key matrix must be  $2 \times 2$ :

```

sage: K = FiniteField(16, "x")
sage: MS = MatrixSpace(K, 1, 2)
sage: key = MS([[K("x^3 + x^2 + x + 1"), K("0")]])
sage: maes.round_key(key, 2)
Traceback (most recent call last):
...
TypeError: secret key must be a 2 x 2 matrix over GF(16)

```

#### **sbox()**

Return the S-box of Mini-AES.

#### EXAMPLES:

```

sage: from sage.crypto.block_cipher.miniaes import MiniAES
sage: maes = MiniAES()
sage: maes.sbox()
(14, 4, 13, 1, 2, 15, 11, 8, 3, 10, 6, 12, 5, 9, 0, 7)

```

#### **shift\_row(block)**

Rotate each row of *block* to the left by different nibble amounts. The first or zero-th row is left unchanged, while the second or row one is rotated left by one nibble. This has the effect of only interchanging

the nibbles in the second row. Let  $b_0, b_1, b_2, b_3$  be four nibbles arranged as the following  $2 \times 2$  matrix

$$\begin{bmatrix} b_0 & b_2 \\ b_1 & b_3 \end{bmatrix}$$

Then the operation of shift-row is the mapping

$$\begin{bmatrix} b_0 & b_2 \\ b_1 & b_3 \end{bmatrix} \mapsto \begin{bmatrix} b_0 & b_2 \\ b_3 & b_1 \end{bmatrix}$$

INPUT:

- `block` – a  $2 \times 2$  matrix with entries over  $\mathbf{F}_{2^4}$

OUTPUT:

- A  $2 \times 2$  matrix resulting from applying shift-row on `block`.

EXAMPLES:

Here we work with elements of the finite field  $\mathbf{F}_{2^4}$ :

```
sage: from sage.crypto.block_cipher.miniaes import MiniAES
sage: maes = MiniAES()
sage: K = FiniteField(16, "x")
sage: MS = MatrixSpace(K, 2, 2)
sage: mat = MS([[K("x^3 + x^2 + x + 1"), K("0")], [K("x^2 + x + 1"), K("x^3 + x")]])
sage: maes.shift_row(mat)
```

```
[x^3 + x^2 + x + 1      0]
[      x^3 + x      x^2 + x + 1]
sage: mat
```

```
[x^3 + x^2 + x + 1      0]
[      x^2 + x + 1      x^3 + x]
```

But we can also work with binary strings:

```
sage: bin = BinaryStrings()
sage: B = bin.encoding("Qt"); B
0101000101110100
sage: B = MS(maes.binary_to_GF(B)); B
```

```
[      x^2 + 1      1]
[x^2 + x + 1      x^2]
sage: maes.shift_row(B)
```

```
[      x^2 + 1      1]
[      x^2 x^2 + x + 1]
```

Here we work with integers  $n$  such that  $0 \leq n \leq 15$ :

```
sage: P = [3, 6, 9, 12]; P
[3, 6, 9, 12]
sage: P = MS(maes.integer_to_GF(P)); P
```

```
[      x + 1      x^2 + x]
[ x^3 + 1 x^3 + x^2]
sage: maes.shift_row(P)
```

```
[      x + 1      x^2 + x]
[x^3 + x^2      x^3 + 1]
```

TESTS:

The input block must be a matrix:

```
sage: from sage.crypto.block_cipher.miniaes import MiniAES
sage: maes = MiniAES()
sage: maes.shift_row("block")
Traceback (most recent call last):
...
TypeError: input block must be a 2 x 2 matrix over GF(16)
```

In addition, the dimensions of the input matrix must be  $2 \times 2$ :

```
sage: K = FiniteField(16, "x")
sage: MS = MatrixSpace(K, 1, 2)
sage: mat = MS([[K("x^3 + x^2 + x + 1"), K("0")]])
sage: maes.shift_row(mat)
Traceback (most recent call last):
...
TypeError: input block must be a 2 x 2 matrix over GF(16)
```





# BLUM-GOLDWASSER PROBABILISTIC ENCRYPTION

The Blum-Goldwasser probabilistic public-key encryption scheme. This scheme was originally described in [Blum-Goldwasser1985]. See also section 8.7.2 of [MenezesEtAl1996] and the [Wikipedia article](#) on this scheme.

REFERENCES:

AUTHORS:

- Mike Hogan and David Joyner (2009-9-19): initial procedural version released as public domain software.
- Minh Van Nguyen (2009-12): integrate into Sage as a class and relicense under the GPLv2+. Complete rewrite of the original version to follow the description contained in [MenezesEtAl1996].

```
class sage.crypto.public_key.blum_goldwasser.BlumGoldwasser
    Bases: sage.crypto.cryptosystem.PublicKeyCryptosystem
```

The Blum-Goldwasser probabilistic public-key encryption scheme.

The Blum-Goldwasser encryption and decryption algorithms as described in `encrypt()` and `decrypt()`, respectively, make use of the least significant bit of a binary string. A related concept is the  $k$  least significant bits of a binary string. For example, given a positive integer  $n$ , let  $b = b_0b_1 \cdots b_{m-1}$  be the binary representation of  $n$  so that  $b$  is a binary string of length  $m$ . Then the least significant bit of  $n$  is  $b_{m-1}$ . If  $0 < k \leq m$ , then the  $k$  least significant bits of  $n$  are  $b_{m-1-k}b_{m-k} \cdots b_{m-1}$ . The least significant bit of an integer is also referred to as its parity bit, because this bit determines whether the integer is even or odd. In the following example, we obtain the least significant bit of an integer:

```
sage: n = 123
sage: b = n.binary(); b
'1111011'
sage: n % 2
1
sage: b[-1]
'1'
```

Now find the 4 least significant bits of the integer  $n = 123$ :

```
sage: b
'1111011'
sage: b[-4:]
'1011'
```

The last two examples could be worked through as follows:

```
sage: from sage.crypto.util import least_significant_bits
sage: least_significant_bits(123, 1)
```

```
[1]
sage: least_significant_bits(123, 4)
[1, 0, 1, 1]
```

**EXAMPLES:**

The following encryption/decryption example is taken from Example 8.57, pages 309–310 of [MenezesEtAl1996]:

```
sage: from sage.crypto.public_key.blum_goldwasser import BlumGoldwasser
sage: bg = BlumGoldwasser(); bg
The Blum-Goldwasser public-key encryption scheme.
sage: p = 499; q = 547
sage: pubkey = bg.public_key(p, q); pubkey
272953
sage: prikey = bg.private_key(p, q); prikey
(499, 547, -57, 52)
sage: P = "10011100000100001100"
sage: C = bg.encrypt(P, pubkey, seed=159201); C
([[0, 0, 1, 0], [0, 0, 0, 0], [1, 1, 0, 0], [1, 1, 1, 0], [0, 1, 0, 0]], 139680)
sage: M = bg.decrypt(C, prikey); M
[[1, 0, 0, 1], [1, 1, 0, 0], [0, 0, 0, 1], [0, 0, 0, 0], [1, 1, 0, 0]]
sage: M = "".join(map(lambda x: str(x), flatten(M))); M
'10011100000100001100'
sage: M == P
True
```

Generate a pair of random public/private keys. Use the public key to encrypt a plaintext. Then decrypt the resulting ciphertext using the private key. Finally, compare the decrypted message with the original plaintext.

```
sage: from sage.crypto.public_key.blum_goldwasser import BlumGoldwasser
sage: from sage.crypto.util import bin_to_ascii
sage: bg = BlumGoldwasser()
sage: pubkey, prikey = bg.random_key(10**4, 10**6)
sage: P = "A fixed plaintext."
sage: C = bg.encrypt(P, pubkey)
sage: M = bg.decrypt(C, prikey)
sage: bin_to_ascii(flatten(M)) == P
True
```

If  $(p, q, a, b)$  is a private key, then  $n = pq$  is the corresponding public key. Furthermore, we have  $\gcd(p, q) = ap + bq = 1$ .

```
sage: p, q, a, b = prikey
sage: pubkey == p * q
True
sage: gcd(p, q) == a*p + b*q == 1
True
```

**decrypt (C, K)**

Apply the Blum-Goldwasser scheme to decrypt the ciphertext  $C$  using the private key  $K$ .

**INPUT:**

- $C$  – a ciphertext resulting from encrypting a plaintext using the Blum-Goldwasser encryption algorithm. The ciphertext  $C$  must be of the form  $C = (c_1, c_2, \dots, c_t, x_{t+1})$ . Each  $c_i$  is a sub-block of binary string and  $x_{t+1}$  is the result of the  $t + 1$ -th iteration of the Blum-Blum-Shub algorithm.
- $K$  – a private key  $(p, q, a, b)$  where  $p$  and  $q$  are distinct Blum primes and  $\gcd(p, q) = ap + bq = 1$ .

**OUTPUT:**

- The plaintext resulting from decrypting the ciphertext  $C$  using the Blum-Goldwasser decryption algorithm.

**ALGORITHM:**

The Blum-Goldwasser decryption algorithm is described in Algorithm 8.56, page 309 of [MenezesEtAl1996]. The algorithm works as follows:

1. Let  $C$  be the ciphertext  $C = (c_1, c_2, \dots, c_t, x_{t+1})$ . Then  $t$  is the number of ciphertext sub-blocks and  $h$  is the length of each binary string sub-block  $c_i$ .
2. Let  $(p, q, a, b)$  be the private key whose corresponding public key is  $n = pq$ . Note that  $\gcd(p, q) = ap + bq = 1$ .
3. Compute  $d_1 = ((p+1)/4)^{t+1} \bmod (p-1)$ .
4. Compute  $d_2 = ((q+1)/4)^{t+1} \bmod (q-1)$ .
5. Let  $u = x_{t+1}^{d_1} \bmod p$ .
6. Let  $v = x_{t+1}^{d_2} \bmod q$ .
7. Compute  $x_0 = vap + ubq \bmod n$ .
8. For  $i$  from 1 to  $t$ , do:
  - (a) Compute  $x_i = x_{i-1}^2 \bmod n$ .
  - (b) Let  $p_i$  be the  $h$  least significant bits of  $x_i$ .
  - (c) Compute  $m_i = p_i \oplus c_i$ .
9. The plaintext is  $m = m_1 m_2 \dots m_t$ .

**EXAMPLES:**

The following decryption example is taken from Example 8.57, pages 309–310 of [MenezesEtAl1996]. Here we decrypt a binary string:

```
sage: from sage.crypto.public_key.blum_goldwasser import BlumGoldwasser
sage: bg = BlumGoldwasser()
sage: p = 499; q = 547
sage: C = ([[0, 0, 1, 0], [0, 0, 0, 0], [1, 1, 0, 0], [1, 1, 1, 0], [0, 1, 0, 0]], 139680)
sage: K = bg.private_key(p, q); K
(499, 547, -57, 52)
sage: P = bg.decrypt(C, K); P
[[1, 0, 0, 1], [1, 1, 0, 0], [0, 0, 0, 1], [0, 0, 0, 0], [1, 1, 0, 0]]
```

Convert the plaintext sub-blocks into a binary string:

```
sage: bin = BinaryStrings()
sage: bin(flatten(P))
10011100000100001100
```

Decrypt a longer ciphertext and convert the resulting plaintext into an ASCII string:

```
sage: from sage.crypto.public_key.blum_goldwasser import BlumGoldwasser
sage: from sage.crypto.util import bin_to_ascii
sage: bg = BlumGoldwasser()
sage: p = 78307; q = 412487
sage: K = bg.private_key(p, q)
sage: C = ([[1, 1, 0, 0, 1, 0, 1, 0, 0, 1, 0, 0, 0, 1, 0, 0], \
... [1, 0, 0, 1, 1, 0, 0, 1, 0, 1, 0, 0, 0, 0, 1, 1], \
... [0, 1, 1, 1, 1, 1, 1, 1, 0, 0, 0, 1, 0, 0, 0, 0], \
... [0, 0, 1, 1, 0, 0, 1, 0, 0, 1, 1, 1, 1, 0, 1, 1], \
```

```

... [1, 0, 0, 1, 0, 1, 0, 1, 1, 1, 0, 0, 1, 0, 0, 0], \
... [0, 0, 1, 0, 0, 1, 0, 0, 1, 0, 0, 0, 0, 1, 0, 1], \
... [1, 1, 1, 0, 0, 1, 1, 1, 0, 1, 0, 0, 1, 0, 0, 0], \
... [1, 1, 0, 0, 0, 0, 0, 1, 1, 1, 0, 0, 0, 0, 0, 1], \
... [0, 1, 0, 1, 0, 0, 0, 1, 0, 1, 0, 1, 0, 1, 0, 0], \
... [1, 0, 1, 1, 0, 1, 0, 0, 1, 1, 0, 1, 0, 1, 0, 1], \
... [1, 1, 1, 0, 0, 1, 1, 1, 0, 1, 0, 1, 0, 1, 0, 1], \
... [1, 1, 1, 1, 1, 1, 1, 1, 1, 0, 1, 1, 0, 0, 1, 0], \
... [0, 1, 1, 1, 1, 1, 1, 1, 0, 0, 0, 0, 0, 1, 1, 1]], 3479653279)
sage: P = bg.decrypt(C, K)
sage: bin_to_ascii(flatten(P))
'Blum-Goldwasser encryption'

```

#### TESTS:

The private key  $K = (p, q, a, b)$  must be such that  $p$  and  $q$  are distinct Blum primes. Even if  $p$  and  $q$  pass this criterion, they must also satisfy the requirement  $\gcd(p, q) = ap + bq = 1$ .

```

sage: from sage.crypto.public_key.blum_goldwasser import BlumGoldwasser
sage: bg = BlumGoldwasser()
sage: C = ([[0, 0, 1, 0], [0, 0, 0, 0], [1, 1, 0, 0], [1, 1, 1, 0], [0, 1, 0, 0]], 139680)
sage: K = (7, 7, 1, 2)
sage: bg.decrypt(C, K)
Traceback (most recent call last):
...
ValueError: p and q must be distinct Blum primes.
sage: K = (7, 23, 1, 2)
sage: bg.decrypt(C, K)
Traceback (most recent call last):
...
ValueError: a and b must satisfy gcd(p, q) = ap + bq = 1.
sage: K = (11, 29, 8, -3)
sage: bg.decrypt(C, K)
Traceback (most recent call last):
...
ValueError: p and q must be distinct Blum primes.

```

#### **encrypt** ( $P, K, \text{seed}=\text{None}$ )

Apply the Blum-Goldwasser scheme to encrypt the plaintext  $P$  using the public key  $K$ .

#### INPUT:

- $P$  – a non-empty string of plaintext. The string "" is an empty string, whereas " " is a string consisting of one white space character. The plaintext can be a binary string or a string of ASCII characters. Where  $P$  is an ASCII string, then  $P$  is first encoded as a binary string prior to encryption.
- $K$  – a public key, which is the product of two Blum primes.
- $\text{seed}$  – (default: None) if  $p$  and  $q$  are Blum primes and  $n = pq$  is a public key, then  $\text{seed}$  is a quadratic residue in the multiplicative group  $(\mathbf{Z}/n\mathbf{Z})^*$ . If  $\text{seed}=\text{None}$ , then the function would generate its own random quadratic residue in  $(\mathbf{Z}/n\mathbf{Z})^*$ . Where a value for  $\text{seed}$  is provided, it is your responsibility to ensure that the seed is a quadratic residue in the multiplicative group  $(\mathbf{Z}/n\mathbf{Z})^*$ .

#### OUTPUT:

- The ciphertext resulting from encrypting  $P$  using the public key  $K$ . The ciphertext  $C$  is of the form  $C = (c_1, c_2, \dots, c_t, x_{t+1})$ . Each  $c_i$  is a sub-block of binary string and  $x_{t+1}$  is the result of the  $t + 1$ -th iteration of the Blum-Blum-Shub algorithm.

#### ALGORITHM:

The Blum-Goldwasser encryption algorithm is described in Algorithm 8.56, page 309 of [MenezesEtAl1996]. The algorithm works as follows:

1. Let  $n$  be a public key, where  $n = pq$  is the product of two distinct Blum primes  $p$  and  $q$ .
2. Let  $k = \lfloor \log_2(n) \rfloor$  and  $h = \lfloor \log_2(k) \rfloor$ .
3. Let  $m = m_1 m_2 \cdots m_t$  be the message (plaintext) where each  $m_i$  is a binary string of length  $h$ .
4. Choose a random seed  $x_0$ , which is a quadratic residue in the multiplicative group  $(\mathbf{Z}/n\mathbf{Z})^*$ . That is, choose a random  $r \in (\mathbf{Z}/n\mathbf{Z})^*$  and compute  $x_0 = r^2 \bmod n$ .
5. For  $i$  from 1 to  $t$ , do:
  - (a) Let  $x_i = x_{i-1}^2 \bmod n$ .
  - (b) Let  $p_i$  be the  $h$  least significant bits of  $x_i$ .
  - (c) Let  $c_i = p_i \oplus m_i$ .
6. Compute  $x_{t+1} = x_t^2 \bmod n$ .
7. The ciphertext is  $c = (c_1, c_2, \dots, c_t, x_{t+1})$ .

The value  $h$  in the algorithm is the sub-block length. If the binary string representing the message cannot be divided into blocks of length  $h$  each, then other sub-block lengths would be used instead. The sub-block lengths to fall back on are in the following order: 16, 8, 4, 2, 1.

EXAMPLES:

The following encryption example is taken from Example 8.57, pages 309–310 of [MenezesEtAl1996]. Here, we encrypt a binary string:

```
sage: from sage.crypto.public_key.blum_goldwasser import BlumGoldwasser
sage: bg = BlumGoldwasser()
sage: p = 499; q = 547; n = p * q
sage: P = "10011100000100001100"
sage: C = bg.encrypt(P, n, seed=159201); C
([[0, 0, 1, 0], [0, 0, 0, 0], [1, 1, 0, 0], [1, 1, 1, 0], [0, 1, 0, 0]], 139680)
```

Convert the ciphertext sub-blocks into a binary string:

```
sage: bin = BinaryStrings()
sage: bin(flatten(C[0]))
00100000110011100100
```

Now encrypt an ASCII string. The result is random; no seed is provided to the encryption function so the function generates its own random seed:

```
sage: from sage.crypto.public_key.blum_goldwasser import BlumGoldwasser
sage: bg = BlumGoldwasser()
sage: K = 32300619509
sage: P = "Blum-Goldwasser encryption"
sage: bg.encrypt(P, K) # random
([[1, 1, 0, 0, 1, 0, 1, 0, 0, 1, 0, 0, 0, 1, 0, 0], \
 [1, 0, 0, 1, 1, 0, 0, 1, 0, 1, 0, 0, 0, 0, 1, 1], \
 [0, 1, 1, 1, 1, 1, 1, 1, 0, 0, 0, 1, 0, 0, 0, 0], \
 [0, 0, 1, 1, 0, 0, 1, 0, 0, 1, 1, 1, 1, 0, 1, 1], \
 [1, 0, 0, 1, 0, 1, 0, 1, 1, 1, 0, 0, 1, 0, 0, 0], \
 [0, 0, 1, 0, 0, 1, 0, 0, 1, 0, 0, 0, 0, 1, 0, 1], \
 [1, 1, 1, 0, 0, 1, 1, 1, 0, 1, 0, 0, 1, 0, 0, 0], \
 [1, 1, 0, 0, 0, 0, 0, 1, 1, 1, 0, 0, 0, 0, 0, 1], \
 [0, 1, 0, 1, 0, 0, 0, 1, 0, 1, 0, 1, 0, 1, 0, 0], \
 [1, 0, 1, 1, 0, 1, 0, 0, 1, 1, 0, 1, 0, 1, 0, 1]], \
```

```
[1, 1, 1, 0, 0, 1, 1, 1, 0, 1, 0, 1, 0, 1, 0, 1], \
[1, 1, 1, 1, 1, 1, 1, 1, 1, 0, 1, 1, 0, 0, 1, 0], \
[0, 1, 1, 1, 1, 1, 1, 1, 0, 0, 0, 0, 0, 0, 1, 1], 3479653279)
```

**TESTS:**

The plaintext cannot be an empty string.

```
sage: from sage.crypto.public_key.blum_goldwasser import BlumGoldwasser
sage: bg = BlumGoldwasser()
sage: bg.encrypt("", 3)
Traceback (most recent call last):
...
ValueError: The plaintext cannot be an empty string.
```

**private\_key**( $p, q$ )

Return the Blum-Goldwasser private key corresponding to the distinct Blum primes  $p$  and  $q$ .

**INPUT:**

- $p$  – a Blum prime.
- $q$  – a Blum prime.

**OUTPUT:**

- The Blum-Goldwasser private key  $(p, q, a, b)$  where  $\gcd(p, q) = ap + bq = 1$ .

Both  $p$  and  $q$  must be distinct Blum primes. Let  $p$  be a positive prime. Then  $p$  is a Blum prime if  $p$  is congruent to 3 modulo 4, i.e.  $p \equiv 3 \pmod{4}$ .

**EXAMPLES:**

Obtain two distinct Blum primes and compute the Blum-Goldwasser private key corresponding to those two Blum primes:

```
sage: from sage.crypto.public_key.blum_goldwasser import BlumGoldwasser
sage: from sage.crypto.util import is_blum_prime
sage: bg = BlumGoldwasser()
sage: P = primes_first_n(10); P
[2, 3, 5, 7, 11, 13, 17, 19, 23, 29]
sage: map(is_blum_prime, P)
[False, True, False, True, True, False, False, True, True, False]
sage: bg.private_key(19, 23)
(19, 23, -6, 5)
```

Choose two distinct random Blum primes, compute the Blum-Goldwasser private key corresponding to those two primes, and test that the resulting private key  $(p, q, a, b)$  satisfies  $\gcd(p, q) = ap + bq = 1$ :

```
sage: from sage.crypto.util import random_blum_prime
sage: p = random_blum_prime(10**4, 10**5)
sage: q = random_blum_prime(10**4, 10**5)
sage: while q == p:
...     q = random_blum_prime(10**4, 10**5)
...
sage: p, q, a, b = bg.private_key(p, q)
sage: gcd(p, q) == a*p + b*q == 1
True
```

**TESTS:**

Both of the input  $p$  and  $q$  must be distinct Blum primes.

```

sage: from sage.crypto.public_key.blum_goldwasser import BlumGoldwasser
sage: bg = BlumGoldwasser()
sage: bg.private_key(78307, 78307)
Traceback (most recent call last):
...
ValueError: p and q must be distinct Blum primes.
sage: bg.private_key(7, 4)
Traceback (most recent call last):
...
ValueError: p and q must be distinct Blum primes.

```

### **public\_key**( $p, q$ )

Return the Blum-Goldwasser public key corresponding to the distinct Blum primes  $p$  and  $q$ .

INPUT:

- $p$  – a Blum prime.
- $q$  – a Blum prime.

OUTPUT:

- The Blum-Goldwasser public key  $n = pq$ .

Both  $p$  and  $q$  must be distinct Blum primes. Let  $p$  be a positive prime. Then  $p$  is a Blum prime if  $p$  is congruent to 3 modulo 4, i.e.  $p \equiv 3 \pmod{4}$ .

EXAMPLES:

Obtain two distinct Blum primes and compute the Blum-Goldwasser public key corresponding to those two Blum primes:

```

sage: from sage.crypto.public_key.blum_goldwasser import BlumGoldwasser
sage: from sage.crypto.util import is_blum_prime
sage: bg = BlumGoldwasser()
sage: P = primes_first_n(10); P
[2, 3, 5, 7, 11, 13, 17, 19, 23, 29]
sage: map(is_blum_prime, P)
[False, True, False, True, True, False, False, True, True, False]
sage: bg.public_key(3, 7)
21

```

Choose two distinct random Blum primes, compute the Blum-Goldwasser public key corresponding to those two primes, and test that the public key factorizes into Blum primes:

```

sage: from sage.crypto.util import random_blum_prime
sage: p = random_blum_prime(10**4, 10**5)
sage: q = random_blum_prime(10**4, 10**5)
sage: while q == p:
...     q = random_blum_prime(10**4, 10**5)
...
sage: n = bg.public_key(p, q)
sage: f = factor(n)
sage: is_blum_prime(f[0][0]); is_blum_prime(f[1][0])
True
True
sage: p * q == f[0][0] * f[1][0]
True

```

TESTS:

The input  $p$  and  $q$  must be distinct Blum primes.

```
sage: from sage.crypto.public_key.blum_goldwasser import BlumGoldwasser
sage: bg = BlumGoldwasser()
sage: bg.public_key(3, 3)
Traceback (most recent call last):
...
ValueError: p and q must be distinct Blum primes.
sage: bg.public_key(23, 29)
Traceback (most recent call last):
...
ValueError: p and q must be distinct Blum primes.
```

**random\_key** (*lbound*, *ubound*, *ntries*=100)

Return a pair of random public and private keys.

INPUT:

- *lbound* – positive integer; the lower bound on how small each random Blum prime  $p$  and  $q$  can be. So we have  $0 < \text{lower\_bound} \leq p, q \leq \text{upper\_bound}$ . The lower bound must be distinct from the upper bound.
- *ubound* – positive integer; the upper bound on how large each random Blum prime  $p$  and  $q$  can be. So we have  $0 < \text{lower\_bound} \leq p, q \leq \text{upper\_bound}$ . The lower bound must be distinct from the upper bound.
- *ntries* – (default: 100) the number of attempts to generate a random public/private key pair. If *ntries* is a positive integer, then perform that many attempts at generating a random public/private key pair.

OUTPUT:

- A random public key and its corresponding private key. Each randomly chosen  $p$  and  $q$  are guaranteed to be Blum primes. The public key is  $n = pq$ , and the private key is  $(p, q, a, b)$  where  $\gcd(p, q) = ap + bq = 1$ .

ALGORITHM:

The key generation algorithm is described in Algorithm 8.55, page 308 of [MenezesEtAl1996]. The algorithm works as follows:

1. Let  $p$  and  $q$  be distinct large random primes, each congruent to 3 modulo 4. That is,  $p$  and  $q$  are Blum primes.
2. Let  $n = pq$  be the product of  $p$  and  $q$ .
3. Use the extended Euclidean algorithm to compute integers  $a$  and  $b$  such that  $\gcd(p, q) = ap + bq = 1$ .
4. The public key is  $n$  and the corresponding private key is  $(p, q, a, b)$ .

---

**Note:** Beware that there might not be any primes between the lower and upper bounds. So make sure that these two bounds are “sufficiently” far apart from each other for there to be primes congruent to 3 modulo 4. In particular, there should be at least two distinct primes within these bounds, each prime being congruent to 3 modulo 4.

---

EXAMPLES:

Choosing a random pair of public and private keys. We then test to see if they satisfy the requirements of the Blum-Goldwasser scheme:

```
sage: from sage.crypto.public_key.blum_goldwasser import BlumGoldwasser
sage: from sage.crypto.util import is_blum_prime
sage: bg = BlumGoldwasser()
```



```
sage: pubkey, prikey = bg.random_key(10**4, 10**5)
sage: p, q, a, b = prikey
sage: is_blum_prime(p); is_blum_prime(q)
True
True
sage: p == q
False
sage: pubkey == p*q
True
sage: gcd(p, q) == a*p + b*q == 1
True
```

#### TESTS:

Make sure that there is at least one Blum prime between the lower and upper bounds. In the following example, we have lbound=24 and ubound=30 with 29 being the only prime within those bounds. But 29 is not a Blum prime.

```
sage: from sage.crypto.public_key.blum_goldwasser import BlumGoldwasser
sage: bg = BlumGoldwasser()
sage: pubkey, privkey = bg.random_key(24, 30)
Traceback (most recent call last):
...
ValueError: No Blum primes within the specified closed interval.
```



# STREAM CRYPTOSYSTEMS

**class** `sage.crypto.stream.LFSRCryptosystem` (*field=None*)

Bases: `sage.crypto.cryptosystem.SymmetricKeyCryptosystem`

Linear feedback shift register cryptosystem class

**encoding** (*M*)

`x.__init__(...)` initializes `x`; see `help(type(x))` for signature

**class** `sage.crypto.stream.ShrinkingGeneratorCryptosystem` (*field=None*)

Bases: `sage.crypto.cryptosystem.SymmetricKeyCryptosystem`

Shrinking generator cryptosystem class

**encoding** (*M*)

`x.__init__(...)` initializes `x`; see `help(type(x))` for signature

`sage.crypto.stream.blum_blum_shub` (*length*, *seed=None*, *p=None*, *q=None*, *lbound=None*,  
*ubound=None*, *ntries=100*)

The Blum-Blum-Shub (BBS) pseudorandom bit generator.

See the original paper by Blum, Blum and Shub [BlumBlumShub1986]. The BBS algorithm is also discussed in section 5.5.2 of [MenezesEtAl1996].

INPUT:

- *length* – positive integer; the number of bits in the output pseudorandom bit sequence.
- *seed* – (default: `None`) if *p* and *q* are Blum primes, then *seed* is a quadratic residue in the multiplicative group  $(\mathbf{Z}/n\mathbf{Z})^*$  where  $n = pq$ . If *seed*=`None`, then the function would generate its own random quadratic residue in  $(\mathbf{Z}/n\mathbf{Z})^*$ . If you provide a value for *seed*, then it is your responsibility to ensure that the seed is a quadratic residue in the multiplicative group  $(\mathbf{Z}/n\mathbf{Z})^*$ .
- *p* – (default: `None`) a large positive prime congruent to 3 modulo 4. Both *p* and *q* must be distinct. If *p*=`None`, then a value for *p* will be generated, where  $0 < \text{lower\_bound} \leq p \leq \text{upper\_bound}$ .
- *q* – (default: `None`) a large positive prime congruence to 3 modulo 4. Both *p* and *q* must be distinct. If *q*=`None`, then a value for *q* will be generated, where  $0 < \text{lower\_bound} \leq q \leq \text{upper\_bound}$ .
- *lbound* – (positive integer, default: `None`) the lower bound on how small each random primes *p* and *q* can be. So we have  $0 < \text{lbound} \leq p, q \leq \text{ubound}$ . The lower bound must be distinct from the upper bound.
- *ubound* – (positive integer, default: `None`) the upper bound on how large each random primes *p* and *q* can be. So we have  $0 < \text{lbound} \leq p, q \leq \text{ubound}$ . The lower bound must be distinct from the upper bound.

- `ntries` – (default: 100) the number of attempts to generate a random Blum prime. If `ntries` is a positive integer, then perform that many attempts at generating a random Blum prime. This might or might not result in a Blum prime.

OUTPUT:

- A pseudorandom bit sequence whose length is specified by `length`.

Here is a common use case for this function. If you want this function to use pre-computed values for  $p$  and  $q$ , you should pass those pre-computed values to this function. In that case, you only need to specify values for `length`, `p` and `q`, and you do not need to worry about doing anything with the parameters `lbound` and `ubound`. The pre-computed values  $p$  and  $q$  must be Blum primes. It is your responsibility to check that both  $p$  and  $q$  are Blum primes.

Here is another common use case. If you want the function to generate its own values for  $p$  and  $q$ , you must specify the lower and upper bounds within which these two primes must lie. In that case, you must specify values for `length`, `lbound` and `ubound`, and you do not need to worry about values for the parameters `p` and `q`. The parameter `ntries` is only relevant when you want this function to generate  $p$  and  $q$ .

---

**Note:** Beware that there might not be any primes between the lower and upper bounds. So make sure that these two bounds are “sufficiently” far apart from each other for there to be primes congruent to 3 modulo 4. In particular, there should be at least two distinct primes within these bounds, each prime being congruent to 3 modulo 4. This function uses the function `random_blum_prime()` to generate random primes that are congruent to 3 modulo 4.

---

ALGORITHM:

The BBS algorithm as described below is adapted from the presentation in Algorithm 5.40, page 186 of [MenezesEtAl1996].

1. Let  $L$  be the desired number of bits in the output bit sequence. That is,  $L$  is the desired length of the bit string.
2. Let  $p$  and  $q$  be two large distinct primes, each congruent to 3 modulo 4.
3. Let  $n = pq$  be the product of  $p$  and  $q$ .
4. Select a random seed value  $s \in (\mathbf{Z}/n\mathbf{Z})^*$ , where  $(\mathbf{Z}/n\mathbf{Z})^*$  is the multiplicative group of  $\mathbf{Z}/n\mathbf{Z}$ .
5. Let  $x_0 = s^2 \bmod n$ .
6. For  $i$  from 1 to  $L$ , do
  - (a) Let  $x_i = x_{i-1}^2 \bmod n$ .
  - (b) Let  $z_i$  be the least significant bit of  $x_i$ .
7. The output pseudorandom bit sequence is  $z_1, z_2, \dots, z_L$ .

EXAMPLES:

A BBS pseudorandom bit sequence with a specified seed:

```
sage: from sage.crypto.stream import blum_blum_shub
sage: blum_blum_shub(length=6, seed=3, p=11, q=19)
110000
```

You could specify the length of the bit string, with given values for  $p$  and  $q$ :

```
sage: blum_blum_shub(length=6, p=11, q=19) # random
001011
```

Or you could specify the length of the bit string, with given values for the lower and upper bounds:

```
sage: blum_blum_shub(length=6, lbound=10**4, ubound=10**5) # random
110111
```

Under some reasonable hypotheses, Blum-Blum-Shub [BlumBlumShub1982] sketch a proof that the period of the BBS stream cipher is equal to  $\lambda(\lambda(n))$ , where  $\lambda(n)$  is the Carmichael function of  $n$ . This is verified below in a few examples by using the function `lfsr_connection_polynomial()` (written by Tim Brock) which computes the connection polynomial of a linear feedback shift register sequence. The degree of that polynomial is the period.

```
sage: from sage.crypto.stream import blum_blum_shub
sage: from sage.crypto.util import carmichael_lambda
sage: carmichael_lambda(carmichael_lambda(7*11))
4
sage: s = [GF(2)(int(str(x))) for x in blum_blum_shub(60, p=7, q=11, seed=13)]
sage: lfsr_connection_polynomial(s)
x^3 + x^2 + x + 1
sage: carmichael_lambda(carmichael_lambda(11*23))
20
sage: s = [GF(2)(int(str(x))) for x in blum_blum_shub(60, p=11, q=23, seed=13)]
sage: lfsr_connection_polynomial(s)
x^19 + x^18 + x^17 + x^16 + x^15 + x^14 + x^13 + x^12 + x^11 + x^10 + x^9 + x^8 + x^7 + x^6 + x^5 + x^4 + x^3 + x^2 + x + 1
```

#### TESTS:

Make sure that there is at least one Blum prime between the lower and upper bounds. In the following example, we have `lbound=24` and `ubound=30` with 29 being the only prime within those bounds. But 29 is not a Blum prime.

```
sage: from sage.crypto.stream import blum_blum_shub
sage: blum_blum_shub(6, lbound=24, ubound=30, ntries=10)
Traceback (most recent call last):
...
ValueError: No Blum primes within the specified closed interval.
```

Both the lower and upper bounds must be greater than 2:

```
sage: blum_blum_shub(6, lbound=2, ubound=3)
Traceback (most recent call last):
...
ValueError: Both the lower and upper bounds must be > 2.
sage: blum_blum_shub(6, lbound=3, ubound=2)
Traceback (most recent call last):
...
ValueError: Both the lower and upper bounds must be > 2.
sage: blum_blum_shub(6, lbound=2, ubound=2)
Traceback (most recent call last):
...
ValueError: Both the lower and upper bounds must be > 2.
```

The lower and upper bounds must be distinct from each other:

```
sage: blum_blum_shub(6, lbound=3, ubound=3)
Traceback (most recent call last):
...
ValueError: The lower and upper bounds must be distinct.
```

The lower bound must be less than the upper bound:

```
sage: blum_blum_shub(6, lbound=4, ubound=3)
Traceback (most recent call last):
...
ValueError: The lower bound must be less than the upper bound.
```

REFERENCES:

# STREAM CIPHERS

```
class sage.crypto.stream_cipher.LFSRCipher(parent, poly, IS)
    Bases: sage.crypto.cipher.SymmetricKeyCipher
```

Create a linear feedback shift register (LFSR) cipher.

INPUT:

- parent - parent
- poly - connection polynomial
- IS - initial state

EXAMPLES:

```
sage: FF = FiniteField(2)
sage: P.<x> = PolynomialRing(FF)
sage: E = LFSRCryptosystem(FF)
sage: E
LFSR cryptosystem over Finite Field of size 2
sage: IS = [ FF(a) for a in [0,1,1,1,0,1,1] ]
sage: g = x^7 + x + 1
sage: e = E((g,IS))
sage: B = BinaryStrings()
sage: m = B.encoding("THECATINTHEHAT")
sage: e(m)
0010001101111010111010101000110000000011010001010101110000101111001001000001111110010010001100
sage: FF = FiniteField(2)
sage: P.<x> = PolynomialRing(FF)
sage: LFSR = LFSRCryptosystem(FF)
sage: e = LFSR((x^2+x+1, [FF(0),FF(1)]))
sage: B = e.domain()
sage: m = B.encoding("The cat in the hat.")
sage: e(m)
001110011101111010111110010011011101010110111010000110011001011010110010000000111001011010101111
sage: m == e(e(m))
True
```

TESTS:

```
sage: FF = FiniteField(2)
sage: P.<x> = PolynomialRing(FF)
sage: E = LFSRCryptosystem(FF)
sage: E == loads(dumps(E))
True
```

**connection\_polynomial()**

The connection polynomial defining the LFSR of the cipher.

EXAMPLE:

```
sage: k = GF(2)
sage: P.<x> = PolynomialRing( k )
sage: LFSR = LFSRCryptosystem( k )
sage: e = LFSR((x^2+x+1,[k(0), k(1)]))
sage: e.connection_polynomial()
x^2 + x + 1
```

**initial\_state()**

The initial state of the LFSR cipher.

EXAMPLE:

```
sage: k = GF(2)
sage: P.<x> = PolynomialRing( k )
sage: LFSR = LFSRCryptosystem( k )
sage: e = LFSR((x^2+x+1,[k(0), k(1)]))
sage: e.initial_state()
[0, 1]
```

**class** sage.crypto.stream\_cipher.**ShrinkingGeneratorCipher**(parent, e1, e2)

Bases: sage.crypto.cipher.SymmetricKeyCipher

Create a shrinking generator cipher.

INPUT:

- parent - parent
- poly - connection polynomial
- IS - initial state

EXAMPLES:

```
sage: FF = FiniteField(2)
sage: P.<x> = PolynomialRing(FF)
sage: LFSR = LFSRCryptosystem(FF)
sage: IS_1 = [ FF(a) for a in [0,1,0,1,0,0,0] ]
sage: e1 = LFSR((x^7 + x + 1,IS_1))
sage: IS_2 = [ FF(a) for a in [0,0,1,0,0,0,1,0,1] ]
sage: e2 = LFSR((x^9 + x^3 + 1,IS_2))
sage: E = ShrinkingGeneratorCryptosystem()
sage: e = E((e1,e2))
sage: e
Shrinking generator cipher on Free binary string monoid
```

**decimating\_cipher()**

The LFSR cipher generating the decimating key stream.

EXAMPLE:

```
sage: FF = FiniteField(2)
sage: P.<x> = PolynomialRing(FF)
sage: LFSR = LFSRCryptosystem(FF)
sage: IS_1 = [ FF(a) for a in [0,1,0,1,0,0,0] ]
sage: e1 = LFSR((x^7 + x + 1,IS_1))
sage: IS_2 = [ FF(a) for a in [0,0,1,0,0,0,1,0,1] ]
sage: e2 = LFSR((x^9 + x^3 + 1,IS_2))
```



```
sage: E = ShrinkingGeneratorCryptosystem()
sage: e = E((e1,e2))
sage: e.decimating_cipher()
LFSR cipher on Free binary string monoid
```

**keystream\_cipher()**

The LFSR cipher generating the output key stream.

EXAMPLE:

```
sage: FF = FiniteField(2)
sage: P.<x> = PolynomialRing(FF)
sage: LFSR = LFSRCryptosystem(FF)
sage: IS_1 = [ FF(a) for a in [0,1,0,1,0,0,0] ]
sage: e1 = LFSR((x^7 + x + 1, IS_1))
sage: IS_2 = [ FF(a) for a in [0,0,1,0,0,0,1,0,1] ]
sage: e2 = LFSR((x^9 + x^3 + 1, IS_2))
sage: E = ShrinkingGeneratorCryptosystem()
sage: e = E((e1,e2))
sage: e.keystream_cipher()
LFSR cipher on Free binary string monoid
```



# LINEAR FEEDBACK SHIFT REGISTER (LFSR) SEQUENCE COMMANDS

Stream ciphers have been used for a long time as a source of pseudo-random number generators.

S. Golomb [G] gives a list of three statistical properties a sequence of numbers  $\mathbf{a} = \{a_n\}_{n=1}^{\infty}$ ,  $a_n \in \{0, 1\}$ , should display to be considered “random”. Define the autocorrelation of  $\mathbf{a}$  to be

$$C(k) = C(k, \mathbf{a}) = \lim_{N \rightarrow \infty} \frac{1}{N} \sum_{n=1}^N (-1)^{a_n + a_{n+k}}.$$

In the case where  $\mathbf{a}$  is periodic with period  $P$  then this reduces to

$$C(k) = \frac{1}{P} \sum_{n=1}^P (-1)^{a_n + a_{n+k}}.$$

Assume  $\mathbf{a}$  is periodic with period  $P$ .

- balance:  $|\sum_{n=1}^P (-1)^{a_n}| \leq 1$ .
- low autocorrelation:

$$C(k) = \begin{cases} 1, & k = 0, \\ \epsilon, & k \neq 0. \end{cases}$$

(For sequences satisfying these first two properties, it is known that  $\epsilon = -1/P$  must hold.)

- proportional runs property: In each period, half the runs have length 1, one-fourth have length 2, etc. Moreover, there are as many runs of 1’s as there are of 0’s.

A general feedback shift register is a map  $f : \mathbf{F}_q^d \rightarrow \mathbf{F}_q^d$  of the form

$$\begin{aligned} f(x_0, \dots, x_{n-1}) &= (x_1, x_2, \dots, x_n), \\ x_n &= C(x_0, \dots, x_{n-1}), \end{aligned}$$

where  $C : \mathbf{F}_q^d \rightarrow \mathbf{F}_q$  is a given function. When  $C$  is of the form

$$C(x_0, \dots, x_{n-1}) = a_0 x_0 + \dots + a_{n-1} x_{n-1},$$

for some given constants  $a_i \in \mathbf{F}_q$ , the map is called a linear feedback shift register (LFSR).

Example of a LFSR Let

$$f(x) = a_0 + a_1 x + \dots + a_n x^n + \dots,$$

$$g(x) = b_0 + b_1x + \dots + b_nx^n + \dots,$$

be given polynomials in  $\mathbf{F}_2[x]$  and let

$$h(x) = \frac{f(x)}{g(x)} = c_0 + c_1x + \dots + c_nx^n + \dots.$$

We can compute a recursion formula which allows us to rapidly compute the coefficients of  $h(x)$  (take  $f(x) = 1$ ):

$$c_n = \sum_{i=1}^n \frac{-b_i}{b_0} c_{n-i}.$$

The coefficients of  $h(x)$  can, under certain conditions on  $f(x)$  and  $g(x)$ , be considered “random” from certain statistical points of view.

Example: For instance, if

$$f(x) = 1, \quad g(x) = x^4 + x + 1,$$

then

$$h(x) = 1 + x + x^2 + x^3 + x^5 + x^7 + x^8 + \dots.$$

The coefficients of  $h$  are

$$\begin{array}{l} 1, 1, 1, 0, 1, 0, 1, 1, 0, 0, 1, 0, 0, 0, 1, \\ 1, 1, 1, 0, 1, 0, 1, 1, 0, 0, 1, 0, 0, 0, 1, \dots \end{array}$$

The sequence of 0, 1's is periodic with period  $P = 2^4 - 1 = 15$  and satisfies Golomb's three randomness conditions. However, this sequence of period 15 can be “cracked” (i.e., a procedure to reproduce  $g(x)$ ) by knowing only 8 terms! This is the function of the Berlekamp-Massey algorithm [M], implemented as `berlekamp_massey.py`.

AUTHORS:

- Timothy Brock

Created 11-24-2005 by wdj. Last updated 12-02-2005.

`sage.crypto.lfsr.lfsr_autocorrelation(L, p, k)`

INPUT:

- $L$  - is a periodic sequence of elements of  $\mathbb{Z}\mathbb{Z}$  or  $\text{GF}(2)$ .  $L$  must have length =  $p$
- $p$  - the period of  $L$
- $k$  -  $k$  is an integer ( $0 \leq k < p$ )

OUTPUT: autocorrelation sequence of  $L$

EXAMPLES:

```
sage: F = GF(2)
sage: o = F(0)
sage: l = F(1)
sage: key = [l,o,o,l]; fill = [l,l,o,l]; n = 20
sage: s = lfsr_sequence(key, fill, n)
sage: lfsr_autocorrelation(s, 15, 7)
4/15
sage: lfsr_autocorrelation(s, int(15), 7)
4/15
```

AUTHORS:

•Timothy Brock (2006-04-17)

`sage.crypto.lfsr.lfsr_connection_polynomial(s)`

INPUT:

•s - a sequence of elements of a finite field (F) of even length

OUTPUT:

•C(x) - the connection polynomial of the minimal LFSR.

This implements the algorithm in section 3 of J. L. Massey's article [M].

EXAMPLE:

```
sage: F = GF(2)
sage: F
Finite Field of size 2
sage: o = F(0); l = F(1)
sage: key = [1,o,o,1]; fill = [1,1,o,1]; n = 20
sage: s = lfsr_sequence(key,fill,n); s
[1, 1, 0, 1, 0, 1, 1, 0, 0, 1, 0, 0, 1, 1, 1, 1, 0, 1, 0]
sage: lfsr_connection_polynomial(s)
x^4 + x + 1
sage: berlekamp_massey(s)
x^4 + x^3 + 1
```

Notice that `berlekamp_massey` returns the reverse of the connection polynomial (and is potentially much faster than this implementation).

AUTHORS:

•Timothy Brock (2006-04-17)

`sage.crypto.lfsr.lfsr_sequence(key,fill,n)`

This function creates an lfsr sequence.

INPUT:

•key - a list of finite field elements,  $[c_0, c_1, \dots, c_k]$ .

•fill - the list of the initial terms of the lfsr sequence,  $[x_0, x_1, \dots, x_k]$ .

•n - number of terms of the sequence that the function returns.

OUTPUT: The lfsr sequence defined by  $x_{n+1} = c_k x_n + \dots + c_0 x_{n-k}$ , for  $n \leq k$ .

EXAMPLES:

```
sage: F = GF(2); l = F(1); o = F(0)
sage: F = GF(2); S = LaurentSeriesRing(F, 'x'); x = S.gen()
sage: fill = [1,1,o,1]; key = [1,o,o,1]; n = 20
sage: L = lfsr_sequence(key,fill,20); L
[1, 1, 0, 1, 0, 1, 1, 0, 0, 1, 0, 0, 0, 1, 1, 1, 1, 0, 1, 0]
sage: g = berlekamp_massey(L); g
x^4 + x^3 + 1
sage: (1/(g.reverse()+O(x^20)))
1 + x + x^2 + x^3 + x^5 + x^7 + x^8 + x^11 + x^15 + x^16 + x^17 + x^18 + O(x^20)
sage: (1+x^2)/(g.reverse()+O(x^20))
1 + x + x^4 + x^8 + x^9 + x^10 + x^11 + x^13 + x^15 + x^16 + x^19 + O(x^20)
sage: (1+x^2+x^3)/(g.reverse()+O(x^20))
1 + x + x^3 + x^5 + x^6 + x^9 + x^13 + x^14 + x^15 + x^16 + x^18 + O(x^20)
sage: fill = [1,1,o,1]; key = [1,o,o,o]; n = 20
sage: L = lfsr_sequence(key,fill,20); L
[1, 1, 0, 1, 1, 1, 0, 1, 1, 1, 0, 1, 1, 1, 0, 1, 1, 1, 0, 1]
```

```
sage: g = berlekamp_massey(L); g
x^4 + 1
sage: (1+x)/(g.reverse()+O(x^20))
1 + x + x^4 + x^5 + x^8 + x^9 + x^12 + x^13 + x^16 + x^17 + O(x^20)
sage: (1+x+x^3)/(g.reverse()+O(x^20))
1 + x + x^3 + x^4 + x^5 + x^7 + x^8 + x^9 + x^11 + x^12 + x^13 + x^15 + x^16 + x^17 + x^19 + O(x^20)
```

**AUTHORS:**

•Timothy Brock (2005-11): with code modified from Python Cookbook,  
<http://aspn.activestate.com/ASPN/Python/Cookbook/>

# UTILITY FUNCTIONS FOR CRYPTOGRAPHY

Miscellaneous utility functions for cryptographic purposes.

AUTHORS:

- Minh Van Nguyen (2009-12): initial version with the following functions: `ascii_integer`, `ascii_to_bin`, `bin_to_ascii`, `has_blum_prime`, `is_blum_prime`, `least_significant_bits`, `random_blum_prime`.

`sage.crypto.util.ascii_integer(B)`

Return the ASCII integer corresponding to the binary string B.

INPUT:

- B – a non-empty binary string or a non-empty list of bits. The number of bits in B must be 8.

OUTPUT:

- The ASCII integer corresponding to the 8-bit block B.

EXAMPLES:

The ASCII integers of some binary strings:

```
sage: from sage.crypto.util import ascii_integer
sage: bin = BinaryStrings()
sage: B = bin.encoding("A"); B
01000001
sage: ascii_integer(B)
65
sage: B = bin.encoding("C"); list(B)
[0, 1, 0, 0, 0, 0, 1, 1]
sage: ascii_integer(list(B))
67
sage: ascii_integer("01000100")
68
sage: ascii_integer([0, 1, 0, 0, 0, 1, 0, 1])
69
```

TESTS:

The input B must be a non-empty string or a non-empty list of bits:

```
sage: from sage.crypto.util import ascii_integer
sage: ascii_integer("")
Traceback (most recent call last):
```

```
...
ValueError: B must consist of 8 bits.
sage: ascii_integer([])
Traceback (most recent call last):
...
ValueError: B must consist of 8 bits.
```

The input B must be an 8-bit string or a list of 8 bits:

```
sage: from sage.crypto.util import ascii_integer
sage: ascii_integer("101")
Traceback (most recent call last):
...
ValueError: B must consist of 8 bits.
sage: ascii_integer([1, 0, 1, 1])
Traceback (most recent call last):
...
ValueError: B must consist of 8 bits.
```

`sage.crypto.util.ascii_to_bin(A)`  
Return the binary representation of the ASCII string A.

INPUT:

- A – a string or list of ASCII characters.

OUTPUT:

- The binary representation of A.

ALGORITHM:

Let  $A = a_0a_1 \cdots a_{n-1}$  be an ASCII string, where each  $a_i$  is an ASCII character. Let  $c_i$  be the ASCII integer corresponding to  $a_i$  and let  $b_i$  be the binary representation of  $c_i$ . The binary representation  $B$  of  $A$  is  $B = b_0b_1 \cdots b_{n-1}$ .

EXAMPLES:

The binary representation of some ASCII strings:

```
sage: from sage.crypto.util import ascii_to_bin
sage: ascii_to_bin("A")
01000001
sage: ascii_to_bin("Abc123")
010000010110001001100011001100010011001000110011
```

The empty string is different from the string with one space character. For the empty string and the empty list, this function returns the same result:

```
sage: from sage.crypto.util import ascii_to_bin
sage: ascii_to_bin("")

sage: ascii_to_bin(" ")
00100000
sage: ascii_to_bin([])
```

This function also accepts a list of ASCII characters. You can also pass in a list of strings:

```
sage: from sage.crypto.util import ascii_to_bin
sage: ascii_to_bin(["A", "b", "c", "1", "2", "3"])
010000010110001001100011001100010011001000110011
sage: ascii_to_bin(["A", "bc", "1", "23"])
010000010110001001100011001100010011001000110011
```



## TESTS:

For a list of ASCII characters or strings, do not mix characters or strings with integers:

```
sage: from sage.crypto.util import ascii_to_bin
sage: ascii_to_bin(["A", "b", "c", 1, 2, 3])
Traceback (most recent call last):
...
TypeError: sequence item 3: expected string, sage.rings.integer.Integer found
sage: ascii_to_bin(["Abc", 1, 2, 3])
Traceback (most recent call last):
...
TypeError: sequence item 1: expected string, sage.rings.integer.Integer found
```

`sage.crypto.util.bin_to_ascii(B)`

Return the ASCII representation of the binary string B.

## INPUT:

- B – a non-empty binary string or a non-empty list of bits. The number of bits in B must be a multiple of 8.

## OUTPUT:

- The ASCII string corresponding to B.

## ALGORITHM:

Consider a block of bits  $B = b_0b_1 \cdots b_{n-1}$  where each sub-block  $b_i$  is a binary string of length 8. Then the total number of bits is a multiple of 8 and is given by  $8n$ . Let  $c_i$  be the integer representation of  $b_i$ . We can consider  $c_i$  as the integer representation of an ASCII character. Then the ASCII representation  $A$  of  $B$  is  $A = a_0a_1 \cdots a_{n-1}$ .

## EXAMPLES:

Convert some ASCII strings to their binary representations and recover the ASCII strings from the binary representations:

```
sage: from sage.crypto.util import ascii_to_bin
sage: from sage.crypto.util import bin_to_ascii
sage: A = "Abc"
sage: B = ascii_to_bin(A); B
010000010110001001100011
sage: bin_to_ascii(B)
'Abc'
sage: bin_to_ascii(B) == A
True

sage: A = "123 \" #"
sage: B = ascii_to_bin(A); B
00110001001100100011001100100000001000100010000000100011
sage: bin_to_ascii(B)
'123 " #'
sage: bin_to_ascii(B) == A
True
```

This function also accepts strings and lists of bits:

```
sage: from sage.crypto.util import bin_to_ascii
sage: bin_to_ascii("010000010110001001100011")
'Abc'
sage: bin_to_ascii([0, 1, 0, 0, 0, 0, 0, 1])
'A'
```

## TESTS:

The number of bits in B must be non-empty and a multiple of 8:

```
sage: from sage.crypto.util import bin_to_ascii
sage: bin_to_ascii("")
Traceback (most recent call last):
...
ValueError: B must be a non-empty binary string.
sage: bin_to_ascii([])
Traceback (most recent call last):
...
ValueError: B must be a non-empty binary string.
sage: bin_to_ascii(" ")
Traceback (most recent call last):
...
ValueError: The number of bits in B must be a multiple of 8.
sage: bin_to_ascii("101")
Traceback (most recent call last):
...
ValueError: The number of bits in B must be a multiple of 8.
sage: bin_to_ascii([1, 0, 1])
Traceback (most recent call last):
...
ValueError: The number of bits in B must be a multiple of 8.
```

sage.crypto.util.**carmichael\_lambda**(n)

Return the Carmichael function of a positive integer n.

The Carmichael function of  $n$ , denoted  $\lambda(n)$ , is the smallest positive integer  $k$  such that  $a^k \equiv 1 \pmod{n}$  for all  $a \in \mathbf{Z}/n\mathbf{Z}$  satisfying  $\gcd(a, n) = 1$ . Thus,  $\lambda(n) = k$  is the exponent of the multiplicative group  $(\mathbf{Z}/n\mathbf{Z})^*$ .

INPUT:

- $n$  – a positive integer.

OUTPUT:

- The Carmichael function of  $n$ .

ALGORITHM:

If  $n = 2, 4$  then  $\lambda(n) = \varphi(n)$ . Let  $p \geq 3$  be an odd prime and let  $k$  be a positive integer. Then  $\lambda(p^k) = p^{k-1}(p-1) = \varphi(p^k)$ . If  $k \geq 3$ , then  $\lambda(2^k) = 2^{k-2}$ . Now consider the case where  $n > 3$  is composite and let  $n = p_1^{k_1} p_2^{k_2} \cdots p_t^{k_t}$  be the prime factorization of  $n$ . Then

$$\lambda(n) = \lambda(p_1^{k_1} p_2^{k_2} \cdots p_t^{k_t}) = \text{lcm}(\lambda(p_1^{k_1}), \lambda(p_2^{k_2}), \dots, \lambda(p_t^{k_t}))$$

EXAMPLES:

The Carmichael function of all positive integers up to and including 10:

```
sage: from sage.crypto.util import carmichael_lambda
sage: map(carmichael_lambda, [1..10])
[1, 1, 2, 2, 4, 2, 6, 2, 6, 4]
```

The Carmichael function of the first ten primes:

```
sage: map(carmichael_lambda, primes_first_n(10))
[1, 2, 4, 6, 10, 12, 16, 18, 22, 28]
```

Cases where the Carmichael function is equivalent to the Euler phi function:

```
sage: carmichael_lambda(2) == euler_phi(2)
True
```

```

sage: carmichael_lambda(4) == euler_phi(4)
True
sage: p = random_prime(1000, lbound=3, proof=True)
sage: k = randint(1, 1000)
sage: carmichael_lambda(p^k) == euler_phi(p^k)
True

```

A case where  $\lambda(n) \neq \varphi(n)$ :

```

sage: k = randint(1, 1000)
sage: carmichael_lambda(2^k) == 2^(k - 2)
True
sage: carmichael_lambda(2^k) == 2^(k - 2) == euler_phi(2^k)
False

```

Verifying the current implementation of the Carmichael function using another implementation. The other implementation that we use for verification is an exhaustive search for the exponent of the multiplicative group  $(\mathbb{Z}/n\mathbb{Z})^*$ .

```

sage: from sage.crypto.util import carmichael_lambda
sage: n = randint(1, 500)
sage: c = carmichael_lambda(n)
sage: def coprime(n):
...     return [i for i in xrange(n) if gcd(i, n) == 1]
...
sage: def znpower(n, k):
...     L = coprime(n)
...     return map(power_mod, L, [k]*len(L), [n]*len(L))
...
sage: def my_carmichael(n):
...     for k in xrange(1, n):
...         L = znpower(n, k)
...         ones = [1] * len(L)
...         T = [L[i] == ones[i] for i in xrange(len(L))]
...         if all(T):
...             return k
...
sage: c == my_carmichael(n)
True

```

Carmichael's theorem states that  $a^{\lambda(n)} \equiv 1 \pmod{n}$  for all elements  $a$  of the multiplicative group  $(\mathbb{Z}/n\mathbb{Z})^*$ . Here, we verify Carmichael's theorem.

```

sage: from sage.crypto.util import carmichael_lambda
sage: n = randint(1, 1000)
sage: c = carmichael_lambda(n)
sage: ZnZ = IntegerModRing(n)
sage: M = ZnZ.list_of_elements_of_multiplicative_group()
sage: ones = [1] * len(M)
sage: P = [power_mod(a, c, n) for a in M]
sage: P == ones
True

```

#### TESTS:

The input  $n$  must be a positive integer:

```

sage: from sage.crypto.util import carmichael_lambda
sage: carmichael_lambda(0)
Traceback (most recent call last):

```

```
...
ValueError: Input n must be a positive integer.
sage: carmichael_lambda(randint(-10, 0))
Traceback (most recent call last):
...
ValueError: Input n must be a positive integer.
```

Bug reported in trac #8283:

```
sage: from sage.crypto.util import carmichael_lambda
sage: type(carmichael_lambda(16))
<type 'sage.rings.integer.Integer'>
```

#### REFERENCES:

`sage.crypto.util.has_blum_prime(lbound, ubound)`

Determine whether or not there is a Blum prime within the specified closed interval.

#### INPUT:

- `lbound` – positive integer; the lower bound on how small a Blum prime can be. The lower bound must be distinct from the upper bound.
- `ubound` – positive integer; the upper bound on how large a Blum prime can be. The lower bound must be distinct from the upper bound.

#### OUTPUT:

- True if there is a Blum prime  $p$  such that  $lbound \leq p \leq ubound$ . False otherwise.

#### ALGORITHM:

Let  $L$  and  $U$  be distinct positive integers. Let  $P$  be the set of all odd primes  $p$  such that  $L \leq p \leq U$ . Our main focus is on Blum primes, i.e. odd primes that are congruent to 3 modulo 4, so we assume that the lower bound  $L > 2$ . The closed interval  $[L, U]$  has a Blum prime if and only if the set  $P$  has a Blum prime.

#### EXAMPLES:

Testing for the presence of Blum primes within some closed intervals. The interval  $[4, 100]$  has a Blum prime, the smallest such prime being 7. The interval  $[24, 28]$  has no primes, hence no Blum primes.

```
sage: from sage.crypto.util import has_blum_prime
sage: from sage.crypto.util import is_blum_prime
sage: has_blum_prime(4, 100)
True
sage: for n in xrange(4, 100):
...     if is_blum_prime(n):
...         print n
...         break
...
7
sage: has_blum_prime(24, 28)
False
```

#### TESTS:

Both the lower and upper bounds must be greater than 2:

```
sage: from sage.crypto.util import has_blum_prime
sage: has_blum_prime(2, 3)
Traceback (most recent call last):
...
ValueError: Both the lower and upper bounds must be > 2.
```

```

sage: has_blum_prime(3, 2)
Traceback (most recent call last):
...
ValueError: Both the lower and upper bounds must be > 2.
sage: has_blum_prime(2, 2)
Traceback (most recent call last):
...
ValueError: Both the lower and upper bounds must be > 2.

```

The lower and upper bounds must be distinct from each other:

```

sage: has_blum_prime(3, 3)
Traceback (most recent call last):
...
ValueError: The lower and upper bounds must be distinct.

```

The lower bound must be less than the upper bound:

```

sage: has_blum_prime(4, 3)
Traceback (most recent call last):
...
ValueError: The lower bound must be less than the upper bound.

```

`sage.crypto.util.is_blum_prime(n)`  
Determine whether or not  $n$  is a Blum prime.

INPUT:

- $n$  a positive prime.

OUTPUT:

- True if  $n$  is a Blum prime; False otherwise.

Let  $n$  be a positive prime. Then  $n$  is a Blum prime if  $n$  is congruent to 3 modulo 4, i.e.  $n \equiv 3 \pmod{4}$ .

EXAMPLES:

Testing some integers to see if they are Blum primes:

```

sage: from sage.crypto.util import is_blum_prime
sage: from sage.crypto.util import random_blum_prime
sage: is_blum_prime(101)
False
sage: is_blum_prime(7)
True
sage: p = random_blum_prime(10**3, 10**5)
sage: is_blum_prime(p)
True

```

`sage.crypto.util.least_significant_bits(n, k)`  
Return the  $k$  least significant bits of  $n$ .

INPUT:

- $n$  – an integer.
- $k$  – a positive integer.

OUTPUT:

- The  $k$  least significant bits of the integer  $n$ . If  $k=1$ , then return the parity bit of the integer  $n$ . Let  $b$  be the binary representation of  $n$ , where  $m$  is the length of the binary string  $b$ . If  $k \geq m$ , then return the binary representation of  $n$ .

**EXAMPLES:**

Obtain the parity bits of some integers:

```
sage: from sage.crypto.util import least_significant_bits
sage: least_significant_bits(0, 1)
[0]
sage: least_significant_bits(2, 1)
[0]
sage: least_significant_bits(3, 1)
[1]
sage: least_significant_bits(-2, 1)
[0]
sage: least_significant_bits(-3, 1)
[1]
```

Obtain the 4 least significant bits of some integers:

```
sage: least_significant_bits(101, 4)
[0, 1, 0, 1]
sage: least_significant_bits(-101, 4)
[0, 1, 0, 1]
sage: least_significant_bits(124, 4)
[1, 1, 0, 0]
sage: least_significant_bits(-124, 4)
[1, 1, 0, 0]
```

The binary representation of 123:

```
sage: n = 123; b = n.binary(); b
'1111011'
sage: least_significant_bits(n, len(b))
[1, 1, 1, 1, 0, 1, 1]
```

`sage.crypto.util.random_blum_prime(lbound, ubound, ntries=100)`

A random Blum prime within the specified bounds.

Let  $p$  be a positive prime. Then  $p$  is a Blum prime if  $p$  is congruent to 3 modulo 4, i.e.  $p \equiv 3 \pmod{4}$ .

**INPUT:**

- `lbound` – positive integer; the lower bound on how small a random Blum prime  $p$  can be. So we have  $0 < \text{lbound} \leq p \leq \text{ubound}$ . The lower bound must be distinct from the upper bound.
- `ubound` – positive integer; the upper bound on how large a random Blum prime  $p$  can be. So we have  $0 < \text{lbound} \leq p \leq \text{ubound}$ . The lower bound must be distinct from the upper bound.
- `ntries` – (default: 100) the number of attempts to generate a random Blum prime. If `ntries` is a positive integer, then perform that many attempts at generating a random Blum prime. This might or might not result in a Blum prime.

**OUTPUT:**

- A random Blum prime within the specified lower and upper bounds.

---

**Note:** Beware that there might not be any primes between the lower and upper bounds. So make sure that these two bounds are “sufficiently” far apart from each other for there to be primes congruent to 3 modulo 4. In particular, there should be at least two distinct Blum primes within the specified bounds.

**EXAMPLES:**

Choose a random prime and check that it is a Blum prime:

```
sage: from sage.crypto.util import random_blum_prime
sage: p = random_blum_prime(10**4, 10**5)
sage: is_prime(p)
True
sage: mod(p, 4) == 3
True
```

**TESTS:**

Make sure that there is at least one Blum prime between the lower and upper bounds. In the following example, we have lbound=24 and ubound=30 with 29 being the only prime within those bounds. But 29 is not a Blum prime.

```
sage: from sage.crypto.util import random_blum_prime
sage: random_blum_prime(24, 30, ntries=10)
Traceback (most recent call last):
...
ValueError: No Blum primes within the specified closed interval.
sage: random_blum_prime(24, 28)
Traceback (most recent call last):
...
ValueError: No Blum primes within the specified closed interval.
```





# BOOLEAN FUNCTIONS

## Boolean functions

Those functions are used for example in LFSR based ciphers like the filter generator or the combination generator.

This module allows to study properties linked to spectral analysis, and also algebraic immunity.

### EXAMPLES:

```
sage: R.<x>=GF(2^8,'a') []
sage: from sage.crypto.boolean_function import BooleanFunction
sage: B = BooleanFunction( x^254 ) # the Boolean function Tr(x^254)
sage: B
Boolean function with 8 variables
sage: B.nonlinearity()
112
sage: B.algebraic_immunity()
4
```

### AUTHOR:

- Yann Laigle-Chapuy (2010-02-26): add basic arithmetic
- Yann Laigle-Chapuy (2009-08-28): first implementation

```
class sage.crypto.boolean_function.BooleanFunction
    Bases: sage.structure.sage_object.SageObject
```

This module implements Boolean functions represented as a truth table.

We can construct a Boolean Function from either:

- an integer - the result is the zero function with  $x$  variables;
- a list - it is expected to be the truth table of the result. Therefore it must be of length a power of 2, and its elements are interpreted as Booleans;
- a string - representing the truth table in hexadecimal;
- a Boolean polynomial - the result is the corresponding Boolean function;
- a polynomial  $P$  over an extension of  $\text{GF}(2)$  - the result is the Boolean function with truth table (  $\text{Tr}(P(x))$  for  $x$  in  $\text{GF}(2^k)$  )

### EXAMPLES:

from the number of variables:

```
sage: from sage.crypto.boolean_function import BooleanFunction
sage: BooleanFunction(5)
Boolean function with 5 variables
```

from a truth table:

```
sage: BooleanFunction([1,0,0,1])
Boolean function with 2 variables
```

note that elements can be of different types:

```
sage: B = BooleanFunction([False, sqrt(2)])
sage: B
Boolean function with 1 variable
sage: [b for b in B]
[False, True]
```

from a string:

```
sage: BooleanFunction("111e")
Boolean function with 4 variables
```

from a `sage.rings.polynomial.pbori.BooleanPolynomial`:

```
sage: R.<x,y,z> = BooleanPolynomialRing(3)
sage: P = x*y
sage: BooleanFunction( P )
Boolean function with 3 variables
```

from a polynomial over a binary field:

```
sage: R.<x> = GF(2^8, 'a')[]
sage: B = BooleanFunction( x^7 )
sage: B
Boolean function with 8 variables
```

two failure cases:

```
sage: BooleanFunction(sqrt(2))
Traceback (most recent call last):
...
TypeError: unable to init the Boolean function

sage: BooleanFunction([1, 0, 1])
Traceback (most recent call last):
...
ValueError: the length of the truth table must be a power of 2
```

**absolut\_indicator()**

Return the absolut indicator of the function. This is the maximal absolut value of the autocorrelation.

EXAMPLES:

```
sage: from sage.crypto.boolean_function import BooleanFunction
sage: B = BooleanFunction("7969817CC5893BA6AC326E47619F5AD0")
sage: B.absolut_indicator()
32
```

**absolute\_autocorrelation()**

Return the absolute autocorrelation of the function.

EXAMPLES:

```
sage: from sage.crypto.boolean_function import BooleanFunction
sage: B = BooleanFunction("7969817CC5893BA6AC326E47619F5AD0")
sage: sorted([_ for _ in B.absolute_autocorrelation().iteritems() ])
[(0, 33), (8, 58), (16, 28), (24, 6), (32, 2), (128, 1)]
```

**absolute\_walsh\_spectrum()**

Return the absolute Walsh spectrum for the function.

EXAMPLES:

```
sage: from sage.crypto.boolean_function import BooleanFunction
sage: B = BooleanFunction("7969817CC5893BA6AC326E47619F5AD0")
sage: sorted([_ for _ in B.absolute_walsh_spectrum().iteritems() ])
[(0, 64), (16, 64)]
```

```
sage: B = BooleanFunction("0113077C165E76A8")
sage: B.absolute_walsh_spectrum()
{8: 64}
```

**algebraic\_immunity(annihilator=False)**

Returns the algebraic immunity of the Boolean function. This is the smallest integer  $i$  such that there exists a non trivial annihilator for  $self$  or  $self$ .

INPUT:

- **annihilator** - a Boolean (default: False), if True, returns also an annihilator of minimal degree.

EXAMPLES:

```
sage: from sage.crypto.boolean_function import BooleanFunction
sage: R.<x0,x1,x2,x3,x4,x5> = BooleanPolynomialRing(6)
sage: B = BooleanFunction(x0*x1 + x1*x2 + x2*x3 + x3*x4 + x4*x5)
sage: B.algebraic_immunity(annihilator=True)
(2, x0*x1 + x1*x2 + x2*x3 + x3*x4 + x4*x5 + 1)
sage: B[0] +=1
sage: B.algebraic_immunity()
2

sage: R.<x> = GF(2^8, 'a') []
sage: B = BooleanFunction(x^31)
sage: B.algebraic_immunity()
4
```

**algebraic\_normal\_form()**

Return the `sage.rings.polynomial.pbori.BooleanPolynomial` corresponding to the algebraic normal form.

EXAMPLES:

```
sage: from sage.crypto.boolean_function import BooleanFunction
sage: B = BooleanFunction([0,1,1,0,1,0,1,1])
sage: P = B.algebraic_normal_form()
sage: P
x0*x1*x2 + x0 + x1*x2 + x1 + x2
sage: [ P(*ZZ(i).digits(base=2, padto=3)) for i in range(8) ]
[0, 1, 1, 0, 1, 0, 1, 1]
```

**annihilator(d, dim=False)**

Return (if it exists) an annihilator of the boolean function of degree at most  $d$ , that is a Boolean polynomial

$g$  such that

$$f(x)g(x) = 0 \forall x.$$

INPUT:

- $d$  – an integer;
- $\dim$  – a Boolean (default: False), if True, return also the dimension of the annihilator vector space.

EXAMPLES:

```
sage: from sage.crypto.boolean_function import BooleanFunction
sage: f = BooleanFunction("7969817CC5893BA6AC326E47619F5AD0")
sage: f.annihilator(1) is None
True
sage: g = BooleanFunction( f.annihilator(3) )
sage: set([ fi*g(i) for i,fi in enumerate(f) ])
set([0])
```

**autocorrelation()**

Return the autocorrelation for the function, defined by

$$\Delta_f(j) = \sum_{i \in \{0,1\}^n} (-1)^{f(i) \oplus f(i \oplus j)}.$$

EXAMPLES:

```
sage: from sage.crypto.boolean_function import BooleanFunction
sage: B = BooleanFunction("03")
sage: B.autocorrelation()
(8, 8, 0, 0, 0, 0, 0, 0)
```

**correlation\_immunity()**

Return the maximum value  $m$  such that the function is correlation immune of order  $m$ .

A Boolean function is said to be correlation immune of order  $m$ , if the output of the function is statistically independent of the combination of any  $m$  of its inputs.

EXAMPLES:

```
sage: from sage.crypto.boolean_function import BooleanFunction
sage: B = BooleanFunction("7969817CC5893BA6AC326E47619F5AD0")
sage: B.correlation_immunity()
2
```

**is\_balanced()**

Return True if the function takes the value True half of the time.

EXAMPLES:

```
sage: from sage.crypto.boolean_function import BooleanFunction
sage: B = BooleanFunction(1)
sage: B.is_balanced()
False
sage: B[0] = True
sage: B.is_balanced()
True
```

**is\_bent()**

Return True if the function is bent.

EXAMPLE:

```
sage: from sage.crypto.boolean_function import BooleanFunction
sage: B = BooleanFunction("0113077C165E76A8")
sage: B.is_bent()
True
```

**is\_symmetric()**

Return True if the function is symmetric, i.e. invariant under permutation of its input bits. Another way to see it is that the output depends only on the Hamming weight of the input.

EXAMPLES:

```
sage: from sage.crypto.boolean_function import BooleanFunction
sage: B = BooleanFunction(5)
sage: B[3] = 1
sage: B.is_symmetric()
False
sage: V_B = [0, 1, 1, 0, 1, 0]
sage: for i in xrange(32): B[i] = V_B[i.popcount()]
sage: B.is_symmetric()
True
```

**nonlinearity()**

Return the nonlinearity of the function. This is the distance to the linear functions, or the number of output ones need to change to obtain a linear function.

EXAMPLE:

```
sage: from sage.crypto.boolean_function import BooleanFunction
sage: B = BooleanFunction(5)
sage: B[1] = B[3] = 1
sage: B.nonlinearity()
2
sage: B = BooleanFunction("0113077C165E76A8")
sage: B.nonlinearity()
28
```

**nvariables()**

The number of variables of this function.

EXAMPLE:

```
sage: from sage.crypto.boolean_function import BooleanFunction
sage: BooleanFunction(4).nvariables()
4
```

**resiliency\_order()**

Return the maximum value  $m$  such that the function is resilient of order  $m$ .

A Boolean function is said to be resilient of order  $m$  if it is balanced and correlation immune of order  $m$ .

If the function is not balanced, we return -1.

EXAMPLES:

```
sage: from sage.crypto.boolean_function import BooleanFunction
sage: B = BooleanFunction("077CE5A2F8831A5DF8831A5D077CE5A2699669966996669999665AA5A55A")
sage: B.resiliency_order()
3
```

**sum\_of\_square\_indicator()**

Return the sum of square indicator of the function.

EXAMPLES:

```
sage: from sage.crypto.boolean_function import BooleanFunction
sage: B = BooleanFunction("7969817CC5893BA6AC326E47619F5AD0")
sage: B.sum_of_square_indicator()
32768
```

**truth\_table** (*format='bin'*)

The truth table of the Boolean function.

INPUT: a string representing the desired format, can be either

- 'bin' (default) : we return a tuple of Boolean values
- 'int' : we return a tuple of 0 or 1 values
- 'hex' : we return a string representing the truth\_table in hexadecimal

EXAMPLE:

```
sage: from sage.crypto.boolean_function import BooleanFunction
sage: R.<x,y,z> = BooleanPolynomialRing(3)
sage: B = BooleanFunction( x*y*z + z + y + 1 )
sage: B.truth_table()
(True, True, False, False, False, True, False)
sage: B.truth_table(format='int')
(1, 1, 0, 0, 0, 0, 1, 0)
sage: B.truth_table(format='hex')
'43'

sage: BooleanFunction('00ab').truth_table(format='hex')
'00ab'

sage: B.truth_table(format='oct')
Traceback (most recent call last):
...
ValueError: unknown output format
```

**walsh\_hadamard\_transform**()

Compute the Walsh Hadamard transform  $W$  of the function  $f$ .

$$W(j) = \sum_{i \in \{0,1\}^n} (-1)^{f(i) \oplus i \cdot j}$$

EXAMPLE:

```
sage: from sage.crypto.boolean_function import BooleanFunction
sage: R.<x> = GF(2^3,'a')[ ]
sage: B = BooleanFunction( x^3 )
sage: B.walsh_hadamard_transform()
(0, 4, 0, -4, 0, -4, 0, -4)
```

**class** sage.crypto.boolean\_function.**BooleanFunctionIterator**

Bases: `object`

Iterator through the values of a Boolean function.

EXAMPLE:

```
sage: from sage.crypto.boolean_function import BooleanFunction
sage: B = BooleanFunction(3)
```

```
sage: type(B.__iter__())
<type 'sage.crypto.boolean_function.BooleanFunctionIterator'>
```

**next()**

x.next() -> the next value, or raise StopIteration

sage.crypto.boolean\_function.**random\_boolean\_function**(*n*)

Returns a random Boolean function with *n* variables.

EXAMPLE:

```
sage: from sage.crypto.boolean_function import random_boolean_function
sage: B = random_boolean_function(9)
sage: B.nvariables()
9
sage: B.nonlinearity()
217          # 32-bit
222          # 64-bit
```

sage.crypto.boolean\_function.**unpickle\_BooleanFunction**(*bool\_list*)

Specific function to unpickle Boolean functions.

EXAMPLE:

```
sage: from sage.crypto.boolean_function import BooleanFunction
sage: B = BooleanFunction([0,1,1,0])
sage: loads(dumps(B)) == B # indirect doctest
True
```





# SMALL SCALE VARIANTS OF THE AES (SR) POLYNOMIAL SYSTEM GENERATOR

Sage supports polynomial system generation for small scale (and full scale) AES variants over  $\mathbb{F}_2$  and  $\mathbb{F}_{2^e}$ . Also, Sage supports both the specification of SR as given in the papers [CMR05] and [CMR06] and a variant of SR\* which is equivalent to AES.

SR is a family of parameterizable variants of the AES suitable as a framework for comparing different cryptanalytic techniques that can be brought to bear on the AES. It is different from *Mini-AES*, whose purpose is as a teaching tool to help beginners understand the basic structure and working of the full AES.

AUTHORS:

- Martin Albrecht (2008,2009-01): usability improvements
- Martin Albrecht (2007-09): initial version
- Niles Johnson (2010-08): Trac #3893: `random_element()` should pass on `*args` and `**kwargs`.

EXAMPLES:

We construct SR(1,1,1,4) and study its properties.

```
sage: sr = mq.SR(1, 1, 1, 4)
```

`n` is the number of rounds, `r` the number of rows in the state array, `c` the number of columns in the state array, and `e` the degree of the underlying field.

```
sage: sr.n, sr.r, sr.c, sr.e
(1, 1, 1, 4)
```

By default variables are ordered reverse to as they appear, e.g.:

```
sage: print sr.R.repr_long()
Polynomial Ring
Base Ring : Finite Field in a of size 2^4
Size : 20 Variables
Block 0 : Ordering : deglex
Names : k100, k101, k102, k103, x100, x101, x102, x103, w100, w101, w102, w103, s000
```

However, this can be prevented by passing in `reverse_variables=False` to the constructor.

For SR(1, 1, 1, 4) the ShiftRows matrix isn't that interesting.:

```
sage: sr.ShiftRows
[1 0 0 0]
[0 1 0 0]
[0 0 1 0]
[0 0 0 1]
```

Also, the MixColumns matrix is the identity matrix.:

```
sage: sr.MixColumns
[1 0 0 0]
[0 1 0 0]
[0 0 1 0]
[0 0 0 1]
```

Lin, however, is not the identity matrix.:

```
sage: sr.Lin
[      a^2 + 1      1      a^3 + a^2      a^2 + 1]
[      a      a      1 a^3 + a^2 + a + 1]
[      a^3 + a      a^2      a^2      1]
[      1      a^3      a + 1      a + 1]
```

M and Mstar are identical for SR(1, 1, 1, 4):

```
sage: sr.M
[      a^2 + 1      1      a^3 + a^2      a^2 + 1]
[      a      a      1 a^3 + a^2 + a + 1]
[      a^3 + a      a^2      a^2      1]
[      1      a^3      a + 1      a + 1]

sage: sr.Mstar
[      a^2 + 1      1      a^3 + a^2      a^2 + 1]
[      a      a      1 a^3 + a^2 + a + 1]
[      a^3 + a      a^2      a^2      1]
[      1      a^3      a + 1      a + 1]
```

However, for larger instances of SR Mstar is not equal to M:

```
sage: sr = mq.SR(10, 4, 4, 8)
sage: sr.Mstar == ~sr.MixColumns * sr.M
True
```

We can compute a Groebner basis for the ideals spanned by SR instances to recover all solutions to the system.:

```
sage: sr = mq.SR(1, 1, 1, 4, gf2=True, polybori=True)
sage: K = sr.base_ring()
sage: a = K.gen()
sage: K = [a]
sage: P = [1]
sage: F, s = sr.polynomial_system(P=P, K=K)
sage: F.groebner_basis()
[k100, k101 + 1, k102, k103 + k003,
 x100 + 1, x101 + k003 + 1, x102 + k003 + 1,
 x103 + k003, w100, w101, w102 + 1, w103 + k003 + 1,
 s000 + 1, s001 + k003, s002 + k003, s003 + k003 + 1,
 k000, k001, k002 + 1]
```

Note that the order of  $k_{000}$ ,  $k_{001}$ ,  $k_{002}$  and  $k_{003}$  is little endian. Thus the result  $k_{002} + 1$ ,  $k_{001}$ ,  $k_{000}$  indicates that the key is either  $a$  or  $a + 1$ . We can verify that both keys encrypt  $P$  to the same ciphertext:

```
sage: sr(P, [a])
[0]
sage: sr(P, [a+1])
[0]
```

All solutions can easily be recovered using the variety function for ideals.:

```
sage: I = F.ideal()
sage: for V in I.variety():
...     for k,v in sorted(V.iteritems()):
...         print k,v
...     print
k003 0
k002 1
k001 0
k000 0
s003 1
s002 0
s001 0
s000 1
w103 1
w102 1
w101 0
w100 0
x103 0
x102 1
x101 1
x100 1
k103 0
k102 0
k101 1
k100 0

k003 1
k002 1
k001 0
k000 0
s003 0
s002 1
s001 1
s000 1
w103 0
w102 1
w101 0
w100 0
x103 1
x102 0
x101 0
x100 1
k103 1
k102 0
k101 1
k100 0
```

We can also verify the correctness of the variety by evaluating all ideal generators on all points.:

```
sage: for V in I.variety():
...     for f in I.gens():
...         if f.subs(V) != 0:
...             print "epic fail"
```

Note that the S-Box object for SR can be constructed with a call to `sr.sbox()`:

```
sage: sr = mq.SR(1,1,1,4, gf2=True, polybori=True)
sage: S = sr.sbox()
```

For example, we can now study the difference distribution matrix of `S`:

```
sage: S.difference_distribution_matrix()
[16 0 0 0 0 0 0 0 0 0 0 0 0 0 0]
[ 0 2 2 2 2 0 0 0 2 0 0 0 2 4 0]
[ 0 2 0 4 2 2 2 0 0 2 0 0 0 0 2]
[ 0 2 4 0 0 2 0 0 2 2 0 2 0 0 2]
[ 0 0 2 0 4 2 0 0 0 0 2 0 2 0 2]
[ 0 0 0 2 0 0 0 2 4 2 0 0 2 0 2]
[ 0 4 0 0 0 2 0 2 0 2 2 0 2 2 0]
[ 0 2 0 0 0 0 2 0 0 0 0 2 4 2 2]
[ 0 2 2 0 0 0 2 2 2 0 2 0 0 0 4]
[ 0 0 2 2 0 0 0 0 0 2 2 4 0 2 0]
[ 0 0 2 0 2 0 2 2 0 4 0 2 2 0 0]
[ 0 0 0 0 2 0 2 0 2 2 4 0 0 2 0]
[ 0 0 0 2 0 4 2 0 2 0 2 2 2 0 0]
[ 0 0 0 0 2 2 0 4 2 0 0 2 0 2 0]
[ 0 0 2 2 0 2 4 2 0 0 0 0 2 2 0]
[ 0 2 0 2 2 0 0 2 0 0 2 2 0 0 4]
```

or use `S` to find alternative polynomial representations for the S-Box.:

```
sage: S.polynomials(degree=3)
[x0*x1 + x1*x2 + x0*x3 + x0*y2 + x1 + y0 + y1 + 1,
 x0*x1 + x0*x2 + x0*y0 + x0*y1 + x0*y2 + x1 + x2 + y0 + y1 + y2,
 x0*x1 + x0*x2 + x0*x3 + x1*x3 + x0*y0 + x1*y0 + x0*y1 + x0*y3,
 x0*x1 + x0*x2 + x0*x3 + x1*x3 + x0*y0 + x1*y1 + x0*y3 + x1 + y0 + y1 + 1,
 x0*x1 + x0*x2 + x0*y2 + x1*y2 + x0*y3 + x0 + x1,
 x0*x3 + x1*x3 + x0*y1 + x0*y2 + x1*y3 + x0 + x1 + x2 + x3 + y0 + y1 + y3 + 1,
 x0*x1 + x1*x3 + x2*x3 + x0*y0 + x0*y2 + x0*y3 + x2 + y0 + y3,
 x0*x1 + x0*x2 + x0*x3 + x1*x3 + x2*y0 + x0*y2 + x0 + x2 + x3 + y3,
 x0*x3 + x1*x3 + x0*y0 + x2*y1 + x0*y2 + x3 + y3,
 x0*x1 + x0*x2 + x0*y0 + x0*y1 + x2*y2 + x0*y3 + x1 + y0 + y1 + 1,
 x0*x3 + x1*x3 + x0*y0 + x0*y1 + x0*y3 + x2*y3 + y0 + y3,
 x0*x1 + x0*x2 + x3*y0 + x0*y1 + x0*y3 + y0,
 x0*y0 + x0*y1 + x3*y1 + x0 + x2 + y0 + y3,
 x0*y0 + x3*y2 + y0,
 x0*x1 + x0*x2 + x0*x3 + x1*x3 + x0*y0 + x0*y2 + x3*y3 + y0,
 x0*x2 + x0*x3 + x0*y1 + y0*y1 + x0*y3 + x2 + x3 + y3,
 x0*x2 + x0*y0 + y0*y2 + x0*y3 + x0 + y0,
 x0*x1 + x0*x2 + x1*x3 + x0*y2 + y0*y3 + y0,
 x0*x1 + x0*y0 + y1*y2 + x0*y3 + x1 + x2 + y0 + 1,
 x0*x2 + x1*x3 + x0*y1 + x0*y2 + x0*y3 + y1*y3 + x0 + y0 + y3,
 x0*x1 + x0*x2 + x0*x3 + x0*y1 + x0*y2 + x0*y3 + y2*y3 + x0 + x1 + x2 + x3 + y1 + y3 + 1,
 x0*x1*x2 + x0*x3 + x0*y0 + x0*y1 + x0*y2 + x0,
 x0*x1*x3 + x0*x2 + x0*x3 + x0*y1 + x0*y3 + x0,
 x0*x1*y0 + x0*x1 + x0*y0 + x0,
 x0*x1*y1,
```

```

x0*x1*y2 + x0*x2 + x0*y2 + x0*y3 + x0,
x0*x1*y3 + x0*x1 + x0*x3 + x0*y0 + x0*y1 + x0*y2 + x0,
x0*x2*x3 + x0*x1 + x0*x3 + x0*y1 + x0*y2 + x0*y3 + x0,
x0*x2*y0 + x0*x1 + x0*x2 + x0*x3 + x0*y1 + x0*y2,
x0*x2*y1 + x0*x2 + x0*x3 + x0*y0 + x0*y1 + x0*y2 + x0,
x0*x2*y2 + x0*x2 + x0*y3 + x0,
x0*x2*y3 + x0*x2 + x0*y3 + x0,
x0*x3*y0 + x0*x1 + x0*x2 + x0*y0 + x0*y1 + x0*y3,
x0*x3*y1 + x0*x2 + x0*y1 + x0*y3 + x0,
x0*x3*y2,
x0*x3*y3 + x0*x1 + x0*y1 + x0*y2 + x0*y3 + x0,
x0*y0*y1 + x0*y1,
x0*y0*y2 + x0*x2 + x0*y3 + x0,
x0*y0*y3 + x0*x1 + x0*x3 + x0*y0 + x0*y1 + x0*y2 + x0*y3 + x0,
x0*y1*y2 + x0*x2 + x0*y3 + x0,
x0*y1*y3 + x0*x3 + x0*y0 + x0*y2 + x0*y3,
x0*y2*y3 + x0*y2,
x1*x2*x3 + x0*x1 + x1*x3 + x0*y0 + x0*y1 + x2 + x3 + y3,
x1*x2*y0 + x0*x1 + x1*x3 + x0*y0 + x0*y1 + x2 + x3 + y3,
x1*x2*y1 + x0*x1 + x1*x3 + x0*y0 + x1 + x2 + x3 + y0 + y1 + y3 + 1,
x1*x2*y2 + x0*x1 + x0*y0 + x0*y1 + x0 + x1 + y0 + y1 + 1,
x1*x2*y3 + x0*x1 + x1*x3 + x0*y0 + x1 + x2 + x3 + y0 + y1 + y3 + 1,
x1*x3*y0 + x0*x1 + x0*x2 + x0*x3 + x1*x3 + x0*y0 + x0*y1 + x0*y3,
x1*x3*y1 + x0*x2 + x0*x3 + x0*y3 + x2 + x3 + y3,
x1*x3*y2 + x0*x2 + x0*x3 + x1*x3 + x0*y1 + x0*y3 + x0,
x1*x3*y3 + x0*x1 + x0*x2 + x0*x3 + x0*y0 + x0*y1 + x0*y3,
x1*y0*y1 + x0*x2 + x0*x3 + x0*y3 + x2 + x3 + y3,
x1*y0*y2 + x0*x2 + x0*x3 + x1*x3 + x0*y1 + x0*y3 + x0,
x1*y0*y3,
x1*y1*y2 + x0*x1 + x0*x2 + x0*x3 + x1*x3 + x0*y0 + x0*y3 + x1 + y0 + y1 + 1,
x1*y1*y3 + x0*x1 + x1*x3 + x0*y0 + x1 + x2 + x3 + y0 + y1 + y3 + 1,
x1*y2*y3 + x0*x1 + x0*x2 + x1*x3 + x0*y0 + x0*y2 + x0*y3 + x0 + x1 + x2 + x3 + y0 + y1 + y3 + 1,
x2*x3*y0 + x0*x1 + x0*x3 + x1*x3 + x0*y2 + x0*y3 + x2 + x3 + y3,
x2*x3*y1 + x0*y1 + x0*y2 + x0*y3 + x3 + y0,
x2*x3*y2 + x1*x3 + x0*y1 + x0 + x2 + x3 + y3,
x2*x3*y3,
x2*y0*y1 + x0*x2 + x0*x3 + x0*y0 + x0*y1 + x0*y2 + x0,
x2*y0*y2 + x0*x2 + x1*x3 + x0*y1 + x0*y3 + x2 + x3 + y3,
x2*y0*y3 + x0*x2 + x0*y3 + x0,
x2*y1*y2 + x0*x1 + x0*x2 + x1*x3 + x0*y0 + x0*y3 + x0 + x1 + x2 + x3 + y0 + y1 + y3 + 1,
x2*y1*y3 + x0*x3 + x1*x3 + x0*y0 + x0*y1 + x0*y3 + y0 + y3,
x2*y2*y3 + x0*x1 + x0*x2 + x1*x3 + x0*y0 + x0*y3 + x0 + x1 + x2 + x3 + y0 + y1 + y3 + 1,
x3*y0*y1 + x0*x3 + x0*y1 + x0 + x2 + x3 + y3,
x3*y0*y2 + x0*y0 + y0,
x3*y0*y3 + x1*x3 + x0*y1 + x0*y2 + x0*y3 + y0,
x3*y1*y2 + x0*x2 + x0*x3 + x0*y3 + x2 + x3 + y3,
x3*y1*y3 + x0*x2 + x0*x3 + x0*y0 + x0*y2 + x0,
x3*y2*y3 + x0*x2 + x0*x3 + x1*x3 + x0*y0 + x0*y1 + x0*y3 + x0 + y0,
y0*y1*y2 + x0*x3 + x0 + x2 + x3 + y3,
y0*y1*y3 + x0*x3 + x0*y0 + x0*y2 + x0*y3,
y0*y2*y3 + x0*x3 + x1*x3 + x0*y0 + x0*y1 + y0,
y1*y2*y3 + x0*x1 + x0*x2 + x1*x3 + x0*y0 + x0*y3 + x0 + x1 + x2 + x3 + y0 + y1 + y3 + 1]

```

**sage:** `S.interpolation_polynomial()`

`(a^2 + 1)*x^14 + x^13 + (a^3 + a^2)*x^11 + (a^2 + 1)*x^7 + a^2 + a`

The `SR_gf2_2` gives an example how use alternative polynomial representations of the S-Box for construction of polynomial systems.

## TESTS:

```
sage: sr == loads(dumps(sr))
True
```

## REFERENCES:

**class** `sage.crypto.mq.sr.AllowZeroInversionsContext` (*sr*)  
Temporarily allow zero inversion.

`sage.crypto.mq.sr.SR` (*n=1, r=1, c=1, e=4, star=False, \*\*kwargs*)  
Return a small scale variant of the AES polynomial system constructor subject to the following conditions:

## INPUT:

- *n* - the number of rounds (default: 1)
- *r* - the number of rows in the state array (default: 1)
- *c* - the number of columns in the state array (default: 1)
- *e* - the exponent of the finite extension field (default: 4)
- *star* - determines if  $SR^*$  or  $SR$  should be constructed (default: `False`)
- *aes\_mode* - as the  $SR$  key schedule specification differs slightly from the AES key schedule, this parameter controls which schedule to use (default: `True`)
- *gf2* - generate polynomial systems over  $\mathbf{F}_2$  rather than over  $\mathbf{F}_{2^e}$  (default: `False`)
- *polybori* - use the `BooleanPolynomialRing` as polynomial representation (default: `True`,  $\mathbf{F}_2$  only)
- *order* - a string to specify the term ordering of the variables (default: `deglex`)
- *postfix* - a string which is appended after the variable name (default: `''`)
- *allow\_zero\_inversions* - a boolean to control whether zero inversions raise an exception (default: `False`)
- *correct\_only* - only include correct inversion polynomials (default: `False`,  $\mathbf{F}_2$  only)
- *biaffine\_only* - only include bilinear and biaffine inversion polynomials (default: `True`,  $\mathbf{F}_2$  only)

## EXAMPLES:

```
sage: sr = mq.SR(1, 1, 1, 4)
sage: ShiftRows = sr.shift_rows_matrix()
sage: MixColumns = sr.mix_columns_matrix()
sage: Lin = sr.lin_matrix()
sage: M = MixColumns * ShiftRows * Lin
sage: print sr.hex_str_matrix(M)
5 1 C 5
2 2 1 F
A 4 4 1
1 8 3 3

sage: sr = mq.SR(1, 2, 1, 4)
sage: ShiftRows = sr.shift_rows_matrix()
sage: MixColumns = sr.mix_columns_matrix()
sage: Lin = sr.lin_matrix()
sage: M = MixColumns * ShiftRows * Lin
sage: print sr.hex_str_matrix(M)
F 3 7 F A 2 B A
A A 5 6 8 8 4 9
```

```

7 8 8 2 D C C 3
4 6 C C 5 E F F
A 2 B A F 3 7 F
8 8 4 9 A A 5 6
D C C 3 7 8 8 2
5 E F F 4 6 C C

```

```

sage: sr = mq.SR(1, 2, 2, 4)
sage: ShiftRows = sr.shift_rows_matrix()
sage: MixColumns = sr.mix_columns_matrix()
sage: Lin = sr.lin_matrix()
sage: M = MixColumns * ShiftRows * Lin
sage: print sr.hex_str_matrix(M)
F 3 7 F 0 0 0 0 0 0 0 0 0 0 A 2 B A
A A 5 6 0 0 0 0 0 0 0 0 0 0 8 8 4 9
7 8 8 2 0 0 0 0 0 0 0 0 0 0 D C C 3
4 6 C C 0 0 0 0 0 0 0 0 0 0 5 E F F
A 2 B A 0 0 0 0 0 0 0 0 0 0 F 3 7 F
8 8 4 9 0 0 0 0 0 0 0 0 0 0 A A 5 6
D C C 3 0 0 0 0 0 0 0 0 0 0 7 8 8 2
5 E F F 0 0 0 0 0 0 0 0 0 0 4 6 C C
0 0 0 0 A 2 B A F 3 7 F 0 0 0 0
0 0 0 0 8 8 4 9 A A 5 6 0 0 0 0
0 0 0 0 D C C 3 7 8 8 2 0 0 0 0
0 0 0 0 5 E F F 4 6 C C 0 0 0 0
0 0 0 0 F 3 7 F A 2 B A 0 0 0 0
0 0 0 0 A A 5 6 8 8 4 9 0 0 0 0
0 0 0 0 7 8 8 2 D C C 3 0 0 0 0
0 0 0 0 4 6 C C 5 E F F 0 0 0 0

```

**class** sage.crypto.mq.sr.**SR\_generic** (*n=1, r=1, c=1, e=4, star=False, \*\*kwargs*)  
 Bases: sage.crypto.mq.mpolynomialssystemgenerator.MPolynomialSystemGenerator

Small Scale Variants of the AES.

EXAMPLES:

```

sage: sr = mq.SR(1, 1, 1, 4)
sage: ShiftRows = sr.shift_rows_matrix()
sage: MixColumns = sr.mix_columns_matrix()
sage: Lin = sr.lin_matrix()
sage: M = MixColumns * ShiftRows * Lin
sage: print sr.hex_str_matrix(M)
5 1 C 5
2 2 1 F
A 4 4 1
1 8 3 3

```

**add\_round\_key** (*d, key*)

Perform the AddRoundKey operation on *d* using *key*.

INPUT:

- *d* - state array or something coercible to a state array
- *key* - state array or something coercible to a state array

EXAMPLE:

```

sage: sr = mq.SR(10, 4, 4, 4)
sage: D = sr.random_state_array()

```

```
sage: K = sr.random_state_array()
sage: sr.add_round_key(D, K) == K + D
True
```

**base\_ring()**

Return the base field of self as determined by `self.e`.

EXAMPLE:

```
sage: sr = mq.SR(10, 2, 2, 4)
sage: sr.base_ring().polynomial()
a^4 + a + 1
```

The Rijndael polynomial:

```
sage: sr = mq.SR(10, 4, 4, 8)
sage: sr.base_ring().polynomial()
a^8 + a^4 + a^3 + a + 1
```

**block\_order()**

Return a block order for self where each round is a block.

EXAMPLE:

```
sage: sr = mq.SR(2, 1, 1, 4)
sage: sr.block_order()
```

Block term order with blocks:

(Degree lexicographic term order of length 16,  
Degree lexicographic term order of length 16,  
Degree lexicographic term order of length 4)

```
sage: P = sr.ring(order='block')
```

```
sage: print P.repr_long()
```

Polynomial Ring

Base Ring : Finite Field in a of size 2^4

Size : 36 Variables

Block 0 : Ordering : deglex

Names : k200, k201, k202, k203, x200, x201, x202, x203, w200, w201, w202, w

Block 1 : Ordering : deglex

Names : k100, k101, k102, k103, x100, x101, x102, x103, w100, w101, w102, w

Block 2 : Ordering : deglex

Names : k000, k001, k002, k003

**hex\_str(M, typ='matrix')**

Return a hex string for the provided AES state array/matrix.

INPUT:

- M - state array

- typ - controls what to return, either 'matrix' or 'vector' (default: 'matrix')

EXAMPLE:

```
sage: sr = mq.SR(2, 2, 2, 4)
sage: k = sr.base_ring()
sage: A = matrix(k, 2, 2, [1, k.gen(), 0, k.gen()^2])
sage: sr.hex_str(A)
' 1 2 \n 0 4 \n'
```



```
sage: sr.hex_str(A, typ='vector')
'1024'
```

### **hex\_str\_matrix**(*M*)

Return a two-dimensional AES-like representation of the matrix *M*.

That is, show the finite field elements as hex strings.

INPUT:

- *M* - an AES state array

EXAMPLE:

```
sage: sr = mq.SR(2, 2, 2, 4)
sage: k = sr.base_ring()
sage: A = matrix(k, 2, 2, [1, k.gen(), 0, k.gen()^2])
sage: sr.hex_str_matrix(A)
' 1 2 \n 0 4 \n'
```

### **hex\_str\_vector**(*M*)

Return a one-dimensional AES-like representation of the matrix *M*.

That is, show the finite field elements as hex strings.

INPUT:

- *M* - an AES state array

EXAMPLE:

```
sage: sr = mq.SR(2, 2, 2, 4)
sage: k = sr.base_ring()
sage: A = matrix(k, 2, 2, [1, k.gen(), 0, k.gen()^2])
sage: sr.hex_str_vector(A)
'1024'
```

### **is\_state\_array**(*d*)

Return True if *d* is a state array, i.e. has the correct dimensions and base field.

EXAMPLE:

```
sage: sr = mq.SR(2, 2, 4, 8)
sage: k = sr.base_ring()
sage: sr.is_state_array( matrix(k, 2, 4) )
True

sage: sr = mq.SR(2, 2, 4, 8)
sage: k = sr.base_ring()
sage: sr.is_state_array( matrix(k, 4, 4) )
False
```

### **key\_schedule**(*kj*, *i*)

Return  $k_i$  for a given *i* and  $k_j$  with  $j = i - 1$ .

EXAMPLES:

```
sage: sr = mq.SR(10, 4, 4, 8, star=True, allow_zero_inversions=True)
sage: ki = sr.state_array()
sage: for i in range(10):
...     ki = sr.key_schedule(ki, i+1)
sage: print sr.hex_str_matrix(ki)
B4 3E 23 6F
```

```
EF 92 E9 8F
5B E2 51 18
CB 11 CF 8E
```

**key\_schedule\_polynomials(*i*)**

Return polynomials for the *i*-th round of the key schedule.

INPUT:

- *i* - round ( $0 \leq i \leq n$ )

EXAMPLE:

```
sage: sr = mq.SR(1, 1, 1, 4, gf2=True, polybori=False)
```

The 0-th subkey is the user provided key, so only conjugacy relations or field polynomials are added.:

```
sage: sr.key_schedule_polynomials(0)
(k000^2 + k000, k001^2 + k001, k002^2 + k002, k003^2 + k003)
```

The 1-th subkey is derived from the user provided key according to the key schedule which is non-linear.:

```
sage: sr.key_schedule_polynomials(1)
(k100 + s000 + s002 + s003,
 k101 + s000 + s001 + s003 + 1,
 k102 + s000 + s001 + s002 + 1,
 k103 + s001 + s002 + s003 + 1,
 k100^2 + k100, k101^2 + k101, k102^2 + k102, k103^2 + k103,
 s000^2 + s000, s001^2 + s001, s002^2 + s002, s003^2 + s003,
 s000*k000 + s000*k003 + s001*k002 + s002*k001 + s003*k000,
 s000*k000 + s000*k001 + s001*k000 + s001*k003 + s002*k002 + s003*k001,
 s000*k001 + s000*k002 + s001*k000 + s001*k001 + s002*k000 + s002*k003 + s003*k002,
 s000*k000 + s000*k001 + s000*k003 + s001*k001 + s002*k000 + s002*k002 + s003*k000 + k000,
 s000*k002 + s001*k000 + s001*k001 + s001*k003 + s002*k001 + s003*k000 + s003*k002 + k001,
 s000*k000 + s000*k001 + s000*k002 + s001*k002 + s002*k000 + s002*k001 + s002*k003 + s003*k003,
 s000*k001 + s001*k000 + s001*k002 + s002*k000 + s003*k001 + s003*k003 + k003,
 s000*k000 + s000*k002 + s000*k003 + s001*k000 + s001*k001 + s002*k002 + s003*k000 + s000,
 s000*k001 + s000*k003 + s001*k001 + s001*k002 + s002*k000 + s002*k003 + s003*k001 + s001,
 s000*k000 + s000*k002 + s001*k000 + s001*k002 + s001*k003 + s002*k000 + s002*k001 + s003*k003,
 s000*k001 + s000*k002 + s001*k000 + s001*k003 + s002*k001 + s003*k003 + s003,
 s000*k002 + s001*k001 + s002*k000 + s003*k003 + 1)
```

**mix\_columns(*d*)**

Perform the MixColumns operation on *d*.

INPUT:

- *d* - state array or something coercible to a state array

EXAMPLES:

```
sage: sr = mq.SR(10, 4, 4, 4)
sage: E = sr.state_array() + 1; E
[1 0 0 0]
[0 1 0 0]
[0 0 1 0]
[0 0 0 1]

sage: sr.mix_columns(E)
[ a a + 1      1      1]
[      1      a a + 1      1]
```

$$\begin{bmatrix} 1 & 1 & a & a + 1 \\ a + 1 & 1 & 1 & a \end{bmatrix}$$

**new\_generator** (\*\*kws)

Return a new SR instance equal to this instance except for the parameters passed explicitly to this function.

INPUT:

- \*\*kws - see the SR constructor for accepted parameters

EXAMPLE:

```
sage: sr = mq.SR(2,1,1,4); sr
SR(2,1,1,4)
sage: sr.ring().base_ring()
Finite Field in a of size 2^4

sage: sr2 = sr.new_generator(gf2=True); sr2
SR(2,1,1,4)
sage: sr2.ring().base_ring()
Finite Field of size 2
sage: sr3 = sr2.new_generator(correct_only=True)
sage: len(sr2.inversion_polynomials_single_sbox())
20
sage: len(sr3.inversion_polynomials_single_sbox())
19
```

**polynomial\_system** (P=None, K=None, C=None)

Return a polynomial system for this small scale AES variant for a given plaintext-key pair.

If neither P, K nor C are provided, a random pair (P, K) will be generated. If P and C are provided no K needs to be provided.

INPUT:

- P - vector, list, or tuple (default: None)
- K - vector, list, or tuple (default: None)
- C - vector, list, or tuple (default: None)

EXAMPLE:

```
sage: sr = mq.SR(1, 1, 1, 4, gf2=True, polybori=True)
sage: P = sr.vector([0, 0, 1, 0])
sage: K = sr.vector([1, 0, 0, 1])
sage: F, s = sr.polynomial_system(P, K)
```

This returns a polynomial system:

```
sage: F
Polynomial Sequence with 36 Polynomials in 20 Variables
```

and a solution:

```
sage: s # random -- maybe we need a better doctest here?
{k000: 1, k001: 0, k003: 1, k002: 0}
```

This solution is not the only solution that we can learn from the Groebner basis of the system.

```
sage: F.groebner_basis() [-3:]
[k000 + 1, k001, k003 + 1]
```

In particular we have two solutions:

```
sage: len(F.ideal().variety())
2
```

In the following example we provide C explicitly:

```
sage: C = sr(P,K)
sage: F,s = sr.polynomial_system(P=P, C=C)
sage: F
Polynomial Sequence with 36 Polynomials in 20 Variables
```

Alternatively, we can use symbols for the P and C. First, we have to create a polynomial ring:

```
sage: sr = mq.SR(1, 1, 1, 4, gf2=True, polybori=True)
sage: R = sr.R
sage: vn = sr.varstrs("P",0,1,4) + R.variable_names() + sr.varstrs("C",0,1,4)
sage: R = BooleanPolynomialRing(len(vn),vn)
sage: sr.R = R
```

Now, we can construct the purely symbolic equation system:

```
sage: C = sr.vars("C",0); C
(C000, C001, C002, C003)
sage: P = sr.vars("P",0)
sage: F,s = sr.polynomial_system(P=P,C=C)
sage: [(k,v) for k,v in sorted(s.iteritems())] # this can be ignored
[(k003, 1), (k002, 1), (k001, 0), (k000, 1)]
sage: F
Polynomial Sequence with 36 Polynomials in 28 Variables
sage: F.part(0)
(P000 + w100 + k000, P001 + w101 + k001, P002 + w102 + k002, P003 + w103 + k003)
sage: F.part(-2)
(k100 + x100 + x102 + x103 + C000, k101 + x100 + x101 + x103 + C001 + 1, ...)
```

We show that the (returned) key is a solution to the returned system:

```
sage: sr = mq.SR(3,4,4,8, star=True, gf2=True, polybori=True)
sage: F,s = sr.polynomial_system()
sage: F.subs(s).groebner_basis() # long time
Polynomial Sequence with 1248 Polynomials in 1248 Variables
```

**random\_element** (*elem\_type*=*'vector'*, \**args*, \*\**kws*)

Return a random element for self. Other arguments and keywords are passed to random\_\* methods.

INPUT:

- elem\_type* - either *'vector'* or *'state array'* (default: *'vector'*)

EXAMPLE:

```
sage: sr = mq.SR()
sage: sr.random_element()
[ a^2]
[ a + 1]
[a^2 + 1]
[ a]
sage: sr.random_element('state_array')
[a^3 + a + 1]
```

Passes extra positional or keyword arguments through:

```

sage: sr.random_element(density=0)
[0]
[0]
[0]
[0]

```

**random\_state\_array** (\*args, \*\*kws)

Return a random element in MatrixSpace(self.base\_ring(), self.r, self.c).

EXAMPLE:

```

sage: sr = mq.SR(2, 2, 2, 4)
sage: sr.random_state_array()
[      a^2      a^3 + a + 1]
[a^3 + a^2 + a + 1      a + 1]

```

**random\_vector** (\*args, \*\*kws)

Return a random vector as it might appear in the algebraic expression of self.

EXAMPLE:

```

sage: sr = mq.SR(2, 2, 2, 4)
sage: sr.random_vector()
[      a^2]
[      a + 1]
[      a^2 + 1]
[      a]
[a^3 + a^2 + a + 1]
[      a^3 + a]
[      a^3]
[      a^3 + a^2]
[      a^3 + a + 1]
[      a^3 + 1]
[      a^3 + a^2 + 1]
[      a^3 + a^2 + a]
[      a + 1]
[      a^2 + 1]
[      a]
[      a^2]

```

---

**Note:**  $\phi$  was already applied to the result.

---

**ring** (order=None, reverse\_variables=None)

Construct a ring as a base ring for the polynomial system.

By default, variables are ordered in the reverse of their natural ordering, i.e. the reverse of as they appear.

INPUT:

- order - a monomial ordering (default: None)
- reverse\_variables - reverse rounds of variables (default: True)

The variable assignment is as follows:

- $k_{i,j,l}$  - subkey round  $i$  word  $j$  conjugate/bit  $l$
- $s_{i,j,l}$  - subkey inverse round  $i$  word  $j$  conjugate/bit  $l$
- $w_{i,j,l}$  - inversion input round  $i$  word  $j$  conjugate/bit  $l$

- $x_{i,j,l}$  - inversion output round  $i$  word  $j$  conjugate/bit  $l$

Note that the variables are ordered in column major ordering in the state array and that the bits are ordered in little endian ordering.

For example, if  $x_{0,1,0}$  is a variable over  $\mathbf{F}_2$  for  $r = 2$  and  $c = 2$  then refers to the *most* significant bit of the entry in the position (1,0) in the state array matrix.

EXAMPLE:

```
sage: sr = mq.SR(2, 1, 1, 4)
sage: P = sr.ring(order='block')
sage: print P.repr_long()
Polynomial Ring
  Base Ring : Finite Field in a of size 2^4
    Size : 36 Variables
  Block 0 : Ordering : deglex
    Names : k200, k201, k202, k203, x200, x201, x202, x203, w200, w201, w202, w
  Block 1 : Ordering : deglex
    Names : k100, k101, k102, k103, x100, x101, x102, x103, w100, w101, w102, w
  Block 2 : Ordering : deglex
    Names : k000, k001, k002, k003
```

**round\_polynomials** ( $i$ , *plaintext*=None, *ciphertext*=None)

Return list of polynomials for a given round  $i$ .

If  $i == 0$  a plaintext must be provided, if  $i == n$  a ciphertext must be provided.

INPUT:

- $i$  - round number
- *plaintext* - optional plaintext (mandatory in first round)
- *ciphertext* - optional ciphertext (mandatory in last round)

OUTPUT: tuple

EXAMPLE:

```
sage: sr = mq.SR(1, 1, 1, 4)
sage: k = sr.base_ring()
sage: p = [k.random_element() for _ in range(sr.r*sr.c)]
sage: sr.round_polynomials(0, plaintext=p)
(w100 + k000 + (a^2 + 1), w101 + k001 + (a), w102 + k002 + (a^2), w103 + k003 + (a + 1))
```

**sbox** (*inversion\_only*=False)

Return an S-Box object for this SR instance.

INPUT:

- *inversion\_only* - do not include the  $\mathbf{F}_2$  affine map when computing the S-Box (default: False)

EXAMPLE:

```
sage: sr = mq.SR(1, 2, 2, 4, allow_zero_inversions=True)
sage: S = sr.sbox(); S
(6, 11, 5, 4, 2, 14, 7, 10, 9, 13, 15, 12, 3, 1, 0, 8)

sage: sr.sub_byte(0)
a^2 + a
sage: sage_eval(str(sr.sub_byte(0)), {'a':2})
6
sage: S(0)
```

6

```

sage: sr.sub_byte(1)
a^3 + a + 1
sage: sage_eval(str(sr.sub_byte(1)), {'a':2})
11
sage: S(1)
11

sage: sr = mq.SR(1,2,2,8, allow_zero_inversions=True)
sage: S = sr.sbox(); S
(99, 124, 119, 123, 242, 107, 111, 197, 48, 1, 103, 43,
254, 215, 171, 118, 202, 130, 201, 125, 250, 89, 71, 240,
173, 212, 162, 175, 156, 164, 114, 192, 183, 253, 147, 38,
54, 63, 247, 204, 52, 165, 229, 241, 113, 216, 49, 21, 4,
199, 35, 195, 24, 150, 5, 154, 7, 18, 128, 226, 235, 39,
178, 117, 9, 131, 44, 26, 27, 110, 90, 160, 82, 59, 214,
179, 41, 227, 47, 132, 83, 209, 0, 237, 32, 252, 177, 91,
106, 203, 190, 57, 74, 76, 88, 207, 208, 239, 170, 251,
67, 77, 51, 133, 69, 249, 2, 127, 80, 60, 159, 168, 81,
163, 64, 143, 146, 157, 56, 245, 188, 182, 218, 33, 16,
255, 243, 210, 205, 12, 19, 236, 95, 151, 68, 23, 196,
167, 126, 61, 100, 93, 25, 115, 96, 129, 79, 220, 34, 42,
144, 136, 70, 238, 184, 20, 222, 94, 11, 219, 224, 50, 58,
10, 73, 6, 36, 92, 194, 211, 172, 98, 145, 149, 228, 121,
231, 200, 55, 109, 141, 213, 78, 169, 108, 86, 244, 234,
101, 122, 174, 8, 186, 120, 37, 46, 28, 166, 180, 198,
232, 221, 116, 31, 75, 189, 139, 138, 112, 62, 181, 102,
72, 3, 246, 14, 97, 53, 87, 185, 134, 193, 29, 158, 225,
248, 152, 17, 105, 217, 142, 148, 155, 30, 135, 233, 206,
85, 40, 223, 140, 161, 137, 13, 191, 230, 66, 104, 65,
153, 45, 15, 176, 84, 187, 22)

sage: sr.sub_byte(0)
a^6 + a^5 + a + 1
sage: sage_eval(str(sr.sub_byte(0)), {'a':2})
99
sage: S(0)
99

sage: sr.sub_byte(1)
a^6 + a^5 + a^4 + a^3 + a^2
sage: sage_eval(str(sr.sub_byte(1)), {'a':2})
124

sage: S(1)
124

sage: sr = mq.SR(1,2,2,4, allow_zero_inversions=True)
sage: S = sr.sbox(inversion_only=True); S
(0, 1, 9, 14, 13, 11, 7, 6, 15, 2, 12, 5, 10, 4, 3, 8)

sage: S(0)
0
sage: S(1)
1

```

```
sage: S(sr.k.gen())
a^3 + 1
```

**sbox\_constant()**

Return the S-Box constant which is added after  $L(x^{-1})$  was performed. That is  $0 \times 63$  if  $e == 8$  or  $0 \times 6$  if  $e == 4$ .

EXAMPLE:

```
sage: sr = mq.SR(10, 1, 1, 8)
sage: sr.sbox_constant()
a^6 + a^5 + a + 1
```

**shift\_rows(d)**

Perform the ShiftRows operation on d.

INPUT:

- d - state array or something coercible to a state array

EXAMPLES:

```
sage: sr = mq.SR(10, 4, 4, 4)
sage: E = sr.state_array() + 1; E
[1 0 0 0]
[0 1 0 0]
[0 0 1 0]
[0 0 0 1]

sage: sr.shift_rows(E)
[1 0 0 0]
[1 0 0 0]
[1 0 0 0]
[1 0 0 0]
```

**state\_array(d=None)**

Convert the parameter to a state array.

INPUT:

- d - a matrix, a list, or a tuple (default: None)

EXAMPLES:

```
sage: sr = mq.SR(2, 2, 2, 4)
sage: k = sr.base_ring()
sage: e1 = [k.fetch_int(e) for e in range(2*2)]; e1
[0, 1, a, a + 1]
sage: e2 = sr.phi( Matrix(k, 2*2, 1, e1) )
sage: sr.state_array(e1) # note the column major ordering
[ 0 a]
[ 1 a + 1]
sage: sr.state_array(e2)
[ 0 a]
[ 1 a + 1]

sage: sr.state_array()
[0 0]
[0 0]
```



**sub\_byte** (*b*)

Perform SubByte on a single byte/halfbyte *b*.

A `ZeroDivision` exception is raised if an attempt is made to perform an inversion on the zero element. This can be disabled by passing `allow_zero_inversion=True` to the constructor. A zero inversion can result in an inconsistent equation system.

INPUT:

- *b* - an element in `self.base_ring()`

EXAMPLE:

The S-Box table for  $\mathbb{F}_{2^4}$ :

```
sage: sr = mq.SR(1, 1, 1, 4, allow_zero_inversions=True)
sage: for e in sr.base_ring():
...     print '% 20s % 20s'%(e, sr.sub_byte(e))
      0                a^2 + a
      a                a^2 + 1
      a^2              a
      a^3              a^3 + 1
      a + 1            a^2
      a^2 + a          a^2 + a + 1
      a^3 + a^2        a + 1
      a^3 + a + 1      a^3 + a^2
      a^2 + 1          a^3 + a^2 + a
      a^3 + a          a^3 + a^2 + a + 1
      a^2 + a + 1      a^3 + a
      a^3 + a^2 + a    0
      a^3 + a^2 + a + 1 a^3
      a^3 + a^2 + 1    1
      a^3 + 1          a^3 + a^2 + 1
      1                a^3 + a + 1
```

**sub\_bytes** (*d*)

Perform the non-linear transform on *d*.

INPUT:

- *d* - state array or something coercible to a state array

EXAMPLE:

```
sage: sr = mq.SR(2, 1, 2, 8, gf2=True)
sage: k = sr.base_ring()
sage: A = Matrix(k, 1, 2, [k(1), k.gen()])
sage: sr.sub_bytes(A)
[ a^6 + a^5 + a^4 + a^3 + a^2 a^6 + a^5 + a^4 + a^2 + a + 1]
```

**varformatstr** (*name*, *n=None*, *rc=None*, *e=None*)

Return a format string which is understood by `print` et al.

If a numerical value is omitted, the default value of `self` is used. The numerical values (*n*, *rc*, *e*) are used to determine the width of the respective fields in the format string.

INPUT:

- *name* - name of the variable
- *n* - number of rounds (default: `None`)
- *rc* - number of rows \* number of cols (default: `None`)

- e - exponent of base field (default: None)

EXAMPLE:

```
sage: sr = mq.SR(1, 2, 2, 4)
sage: sr.varformatstr('x')
'x%01d%01d%01d'
sage: sr.varformatstr('x', n=1000)
'x%03d%03d%03d'
```

**variable\_dict()**

Return a dictionary to access variables in `self.R` by their names.

EXAMPLE:

```
sage: sr = mq.SR(1, 1, 1, 4)
sage: sr.variable_dict()
{'x101': x101, 'x100': x100, 'x103': x103, 'x102': x102,
 's002': s002, 'w100': w100, 'w101': w101, 'w102': w102,
 'w103': w103, 'k100': k100, 'k101': k101, 'k102': k102,
 'k103': k103, 's003': s003, 's001': s001, 'k002': k002,
 'k001': k001, 'k000': k000, 'k003': k003, 's000': s000}

sage: sr = mq.SR(1, 1, 1, 4, gf2=True)
sage: sr.variable_dict()
{'x101': x101, 'x100': x100, 'x103': x103, 'x102': x102,
 's002': s002, 'w100': w100, 'w101': w101, 'w102': w102,
 'w103': w103, 'k100': k100, 'k101': k101, 'k102': k102,
 'k103': k103, 's003': s003, 's001': s001, 'k002': k002,
 'k001': k001, 'k000': k000, 'k003': k003, 's000': s000}
```

**vars** (*name, nr, rc=None, e=None*)

Return a list of variables in `self`.

INPUT:

- name - variable name
- nr - number of round to create variable strings for
- rc - number of rounds \* number of columns in the state array (default: None)
- e - exponent of base field (default: None)

EXAMPLE:

```
sage: sr = mq.SR(10, 1, 2, 4)
sage: sr.vars('x', 2)
(x200, x201, x202, x203, x210, x211, x212, x213)
```

**varstr** (*name, nr, rc, e*)

Return a string representing a variable for the small scale AES subject to the given constraints.

INPUT:

- name - variable name
- nr - number of round to create variable strings for
- rc - row\*column index in state array
- e - exponent of base field

EXAMPLE:

```

sage: sr = mq.SR(10, 1, 2, 4)
sage: sr.varstr('x', 2, 1, 1)
'x211'

```

**varstrs** (*name*, *nr*, *rc*=None, *e*=None)

Return a list of strings representing variables in *self*.

INPUT:

- *name* - variable name
- *nr* - number of round to create variable strings for
- *rc* - number of rows \* number of columns in the state array (default: None)
- *e* - exponent of base field (default: None)

EXAMPLE:

```

sage: sr = mq.SR(10, 1, 2, 4)
sage: sr.varstrs('x', 2)
('x200', 'x201', 'x202', 'x203', 'x210', 'x211', 'x212', 'x213')

```

**class** sage.crypto.mq.sr.**SR\_gf2** (*n*=1, *r*=1, *c*=1, *e*=4, *star*=False, *\*\*kwargs*)

Bases: sage.crypto.mq.sr.SR\_generic

Small Scale Variants of the AES polynomial system constructor over  $F_2$ . See help for SR.

EXAMPLE:

```

sage: sr = mq.SR(gf2=True)
sage: sr
SR(1,1,1,4)

```

**antiphi** (*l*)

The operation  $\phi^{-1}$  from [MR02] or the inverse of *self*.phi.

INPUT:

- *l* - a vector in the sense of *self*.is\_vector

EXAMPLE:

```

sage: sr = mq.SR(gf2=True)
sage: A = sr.random_state_array()
sage: A
[a^2]
sage: sr.antiphi(sr.phi(A)) == A
True

```

**field\_polynomials** (*name*, *i*, *l*=None)

Return list of field polynomials for a given round *i* and name *name*.

INPUT:

- *name* - variable name
- *i* - round number
- *l* - length of variable list (default: None = *r*\**c*)

EXAMPLE:

```
sage: sr = mq.SR(3, 1, 1, 8, gf2=True, polybori=False)
sage: sr.field_polynomials('x', 2)
[x200^2 + x200, x201^2 + x201,
 x202^2 + x202, x203^2 + x203,
 x204^2 + x204, x205^2 + x205,
 x206^2 + x206, x207^2 + x207]

sage: sr = mq.SR(3, 1, 1, 8, gf2=True, polybori=True)
sage: sr.field_polynomials('x', 2)
[]
```

**inversion\_polynomials** (*xi, wi, length*)

Return polynomials to represent the inversion in the AES S-Box.

INPUT:

- xi - output variables
- wi - input variables
- length - length of both lists

EXAMPLE:

```
sage: sr = mq.SR(1, 1, 1, 8, gf2=True)
sage: xi = sr.vars('x', 1)
sage: wi = sr.vars('w', 1)
sage: sr.inversion_polynomials(xi, wi, len(xi))[:3]
[x100*w100 + x100*w102 + x100*w103 + x100*w107 + x101*w101 + x101*w102 + x101*w106 + x102*w101
 x100*w101 + x100*w103 + x100*w104 + x101*w100 + x101*w102 + x101*w103 + x101*w107 + x102*w101
 x100*w102 + x100*w104 + x100*w105 + x101*w101 + x101*w103 + x101*w104 + x102*w100 + x102*w101]
```

**inversion\_polynomials\_single\_sbox** (*x=None, w=None, biaffine\_only=None, correct\_only=None*)

Return inversion polynomials of a single S-Box.

INPUT:

- xi - output variables
- wi - input variables
- length - length of both lists

EXAMPLES:

```
sage: sr = mq.SR(1, 1, 1, 8, gf2=True)
sage: len(sr.inversion_polynomials_single_sbox())
24
sage: len(sr.inversion_polynomials_single_sbox(correct_only=True))
23
sage: len(sr.inversion_polynomials_single_sbox(biaffine_only=False))
40
sage: len(sr.inversion_polynomials_single_sbox(biaffine_only=False, correct_only=True))
39

sage: sr = mq.SR(1, 1, 1, 8, gf2=True)
sage: l0 = sr.inversion_polynomials_single_sbox(); len(l0)
24
sage: l1 = sr.inversion_polynomials_single_sbox(correct_only=True); len(l1)
23
```

```

sage: l2 = sr.inversion_polynomials_single_sbox(biaffine_only=False); len(l2)
40
sage: l3 = sr.inversion_polynomials_single_sbox(biaffine_only=False, correct_only=True); len(l3)
39

sage: set(l10) == set(sr._inversion_polynomials_single_sbox())
True
sage: set(l11) == set(sr._inversion_polynomials_single_sbox(correct_only=True))
True
sage: set(l12) == set(sr._inversion_polynomials_single_sbox(biaffine_only=False))
True
sage: set(l13) == set(sr._inversion_polynomials_single_sbox(biaffine_only=False, correct_only=True))
True

sage: sr = mq.SR(1, 1, 1, 4, gf2=True)
sage: l10 = sr.inversion_polynomials_single_sbox(); len(l10)
12
sage: l11 = sr.inversion_polynomials_single_sbox(correct_only=True); len(l11)
11
sage: l12 = sr.inversion_polynomials_single_sbox(biaffine_only=False); len(l12)
20
sage: l13 = sr.inversion_polynomials_single_sbox(biaffine_only=False, correct_only=True); len(l13)
19

sage: set(l10) == set(sr._inversion_polynomials_single_sbox())
True
sage: set(l11) == set(sr._inversion_polynomials_single_sbox(correct_only=True))
True
sage: set(l12) == set(sr._inversion_polynomials_single_sbox(biaffine_only=False))
True
sage: set(l13) == set(sr._inversion_polynomials_single_sbox(biaffine_only=False, correct_only=True))
True

```

**is\_vector** (*d*)

Return True if the given matrix satisfies the conditions for a vector as it appears in the algebraic expression of self.

INPUT:

- *d* - matrix

EXAMPLE:

```

sage: sr = mq.SR(gf2=True)
sage: sr
SR(1,1,1,4)
sage: k = sr.base_ring()
sage: A = Matrix(k, 1, 1, [k.gen()])
sage: B = sr.vector(A)
sage: sr.is_vector(A)
False
sage: sr.is_vector(B)
True

```

**lin\_matrix** (*length=None*)

Return the Lin matrix.

If no length is provided, the standard state space size is used. The key schedule calls this method with an explicit length argument because only `self.r` S-Box applications are performed in the key schedule.

INPUT:

- length - length of state space (default: None)

EXAMPLE:

```
sage: sr = mq.SR(1, 1, 1, 4, gf2=True)
sage: sr.lin_matrix()
[1 0 1 1]
[1 1 0 1]
[1 1 1 0]
[0 1 1 1]
```

**mix\_columns\_matrix()**

Return the MixColumns matrix.

EXAMPLE:

```
sage: sr = mq.SR(1, 2, 2, 4, gf2=True)
sage: s = sr.random_state_array()
sage: r1 = sr.mix_columns(s)
sage: r2 = sr.state_array(sr.mix_columns_matrix() * sr.vector(s))
sage: r1 == r2
True
```

**phi** (*l*, *diffusion\_matrix=False*)

The operation  $\phi$  from [MR02]

Given a list/matrix of elements in  $\mathbf{F}_{2^e}$ , return a matching list/matrix of elements in  $\mathbf{F}_2$ .

INPUT:

- l* - element to perform  $\phi$  on.
- diffusion\_matrix* - if True, the given matrix *l* is transformed to a matrix which performs the same operation over  $\mathbf{F}_2$  as *l* over  $\mathbf{F}_{2^n}$  (default: False).

EXAMPLE:

```
sage: sr = mq.SR(2, 1, 2, 4, gf2=True)
sage: k = sr.base_ring()
sage: A = matrix(k, 1, 2, [k.gen(), 0] )
sage: sr.phi(A)
[0 0]
[0 0]
[1 0]
[0 0]
```

**shift\_rows\_matrix()**

Return the ShiftRows matrix.

EXAMPLE:

```
sage: sr = mq.SR(1, 2, 2, 4, gf2=True)
sage: s = sr.random_state_array()
sage: r1 = sr.shift_rows(s)
sage: r2 = sr.state_array( sr.shift_rows_matrix() * sr.vector(s) )
sage: r1 == r2
True
```

**vector** (*d=None*)

Constructs a vector suitable for the algebraic representation of SR.

INPUT:

- d - values for vector (default: None)

EXAMPLE:

```
sage: sr = mq.SR(gf2=True)
sage: sr
SR(1, 1, 1, 4)
sage: k = sr.base_ring()
sage: A = Matrix(k, 1, 1, [k.gen()])
sage: sr.vector(A)
[0]
[0]
[1]
[0]
```

```
class sage.crypto.mq.sr.SR_gf2_2(n=1, r=1, c=1, e=4, star=False, **kwargs)
Bases: sage.crypto.mq.sr.SR_gf2
```

This is an example how to customize the SR constructor.

In this example, we replace the S-Box inversion polynomials by the polynomials generated by the S-Box class.

```
inversion_polynomials_single_sbox(x=None, w=None, biaffine_only=None, correct_only=None, groebner=False)
```

Return inversion polynomials of a single S-Box.

INPUT:

- x - output variables (default: None)
- w - input variables (default: None)
- biaffine\_only - ignored (always False)
- correct\_only - ignored (always True)
- groebner - precompute the Groebner basis for this S-Box (default: False).

EXAMPLES:

```
sage: from sage.crypto.mq.sr import SR_gf2_2
sage: e = 4
sage: sr = SR_gf2_2(1, 1, 1, e)
sage: P = PolynomialRing(GF(2), ['x%d'%i for i in range(e)] + ['w%d'%i for i in range(e)], order='lex')
sage: X, W = P.gens()[:e], P.gens()[e:]
sage: sr.inversion_polynomials_single_sbox(X, W, groebner=True)
[x0 + w0*w1*w2 + w0*w1 + w0*w2 + w0*w3 + w0 + w1 + w2,
 x1 + w0*w1*w3 + w0*w3 + w0 + w1*w3 + w1 + w2*w3,
 x2 + w0*w2*w3 + w0*w2 + w0 + w1*w2 + w1*w3 + w2*w3,
 x3 + w0*w1*w2 + w0 + w1*w2*w3 + w1*w2 + w1*w3 + w1 + w2 + w3]

sage: from sage.crypto.mq.sr import SR_gf2_2
sage: e = 4
sage: sr = SR_gf2_2(1, 1, 1, e)
sage: sr.inversion_polynomials_single_sbox()
[w3*w1 + w3*w0 + w3*x2 + w3*x1 + w3 + w2*w1 + w1 + x3 + x2 + x1,
 w3*w2 + w3*w1 + w3*x3 + w2 + w1 + x3,
 w3*w2 + w3*w1 + w3*x2 + w3 + w2*x3 + x2 + x1,
 w3*w2 + w3*w1 + w3*x3 + w3*x2 + w3*x1 + w3 + w2*x2 + w0 + x3 + x2 + x1 + x0,
 w3*w2 + w3*w1 + w3*x1 + w3*x0 + w2*x1 + w0 + x3 + x0,
 w3*w2 + w3*w1 + w3*w0 + w3*x2 + w3*x1 + w2*w0 + w2*x0 + w0 + x3 + x2 + x1 + x0,
 w3*w2 + w3*x1 + w3 + w2*w0 + w1*w0 + w1 + x3 + x2,
 w3*w2 + w3*w1 + w3*x1 + w1*x3 + x3 + x2 + x1,
 w3*x3 + w3*x2 + w3*x0 + w3 + w1*x2 + w1 + w0 + x2 + x0,
```

```

w3*w2 + w3*w1 + w3*x2 + w3*x1 + w1*x1 + w1 + w0 + x2 + x0,
w3*w2 + w3*w1 + w3*w0 + w3*x3 + w3*x1 + w2*w0 + w1*x0 + x3 + x2,
w3*w2 + w3*w1 + w3*x2 + w3*x1 + w3*x0 + w3 + w1 + w0*x3 + x3 + x2,
w3*w2 + w3*w1 + w3*w0 + w3*x3 + w3 + w2*w0 + w1 + w0*x2 + x3 + x2,
w3*w0 + w3*x2 + w2*w0 + w0*x1 + w0 + x3 + x1 + x0,
w3*w0 + w3*x3 + w3*x0 + w2*w0 + w1 + w0*x0 + w0 + x3 + x2,
w3*w2 + w3 + w1 + x3*x2 + x3 + x1,
w3*w2 + w3*x3 + w1 + x3*x1 + x3 + x2,
w3*w2 + w3*w0 + w3*x3 + w3*x2 + w3*x1 + w0 + x3*x0 + x1 + x0,
w3*w2 + w3*w1 + w3*w0 + w3*x3 + w1 + w0 + x2*x1 + x2 + x0,
w3*w2 + w2*w0 + w1 + x3 + x2*x0,
w3*x3 + w3*x1 + w2*w0 + w1 + x3 + x2 + x1*x0 + x1]

```

**TESTS:**

Note that `biaffine_only` and `correct_only` are always ignored. The former is always false while the second is always true. They are only accepted for compatibility with the base class.

```

sage: from sage.crypto.mq.sr import SR_gf2_2 sage: e = 4 sage: sr =
SR_gf2_2(1, 1, 1, e) sage: l = sr.inversion_polynomials_single_sbox() sage: l ==
sr.inversion_polynomials_single_sbox(biaffine_only=True, correct_only=False) True

```

**class** `sage.crypto.mq.sr.SR_gf2n` ( $n=1, r=1, c=1, e=4, star=False, **kwargs$ )

Bases: `sage.crypto.mq.sr.SR_generic`

Small Scale Variants of the AES polynomial system constructor over  $\mathbb{F}_{2^n}$ .

**antiphi** ( $l$ )

The operation  $\phi^{-1}$  from [MR02] or the inverse of `self.phi`.

INPUT:

- `l` - a vector in the sense of `self.is_vector`

EXAMPLE:

```

sage: sr = mq.SR()
sage: A = sr.random_state_array()
sage: A
[a^2]
sage: sr.antiphi(sr.phi(A)) == A
True

```

**field\_polynomials** ( $name, i, l=None$ )

Return list of conjugacy polynomials for a given round  $i$  and name `name`.

INPUT:

- `name` - variable name
- `i` - round number
- `l` -  $r*c$  (default: None)

EXAMPLE:

```

sage: sr = mq.SR(3, 1, 1, 8)
sage: sr.field_polynomials('x', 2)
[x200^2 + x201,
x201^2 + x202,
x202^2 + x203,
x203^2 + x204,
x204^2 + x205,

```



```
x205^2 + x206,
x206^2 + x207,
x207^2 + x200]
```

**inversion\_polynomials** (*xi, wi, length*)

Return polynomials to represent the inversion in the AES S-Box.

INPUT:

- *xi* - output variables
- *wi* - input variables
- *length* - length of both lists

EXAMPLE:

```
sage: sr = mq.SR(1, 1, 1, 8)
sage: R = sr.ring()
sage: xi = Matrix(R, 8, 1, sr.vars('x', 1))
sage: wi = Matrix(R, 8, 1, sr.vars('w', 1))
sage: sr.inversion_polynomials(xi, wi, 8)
[x100*w100 + 1,
x101*w101 + 1,
x102*w102 + 1,
x103*w103 + 1,
x104*w104 + 1,
x105*w105 + 1,
x106*w106 + 1,
x107*w107 + 1]
```

**is\_vector** (*d*)

Return True if *d* can be used as a vector for self.

EXAMPLE:

```
sage: sr = mq.SR()
sage: sr
SR(1,1,1,4)
sage: k = sr.base_ring()
sage: A = Matrix(k, 1, 1, [k.gen()])
sage: B = sr.vector(A)
sage: sr.is_vector(A)
False
sage: sr.is_vector(B)
True
```

**lin\_matrix** (*length=None*)

Return the Lin matrix.

If no *length* is provided, the standard state space size is used. The key schedule calls this method with an explicit *length* argument because only *self.r* S-Box applications are performed in the key schedule.

INPUT:

- *length* - length of state space (default: None)

EXAMPLE:

```
sage: sr = mq.SR(1, 1, 1, 4)
sage: sr.lin_matrix()
[      a^2 + 1      1      a^3 + a^2      a^2 + 1]
[      a      a      1 a^3 + a^2 + a + 1]
```

$$\begin{bmatrix} a^3 + a & a^2 & a^2 & 1 \\ 1 & a^3 & a + 1 & a + 1 \end{bmatrix}$$
**mix\_columns\_matrix()**

Return the MixColumns matrix.

EXAMPLE:

```
sage: sr = mq.SR(1, 2, 2, 4)
sage: s = sr.random_state_array()
sage: r1 = sr.mix_columns(s)
sage: r2 = sr.state_array(sr.mix_columns_matrix() * sr.vector(s))
sage: r1 == r2
True
```

**phi(l)**

The operation  $\phi$  from [MR02]

Projects state arrays to their algebraic representation.

INPUT:

- l - element to perform  $\phi$  on.

EXAMPLE:

```
sage: sr = mq.SR(2, 1, 2, 4)
sage: k = sr.base_ring()
sage: A = matrix(k, 1, 2, [k.gen(), 0] )
sage: sr.phi(A)
[      a      0]
[     a^2      0]
[   a + 1      0]
[a^2 + 1      0]
```

**shift\_rows\_matrix()**

Return the ShiftRows matrix.

EXAMPLE:

```
sage: sr = mq.SR(1, 2, 2, 4)
sage: s = sr.random_state_array()
sage: r1 = sr.shift_rows(s)
sage: r2 = sr.state_array( sr.shift_rows_matrix() * sr.vector(s) )
sage: r1 == r2
True
```

**vector(d=None)**

Constructs a vector suitable for the algebraic representation of SR, i.e. BES.

INPUT:

- d - values for vector, must be understood by self.phi (default:None)

EXAMPLE:

```
sage: sr = mq.SR()
sage: sr
SR(1,1,1,4)
sage: k = sr.base_ring()
sage: A = Matrix(k, 1, 1, [k.gen()])
sage: sr.vector(A)
[      a]
```

```
[      a^2]
[  a + 1]
[a^2 + 1]
```

```
sage.crypto.mq.sr.test_consistency(max_n=2, **kwargs)
```

Test all combinations of  $r$ ,  $c$ ,  $e$  and  $n$  in  $(1, 2)$  for consistency of random encryptions and their polynomial systems.  $\mathbf{F}_2$  and  $\mathbf{F}_{2^e}$  systems are tested. This test takes a while.

INPUT:

- `max_n` – maximal number of rounds to consider (default: 2)
- `kwargs` – are passed to the SR constructor

TESTS:

The following test called with `max_n = 2` requires a LOT of RAM (much more than 2GB). Since this might cause the doctest to fail on machines with “only” 2GB of RAM, we test `max_n = 1`, which has a more reasonable memory usage.

```
sage: from sage.crypto.mq.sr import test_consistency
sage: test_consistency(1) # long time (65s on sage.math, 2012)
True
```



# S-BOXES AND THEIR ALGEBRAIC REPRESENTATIONS

```
class sage.crypto.mq.sbox.SBox(*args, **kwargs)
    Bases: sage.structure.sage_object.SageObject
```

A substitution box or S-box is one of the basic components of symmetric key cryptography. In general, an S-box takes  $m$  input bits and transforms them into  $n$  output bits. This is called an  $m \times n$  S-box and is often implemented as a lookup table. These S-boxes are carefully chosen to resist linear and differential cryptanalysis [Heys02].

This module implements an S-box class which allows an algebraic treatment.

EXAMPLE:

We consider the S-box of the block cipher PRESENT [PRESENT07]:

```
sage: S = mq.SBox(12, 5, 6, 11, 9, 0, 10, 13, 3, 14, 15, 8, 4, 7, 1, 2); S
(12, 5, 6, 11, 9, 0, 10, 13, 3, 14, 15, 8, 4, 7, 1, 2)
sage: S(1)
5
```

Note that by default bits are interpreted in big endian order. This is not consistent with the rest of Sage, which has a strong bias towards little endian, but is consistent with most cryptographic literature:

```
sage: S([0, 0, 0, 1])
[0, 1, 0, 1]

sage: S = mq.SBox(12, 5, 6, 11, 9, 0, 10, 13, 3, 14, 15, 8, 4, 7, 1, 2, big_endian=False)
sage: S(1)
5
sage: S([0, 0, 0, 1])
[1, 1, 0, 0]
```

Now we construct an SBox object for the 4-bit small scale AES S-Box (cf. `sage.crypto.mq.sr`):

```
sage: sr = mq.SR(1, 1, 1, 4, allow_zero_inversions=True)
sage: S = mq.SBox([sr.sub_byte(e) for e in list(sr.k)])
sage: S
(6, 5, 2, 9, 4, 7, 3, 12, 14, 15, 10, 0, 8, 1, 13, 11)
```

REFERENCES:

```
cnf (xi=None, yi=None, format=None)
```

Return a representation of this S-Box in conjunctive normal form.

This function examines the truth tables for each output bit of the S-Box and thus has complexity  $n * 2^m$  for an  $m \times n$  S-Box.

INPUT:

- `xi` - indices for the input variables (default: `1...m`)
- `yi` - indices for the output variables (default: `m+1...m+n`)
- `format` - output format, see below (default: `None`)

FORMATS:

- `None` - return a list of tuples of integers where each tuple represents a clause, the absolute value of an integer represents a variable and the sign of an integer indicates inversion.
- `symbolic` - a string that can be parsed by the `SymbolicLogic` package.
- `dimacs` - a string in DIMACS format which is the gold standard for SAT-solver input (cf. <http://www.satlib.org/>).
- `dimacs_headless` - a string in DIMACS format, but without the header. This is useful for concatenation of outputs.

EXAMPLE:

We give a very small example to explain the output format:

```
sage: S = mq.SBox(1,2,0,3); S
(1, 2, 0, 3)
sage: cnf = S.cnf(); cnf
[(1, 2, -3), (1, 2, 4),
 (1, -2, 3), (1, -2, -4),
 (-1, 2, -3), (-1, 2, -4),
 (-1, -2, 3), (-1, -2, 4)]
```

This output completely describes the S-Box. For instance, we can check that  $S([0,1]) \rightarrow [1,0]$  satisfies every clause if the first input bit corresponds to the index 1 and the last output bit corresponds to the index 3 in the output.

We can convert this representation to the DIMACS format:

```
sage: print S.cnf(format='dimacs')
p cnf 4 8
1 2 -3 0
1 2 4 0
1 -2 3 0
1 -2 -4 0
-1 2 -3 0
-1 2 -4 0
-1 -2 3 0
-1 -2 4 0
```

For concatenation we can strip the header:

```
sage: print S.cnf(format='dimacs_headless')
1 2 -3 0
1 2 4 0
1 -2 3 0
1 -2 -4 0
-1 2 -3 0
-1 2 -4 0
-1 -2 3 0
-1 -2 4 0
```

This might be helpful in combination with the `xi` and `yi` parameter to assign indices manually:

```

sage: print S.cnf(xi=[10,20],yi=[30,40], format='dimacs_headless')
10 20 -30 0
10 20 40 0
10 -20 30 0
10 -20 -40 0
-10 20 -30 0
-10 20 -40 0
-10 -20 30 0
-10 -20 40 0

```

We can also return a string which is parse-able by the SymbolicLogic package:

```

sage: log = SymbolicLogic()
sage: s = log.statement(S.cnf(format='symbolic'))
sage: log.truthtable(s)[1:]
[['False', 'False', 'False', 'False', 'False'],
 ['False', 'False', 'False', 'True', 'False'],
 ['False', 'False', 'True', 'False', 'False'],
 ['False', 'False', 'True', 'True', 'True'],
 ['False', 'True', 'False', 'False', 'True'],
 ['False', 'True', 'False', 'True', 'True'],
 ['False', 'True', 'True', 'False', 'True'],
 ['False', 'True', 'True', 'True', 'True'],
 ['True', 'False', 'False', 'False', 'True'],
 ['True', 'False', 'False', 'True', 'True'],
 ['True', 'False', 'True', 'False', 'True'],
 ['True', 'False', 'True', 'True', 'True'],
 ['True', 'True', 'False', 'False', 'True'],
 ['True', 'True', 'False', 'True', 'True'],
 ['True', 'True', 'True', 'False', 'True'],
 ['True', 'True', 'True', 'True', 'True']]

```

This function respects endianness of the S-Box:

```

sage: S = mq.SBox(1,2,0,3, big_endian=False); S
(1, 2, 0, 3)
sage: cnf = S.cnf(); cnf
[(1, 2, -4), (1, 2, 3),
 (-1, 2, 4), (-1, 2, -3),
 (1, -2, -4), (1, -2, -3),
 (-1, -2, 4), (-1, -2, 3)]

```

S-Boxes with  $m \neq n$  also work:

```

sage: o = range(8) + range(8) sage: shuffle(o) sage: S = mq.SBox(o) sage: S.is_permutation()
False

```

```

sage: len(S.cnf()) == 3*2^4 True

```

TESTS:

```

sage: S = mq.SBox(1,2,0,3, big_endian=False) sage: S.cnf([1000,1001,1002],
[2000,2001,2002]) Traceback (most recent call last): ... TypeError: first arg required to
have length 2, got 3 instead.

```

#### **difference\_distribution\_matrix()**

Return difference distribution matrix A for this S-box.

The rows of A encode the differences  $\Delta a$  of the input and the columns encode the difference  $\Delta a$   $\circ$  for the output. The bits are ordered according to the endianness of this S-box. The value at A[ $\Delta a$

`I, Delta O]` encodes how often Delta O is the actual output difference given Delta I as input difference.

See [Heys02] for an introduction to differential cryptanalysis.

EXAMPLE:

```
sage: S = mq.SBox(7,6,0,4,2,5,1,3)
sage: S.difference_distribution_matrix()
[8 0 0 0 0 0 0 0]
[0 2 2 0 2 0 0 2]
[0 0 2 2 0 0 2 2]
[0 2 0 2 2 0 2 0]
[0 2 0 2 0 2 0 2]
[0 0 2 2 2 2 0 0]
[0 2 2 0 0 2 2 0]
[0 0 0 0 2 2 2 2]
```

**from\_bits** (*x*, *n=None*)

Return integer for bitstring *x* of length *n*.

INPUT:

- *x* - a bitstring
- *n* - bit length (optional)

EXAMPLE:

```
sage: S = mq.SBox(7,6,0,4,2,5,1,3)
sage: S.from_bits( [1,1,0])
6

sage: S( S.from_bits( [1,1,0] ) )
1
sage: S.from_bits( S( [1,1,0] ) )
1
```

**interpolation\_polynomial** (*k=None*)

Return a univariate polynomial over an extension field representing this S-box.

If *m* is the input length of this S-box then the extension field is of degree *m*.

If the output length does not match the input length then a `TypeError` is raised.

INPUT:

- *k* - an instance of  $\mathbb{F}_{2^m}$  (default: `None`)

EXAMPLE:

```
sage: S = mq.SBox(7,6,0,4,2,5,1,3)
sage: f = S.interpolation_polynomial()
sage: f
x^6 + a*x^5 + (a + 1)*x^4 + (a^2 + a + 1)*x^3
+ (a^2 + 1)*x^2 + (a + 1)*x + a^2 + a + 1

sage: a = f.base_ring().gen()

sage: f(0), S(0)
(a^2 + a + 1, 7)

sage: f(a^2 + 1), S(5)
(a^2 + 1, 5)
```



**is\_permutation()**

Return True if this S-Box is a permutation.

EXAMPLE:

```
sage: S = mq.SBox(7,6,0,4,2,5,1,3)
sage: S.is_permutation()
True

sage: S = mq.SBox(3,2,0,0,2,1,1,3)
sage: S.is_permutation()
False
```

**linear\_approximation\_matrix()**

Return linear approximation matrix A for this S-box.

Let  $i_b$  be the  $b$ -th bit of  $i$  and  $o_b$  the  $b$ -th bit of  $o$ . Then  $v = A[i, o]$  encodes the bias of the equation  $\sum(i_b * x_i) = \sum(o_b * y_i)$  if  $x_i$  and  $y_i$  represent the input and output variables of the S-box.

See [Heys02] for an introduction to linear cryptanalysis.

EXAMPLE:

```
sage: S = mq.SBox(7,6,0,4,2,5,1,3)
sage: S.linear_approximation_matrix()
[ 4  0  0  0  0  0  0  0]
[ 0  0  0  0  2  2  2 -2]
[ 0  0 -2 -2 -2  2  0  0]
[ 0  0 -2  2  0  0 -2 -2]
[ 0  2  0  2 -2  0  2  0]
[ 0 -2  0  2  0  2  0  2]
[ 0 -2 -2  0  0 -2  2  0]
[ 0 -2  2  0 -2  0  0 -2]
```

According to this matrix the first bit of the input is equal to the third bit of the output 6 out of 8 times:

```
sage: for i in xrange(8): print S.to_bits(i)[0] == S.to_bits(S(i))[2]
False
True
True
True
False
True
True
True
```

**maximal\_difference\_probability()**

Return the difference probability of the difference with the highest probability in the range between 0.0 and 1.0 indicating 0% or 100% respectively.

EXAMPLE:

```
sage: S = mq.SBox(7,6,0,4,2,5,1,3)
sage: S.maximal_difference_probability()
0.25
```

**maximal\_difference\_probability\_absolute()**

Return the difference probability of the difference with the highest probability in absolute terms, i.e. how often it occurs in total.

EXAMPLE:

```
sage: S = mq.SBox(7,6,0,4,2,5,1,3)
sage: S.maximal_difference_probability_absolute()
2
```

---

**Note:** This code is mainly called internally.

---

**maximal\_linear\_bias\_absolute()**

Return maximal linear bias, i.e. how often the linear approximation with the highest bias is true or false minus  $2^{n-1}$ .

EXAMPLE:

```
sage: S = mq.SBox(7,6,0,4,2,5,1,3)
sage: S.maximal_linear_bias_absolute()
2
```

**maximal\_linear\_bias\_relative()**

Return maximal bias of all linear approximations of this S-box.

EXAMPLE:

```
sage: S = mq.SBox(7,6,0,4,2,5,1,3)
sage: S.maximal_linear_bias_relative()
0.25
```

**polynomials(X=None, Y=None, degree=2, groebner=False)**

Return a list of polynomials satisfying this S-box.

First, a simple linear fitting is performed for the given degree (cf. for example [BC03]). If `groebner=True` a Groebner basis is also computed for the result of that process.

INPUT:

- X - input variables
- Y - output variables
- degree - integer > 0 (default: 2)
- groebner - calculate a reduced Groebner basis of the spanning polynomials to obtain more polynomials (default: False)

EXAMPLES:

```
sage: S = mq.SBox(7,6,0,4,2,5,1,3)
sage: P = S.ring()
```

By default, this method returns an indirect representation:

```
sage: S.polynomials()
[x0*x2 + x1 + y1 + 1,
 x0*x1 + x1 + x2 + y0 + y1 + y2 + 1,
 x0*y1 + x0 + x2 + y0 + y2,
 x0*y0 + x0*y2 + x1 + x2 + y0 + y1 + y2 + 1,
 x1*x2 + x0 + x1 + x2 + y2 + 1,
 x0*y0 + x1*y0 + x0 + x2 + y1 + y2,
 x0*y0 + x1*y1 + x1 + y1 + 1,
 x1*y2 + x1 + x2 + y0 + y1 + y2 + 1,
 x0*y0 + x2*y0 + x1 + x2 + y1 + 1,
 x2*y1 + x0 + y1 + y2,
 x2*y2 + x1 + y1 + 1,
```

```

y0*y1 + x0 + x2 + y0 + y1 + y2,
y0*y2 + x1 + x2 + y0 + y1 + 1,
y1*y2 + x2 + y0]

```

We can get a direct representation by computing a lexicographical Groebner basis with respect to the right variable ordering, i.e. a variable ordering where the output bits are greater than the input bits:

```

sage: P.<y0,y1,y2,x0,x1,x2> = PolynomialRing(GF(2),6,order='lex')
sage: S.polynomials([x0,x1,x2],[y0,y1,y2], groebner=True)
[y0 + x0*x1 + x0*x2 + x0 + x1*x2 + x1 + 1,
 y1 + x0*x2 + x1 + 1,
 y2 + x0 + x1*x2 + x1 + x2 + 1]

```

## REFERENCES:

### **ring()**

Create, return and cache a polynomial ring for S-box polynomials.

#### EXAMPLE:

```

sage: S = mq.SBox(7,6,0,4,2,5,1,3)
sage: S.ring()
Multivariate Polynomial Ring in x0, x1, x2, y0, y1, y2 over Finite Field of size 2

```

### **solutions** (*X=None, Y=None*)

Return a dictionary of solutions to this S-box.

#### INPUT:

- X - input variables (default: None)
- Y - output variables (default: None)

#### EXAMPLE:

```

sage: S = mq.SBox([7,6,0,4,2,5,1,3])
sage: F = S.polynomials()
sage: s = S.solutions()
sage: any(f.subs(_s) for f in F for _s in s)
False

```

### **to\_bits** (*x, n=None*)

Return bitstring of length n for integer x. The returned bitstring is guaranteed to have length n.

#### INPUT:

- x - an integer
- n - bit length (optional)

#### EXAMPLE:

```

sage: S = mq.SBox(7,6,0,4,2,5,1,3)
sage: S.to_bits(6)
[1, 1, 0]

sage: S.to_bits( S(6) )
[0, 0, 1]

sage: S( S.to_bits( 6 ) )
[0, 0, 1]

```



# HARD LATTICE GENERATOR

This module contains lattice related functions relevant in cryptography.

Feel free to add more functionality.

## AUTHORS:

- Richard Lindner <[rlindner@cdc.informatik.tu-darmstadt.de](mailto:rlindner@cdc.informatik.tu-darmstadt.de)>
- Michael Schneider <[mischnei@cdc.informatik.tu-darmstadt.de](mailto:mischnei@cdc.informatik.tu-darmstadt.de)>

```
sage.crypto.lattice.gen_lattice(type='modular', n=4, m=8, q=11, seed=None, quotient=None, dual=False, ntl=False, lattice=False)
```

This function generates different types of integral lattice bases of row vectors relevant in cryptography.

Randomness can be set either with seed, or by using `sage.misc.randstate.set_random_seed()`.

## INPUT:

### •type - one of the following strings

- 'modular' (default). A class of lattices for which asymptotic worst-case to average-case connections hold. For more refer to [A96].
- 'random' - Special case of modular ( $n=1$ ). A dense class of lattice used for testing basis reduction algorithms proposed by Goldstein and Mayer [GM02].
- 'ideal' - Special case of modular. Allows for a more compact representation proposed by [LM06].
- 'cyclotomic' - Special case of ideal. Allows for efficient processing proposed by [LM06].
- n - Determinant size, primal:  $\det(L) = q^n$ , dual:  $\det(L) = q^{m-n}$ . For ideal lattices this is also the degree of the quotient polynomial.
- m - Lattice dimension,  $L \subseteq \mathbb{Z}^m$ .
- q - Coefficient size,  $q * \mathbb{Z}^m \subseteq L$ .
- seed - Randomness seed.
- quotient - For the type ideal, this determines the quotient polynomial. Ignored for all other types.
- dual - Set this flag if you want a basis for  $q * \text{dual}(L)$ , for example for Regev's LWE bases [R05].

- `ntl` - Set this flag if you want the lattice basis in NTL readable format.
- `lattice` - Set this flag if you want a `FreeModule_submodule_with_basis_integer` object instead of an integer matrix representing the basis.

**OUTPUT:** **B** a unique size-reduced triangular (primal: `lower_left`, dual: `lower_right`) basis of row vectors for the lattice in question.

EXAMPLES:

- Modular basis

```
sage: sage.crypto.gen_lattice(m=10, seed=42)
[11  0  0  0  0  0  0  0  0  0]
[ 0 11  0  0  0  0  0  0  0  0]
[ 0  0 11  0  0  0  0  0  0  0]
[ 0  0  0 11  0  0  0  0  0  0]
[ 2  4  3  5  1  0  0  0  0  0]
[ 1 -5 -4  2  0  1  0  0  0  0]
[-4  3 -1  1  0  0  1  0  0  0]
[-2 -3 -4 -1  0  0  0  1  0  0]
[-5 -5  3  3  0  0  0  0  1  0]
[-4 -3  2 -5  0  0  0  0  0  1]
```

- Random basis

```
sage: sage.crypto.gen_lattice(type='random', n=1, m=10, q=11^4, seed=42)
[14641  0  0  0  0  0  0  0  0  0]
[  431  1  0  0  0  0  0  0  0  0]
[-4792  0  1  0  0  0  0  0  0  0]
[ 1015  0  0  1  0  0  0  0  0  0]
[-3086  0  0  0  1  0  0  0  0  0]
[-5378  0  0  0  0  1  0  0  0  0]
[  4769  0  0  0  0  0  1  0  0  0]
[-1159  0  0  0  0  0  0  1  0  0]
[  3082  0  0  0  0  0  0  0  1  0]
[-4580  0  0  0  0  0  0  0  0  1]
```

- Ideal bases with quotient  $x^n-1$ ,  $m=2*n$  are NTRU bases

```
sage: sage.crypto.gen_lattice(type='ideal', seed=42, quotient=x^4-1)
[11  0  0  0  0  0  0  0]
[ 0 11  0  0  0  0  0  0]
[ 0  0 11  0  0  0  0  0]
[ 0  0  0 11  0  0  0  0]
[ 4 -2 -3 -3  1  0  0  0]
[-3  4 -2 -3  0  1  0  0]
[-3 -3  4 -2  0  0  1  0]
[-2 -3 -3  4  0  0  0  1]
```

- Cyclotomic bases with  $n=2^k$  are SWIFFT bases

```
sage: sage.crypto.gen_lattice(type='cyclotomic', seed=42)
[11  0  0  0  0  0  0  0]
[ 0 11  0  0  0  0  0  0]
```

```
[ 0  0 11  0  0  0  0  0]
[ 0  0  0 11  0  0  0  0]
[ 4 -2 -3 -3  1  0  0  0]
[ 3  4 -2 -3  0  1  0  0]
[ 3  3  4 -2  0  0  1  0]
[ 2  3  3  4  0  0  0  1]
```

- Dual modular bases are related to Regev's famous public-key encryption [R05]

```
sage: sage.crypto.gen_lattice(type='modular', m=10, seed=42, dual=True)
[ 0  0  0  0  0  0  0  0  0 11]
[ 0  0  0  0  0  0  0  0  0 11 0]
[ 0  0  0  0  0  0  0  0 11  0 0]
[ 0  0  0  0  0  0  0 11  0  0 0]
[ 0  0  0  0  0  0 11  0  0  0 0]
[ 0  0  0  0 11  0  0  0  0  0 0]
[ 0  0  0  1 -5 -2 -1  1 -3  5]
[ 0  0  1  0 -3  4  1  4 -3 -2]
[ 0  1  0  0 -4  5 -3  3  5  3]
[ 1  0  0  0 -2 -1  4  2  5  4]
```

- Relation of primal and dual bases

```
sage: B_primal=sage.crypto.gen_lattice(m=10, q=11, seed=42)
sage: B_dual=sage.crypto.gen_lattice(m=10, q=11, seed=42, dual=True)
sage: B_dual_alt=transpose(11*B_primal.inverse()).change_ring(ZZ)
sage: B_dual_alt.hermite_form() == B_dual.hermite_form()
True
```

## TESTS:

We are testing output format choices:

```
sage: sage.crypto.gen_lattice(m=10, q=11, seed=42)
[11  0  0  0  0  0  0  0  0  0]
[ 0 11  0  0  0  0  0  0  0  0]
[ 0  0 11  0  0  0  0  0  0  0]
[ 0  0  0 11  0  0  0  0  0  0]
[ 2  4  3  5  1  0  0  0  0  0]
[ 1 -5 -4  2  0  1  0  0  0  0]
[-4  3 -1  1  0  0  1  0  0  0]
[-2 -3 -4 -1  0  0  0  1  0  0]
[-5 -5  3  3  0  0  0  0  1  0]
[-4 -3  2 -5  0  0  0  0  0  1]

sage: sage.crypto.gen_lattice(m=10, q=11, seed=42, ntl=True)
[
[11 0 0 0 0 0 0 0 0 0]
[0 11 0 0 0 0 0 0 0 0]
[0 0 11 0 0 0 0 0 0 0]
[0 0 0 11 0 0 0 0 0 0]
[2 4 3 5 1 0 0 0 0 0]
[1 -5 -4 2 0 1 0 0 0 0]
[-4 3 -1 1 0 0 1 0 0 0]
[-2 -3 -4 -1 0 0 0 1 0 0]
]
```

```
[-5 -5 3 3 0 0 0 0 1 0]
[-4 -3 2 -5 0 0 0 0 0 1]
]
```

```
sage: sage.crypto.gen_lattice(m=10, q=11, seed=42, lattice=True)
```

Free module of degree 10 and rank 10 over Integer Ring

User basis matrix:

```
[ 0  0  1  1  0 -1 -1 -1  1  0]
[-1  1  0  1  0  1  1  0  1  1]
[-1  0  0  0 -1  1  1 -2  0  0]
[-1 -1  0  1  1  0  0  1  1 -1]
[ 1  0 -1  0  0  0 -2 -2  0  0]
[ 2 -1  0  0  1  0  1  0  0 -1]
[-1  1 -1  0  1 -1  1  0 -1 -2]
[ 0  0 -1  3  0  0  0 -1 -1 -1]
[ 0 -1  0 -1  2  0 -1  0  0  2]
[ 0  1  1  0  1  1 -2  1 -1 -2]
```

REFERENCES:



# (RING-)LWE ORACLE GENERATORS

The Learning with Errors problem (LWE) is solving linear systems of equations where the right hand side has been disturbed ‘slightly’ where ‘slightly’ is made precise by a noise distribution - typically a discrete Gaussian distribution. See [Reg09] for details.

The Ring Learning with Errors problem (LWE) is solving a set of univariate polynomial equations - typically in a cyclotomic field - where the right hand side was disturbed ‘slightly’. See [LPR10] for details.

This module implements generators of LWE samples where parameters are chosen following proposals in the cryptographic literature.

## EXAMPLES:

We get 30 samples from an LWE oracle parameterised by security parameter  $n=20$  and where the modulus and the standard deviation of the noise are chosen as in [Reg09]:

```
sage: from sage.crypto.lwe import samples
sage: samples(30, 20, 'Regev')
[((360, 264, 123, 368, 398, 392, 41, 84, 25, 389, 311, 68, 322, 41, 161, 372, 222, 153, 243, 381), 123),
...
((138, 198, 204, 235, 339, 168, 269, 276, 392, 243, 86, 18, 378, 20, 369, 141, 108, 151, 336, 141), 123)]
```

We may also pass classes to the samples function, which is useful for users implementing their own oracles:

```
sage: from sage.crypto.lwe import samples, LindnerPeikert
sage: samples(30, 20, LindnerPeikert)
[((350, 835, 2023, 1785, 1958, 1818, 1130, 1285, 1331, 284, 2048, 441, 1581, 1406, 1185, 1724, 1397,
...
((1918, 1823, 1598, 18, 588, 1093, 744, 1934, 689, 1327, 1632, 1867, 228, 378, 798, 511, 274, 1001, 123)]
```

Finally, `samples()` also accepts instances of classes:

```
sage: from sage.crypto.lwe import LindnerPeikert
sage: lwe = LindnerPeikert(20)
sage: samples(30, 20, lwe)
[((1817, 1322, 818, 1232, 354, 639, 1770, 754, 1366, 1731, 649, 162, 483, 1741, 1942, 1232, 1424, 103,
...
((2021, 829, 572, 1698, 1025, 170, 598, 1193, 1268, 607, 1502, 1984, 1655, 206, 958, 334, 1213, 1413, 123)]
```

Note that Ring-LWE samples are returned as vectors:

```
sage: from sage.crypto.lwe import DiscreteGaussianPolynomialSamplerRejection, RingLWE
sage: D = DiscreteGaussianPolynomialSamplerRejection(euler_phi(16), 5)
sage: ringlwe = RingLWE(16, 257, D, secret_dist='uniform')
sage: samples(30, euler_phi(16), ringlwe)
```

```
[((158, 49, 174, 179, 109, 92, 234, 41), (200, 159, 131, 197, 241, 172, 1, 107)),
...
((80, 227, 249, 205, 149, 92, 46, 68), (69, 256, 29, 219, 218, 34, 182, 178))]
```

One technical issue when working with these generators is that by default they return vectors and scalars over/in rings modulo some  $q$ . These are represented as elements in  $(0, q - 1)$  by Sage. However, it usually is more natural to think of these entries as integers in  $(-q/2, q/2)$ . To allow for this, this module provides the option to balance the representation. In this case vectors and scalars over/in the integers are returned:

```
sage: from sage.crypto.lwe import samples
sage: samples(30, 20, 'Regev', balanced=True)
[(-38, 59, -33, -80, 165, -55, -46, -49, -113, 135, -32, 185, -80, -184, 127, 153, 162, -31, 115, 17,
...
(-165, -187, -87, 188, 160, -118, -7, 107, -77, -107, -109, 77, 63, -66, -55, -75, -12, 90, 58, -188,
...
```

**AUTHORS:**

- Martin Albrecht
- Robert Fitzpatrick
- Daniel Cabracas
- Florian Göpfert
- Michael Schneider

REFERENCES:

sage.crypto.lwe.**DiscreteGaussianPolynomialSampler**  
alias of [DiscreteGaussianPolynomialSamplerRejection](#)

```
class sage.crypto.lwe.DiscreteGaussianPolynomialSamplerRejection(n, stddev, pre-
precision=53,
tailcut=4,
D=<class
'sage.crypto.lwe.DiscreteGaussianSample
```

Bases: sage.structure.sage\_object.SageObject

Discrete Gaussian sampler for polynomials.

EXAMPLE:

```
sage: from sage.crypto.lwe import DiscreteGaussianPolynomialSamplerRejection
sage: DiscreteGaussianPolynomialSamplerRejection(8, 3.0)()
x^7 - x^6 - 2*x^4 + 2*x^3 - x^2 + x - 1
sage: gs = DiscreteGaussianPolynomialSamplerRejection(8, 3.0, precision=100, tailcut=1.0)
sage: [gs() for _ in xrange(3)]
[-x^7 + x^6 + 2*x^5 + 2*x^4 - x^3 - x^2 - 1,
 x^7 - 2*x^6 + 2*x^5 + x^4 - x^3 + 2*x^2 - x + 2,
 x^5 + 2*x^3 + 2*x + 1]
```

**\_\_init\_\_** (*n*, *stddev*, *precision*=53, *tailcut*=4, *D*=<class 'sage.crypto.lwe.DiscreteGaussianSamplerRejection'>)  
Construct a sampler for univariate polynomials of degree  $n-1$  where coefficients are drawn independently with standard deviation *stddev* using *D*.

INPUT:

- `n` - number of coefficients to be sampled
- `stddev` - standard deviation
- `precision` - precision used for internal computations (default: 53)

- tailcut - cut the tail at tailcut standard deviations (default: 4)
- D - a discrete Gaussian sampler (default: `DiscreteGaussianSampler`)

EXAMPLE:

```
sage: from sage.crypto.lwe import DiscreteGaussianPolynomialSamplerRejection
sage: DiscreteGaussianPolynomialSamplerRejection(8, 3.0) ()
x^7 - x^6 - 2*x^4 + 2*x^3 - x^2 + x - 1
sage: gs = DiscreteGaussianPolynomialSamplerRejection(8, 3.0, precision=100, tailcut=1.0)
sage: [gs() for _ in xrange(3)]
[-x^7 + x^6 + 2*x^5 + 2*x^4 - x^3 - x^2 - 1,
 x^7 - 2*x^6 + 2*x^5 + x^4 - x^3 + 2*x^2 - x + 2,
 x^5 + 2*x^3 + 2*x + 1]
```

`__call__()`

Return a new sample.

EXAMPLE:

```
sage: from sage.crypto.lwe import DiscreteGaussianPolynomialSamplerRejection
sage: sampler = DiscreteGaussianPolynomialSamplerRejection(8, 12.0)
sage: sampler()
x^7 - 9*x^5 + 2*x^4 + 8*x^3 - 5*x^2 + 7*x - 5
```

`sage.crypto.lwe.DiscreteGaussianSampler`  
alias of `DiscreteGaussianSamplerRejection`

**class** `sage.crypto.lwe.DiscreteGaussianSamplerRejection` (*stddev*, *precision=53*, *tailcut=4*)

Bases: `sage.structure.sage_object.SageObject`

Discrete Gaussian sampler using rejection sampling.

EXAMPLE:

```
sage: from sage.crypto.lwe import DiscreteGaussianSamplerRejection
sage: DiscreteGaussianSamplerRejection(3.0) ()
-1
sage: gs = DiscreteGaussianSamplerRejection(3.0, precision=100, tailcut=1.0)
sage: all(gs() <= 3.0 for _ in xrange(1000))
True
```

`__init__` (*stddev*, *precision=53*, *tailcut=4*)

Construct a new discrete Gaussian sampler.

INPUT:

- stddev - standard deviation
- precision - precision used for internal computations (default: 53)
- tailcut - cut the tail at tailcut standard deviations (default: 4)

EXAMPLE:

```
sage: from sage.crypto.lwe import DiscreteGaussianSamplerRejection
sage: gs = DiscreteGaussianSamplerRejection(3.0)
sage: sqrt(variance([gs() for _ in xrange(1000)])) .n()
2.965...
```

`__call__()`

Return a new sample.

EXAMPLE:

```
sage: from sage.crypto.lwe import DiscreteGaussianSamplerRejection
sage: sampler = DiscreteGaussianSamplerRejection(12.0)
sage: sampler()
-5
```

```
class sage.crypto.lwe.LWE(n, q, D, secret_dist='uniform', m=None)
    Bases: sage.structure.sage_object.SageObject
```

Learning with Errors (LWE) oracle.

```
__init__(n, q, D, secret_dist='uniform', m=None)
    Construct an LWE oracle in dimension  $n$  over a ring of order  $q$  with noise distribution  $D$ .
```

INPUT:

- $n$  - dimension (integer  $> 0$ )
- $q$  - modulus typically  $> n$  (integer  $> 0$ )
- $D$  - an error distribution such as an instance of `DiscreteGaussianSamplerRejection` or `UniformSampler`
- `secret_dist` - distribution of the secret (default: 'uniform'); one of
  - "uniform" - secret follows the uniform distribution in  $\mathbb{Z}/q\mathbb{Z}$
  - "noise" - secret follows the noise distribution
  - $(lb, ub)$  - the secret is chosen uniformly from  $[lb, \dots, ub]$  including both endpoints
- $m$  - number of allowed samples or `None` if no such limit exists (default: `None`)

EXAMPLE:

First, we construct a noise distribution with standard deviation 3.0:

```
sage: from sage.crypto.lwe import DiscreteGaussianSampler
sage: D = DiscreteGaussianSampler(3.0)
```

Next, we construct our oracle:

```
sage: from sage.crypto.lwe import LWE
sage: lwe = LWE(n=20, q=next_prime(400), D=D); lwe
LWE(20, 401, DiscreteGaussianSamplerRejection(3.000000, 53, 4), 'uniform', None)
```

and sample 1000 samples:

```
sage: L = [lwe() for _ in range(1000)]
```

To test the oracle, we use the internal secret to evaluate the samples in the secret:

```
sage: S = [ZZ(a.dot_product(lwe._LWE__s) - c) for (a,c) in L]
```

However, while Sage represents finite field elements between 0 and  $q-1$  we rely on a balanced representation of those elements here. Hence, we fix the representation and recover the correct standard deviation of the noise:

```
sage: sqrt(variance([e if e <= 200 else e-401 for e in S]).n())
3.0...
```

If  $m$  is not `None` the number of available samples is restricted:

```

sage: from sage.crypto.lwe import LWE
sage: lwe = LWE(n=20, q=next_prime(400), D=D, m=30)
sage: _ = [lwe() for _ in range(30)]
sage: lwe() # 31
Traceback (most recent call last):
...
IndexError: Number of available samples exhausted.

```

`__call__()`

EXAMPLE:

```

sage: from sage.crypto.lwe import DiscreteGaussianSampler, LWE
sage: LWE(10, 401, DiscreteGaussianSampler(3))()
((309, 347, 198, 194, 336, 360, 264, 123, 368, 398), 198)

```

**class** `sage.crypto.lwe.LindnerPeikert` (*n*, *delta*=0.01, *m*=None)

Bases: `sage.crypto.lwe.LWE`

LWE oracle with parameters as in [LP11].

`__init__` (*n*, *delta*=0.01, *m*=None)

Construct LWE instance parameterised by security parameter *n* where the modulus *q* and the stddev of the noise is chosen as in [LP11].

INPUT:

- *n* - security parameter (integer > 0)
- *delta* - error probability per symbol (default: 0.01)
- *m* - number of allowed samples or None in which case  $m=2*n + 128$  as in [LP11] (default: None)

EXAMPLES:

```

sage: from sage.crypto.lwe import LindnerPeikert
sage: LindnerPeikert(n=20)
LWE(20, 2053, DiscreteGaussianSamplerRejection(3.600954, 53, 4), 'noise', 168)

```

**class** `sage.crypto.lwe.Regev` (*n*, *secret\_dist*='uniform', *m*=None)

Bases: `sage.crypto.lwe.LWE`

LWE oracle with parameters as in [Reg09].

`__init__` (*n*, *secret\_dist*='uniform', *m*=None)

Construct LWE instance parameterised by security parameter *n* where the modulus *q* and the stddev of the noise are chosen as in [Reg09].

INPUT:

- *n* - security parameter (integer > 0)
- *secret\_dist* - distribution of the secret. See documentation of `LWE` for details (default='uniform')
- *m* - number of allowed samples or None if no such limit exists (default: None)

EXAMPLES:

```

sage: from sage.crypto.lwe import Regev
sage: Regev(n=20)
LWE(20, 401, DiscreteGaussianSamplerRejection(1.915069, 401, 4), 'uniform', None)

```

**class** `sage.crypto.lwe.RingLWE` (*N*, *q*, *D*, *poly*=None, *secret\_dist*='uniform', *m*=None)

Bases: `sage.structure.sage_object.SageObject`

Ring Learning with Errors oracle.

**\_\_init\_\_**(*N, q, D, poly=None, secret\_dist='uniform', m=None*)

Construct a Ring-LWE oracle in dimension  $n=\phi(N)$  over a ring of order  $q$  with noise distribution  $D$ .

INPUT:

- $N$  - index of cyclotomic polynomial (integer  $> 0$ , must be power of 2)
- $q$  - modulus typically  $> N$  (integer  $> 0$ )
- $D$  - an error distribution such as an instance of `DiscreteGaussianPolynomialSamplerRejection` or `UniformSampler`
- $poly$  - a polynomial of degree  $\phi(N)$ . If `None` the cyclotomic polynomial used (default: `None`).
- $secret\_dist$  - distribution of the secret. See documentation of `LWE` for details (default: `'uniform'`)
- $m$  - number of allowed samples or `None` if no such limit exists (default: `None`)

EXAMPLE:

```
sage: from sage.crypto.lwe import DiscreteGaussianPolynomialSampler, RingLWE
sage: D = DiscreteGaussianPolynomialSampler(n=euler_phi(20), stddev=3.0)
sage: RingLWE(N=20, q=next_prime(800), D=D);
RingLWE(20, 809, DiscreteGaussianPolynomialSamplerRejection(8, 3.000000, 53, 4), x^8 - x^6 +
```

**\_\_call\_\_**()

EXAMPLE:

```
sage: from sage.crypto.lwe import DiscreteGaussianPolynomialSampler, RingLWE
sage: N = 16
sage: n = euler_phi(N)
sage: D = DiscreteGaussianPolynomialSampler(n, 5)
sage: ringlwe = RingLWE(N, 257, D, secret_dist='uniform')
sage: ringlwe()
((228, 149, 226, 198, 38, 222, 222, 127), (177, 138, 68, 134, 74, 162, 203, 243))
```

**class** `sage.crypto.lwe.RingLWEConverter`(*ringlwe*)

Bases: `sage.structure.sage_object.SageObject`

Wrapper callable to convert Ring-LWE oracles into LWE oracles by disregarding the additional structure.

**\_\_init\_\_**(*ringlwe*)

INPUT:

- *ringlwe* - an instance of a `RingLWE`

EXAMPLE:

```
sage: from sage.crypto.lwe import DiscreteGaussianPolynomialSampler, RingLWE, RingLWEConverter
sage: D = DiscreteGaussianPolynomialSampler(euler_phi(16), 5)
sage: lwe = RingLWEConverter(RingLWE(16, 257, D, secret_dist='uniform'))
sage: set_random_seed(1337)
sage: lwe()
((130, 32, 216, 3, 125, 58, 197, 171), 182)
```

**\_\_call\_\_**()

EXAMPLE:

```
sage: from sage.crypto.lwe import DiscreteGaussianPolynomialSampler, RingLWE, RingLWEConverter
sage: D = DiscreteGaussianPolynomialSampler(euler_phi(16), 5)
sage: lwe = RingLWEConverter(RingLWE(16, 257, D, secret_dist='uniform'))
sage: set_random_seed(1337)
```

```
sage: lwe()
      ((130, 32, 216, 3, 125, 58, 197, 171), 182)
```

**class** `sage.crypto.lwe.RingLindnerPeikert` (*N*, *delta*=0.01, *m*=None)

Bases: `sage.crypto.lwe.RingLWE`

Ring-LWE oracle with parameters as in [LP11].

**\_\_init\_\_** (*N*, *delta*=0.01, *m*=None)

Construct a Ring-LWE oracle in dimension  $n=\phi(N)$  where the modulus  $q$  and the stddev of the noise is chosen as in [LP11].

INPUT:

- *N* - index of cyclotomic polynomial (integer > 0, must be power of 2)
- *delta* - error probability per symbol (default: 0.01)
- *m* - number of allowed samples or None in which case  $3*n$  is used (default: None)

EXAMPLES:

```
sage: from sage.crypto.lwe import RingLindnerPeikert
```

```
sage: RingLindnerPeikert(N=16)
```

```
RingLWE(16, 1031, DiscreteGaussianPolynomialSamplerRejection(8, 2.803372, 53, 4), x^8 + 1, '
```

**class** `sage.crypto.lwe.UniformNoiseLWE` (*n*, *instance*='key', *m*=None)

Bases: `sage.crypto.lwe.LWE`

LWE oracle with uniform secret with parameters as in [CGW13].

**\_\_init\_\_** (*n*, *instance*='key', *m*=None)

Construct LWE instance parameterised by security parameter  $n$  where all other parameters are chosen as in [CGW13].

INPUT:

- *n* - security parameter (integer  $\geq 89$ )
- *instance* - one of
  - “key” - the LWE-instance that hides the secret key is generated
  - “encrypt” - the LWE-instance that hides the message is generated (default: key)
- *m* - number of allowed samples or None in which case  $m$  is chosen as in [CGW13]. (default: None)

EXAMPLES:

```
sage: from sage.crypto.lwe import UniformNoiseLWE
```

```
sage: UniformNoiseLWE(89)
```

```
LWE(89, 154262477, UniformSampler(0, 351), 'noise', 131)
```

```
sage: UniformNoiseLWE(89, instance='encrypt')
```

```
LWE(131, 154262477, UniformSampler(0, 497), 'noise', 181)
```

**class** `sage.crypto.lwe.UniformPolynomialSampler` (*n*, *lower\_bound*, *upper\_bound*)

Bases: `sage.structure.sage_object.SageObject`

Uniform sampler for polynomials.

EXAMPLE:

```
sage: from sage.crypto.lwe import UniformPolynomialSampler
sage: UniformPolynomialSampler(8, -2, 2)()
-2*x^7 + x^6 - 2*x^5 - x^3 - 2*x^2 - 2
```

**\_\_init\_\_**(*n*, *lower\_bound*, *upper\_bound*)

Construct a sampler for univariate polynomials of degree  $n-1$  where coefficients are drawn uniformly at random between *lower\_bound* and *upper\_bound* (both endpoints inclusive).

INPUT:

- *n* - number of coefficients to be sampled
- *lower\_bound* - integer
- *upper\_bound* - integer

EXAMPLE:

```
sage: from sage.crypto.lwe import UniformPolynomialSampler
sage: UniformPolynomialSampler(10, -10, 10)
UniformPolynomialSampler(10, -10, 10)
```

**\_\_call\_\_**()

Return a new sample.

EXAMPLE:

```
sage: from sage.crypto.lwe import UniformPolynomialSampler
sage: sampler = UniformPolynomialSampler(8, -12, 12)
sage: sampler()
-10*x^7 + 5*x^6 - 8*x^5 + x^4 - 4*x^3 - 11*x^2 - 10
```

**class** `sage.crypto.lwe.UniformSampler`(*lower\_bound*, *upper\_bound*)

Bases: `sage.structure.sage_object.SageObject`

Uniform sampling in a range of integers.

EXAMPLE:

```
sage: from sage.crypto.lwe import UniformSampler
sage: sampler = UniformSampler(-2, 2); sampler
UniformSampler(-2, 2)
sage: sampler()
-2
```

**\_\_init\_\_**(*lower\_bound*, *upper\_bound*)

Construct a uniform sampler with bounds *lower\_bound* and *upper\_bound* (both endpoints inclusive).

INPUT:

- *lower\_bound* - integer
- *upper\_bound* - integer

EXAMPLE:

```
sage: from sage.crypto.lwe import UniformSampler
sage: UniformSampler(-2, 2)
UniformSampler(-2, 2)
```

**\_\_call\_\_**()

Return a new sample.



EXAMPLE:

```
sage: from sage.crypto.lwe import UniformSampler
sage: sampler = UniformSampler(-12, 12)
sage: sampler()
-10
```

`sage.crypto.lwe.balance_sample(s, q=None)`

Given  $(a, c) = s$  return a tuple  $(a', c')$  where  $a'$  is an integer vector with entries between  $-q/2$  and  $q/2$  and  $c$  is also within these bounds.

If  $q$  is given  $(a, c) = s$  may live in the integers. If  $q$  is not given, then  $(a, c)$  are assumed to live in  $\mathbb{Z}/q\mathbb{Z}$ .

INPUT:

- $s$  - sample of the form  $(a, c)$  where  $a$  is a vector and  $c$  is a scalar
- $q$  - modulus (default: None)

EXAMPLE:

```
sage: from sage.crypto.lwe import balance_sample, samples, Regev
sage: map(balance_sample, samples(10, 5, Regev))
[((-9, -4, -4, 4, -4), 6), ((-3, -10, 8, -3, -1), -10), ((-6, -12, -3, -2, -6), -6),
...
((-1, -8, -11, 13, 4), -6), ((10, 11, -3, -13, 0), 6), ((6, -1, 2, -11, 14), 2)]

sage: from sage.crypto.lwe import balance_sample, DiscreteGaussianPolynomialSampler, RingLWE, sa
sage: D = DiscreteGaussianPolynomialSampler(8, 5)
sage: rlwe = RingLWE(20, 257, D)
sage: map(balance_sample, samples(10, 8, rlwe))
[((5, -55, -31, -90, 6, 100, -46, -107), (6, -64, -40, 117, 27, 54, -98, -56)),
((109, -106, 28, 77, -14, -109, 115, 34), (82, 17, -89, 62, 1, -77, 128, 64)),
...
((-32, 51, -110, -106, 35, -82, 14, -113), (126, -120, 126, 119, 101, 3, -122, -75))]
```

---

**Note:** This function is useful to convert between Sage’s standard representation of elements in  $\mathbb{Z}/q\mathbb{Z}$  as integers between 0 and  $q-1$  and the usual representation of such elements in lattice cryptography as integers between  $-q/2$  and  $q/2$ .

---

`sage.crypto.lwe.samples(m, n, lwe, seed=None, balanced=False, **kws)`

Return  $m$  LWE samples.

INPUT:

- $m$  - the number of samples (integer  $> 0$ )
- $n$  - the security parameter (integer  $> 0$ )
- $lwe$  - either
  - a subclass of `LWE` such as `Regev` or `LindnerPeikert`
  - an instance of `LWE` or any subclass
  - the name of any such class (e.g., “Regev”, “LindnerPeikert”)
- $seed$  - seed to be used for generation or `None` if no specific seed shall be set (default: `None`)
- $balanced$  - use function `balance_sample()` to return balanced representations of finite field elements (default: `False`)

```
sage: from sage.crypto.lwe import samples, Regev
sage: samples(2, 20, Regev, seed=1337)
[[ (199, 388, 337, 53, 200, 284, 336, 215, 75, 14, 274, 234, 97, 255, 246, 153, 268, 218, 396, 35),
  (286, 42, 175, 155, 190, 275, 114, 280, 45, 218, 304, 386, 98, 235, 77, 0, 65, 20, 163, 14), 33]

sage: from sage.crypto.lwe import samples, Regev
sage: samples(2, 20, Regev, balanced=True, seed=1337)
[[ (199, -13, -64, 53, 200, -117, -65, -186, 75, 14, -127, -167, 97, -146, -155, 153, -133, -183,
  (-115, 42, 175, 155, 190, -126, 114, -121, 45, -183, -97, -15, 98, -166, 77, 0, 65, 20, 163, 14)

sage: from sage.crypto.lwe import samples
sage: samples(2, 20, 'LindnerPeikert')
[[ (1302, 718, 1397, 147, 278, 979, 1185, 133, 902, 1180, 1264, 734, 2029, 314, 428, 18, 707, 202,
  (2015, 1278, 455, 429, 1391, 186, 149, 1199, 220, 1629, 843, 719, 1744, 1568, 674, 1462, 1549,
```

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