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# **Sage Reference Manual: Numerical Optimization**

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**The Sage Development Team**

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# KNAPSACK PROBLEMS

This module implements a number of solutions to various knapsack problems, otherwise known as linear integer programming problems. Solutions to the following knapsack problems are implemented:

- Solving the subset sum problem for super-increasing sequences.
- General case using Linear Programming

AUTHORS:

- Minh Van Nguyen (2009-04): initial version
- Nathann Cohen (2009-08): Linear Programming version

## 1.1 Definition of Knapsack problems

You have already had a knapsack problem, so you should know, but in case you do not, a knapsack problem is what happens when you have hundred of items to put into a bag which is too small, and you want to pack the most useful of them.

When you formally write it, here is your problem:

- Your bag can contain a weight of at most  $W$ .
- Each item  $i$  has a weight  $w_i$ .
- Each item  $i$  has a usefulness  $u_i$ .

You then want to maximize the total usefulness of the items you will store into your bag, while keeping sure the weight of the bag will not go over  $W$ .

As a linear program, this problem can be represented this way (if you define  $b_i$  as the binary variable indicating whether the item  $i$  is to be included in your bag):

$$\begin{aligned} \text{Maximize: } & \sum_i b_i u_i \\ \text{Such that: } & \sum_i b_i w_i \leq W \\ & \forall i, b_i \text{ binary variable} \end{aligned}$$

(For more information, see the [Wikipedia article Knapsack\\_problem](#))

## 1.2 Examples

If your knapsack problem is composed of three items (weight, value) defined by (1,2), (1.5,1), (0.5,3), and a bag of maximum weight 2, you can easily solve it this way:

```
sage: from sage.numerical.knapsack import knapsack
sage: knapsack( [(1,2), (1.5,1), (0.5,3)], max=2)
[5.0, [(1, 2), (0.5000000000000000, 3)]]
```

## 1.3 Super-increasing sequences

We can test for whether or not a sequence is super-increasing:

```
sage: from sage.numerical.knapsack import Superincreasing
sage: L = [1, 2, 5, 21, 69, 189, 376, 919]
sage: seq = Superincreasing(L)
sage: seq
Super-increasing sequence of length 8
sage: seq.is_superincreasing()
True
sage: Superincreasing().is_superincreasing([1,3,5,7])
False
```

Solving the subset sum problem for a super-increasing sequence and target sum:

```
sage: L = [1, 2, 5, 21, 69, 189, 376, 919]
sage: Superincreasing(L).subset_sum(98)
[69, 21, 5, 2, 1]
```

```
class sage.numerical.knapsack.Superincreasing(seq=None)
Bases: sage.structure.sage_object.SageObject
```

A class for super-increasing sequences.

Let  $L = (a_1, a_2, a_3, \dots, a_n)$  be a non-empty sequence of non-negative integers. Then  $L$  is said to be super-increasing if each  $a_i$  is strictly greater than the sum of all previous values. That is, for each  $a_i \in L$  the sequence  $L$  must satisfy the property

$$a_i > \sum_{k=1}^{i-1} a_k$$

in order to be called a super-increasing sequence, where  $|L| \geq 2$ . If  $L$  has only one element, it is also defined to be a super-increasing sequence.

If `seq` is `None`, then construct an empty sequence. By definition, this empty sequence is not super-increasing.

INPUT:

- `seq` – (default: `None`) a non-empty sequence.

EXAMPLES:

```
sage: from sage.numerical.knapsack import Superincreasing
sage: L = [1, 2, 5, 21, 69, 189, 376, 919]
sage: Superincreasing(L).is_superincreasing()
True
sage: Superincreasing().is_superincreasing([1,3,5,7])
```

```
False
sage: seq = Superincreasing(); seq
An empty sequence.
sage: seq = Superincreasing([1, 3, 6]); seq
Super-increasing sequence of length 3
sage: seq = Superincreasing(list([1, 2, 5, 21, 69, 189, 376, 919])); seq
Super-increasing sequence of length 8
```

**is\_superincreasing** (*seq=None*)

Determine whether or not *seq* is super-increasing.

If *seq=None* then determine whether or not *self* is super-increasing.

Let  $L = (a_1, a_2, a_3, \dots, a_n)$  be a non-empty sequence of non-negative integers. Then  $L$  is said to be super-increasing if each  $a_i$  is strictly greater than the sum of all previous values. That is, for each  $a_i \in L$  the sequence  $L$  must satisfy the property

$$a_i > \sum_{k=1}^{i-1} a_k$$

in order to be called a super-increasing sequence, where  $|L| \geq 2$ . If  $L$  has exactly one element, then it is also defined to be a super-increasing sequence.

INPUT:

- *seq* – (default: None) a sequence to test

OUTPUT:

- If *seq* is None, then test *self* to determine whether or not it is super-increasing. In that case, return True if *self* is super-increasing; False otherwise.
- If *seq* is not None, then test *seq* to determine whether or not it is super-increasing. Return True if *seq* is super-increasing; False otherwise.

EXAMPLES:

By definition, an empty sequence is not super-increasing:

```
sage: from sage.numerical.knapsack import Superincreasing
sage: Superincreasing().is_superincreasing([])
False
sage: Superincreasing().is_superincreasing()
False
sage: Superincreasing().is_superincreasing(tuple())
False
sage: Superincreasing().is_superincreasing(())
False
```

But here is an example of a super-increasing sequence:

```
sage: L = [1, 2, 5, 21, 69, 189, 376, 919]
sage: Superincreasing(L).is_superincreasing()
True
sage: L = (1, 2, 5, 21, 69, 189, 376, 919)
sage: Superincreasing(L).is_superincreasing()
True
```

A super-increasing sequence can have zero as one of its elements:

```
sage: L = [0, 1, 2, 4]
sage: Superincreasing(L).is_superincreasing()
True
```

A super-increasing sequence can be of length 1:

```
sage: Superincreasing([randint(0, 100)]).is_superincreasing()
True
```

TESTS:

The sequence must contain only integers:

```
sage: from sage.numerical.knapsack import Superincreasing
sage: L = [1.0, 2.1, pi, 21, 69, 189, 376, 919]
sage: Superincreasing(L).is_superincreasing()
Traceback (most recent call last):
...
TypeError: Element e (= 1.000000000000000) of seq must be a non-negative integer.
sage: L = [1, 2.1, pi, 21, 69, 189, 376, 919]
sage: Superincreasing(L).is_superincreasing()
Traceback (most recent call last):
...
TypeError: Element e (= 2.100000000000000) of seq must be a non-negative integer.
```

#### **largest\_less\_than(N)**

Return the largest integer in the sequence `self` that is less than or equal to `N`.

This function narrows down the candidate solution using a binary trim, similar to the way binary search halves the sequence at each iteration.

INPUT:

- `N` – integer; the target value to search for.

OUTPUT:

The largest integer in `self` that is less than or equal to `N`. If no solution exists, then return `None`.

EXAMPLES:

When a solution is found, return it:

```
sage: from sage.numerical.knapsack import Superincreasing
sage: L = [2, 3, 7, 25, 67, 179, 356, 819]
sage: Superincreasing(L).largest_less_than(207)
179
sage: L = (2, 3, 7, 25, 67, 179, 356, 819)
sage: Superincreasing(L).largest_less_than(2)
2
```

But if no solution exists, return `None`:

```
sage: L = [2, 3, 7, 25, 67, 179, 356, 819]
sage: Superincreasing(L).largest_less_than(-1) is None
True
```

TESTS:

The target `N` must be an integer:

```
sage: from sage.numerical.knapsack import Superincreasing
sage: L = [2, 3, 7, 25, 67, 179, 356, 819]
```



```
sage: Superincreasing(L).largest_less_than(2.30)
Traceback (most recent call last):
...
TypeError: N (= 2.3000000000000000) must be an integer.
```

The sequence that `self` represents must also be non-empty:

```
sage: Superincreasing([]).largest_less_than(2)
Traceback (most recent call last):
...
ValueError: seq must be a super-increasing sequence
sage: Superincreasing(list()).largest_less_than(2)
Traceback (most recent call last):
...
ValueError: seq must be a super-increasing sequence
```

### **subset\_sum(N)**

Solving the subset sum problem for a super-increasing sequence.

Let  $S = (s_1, s_2, s_3, \dots, s_n)$  be a non-empty sequence of non-negative integers, and let  $N \in \mathbf{Z}$  be non-negative. The subset sum problem asks for a subset  $A \subseteq S$  all of whose elements sum to  $N$ . This method specializes the subset sum problem to the case of super-increasing sequences. If a solution exists, then it is also a super-increasing sequence.

---

**Note:** This method only solves the subset sum problem for super-increasing sequences. In general, solving the subset sum problem for an arbitrary sequence is known to be computationally hard.

---

INPUT:

- $N$  – a non-negative integer.

OUTPUT:

- A non-empty subset of `self` whose elements sum to  $N$ . This subset is also a super-increasing sequence. If no such subset exists, then return the empty list.

ALGORITHMS:

The algorithm used is adapted from page 355 of [HPS08].

EXAMPLES:

Solving the subset sum problem for a super-increasing sequence and target sum:

```
sage: from sage.numerical.knapsack import Superincreasing
sage: L = [1, 2, 5, 21, 69, 189, 376, 919]
sage: Superincreasing(L).subset_sum(98)
[69, 21, 5, 2, 1]
```

TESTS:

The target  $N$  must be a non-negative integer:

```
sage: from sage.numerical.knapsack import Superincreasing
sage: L = [0, 1, 2, 4]
sage: Superincreasing(L).subset_sum(-6)
Traceback (most recent call last):
...
TypeError: N (= -6) must be a non-negative integer.
sage: Superincreasing(L).subset_sum(-6.2)
Traceback (most recent call last):
```

```
...
TypeError: N (= -6.200000000000000) must be a non-negative integer.
```

The sequence that `self` represents must only contain non-negative integers:

```
sage: L = [-10, -9, -8, -7, -6, -5, -4, -3, -2, -1, 0, 1]
sage: Superincreasing(L).subset_sum(1)
Traceback (most recent call last):
...
TypeError: Element e (= -10) of seq must be a non-negative integer.
```

#### REFERENCES:

`sage.numerical.knapsack.knapsack` (*seq*, *binary=True*, *max=1*, *value\_only=False*, *solver=None*, *verbose=0*)

Solves the knapsack problem

For more information on the knapsack problem, see the documentation of the [knapsack module](#) or the [Wikipedia article Knapsack\\_problem](#).

#### INPUT:

- `seq` – Two different possible types:
  - A sequence of tuples (`weight`, `value`, `something1`, `something2`, ...). Note that only the first two coordinates (`weight` and `values`) will be taken into account. The rest (if any) will be ignored. This can be useful if you need to attach some information to the items.
  - A sequence of reals (a value of 1 is assumed).
- `binary` – When set to `True`, an item can be taken 0 or 1 time. When set to `False`, an item can be taken any amount of times (while staying integer and positive).
- `max` – Maximum admissible weight.
- `value_only` – When set to `True`, only the maximum useful value is returned. When set to `False`, both the maximum useful value and an assignment are returned.
- `solver` – (default: `None`) Specify a Linear Program (LP) solver to be used. If set to `None`, the default one is used. For more information on LP solvers and which default solver is used, see the documentation of class [MixedIntegerLinearProgram](#).
- `verbose` – integer (default: 0). Sets the level of verbosity. Set to 0 by default, which means quiet.

#### OUTPUT:

If `value_only` is set to `True`, only the maximum useful value is returned. Else (the default), the function returns a pair `[value, list]`, where `list` can be of two types according to the type of `seq`:

- The list of tuples  $(w_i, u_i, \dots)$  occurring in the solution.
- A list of reals where each real is repeated the number of times it is taken into the solution.

#### EXAMPLES:

If your knapsack problem is composed of three items (`weight`, `value`) defined by  $(1, 2)$ ,  $(1.5, 1)$ ,  $(0.5, 3)$ , and a bag of maximum weight 2, you can easily solve it this way:

```
sage: from sage.numerical.knapsack import knapsack
sage: knapsack([ (1,2), (1.5,1), (0.5,3) ], max=2)
[5.0, [(1, 2), (0.5000000000000000, 3)]]

sage: knapsack([ (1,2), (1.5,1), (0.5,3) ], max=2, value_only=True)
5.0
```

Besides weight and value, you may attach any data to the items:

```
sage: from sage.numerical.knapsack import knapsack
sage: knapsack( [(1, 2, 'spam'), (0.5, 3, 'a', 'lot')])
[3.0, [(0.5000000000000000, 3, 'a', 'lot')]]
```

In the case where all the values (usefulness) of the items are equal to one, you do not need embarrass yourself with the second values, and you can just type for items (1,1), (1.5,1), (0.5,1) the command:

```
sage: from sage.numerical.knapsack import knapsack
sage: knapsack([1,1.5,0.5], max=2, value_only=True)
2.0
```



# MIXED INTEGER LINEAR PROGRAMMING

## Mixed Integer Linear Programming

A linear program (LP) is an [optimization problem](#) in the following form

$$\max\{c^T x \mid Ax \leq b, x \geq 0\}$$

with given  $A \in \mathbb{R}^{m,n}$ ,  $b \in \mathbb{R}^m$ ,  $c \in \mathbb{R}^n$  and unknown  $x \in \mathbb{R}^n$ . If some or all variables in the vector  $x$  are restricted over the integers  $\mathbb{Z}$ , the problem is called mixed integer linear program ([MILP](#)). A wide variety of problems in optimization can be formulated in this standard form. Then, solvers are able to calculate a solution.

Imagine you want to solve the following linear system of three equations:

- $w_0 + w_1 + w_2 - 14w_3 = 0$
- $w_1 + 2w_2 - 8w_3 = 0$
- $2w_2 - 3w_3 = 0$

and this additional inequality:

- $w_0 - w_1 - w_2 \geq 0$

where all  $w_i \in \mathbb{Z}^+$ . You know that the trivial solution is  $w_i = 0$ , but what is the first non-trivial one with  $w_3 \geq 1$ ?

A mixed integer linear program can give you an answer:

1. You have to create an instance of `MixedIntegerLinearProgram` and – in our case – specify that it is a minimization.
2. Create an dictionary `w` of integer variables `w` via `w = p.new_variable(integer=True)` (note that **by default all variables are non-negative**, cf `new_variable()`).
3. Add those three equations as equality constraints via `add_constraint`.
4. Also add the inequality constraint.
5. Add an inequality constraint  $w_3 \geq 1$  to exclude the trivial solution.
6. By default, all variables are non-negative. We remove that constraint via `p.set_min(variable, None)`, see `set_min`.
7. Specify the objective function via `set_objective`. In our case that is just  $w_3$ . If it is a pure constraint satisfaction problem, specify it as `None`.
8. To check if everything is set up correctly, you can print the problem via `show`.

9. `Solve` it and print the solution.

The following example shows all these steps:

```
sage: p = MixedIntegerLinearProgram(maximization=False, solver = "GLPK")
sage: w = p.new_variable(integer=True, nonnegative=True)
sage: p.add_constraint(w[0] + w[1] + w[2] - 14*w[3] == 0)
sage: p.add_constraint(w[1] + 2*w[2] - 8*w[3] == 0)
sage: p.add_constraint(2*w[2] - 3*w[3] == 0)
sage: p.add_constraint(w[0] - w[1] - w[2] >= 0)
sage: p.add_constraint(w[3] >= 1)
sage: _ = [ p.set_min(w[i], None) for i in range(1,4) ]
sage: p.set_objective(w[3])
sage: p.show()
Minimization:
    x_3
Constraints:
    0.0 <= x_0 + x_1 + x_2 - 14.0 x_3 <= 0.0
    0.0 <= x_1 + 2.0 x_2 - 8.0 x_3 <= 0.0
    0.0 <= 2.0 x_2 - 3.0 x_3 <= 0.0
    - x_0 + x_1 + x_2 <= 0.0
    - x_3 <= -1.0
Variables:
    x_0 is an integer variable (min=0.0, max=+oo)
    x_1 is an integer variable (min=-oo, max=+oo)
    x_2 is an integer variable (min=-oo, max=+oo)
    x_3 is an integer variable (min=-oo, max=+oo)
sage: print 'Objective Value:', p.solve()
Objective Value: 2.0
sage: for i, v in p.get_values(w).iteritems():\
    print 'w_%s = %s' % (i, int(round(v)))

w_0 = 15
w_1 = 10
w_2 = 3
w_3 = 2
```

Different backends compute with different base fields, for example:

```
sage: p = MixedIntegerLinearProgram(solver = 'GLPK')
sage: p.base_ring()
Real Double Field
sage: x = p.new_variable(real=True, nonnegative=True)
sage: 0.5 + 3/2*x[1]
0.5 + 1.5*x_0

sage: p = MixedIntegerLinearProgram(solver = 'ppl')
sage: p.base_ring()
Rational Field
sage: x = p.new_variable(nonnegative=True)
sage: 0.5 + 3/2*x[1]
1/2 + 3/2*x_0
```

AUTHORS:

- Risan (2012/02): added extension for exact computation

## 2.1 Index of functions and methods

Below are listed the methods of `MixedIntegerLinearProgram`. This module also implements the `MIPSolverException` exception, as well as the `MIPVariable` class.

<code>add_constraint()</code>	Adds a constraint to the <code>MixedIntegerLinearProgram</code>
<code>base_ring()</code>	Return the base ring
<code>constraints()</code>	Returns a list of constraints, as 3-tuples
<code>get_backend()</code>	Returns the backend instance used
<code>get_max()</code>	Returns the maximum value of a variable
<code>get_min()</code>	Returns the minimum value of a variable
<code>get_values()</code>	Return values found by the previous call to <code>solve()</code>
<code>is_binary()</code>	Tests whether the variable <code>e</code> is binary
<code>is_integer()</code>	Tests whether the variable is an integer
<code>is_real()</code>	Tests whether the variable is real
<code>linear_constraints_parent()</code>	Return the parent for all linear constraints
<code>linear_function()</code>	Construct a new linear function
<code>linear_functions_parent()</code>	Return the parent for all linear functions
<code>new_variable()</code>	Returns an instance of <code>MIPVariable</code> associated
<code>number_of_constraints()</code>	Returns the number of constraints assigned so far
<code>number_of_variables()</code>	Returns the number of variables used so far
<code>polyhedron()</code>	Returns the polyhedron defined by the Linear Program
<code>remove_constraint()</code>	Removes a constraint from self
<code>remove_constraints()</code>	Remove several constraints
<code>set_binary()</code>	Sets a variable or a <code>MIPVariable</code> as binary
<code>set_integer()</code>	Sets a variable or a <code>MIPVariable</code> as integer
<code>set_max()</code>	Sets the maximum value of a variable
<code>set_min()</code>	Sets the minimum value of a variable
<code>set_objective()</code>	Sets the objective of the <code>MixedIntegerLinearProgram</code>
<code>set_problem_name()</code>	Sets the name of the <code>MixedIntegerLinearProgram</code>
<code>set_real()</code>	Sets a variable or a <code>MIPVariable</code> as real
<code>show()</code>	Displays the <code>MixedIntegerLinearProgram</code> in a human-readable
<code>solve()</code>	Solves the <code>MixedIntegerLinearProgram</code>
<code>solver_parameter()</code>	Return or define a solver parameter
<code>sum()</code>	Efficiently computes the sum of a sequence of <code>LinearFunction</code> elements
<code>write_lp()</code>	Write the linear program as a LP file
<code>write_mps()</code>	Write the linear program as a MPS file

## 2.2 Classes and methods

**exception** `sage.numerical.mip.MIPSolverException` (*value*)

Bases: `exceptions.RuntimeError`

Exception raised when the solver fails.

**class** `sage.numerical.mip.MIPVariable`

Bases: `sage.structure.sage_object.SageObject`

`MIPVariable` is a variable used by the class `MixedIntegerLinearProgram`.

**depth()**

Returns the current variable's depth.

EXAMPLE:

```
sage: p = MixedIntegerLinearProgram()
sage: v = p.new_variable(nonnegative=True)
sage: p.set_objective(v[0] + v[1])
sage: v.depth()
1
```

**items()**

Returns the pairs (keys,value) contained in the dictionary.

EXAMPLE:

```
sage: p = MixedIntegerLinearProgram()
sage: v = p.new_variable(nonnegative=True)
sage: p.set_objective(v[0] + v[1])
sage: v.items()
[(0, x_0), (1, x_1)]
```

**keys()**

Returns the keys already defined in the dictionary.

EXAMPLE:

```
sage: p = MixedIntegerLinearProgram()
sage: v = p.new_variable(nonnegative=True)
sage: p.set_objective(v[0] + v[1])
sage: v.keys()
[0, 1]
```

**set\_max**(*max*)

Sets an upper bound on the variable.

INPUT:

- max* – an upper bound, or None to mean that the variable is unbounded.

EXAMPLES:

```
sage: p = MixedIntegerLinearProgram()
sage: v = p.new_variable(real=True, nonnegative=True, dim=2)
sage: p.get_max(v)
sage: p.get_max(v[0])
sage: p.get_max(v[0][0])
sage: p.set_max(v, 4)
sage: p.get_max(v)
4
sage: p.get_max(v[0])
4
sage: p.get_max(v[0][0])
4.0
```

**set\_min**(*min*)

Sets a lower bound on the variable.

INPUT:

- min* – a lower bound, or None to mean that the variable is unbounded.

EXAMPLES:

```
sage: p = MixedIntegerLinearProgram()
sage: v = p.new_variable(real=True, nonnegative=True, dim=2)
```



```

sage: p.get_min(v)
0
sage: p.get_min(v[0])
0
sage: p.get_min(v[0][0])
0.0
sage: p.set_min(v, 4)
sage: p.get_min(v)
4
sage: p.get_min(v[0])
4
sage: p.get_min(v[0][0])
4.0

```

**values()**

Returns the symbolic variables associated to the current dictionary.

**EXAMPLE:**

```

sage: p = MixedIntegerLinearProgram()
sage: v = p.new_variable(nonnegative=True)
sage: p.set_objective(v[0] + v[1])
sage: v.values()
[x_0, x_1]

```

**class sage.numerical.mip.MixedIntegerLinearProgram**

Bases: `sage.structure.sage_object.SageObject`

The `MixedIntegerLinearProgram` class is the link between Sage, linear programming (LP) and mixed integer programming (MIP) solvers.

A Mixed Integer Linear Program (MILP) consists of variables, linear constraints on these variables, and an objective function which is to be maximised or minimised under these constraints.

See the [Wikipedia article Linear programming](#) for further information on linear programming, and the [MILP module](#) for its use in Sage.

**INPUT:**

- **solver** – selects a solver:
  - **GLPK** (`solver="GLPK"`). See the [GLPK](#) web site.
  - **COIN Branch and Cut** (`solver="Coin"`). See the [COIN-OR](#) web site.
  - **CPLEX** (`solver="CPLEX"`). See the [CPLEX](#) web site.
  - **Gurobi** (`solver="Gurobi"`). See the [Gurobi](#) web site.
  - **CVXOPT** (`solver="CVXOPT"`). See the [CVXOPT](#) web site.
  - **PPL** (`solver="PPL"`). See the [PPL](#) web site.
  - If `solver=None` (default), the default solver is used (see `default_mip_solver()`)
- **maximization**
  - When set to `True` (default), the `MixedIntegerLinearProgram` is defined as a maximization.
  - When set to `False`, the `MixedIntegerLinearProgram` is defined as a minimization.
- **constraint\_generation** – Only used when `solver=None`.

–When set to `True`, after solving the `MixedIntegerLinearProgram`, it is possible to add a constraint, and then solve it again. The effect is that solvers that do not support this feature will not be used.

–Defaults to `False`.

**Warning:** All LP variables are non-negative by default (see `new_variable()` and `set_min()`).

#### See Also:

- `default_mip_solver()` – Returns/Sets the default MIP solver.

#### EXAMPLES:

Computation of a maximum stable set in Petersen’s graph:

```
sage: g = graphs.PetersenGraph()
sage: p = MixedIntegerLinearProgram(maximization=True)
sage: b = p.new_variable(binary=True)
sage: p.set_objective(sum([b[v] for v in g]))
sage: for (u,v) in g.edges(labels=None):
....:     p.add_constraint(b[u] + b[v], max=1)
sage: p.solve(objective_only=True)
4.0
```

#### TESTS:

Check that [trac ticket #16497](#) is fixed:

```
sage: from sage.numerical.mip import MixedIntegerLinearProgram
sage: for type in ["binary", "integer"]:
....:     k = 3
....:     items = [1/5, 1/3, 2/3, 3/4, 5/7]
....:     maximum=1
....:     p=MixedIntegerLinearProgram()
....:     box=p.new_variable(nonnegative=True, **{type:True})
....:     for b in range(k):
....:         p.add_constraint(p.sum([items[i]*box[i,b] for i in range(len(items))]) <= maximum)
....:     for i in range(len(items)):
....:         p.add_constraint(p.sum([box[i,b] for b in range(k)]) == 1)
....:     p.set_objective(None)
....:     _ = p.solve()
....:     box=p.get_values(box)
....:     print(all(v in ZZ for v in box.values()))
True
True
```

**`add_constraint`** (*linear\_function*, *max=None*, *min=None*, *name=None*)

Adds a constraint to the `MixedIntegerLinearProgram`.

INPUT:

- `linear_function`** – Two different types of arguments are possible:

- A linear function. In this case, arguments `min` or `max` have to be specified.
- A linear constraint of the form  $A \leq B$ ,  $A \geq B$ ,  $A \leq B \leq C$ ,  $A \geq B \geq C$  or  $A == B$ . In this case, arguments `min` and `max` will be ignored.

- `max`** – An upper bound on the constraint (set to `None` by default). This must be a numerical value.

- min – A lower bound on the constraint. This must be a numerical value.
- name – A name for the constraint.

To set a lower and/or upper bound on the variables use the methods `set_min` and/or `set_max` of `MixedIntegerLinearProgram`.

EXAMPLE:

Consider the following linear program:

```
Maximize:
  x + 5 * y
Constraints:
  x + 0.2 y      <= 4
  1.5 * x + 3 * y <= 4
Variables:
  x is Real (min = 0, max = None)
  y is Real (min = 0, max = None)
```

It can be solved as follows:

```
sage: p = MixedIntegerLinearProgram(maximization=True)
sage: x = p.new_variable(nonnegative=True)
sage: p.set_objective(x[1] + 5*x[2])
sage: p.add_constraint(x[1] + 0.2*x[2], max=4)
sage: p.add_constraint(1.5*x[1] + 3*x[2], max=4)
sage: round(p.solve(), 6)
6.666667
```

There are two different ways to add the constraint  $x[5] + 3x[7] \leq x[6] + 3$  to a `MixedIntegerLinearProgram`.

The first one consists in giving `add_constraint` this very expression:

```
sage: p.add_constraint( x[5] + 3*x[7] <= x[6] + 3 )
```

The second (slightly more efficient) one is to use the arguments `min` or `max`, which can only be numerical values:

```
sage: p.add_constraint( x[5] + 3*x[7] - x[6], max = 3 )
```

One can also define double-bounds or equality using symbols `<=`, `>=` and `==`:

```
sage: p.add_constraint( x[5] + 3*x[7] == x[6] + 3 )
sage: p.add_constraint( x[5] + 3*x[7] <= x[6] + 3 <= x[8] + 27 )
```

The previous program can be rewritten:

```
sage: p = MixedIntegerLinearProgram(maximization=True)
sage: x = p.new_variable(nonnegative=True)
sage: p.set_objective(x[1] + 5*x[2])
sage: p.add_constraint(x[1] + 0.2*x[2] <= 4)
sage: p.add_constraint(1.5*x[1] + 3*x[2] <= 4)
sage: round(p.solve(), 5)
6.66667
```

TESTS:

Complex constraints:

```
sage: p = MixedIntegerLinearProgram(solver = "GLPK")
sage: b = p.new_variable(nonnegative=True)
```

```
sage: p.add_constraint( b[8] - b[15] <= 3*b[8] + 9)
sage: p.show()
Maximization:
```

```
Constraints:
-2.0 x_0 - x_1 <= 9.0
```

```
Variables:
x_0 is a continuous variable (min=0.0, max=+oo)
x_1 is a continuous variable (min=0.0, max=+oo)
```

Empty constraint:

```
sage: p=MixedIntegerLinearProgram()
sage: p.add_constraint(sum([]),min=2)
```

Min/Max are numerical

```
sage: v = p.new_variable(nonnegative=True)
sage: p.add_constraint(v[3] + v[5], min = v[6])
Traceback (most recent call last):
...
ValueError: min and max arguments are required to be numerical
sage: p.add_constraint(v[3] + v[5], max = v[6])
Traceback (most recent call last):
...
ValueError: min and max arguments are required to be numerical
```

Do not add redundant elements (notice only one copy of each constraint is added):

```
sage: lp = MixedIntegerLinearProgram(solver = "GLPK", check_redundant=True)
sage: for each in xrange(10): lp.add_constraint(lp[0]-lp[1],min=1)
sage: lp.show()
Maximization:
```

```
Constraints:
1.0 <= x_0 - x_1
Variables:
x_0 is a continuous variable (min=0.0, max=+oo)
x_1 is a continuous variable (min=0.0, max=+oo)
```

We check for constant multiples of constraints as well:

```
sage: for each in xrange(10): lp.add_constraint(2*lp[0]-2*lp[1],min=2)
sage: lp.show()
Maximization:
```

```
Constraints:
1.0 <= x_0 - x_1
Variables:
x_0 is a continuous variable (min=0.0, max=+oo)
x_1 is a continuous variable (min=0.0, max=+oo)
```

But if the constant multiple is negative, we should add it anyway (once):

```
sage: for each in xrange(10): lp.add_constraint(-2*lp[0]+2*lp[1],min=-2)
sage: lp.show()
Maximization:
```

```
Constraints:
1.0 <= x_0 - x_1
```

```

x_0 - x_1 <= 1.0
Variables:
  x_0 is a continuous variable (min=0.0, max=+oo)
  x_1 is a continuous variable (min=0.0, max=+oo)

```

**TESTS:**

Catch True / False as INPUT ([trac ticket #13646](#)):

```

sage: p = MixedIntegerLinearProgram()
sage: x = p.new_variable(nonnegative=True)
sage: p.add_constraint(True)
Traceback (most recent call last):
...
ValueError: argument must be a linear function or constraint, got True

```

**base\_ring()**

Return the base ring.

**OUTPUT:**

A ring. The coefficients that the chosen solver supports.

**EXAMPLES:**

```

sage: p = MixedIntegerLinearProgram(solver='GLPK')
sage: p.base_ring()
Real Double Field
sage: p = MixedIntegerLinearProgram(solver='ppl')
sage: p.base_ring()
Rational Field

```

**constraints (indices=None)**

Returns a list of constraints, as 3-tuples.

**INPUT:**

- **indices** – select which constraint(s) to return
  - If **indices** = None, the method returns the list of all the constraints.
  - If **indices** is an integer *i*, the method returns constraint *i*.
  - If **indices** is a list of integers, the method returns the list of the corresponding constraints.

**OUTPUT:**

Each constraint is returned as a triple `lower_bound, (indices, coefficients), upper_bound`. For each of those entries, the corresponding linear function is the one associating to variable `indices[i]` the coefficient `coefficients[i]`, and 0 to all the others.

`lower_bound` and `upper_bound` are numerical values.

**EXAMPLE:**

First, let us define a small LP:

```

sage: p = MixedIntegerLinearProgram()
sage: p.add_constraint(p[0] - p[2], min = 1, max = 4)
sage: p.add_constraint(p[0] - 2*p[1], min = 1)

```

To obtain the list of all constraints:

```
sage: p.constraints()          # not tested
[(1.0, ([1, 0], [-1.0, 1.0]), 4.0), (1.0, ([2, 0], [-2.0, 1.0]), None)]
```

Or constraint 0 only:

```
sage: p.constraints(0)        # not tested
(1.0, ([1, 0], [-1.0, 1.0]), 4.0)
```

A list of constraints containing only 1:

```
sage: p.constraints([1])      # not tested
[(1.0, ([2, 0], [-2.0, 1.0]), None)]
```

TESTS:

As the ordering of the variables in each constraint depends on the solver used, we define a short function reordering it before it is printed. The output would look the same without this function applied:

```
sage: def reorder_constraint((lb, (ind, coef), ub)):
...     d = dict(zip(ind, coef))
...     ind.sort()
...     return (lb, (ind, [d[i] for i in ind]), ub)
```

Running the examples from above, reordering applied:

```
sage: p = MixedIntegerLinearProgram(solver = "GLPK")
sage: p.add_constraint(p[0] - p[2], min = 1, max = 4)
sage: p.add_constraint(p[0] - 2*p[1], min = 1)
sage: sorted(map(reorder_constraint, p.constraints()))
[(1.0, ([0, 1], [1.0, -1.0]), 4.0), (1.0, ([0, 2], [1.0, -2.0]), None)]
sage: reorder_constraint(p.constraints(0))
(1.0, ([0, 1], [1.0, -1.0]), 4.0)
sage: sorted(map(reorder_constraint, p.constraints([1])))
[(1.0, ([0, 2], [1.0, -2.0]), None)]
```

**get\_backend()**

Returns the backend instance used.

This might be useful when access to additional functions provided by the backend is needed.

EXAMPLE:

This example uses the simplex algorithm and prints information:

```
sage: p = MixedIntegerLinearProgram(solver="GLPK")
sage: x, y = p[0], p[1]
sage: p.add_constraint(2*x + 3*y, max = 6)
sage: p.add_constraint(3*x + 2*y, max = 6)
sage: p.set_objective(x + y + 7)
sage: b = p.get_backend()
sage: b.solver_parameter("simplex_or_intopt", "simplex_only")
sage: b.solver_parameter("verbosity_simplex", "GLP_MSG_ALL")
sage: p.solve() # tol 0.00001
GLPK Simplex Optimizer, v4.44
2 rows, 2 columns, 4 non-zeros
*      0: obj =   7.000000000e+00   infeas =   0.000e+00 (0)
*      2: obj =   9.400000000e+00   infeas =   0.000e+00 (0)
OPTIMAL SOLUTION FOUND
9.4
```

**get\_max(v)**

Returns the maximum value of a variable.

INPUT:

- `v` – a variable.

OUTPUT:

Maximum value of the variable, or `None` if the variable has no upper bound.

EXAMPLE:

```
sage: p = MixedIntegerLinearProgram()
sage: v = p.new_variable(nonnegative=True)
sage: p.set_objective(v[1])
sage: p.get_max(v[1])
sage: p.set_max(v[1], 6)
sage: p.get_max(v[1])
6.0
```

**get\_min**(`v`)

Returns the minimum value of a variable.

INPUT:

- `v` – a variable

OUTPUT:

Minimum value of the variable, or `None` if the variable has no lower bound.

EXAMPLE:

```
sage: p = MixedIntegerLinearProgram()
sage: v = p.new_variable(nonnegative=True)
sage: p.set_objective(v[1])
sage: p.get_min(v[1])
0.0
sage: p.set_min(v[1], 6)
sage: p.get_min(v[1])
6.0
sage: p.set_min(v[1], None)
sage: p.get_min(v[1])
```

**get\_values**(`*lists`)

Return values found by the previous call to `solve()`.

INPUT:

- Any instance of `MIPVariable` (or one of its elements), or lists of them.

OUTPUT:

- Each instance of `MIPVariable` is replaced by a dictionary containing the numerical values found for each corresponding variable in the instance.
- Each element of an instance of a `MIPVariable` is replaced by its corresponding numerical value.

---

**Note:** While a variable may be declared as binary or integer, its value as returned by the solver is of type `float`.

---

EXAMPLE:

```
sage: p = MixedIntegerLinearProgram()
sage: x = p.new_variable(nonnegative=True)
sage: y = p.new_variable(nonnegative=True)
sage: p.set_objective(x[3] + 3*y[2,9] + x[5])
sage: p.add_constraint(x[3] + y[2,9] + 2*x[5], max=2)
sage: p.solve()
6.0
```

To return the optimal value of  $y[2, 9]$ :

```
sage: p.get_values(y[2,9])
2.0
```

To get a dictionary identical to  $x$  containing optimal values for the corresponding variables

```
sage: x_sol = p.get_values(x)
sage: x_sol.keys()
[3, 5]
```

Obviously, it also works with variables of higher dimension:

```
sage: y_sol = p.get_values(y)
```

We could also have tried

```
sage: [x_sol, y_sol] = p.get_values(x, y)
```

Or:

```
sage: [x_sol, y_sol] = p.get_values([x, y])
```

TESTS:

When ‘dim’ will be removed, also remove from this function the code that uses it:

```
sage: p = MixedIntegerLinearProgram()
sage: b = p.new_variable(dim=2)
doctest:...: DeprecationWarning: The 'dim' argument will soon disappear. Fortunately variable
See http://trac.sagemath.org/15489 for details.
sage: p.add_constraint(b[1][2] + b[2][3] == 0)
sage: _ = p.solve()
sage: _ = p.get_values(b)
```

**is\_binary**( $e$ )

Tests whether the variable  $e$  is binary. Variables are real by default.

INPUT:

- $e$  – A variable (not a `MIPVariable`, but one of its elements.)

OUTPUT:

True if the variable  $e$  is binary; False otherwise.

EXAMPLE:

```
sage: p = MixedIntegerLinearProgram()
sage: v = p.new_variable(nonnegative=True)
sage: p.set_objective(v[1])
sage: p.is_binary(v[1])
False
sage: p.set_binary(v[1])
```



```
sage: p.is_binary(v[1])
True
```

### **is\_integer(*e*)**

Tests whether the variable is an integer. Variables are real by default.

INPUT:

- *e* – A variable (not a MIPVariable, but one of its elements.)

OUTPUT:

True if the variable *e* is an integer; False otherwise.

EXAMPLE:

```
sage: p = MixedIntegerLinearProgram()
sage: v = p.new_variable(nonnegative=True)
sage: p.set_objective(v[1])
sage: p.is_integer(v[1])
False
sage: p.set_integer(v[1])
sage: p.is_integer(v[1])
True
```

### **is\_real(*e*)**

Tests whether the variable is real.

INPUT:

- *e* – A variable (not a MIPVariable, but one of its elements.)

OUTPUT:

True if the variable is real; False otherwise.

EXAMPLE:

```
sage: p = MixedIntegerLinearProgram()
sage: v = p.new_variable(nonnegative=True)
sage: p.set_objective(v[1])
sage: p.is_real(v[1])
True
sage: p.set_binary(v[1])
sage: p.is_real(v[1])
False
sage: p.set_real(v[1])
sage: p.is_real(v[1])
True
```

### **linear\_constraints\_parent()**

Return the parent for all linear constraints

See [linear\\_functions](#) for more details.

EXAMPLES:

```
sage: p = MixedIntegerLinearProgram()
sage: p.linear_constraints_parent()
Linear constraints over Real Double Field
```

### **linear\_function(*x*)**

Construct a new linear function

EXAMPLES:

```
sage: p = MixedIntegerLinearProgram()
sage: p.linear_function({1:3, 4:5})
3*x_1 + 5*x_4
```

This is equivalent to:

```
sage: p({1:3, 4:5})
3*x_1 + 5*x_4
```

#### **linear\_functions\_parent()**

Return the parent for all linear functions

EXAMPLES:

```
sage: p = MixedIntegerLinearProgram()
sage: p.linear_functions_parent()
Linear functions over Real Double Field
```

#### **new\_variable (real=False, binary=False, integer=False, nonnegative=None, dim=1, name='')**

Returns an instance of `MIPVariable` associated to the current instance of `MixedIntegerLinearProgram`.

A new variable `x` is defined by:

```
sage: p = MixedIntegerLinearProgram()
sage: x = p.new_variable(nonnegative=True)
```

It behaves exactly as an usual dictionary would. It can use any key argument you may like, as `x[5]` or `x["b"]`, and has methods `items()` and `keys()`.

INPUT:

- `dim (integer)` – Defines the dimension of the dictionary. If `x` has dimension 2, its fields will be of the form `x[key1][key2]`. Deprecated.
- `binary, integer, real (boolean)` – Set one of these arguments to `True` to ensure that the variable gets the corresponding type.
- `nonnegative (boolean)` – whether the variable should be assumed to be nonnegative. Rather useless for the binary type.
- `name (string)` – Associates a name to the variable. This is only useful when exporting the linear program to a file using `write_mps` or `write_lp`, and has no other effect.

See Also:

- `set_min()`, meth:*get<sub>m</sub>in* – set/get the lower bound of a variable. Note that by default, all variables are non-negative.
- `set_max()`, meth:*get<sub>m</sub>ax* – set/get the upper bound of a variable.

EXAMPLE:

```
sage: p = MixedIntegerLinearProgram()
```

To define two dictionaries of variables, the first being of real type, and the second of integer type ::

```
sage: x = p.new_variable(real=True, nonnegative=True)
sage: y = p.new_variable(integer=True, nonnegative=True)
sage: p.add_constraint(x[2] + y[3,5], max=2)
```

```
sage: p.is_integer(x[2])
False
sage: p.is_integer(y[3,5])
True
```

An exception is raised when two types are supplied

```
sage: z = p.new_variable(real = True, integer = True)
Traceback (most recent call last):
...
ValueError: Exactly one of the available types has to be True
```

Unbounded variables:

```
sage: p = MixedIntegerLinearProgram()
sage: x = p.new_variable(real=True, nonnegative=False)
sage: y = p.new_variable(integer=True, nonnegative=False)
sage: p.add_constraint(x[0]+x[3] <= 8)
sage: p.add_constraint(y[0] >= y[1])
sage: p.show()
Maximization:
```

Constraints:

```
x_0 + x_1 <= 8.0
- x_2 + x_3 <= 0.0
```

Variables:

```
x_0 is a continuous variable (min=-oo, max=+oo)
x_1 is a continuous variable (min=-oo, max=+oo)
x_2 is an integer variable (min=-oo, max=+oo)
x_3 is an integer variable (min=-oo, max=+oo)
```

TESTS:

Default behaviour ([trac ticket #15521](#)):

```
sage: x = p.new_variable(nonnegative=True)
sage: p.get_min(x[0])
0.0
```

**number\_of\_constraints()**

Returns the number of constraints assigned so far.

EXAMPLE:

```
sage: p = MixedIntegerLinearProgram()
sage: p.add_constraint(p[0] - p[2], min = 1, max = 4)
sage: p.add_constraint(p[0] - 2*p[1], min = 1)
sage: p.number_of_constraints()
2
```

**number\_of\_variables()**

Returns the number of variables used so far.

Note that this is backend-dependent, i.e. we count solver's variables rather than user's variables. An example of the latter can be seen below: Gurobi converts double inequalities, i.e. inequalities like  $m \leq c^T x \leq M$ , with  $m < M$ , into equations, by adding extra variables:  $c^T x + y = M$ ,  $0 \leq y \leq M - m$ .

EXAMPLE:

```
sage: p = MixedIntegerLinearProgram()
sage: p.add_constraint(p[0] - p[2], max = 4)
```

```

sage: p.number_of_variables()
2
sage: p.add_constraint(p[0] - 2*p[1], min = 1)
sage: p.number_of_variables()
3
sage: p = MixedIntegerLinearProgram(solver="glpk")
sage: p.add_constraint(p[0] - p[2], min = 1, max = 4)
sage: p.number_of_variables()
2
sage: p = MixedIntegerLinearProgram(solver="gurobi") # optional - Gurobi
sage: p.add_constraint(p[0] - p[2], min = 1, max = 4) # optional - Gurobi
sage: p.number_of_variables() # optional - Gurobi
3

```

**polyhedron** (\*\*kws)

Returns the polyhedron defined by the Linear Program.

INPUT:

All arguments given to this method are forwarded to the constructor of the `Polyhedron()` class.

OUTPUT:

A `Polyhedron()` object whose  $i$ -th variable represents the  $i$ -th variable of `self`.

**Warning:** The polyhedron is built from the variables stored by the LP solver (i.e. the output of `show()`). While they usually match the ones created explicitly when defining the LP, a solver like Gurobi has been known to introduce additional variables to store constraints of the type `lower_bound <= linear_function <= upper bound`. You should be fine if you did not install Gurobi or if you do not use it as a solver, but keep an eye on the number of variables in the polyhedron, or on the output of `show()`. Just in case.

EXAMPLES:

A LP on two variables:

```

sage: p = MixedIntegerLinearProgram()
sage: p.add_constraint(2*p['x'] + p['y'] <= 1)
sage: p.add_constraint(3*p['y'] + p['x'] <= 2)
sage: P = p.polyhedron(); P
A 2-dimensional polyhedron in QQ^2 defined as the convex hull of 4 vertices

```

3-D Polyhedron:

```

sage: p = MixedIntegerLinearProgram()
sage: p.add_constraint(2*p['x'] + p['y'] + 3*p['z'] <= 1)
sage: p.add_constraint(2*p['y'] + p['z'] + 3*p['x'] <= 1)
sage: p.add_constraint(2*p['z'] + p['x'] + 3*p['y'] <= 1)
sage: P = p.polyhedron(); P
A 3-dimensional polyhedron in QQ^3 defined as the convex hull of 8 vertices

```

An empty polyhedron:

```

sage: p = MixedIntegerLinearProgram()
sage: p.add_constraint(2*p['x'] + p['y'] + 3*p['z'] <= 1)
sage: p.add_constraint(2*p['y'] + p['z'] + 3*p['x'] <= 1)
sage: p.add_constraint(2*p['z'] + p['x'] + 3*p['y'] >= 2)
sage: P = p.polyhedron(); P
The empty polyhedron in QQ^3

```

An unbounded polyhedron:

```
sage: p = MixedIntegerLinearProgram()
sage: p.add_constraint(2*p['x'] + p['y'] - p['z'] <= 1)
sage: P = p.polyhedron(); P
```

A 3-dimensional polyhedron in  $\mathbb{Q}\mathbb{Q}^3$  defined as the convex hull of 3 vertices and 3 rays

A square (see [trac ticket #14395](#))

```
sage: p = MixedIntegerLinearProgram()
sage: x,y = p['x'], p['y']
sage: p.set_min(x, None)
sage: p.set_min(y, None)
sage: p.add_constraint( x <= 1 )
sage: p.add_constraint( x >= -1 )
sage: p.add_constraint( y <= 1 )
sage: p.add_constraint( y >= -1 )
sage: p.polyhedron()
```

A 2-dimensional polyhedron in  $\mathbb{Q}\mathbb{Q}^2$  defined as the convex hull of 4 vertices

### **remove\_constraint** (*i*)

Removes a constraint from self.

INPUT:

- *i* – Index of the constraint to remove.

EXAMPLE:

```
sage: p = MixedIntegerLinearProgram()
sage: x, y = p[0], p[1]
sage: p.add_constraint(x + y, max = 10)
sage: p.add_constraint(x - y, max = 0)
sage: p.add_constraint(x, max = 4)
sage: p.show()
```

Maximization:

```
Constraints:
  x_0 + x_1 <= 10.0
  x_0 - x_1 <= 0.0
  x_0 <= 4.0
...
```

```
sage: p.remove_constraint(1)
```

```
sage: p.show()
```

Maximization:

```
Constraints:
  x_0 + x_1 <= 10.0
  x_0 <= 4.0
...
```

```
sage: p.number_of_constraints()
```

```
2
```

### **remove\_constraints** (*constraints*)

Remove several constraints.

INPUT:

- *constraints* – an iterable containing the indices of the rows to remove.

EXAMPLE:

```
sage: p = MixedIntegerLinearProgram()
sage: x, y = p[0], p[1]
sage: p.add_constraint(x + y, max = 10)
sage: p.add_constraint(x - y, max = 0)
sage: p.add_constraint(x, max = 4)
sage: p.show()
Maximization:
```

```
Constraints:
  x_0 + x_1 <= 10.0
  x_0 - x_1 <= 0.0
  x_0 <= 4.0
...
```

```
sage: p.remove_constraints([0, 1])
sage: p.show()
Maximization:
```

```
Constraints:
  x_0 <= 4.0
...
```

```
sage: p.number_of_constraints()
1
```

When checking for redundant constraints, make sure you remove only the constraints that were actually added. Problems could arise if you have a function that builds lps non-interactively, but it fails to check whether adding a constraint actually increases the number of constraints. The function might later try to remove constraints that are not actually there:

```
sage: p = MixedIntegerLinearProgram(check_redundant=True)
sage: x, y = p[0], p[1]
sage: p.add_constraint(x + y, max = 10)
sage: for each in xrange(10): p.add_constraint(x - y, max = 10)
sage: p.add_constraint(x, max = 4)
sage: p.number_of_constraints()
3
sage: p.remove_constraints(range(1, 9))
Traceback (most recent call last):
...
IndexError: pop index out of range
sage: p.remove_constraint(1)
sage: p.number_of_constraints()
2
```

We should now be able to add the old constraint back in:

```
sage: for each in xrange(10): p.add_constraint(x - y, max = 10)
sage: p.number_of_constraints()
3
```

### **set\_binary**(*ee*)

Sets a variable or a MIPVariable as binary.

INPUT:

- *ee* – An instance of MIPVariable or one of its elements.

EXAMPLE:

```
sage: p = MixedIntegerLinearProgram()
sage: x = p.new_variable(nonnegative=True)
```

With the following instruction, all the variables from `x` will be binary:

```
sage: p.set_binary(x)
sage: p.set_objective(x[0] + x[1])
sage: p.add_constraint(-3*x[0] + 2*x[1], max=2)
```

It is still possible, though, to set one of these variables as integer while keeping the others as they are:

```
sage: p.set_integer(x[3])
```

TESTS:

When ‘dim’ will be removed, also remove all the `is_*` and `set_*` functions the code that uses it:

```
sage: p = MixedIntegerLinearProgram()
sage: b = p.new_variable(dim=2)
sage: p.add_constraint(b[1][2] + b[2][3] == 0)
sage: p.set_binary(b)
```

### `set_integer(ee)`

Sets a variable or a `MIPVariable` as integer.

INPUT:

- `ee` – An instance of `MIPVariable` or one of its elements.

EXAMPLE:

```
sage: p = MixedIntegerLinearProgram()
sage: x = p.new_variable(nonnegative=True)
```

With the following instruction, all the variables from `x` will be integers:

```
sage: p.set_integer(x)
sage: p.set_objective(x[0] + x[1])
sage: p.add_constraint(-3*x[0] + 2*x[1], max=2)
```

It is still possible, though, to set one of these variables as binary while keeping the others as they are:

```
sage: p.set_binary(x[3])
```

### `set_max(v, max)`

Sets the maximum value of a variable.

INPUT

- `v` – a variable.
- `max` – the maximum value the variable can take. When `max=None`, the variable has no upper bound.

EXAMPLE:

```
sage: p = MixedIntegerLinearProgram()
sage: v = p.new_variable(nonnegative=True)
sage: p.set_objective(v[1])
sage: p.get_max(v[1])
sage: p.set_max(v[1], 6)
sage: p.get_max(v[1])
6.0
```

With a `MIPVariable` as an argument:

```
sage: vv = p.new_variable(real=True, nonnegative=False)
sage: p.get_max(vv)
```

```
sage: p.get_max(vv[0])
sage: p.set_max(vv, 5)
sage: p.get_max(vv[0])
5.0
sage: p.get_max(vv[9])
5.0
```

**set\_min**(*v*, *min*)

Sets the minimum value of a variable.

INPUT:

- *v* – a variable.
- *min* – the minimum value the variable can take. When *min*=None, the variable has no lower bound.

See Also:

- `get_min()` – get the minimum value of a variable.

EXAMPLE:

```
sage: p = MixedIntegerLinearProgram()
sage: v = p.new_variable(nonnegative=True)
sage: p.set_objective(v[1])
sage: p.get_min(v[1])
0.0
sage: p.set_min(v[1], 6)
sage: p.get_min(v[1])
6.0
sage: p.set_min(v[1], None)
sage: p.get_min(v[1])
```

With a `MIPVariable` as an argument:

```
sage: vv = p.new_variable(real=True, nonnegative=False)
sage: p.get_min(vv)
sage: p.get_min(vv[0])
sage: p.set_min(vv, 5)
sage: p.get_min(vv[0])
5.0
sage: p.get_min(vv[9])
5.0
```

**set\_objective**(*obj*)

Sets the objective of the `MixedIntegerLinearProgram`.

INPUT:

- *obj* – A linear function to be optimized. ( can also be set to None or 0 when just looking for a feasible solution )

EXAMPLE:

Let's solve the following linear program:

```
Maximize:
  x + 5 * y
Constraints:
  x + 0.2 y      <= 4
  1.5 * x + 3 * y <= 4
Variables:
```



```
x is Real (min = 0, max = None)
y is Real (min = 0, max = None)
```

This linear program can be solved as follows:

```
sage: p = MixedIntegerLinearProgram(maximization=True)
sage: x = p.new_variable(nonnegative=True)
sage: p.set_objective(x[1] + 5*x[2])
sage: p.add_constraint(x[1] + 2/10*x[2], max=4)
sage: p.add_constraint(1.5*x[1]+3*x[2], max=4)
sage: round(p.solve(),5)
6.66667
sage: p.set_objective(None)
sage: _ = p.solve()
```

### **set\_problem\_name** (*name*)

Sets the name of the `MixedIntegerLinearProgram`.

INPUT:

- *name* – A string representing the name of the `MixedIntegerLinearProgram`.

EXAMPLE:

```
sage: p=MixedIntegerLinearProgram()
sage: p.set_problem_name("Test program")
sage: p
Mixed Integer Program "Test program" ( maximization, 0 variables, 0 constraints )
```

### **set\_real** (*ee*)

Sets a variable or a `MIPVariable` as real.

INPUT:

- *ee* – An instance of `MIPVariable` or one of its elements.

EXAMPLE:

```
sage: p = MixedIntegerLinearProgram()
sage: x = p.new_variable(nonnegative=True)
```

With the following instruction, all the variables from *x* will be real:

```
sage: p.set_real(x)
sage: p.set_objective(x[0] + x[1])
sage: p.add_constraint(-3*x[0] + 2*x[1], max=2)
```

It is still possible, though, to set one of these variables as binary while keeping the others as they are::

```
sage: p.set_binary(x[3])
```

### **show** ()

Displays the `MixedIntegerLinearProgram` in a human-readable way.

EXAMPLES:

When constraints and variables have names

```
sage: p = MixedIntegerLinearProgram(solver = "GLPK")
sage: x = p.new_variable(name="Hey")
sage: p.set_objective(x[1] + x[2])
```

```

sage: p.add_constraint(-3*x[1] + 2*x[2], max=2, name="Constraint_1")
sage: p.show()
Maximization:
  Hey[1] + Hey[2]
Constraints:
  Constraint_1: -3.0 Hey[1] + 2.0 Hey[2] <= 2.0
Variables:
  Hey[1] is a continuous variable (min=0.0, max=+oo)
  Hey[2] is a continuous variable (min=0.0, max=+oo)

```

Without any names

```

sage: p = MixedIntegerLinearProgram(solver = "GLPK")
sage: x = p.new_variable(nonnegative=True)
sage: p.set_objective(x[1] + x[2])
sage: p.add_constraint(-3*x[1] + 2*x[2], max=2)
sage: p.show()
Maximization:
  x_0 + x_1
Constraints:
  -3.0 x_0 + 2.0 x_1 <= 2.0
Variables:
  x_0 is a continuous variable (min=0.0, max=+oo)
  x_1 is a continuous variable (min=0.0, max=+oo)

```

With Q coefficients:

```

sage: p = MixedIntegerLinearProgram(solver='ppl')
sage: x = p.new_variable(nonnegative=True)
sage: p.set_objective(x[1] + 1/2*x[2])
sage: p.add_constraint(-3/5*x[1] + 2/7*x[2], max=2/5)
sage: p.show()
Maximization:
  x_0 + 1/2 x_1
Constraints:
  constraint_0: -3/5 x_0 + 2/7 x_1 <= 2/5
Variables:
  x_0 is a continuous variable (min=0, max=+oo)
  x_1 is a continuous variable (min=0, max=+oo)

```

**solve** (*log=None, objective\_only=False*)

Solves the MixedIntegerLinearProgram.

INPUT:

- *log* – integer (default: None) The verbosity level. Indicates whether progress should be printed during computation. The solver is initialized to report no progress.
- *objective\_only* – Boolean variable.
  - When set to True, only the objective function is returned.
  - When set to False (default), the optimal numerical values are stored (takes computational time).

OUTPUT:

The optimal value taken by the objective function.

**Warning:** By default, all variables of a LP are assumed to be non-negative. See `set_min()` to change it.

## EXAMPLES:

Consider the following linear program:

```
Maximize:
  x + 5 * y
Constraints:
  x + 0.2 y          <= 4
  1.5 * x + 3 * y <= 4
Variables:
  x is Real (min = 0, max = None)
  y is Real (min = 0, max = None)
```

This linear program can be solved as follows:

```
sage: p = MixedIntegerLinearProgram(maximization=True)
sage: x = p.new_variable(nonnegative=True)
sage: p.set_objective(x[1] + 5*x[2])
sage: p.add_constraint(x[1] + 0.2*x[2], max=4)
sage: p.add_constraint(1.5*x[1] + 3*x[2], max=4)
sage: round(p.solve(), 6)
6.666667
sage: x = p.get_values(x)
sage: round(x[1], 6)
0.0
sage: round(x[2], 6)
1.333333
```

Computation of a maximum stable set in Petersen's graph::

```
sage: g = graphs.PetersenGraph()
sage: p = MixedIntegerLinearProgram(maximization=True)
sage: b = p.new_variable(nonnegative=True)
sage: p.set_objective(sum([b[v] for v in g]))
sage: for (u,v) in g.edges(labels=None):
...     p.add_constraint(b[u] + b[v], max=1)
sage: p.set_binary(b)
sage: p.solve(objective_only=True)
4.0
```

Constraints in the objective function are respected:

```
sage: p = MixedIntegerLinearProgram()
sage: x, y = p[0], p[1]
sage: p.add_constraint(2*x + 3*y, max = 6)
sage: p.add_constraint(3*x + 2*y, max = 6)
sage: p.set_objective(x + y + 7)
sage: p.set_integer(x); p.set_integer(y)
sage: p.solve()
9.0
```

**solver\_parameter** (*name*, *value=None*)

Return or define a solver parameter

The solver parameters are by essence solver-specific, which means their meaning heavily depends on the solver used.

(If you do not know which solver you are using, then you use use GLPK).

Aliases:

Very common parameters have aliases making them solver-independent. For example, the following:

```
sage: p = MixedIntegerLinearProgram(solver = "GLPK")
sage: p.solver_parameter("timelimit", 60)
```

Sets the solver to stop its computations after 60 seconds, and works with GLPK, CPLEX and Gurobi.

- "timelimit" – defines the maximum time spent on a computation. Measured in seconds.

Solver-specific parameters:

- GLPK : We have implemented very close to comprehensive coverage of the GLPK solver parameters for the simplex and integer optimization methods. For details, see the documentation of `GLPKBackend.solver_parameter`.
- CPLEX's parameters are identified by a string. Their list is available on [ILOG's website](http://www.ilog.com).

The command

```
sage: p = MixedIntegerLinearProgram(solver = "CPLEX") # optional - CPLEX
sage: p.solver_parameter("CPX_PARAM_TILIM", 60)      # optional - CPLEX
```

works as intended.

- Gurobi's parameters should all be available through this method. Their list is available on Gurobi's website <http://www.gurobi.com/documentation/5.5/reference-manual/node798>.

INPUT:

- name (string) – the parameter
- value – the parameter's value if it is to be defined, or None (default) to obtain its current value.

EXAMPLE:

```
sage: p = MixedIntegerLinearProgram(solver = "GLPK")
sage: p.solver_parameter("timelimit", 60)
sage: p.solver_parameter("timelimit")
60.0
```

**sum**(L)

Efficiently computes the sum of a sequence of `LinearFunction` elements

INPUT:

- mip – the `MixedIntegerLinearProgram` parent.
- L – list of `LinearFunction` instances.

---

**Note:** The use of the regular `sum` function is not recommended as it is much less efficient than this one

---

EXAMPLES:

```
sage: p = MixedIntegerLinearProgram()
sage: v = p.new_variable(nonnegative=True)
```

The following command:

```
sage: s = p.sum([v[i] for i in xrange(90)])
```

is much more efficient than:

```
sage: s = sum([v[i] for i in xrange(90)])
```

**write\_lp**(filename)

Write the linear program as a LP file.

This function export the problem as a LP file.

INPUT:

- filename – The file in which you want the problem to be written.

EXAMPLE:

```
sage: p = MixedIntegerLinearProgram(solver="GLPK")
sage: x = p.new_variable(nonnegative=True)
sage: p.set_objective(x[1] + x[2])
sage: p.add_constraint(-3*x[1] + 2*x[2], max=2)
sage: p.write_lp(os.path.join(SAGE_TMP, "lp_problem.lp"))
Writing problem data to ...
9 lines were written
```

For more information about the LP file format : <http://lpsolve.sourceforge.net/5.5/lp-format.htm>

**write\_mps**(filename, modern=True)

Write the linear program as a MPS file.

This function export the problem as a MPS file.

INPUT:

- filename – The file in which you want the problem to be written.
- modern – Lets you choose between Fixed MPS and Free MPS
  - True – Outputs the problem in Free MPS
  - False – Outputs the problem in Fixed MPS

EXAMPLE:

```
sage: p = MixedIntegerLinearProgram(solver="GLPK")
sage: x = p.new_variable(nonnegative=True)
sage: p.set_objective(x[1] + x[2])
sage: p.add_constraint(-3*x[1] + 2*x[2], max=2, name="OneConstraint")
sage: p.write_mps(os.path.join(SAGE_TMP, "lp_problem.mps"))
Writing problem data to ...
17 records were written
```

For information about the MPS file format : [http://en.wikipedia.org/wiki/MPS\\_%28format%29](http://en.wikipedia.org/wiki/MPS_%28format%29)

sage.numerical.mip.**Sum**(x)

Only for legacy support, use `MixedIntegerLinearProgram.sum()` instead.

EXAMPLES:

```
sage: from sage.numerical.mip import Sum
sage: Sum([])
doctest:...: DeprecationWarning: use MixedIntegerLinearProgram.sum() instead
See http://trac.sagemath.org/13646 for details.

sage: p = MixedIntegerLinearProgram()
sage: x = p.new_variable(nonnegative=True)
sage: Sum([ x[0]+x[1], x[1]+x[2], x[2]+x[3] ]) # deprecation is only shown once
x_0 + 2*x_1 + 2*x_2 + x_3
```



# LINEAR FUNCTIONS AND CONSTRAINTS

## Linear Functions and Constraints

This module implements linear functions (see [LinearFunction](#)) in formal variables and chained (in)equalities between them (see [LinearConstraint](#)). By convention, these are always written as either equalities or less-or-equal. For example:

```
sage: p = MixedIntegerLinearProgram()
sage: x = p.new_variable()
doctest:...: DeprecationWarning: The default value of 'nonnegative' will change, to False instead of
See http://trac.sagemath.org/15521 for details.
sage: f = 1 + x[1] + 2*x[2]; f      # a linear function
1 + x_0 + 2*x_1
sage: type(f)
<type 'sage.numerical.linear_functions.LinearFunction'>

sage: c = (0 <= f); c      # a constraint
0 <= 1 + x_0 + 2*x_1
sage: type(c)
<type 'sage.numerical.linear_functions.LinearConstraint'>
```

Note that you can use this module without any reference to linear programming, it only implements linear functions over a base ring and constraints. However, for ease of demonstration we will always construct them out of linear programs (see [mip](#)).

Constraints can be equations or (non-strict) inequalities. They can be chained:

```
sage: p = MixedIntegerLinearProgram()
sage: x = p.new_variable()
sage: x[0] == x[1] == x[2] == x[3]
x_0 == x_1 == x_2 == x_3

sage: ieq_01234 = x[0] <= x[1] <= x[2] <= x[3] <= x[4]
sage: ieq_01234
x_0 <= x_1 <= x_2 <= x_3 <= x_4
```

If necessary, the direction of inequality is flipped to always write inequalities as less or equal:

```
sage: x[5] >= ieq_01234
x_0 <= x_1 <= x_2 <= x_3 <= x_4 <= x_5

sage: (x[5] <= x[6]) >= ieq_01234
```

```
x_0 <= x_1 <= x_2 <= x_3 <= x_4 <= x_5 <= x_6
sage: (x[5]<=x[6]) <= ieq_01234
x_5 <= x_6 <= x_0 <= x_1 <= x_2 <= x_3 <= x_4
```

TESTS:

See [trac ticket #12091](#)

```
sage: p = MixedIntegerLinearProgram()
sage: b = p.new_variable()
sage: b[0] <= b[1] <= 2
x_0 <= x_1 <= 2
sage: list(b[0] <= b[1] <= 2)
[x_0, x_1, 2]
sage: 1 >= b[1] >= 2*b[0]
2*x_0 <= x_1 <= 1
sage: b[2] >= b[1] >= 2*b[0]
2*x_0 <= x_1 <= x_2
```

**class** `sage.numerical.linear_functions.LinearConstraint`

Bases: `sage.structure.element.Element`

A class to represent formal Linear Constraints.

A Linear Constraint being an inequality between two linear functions, this class lets the user write `LinearFunction1 <= LinearFunction2` to define the corresponding constraint, which can potentially involve several layers of such inequalities ( $(A <= B <= C)$ ), or even equalities like  $A == B$ .

Trivial constraints (meaning that they have only one term and no relation) are also allowed. They are required for the coercion system to work.

**Warning:** This class has no reason to be instantiated by the user, and is meant to be used by instances of `MixedIntegerLinearProgram`.

INPUT:

- `parent` – the parent, a `LinearConstraintsParent_class`
- `terms` – a list/tuple/iterable of two or more linear functions (or things that can be converted into linear functions).
- `equality` – boolean (default: `False`). Whether the terms are the entries of a chained less-or-equal (`<=`) inequality or a chained equality.

EXAMPLE:

```
sage: p = MixedIntegerLinearProgram()
sage: b = p.new_variable()
sage: b[2]+2*b[3] <= b[8]-5
x_0 + 2*x_1 <= -5 + x_2
```

**equals** (*left, right*)

Compare left and right.

OUTPUT:

Boolean. Whether all terms of `left` and `right` are equal. Note that this is stronger than mathematical equivalence of the relations.

EXAMPLES:



```

sage: p = MixedIntegerLinearProgram()
sage: x = p.new_variable()
sage: (x[1] + 1 >= 2).equals(3/3 + 1*x[1] + 0*x[2] >= 8/4)
True
sage: (x[1] + 1 >= 2).equals(x[1] + 1-1 >= 1-1)
False

```

**equations()**

Iterate over the unchained(!) equations

OUTPUT:

An iterator over pairs (lhs, rhs) such that the individual equations are lhs == rhs.

EXAMPLES:

```

sage: p = MixedIntegerLinearProgram()
sage: b = p.new_variable()
sage: eqns = 1 == b[0] == b[2] == 3 == b[3]; eqns
1 == x_0 == x_1 == 3 == x_2

sage: for lhs, rhs in eqns.equations():
...     print str(lhs) + ' == ' + str(rhs)
1 == x_0
x_0 == x_1
x_1 == 3
3 == x_2

```

**inequalities()**

Iterate over the unchained(!) inequalities

OUTPUT:

An iterator over pairs (lhs, rhs) such that the individual equations are lhs <= rhs.

EXAMPLES:

```

sage: p = MixedIntegerLinearProgram()
sage: b = p.new_variable()
sage: ieq = 1 <= b[0] <= b[2] <= 3 <= b[3]; ieq
1 <= x_0 <= x_1 <= 3 <= x_2

sage: for lhs, rhs in ieq.inequalities():
...     print str(lhs) + ' <= ' + str(rhs)
1 <= x_0
x_0 <= x_1
x_1 <= 3
3 <= x_2

```

**is\_equation()**

Whether the constraint is a chained equation

OUTPUT:

Boolean.

EXAMPLES:

```

sage: p = MixedIntegerLinearProgram()
sage: b = p.new_variable()
sage: (b[0] == b[1]).is_equation()
True

```

```
sage: (b[0] <= b[1]).is_equation()
False
```

**is\_less\_or\_equal()**

Whether the constraint is a chained less-or\_equal inequality

OUTPUT:

Boolean.

EXAMPLES:

```
sage: p = MixedIntegerLinearProgram()
sage: b = p.new_variable()
sage: (b[0] == b[1]).is_less_or_equal()
False
sage: (b[0] <= b[1]).is_less_or_equal()
True
```

**is\_trivial()**

Test whether the constraint is trivial.

EXAMPLES:

```
sage: p = MixedIntegerLinearProgram()
sage: LC = p.linear_constraints_parent()
sage: ieq = LC(1,2); ieq
1 <= 2
sage: ieq.is_trivial()
False

sage: ieq = LC(1); ieq
trivial constraint starting with 1
sage: ieq.is_trivial()
True
```

`sage.numerical.linear_functions.LinearConstraintsParent` (*linear\_functions\_parent*)

Return the parent for linear functions over `base_ring`.

The output is cached, so only a single parent is ever constructed for a given base ring.

INPUT:

- `linear_functions_parent` – a `LinearFunctionsParent_class`. The type of linear functions that the constraints are made out of.

OUTPUT:

The parent of the linear constraints with the given linear functions.

EXAMPLES:

```
sage: from sage.numerical.linear_functions import ... LinearFunctionsParent, I
sage: LF = LinearFunctionsParent(QQ)
sage: LinearConstraintsParent(LF)
Linear constraints over Rational Field
```

**class** `sage.numerical.linear_functions.LinearConstraintsParent_class`

Bases: `sage.structure.parent.Parent`

Parent for `LinearConstraint`

**Warning:** This class has no reason to be instantiated by the user, and is meant to be used by instances of `MixedIntegerLinearProgram`. Also, use the `LinearConstraintsParent()` factory function.

INPUT/OUTPUT:

See `LinearFunctionsParent()`

EXAMPLES:

```
sage: p = MixedIntegerLinearProgram()
sage: LC = p.linear_constraints_parent(); LC
Linear constraints over Real Double Field
sage: from sage.numerical.linear_functions import LinearConstraintsParent
sage: LinearConstraintsParent(p.linear_functions_parent()) is LC
True
```

**linear\_functions\_parent()**

Return the parent for the linear functions

EXAMPLES:

```
sage: LC = MixedIntegerLinearProgram().linear_constraints_parent()
sage: LC.linear_functions_parent()
Linear functions over Real Double Field
```

**class** `sage.numerical.linear_functions.LinearFunction`

Bases: `sage.structure.element.ModuleElement`

An elementary algebra to represent symbolic linear functions.

**Warning:** You should never instantiate `LinearFunction` manually. Use the element constructor in the parent instead. For convenience, you can also call the `MixedIntegerLinearProgram` instance directly.

EXAMPLES:

For example, do this:

```
sage: p = MixedIntegerLinearProgram()
sage: p({0 : 1, 3 : -8})
x_0 - 8*x_3
```

or this:

```
sage: parent = p.linear_functions_parent()
sage: parent({0 : 1, 3 : -8})
x_0 - 8*x_3
```

instead of this:

```
sage: from sage.numerical.linear_functions import LinearFunction
sage: LinearFunction(p.linear_functions_parent(), {0 : 1, 3 : -8})
x_0 - 8*x_3
```

**dict()**

Returns the dictionary corresponding to the Linear Function.

OUTPUT:

The linear function is represented as a dictionary. The value are the coefficient of the variable represented by the keys ( which are integers ). The key `-1` corresponds to the constant term.

EXAMPLE:

```
sage: p = MixedIntegerLinearProgram()
sage: lf = p({0 : 1, 3 : -8})
sage: lf.dict()
{0: 1.0, 3: -8.0}
```

**equals** (*left, right*)

Logically compare left and right.

OUTPUT:

Boolean.

EXAMPLES:

```
sage: p = MixedIntegerLinearProgram()
sage: x = p.new_variable()
sage: (x[1] + 1).equals(3/3 + 1*x[1] + 0*x[2])
True
```

**is\_zero** ()

Test whether self is zero.

OUTPUT:

Boolean.

EXAMPLES:

```
sage: p = MixedIntegerLinearProgram()
sage: x = p.new_variable()
sage: (x[1] - x[1] + 0*x[2]).is_zero()
True
```

**iteritems** ()

Iterate over the index, coefficient pairs

OUTPUT:

An iterator over the (key, coefficient) pairs. The keys are integers indexing the variables. The key -1 corresponds to the constant term.

EXAMPLES:

```
sage: p = MixedIntegerLinearProgram(solver = 'ppl')
sage: x = p.new_variable()
sage: f = 0.5 + 3/2*x[1] + 0.6*x[3]
sage: for id, coeff in f.iteritems():
...     print 'id =', id, 'coeff =', coeff
id = 0    coeff = 3/2
id = 1    coeff = 3/5
id = -1   coeff = 1/2
```

`sage.numerical.linear_functions.LinearFunctionsParent` (*base\_ring*)

Return the parent for linear functions over *base\_ring*.

The output is cached, so only a single parent is ever constructed for a given base ring.

INPUT:

- *base\_ring* – a ring. The coefficient ring for the linear functions.

OUTPUT:

The parent of the linear functions over `base_ring`.

EXAMPLES:

```
sage: from sage.numerical.linear_functions import LinearFunctionsParent
sage: LinearFunctionsParent(QQ)
Linear functions over Rational Field
```

**class** `sage.numerical.linear_functions.LinearFunctionsParent_class`

Bases: `sage.structure.parent.Parent`

The parent for all linear functions over a fixed base ring.

**Warning:** You should use `LinearFunctionsParent()` to construct instances of this class.

INPUT/OUTPUT:

See `LinearFunctionsParent()`

EXAMPLES:

```
sage: from sage.numerical.linear_functions import LinearFunctionsParent_class
sage: LinearFunctionsParent_class
<type 'sage.numerical.linear_functions.LinearFunctionsParent_class'>
```

**set\_multiplication\_symbol** (*symbol*='\*')

Set the multiplication symbol when pretty-printing linear functions.

INPUT:

- *symbol* – string, default: `'*'`. The multiplication symbol to be used.

EXAMPLES:

```
sage: p = MixedIntegerLinearProgram()
sage: x = p.new_variable()
sage: f = -1-2*x[0]-3*x[1]
sage: LF = f.parent()
sage: LF._get_multiplication_symbol()
'*'
sage: f
-1 - 2*x_0 - 3*x_1
sage: LF.set_multiplication_symbol(' ')
sage: f
-1 - 2 x_0 - 3 x_1
sage: LF.set_multiplication_symbol()
sage: f
-1 - 2*x_0 - 3*x_1
```

**sage.numerical.linear\_functions.is\_LinearConstraint** (*x*)

Test whether *x* is a linear constraint

INPUT:

- *x* – anything.

OUTPUT:

Boolean.

EXAMPLES:

```
sage: p = MixedIntegerLinearProgram()
sage: x = p.new_variable()
sage: ieq = (x[0] <= x[1])
sage: from sage.numerical.linear_functions import is_LinearConstraint
sage: is_LinearConstraint(ieq)
True
sage: is_LinearConstraint('a string')
False
```

`sage.numerical.linear_functions.is_LinearFunction(x)`

Test whether  $x$  is a linear function

INPUT:

- $x$  – anything.

OUTPUT:

Boolean.

EXAMPLES:

```
sage: p = MixedIntegerLinearProgram()
sage: x = p.new_variable()
sage: from sage.numerical.linear_functions import is_LinearFunction
sage: is_LinearFunction(x[0] - 2*x[2])
True
sage: is_LinearFunction('a string')
False
```

# NUMERICAL ROOT FINDING AND OPTIMIZATION

AUTHOR:

- William Stein (2007): initial version
- Nathann Cohen (2008) : Bin Packing

## 4.1 Functions and Methods

`sage.numerical.optimize.binpacking` (*items*, *maximum*=*1*, *k*=*None*)

Solves the bin packing problem.

The Bin Packing problem is the following :

Given a list of items of weights  $p_i$  and a real value  $K$ , what is the least number of bins such that all the items can be put in the bins, while keeping sure that each bin contains a weight of at most  $K$  ?

For more informations : [http://en.wikipedia.org/wiki/Bin\\_packing\\_problem](http://en.wikipedia.org/wiki/Bin_packing_problem)

**Two version of this problem are solved by this algorithm :**

- Is it possible to put the given items in  $L$  bins ?
- What is the assignment of items using the least number of bins with the given list of items ?

INPUT:

- *items* – A list of real values (the items' weight)
- *maximum* – The maximal size of a bin
- *k* – Number of bins
  - When set to an integer value, the function returns a partition of the items into  $k$  bins if possible, and raises an exception otherwise.
  - When set to `None`, the function returns a partition of the items using the least number possible of bins.

OUTPUT:

A list of lists, each member corresponding to a box and containing the list of the weights inside it. If there is no solution, an exception is raised (this can only happen when *k* is specified or if *maximum* is less than the size of one item).

## EXAMPLES:

Trying to find the minimum amount of boxes for 5 items of weights  $1/5, 1/4, 2/3, 3/4, 5/7$ :

```
sage: from sage.numerical.optimize import binpacking
sage: values = [1/5, 1/3, 2/3, 3/4, 5/7]
sage: bins = binpacking(values)
sage: len(bins)
3
```

Checking the bins are of correct size

```
sage: all([ sum(b) <= 1 for b in bins ])
True
```

Checking every item is in a bin

```
sage: b1, b2, b3 = bins
sage: all([ (v in b1 or v in b2 or v in b3) for v in values ])
True
```

One way to use only three boxes (which is best possible) is to put  $1/5 + 3/4$  together in a box,  $1/3 + 2/3$  in another, and  $5/7$  by itself in the third one.

Of course, we can also check that there is no solution using only two boxes

```
sage: from sage.numerical.optimize import binpacking
sage: binpacking([0.2, 0.3, 0.8, 0.9], k=2)
Traceback (most recent call last):
...
ValueError: This problem has no solution !
```

`sage.numerical.optimize.find_fit(data, model, initial_guess=None, parameters=None, variables=None, solution_dict=False)`

Finds numerical estimates for the parameters of the function model to give a best fit to data.

## INPUT:

- **data** – A two dimensional table of floating point numbers of the form  $[[x_{1,1}, x_{1,2}, \dots, x_{1,k}, f_1], [x_{2,1}, x_{2,2}, \dots, x_{2,k}, f_2], \dots, [x_{n,1}, x_{n,2}, \dots, x_{n,k}, f_n]]$  given as either a list of lists, matrix, or numpy array.
- **model** – Either a symbolic expression, symbolic function, or a Python function. `model` has to be a function of the variables  $(x_1, x_2, \dots, x_k)$  and free parameters  $(a_1, a_2, \dots, a_l)$ .
- **initial\_guess** – (default: None) Initial estimate for the parameters  $(a_1, a_2, \dots, a_l)$ , given as either a list, tuple, vector or numpy array. If None, the default estimate for each parameter is 1.
- **parameters** – (default: None) A list of the parameters  $(a_1, a_2, \dots, a_l)$ . If `model` is a symbolic function it is ignored, and the free parameters of the symbolic function are used.
- **variables** – (default: None) A list of the variables  $(x_1, x_2, \dots, x_k)$ . If `model` is a symbolic function it is ignored, and the variables of the symbolic function are used.
- **solution\_dict** – (default: False) if True, return the solution as a dictionary rather than an equation.

## EXAMPLES:

First we create some data points of a sine function with some random perturbations:

```
sage: data = [(i, 1.2 * sin(0.5*i-0.2) + 0.1 * normalvariate(0, 1)) for i in xrange(0, 4*pi, 0.1)]
sage: var('a, b, c, x')
(a, b, c, x)
```



We define a function with free parameters  $a$ ,  $b$  and  $c$ :

```
sage: model(x) = a * sin(b * x - c)
```

We search for the parameters that give the best fit to the data:

```
sage: find_fit(data, model)
[a == 1.21..., b == 0.49..., c == 0.19...]
```

We can also use a Python function for the model:

```
sage: def f(x, a, b, c): return a * sin(b * x - c)
sage: fit = find_fit(data, f, parameters = [a, b, c], variables = [x], solution_dict = True)
sage: fit[a], fit[b], fit[c]
(1.21..., 0.49..., 0.19...)
```

We search for a formula for the  $n$ -th prime number:

```
sage: dataprime = [(i, nth_prime(i)) for i in xrange(1, 5000, 100)]
sage: find_fit(dataprime, a * x * log(b * x), parameters = [a, b], variables = [x])
[a == 1.11..., b == 1.24...]
```

#### ALGORITHM:

Uses `scipy.optimize.leastsq` which in turn uses MINPACK's `lmdif` and `lmdcr` algorithms.

```
sage.numerical.optimize.find_local_maximum(f, a, b, tol=1.48e-08, maxfun=500)
```

Numerically find a local maximum of the expression  $f$  on the interval  $[a, b]$  (or  $[b, a]$ ) along with the point at which the maximum is attained.

Note that this function only finds a *local* maximum, and not the global maximum on that interval – see the examples with `find_local_maximum()`.

See the documentation for `find_local_maximum()` for more details and possible workarounds for finding the global minimum on an interval.

#### EXAMPLES:

```
sage: f = lambda x: x*cos(x)
sage: find_local_maximum(f, 0, 5)
(0.561096338191..., 0.8603335890...)
sage: find_local_maximum(f, 0, 5, tol=0.1, maxfun=10)
(0.561090323458..., 0.857926501456...)
sage: find_local_maximum(8*e^(-x)*sin(x) - 1, 0, 7)
(1.579175535558..., 0.7853981...)
```

```
sage.numerical.optimize.find_local_minimum(f, a, b, tol=1.48e-08, maxfun=500)
```

Numerically find a local minimum of the expression  $f$  on the interval  $[a, b]$  (or  $[b, a]$ ) and the point at which it attains that minimum. Note that  $f$  must be a function of (at most) one variable.

Note that this function only finds a *local* minimum, and not the global minimum on that interval – see the examples below.

#### INPUT:

- $f$  – a function of at most one variable.
- $a, b$  – endpoints of interval on which to minimize self.
- `tol` – the convergence tolerance
- `maxfun` – maximum function evaluations

#### OUTPUT:

- `minval` – (float) the minimum value that self takes on in the interval  $[a, b]$
- `x` – (float) the point at which self takes on the minimum value

## EXAMPLES:

```
sage: f = lambda x: x*cos(x)
sage: find_local_minimum(f, 1, 5)
(-3.28837139559..., 3.4256184695...)
sage: find_local_minimum(f, 1, 5, tol=1e-3)
(-3.28837136189098..., 3.42575079030572...)
sage: find_local_minimum(f, 1, 5, tol=1e-2, maxfun=10)
(-3.28837084598..., 3.4250840220...)
sage: show(plot(f, 0, 20))
sage: find_local_minimum(f, 1, 15)
(-9.4772942594..., 9.5293344109...)
```

Only local minima are found; if you enlarge the interval, the returned minimum may be *larger*! See [trac ticket #2607](#).

```
sage: f(x) = -x*sin(x^2)
sage: find_local_minimum(f, -2.5, -1)
(-2.182769784677722, -2.1945027498534686)
```

Enlarging the interval returns a larger minimum:

```
sage: find_local_minimum(f, -2.5, 2)
(-1.3076194129914434, 1.3552111405712108)
```

One work-around is to plot the function and grab the minimum from that, although the plotting code does not necessarily do careful numerics (observe the small number of decimal places that we actually test):

```
sage: plot(f, (x, -2.5, -1)).ymin()
-2.1827...
sage: plot(f, (x, -2.5, 2)).ymin()
-2.1827...
```

## ALGORITHM:

Uses `scipy.optimize.fminbound` which uses Brent's method.

## AUTHOR:

- William Stein (2007-12-07)

```
sage.numerical.optimize.find_root(f, a, b, xtol=1e-12, rtol=4.5e-16, maxiter=100,
                                   full_output=False)
```

Numerically find a root of `f` on the closed interval  $[a, b]$  (or  $[b, a]$ ) if possible, where `f` is a function in the one variable. Note: this function only works in fixed (machine) precision, it is not possible to get arbitrary precision approximations with it.

## INPUT:

- `f` – a function of one variable or symbolic equality
- `a, b` – endpoints of the interval
- `xtol, rtol` – the routine converges when a root is known to lie within `xtol` of the value return. Should be  $\geq 0$ . The routine modifies this to take into account the relative precision of doubles.
- `maxiter` – integer; if convergence is not achieved in `maxiter` iterations, an error is raised. Must be  $\geq 0$ .

- `full_output` – bool (default: False), if True, also return object that contains information about convergence.

## EXAMPLES:

An example involving an algebraic polynomial function:

```
sage: R.<x> = QQ[]
sage: f = (x+17)*(x-3)*(x-1/8)^3
sage: find_root(f, 0, 4)
2.999999999999995
sage: find_root(f, 0, 1) # note -- precision of answer isn't very good on some machines.
0.124999...
sage: find_root(f, -20, -10)
-17.0
```

In Pomerance's book on primes he asserts that the famous Riemann Hypothesis is equivalent to the statement that the function  $f(x)$  defined below is positive for all  $x \geq 2.01$ :

```
sage: def f(x):
...     return sqrt(x) * log(x) - abs(Li(x) - prime_pi(x))
```

We find where  $f$  equals, i.e., what value that is slightly smaller than 2.01 that could have been used in the formulation of the Riemann Hypothesis:

```
sage: find_root(f, 2, 4, rtol=0.0001)
2.0082...
```

This agrees with the plot:

```
sage: plot(f, 2, 2.01)
```

`sage.numerical.optimize.linear_program(c, G, h, A=None, b=None, solver=None)`

Solves the dual linear programs:

- Minimize  $c'x$  subject to  $Gx + s = h$ ,  $Ax = b$ , and  $s \geq 0$  where  $'$  denotes transpose.
- Maximize  $-h'z - b'y$  subject to  $G'z + A'y + c = 0$  and  $z \geq 0$ .

## INPUT:

- `c` – a vector
- `G` – a matrix
- `h` – a vector
- `A` – a matrix
- `b` – a vector
- **`solver` (optional)** — solver to use. If None, the cvxopt's lp-solver is used. If it is 'glpk', then glpk's solver is used.

These can be over any field that can be turned into a floating point number.

## OUTPUT:

A dictionary `sol` with keys `x`, `s`, `y`, `z` corresponding to the variables above:

- `sol['x']` – the solution to the linear program
- `sol['s']` – the slack variables for the solution
- `sol['z'], sol['y']` – solutions to the dual program

## EXAMPLES:

First, we minimize  $-4x_1 - 5x_2$  subject to  $2x_1 + x_2 \leq 3$ ,  $x_1 + 2x_2 \leq 3$ ,  $x_1 \geq 0$ , and  $x_2 \geq 0$ :

```
sage: c=vector(RDF, [-4, -5])
sage: G=matrix(RDF, [[2, 1], [1, 2], [-1, 0], [0, -1]])
sage: h=vector(RDF, [3, 3, 0, 0])
sage: sol=linear_program(c, G, h)
sage: sol['x']
(0.999..., 1.000...)
```

Next, we maximize  $x + y - 50$  subject to  $50x + 24y \leq 2400$ ,  $30x + 33y \leq 2100$ ,  $x \geq 45$ , and  $y \geq 5$ :

```
sage: v=vector([-1.0, -1.0, -1.0])
sage: m=matrix([[50.0, 24.0, 0.0], [30.0, 33.0, 0.0], [-1.0, 0.0, 0.0], [0.0, -1.0, 0.0], [0.0, 0.0, 1.0], [0.0, 0.0, 1.0]])
sage: h=vector([2400.0, 2100.0, -45.0, -5.0, 1.0, -1.0])
sage: sol=linear_program(v, m, h)
sage: sol['x']
(45.000000..., 6.2499999..., 1.00000000...)
sage: sol=linear_program(v, m, h, solver='glpk')
GLPK Simplex Optimizer...
OPTIMAL SOLUTION FOUND
sage: sol['x']
(45.0..., 6.25, 1.0...)
```

```
sage.numerical.optimize.minimize(func, x0, gradient=None, hessian=None, algorithm='default', **args)
```

This function is an interface to a variety of algorithms for computing the minimum of a function of several variables.

## INPUT:

- **func** – Either a symbolic function or a Python function whose argument is a tuple with  $n$  components
- **x0** – Initial point for finding minimum.
- **gradient** – Optional gradient function. This will be computed automatically for symbolic functions. For Python functions, it allows the use of algorithms requiring derivatives. It should accept a tuple of arguments and return a NumPy array containing the partial derivatives at that point.
- **hessian** – Optional hessian function. This will be computed automatically for symbolic functions. For Python functions, it allows the use of algorithms requiring derivatives. It should accept a tuple of arguments and return a NumPy array containing the second partial derivatives of the function.
- **algorithm** – String specifying algorithm to use. Options are 'default' (for Python functions, the simplex method is the default) (for symbolic functions bfgs is the default):
  - 'simplex'
  - 'powell'
  - 'bfgs' – (Broyden-Fletcher-Goldfarb-Shanno) requires gradient
  - 'cg' – (conjugate-gradient) requires gradient
  - 'ncg' – (newton-conjugate gradient) requires gradient and hessian

## EXAMPLES:

```
sage: vars=var('x y z')
sage: f=100*(y-x^2)^2+(1-x)^2+100*(z-y^2)^2+(1-y)^2
sage: minimize(f, [.1, .3, .4], disp=0)
(1.00..., 1.00..., 1.00...)
```

```
sage: minimize(f, [.1, .3, .4], algorithm="nbg", disp=0)
(0.9999999..., 0.9999999..., 0.9999999...)
```

Same example with just Python functions:

```
sage: def rosen(x): # The Rosenbrock function
...     return sum(100.0r*(x[1r:]-x[:-1r]**2.0r)**2.0r + (1r-x[:-1r])**2.0r)
sage: minimize(rosen, [.1, .3, .4], disp=0)
(1.00..., 1.00..., 1.00...)
```

Same example with a pure Python function and a Python function to compute the gradient:

```
sage: def rosen(x): # The Rosenbrock function
...     return sum(100.0r*(x[1r:]-x[:-1r]**2.0r)**2.0r + (1r-x[:-1r])**2.0r)
sage: import numpy
sage: from numpy import zeros
sage: def rosen_der(x):
...     xm = x[1r:-1r]
...     xm_m1 = x[:-2r]
...     xm_p1 = x[2r:]
...     der = zeros(x.shape, dtype=float)
...     der[1r:-1r] = 200r*(xm-xm_m1**2r) - 400r*(xm_p1 - xm**2r)*xm - 2r*(1r-xm)
...     der[0] = -400r*x[0r]*(x[1r]-x[0r]**2r) - 2r*(1r-x[0r])
...     der[-1] = 200r*(x[-1r]-x[-2r]**2r)
...     return der
sage: minimize(rosen, [.1, .3, .4], gradient=rosen_der, algorithm="bfgs", disp=0)
(1.00..., 1.00..., 1.00...)
```

`sage.numerical.optimize.minimize_constrained` (*func*, *cons*, *x0*, *gradient=None*, *algorithm='default'*, *\*\*args*)

Minimize a function with constraints.

INPUT:

- *func* – Either a symbolic function, or a Python function whose argument is a tuple with  $n$  components
- *cons* – constraints. This should be either a function or list of functions that must be positive. Alternatively, the constraints can be specified as a list of intervals that define the region we are minimizing in. If the constraints are specified as functions, the functions should be functions of a tuple with  $n$  components (assuming  $n$  variables). If the constraints are specified as a list of intervals and there are no constraints for a given variable, that component can be (None, None).
- *x0* – Initial point for finding minimum
- *algorithm* – Optional, specify the algorithm to use:
  - 'default' – default choices
  - 'l-bfgs-b' – only effective if you specify bound constraints. See [ZBN97].
- *gradient* – Optional gradient function. This will be computed automatically for symbolic functions. This is only used when the constraints are specified as a list of intervals.

EXAMPLES:

Let us maximize  $x + y - 50$  subject to the following constraints:  $50x + 24y \leq 2400$ ,  $30x + 33y \leq 2100$ ,  $x \geq 45$ , and  $y \geq 5$ :

```
sage: y = var('y')
sage: f = lambda p: -p[0]-p[1]+50
sage: c_1 = lambda p: p[0]-45
sage: c_2 = lambda p: p[1]-5
```

```
sage: c_3 = lambda p: -50*p[0]-24*p[1]+2400
sage: c_4 = lambda p: -30*p[0]-33*p[1]+2100
sage: a = minimize_constrained(f, [c_1, c_2, c_3, c_4], [2, 3])
sage: a
(45.0, 6.25)
```

Let's find a minimum of  $\sin(xy)$ :

```
sage: x, y = var('x y')
sage: f = sin(x*y)
sage: minimize_constrained(f, [(None, None), (4, 10)], [5, 5])
(4.8..., 4.8...)
```

Check, if L-BFGS-B finds the same minimum:

```
sage: minimize_constrained(f, [(None, None), (4, 10)], [5, 5], algorithm='l-bfgs-b')
(4.7..., 4.9...)
```

Rosenbrock function, [[http://en.wikipedia.org/wiki/Rosenbrock\\_function](http://en.wikipedia.org/wiki/Rosenbrock_function)]:

```
sage: from scipy.optimize import rosen, rosen_der
sage: minimize_constrained(rosen, [(-50, -10), (5, 10)], [1, 1], gradient=rosen_der, algorithm='l-bfgs-b')
(-10.0, 10.0)
sage: minimize_constrained(rosen, [(-50, -10), (5, 10)], [1, 1], algorithm='l-bfgs-b')
(-10.0, 10.0)
```

REFERENCES:

# INTERACTIVE SIMPLEX METHOD

The purpose of this module is to support learning and exploring of the simplex method. While it does allow solving Linear Programming Problems (LPPs) in a number of ways, it may require explicit (and correct!) description of steps and is likely to be much slower than “regular” LP solvers. Therefore, do not use this module if all you want is the result. If, however, you want to learn how the simplex method works and see what happens in different situations using different strategies, but don’t want to deal with tedious arithmetic, this module is for you!

Historically it was created to complement the Math 373 course on Mathematical Programming and Optimization at the University of Alberta, Edmonton, Canada.

AUTHORS:

- Andrey Novoseltsev (2013-03-16): initial version.

EXAMPLES:

Most of the module functionality is demonstrated on the following problem.

---

## Corn & Barley

A farmer has 1000 acres available to grow corn and barley. Corn has a net profit of 10 dollars per acre while barley has a net profit of 5 dollars per acre. The farmer has 1500 kg of fertilizer available with 3 kg per acre needed for corn and 1 kg per acre needed for barley. The farmer wants to maximize profit. (Sometimes we also add one more constraint to make the initial dictionary infeasible: the farmer has to use at least 40% of the available land.)

---

Using variables  $C$  and  $B$  for land used to grow corn and barley respectively, in acres, we can construct the following LP problem:

```
sage: A = ([1, 1], [3, 1])
sage: b = (1000, 1500)
sage: c = (10, 5)
sage: P = LPPProblem(A, b, c, ["C", "B"], variable_type=">=")
sage: P
LP problem (use typeset mode to see details)
```

It is recommended to copy-paste such examples into your own worksheet, so that you can run these commands with typeset mode on and get

$$\begin{aligned} \max \quad & 10C + 5B \\ & C + B \leq 1000 \\ & 3C + B \leq 1500 \\ & C, B \geq 0 \end{aligned}$$

Since it has only two variables, we can solve it graphically:

```
sage: P.plot()
```

The simplex method can be applied only to `problems in standard form`, which can be created either directly

```
sage: LPPProblemStandardForm(A, b, c, ["C", "B"])
LP problem (use typeset mode to see details)
```

or from an already constructed problem of “general type”:

```
sage: P = P.standard_form()
```

In this case the problem does not require any modifications to be written in standard form, but this step is still necessary to enable methods related to the simplex method.

The simplest way to use the simplex method is:

```
sage: P.run_simplex_method()
'...'
```

(This method produces quite long formulas which have been omitted here.) But, of course, it is much more fun to do most of the steps by hand. Let’s start by creating the initial dictionary:

```
sage: D = P.initial_dictionary()
sage: D
LP problem dictionary (use typeset mode to see details)
```

Using typeset mode as recommended, you’ll see

$x_3 = 1000 - C - B$
$x_4 = 1500 - 3C - B$
$z = 0 + 10C + 5B$

With the initial or any other dictionary you can perform a number of checks:

```
sage: D.is_feasible()
True
sage: D.is_optimal()
False
```

You can look at many of its pieces and associated data:

```
sage: D.basic_variables()
(x3, x4)
sage: D.basic_solution()
(0, 0)
sage: D.objective_value()
0
```

Most importantly, you can perform steps of the simplex method by picking an entering variable, a leaving variable, and updating the dictionary:

```
sage: D.enter("C")
sage: D.leave(4)
sage: D.update()
```

If everything was done correctly, the new dictionary is still feasible and the objective value did not decrease:



```
sage: D.is_feasible()
True
sage: D.objective_value()
5000
```

If you are unsure about picking entering and leaving variables, you can use helper methods that will try their best to tell you what are your next options:

```
sage: D.possible_entering()
[B]
sage: D.possible_leaving()
Traceback (most recent call last):
...
ValueError: leaving variables can be determined
for feasible dictionaries with a set entering variable
or for dual feasible dictionaries
```

It is also possible to obtain `feasible sets` and `final dictionaries` of problems, work with `revised dictionaries`, and use the dual simplex method!

---

**Note:** Currently this does not have a display format for the terminal.

---

**class** `sage.numerical.interactive_simplex_method.LPAbstractDictionary`

Bases: `sage.structure.sage_object.SageObject`

Abstract base class for dictionaries for LP problems.

Instantiating this class directly is meaningless, see `LPDictionary` and `LPRevisedDictionary` for useful extensions.

**base\_ring()**

Return the base ring of `self`, i.e. the ring of coefficients.

OUTPUT:

•a ring

EXAMPLES:

```
sage: A = ([1, 1], [3, 1])
sage: b = (1000, 1500)
sage: c = (10, 5)
sage: P = LPPProblemStandardForm(A, b, c)
sage: D = P.initial_dictionary()
sage: D.base_ring()
Rational Field
sage: D = P.revised_dictionary()
sage: D.base_ring()
Rational Field
```

**basic\_solution** (*include\_slack\_variables=False*)

Return the basic solution of `self`.

The basic solution associated to a dictionary is obtained by setting to zero all `nonbasic_variables()`, in which case `basic_variables()` have to be equal to `constant_terms()` in equations. It may refer to values of `decision_variables()` only or include `slack_variables()` as well.

INPUT:

- `include_slack_variables` – (default: `False`) if `True`, values of slack variables will be appended at the end

OUTPUT:

- a vector

EXAMPLES:

```
sage: A = ([1, 1], [3, 1])
sage: b = (1000, 1500)
sage: c = (10, 5)
sage: P = LPProblemStandardForm(A, b, c)
sage: D = P.initial_dictionary()
sage: D.basic_solution()
(0, 0)
sage: D.basic_solution(True)
(0, 0, 1000, 1500)
sage: D = P.revised_dictionary()
sage: D.basic_solution()
(0, 0)
sage: D.basic_solution(True)
(0, 0, 1000, 1500)
```

**`coordinate_ring()`**

Return the coordinate ring of `self`.

OUTPUT:

- a polynomial ring in `auxiliary_variable()`, `decision_variables()`, and `slack_variables()` of `self` over the `base_ring()`

EXAMPLES:

```
sage: A = ([1, 1], [3, 1])
sage: b = (1000, 1500)
sage: c = (10, 5)
sage: P = LPProblemStandardForm(A, b, c)
sage: D = P.initial_dictionary()
sage: D.coordinate_ring()
Multivariate Polynomial Ring in x0, x1, x2, x3, x4
over Rational Field
sage: D = P.revised_dictionary()
sage: D.coordinate_ring()
Multivariate Polynomial Ring in x0, x1, x2, x3, x4
over Rational Field
```

**`dual_ratios()`**

Return ratios used to determine the entering variable based on leaving.

OUTPUT:

- A list of pairs  $(r_j, x_j)$  where  $x_j$  is a non-basic variable and  $r_j = c_j/a_{ij}$  is the ratio of the objective coefficient  $c_j$  to the coefficient  $a_{ij}$  of  $x_j$  in the relation for the leaving variable  $x_i$ :

$$x_i = b_i - \cdots - a_{ij}x_j - \cdots .$$

The order of pairs matches the order of `nonbasic_variables()`, but only  $x_j$  with negative  $a_{ij}$  are considered.

EXAMPLES:

```

sage: A = ([1, 1], [3, 1], [-1, -1])
sage: b = (1000, 1500, -400)
sage: c = (10, 5)
sage: P = LPPProblemStandardForm(A, b, c)
sage: D = P.dictionary(2, 3, 5)
sage: D.leave(3)
sage: D.dual_ratios()
[(5/2, x1), (5, x4)]
sage: D = P.revised_dictionary(2, 3, 5)
sage: D.leave(3)
sage: D.dual_ratios()
[(5/2, x1), (5, x4)]

```

**enter(v)**

Set  $v$  as the entering variable of `self`.

INPUT:

- $v$  – a non-basic variable of `self`, can be given as a string, an actual variable, or an integer interpreted as the index of a variable

OUTPUT:

- none, but the selected variable will be used as entering by methods that require an entering variable and the corresponding column will be typeset in green

EXAMPLES:

```

sage: A = ([1, 1], [3, 1])
sage: b = (1000, 1500)
sage: c = (10, 5)
sage: P = LPPProblemStandardForm(A, b, c)
sage: D = P.initial_dictionary()
sage: D.enter("x1")

```

We can also use indices of variables:

```
sage: D.enter(1)
```

Or variable names without quotes after injecting them:

```

sage: P.inject_variables()
Defining x0, x1, x2, x3, x4
sage: D.enter(x1)

```

The same works for revised dictionaries as well:

```

sage: D = P.revised_dictionary()
sage: D.enter(x1)

```

**is\_dual\_feasible()**

Check if `self` is dual feasible.

OUTPUT:

- True if all `objective_coefficients()` are non-positive, False otherwise

EXAMPLES:

```

sage: A = ([1, 1], [3, 1])
sage: b = (1000, 1500)
sage: c = (10, 5)

```

```
sage: P = LPProblemStandardForm(A, b, c)
sage: D = P.initial_dictionary()
sage: D.is_dual_feasible()
False
sage: D = P.revised_dictionary()
sage: D.is_dual_feasible()
False
```

**is\_feasible()**

Check if self is feasible.

OUTPUT:

- True if all `constant_terms()` are non-negative, “False” otherwise

EXAMPLES:

```
sage: A = ([1, 1], [3, 1])
sage: b = (1000, 1500)
sage: c = (10, 5)
sage: P = LPProblemStandardForm(A, b, c)
sage: D = P.initial_dictionary()
sage: D.is_feasible()
True
sage: D = P.revised_dictionary()
sage: D.is_feasible()
True
```

**is\_optimal()**

Check if self is optimal.

OUTPUT:

- True if self `is_feasible()` and `is_dual_feasible()` (i.e. all `constant_terms()` are non-negative and all `objective_coefficients()` are non-positive), False otherwise.

EXAMPLES:

```
sage: A = ([1, 1], [3, 1])
sage: b = (1000, 1500)
sage: c = (10, 5)
sage: P = LPProblemStandardForm(A, b, c)
sage: D = P.initial_dictionary()
sage: D.is_optimal()
False
sage: D = P.revised_dictionary()
sage: D.is_optimal()
False
sage: D = P.revised_dictionary(1, 2)
sage: D.is_optimal()
True
```

**leave(v)**

Set v as the leaving variable of self.

INPUT:

- v – a basic variable of self, can be given as a string, an actual variable, or an integer interpreted as the index of a variable

OUTPUT:

- none, but the selected variable will be used as leaving by methods that require a leaving variable and the corresponding row will be typeset in red

EXAMPLES:

```
sage: A = ([1, 1], [3, 1])
sage: b = (1000, 1500)
sage: c = (10, 5)
sage: P = LPProblemStandardForm(A, b, c)
sage: D = P.initial_dictionary()
sage: D.leave("x4")
```

We can also use indices of variables:

```
sage: D.leave(4)
```

Or variable names without quotes after injecting them:

```
sage: P.inject_variables()
Defining x0, x1, x2, x3, x4
sage: D.leave(x4)
```

The same works for revised dictionaries as well:

```
sage: D = P.revised_dictionary()
sage: D.leave(x4)
```

#### **possible\_dual\_simplex\_method\_steps()**

Return possible dual simplex method steps for `self`.

OUTPUT:

- A list of pairs (`leaving`, `entering`), where `leaving` is a basic variable that may `leave()` and `entering` is a list of non-basic variables that may `enter()` when `leaving` leaves. Note that `entering` may be empty, indicating that the problem is infeasible (since the dual one is unbounded).

EXAMPLES:

```
sage: A = ([1, 1], [3, 1])
sage: b = (1000, 1500)
sage: c = (10, 5)
sage: P = LPProblemStandardForm(A, b, c)
sage: D = P.dictionary(2, 3)
sage: D.possible_dual_simplex_method_steps()
[(x3, [x1])]
sage: D = P.revised_dictionary(2, 3)
sage: D.possible_dual_simplex_method_steps()
[(x3, [x1])]
```

#### **possible\_entering()**

Return possible entering variables for `self`.

OUTPUT:

- a list of non-basic variables of `self` that can `enter()` on the next step of the (dual) simplex method

EXAMPLES:

```
sage: A = ([1, 1], [3, 1])
sage: b = (1000, 1500)
sage: c = (10, 5)
sage: P = LPProblemStandardForm(A, b, c)
sage: D = P.initial_dictionary()
```

```
sage: D.possible_entering()
[x1, x2]
sage: D = P.revised_dictionary()
sage: D.possible_entering()
[x1, x2]
```

**possible\_leaving()**

Return possible leaving variables for self.

OUTPUT:

- a list of basic variables of self that can `leave()` on the next step of the (dual) simplex method

EXAMPLES:

```
sage: A = ([1, 1], [3, 1])
sage: b = (1000, 1500)
sage: c = (10, 5)
sage: P = LPPProblemStandardForm(A, b, c)
sage: D = P.initial_dictionary()
sage: D.enter(1)
sage: D.possible_leaving()
[x4]
sage: D = P.revised_dictionary()
sage: D.enter(1)
sage: D.possible_leaving()
[x4]
```

**possible\_simplex\_method\_steps()**

Return possible simplex method steps for self.

OUTPUT:

- A list of pairs (entering, leaving), where entering is a non-basic variable that may `enter()` and leaving is a list of basic variables that may `leave()` when entering enters. Note that leaving may be empty, indicating that the problem is unbounded.

EXAMPLES:

```
sage: A = ([1, 1], [3, 1])
sage: b = (1000, 1500)
sage: c = (10, 5)
sage: P = LPPProblemStandardForm(A, b, c)
sage: D = P.initial_dictionary()
sage: D.possible_simplex_method_steps()
[(x1, [x4]), (x2, [x3])]
sage: D = P.revised_dictionary()
sage: D.possible_simplex_method_steps()
[(x1, [x4]), (x2, [x3])]
```

**ratios()**

Return ratios used to determine the leaving variable based on entering.

OUTPUT:

- A list of pairs  $(r_i, x_i)$  where  $x_i$  is a basic variable and  $r_i = b_i/a_{ik}$  is the ratio of the constant term  $b_i$  to the coefficient  $a_{ik}$  of the entering variable  $x_k$  in the relation for  $x_i$ :

$$x_i = b_i - \cdots - a_{ik}x_k - \cdots.$$

The order of pairs matches the order of `basic_variables()`, but only  $x_i$  with positive  $a_{ik}$  are considered.

## EXAMPLES:

```

sage: A = ([1, 1], [3, 1])
sage: b = (1000, 1500)
sage: c = (10, 5)
sage: P = LPProblemStandardForm(A, b, c)
sage: D = P.initial_dictionary()
sage: D.enter(1)
sage: D.ratios()
[(1000, x3), (500, x4)]
sage: D = P.revised_dictionary()
sage: D.enter(1)
sage: D.ratios()
[(1000, x3), (500, x4)]

```

**class** `sage.numerical.interactive_simplex_method.LPDictionary`(*A*, *b*, *c*, *objective\_value*, *basic\_variables*, *nonbasic\_variables*, *objective\_variable*)

Bases: `sage.numerical.interactive_simplex_method.LPAbstractDictionary`

Construct a dictionary for an LP problem.

A dictionary consists of the following data:

$x_B = b - Ax_N$
$z = z^* + cx_N$

## INPUT:

- *A* – a matrix of relation coefficients
- *b* – a vector of relation constant terms
- *c* – a vector of objective coefficients
- *objective\_value* – current value of the objective  $z^*$
- *basic\_variables* – a list of basic variables  $x_B$
- *nonbasic\_variables* – a list of non-basic variables  $x_N$
- *objective\_variable* – an objective variable  $z$

## OUTPUT:

- a dictionary for an LP problem

---

**Note:** This constructor does not check correctness of input, as it is intended to be used internally by `LPProblemStandardForm`.

---

## EXAMPLES:

The intended way to use this class is indirect:

```

sage: A = ([1, 1], [3, 1])
sage: b = (1000, 1500)
sage: c = (10, 5)
sage: P = LPProblemStandardForm(A, b, c)
sage: D = P.initial_dictionary()
sage: D
LP problem dictionary (use typeset mode to see details)

```

But if you want you can create a dictionary without starting with an LP problem, here is construction of the same dictionary as above:

```
sage: A = matrix(QQ, ([1, 1], [3, 1]))
sage: b = vector(QQ, (1000, 1500))
sage: c = vector(QQ, (10, 5))
sage: R = PolynomialRing(QQ, "x1, x2, x3, x4", order="neglex")
sage: from sage.numerical.interactive_simplex_method \
....:     import LPDictionary
sage: D2 = LPDictionary(A, b, c, 0, R.gens()[2:], R.gens()[2:], "z")
sage: D2 == D
True
```

**ELLUL** (*entering, leaving*)

Perform the Enter-Leave-LaTeX-Update-LaTeX step sequence on `self`.

INPUT:

- `entering` – the entering variable
- `leaving` – the leaving variable

OUTPUT:

- a string with LaTeX code for `self` before and after update

EXAMPLES:

```
sage: A = ([1, 1], [3, 1])
sage: b = (1000, 1500)
sage: c = (10, 5)
sage: P = LPProblemStandardForm(A, b, c)
sage: D = P.initial_dictionary()
sage: D.ELLUL("x1", "x4")
\renewcommand{\arraystretch}{1.5} \setlength{\arraycolsep}{0.125em}
\begin{array}{|rcrcrcr|}
\hline
x_{3} & = & 1000 & - & x_{1} & - & x_{2} \\
\color{red}x_{4} & \color{red}= & \color{red}1500 & \color{red}- & \color{blue}3 x_{1} & \color{red}& 
\hline
z & = & 0 & + & 10 x_{1} & + & 5 x_{2} \\
\hline
\multicolumn{2}{c}{c} & & & & & [-3ex]
\hline
x_{3} & = & 500 & + & \frac{1}{3} x_{4} & - & \frac{2}{3} x_{2} \\
x_{1} & = & 500 & - & \frac{1}{3} x_{4} & - & \frac{1}{3} x_{2} \\
\hline
z & = & 5000 & - & \frac{10}{3} x_{4} & + & \frac{5}{3} x_{2} \\
\hline
\end{array}
```



This is how the above output looks when rendered:

$x_3 = 1000 - x_1 - x_2$
$x_4 = 1500 - 3x_1 - x_2$
$z = 0 + 10x_1 + 5x_2$
$x_3 = 500 + \frac{1}{3}x_4 - \frac{2}{3}x_2$
$x_1 = 500 - \frac{1}{3}x_4 - \frac{1}{3}x_2$
$z = 5000 - \frac{10}{3}x_4 + \frac{5}{3}x_2$

The column of the entering variable is green, while the row of the leaving variable is red in the original dictionary state on the top. The new state after the update step is shown on the bottom.

#### **basic\_variables()**

Return the basic variables of `self`.

OUTPUT:

•a vector

EXAMPLES:

```
sage: A = ([1, 1], [3, 1])
sage: b = (1000, 1500)
sage: c = (10, 5)
sage: P = LPPProblemStandardForm(A, b, c)
sage: D = P.initial_dictionary()
sage: D.basic_variables()
(x3, x4)
```

#### **constant\_terms()**

Return the constant terms of relations of `self`.

OUTPUT:

•a vector.

EXAMPLES:

```
sage: A = ([1, 1], [3, 1])
sage: b = (1000, 1500)
sage: c = (10, 5)
sage: P = LPPProblemStandardForm(A, b, c)
sage: D = P.initial_dictionary()
sage: D.constant_terms()
(1000, 1500)
```

#### **entering\_coefficients()**

Return coefficients of the entering variable.

OUTPUT:

•a vector

EXAMPLES:

```
sage: A = ([1, 1], [3, 1])
sage: b = (1000, 1500)
sage: c = (10, 5)
```

```
sage: P = LPProblemStandardForm(A, b, c)
sage: D = P.initial_dictionary()
sage: D.enter(1)
sage: D.entering_coefficients()
(1, 3)
```

**leaving\_coefficients()**

Return coefficients of the leaving variable.

OUTPUT:

•a vector

EXAMPLES:

```
sage: A = ([1, 1], [3, 1])
sage: b = (1000, 1500)
sage: c = (10, 5)
sage: P = LPProblemStandardForm(A, b, c)
sage: D = P.dictionary(2, 3)
sage: D.leave(3)
sage: D.leaving_coefficients()
(-2, -1)
```

**nonbasic\_variables()**

Return non-basic variables of self.

OUTPUT:

•a vector

EXAMPLES:

```
sage: A = ([1, 1], [3, 1])
sage: b = (1000, 1500)
sage: c = (10, 5)
sage: P = LPProblemStandardForm(A, b, c)
sage: D = P.initial_dictionary()
sage: D.nonbasic_variables()
(x1, x2)
```

**objective\_coefficients()**

Return coefficients of the objective of self.

OUTPUT:

•a vector

EXAMPLES:

```
sage: A = ([1, 1], [3, 1])
sage: b = (1000, 1500)
sage: c = (10, 5)
sage: P = LPProblemStandardForm(A, b, c)
sage: D = P.initial_dictionary()
sage: D.objective_coefficients()
(10, 5)
```

**objective\_value()**

Return the value of the objective at the `basic_solution()` of self.

OUTPUT:

- a number

EXAMPLES:

```
sage: A = ([1, 1], [3, 1])
sage: b = (1000, 1500)
sage: c = (10, 5)
sage: P = LPProblemStandardForm(A, b, c)
sage: D = P.initial_dictionary()
sage: D.objective_value()
0
```

**update()**

Update self using previously set entering and leaving variables.

EXAMPLES:

```
sage: A = ([1, 1], [3, 1])
sage: b = (1000, 1500)
sage: c = (10, 5)
sage: P = LPProblemStandardForm(A, b, c)
sage: D = P.initial_dictionary()
sage: D.objective_value()
0
sage: D.enter("x1")
sage: D.leave("x4")
sage: D.update()
sage: D.objective_value()
5000
```

```
class sage.numerical.interactive_simplex_method.LPProblem(A, b, c, x='x', con-
    straint_type='<=',
    variable_type='', prob-
    lem_type='max', pre-
    fix='x', base_ring=None)
```

Bases: `sage.structure.sage_object.SageObject`

Construct an LP (Linear Programming) problem.

This class supports LP problems with “variables on the left” constraints.

INPUT:

- A** – a matrix of constraint coefficients
- b** – a vector of constraint constant terms
- c** – a vector of objective coefficients
- x** – (default: "x") a vector of decision variables or a string giving the base name
- constraint\_type** – (default: "<=") a string specifying constraint type(s): either "<=", ">=", "==", or a list of them
- variable\_type** – (default: "") a string specifying variable type(s): either ">=", "<=", "" (the empty string), or a list of them, corresponding, respectively, to non-negative, non-positive, and free variables
- problem\_type** – (default: "max") a string specifying the problem type: "max", "min", "-max", or "-min"
- prefix** – (default: parameter `x` if it was given as a string, string "x" otherwise) a string giving the base name of automatically created variables in `standard_form()`

- `base_ring` – (default: the fraction field of a common ring for all input coefficients) a field to which all input coefficients will be converted

EXAMPLES:

We will construct the following problem:

$$\begin{aligned} \max \quad & 10C + 5B \\ & C + B \leq 1000 \\ & 3C + B \leq 1500 \\ & C, B \geq 0 \end{aligned}$$

```
sage: A = ([1, 1], [3, 1])
sage: b = (1000, 1500)
sage: c = (10, 5)
sage: P = LPPProblem(A, b, c, ["C", "B"], variable_type=">=")
```

Same problem, but more explicitly:

```
sage: P = LPPProblem(A, b, c, ["C", "B"],
....:      constraint_type="<=", variable_type=">=")
```

Even more explicitly:

```
sage: P = LPPProblem(A, b, c, ["C", "B"], problem_type="max",
....:      constraint_type=["<=", "<="], variable_type=[">=", ">="])
```

Using the last form you should be able to represent any LP problem, as long as all like terms are collected and in constraints variables and constants are on different sides.

**A()**

Return coefficients of constraints of `self`, i.e.  $A$ .

OUTPUT:

- a matrix

EXAMPLES:

```
sage: A = ([1, 1], [3, 1])
sage: b = (1000, 1500)
sage: c = (10, 5)
sage: P = LPPProblem(A, b, c, ["C", "B"], variable_type=">=")
sage: P.constraint_coefficients()
[1 1]
[3 1]
sage: P.A()
[1 1]
[3 1]
```

**Abcx()**

Return  $A$ ,  $b$ ,  $c$ , and  $x$  of `self` as a tuple.

OUTPUT:

- a tuple

EXAMPLES:

```
sage: A = ([1, 1], [3, 1])
sage: b = (1000, 1500)
sage: c = (10, 5)
sage: P = LPPProblem(A, b, c, ["C", "B"], variable_type=">=")
```

```
sage: P.Abcx()
(
[1 1]
[3 1], (1000, 1500), (10, 5), (C, B)
)
```

**b()**

Return constant terms of constraints of `self`, i.e. `b`.

OUTPUT:

•a vector

EXAMPLES:

```
sage: A = ([1, 1], [3, 1])
sage: b = (1000, 1500)
sage: c = (10, 5)
sage: P = LPPProblem(A, b, c, ["C", "B"], variable_type=">=")
sage: P.constant_terms()
(1000, 1500)
sage: P.b()
(1000, 1500)
```

**base\_ring()**

Return the base ring of `self`.

---

**Note:** The base ring of LP problems is always a field.

---

OUTPUT:

•a ring

EXAMPLES:

```
sage: A = ([1, 1], [3, 1])
sage: b = (1000, 1500)
sage: c = (10, 5)
sage: P = LPPProblem(A, b, c, ["C", "B"], variable_type=">=")
sage: P.base_ring()
Rational Field

sage: c = (10, 5.)
sage: P = LPPProblem(A, b, c, ["C", "B"], variable_type=">=")
sage: P.base_ring()
Real Field with 53 bits of precision
```

**c()**

Return coefficients of the objective of `self`, i.e. `c`.

OUTPUT:

•a vector

EXAMPLES:

```
sage: A = ([1, 1], [3, 1])
sage: b = (1000, 1500)
sage: c = (10, 5)
sage: P = LPPProblem(A, b, c, ["C", "B"], variable_type=">=")
sage: P.objective_coefficients()
```

```
(10, 5)
sage: P.c()
(10, 5)
```

**constant\_terms()**

Return constant terms of constraints of `self`, i.e. `b`.

OUTPUT:

•a vector

EXAMPLES:

```
sage: A = ([1, 1], [3, 1])
sage: b = (1000, 1500)
sage: c = (10, 5)
sage: P = LPProblem(A, b, c, ["C", "B"], variable_type=">=")
sage: P.constant_terms()
(1000, 1500)
sage: P.b()
(1000, 1500)
```

**constraint\_coefficients()**

Return coefficients of constraints of `self`, i.e. `A`.

OUTPUT:

•a matrix

EXAMPLES:

```
sage: A = ([1, 1], [3, 1])
sage: b = (1000, 1500)
sage: c = (10, 5)
sage: P = LPProblem(A, b, c, ["C", "B"], variable_type=">=")
sage: P.constraint_coefficients()
[1 1]
[3 1]
sage: P.A()
[1 1]
[3 1]
```

**decision\_variables()**

Return decision variables of `self`, i.e. `x`.

OUTPUT:

•a vector

EXAMPLES:

```
sage: A = ([1, 1], [3, 1])
sage: b = (1000, 1500)
sage: c = (10, 5)
sage: P = LPProblem(A, b, c, ["C", "B"], variable_type=">=")
sage: P.decision_variables()
(C, B)
sage: P.x()
(C, B)
```

**dual** (`y=None`)

Construct the dual LP problem for `self`.

INPUT:

- `y` – (default: "x" if the prefix of `self` is "y", "y" otherwise) a vector of dual decision variables or a string giving the base name

OUTPUT:

- an `LPPProblem`

EXAMPLES:

```
sage: A = ([1, 1], [3, 1])
sage: b = (1000, 1500)
sage: c = (10, 5)
sage: P = LPPProblem(A, b, c, ["C", "B"], variable_type=">=")
sage: DP = P.dual()
sage: DP.b() == P.c()
True
sage: DP.dual(["C", "B"]) == P
True
```

**`feasible_set()`**

Return the feasible set of `self`.

OUTPUT:

- a `Polyhedron`

EXAMPLES:

```
sage: A = ([1, 1], [3, 1])
sage: b = (1000, 1500)
sage: c = (10, 5)
sage: P = LPPProblem(A, b, c, ["C", "B"], variable_type=">=")
sage: P.feasible_set()
A 2-dimensional polyhedron in QQ^2
defined as the convex hull of 4 vertices
```

**`is_bounded()`**

Check if `self` is bounded.

OUTPUT:

- True if `self` is bounded, False otherwise

EXAMPLES:

```
sage: A = ([1, 1], [3, 1])
sage: b = (1000, 1500)
sage: c = (10, 5)
sage: P = LPPProblem(A, b, c, ["C", "B"], variable_type=">=")
sage: P.is_bounded()
True
```

**`is_feasible()`**

Check if `self` is feasible.

OUTPUT:

- True if `self` is feasible, False otherwise

EXAMPLES:

```
sage: A = ([1, 1], [3, 1])
sage: b = (1000, 1500)
```

```
sage: c = (10, 5)
sage: P = LPProblem(A, b, c, ["C", "B"], variable_type=">=")
sage: P.is_feasible()
True
```

**m()**

Return the number of constraints of `self`, i.e.  $m$ .

OUTPUT:

•an integer

EXAMPLES:

```
sage: A = ([1, 1], [3, 1])
sage: b = (1000, 1500)
sage: c = (10, 5)
sage: P = LPProblem(A, b, c, ["C", "B"], variable_type=">=")
sage: P.n_constraints()
2
sage: P.m()
2
```

**n()**

Return the number of decision variables of `self`, i.e.  $n$ .

OUTPUT:

•an integer

EXAMPLES:

```
sage: A = ([1, 1], [3, 1])
sage: b = (1000, 1500)
sage: c = (10, 5)
sage: P = LPProblem(A, b, c, ["C", "B"], variable_type=">=")
sage: P.n_variables()
2
sage: P.n()
2
```

**n\_constraints()**

Return the number of constraints of `self`, i.e.  $m$ .

OUTPUT:

•an integer

EXAMPLES:

```
sage: A = ([1, 1], [3, 1])
sage: b = (1000, 1500)
sage: c = (10, 5)
sage: P = LPProblem(A, b, c, ["C", "B"], variable_type=">=")
sage: P.n_constraints()
2
sage: P.m()
2
```

**n\_variables()**

Return the number of decision variables of `self`, i.e.  $n$ .

OUTPUT:



- an integer

## EXAMPLES:

```
sage: A = ([1, 1], [3, 1])
sage: b = (1000, 1500)
sage: c = (10, 5)
sage: P = LPProblem(A, b, c, ["C", "B"], variable_type=">=")
sage: P.n_variables()
2
sage: P.n()
2
```

**objective\_coefficients()**

Return coefficients of the objective of `self`, i.e. `c`.

## OUTPUT:

- a vector

## EXAMPLES:

```
sage: A = ([1, 1], [3, 1])
sage: b = (1000, 1500)
sage: c = (10, 5)
sage: P = LPProblem(A, b, c, ["C", "B"], variable_type=">=")
sage: P.objective_coefficients()
(10, 5)
sage: P.c()
(10, 5)
```

**optimal\_solution()**

Return an optimal solution of `self`.

## OUTPUT:

- a vector or `None` if there are no optimal solutions

## EXAMPLES:

```
sage: A = ([1, 1], [3, 1])
sage: b = (1000, 1500)
sage: c = (10, 5)
sage: P = LPProblem(A, b, c, ["C", "B"], variable_type=">=")
sage: P.optimal_solution()
(250, 750)
```

**optimal\_value()**

Return the optimal value for `self`.

## OUTPUT:

- a number if the problem is bounded,  $\pm\infty$  if it is unbounded, or `None` if it is infeasible

## EXAMPLES:

```
sage: A = ([1, 1], [3, 1])
sage: b = (1000, 1500)
sage: c = (10, 5)
sage: P = LPProblem(A, b, c, ["C", "B"], variable_type=">=")
sage: P.optimal_value()
6250
```

**plot** (\*args, \*\*kws)

Return a plot for solving `self` graphically.

INPUT:

- `xmin, xmax, ymin, ymax` – bounds for the axes, if not given, an attempt will be made to pick reasonable values
- `alpha` – (default: 0.2) determines how opaque are shadows

OUTPUT:

- a plot

This only works for problems with two decision variables. On the plot the black arrow indicates the direction of growth of the objective. The lines perpendicular to it are level curves of the objective. If there are optimal solutions, the arrow originates in one of them and the corresponding level curve is solid: all points of the feasible set on it are optimal solutions. Otherwise the arrow is placed in the center. If the problem is infeasible or the objective is zero, a plot of the feasible set only is returned.

EXAMPLES:

```
sage: A = ([1, 1], [3, 1])
sage: b = (1000, 1500)
sage: c = (10, 5)
sage: P = LPPProblem(A, b, c, ["C", "B"], variable_type=">=")
sage: p = P.plot()
sage: p.show()
```

In this case the plot works better with the following axes ranges:

```
sage: p = P.plot(0, 1000, 0, 1500)
sage: p.show()
```

TESTS:

We check that zero objective can be dealt with:

```
sage: LPPProblem(A, b, (0, 0), ["C", "B"], variable_type=">=").plot()
```

**plot\_feasible\_set** (`xmin=None, xmax=None, ymin=None, ymax=None, alpha=0.2`)

Return a plot of the feasible set of `self`.

INPUT:

- `xmin, xmax, ymin, ymax` – bounds for the axes, if not given, an attempt will be made to pick reasonable values
- `alpha` – (default: 0.2) determines how opaque are shadows

OUTPUT:

- a plot

This only works for a problem with two decision variables. The plot shows boundaries of constraints with a shadow on one side for inequalities. If the `feasible_set()` is not empty and at least part of it is in the given boundaries, it will be shaded gray and  $F$  will be placed in its middle.

EXAMPLES:

```
sage: A = ([1, 1], [3, 1])
sage: b = (1000, 1500)
sage: c = (10, 5)
sage: P = LPPProblem(A, b, c, ["C", "B"], variable_type=">=")
```

```
sage: p = P.plot_feasible_set()
sage: p.show()
```

In this case the plot works better with the following axes ranges:

```
sage: p = P.plot_feasible_set(0, 1000, 0, 1500)
sage: p.show()
```

### **standard\_form()**

Construct the LP problem in standard form equivalent to `self`.

OUTPUT:

•an `LPPProblemStandardForm`

EXAMPLES:

```
sage: A = ([1, 1], [3, 1])
sage: b = (1000, 1500)
sage: c = (10, 5)
sage: P = LPPProblem(A, b, c, ["C", "B"], variable_type=">=")
sage: DP = P.dual()
sage: DPSF = DP.standard_form()
sage: DPSF.b()
(-10, -5)
```

### **x()**

Return decision variables of `self`, i.e.  $x$ .

OUTPUT:

•a vector

EXAMPLES:

```
sage: A = ([1, 1], [3, 1])
sage: b = (1000, 1500)
sage: c = (10, 5)
sage: P = LPPProblem(A, b, c, ["C", "B"], variable_type=">=")
sage: P.decision_variables()
(C, B)
sage: P.x()
(C, B)
```

```
class sage.numerical.interactive_simplex_method.LPPProblemStandardForm(A, b, c,
                                                                    x='x',
                                                                    prob-
                                                                    lem_type='max',
                                                                    slack_variables=None,
                                                                    auxil-
                                                                    iary_variable=None,
                                                                    objec-
                                                                    tive='z',
                                                                    base_ring=None)
```

Bases: `sage.numerical.interactive_simplex_method.LPPProblem`

Construct an LP (Linear Programming) problem in standard form.

The used standard form is:

$$\begin{aligned} &\pm \max cx \\ &Ax \leq b \\ &x \geq 0 \end{aligned}$$

INPUT:

- **A** – a matrix of constraint coefficients
- **b** – a vector of constraint constant terms
- **c** – a vector of objective coefficients
- **x** – (default: "x") a vector of decision variables or a string the base name giving
- **problem\_type** – (default: "max") a string specifying the problem type: either "max" or "-max"
- **slack\_variables** – (default: same as x parameter, if it was given as a string, otherwise string "x") a vector of slack variables or a sting giving the base name
- **auxiliary\_variable** – (default: same as x parameter with adjoined "0" if it was given as a string, otherwise "x0") the auxiliary name, expected to be the same as the first decision variable for auxiliary problems
- **objective** – (default: "z") the objective variable (used for the initial dictionary)
- **base\_ring** – (default: the fraction field of a common ring for all input coefficients) a field to which all input coefficients will be converted

EXAMPLES:

```
sage: A = ([1, 1], [3, 1])
sage: b = (1000, 1500)
sage: c = (10, 5)
sage: P = LPPProblemStandardForm(A, b, c)
```

Unlike `LPPProblem`, this class does not allow you to adjust types of constraints (they are always " $\leq$ ") and variables (they are always " $\geq$ "), and the problem type may only be "max" or "-max". You may give custom names to slack and auxiliary variables, but in most cases defaults should work:

```
sage: P.decision_variables()
(x1, x2)
sage: P.slack_variables()
(x3, x4)
```

**auxiliary\_problem()**

Construct the auxiliary problem for self.

OUTPUT:

- an LP problem in standard form

The auxiliary problem with the auxiliary variable  $x_0$  is

$$\begin{aligned} &\max -x_0 \\ &-x_0 + A_i x \leq b_i \text{ for all } i \quad . \\ &x \geq 0 \end{aligned}$$

Such problems are used when the `initial_dictionary()` is infeasible.

EXAMPLES:

```

sage: A = ([1, 1], [3, 1], [-1, -1])
sage: b = (1000, 1500, -400)
sage: c = (10, 5)
sage: P = LPPProblemStandardForm(A, b, c)
sage: AP = P.auxiliary_problem()

```

**auxiliary\_variable()**

Return the auxiliary variable of self.

Note that the auxiliary variable may or may not be among `decision_variables()`.

OUTPUT:

- a variable of the `coordinate_ring()` of self

EXAMPLES:

```

sage: A = ([1, 1], [3, 1], [-1, -1])
sage: b = (1000, 1500, -400)
sage: c = (10, 5)
sage: P = LPPProblemStandardForm(A, b, c)
sage: P.auxiliary_variable()
x0
sage: P.decision_variables()
(x1, x2)
sage: AP = P.auxiliary_problem()
sage: AP.auxiliary_variable()
x0
sage: AP.decision_variables()
(x0, x1, x2)

```

**coordinate\_ring()**

Return the coordinate ring of self.

OUTPUT:

- a polynomial ring over the `base_ring()` of self in the `auxiliary_variable()`, `decision_variables()`, and `slack_variables()` with “neglex” order

EXAMPLES:

```

sage: A = ([1, 1], [3, 1], [-1, -1])
sage: b = (1000, 1500, -400)
sage: c = (10, 5)
sage: P = LPPProblemStandardForm(A, b, c)
sage: P.coordinate_ring()
Multivariate Polynomial Ring in x0, x1, x2, x3, x4, x5
over Rational Field
sage: P.base_ring()
Rational Field
sage: P.auxiliary_variable()
x0
sage: P.decision_variables()
(x1, x2)
sage: P.slack_variables()
(x3, x4, x5)

```

**dictionary(\*x\_B)**

Construct a dictionary for self with given basic variables.

INPUT:

- basic variables for the dictionary to be constructed

OUTPUT:

- a `dictionary`

---

**Note:** This is a synonym for `self.revised_dictionary(x_B).dictionary()`, but basic variables are mandatory.

---

EXAMPLES:

```
sage: A = ([1, 1], [3, 1])
sage: b = (1000, 1500)
sage: c = (10, 5)
sage: P = LPPProblemStandardForm(A, b, c)
sage: D = P.dictionary("x1", "x2")
sage: D.basic_variables()
(x1, x2)
```

**feasible\_dictionary** (*auxiliary\_dictionary*)

Construct a feasible dictionary for `self`.

INPUT:

- `auxiliary_dictionary` – an optimal dictionary for the `auxiliary_problem()` of `self` with the optimal value 0 and a non-basic auxiliary variable

OUTPUT:

- a feasible `dictionary` for `self`

EXAMPLES:

```
sage: A = ([1, 1], [3, 1], [-1, -1])
sage: b = (1000, 1500, -400)
sage: c = (10, 5)
sage: P = LPPProblemStandardForm(A, b, c)
sage: AP = P.auxiliary_problem()
sage: D = AP.initial_dictionary()
sage: D.enter(0)
sage: D.leave(5)
sage: D.update()
sage: D.enter(1)
sage: D.leave(0)
sage: D.update()
sage: D.is_optimal()
True
sage: D.objective_value()
0
sage: D.basic_solution()
(0, 400, 0)
sage: D = P.feasible_dictionary(D)
sage: D.is_optimal()
False
sage: D.is_feasible()
True
sage: D.objective_value()
4000
sage: D.basic_solution()
(400, 0)
```

**final\_dictionary()**

Return the final dictionary of the simplex method applied to `self`.

See `run_simplex_method()` for the description of possibilities.

OUTPUT:

•a dictionary

EXAMPLES:

```
sage: A = ([1, 1], [3, 1])
sage: b = (1000, 1500)
sage: c = (10, 5)
sage: P = LPPProblemStandardForm(A, b, c)
sage: D = P.final_dictionary()
sage: D.is_optimal()
True
```

TESTS:

```
sage: P.final_dictionary() is P.final_dictionary()
False
```

**final\_revised\_dictionary()**

Return the final dictionary of the revised simplex method applied to `self`.

See `run_revised_simplex_method()` for the description of possibilities.

OUTPUT:

•a revised dictionary

EXAMPLES:

```
sage: A = ([1, 1], [3, 1])
sage: b = (1000, 1500)
sage: c = (10, 5)
sage: P = LPPProblemStandardForm(A, b, c)
sage: D = P.final_revised_dictionary()
sage: D.is_optimal()
True
```

TESTS:

```
sage: P.final_revised_dictionary() is P.final_revised_dictionary()
False
```

**initial\_dictionary()**

Construct the initial dictionary of `self`.

The initial dictionary “defines” `slack_variables()` in terms of the `decision_variables()`, i.e. it has slack variables as basic ones.

OUTPUT:

•a dictionary

EXAMPLES:

```
sage: A = ([1, 1], [3, 1])
sage: b = (1000, 1500)
sage: c = (10, 5)
sage: P = LPPProblemStandardForm(A, b, c)
sage: D = P.initial_dictionary()
```

**inject\_variables** (*scope=None, verbose=True*)

Inject variables of self into scope.

INPUT:

- scope – namespace (default: global)
- verbose – if True (default), names of injected variables will be printed

OUTPUT:

- none

EXAMPLES:

```
sage: A = ([1, 1], [3, 1])
sage: b = (1000, 1500)
sage: c = (10, 5)
sage: P = LPPProblemStandardForm(A, b, c)
sage: P.inject_variables()
Defining x0, x1, x2, x3, x4
sage: 3*x1 + x2
x2 + 3*x1
```

**revised\_dictionary** (\**x\_B*)

Construct a revised dictionary for self.

INPUT:

- basic variables for the dictionary to be constructed; if not given, `slack_variables()` will be used, perhaps with the `auxiliary_variable()` to give a feasible dictionary

OUTPUT:

- a revised dictionary

EXAMPLES:

```
sage: A = ([1, 1], [3, 1])
sage: b = (1000, 1500)
sage: c = (10, 5)
sage: P = LPPProblemStandardForm(A, b, c)
sage: D = P.revised_dictionary("x1", "x2")
sage: D.basic_variables()
(x1, x2)
```

If basic variables are not given the initial dictionary is constructed:

```
sage: P.revised_dictionary().basic_variables()
(x3, x4)
sage: P.initial_dictionary().basic_variables()
(x3, x4)
```

Unless it is infeasible, in which case a feasible dictionary for the auxiliary problem is constructed:

```
sage: A = ([1, 1], [3, 1], [-1, -1])
sage: b = (1000, 1500, -400)
sage: P = LPPProblemStandardForm(A, b, c)
sage: P.initial_dictionary().is_feasible()
False
sage: P.revised_dictionary().basic_variables()
(x3, x4, x0)
```



**run\_revised\_simplex\_method()**

Apply the revised simplex method to solve `self` and show the steps.

OUTPUT:

- a string with L<sup>A</sup>T<sub>E</sub>X code of intermediate dictionaries

---

**Note:** You can access the `final_revised_dictionary()`, which can be one of the following:

- an optimal dictionary with the `auxiliary_variable()` among `basic_variables()` and a non-zero optimal value indicating that `self` is infeasible;
  - a non-optimal dictionary that has marked entering variable for which there is no choice of the leaving variable, indicating that `self` is unbounded;
  - an optimal dictionary.
- 

EXAMPLES:

```
sage: A = ([1, 1], [3, 1], [-1, -1])
sage: b = (1000, 1500, -400)
sage: c = (10, 5)
sage: P = LPPProblemStandardForm(A, b, c)
sage: P.run_revised_simplex_method()
\renewcommand{\arraystretch}{1.500000}
\begin{array}{l}
...
\text{Entering: } x_{1}$. Leaving: } x_{0}$.}\\
...
\text{Entering: } x_{5}$. Leaving: } x_{4}$.}\\
...
\text{Entering: } x_{2}$. Leaving: } x_{3}$.}\\
...
\text{The optimal value: } \$6250$. An optimal solution: } \$\left(250,\,750\right)$.}
\end{array}
```

**run\_simplex\_method()**

Apply the simplex method to solve `self` and show the steps.

OUTPUT:

- a string with L<sup>A</sup>T<sub>E</sub>X code of intermediate dictionaries

---

**Note:** You can access the `final_dictionary()`, which can be one of the following:

- an optimal dictionary for the `auxiliary_problem()` with a non-zero optimal value indicating that `self` is infeasible;
  - a non-optimal dictionary for `self` that has marked entering variable for which there is no choice of the leaving variable, indicating that `self` is unbounded;
  - an optimal dictionary for `self`.
- 

EXAMPLES:

```
sage: A = ([1, 1], [3, 1], [-1, -1])
sage: b = (1000, 1500, -400)
sage: c = (10, 5)
sage: P = LPPProblemStandardForm(A, b, c)
sage: P.run_simplex_method() # not tested
```

You should use the typeset mode as the command above generates long  $\LaTeX$  code:

```
sage: print P.run_simplex_method()
\begin{gather*}
...
\text{The initial dictionary is infeasible, solving auxiliary problem.}\displaybreak[0]\\
...
\text{Entering: } x_{0}\$. Leaving: } x_{5}\$.}\displaybreak[0]\\
...
\text{Entering: } x_{1}\$. Leaving: } x_{0}\$.}\displaybreak[0]\\
...
\text{Back to the original problem.}\displaybreak[0]\\
...
\text{Entering: } x_{5}\$. Leaving: } x_{4}\$.}\displaybreak[0]\\
...
\text{Entering: } x_{2}\$. Leaving: } x_{3}\$.}\displaybreak[0]\\
...
\text{The optimal value: } \$6250\$. An optimal solution: } \$\left(250,\,750\right)\$.}
\end{gather*}
```

#### **slack\_variables()**

Return slack variables of self.

Slack variables are differences between the constant terms and left hand sides of the constraints.

If you want to give custom names to slack variables, you have to do so during construction of the problem.

OUTPUT:

- a tuple

EXAMPLES:

```
sage: A = ([1, 1], [3, 1])
sage: b = (1000, 1500)
sage: c = (10, 5)
sage: P = LPProblemStandardForm(A, b, c)
sage: P.slack_variables()
(x3, x4)
sage: P = LPProblemStandardForm(A, b, c, ["C", "B"],
...:     slack_variables=["L", "F"])
sage: P.slack_variables()
(L, F)
```

**class** sage.numerical.interactive\_simplex\_method.**LPRevisedDictionary** (*problem*,  
*basic\_variables*)  
Bases: sage.numerical.interactive\_simplex\_method.LPAbstractDictionary

Construct a revised dictionary for an LP problem.

INPUT:

- problem* – an LP problem in standard form
- basic\_variables* – a list of basic variables or their indices

OUTPUT:

- a revised dictionary for an LP problem

A revised dictionary encodes the same relations as a *regular dictionary*, but stores only what is “necessary to efficiently compute data for the simplex method”.

Let the original problem be

$$\begin{aligned} & \pm \max cx \\ & Ax \leq b \\ & x \geq 0 \end{aligned}$$

Let  $\bar{x}$  be the vector of `decision_variables()`  $x$  followed by the `slack_variables()`. Let  $\bar{c}$  be the vector of `objective_coefficients()`  $c$  followed by zeroes for all slack variables. Let  $\bar{A} = (A|I)$  be the matrix of `constraint_coefficients()`  $A$  augmented by the identity matrix as columns corresponding to the slack variables. Then the problem above can be written as

$$\begin{aligned} & \pm \max \bar{c}\bar{x} \\ & \bar{A}\bar{x} = b \\ & \bar{x} \geq 0 \end{aligned}$$

and any dictionary is a system of equations equivalent to  $\bar{A}\bar{x} = b$ , but resolved for `basic_variables()`  $x_B$  in terms of `nonbasic_variables()`  $x_N$  together with the expression for the objective in terms of  $x_N$ . Let  $c_B()$  and  $c_N()$  be vectors “splitting  $\bar{c}$  into basic and non-basic parts”. Let  $B()$  and  $A_N()$  be the splitting of  $\bar{A}$ . Then the corresponding dictionary is

$x_B = B^{-1}b - B^{-1}A_Nx_N$
$z = yb + (c_N - y^T A_N)x_N$

where  $y = c_B^T B^{-1}$ . To proceed with the simplex method, it is not necessary to compute all entries of this dictionary. On the other hand, any entry is easy to compute, if you know  $B^{-1}$ , so we keep track of it through the update steps.

#### EXAMPLES:

```
sage: A = ([1, 1], [3, 1])
sage: b = (1000, 1500)
sage: c = (10, 5)
sage: P = LPPProblemStandardForm(A, b, c)
sage: from sage.numerical.interactive_simplex_method \
....:     import LPRevisedDictionary
sage: D = LPRevisedDictionary(P, [1, 2])
sage: D.basic_variables()
(x1, x2)
sage: D
LP problem dictionary (use typeset mode to see details)
```

The same dictionary can be constructed through the problem:

```
sage: P.revised_dictionary(1, 2) == D
True
```

When this dictionary is typeset, you will see two tables like these ones:

$x_B$	$c_B$	$B^{-1}$		$y$	$B^{-1}b$
$x_1$	10	$-\frac{1}{2}$	$\frac{1}{2}$	$\frac{5}{2}$	250
$x_2$	5	$\frac{3}{2}$	$-\frac{1}{2}$	$\frac{5}{2}$	750

$x_N$	$x_3$	$x_4$
$c_N^T$	0	0
$y^T A_N$	$\frac{5}{2}$	$\frac{5}{2}$
$c_N^T - y^T A_N$	$-\frac{5}{2}$	$-\frac{5}{2}$

More details will be shown if entering and leaving variables are set, but in any case the top table shows  $B^{-1}$  and a few extra columns, while the bottom one shows several rows: these are related to columns and rows of dictionary entries.

**A(v)**

Return the column of constraint coefficients corresponding to  $v$ .

INPUT:

- $v$  – a variable, its name, or its index

OUTPUT:

- a vector

EXAMPLES:

```
sage: A = ([1, 1], [3, 1])
sage: b = (1000, 1500)
sage: c = (10, 5)
sage: P = LPPProblemStandardForm(A, b, c)
sage: D = P.revised_dictionary()
sage: D.A(1)
(1, 3)
sage: D.A(0)
(-1, -1)
sage: D.A("x3")
(1, 0)
```

**A\_N()**

Return the  $A_N$  matrix, constraint coefficients of non-basic variables.

OUTPUT:

- a matrix

EXAMPLES:

```
sage: A = ([1, 1], [3, 1])
sage: b = (1000, 1500)
sage: c = (10, 5)
sage: P = LPPProblemStandardForm(A, b, c)
sage: D = P.revised_dictionary()
sage: D.A_N()
[1 1]
[3 1]
```

**B()**

Return the  $B$  matrix, i.e. constraint coefficients of basic variables.

OUTPUT:

- a matrix

EXAMPLES:

```
sage: A = ([1, 1], [3, 1])
sage: b = (1000, 1500)
sage: c = (10, 5)
sage: P = LPPProblemStandardForm(A, b, c)
sage: D = P.revised_dictionary(1, 2)
sage: D.B()
[1 1]
[3 1]
```

**B\_inverse()**

Return the inverse of the `B()` matrix.

This inverse matrix is stored and computed during dictionary update in a more efficient way than generic inversion.

OUTPUT:

•a matrix

EXAMPLES:

```
sage: A = ([1, 1], [3, 1])
sage: b = (1000, 1500)
sage: c = (10, 5)
sage: P = LPProblemStandardForm(A, b, c)
sage: D = P.revised_dictionary(1, 2)
sage: D.B_inverse()
[-1/2  1/2]
[ 3/2 -1/2]
```

**E()**

Return the eta matrix between `self` and the next dictionary.

OUTPUT:

•a matrix

If  $B_{\text{old}}$  is the current matrix  $B$  and  $B_{\text{new}}$  is the  $B$  matrix of the next dictionary (after the update step), then  $B_{\text{new}} = B_{\text{old}}E$ .

EXAMPLES:

```
sage: A = ([1, 1], [3, 1])
sage: b = (1000, 1500)
sage: c = (10, 5)
sage: P = LPProblemStandardForm(A, b, c)
sage: D = P.revised_dictionary()
sage: D.enter(1)
sage: D.leave(4)
sage: D.E()
[1 1]
[0 3]
```

**E\_inverse()**

Return the inverse of the matrix `E()`.

This inverse matrix is computed in a more efficient way than generic inversion.

OUTPUT:

•a matrix

EXAMPLES:

```
sage: A = ([1, 1], [3, 1])
sage: b = (1000, 1500)
sage: c = (10, 5)
sage: P = LPProblemStandardForm(A, b, c)
sage: D = P.revised_dictionary()
sage: D.enter(1)
sage: D.leave(4)
sage: D.E_inverse()
```

```
[ 1 -1/3]
[ 0  1/3]
```

**basic\_indices()**

Return the basic indices of `self`.

---

**Note:** Basic indices are indices of `basic_variables()` in the list of generators of the `coordinate_ring()` of the `problem()` of `self`, they may not coincide with the indices of variables which are parts of their names. (They will for the default indexed names.)

---

OUTPUT:

•a list.

EXAMPLES:

```
sage: A = ([1, 1], [3, 1])
sage: b = (1000, 1500)
sage: c = (10, 5)
sage: P = LPPProblemStandardForm(A, b, c)
sage: D = P.revised_dictionary()
sage: D.basic_indices()
[3, 4]
```

**basic\_variables()**

Return the basic variables of `self`.

OUTPUT:

•a vector

EXAMPLES:

```
sage: A = ([1, 1], [3, 1])
sage: b = (1000, 1500)
sage: c = (10, 5)
sage: P = LPPProblemStandardForm(A, b, c)
sage: D = P.revised_dictionary()
sage: D.basic_variables()
(x3, x4)
```

**c\_B()**

Return the  $c_B$  vector, objective coefficients of basic variables.

OUTPUT:

•a vector

EXAMPLES:

```
sage: A = ([1, 1], [3, 1])
sage: b = (1000, 1500)
sage: c = (10, 5)
sage: P = LPPProblemStandardForm(A, b, c)
sage: D = P.revised_dictionary(1, 2)
sage: D.c_B()
(10, 5)
```

**c\_N()**

Return the  $c_N$  vector, objective coefficients of non-basic variables.

OUTPUT:

•a vector

EXAMPLES:

```
sage: A = ([1, 1], [3, 1])
sage: b = (1000, 1500)
sage: c = (10, 5)
sage: P = LPProblemStandardForm(A, b, c)
sage: D = P.revised_dictionary()
sage: D.c_N()
(10, 5)
```

**constant\_terms()**

Return constant terms in the relations of self.

OUTPUT:

•a vector

EXAMPLES:

```
sage: A = ([1, 1], [3, 1])
sage: b = (1000, 1500)
sage: c = (10, 5)
sage: P = LPProblemStandardForm(A, b, c)
sage: D = P.revised_dictionary()
sage: D.constant_terms()
(1000, 1500)
```

**dictionary()**

Return a regular LP dictionary matching self.

OUTPUT:

•an LP dictionary

EXAMPLES:

```
sage: A = ([1, 1], [3, 1], [-1, -1])
sage: b = (1000, 1500, -400)
sage: c = (10, 5)
sage: P = LPProblemStandardForm(A, b, c)
sage: D = P.revised_dictionary()
sage: D.dictionary()
LP problem dictionary (use typeset mode to see details)
```

**entering\_coefficients()**

Return coefficients of the entering variable.

OUTPUT:

•a vector

EXAMPLES:

```
sage: A = ([1, 1], [3, 1])
sage: b = (1000, 1500)
sage: c = (10, 5)
sage: P = LPProblemStandardForm(A, b, c)
sage: D = P.revised_dictionary()
sage: D.enter(1)
```

```
sage: D.entering_coefficients()
(1, 3)
```

**leaving\_coefficients()**

Return coefficients of the leaving variable.

OUTPUT:

•a vector

EXAMPLES:

```
sage: A = ([1, 1], [3, 1])
sage: b = (1000, 1500)
sage: c = (10, 5)
sage: P = LPProblemStandardForm(A, b, c)
sage: D = P.revised_dictionary(2, 3)
sage: D.leave(3)
sage: D.leaving_coefficients()
(-2, -1)
```

**nonbasic\_indices()**

Return the non-basic indices of self.

---

**Note:** Non-basic indices are indices of `nonbasic_variables()` in the list of generators of the `coordinate_ring()` of the `problem()` of self, they may not coincide with the indices of variables which are parts of their names. (They will for the default indexed names.)

---

OUTPUT:

•a list

EXAMPLES:

```
sage: A = ([1, 1], [3, 1])
sage: b = (1000, 1500)
sage: c = (10, 5)
sage: P = LPProblemStandardForm(A, b, c)
sage: D = P.revised_dictionary()
sage: D.nonbasic_indices()
[1, 2]
```

**nonbasic\_variables()**

Return non-basic variables of self.

OUTPUT:

•a vector

EXAMPLES:

```
sage: A = ([1, 1], [3, 1])
sage: b = (1000, 1500)
sage: c = (10, 5)
sage: P = LPProblemStandardForm(A, b, c)
sage: D = P.revised_dictionary()
sage: D.nonbasic_variables()
(x1, x2)
```



**objective\_coefficients()**

Return coefficients of the objective of `self`.

OUTPUT:

•a vector

These are coefficients of non-basic variables when basic variables are eliminated.

EXAMPLES:

```
sage: A = ([1, 1], [3, 1])
sage: b = (1000, 1500)
sage: c = (10, 5)
sage: P = LPProblemStandardForm(A, b, c)
sage: D = P.revised_dictionary()
sage: D.objective_coefficients()
(10, 5)
```

**objective\_value()**

Return the value of the objective at the basic solution of `self`.

OUTPUT:

•a number

EXAMPLES:

```
sage: A = ([1, 1], [3, 1])
sage: b = (1000, 1500)
sage: c = (10, 5)
sage: P = LPProblemStandardForm(A, b, c)
sage: D = P.revised_dictionary()
sage: D.objective_value()
0
```

**problem()**

Return the original problem.

OUTPUT:

•an LP problem in standard form

EXAMPLES:

```
sage: A = ([1, 1], [3, 1])
sage: b = (1000, 1500)
sage: c = (10, 5)
sage: P = LPProblemStandardForm(A, b, c)
sage: D = P.revised_dictionary()
sage: D.problem() is P
True
```

**update()**

Update `self` using previously set entering and leaving variables.

EXAMPLES:

```
sage: A = ([1, 1], [3, 1])
sage: b = (1000, 1500)
sage: c = (10, 5)
sage: P = LPProblemStandardForm(A, b, c)
sage: D = P.revised_dictionary()
sage: D.objective_value()
```

```
0
sage: D.enter("x1")
sage: D.leave("x4")
sage: D.update()
sage: D.objective_value()
5000
```

**x\_B()**

Return the basic variables of `self`.

OUTPUT:

•a vector

EXAMPLES:

```
sage: A = ([1, 1], [3, 1])
sage: b = (1000, 1500)
sage: c = (10, 5)
sage: P = LPProblemStandardForm(A, b, c)
sage: D = P.revised_dictionary()
sage: D.basic_variables()
(x3, x4)
```

**x\_N()**

Return non-basic variables of `self`.

OUTPUT:

•a vector

EXAMPLES:

```
sage: A = ([1, 1], [3, 1])
sage: b = (1000, 1500)
sage: c = (10, 5)
sage: P = LPProblemStandardForm(A, b, c)
sage: D = P.revised_dictionary()
sage: D.nonbasic_variables()
(x1, x2)
```

**y()**

Return the  $y$  vector, the product of `c_B()` and `B_inverse()`.

OUTPUT:

•a vector

EXAMPLES:

```
sage: A = ([1, 1], [3, 1])
sage: b = (1000, 1500)
sage: c = (10, 5)
sage: P = LPProblemStandardForm(A, b, c)
sage: D = P.revised_dictionary()
sage: D.y()
(0, 0)
```

`sage.numerical.interactive_simplex_method.random_dictionary`(*m*, *n*, *bound*=5, *special\_probability*=0.2)

Construct a random dictionary.

INPUT:

- `m` – the number of constraints/basic variables
- `n` – the number of decision/non-basic variables
- `bound` – (default: 5) a bound on dictionary entries
- `special_probability` – (default: 0.2) probability of constructing a potentially infeasible or potentially optimal dictionary

OUTPUT:

- an LP problem dictionary

EXAMPLES:

```
sage: from sage.numerical.interactive_simplex_method \
....:     import random_dictionary
sage: random_dictionary(3, 4)
LP problem dictionary (use typeset mode to see details)
```

```
sage.numerical.interactive_simplex_method.variable(R, v)
Interpret v as a variable of R.
```

INPUT:

- `R` – a polynomial ring
- `v` – a variable of `R` or convertible into `R`, a string with the name of a variable of `R` or an index of a variable in `R`

OUTPUT:

- a variable of `R`

EXAMPLES:

```
sage: from sage.numerical.interactive_simplex_method \
....:     import variable
sage: R = PolynomialRing(QQ, "x3, y5, x5, y")
sage: R.inject_variables()
Defining x3, y5, x5, y
sage: variable(R, "x3")
x3
sage: variable(R, x3)
x3
sage: variable(R, 3)
x3
sage: variable(R, 0)
Traceback (most recent call last):
...
ValueError: there is no variable with the given index
sage: variable(R, 5)
Traceback (most recent call last):
...
ValueError: the given index is ambiguous
sage: variable(R, 2 * x3)
Traceback (most recent call last):
...
ValueError: cannot interpret given data as a variable
sage: variable(R, "z")
Traceback (most recent call last):
...
ValueError: cannot interpret given data as a variable
```



# LP SOLVER BACKENDS

## 6.1 Generic Backend for LP solvers

### Generic Backend for LP solvers

This class only lists the methods that should be defined by any interface with a LP Solver. All these methods immediately raise `NotImplementedError` exceptions when called, and are obviously meant to be replaced by the solver-specific method. This file can also be used as a template to create a new interface : one would only need to replace the occurrences of `"Nonexistent_LP_solver"` by the solver's name, and replace `GenericBackend` by `SolverName(GenericBackend)` so that the new solver extends this class.

#### AUTHORS:

- Nathann Cohen (2010-10) : initial implementation
- Risan (2012-02) : extension for PPL backend
- Ingolfur Edvardsson (2014-06): extension for CVXOPT backend

**class** `sage.numerical.backends.generic_backend.GenericBackend`

Bases: `object`

`x.__init__(...)` initializes x; see `help(type(x))` for signature

**add\_col** (*indices, coeffs*)

Add a column.

INPUT:

- *indices* (list of integers) – this list contains the indices of the constraints in which the variable's coefficient is nonzero
- *coeffs* (list of real values) – associates a coefficient to the variable in each of the constraints in which it appears. Namely, the *i*th entry of *coeffs* corresponds to the coefficient of the variable in the constraint represented by the *i*th entry in *indices*.

---

**Note:** *indices* and *coeffs* are expected to be of the same length.

---

#### EXAMPLE:

```
sage: from sage.numerical.backends.generic_backend import get_solver
sage: p = get_solver(solver = "Nonexistent_LP_solver") # optional - Nonexistent_LP_solver
sage: p.ncols()                                     # optional - Nonexistent_LP_solver
0
sage: p.nrows()                                     # optional - Nonexistent_LP_solver
0
```

```

sage: p.add_linear_constraints(5, 0, None)           # optional - Nonexistent_LP_solver
sage: p.add_col(range(5), range(5))                # optional - Nonexistent_LP_solver
sage: p.nrows()                                     # optional - Nonexistent_LP_solver
5

```

**add\_linear\_constraint** (*coefficients, lower\_bound, upper\_bound, name=None*)

Add a linear constraint.

INPUT:

- *coefficients* an iterable with  $(c, v)$  pairs where  $c$  is a variable index (integer) and  $v$  is a value (real value).
- *lower\_bound* - a lower bound, either a real value or None
- *upper\_bound* - an upper bound, either a real value or None
- *name* - an optional name for this row (default: None)

EXAMPLE:

```

sage: from sage.numerical.backends.generic_backend import get_solver
sage: p = get_solver(solver = "Nonexistent_LP_solver") # optional - Nonexistent_LP_solver
sage: p.add_variables(5)                               # optional - Nonexistent_LP_solver
4
sage: p.add_linear_constraint(zip(range(5), range(5)), 2.0, 2.0) # optional - Nonexistent_LP_solver
sage: p.row(0)                                         # optional - Nonexistent_LP_solver
([4, 3, 2, 1], [4.0, 3.0, 2.0, 1.0])                  # optional - Nonexistent_LP_solver
sage: p.row_bounds(0)                                 # optional - Nonexistent_LP_solver
(2.0, 2.0)
sage: p.add_linear_constraint(zip(range(5), range(5)), 1.0, 1.0, name='foo') # optional - M
sage: p.row_name(-1)                                   # optional - M
"foo"

```

**add\_linear\_constraints** (*number, lower\_bound, upper\_bound, names=None*)

Add constraints.

INPUT:

- *number* (integer) – the number of constraints to add.
- *lower\_bound* - a lower bound, either a real value or None
- *upper\_bound* - an upper bound, either a real value or None
- *names* - an optional list of names (default: None)

EXAMPLE:

```

sage: from sage.numerical.backends.generic_backend import get_solver
sage: p = get_solver(solver = "Nonexistent_LP_solver") # optional - Nonexistent_LP_solver
sage: p.add_variables(5)                               # optional - Nonexistent_LP_solver
5
sage: p.add_linear_constraints(5, None, 2)              # optional - Nonexistent_LP_solver
sage: p.row(4)                                         # optional - Nonexistent_LP_solver
([], [])
sage: p.row_bounds(4)                                 # optional - Nonexistent_LP_solver
(None, 2.0)

```

**add\_variable** (*lower\_bound=None, upper\_bound=None, binary=False, continuous=True, integer=False, obj=None, name=None*)

Add a variable.

This amounts to adding a new column to the matrix. By default, the variable is both positive and real.

INPUT:

- `lower_bound` - the lower bound of the variable (default: 0)
- `upper_bound` - the upper bound of the variable (default: None)
- `binary` - True if the variable is binary (default: False).
- `continuous` - True if the variable is binary (default: True).
- `integer` - True if the variable is binary (default: False).
- `obj` - (optional) coefficient of this variable in the objective function (default: 0.0)
- `name` - an optional name for the newly added variable (default: None).

OUTPUT: The index of the newly created variable

EXAMPLE:

```
sage: from sage.numerical.backends.generic_backend import get_solver
sage: p = get_solver(solver = "Nonexistent_LP_solver") # optional - Nonexistent_LP_solver
sage: p.ncols() # optional - Nonexistent_LP_solver
0
sage: p.add_variable() # optional - Nonexistent_LP_solver
0
sage: p.ncols() # optional - Nonexistent_LP_solver
1
sage: p.add_variable(binary=True) # optional - Nonexistent_LP_solver
1
sage: p.add_variable(lower_bound=-2.0, integer=True) # optional - Nonexistent_LP_solver
2
sage: p.add_variable(continuous=True, integer=True) # optional - Nonexistent_LP_solver
Traceback (most recent call last):
...
ValueError: ...
sage: p.add_variable(name='x', obj=1.0) # optional - Nonexistent_LP_solver
3
sage: p.col_name(3) # optional - Nonexistent_LP_solver
'x'
sage: p.objective_coefficient(3) # optional - Nonexistent_LP_solver
1.0
```

**add\_variables** (*n*, *lower\_bound*=None, *upper\_bound*=None, *binary*=False, *continuous*=True, *integer*=False, *obj*=None, *names*=None)

Add *n* variables.

This amounts to adding new columns to the matrix. By default, the variables are both positive and real.

INPUT:

- *n* - the number of new variables (must be > 0)
- `lower_bound` - the lower bound of the variable (default: 0)
- `upper_bound` - the upper bound of the variable (default: None)
- `binary` - True if the variable is binary (default: False).
- `continuous` - True if the variable is binary (default: True).
- `integer` - True if the variable is binary (default: False).
- `obj` - (optional) coefficient of all variables in the objective function (default: 0.0)

- names - optional list of names (default: None)

OUTPUT: The index of the variable created last.

EXAMPLE:

```
sage: from sage.numerical.backends.generic_backend import get_solver
sage: p = get_solver(solver = "Nonexistent_LP_solver") # optional - Nonexistent_LP_solver
sage: p.ncols() # optional - Nonexistent_LP_solver
0
sage: p.add_variables(5) # optional - Nonexistent_LP_solver
4
sage: p.ncols() # optional - Nonexistent_LP_solver
5
sage: p.add_variables(2, lower_bound=-2.0, integer=True, names=['a','b']) # optional - Nonexistent_LP_solver
6
```

**base\_ring()**

**col\_bounds** (index)

Return the bounds of a specific variable.

INPUT:

- index (integer) – the variable’s id.

OUTPUT:

A pair (lower\_bound, upper\_bound). Each of them can be set to None if the variable is not bounded in the corresponding direction, and is a real value otherwise.

EXAMPLE:

```
sage: from sage.numerical.backends.generic_backend import get_solver
sage: p = get_solver(solver = "Nonexistent_LP_solver") # optional - Nonexistent_LP_solver
sage: p.add_variable() # optional - Nonexistent_LP_solver
1
sage: p.col_bounds(0) # optional - Nonexistent_LP_solver
(0.0, None)
sage: p.variable_upper_bound(0, 5) # optional - Nonexistent_LP_solver
sage: p.col_bounds(0) # optional - Nonexistent_LP_solver
(0.0, 5.0)
```

**col\_name** (index)

Return the index th col name

INPUT:

- index (integer) – the col’s id
- name (char \*) – its name. When set to NULL (default), the method returns the current name.

EXAMPLE:

```
sage: from sage.numerical.backends.generic_backend import get_solver
sage: p = get_solver(solver = "Nonexistent_LP_solver") # optional - Nonexistent_LP_solver
sage: p.add_variable(name="I am a variable") # optional - Nonexistent_LP_solver
1
sage: p.col_name(0) # optional - Nonexistent_LP_solver
'I am a variable'
```

**get\_objective\_value()**

Return the value of the objective function.



**Note:** Behaviour is undefined unless `solve` has been called before.

EXAMPLE:

```
sage: from sage.numerical.backends.generic_backend import get_solver
sage: p = get_solver(solver = "Nonexistent_LP_solver") # optional - Nonexistent_LP_solver
sage: p.add_variables(2)                               # optional - Nonexistent_LP_solver
2
sage: p.add_linear_constraint([(0,1), (1,2)], None, 3) # optional - Nonexistent_LP_solver
sage: p.set_objective([2, 5])                          # optional - Nonexistent_LP_solver
sage: p.solve()                                         # optional - Nonexistent_LP_solver
0
sage: p.get_objective_value()                          # optional - Nonexistent_LP_solver
7.5
sage: p.get_variable_value(0)                         # optional - Nonexistent_LP_solver
0.0
sage: p.get_variable_value(1)                         # optional - Nonexistent_LP_solver
1.5
```

**get\_variable\_value** (*variable*)

Return the value of a variable given by the solver.

**Note:** Behaviour is undefined unless `solve` has been called before.

EXAMPLE:

```
sage: from sage.numerical.backends.generic_backend import get_solver
sage: p = get_solver(solver = "Nonexistent_LP_solver") # optional - Nonexistent_LP_solver
sage: p.add_variables(2)                               # optional - Nonexistent_LP_solver
2
sage: p.add_linear_constraint([(0,1), (1, 2)], None, 3) # optional - Nonexistent_LP_solver
sage: p.set_objective([2, 5])                          # optional - Nonexistent_LP_solver
sage: p.solve()                                         # optional - Nonexistent_LP_solver
0
sage: p.get_objective_value()                          # optional - Nonexistent_LP_solver
7.5
sage: p.get_variable_value(0)                         # optional - Nonexistent_LP_solver
0.0
sage: p.get_variable_value(1)                         # optional - Nonexistent_LP_solver
1.5
```

**is\_maximization** ()

Test whether the problem is a maximization

EXAMPLE:

```
sage: from sage.numerical.backends.generic_backend import get_solver
sage: p = get_solver(solver = "Nonexistent_LP_solver") # optional - Nonexistent_LP_solver
sage: p.is_maximization()                             # optional - Nonexistent_LP_solver
True
sage: p.set_sense(-1)                                  # optional - Nonexistent_LP_solver
sage: p.is_maximization()                             # optional - Nonexistent_LP_solver
False
```

**is\_variable\_binary** (*index*)

Test whether the given variable is of binary type.

INPUT:

- index (integer) – the variable’s id

EXAMPLE:

```
sage: from sage.numerical.backends.generic_backend import get_solver
sage: p = get_solver(solver = "Nonexistent_LP_solver") # optional - Nonexistent_LP_solver
sage: p.ncols() # optional - Nonexistent_LP_solver
0
sage: p.add_variable() # optional - Nonexistent_LP_solver
1
sage: p.set_variable_type(0,0) # optional - Nonexistent_LP_solver
sage: p.is_variable_binary(0) # optional - Nonexistent_LP_solver
True
```

**is\_variable\_continuous** (index)

Test whether the given variable is of continuous/real type.

INPUT:

- index (integer) – the variable’s id

EXAMPLE:

```
sage: from sage.numerical.backends.generic_backend import get_solver
sage: p = get_solver(solver = "Nonexistent_LP_solver") # optional - Nonexistent_LP_solver
sage: p.ncols() # optional - Nonexistent_LP_solver
0
sage: p.add_variable() # optional - Nonexistent_LP_solver
1
sage: p.is_variable_continuous(0) # optional - Nonexistent_LP_solver
True
sage: p.set_variable_type(0,1) # optional - Nonexistent_LP_solver
sage: p.is_variable_continuous(0) # optional - Nonexistent_LP_solver
False
```

**is\_variable\_integer** (index)

Test whether the given variable is of integer type.

INPUT:

- index (integer) – the variable’s id

EXAMPLE:

```
sage: from sage.numerical.backends.generic_backend import get_solver
sage: p = get_solver(solver = "Nonexistent_LP_solver") # optional - Nonexistent_LP_solver
sage: p.ncols() # optional - Nonexistent_LP_solver
0
sage: p.add_variable() # optional - Nonexistent_LP_solver
1
sage: p.set_variable_type(0,1) # optional - Nonexistent_LP_solver
sage: p.is_variable_integer(0) # optional - Nonexistent_LP_solver
True
```

**ncols** ()

Return the number of columns/variables.

EXAMPLE:

```
sage: from sage.numerical.backends.generic_backend import get_solver
sage: p = get_solver(solver = "Nonexistent_LP_solver") # optional - Nonexistent_LP_solver
sage: p.ncols() # optional - Nonexistent_LP_solver
0
```

```

sage: p.add_variables(2)                                # optional - Nonexistent_LP_solver
2
sage: p.ncols()                                         # optional - Nonexistent_LP_solver
2

```

**nrows()**

Return the number of rows/constraints.

EXAMPLE:

```

sage: from sage.numerical.backends.generic_backend import get_solver
sage: p = get_solver(solver = "Nonexistent_LP_solver") # optional - Nonexistent_LP_solver
sage: p.nrows()                                       # optional - Nonexistent_LP_solver
0
sage: p.add_linear_constraints(2, 2.0, None)          # optional - Nonexistent_LP_solver
sage: p.nrows()                                       # optional - Nonexistent_LP_solver
2

```

**objective\_coefficient** (*variable*, *coeff=None*)

Set or get the coefficient of a variable in the objective function

INPUT:

- *variable* (integer) – the variable’s id
- *coeff* (double) – its coefficient

EXAMPLE:

```

sage: from sage.numerical.backends.generic_backend import get_solver
sage: p = get_solver(solver = "Nonexistent_LP_solver") # optional - Nonexistent_LP_solver
sage: p.add_variable()                                # optional - Nonexistent_LP_solver
1
sage: p.objective_coefficient(0)                      # optional - Nonexistent_LP_solver
0.0
sage: p.objective_coefficient(0,2)                   # optional - Nonexistent_LP_solver
sage: p.objective_coefficient(0)                     # optional - Nonexistent_LP_solver
2.0

```

**problem\_name** (*name='NULL'*)

Return or define the problem’s name

INPUT:

- *name* (char \*) – the problem’s name. When set to NULL (default), the method returns the problem’s name.

EXAMPLE:

```

sage: from sage.numerical.backends.generic_backend import get_solver
sage: p = get_solver(solver = "Nonexistent_LP_solver") # optional - Nonexistent_LP_solver
sage: p.problem_name("There once was a french fry") # optional - Nonexistent_LP_solver
sage: print p.get_problem_name()                   # optional - Nonexistent_LP_solver
There once was a french fry

```

**remove\_constraint** (*i*)

Remove a constraint.

INPUT:

- *i* -- index of the constraint to remove.

EXAMPLE:

```
sage: from sage.numerical.backends.generic_backend import get_solver
sage: p = get_solver(solver = "Nonexistent_LP_solver") # optional - Nonexistent_LP_solver
sage: p.add_constraint(p[0] + p[1], max = 10)         # optional - Nonexistent_LP_solver
sage: p.remove_constraint(0)                         # optional - Nonexistent_LP_solver
```

**remove\_constraints** (*constraints*)

Remove several constraints.

INPUT:

- *constraints* – an iterable containing the indices of the rows to remove.

EXAMPLE:

```
sage: from sage.numerical.backends.generic_backend import get_solver
sage: p = get_solver(solver = "Nonexistent_LP_solver") # optional - Nonexistent_LP_solver
sage: p.add_constraint(p[0] + p[1], max = 10)         # optional - Nonexistent_LP_solver
sage: p.remove_constraints([0])                      # optional - Nonexistent_LP_solver
```

**row** (*i*)

Return a row

INPUT:

- *index* (integer) – the constraint's id.

OUTPUT:

A pair (*indices*, *coeffs*) where *indices* lists the entries whose coefficient is nonzero, and to which *coeffs* associates their coefficient on the model of the `add_linear_constraint` method.

EXAMPLE:

```
sage: from sage.numerical.backends.generic_backend import get_solver
sage: p = get_solver(solver = "Nonexistent_LP_solver") # optional - Nonexistent_LP_solver
sage: p.add_variables(5)                               # optional - Nonexistent_LP_solver
5
sage: p.add_linear_constraint(zip(range(5), range(5)), 2, 2) # optional - Nonexistent_LP_solver
sage: p.row(0)                                         # optional - Nonexistent_LP_solver
([4, 3, 2, 1], [4.0, 3.0, 2.0, 1.0])
sage: p.row_bounds(0)                                # optional - Nonexistent_LP_solver
(2.0, 2.0)
```

**row\_bounds** (*index*)

Return the bounds of a specific constraint.

INPUT:

- *index* (integer) – the constraint's id.

OUTPUT:

A pair (*lower\_bound*, *upper\_bound*). Each of them can be set to `None` if the constraint is not bounded in the corresponding direction, and is a real value otherwise.

EXAMPLE:

```
sage: from sage.numerical.backends.generic_backend import get_solver
sage: p = get_solver(solver = "Nonexistent_LP_solver") # optional - Nonexistent_LP_solver
sage: p.add_variables(5)                               # optional - Nonexistent_LP_solver
5
sage: p.add_linear_constraint(range(5), range(5), 2, 2) # optional - Nonexistent_LP_solver
```

```

sage: p.row(0) # optional - Nonexistent_LP_solver
([4, 3, 2, 1], [4.0, 3.0, 2.0, 1.0])
sage: p.row_bounds(0) # optional - Nonexistent_LP_solver
(2.0, 2.0)

```

**row\_name** (*index*)

Return the *index* th row name

INPUT:

- *index* (integer) – the row's id

EXAMPLE:

```

sage: from sage.numerical.backends.generic_backend import get_solver
sage: p = get_solver(solver = "Nonexistent_LP_solver") # optional - Nonexistent_LP_solver
sage: p.add_linear_constraints(1, 2, None, name="Empty constraint 1") # optional - Nonexistent_LP_solver
sage: p.row_name(0) # optional - Nonexistent_LP_solver
'Empty constraint 1'

```

**set\_objective** (*coeff*, *d=0.0*)

Set the objective function.

INPUT:

- *coeff* – a list of real values, whose *ith* element is the coefficient of the *ith* variable in the objective function.
- *d* (double) – the constant term in the linear function (set to 0 by default)

EXAMPLE:

```

sage: from sage.numerical.backends.generic_backend import get_solver
sage: p = get_solver(solver = "Nonexistent_LP_solver") # optional - Nonexistent_LP_solver
sage: p.add_variables(5) # optional - Nonexistent_LP_solver
5
sage: p.set_objective([1, 1, 2, 1, 3]) # optional - Nonexistent_LP_solver
sage: map(lambda x : p.objective_coefficient(x), range(5)) # optional - Nonexistent_LP_solver
[1.0, 1.0, 2.0, 1.0, 3.0]

```

Constants in the objective function are respected:

```

sage: p = MixedIntegerLinearProgram(solver='Nonexistent_LP_solver') # optional - Nonexistent_LP_solver
sage: x,y = p[0], p[1] # optional - Nonexistent_LP_solver
sage: p.add_constraint(2*x + 3*y, max = 6) # optional - Nonexistent_LP_solver
sage: p.add_constraint(3*x + 2*y, max = 6) # optional - Nonexistent_LP_solver
sage: p.set_objective(x + y + 7) # optional - Nonexistent_LP_solver
sage: p.set_integer(x); p.set_integer(y) # optional - Nonexistent_LP_solver
sage: p.solve() # optional - Nonexistent_LP_solver
9.0

```

**set\_sense** (*sense*)

Set the direction (maximization/minimization).

INPUT:

- *sense* (integer) :
  - +1 => Maximization
  - -1 => Minimization

EXAMPLE:

```
sage: from sage.numerical.backends.generic_backend import get_solver
sage: p = get_solver(solver = "Nonexistent_LP_solver") # optional - Nonexistent_LP_solver
sage: p.is_maximization() # optional - Nonexistent_LP_solver
True
sage: p.set_sense(-1) # optional - Nonexistent_LP_solver
sage: p.is_maximization() # optional - Nonexistent_LP_solver
False
```

**set\_variable\_type** (*variable*, *vtype*)

Set the type of a variable

INPUT:

- *variable* (integer) – the variable's id
- *vtype* (integer) :
  - 1 Integer
  - 0 Binary
  - -1 Continuous

EXAMPLE:

```
sage: from sage.numerical.backends.generic_backend import get_solver
sage: p = get_solver(solver = "Nonexistent_LP_solver") # optional - Nonexistent_LP_solver
sage: p.ncols() # optional - Nonexistent_LP_solver
0
sage: p.add_variable() # optional - Nonexistent_LP_solver
1
sage: p.set_variable_type(0,1) # optional - Nonexistent_LP_solver
sage: p.is_variable_integer(0) # optional - Nonexistent_LP_solver
True
```

**set\_verbosity** (*level*)

Set the log (verbosity) level

INPUT:

- *level* (integer) – From 0 (no verbosity) to 3.

EXAMPLE:

```
sage: from sage.numerical.backends.generic_backend import get_solver
sage: p = get_solver(solver = "Nonexistent_LP_solver") # optional - Nonexistent_LP_solver
sage: p.set_verbosity(2) # optional - Nonexistent_LP_solver
```

**solve** ()

Solve the problem.

---

**Note:** This method raises `MIPSolverException` exceptions when the solution can not be computed for any reason (none exists, or the LP solver was not able to find it, etc...)

---

EXAMPLE:

```
sage: from sage.numerical.backends.generic_backend import get_solver
sage: p = get_solver(solver = "Nonexistent_LP_solver") # optional - Nonexistent_LP_solver
sage: p.add_linear_constraints(5, 0, None) # optional - Nonexistent_LP_solver
sage: p.add_col(range(5), range(5)) # optional - Nonexistent_LP_solver
sage: p.solve() # optional - Nonexistent_LP_solver
```

```

0
sage: p.objective_coefficient(0,1)                # optional - Nonexistent_LP_solver
sage: p.solve()                                  # optional - Nonexistent_LP_solver
Traceback (most recent call last):
...
MIPSolverException: ...

```

**solver\_parameter** (*name*, *value=None*)

Return or define a solver parameter

INPUT:

- *name* (string) – the parameter
- *value* – the parameter’s value if it is to be defined, or None (default) to obtain its current value.

---

**Note:** The list of available parameters is available at `solver_parameter()`.

---

EXAMPLE:

```

sage: from sage.numerical.backends.generic_backend import get_solver
sage: p = get_solver(solver = "Nonexistent_LP_solver") # optional - Nonexistent_LP_solver
sage: p.solver_parameter("timelimit")                 # optional - Nonexistent_LP_solver
sage: p.solver_parameter("timelimit", 60)             # optional - Nonexistent_LP_solver
sage: p.solver_parameter("timelimit")                 # optional - Nonexistent_LP_solver

```

**variable\_lower\_bound** (*index*, *value=None*)

Return or define the lower bound on a variable

INPUT:

- *index* (integer) – the variable’s id
- *value* – real value, or None to mean that the variable has not lower bound. When set to None (default), the method returns the current value.

EXAMPLE:

```

sage: from sage.numerical.backends.generic_backend import get_solver
sage: p = get_solver(solver = "Nonexistent_LP_solver") # optional - Nonexistent_LP_solver
sage: p.add_variable()                                # optional - Nonexistent_LP_solver
1
sage: p.col_bounds(0)                                 # optional - Nonexistent_LP_solver
(0.0, None)
sage: p.variable_lower_bound(0, 5)                   # optional - Nonexistent_LP_solver
sage: p.col_bounds(0)                                 # optional - Nonexistent_LP_solver
(5.0, None)

```

**variable\_upper\_bound** (*index*, *value=None*)

Return or define the upper bound on a variable

INPUT:

- *index* (integer) – the variable’s id
- *value* – real value, or None to mean that the variable has not upper bound. When set to None (default), the method returns the current value.

EXAMPLE:

```
sage: from sage.numerical.backends.generic_backend import get_solver
sage: p = get_solver(solver = "Nonexistent_LP_solver") # optional - Nonexistent_LP_solver
sage: p.add_variable()                               # optional - Nonexistent_LP_solver
1
sage: p.col_bounds(0)                                # optional - Nonexistent_LP_solver
(0.0, None)
sage: p.variable_upper_bound(0, 5)                   # optional - Nonexistent_LP_solver
sage: p.col_bounds(0)                                # optional - Nonexistent_LP_solver
(0.0, 5.0)
```

**write\_lp** (*name*)

Write the problem to a .lp file

INPUT:

- filename (string)

EXAMPLE:

```
sage: from sage.numerical.backends.generic_backend import get_solver
sage: p = get_solver(solver = "Nonexistent_LP_solver") # optional - Nonexistent_LP_solver
sage: p.add_variables(2)                               # optional - Nonexistent_LP_solver
2
sage: p.add_linear_constraint([(0, 1], (1, 2)], None, 3) # optional - Nonexistent_LP_solver
sage: p.set_objective([2, 5])                          # optional - Nonexistent_LP_solver
sage: p.write_lp(os.path.join(SAGE_TMP, "lp_problem.lp")) # optional - Nonexistent_LP_solver
```

**write\_mps** (*name*, *modern*)

Write the problem to a .mps file

INPUT:

- filename (string)

EXAMPLE:

```
sage: from sage.numerical.backends.generic_backend import get_solver
sage: p = get_solver(solver = "Nonexistent_LP_solver") # optional - Nonexistent_LP_solver
sage: p.add_variables(2)                               # optional - Nonexistent_LP_solver
2
sage: p.add_linear_constraint([(0, 1), (1, 2)], None, 3) # optional - Nonexistent_LP_solver
sage: p.set_objective([2, 5])                          # optional - Nonexistent_LP_solver
sage: p.write_lp(os.path.join(SAGE_TMP, "lp_problem.lp")) # optional - Nonexistent_LP_solver
```

**zero** ()

sage.numerical.backends.generic\_backend.**default\_mip\_solver** (*solver=None*)

Returns/Sets the default MILP Solver used by Sage

INPUT:

- solver – defines the solver to use:
  - GLPK (solver="GLPK"). See the [GLPK](#) web site.
  - COIN Branch and Cut (solver="Coin"). See the [COIN-OR](#) web site.
  - CPLEX (solver="CPLEX"). See the [CPLEX](#) web site.
  - CVXOPT (solver="CVXOPT"). See the [CVXOPT](#) web site.
  - PPL (solver="PPL"). See the [PPL](#) web site.



–Gurobi (`solver="Gurobi"`). See the [Gurobi](#) web site.

`solver` should then be equal to one of "GLPK", "Coin", "CPLEX", "CVXOPT", "Gurobi" or "PPL".

–If `solver=None` (default), the current default solver's name is returned.

OUTPUT:

This function returns the current default solver's name if `solver = None` (default). Otherwise, it sets the default solver to the one given. If this solver does not exist, or is not available, a `ValueError` exception is raised.

EXAMPLE:

```
sage: former_solver = default_mip_solver()
sage: default_mip_solver("GLPK")
sage: default_mip_solver()
'Glpk'
sage: default_mip_solver("PPL")
sage: default_mip_solver()
'Ppl'
sage: default_mip_solver("GUROBI")
Traceback (most recent call last):
...
ValueError: Gurobi is not available. Please refer to the documentation to install it.
sage: default_mip_solver("Yeahhhhhhhhhhh")
Traceback (most recent call last):
...
ValueError: 'solver' should be set to 'GLPK', 'Coin', 'CPLEX', 'Gurobi', 'CVXOPT', 'PPL' or None
sage: default_mip_solver(former_solver)
```

`sage.numerical.backends.generic_backend.get_solver` (*constraint\_generation=False*,  
*solver=None*)

Return a solver according to the given preferences

INPUT:

- `solver` – 6 solvers should be available through this class:

- GLPK (`solver="GLPK"`). See the [GLPK](#) web site.

- COIN Branch and Cut (`solver="Coin"`). See the [COIN-OR](#) web site.

- CPLEX (`solver="CPLEX"`). See the [CPLEX](#) web site.

- CVXOPT (`solver="CVXOPT"`). See the [CVXOPT](#) web site.

- Gurobi (`solver="Gurobi"`). See the [Gurobi](#) web site.

- PPL (`solver="PPL"`). See the [PPL](#) web site.

`solver` should then be equal to one of "GLPK", "Coin", "CPLEX", "CVXOPT", "Gurobi", "PPL", or None. If `solver=None` (default), the default solver is used (see `default_mip_solver` method).

- `constraint_generation` – Only used when `solver=None`.

- When set to True, after solving the `MixedIntegerLinearProgram`, it is possible to add a constraint, and then solve it again. The effect is that solvers that do not support this feature will not be used.

- Defaults to False.

See Also:

- `default_mip_solver()` – Returns/Sets the default MIP solver.

EXAMPLE:

```
sage: from sage.numerical.backends.generic_backend import get_solver
sage: p = get_solver()
```

## 6.2 GLPK Backend

GLPK Backend

AUTHORS:

- Nathann Cohen (2010-10): initial implementation
- John Perry (2012-01): `glp_simplex` preprocessing
- John Perry and Raniere Gaia Silva (2012-03): solver parameters
- Christian Kuper (2012-10): Additions for sensitivity analysis

**class** `sage.numerical.backends.glpk_backend.GLPKBackend`  
Bases: `sage.numerical.backends.generic_backend.GenericBackend`

`x.__init__(...)` initializes `x`; see `help(type(x))` for signature

**add\_col** (*indices, coeffs*)  
Add a column.

INPUT:

- *indices* (list of integers) – this list contains the indices of the constraints in which the variable's coefficient is nonzero
- *coeffs* (list of real values) – associates a coefficient to the variable in each of the constraints in which it appears. Namely, the *i*th entry of *coeffs* corresponds to the coefficient of the variable in the constraint represented by the *i*th entry in *indices*.

---

**Note:** *indices* and *coeffs* are expected to be of the same length.

---

EXAMPLE:

```
sage: from sage.numerical.backends.generic_backend import get_solver
sage: p = get_solver(solver = "GLPK")
sage: p.ncols()
0
sage: p.nrows()
0
sage: p.add_linear_constraints(5, 0, None)
sage: p.add_col(range(5), range(5))
sage: p.nrows()
5
```

**add\_linear\_constraint** (*coefficients, lower\_bound, upper\_bound, name=None*)  
Add a linear constraint.

INPUT:

- *coefficients* an iterable with  $(c, v)$  pairs where  $c$  is a variable index (integer) and  $v$  is a value (real value).

- `lower_bound` - a lower bound, either a real value or `None`
- `upper_bound` - an upper bound, either a real value or `None`
- `name` - an optional name for this row (default: `None`)

EXAMPLE:

```
sage: from sage.numerical.backends.generic_backend import get_solver
sage: p = get_solver(solver = "GLPK")
sage: p.add_variables(5)
4
sage: p.add_linear_constraint( zip(range(5), range(5)), 2.0, 2.0)
sage: p.row(0)
([4, 3, 2, 1], [4.0, 3.0, 2.0, 1.0])
sage: p.row_bounds(0)
(2.0, 2.0)
sage: p.add_linear_constraint( zip(range(5), range(5)), 1.0, 1.0, name='foo')
sage: p.row_name(1)
'foo'
```

**`add_linear_constraints`** (*number*, *lower\_bound*, *upper\_bound*, *names=None*)

Add '*number*' linear constraints.

INPUT:

- *number* (integer) – the number of constraints to add.
- `lower_bound` - a lower bound, either a real value or `None`
- `upper_bound` - an upper bound, either a real value or `None`
- `names` - an optional list of names (default: `None`)

EXAMPLE:

```
sage: from sage.numerical.backends.generic_backend import get_solver
sage: p = get_solver(solver = "GLPK")
sage: p.add_variables(5)
4
sage: p.add_linear_constraints(5, None, 2)
sage: p.row(4)
([], [])
sage: p.row_bounds(4)
(None, 2.0)
sage: p.add_linear_constraints(2, None, 2, names=['foo', 'bar'])
```

**`add_variable`** (*lower\_bound=0.0*, *upper\_bound=None*, *binary=False*, *continuous=False*, *integer=False*, *obj=0.0*, *name=None*)

Add a variable.

This amounts to adding a new column to the matrix. By default, the variable is both positive, real and the coefficient in the objective function is 0.0.

INPUT:

- `lower_bound` - the lower bound of the variable (default: 0)
- `upper_bound` - the upper bound of the variable (default: `None`)
- `binary` - True if the variable is binary (default: `False`).
- `continuous` - True if the variable is binary (default: `True`).
- `integer` - True if the variable is binary (default: `False`).

- obj - (optional) coefficient of this variable in the objective function (default: 0.0)
- name - an optional name for the newly added variable (default: None).

OUTPUT: The index of the newly created variable

EXAMPLE:

```
sage: from sage.numerical.backends.generic_backend import get_solver
sage: p = get_solver(solver = "GLPK")
sage: p.ncols()
0
sage: p.add_variable()
0
sage: p.ncols()
1
sage: p.add_variable(binary=True)
1
sage: p.add_variable(lower_bound=-2.0, integer=True)
2
sage: p.add_variable(continuous=True, integer=True)
Traceback (most recent call last):
...
ValueError: ...
sage: p.add_variable(name='x', obj=1.0)
3
sage: p.col_name(3)
'x'
sage: p.objective_coefficient(3)
1.0
```

**add\_variables** (*number*, *lower\_bound*=0.0, *upper\_bound*=None, *binary*=False, *continuous*=False, *integer*=False, *obj*=0.0, *names*=None)

Add number new variables.

This amounts to adding new columns to the matrix. By default, the variables are both positive, real and their coefficient in the objective function is 0.0.

INPUT:

- n - the number of new variables (must be > 0)
- lower\_bound - the lower bound of the variable (default: 0)
- upper\_bound - the upper bound of the variable (default: None)
- binary - True if the variable is binary (default: False).
- continuous - True if the variable is binary (default: True).
- integer - True if the variable is binary (default: False).
- obj - (optional) coefficient of all variables in the objective function (default: 0.0)
- names - optional list of names (default: None)

OUTPUT: The index of the variable created last.

EXAMPLE:

```
sage: from sage.numerical.backends.generic_backend import get_solver
sage: p = get_solver(solver = "GLPK")
sage: p.ncols()
0
sage: p.add_variables(5)
```

```

4
sage: p.ncols()
5
sage: p.add_variables(2, lower_bound=-2.0, integer=True, names=['a', 'b'])
6

```

**col\_bounds** (*index*)

Return the bounds of a specific variable.

INPUT:

- *index* (integer) – the variable's id.

OUTPUT:

A pair (*lower\_bound*, *upper\_bound*). Each of them can be set to *None* if the variable is not bounded in the corresponding direction, and is a real value otherwise.

EXAMPLE:

```

sage: from sage.numerical.backends.generic_backend import get_solver
sage: p = get_solver(solver = "GLPK")
sage: p.add_variable()
0
sage: p.col_bounds(0)
(0.0, None)
sage: p.variable_upper_bound(0, 5)
sage: p.col_bounds(0)
(0.0, 5.0)

```

**col\_name** (*index*)

Return the *index* th col name

INPUT:

- *index* (integer) – the col's id

EXAMPLE:

```

sage: from sage.numerical.backends.generic_backend import get_solver
sage: p = get_solver(solver = "GLPK")
sage: p.add_variable(name='I am a variable')
0
sage: p.col_name(0)
'I am a variable'

```

**copy** ()

Returns a copy of self.

EXAMPLE:

```

sage: from sage.numerical.backends.generic_backend import get_solver
sage: p = MixedIntegerLinearProgram(solver = "GLPK")
sage: b = p.new_variable()
sage: p.add_constraint(b[1] + b[2] <= 6)
sage: p.set_objective(b[1] + b[2])
sage: copy(p).solve()
6.0

```

**get\_col\_dual** (*variable*)

Returns the dual value (reduced cost) of a variable

The dual value is the reduced cost of a variable. The reduced cost is the amount by which the objective coefficient of a non basic variable has to change to become a basic variable.

INPUT:

- variable – The number of the variable

---

**Note:** Behaviour is undefined unless `solve` has been called before. If the simplex algorithm has not been used for solving just a 0.0 will be returned.

---

EXAMPLE:

```
sage: from sage.numerical.backends.generic_backend import get_solver
sage: p = get_solver(solver = "GLPK")
sage: p.add_variables(3)
2
sage: p.add_linear_constraint(zip([0, 1, 2], [8, 6, 1]), None, 48)
sage: p.add_linear_constraint(zip([0, 1, 2], [4, 2, 1.5]), None, 20)
sage: p.add_linear_constraint(zip([0, 1, 2], [2, 1.5, 0.5]), None, 8)
sage: p.set_objective([60, 30, 20])
sage: import sage.numerical.backends.glpk_backend as backend
sage: p.solver_parameter(backend.glp_simplex_or_intopt, backend.glp_simplex_only)
sage: p.solve()
0
sage: p.get_col_dual(1)
-5.0
```

**get\_objective\_value()**

Returns the value of the objective function.

---

**Note:** Behaviour is undefined unless `solve` has been called before.

---

EXAMPLE:

```
sage: from sage.numerical.backends.generic_backend import get_solver
sage: p = get_solver(solver = "GLPK")
sage: p.add_variables(2)
1
sage: p.add_linear_constraint([0, 1], [1, 2], None, 3)
sage: p.set_objective([2, 5])
sage: p.solve()
0
sage: p.get_objective_value()
7.5
sage: p.get_variable_value(0)
0.0
sage: p.get_variable_value(1)
1.5
```

**get\_row\_dual(variable)**

Returns the dual value of a constraint.

The dual value of the *ith* row is also the value of the *ith* variable of the dual problem.

The dual value of a constraint is the shadow price of the constraint. The shadow price is the amount by which the objective value will change if the constraints bounds change by one unit under the precondition that the basis remains the same.

INPUT:

•`variable` – The number of the constraint

---

**Note:** Behaviour is undefined unless `solve` has been called before. If the simplex algorithm has not been used for solving 0.0 will be returned.

---

EXAMPLE:

```
sage: from sage.numerical.backends.generic_backend import get_solver
sage: lp = get_solver(solver = "GLPK")
sage: lp.add_variables(3)
2
sage: lp.add_linear_constraint(zip([0, 1, 2], [8, 6, 1]), None, 48)
sage: lp.add_linear_constraint(zip([0, 1, 2], [4, 2, 1.5]), None, 20)
sage: lp.add_linear_constraint(zip([0, 1, 2], [2, 1.5, 0.5]), None, 8)
sage: lp.set_objective([60, 30, 20])
sage: import sage.numerical.backends.glpk_backend as backend
sage: lp.solver_parameter(backend.glp_simplex_or_intopt, backend.glp_simplex_only)
sage: lp.solve()
0
sage: lp.get_row_dual(0)      # tolerance 0.00001
0.0
sage: lp.get_row_dual(1)      # tolerance 0.00001
10.0
```

**get\_variable\_value** (*variable*)

Returns the value of a variable given by the solver.

---

**Note:** Behaviour is undefined unless `solve` has been called before.

---

EXAMPLE:

```
sage: from sage.numerical.backends.generic_backend import get_solver
sage: p = get_solver(solver = "GLPK")
sage: p.add_variables(2)
1
sage: p.add_linear_constraint([[0, 1], [1, 2]], None, 3)
sage: p.set_objective([2, 5])
sage: p.solve()
0
sage: p.get_objective_value()
7.5
sage: p.get_variable_value(0)
0.0
sage: p.get_variable_value(1)
1.5
```

**is\_maximization** ()

Test whether the problem is a maximization

EXAMPLE:

```
sage: from sage.numerical.backends.generic_backend import get_solver
sage: p = get_solver(solver = "GLPK")
sage: p.is_maximization()
True
sage: p.set_sense(-1)
sage: p.is_maximization()
False
```

**is\_variable\_binary** (*index*)

Test whether the given variable is of binary type.

INPUT:

- *index* (integer) – the variable's id

EXAMPLE:

```
sage: from sage.numerical.backends.generic_backend import get_solver
sage: p = get_solver(solver = "GLPK")
sage: p.ncols()
0
sage: p.add_variable()
0
sage: p.set_variable_type(0,0)
sage: p.is_variable_binary(0)
True
```

**is\_variable\_continuous** (*index*)

Test whether the given variable is of continuous/real type.

INPUT:

- *index* (integer) – the variable's id

EXAMPLE:

```
sage: from sage.numerical.backends.generic_backend import get_solver
sage: p = get_solver(solver = "GLPK")
sage: p.ncols()
0
sage: p.add_variable()
0
sage: p.is_variable_continuous(0)
True
sage: p.set_variable_type(0,1)
sage: p.is_variable_continuous(0)
False
```

**is\_variable\_integer** (*index*)

Test whether the given variable is of integer type.

INPUT:

- *index* (integer) – the variable's id

EXAMPLE:

```
sage: from sage.numerical.backends.generic_backend import get_solver
sage: p = get_solver(solver = "GLPK")
sage: p.ncols()
0
sage: p.add_variable()
0
sage: p.set_variable_type(0,1)
sage: p.is_variable_integer(0)
True
```

**ncols** ()

Return the number of columns/variables.

EXAMPLE:



```

sage: from sage.numerical.backends.generic_backend import get_solver
sage: p = get_solver(solver = "GLPK")
sage: p.ncols()
0
sage: p.add_variables(2)
1
sage: p.ncols()
2

```

**nrows()**

Return the number of rows/constraints.

EXAMPLE:

```

sage: from sage.numerical.backends.generic_backend import get_solver
sage: p = get_solver(solver = "GLPK")
sage: p.nrows()
0
sage: p.add_linear_constraints(2, 2, None)
sage: p.nrows()
2

```

**objective\_coefficient** (*variable*, *coeff=None*)

Set or get the coefficient of a variable in the objective function

INPUT:

- *variable* (integer) – the variable's id
- *coeff* (double) – its coefficient or None for reading (default: None)

EXAMPLE:

```

sage: from sage.numerical.backends.generic_backend import get_solver
sage: p = get_solver(solver = "GLPK")
sage: p.add_variable()
0
sage: p.objective_coefficient(0)
0.0
sage: p.objective_coefficient(0,2)
sage: p.objective_coefficient(0)
2.0

```

**print\_ranges** (*filename='NULL'*)

Print results of a sensitivity analysis

If no filename is given as an input the results of the sensitivity analysis are displayed on the screen. If a filename is given they are written to a file.

INPUT:

- *filename* – (optional) name of the file

OUTPUT:

Zero if the operations was successful otherwise nonzero.

---

**Note:** This method is only effective if an optimal solution has been found for the lp using the simplex algorithm. In all other cases an error message is printed.

---

EXAMPLE:

```

sage: from sage.numerical.backends.generic_backend import get_solver
sage: p = get_solver(solver = "GLPK")
sage: p.add_variables(2)
1
sage: p.add_linear_constraint(zip([0, 1], [1, 2]), None, 3)
sage: p.set_objective([2, 5])
sage: import sage.numerical.backends.glpk_backend as backend
sage: p.solver_parameter(backend.glp_simplex_or_intopt, backend.glp_simplex_only)
sage: p.print_ranges()
glp_print_ranges: optimal basic solution required
1
sage: p.solve()
0
sage: p.print_ranges()
Write sensitivity analysis report to...
GLPK ... - SENSITIVITY ANALYSIS REPORT

```

Problem:  
Objective: 7.5 (MAXimum)

No.	Row name	St	Activity	Slack Marginal	Lower bound Upper bound	Activity range	Obj coef range
1		NU	3.00000	. 2.50000	-Inf 3.00000	. +Inf	-2.5000 +Inf

GLPK ... - SENSITIVITY ANALYSIS REPORT

Problem:  
Objective: 7.5 (MAXimum)

No.	Column name	St	Activity	Obj coef Marginal	Lower bound Upper bound	Activity range	Obj coef range
1		NL	.	2.00000 -.50000	. +Inf	-Inf 3.00000	-Inf 2.50000
2		BS	1.50000	5.00000 .	. +Inf	-Inf 1.50000	4.0000 +Inf

End of report

0

**problem\_name** (name='NULL')

Return or define the problem's name

INPUT:

- name (char \*) – the problem's name. When set to NULL (default), the method returns the problem's name.

EXAMPLE:

```

sage: from sage.numerical.backends.generic_backend import get_solver
sage: p = get_solver(solver = "GLPK")
sage: p.problem_name("There once was a french fry")
sage: print p.problem_name()
There once was a french fry

```

**remove\_constraint** (*i*)

Remove a constraint from self.

INPUT:

- *i* – index of the constraint to remove

EXAMPLE:

```
sage: p = MixedIntegerLinearProgram(solver='GLPK')
sage: x,y = p[0], p[1]
doctest:...: DeprecationWarning: The default value of 'nonnegative' will change, to False in
See http://trac.sagemath.org/15521 for details.
sage: p.add_constraint(2*x + 3*y, max = 6)
sage: p.add_constraint(3*x + 2*y, max = 6)
sage: p.set_objective(x + y + 7)
sage: p.set_integer(x); p.set_integer(y)
sage: p.solve()
9.0
sage: p.remove_constraint(0)
sage: p.solve()
10.0
```

Removing fancy constraints does not make Sage crash:

```
sage: MixedIntegerLinearProgram(solver = "GLPK").remove_constraint(-2)
Traceback (most recent call last):
...
ValueError: The constraint's index i must satisfy 0 <= i < number_of_constraints
```

**remove\_constraints** (*constraints*)

Remove several constraints.

INPUT:

- *constraints* – an iterable containing the indices of the rows to remove.

EXAMPLE:

```
sage: p = MixedIntegerLinearProgram(solver='GLPK')
sage: x,y = p[0], p[1]
sage: p.add_constraint(2*x + 3*y, max = 6)
sage: p.add_constraint(3*x + 2*y, max = 6)
sage: p.set_objective(x + y + 7)
sage: p.set_integer(x); p.set_integer(y)
sage: p.solve()
9.0
sage: p.remove_constraints([1])
sage: p.solve()
10.0
sage: p.get_values([x,y])
[3.0, 0.0]
```

TESTS:

Removing fancy constraints does not make Sage crash:

```
sage: MixedIntegerLinearProgram(solver= "GLPK").remove_constraints([0, -2])
Traceback (most recent call last):
...
ValueError: The constraint's index i must satisfy 0 <= i < number_of_constraints
```

**row**(*index*)

Return a row

INPUT:

- *index* (integer) – the constraint's id.

OUTPUT:

A pair (*indices*, *coeffs*) where *indices* lists the entries whose coefficient is nonzero, and to which *coeffs* associates their coefficient on the model of the `add_linear_constraint` method.

EXAMPLE:

```
sage: from sage.numerical.backends.generic_backend import get_solver
sage: p = get_solver(solver = "GLPK")
sage: p.add_variables(5)
4
sage: p.add_linear_constraint(zip(range(5), range(5)), 2, 2)
sage: p.row(0)
([4, 3, 2, 1], [4.0, 3.0, 2.0, 1.0])
sage: p.row_bounds(0)
(2.0, 2.0)
```

**row\_bounds**(*index*)

Return the bounds of a specific constraint.

INPUT:

- *index* (integer) – the constraint's id.

OUTPUT:

A pair (*lower\_bound*, *upper\_bound*). Each of them can be set to `None` if the constraint is not bounded in the corresponding direction, and is a real value otherwise.

EXAMPLE:

```
sage: from sage.numerical.backends.generic_backend import get_solver
sage: p = get_solver(solver = "GLPK")
sage: p.add_variables(5)
4
sage: p.add_linear_constraint(zip(range(5), range(5)), 2, 2)
sage: p.row(0)
([4, 3, 2, 1], [4.0, 3.0, 2.0, 1.0])
sage: p.row_bounds(0)
(2.0, 2.0)
```

**row\_name**(*index*)Return the *index* th row name

INPUT:

- *index* (integer) – the row's id

EXAMPLE:

```
sage: from sage.numerical.backends.generic_backend import get_solver
sage: p = get_solver(solver = "GLPK")
sage: p.add_linear_constraints(1, 2, None, names=['Empty constraint 1'])
sage: p.row_name(0)
'Empty constraint 1'
```

**set\_objective** (*coeff*, *d=0.0*)

Set the objective function.

INPUT:

- *coeff* - a list of real values, whose *i*th element is the coefficient of the *i*th variable in the objective function.
- *d* (double) – the constant term in the linear function (set to 0 by default)

EXAMPLE:

```
sage: from sage.numerical.backends.generic_backend import get_solver
sage: p = get_solver(solver = "GLPK")
sage: p.add_variables(5)
4
sage: p.set_objective([1, 1, 2, 1, 3])
sage: map(lambda x :p.objective_coefficient(x), range(5))
[1.0, 1.0, 2.0, 1.0, 3.0]
```

**set\_sense** (*sense*)

Set the direction (maximization/minimization).

INPUT:

- *sense* (integer) :
  - +1 => Maximization
  - 1 => Minimization

EXAMPLE:

```
sage: from sage.numerical.backends.generic_backend import get_solver
sage: p = get_solver(solver = "GLPK")
sage: p.is_maximization()
True
sage: p.set_sense(-1)
sage: p.is_maximization()
False
```

**set\_variable\_type** (*variable*, *vtype*)

Set the type of a variable

INPUT:

- *variable* (integer) – the variable's id
- *vtype* (integer) :
  - 1 Integer
  - 0 Binary
  - 1 Real

EXAMPLE:

```
sage: from sage.numerical.backends.generic_backend import get_solver
sage: p = get_solver(solver = "GLPK")
sage: p.ncols()
0
sage: p.add_variable()
0
sage: p.set_variable_type(0,1)
```

```
sage: p.is_variable_integer(0)
True
```

**set\_verbosity** (*level*)

Set the verbosity level

INPUT:

- *level* (integer) – From 0 (no verbosity) to 3.

EXAMPLE:

```
sage: from sage.numerical.backends.generic_backend import get_solver
sage: p = get_solver(solver = "GLPK")
sage: p.set_verbosity(2)
```

**solve** ()

Solve the problem.

---

**Note:** This method raises `MIPSolverException` exceptions when the solution can not be computed for any reason (none exists, or the LP solver was not able to find it, etc...)

---

EXAMPLE:

```
sage: from sage.numerical.backends.generic_backend import get_solver
sage: p = get_solver(solver = "GLPK")
sage: p.add_linear_constraints(5, 0, None)
sage: p.add_col(range(5), range(5))
sage: p.solve()
0
sage: p.objective_coefficient(0,1)
sage: p.solve()
Traceback (most recent call last):
...
MIPSolverException: ...
```

**Warning:** Sage uses GLPK's `glp_intopt` to find solutions. This routine sometimes FAILS CATASTROPHICALLY when given a system it cannot solve. (Ticket #12309.) Here, “catastrophic” can mean either “infinite loop” or segmentation fault. Upstream considers this behavior “essentially innate” to their design, and suggests preprocessing it with `glpk_simplex` first. Thus, if you suspect that your system is infeasible, set the preprocessing option first.

EXAMPLE:

```
sage: lp = MixedIntegerLinearProgram(solver = "GLPK")
sage: lp.add_constraint( lp[1] +lp[2] -2.0 *lp[3] , max =-1.0)
sage: lp.add_constraint( lp[0] -4.0/3 *lp[1] +1.0/3 *lp[2] , max =-1.0/3)
sage: lp.add_constraint( lp[0] +0.5 *lp[1] -0.5 *lp[2] +0.25 *lp[3] , max =-0.25)
sage: lp.solve()
0.0
sage: lp.add_constraint( lp[0] +4.0 *lp[1] -lp[2] +lp[3] , max =-1.0)
sage: lp.solve()
Traceback (most recent call last):
...
RuntimeError: GLPK : Signal sent, try preprocessing option
sage: lp.solver_parameter("simplex_or_intopt", "simplex_then_intopt")
sage: lp.solve()
Traceback (most recent call last):
```

```
...
MIPSolverException: 'GLPK' : Simplex cannot find a feasible solution'
```

The user can ask sage to solve via `simplex` or `intopt`. The default solver is `intopt`, so we get integer solutions.

EXAMPLE:

```
sage: lp = MixedIntegerLinearProgram(solver = 'GLPK', maximization = False)
sage: x, y = lp[0], lp[1]
sage: lp.add_constraint(-2*x + y <= 1)
sage: lp.add_constraint(x - y <= 1)
sage: lp.add_constraint(x + y >= 2)
sage: lp.set_objective(x + y)
sage: lp.set_integer(x)
sage: lp.set_integer(y)
sage: lp.solve()
2.0
sage: lp.get_values([x, y])
[1.0, 1.0]
```

If we switch to `simplex`, we get continuous solutions.

EXAMPLE:

```
sage: lp.solver_parameter("simplex_or_intopt", "simplex_only") # use simplex only
sage: lp.solve()
2.0
sage: lp.get_values([x, y])
[1.5, 0.5]
```

**solver\_parameter** (*name*, *value=None*)

Return or define a solver parameter

INPUT:

- *name* (string) – the parameter
- *value* – the parameter's value if it is to be defined, or `None` (default) to obtain its current value.

You can supply the name of a parameter and its value using either a string or a `glp_` constant (which are defined as Cython variables of this module).

In most cases, you can use the same name for a parameter as that given in the GLPK documentation, which is available by downloading GLPK from <<http://www.gnu.org/software/glpk/>>. The exceptions relate to parameters common to both methods; these require you to append `_simplex` or `_intopt` to the name to resolve ambiguity, since the interface allows access to both.

We have also provided more meaningful names, to assist readability.

Parameter **names** are specified in lower case. To use a constant instead of a string, prepend `glp_` to the name. For example, both `glp_gmi_cuts` or `"gmi_cuts"` control whether to solve using Gomory cuts.

Parameter **values** are specified as strings in upper case, or as constants in lower case. For example, both `glp_on` and `"GLP_ON"` specify the same thing.

Naturally, you can use `True` and `False` in cases where `glp_on` and `glp_off` would be used.

A list of parameter names, with their possible values:

**General-purpose parameters:**

<code>timelimit</code>	specify the time limit IN SECONDS. This affects both simplex and intopt.
<code>timelimit_simplex</code> and <code>timelimit_intopt</code>	specify the time limit IN MILLISECONDS. (This is glpk's default.)
<code>simplex_or_intopt</code>	whether to use the simplex or intopt routines in GLPK. This is controlled by using <code>glp_simplex_only</code> , <code>glp_intopt_only</code> , and <code>glp_simplex_then_intopt</code> . The latter is useful to deal with a problem in GLPK where problems with no solution hang when using integer optimization; if you specify <code>glp_simplex_then_intopt</code> , sage will try simplex first, then perform integer optimization only if a solution of the LP relaxation exists.
<code>verbosity_intopt</code> and <code>verbosity_simplex</code>	one of <code>GLP_MSG_OFF</code> , <code>GLP_MSG_ERR</code> , <code>GLP_MSG_ON</code> , or <code>GLP_MSG_ALL</code> . The default is <code>GLP_MSG_OFF</code> .
<code>output_frequency_intopt</code> and <code>output_frequency_simplex</code>	the output frequency, in milliseconds. Default is 5000.
<code>output_delay_intopt</code> and <code>output_delay_simplex</code>	the output delay, in milliseconds, regarding the use of the simplex method on the LP relaxation. Default is 10000.

**intopt-specific parameters:**



branching	<ul style="list-style-type: none"> <li>•GLP_BR_FFV first fractional variable</li> <li>•GLP_BR_LFV last fractional variable</li> <li>•GLP_BR_MFV most fractional variable</li> <li>•GLP_BR_DTH Driebeck-Tomlin heuristic (default)</li> <li>•GLP_BR_PCH hybrid pseudocost heuristic</li> </ul>
backtracking	<ul style="list-style-type: none"> <li>•GLP_BT_DFS depth first search</li> <li>•GLP_BT_BFS breadth first search</li> <li>•GLP_BT_BLB best local bound (default)</li> <li>•GLP_BT_BPH best projection heuristic</li> </ul>
preprocessing	<ul style="list-style-type: none"> <li>•GLP_PP_NONE</li> <li>•GLP_PP_ROOT preprocessing only at root level</li> <li>•GLP_PP_ALL (default)</li> </ul>
feasibility_pump	GLP_ON or GLP_OFF (default)
gomory_cuts	GLP_ON or GLP_OFF (default)
mixed_int_rounding_cuts	GLP_ON or GLP_OFF (default)
mixed_cover_cuts	GLP_ON or GLP_OFF (default)
clique_cuts	GLP_ON or GLP_OFF (default)
absolute_tolerance	(double) used to check if optimal solution to LP relaxation is integer feasible. GLPK manual advises, “do not change... without detailed understanding of its purpose.”
relative_tolerance	(double) used to check if objective value in LP relaxation is not better than best known integer solution. GLPK manual advises, “do not change... without detailed understanding of its purpose.”
mip_gap_tolerance	(double) relative mip gap tolerance. Default is 0.0.
presolve_intopt	GLP_ON (default) or GLP_OFF.
binarize	GLP_ON or GLP_OFF (default)

**simplex-specific parameters:**

primal_v_dual	<ul style="list-style-type: none"> <li>•GLP_PRIMAL (default)</li> <li>•GLP_DUAL</li> <li>•GLP_DUALP</li> </ul>
pricing	<ul style="list-style-type: none"> <li>•GLP_PT_STD standard (textbook)</li> <li>•GLP_PT_PSE projected steepest edge (default)</li> </ul>
ratio_test	<ul style="list-style-type: none"> <li>•GLP_RT_STD standard (textbook)</li> <li>•GLP_RT_HAR Harris' two-pass ratio test (default)</li> </ul>
tolerance_primal	(double) tolerance used to check if basic solution is primal feasible. GLPK manual advises, "do not change... without detailed understanding of its purpose."
tolerance_dual	(double) tolerance used to check if basic solution is dual feasible. GLPK manual advises, "do not change... without detailed understanding of its purpose."
tolerance_pivot	(double) tolerance used to choose pivot. GLPK manual advises, "do not change... without detailed understanding of its purpose."
obj_lower_limit	(double) lower limit of the objective function. The default is <code>-DBL_MAX</code> .
obj_upper_limit	(double) upper limit of the objective function. The default is <code>DBL_MAX</code> .
iteration_limit	(int) iteration limit of the simplex algorithm. The default is <code>INT_MAX</code> .
presolve_simplex	GLP_ON or GLP_OFF (default).

**Note:** The coverage for GLPK's control parameters for simplex and integer optimization is nearly complete. The only thing lacking is a wrapper for callback routines.

To date, no attempt has been made to expose the interior point methods.

EXAMPLE:

```
sage: from sage.numerical.backends.generic_backend import get_solver
sage: p = get_solver(solver = "GLPK")
sage: p.solver_parameter("timelimit", 60)
sage: p.solver_parameter("timelimit")
60.0
```

•Don't forget the difference between `timelimit` and `timelimit_intopt`

```
sage: p.solver_parameter("timelimit_intopt")
60000
```

If you don't care for an integer answer, you can ask for an lp relaxation instead. The default solver performs integer optimization, but you can switch to the standard simplex algorithm through the `glp_simplex_or_intopt` parameter.

EXAMPLE:

```
sage: lp = MixedIntegerLinearProgram(solver = 'GLPK', maximization = False)
sage: x, y = lp[0], lp[1]
sage: lp.add_constraint(-2*x + y <= 1)
sage: lp.add_constraint(x - y <= 1)
sage: lp.add_constraint(x + y >= 2)
sage: lp.set_integer(x); lp.set_integer(y)
sage: lp.set_objective(x + y)
sage: lp.solve()
2.0
sage: lp.get_values([x,y])
[1.0, 1.0]
sage: import sage.numerical.backends.glpk_backend as backend
sage: lp.solver_parameter(backend.glp_simplex_or_intopt, backend.glp_simplex_only)
sage: lp.solve()
2.0
sage: lp.get_values([x,y])
[1.5, 0.5]
```

You can get glpk to spout all sorts of information at you. The default is to turn this off, but sometimes (debugging) it's very useful:

```
sage: lp.solver_parameter(backend.glp_simplex_or_intopt, backend.glp_simplex_then_intopt)
sage: lp.solver_parameter(backend.glp_mir_cuts, backend.glp_on)
sage: lp.solver_parameter(backend.glp_msg_lev_intopt, backend.glp_msg_all)
sage: lp.solver_parameter(backend.glp_mir_cuts)
1
```

If you actually try to solve `lp`, you will get a lot of detailed information.

**variable\_lower\_bound** (*index*, *value=False*)

Return or define the lower bound on a variable

INPUT:

- *index* (integer) – the variable's id
- *value* – real value, or None to mean that the variable has not lower bound. When set to False (default), the method returns the current value.

EXAMPLE:

```
sage: from sage.numerical.backends.generic_backend import get_solver
sage: p = get_solver(solver = "GLPK")
sage: p.add_variable()
0
sage: p.col_bounds(0)
(0.0, None)
sage: p.variable_lower_bound(0, 5)
sage: p.col_bounds(0)
(5.0, None)
```

TESTS:

trac ticket #14581:

```
sage: P = MixedIntegerLinearProgram(solver="GLPK")
sage: x = P["x"]
sage: P.set_min(x, 5)
sage: P.set_min(x, 0)
sage: P.get_min(x)
0.0
```

**variable\_upper\_bound** (*index*, *value=False*)

Return or define the upper bound on a variable

INPUT:

- *index* (integer) – the variable's id
- *value* – real value, or None to mean that the variable has not upper bound. When set to False (default), the method returns the current value.

EXAMPLE:

```
sage: from sage.numerical.backends.generic_backend import get_solver
sage: p = get_solver(solver = "GLPK")
sage: p.add_variable()
0
sage: p.col_bounds(0)
(0.0, None)
sage: p.variable_upper_bound(0, 5)
sage: p.col_bounds(0)
(0.0, 5.0)
```

TESTS:

trac ticket #14581:

```
sage: P = MixedIntegerLinearProgram(solver="GLPK")
sage: x = P["x"]
sage: P.set_max(x, 0)
sage: P.get_max(x)
0.0
```

**write\_lp** (*filename*)

Write the problem to a .lp file

INPUT:

- *filename* (string)

EXAMPLE:

```
sage: from sage.numerical.backends.generic_backend import get_solver
sage: p = get_solver(solver = "GLPK")
sage: p.add_variables(2)
1
sage: p.add_linear_constraint([[0, 1], [1, 2]], None, 3)
sage: p.set_objective([2, 5])
sage: p.write_lp(os.path.join(SAGE_TMP, "lp_problem.lp"))
Writing problem data to ...
9 lines were written
```

**write\_mps** (*filename*, *modern*)

Write the problem to a .mps file

INPUT:

- *filename* (string)

EXAMPLE:

```
sage: from sage.numerical.backends.generic_backend import get_solver
sage: p = get_solver(solver = "GLPK")
sage: p.add_variables(2)
1
```

```

sage: p.add_linear_constraint([[0, 1], [1, 2]], None, 3)
sage: p.set_objective([2, 5])
sage: p.write_mps(os.path.join(SAGE_TMP, "lp_problem.mps"), 2)
Writing problem data to...
17 records were written

```

## 6.3 GLPK Backend for access to GLPK graph functions

GLPK Backend for access to GLPK graph functions

AUTHORS:

- Christian Kuper (2012-11): Initial implementation

### 6.3.1 Methods index

Graph creation and modification operations:

<code>add_vertex()</code>	Adds an isolated vertex to the graph.
<code>add_vertices()</code>	Adds vertices from an iterable container of vertices.
<code>set_vertex_demand()</code>	Sets the vertex parameters.
<code>set_vertices_demand()</code>	Sets the parameters of selected vertices.
<code>get_vertex()</code>	Returns a specific vertex as a <code>dict</code> Object.
<code>get_vertices()</code>	Returns a dictionary of the dictionaries associated to each vertex.
<code>vertices()</code>	Returns a list of all vertices.
<code>delete_vertex()</code>	Removes a vertex from the graph.
<code>delete_vertices()</code>	Removes vertices from the graph.
<code>add_edge()</code>	Adds an edge between vertices <code>u</code> and <code>v</code> .
<code>add_edges()</code>	Adds edges to the graph.
<code>get_edge()</code>	Returns an edge connecting two vertices.
<code>edges()</code>	Returns a list of all edges in the graph.
<code>delete_edge()</code>	Deletes an edge from the graph.
<code>delete_edges()</code>	Deletes edges from the graph.

Graph writing operations:

<code>write_graph()</code>	Writes the graph to a plain text file.
<code>write_ccdata()</code>	Writes the graph to a text file in DIMACS format.
<code>write_mincost()</code>	Writes the mincost flow problem data to a text file in DIMACS format.
<code>write_maxflow()</code>	Writes the maximum flow problem data to a text file in DIMACS format.

Network optimization operations:

<code>mincost_okalg()</code>	Finds solution to the mincost problem with the out-of-kilter algorithm.
<code>maxflow_ffalg()</code>	Finds solution to the maxflow problem with Ford-Fulkerson algorithm.
<code>cpp()</code>	Solves the critical path problem of a project network.

### 6.3.2 Classes and methods

**class** `sage.numerical.backends.glpk_graph_backend.GLPKGraphBackend`

Bases: `object`

GLPK Backend for access to GLPK graph functions

The constructor can either be called without arguments (which results in an empty graph) or with arguments to read graph data from a file.

INPUT:

- `data` – a filename or a Graph object.
- `format` – when `data` is a filename, specifies the format of the data read from a file. The `format` parameter is a string and can take values as described in the table below.

**Format parameters:**

plain	Read data from a plain text file containing the following information: <code>nv na</code> <code>i[1] j[1]</code> <code>i[2] j[2]</code> ... <code>i[na] j[na]</code> where: • <code>nv</code> is the number of vertices (nodes); • <code>na</code> is the number of arcs; • <code>i[k]</code> , $k = 1, \dots, na$ , is the index of tail vertex of arc $k$ ; • <code>j[k]</code> , $k = 1, \dots, na$ , is the index of head vertex of arc $k$ .
dimacs	Read data from a plain ASCII text file in DIMACS format. A discription of the DIMACS format can be found at <a href="http://dimacs.rutgers.edu/Challenges/">http://dimacs.rutgers.edu/Challenges/</a> .
mincost	Reads the mincost flow problem data from a text file in DIMACS format
maxflow	Reads the maximum flow problem data from a text file in DIMACS format

**Note:** When `data` is a Graph, the following restrictions are applied.

- `vertices` – the value of the demand of each vertex (see `set_vertex_demand()`) is obtained from the numerical value associated with the key “rhs” if it is a dictionary.
- `edges` – The edge values used in the algorithms are read from the edges labels (and left undefined if the edge labels are equal to None). To be defined, the labels must be dict objects with keys “low”, “cap” and “cost”. See `get_edge()` for details.

EXAMPLES:

The following example creates an empty graph:

```
sage: from sage.numerical.backends.glpk_graph_backend import GLPKGraphBackend
sage: gbe = GLPKGraphBackend()
```

The following example creates an empty graph, adds some data, saves the data to a file and loads it:

```
sage: from sage.numerical.backends.glpk_graph_backend import GLPKGraphBackend
sage: gbe = GLPKGraphBackend()
sage: gbe.add_vertices([None, None])
sage: ['0', '1']
sage: a = gbe.add_edge('0', '1')
sage: gbe.write_graph(SAGE_TMP+"/graph.txt")
```

```

Writing graph to ...
2 lines were written
0
sage: gbe1 = GLPKGraphBackend(SAGE_TMP+"/graph.txt", "plain")
Reading graph from ...
Graph has 2 vertices and 1 arc
2 lines were read

```

The following example imports a Sage Graph and then uses it to solve a maxflow problem:

```

sage: from sage.numerical.backends.glpk_graph_backend import GLPKGraphBackend
sage: g = graphs.PappusGraph()
sage: for ed in g.edges():
...     g.set_edge_label(ed[0], ed[1], {"cap":1})
sage: gbe = GLPKGraphBackend(g)
sage: gbe.maxflow_ffalg('1', '2')
3.0

```

**add\_edge** (*u*, *v*, *params*=None)

Adds an edge between vertices *u* and *v*.

Allows adding an edge and optionally providing parameters used by the algorithms. If a vertex does not exist it is created.

INPUT:

- *u* – The name (as `str`) of the tail vertex
- *v* – The name (as `str`) of the head vertex
- *params* – An optional dict containing the edge parameters used for the algorithms. The following keys are used:
  - `low` – The minimum flow through the edge
  - `cap` – The maximum capacity of the edge
  - `cost` – The cost of transporting one unit through the edge

EXAMPLE:

```

sage: from sage.numerical.backends.glpk_graph_backend import GLPKGraphBackend
sage: gbe = GLPKGraphBackend()
sage: gbe.add_edge("A", "B", {"low":0.0, "cap":10.0, "cost":5})
sage: gbe.vertices()
['A', 'B']
sage: for ed in gbe.edges():
...     print ed[0], ed[1], ed[2]['cap'], ed[2]['cost'], ed[2]['low']
A B 10.0 5.0 0.0
sage: gbe.add_edge("B", "C", {"low":0.0, "cap":10.0, "cost":5})
Traceback (most recent call last):
...
TypeError: Invalid edge parameter.

```

**add\_edges** (*edges*)

Adds edges to the graph.

INPUT:

- *edges* – An iterable container of pairs of the form (*u*, *v*), where *u* is name (as `str`) of the tail vertex and *v* is the name (as `str`) of the head vertex or an iterable container of triples of the form (*u*, *v*, *params*) where *params* is a dict as described in `add_edge`.

EXAMPLE:

```
sage: from sage.numerical.backends.glpk_graph_backend import GLPKGraphBackend
sage: gbe = GLPKGraphBackend()
sage: edges = [("A", "B", {"low":0.0, "cap":10.0, "cost":5})]
sage: edges.append(("B", "C"))
sage: gbe.add_edges(edges)
sage: for ed in gbe.edges():
...     print ed[0], ed[1], ed[2]['cap'], ed[2]['cost'], ed[2]['low']
A B 10.0 5.0 0.0
B C 0.0 0.0 0.0
sage: edges = [("C", "D", {"low":0.0, "cap":10.0, "cost":5})]
sage: edges.append(("C", "E", 5))
sage: gbe.add_edges(edges)
Traceback (most recent call last):
...
TypeError: Argument 'params' has incorrect type ...
sage: for ed in gbe.edges():
...     print ed[0], ed[1], ed[2]['cap'], ed[2]['cost'], ed[2]['low']
A B 10.0 5.0 0.0
B C 0.0 0.0 0.0
C D 10.0 5.0 0.0
```

**add\_vertex** (*name*='NULL')

Adds an isolated vertex to the graph.

If the vertex already exists, nothing is done.

INPUT:

- name** – String of max 255 chars length. If no name is specified, then the vertex will be represented by the string representation of the ID of the vertex or - if this already exists - a string representation of the least integer not already representing a vertex.

OUTPUT:

If no name is passed as an argument, the new vertex name is returned. None otherwise.

EXAMPLE:

```
sage: from sage.numerical.backends.glpk_graph_backend import GLPKGraphBackend
sage: gbe = GLPKGraphBackend()
sage: gbe.add_vertex()
'0'
sage: gbe.add_vertex("2")
sage: gbe.add_vertex()
'1'
```

**add\_vertices** (*vertices*)

Adds vertices from an iterable container of vertices.

Vertices that already exist in the graph will not be added again.

INPUT:

- vertices** – iterator of vertex labels (str). A label can be None.

OUTPUT:

Generated names of new vertices if there is at least one None value present in vertices. None otherwise.

EXAMPLE:



```

sage: from sage.numerical.backends.glpk_graph_backend import GLPKGraphBackend
sage: gbe = GLPKGraphBackend()
sage: vertices = [None for i in range(3)]
sage: gbe.add_vertices(vertices)
['0', '1', '2']
sage: gbe.add_vertices(['A', 'B', None])
['5']
sage: gbe.add_vertices(['A', 'B', 'C'])
sage: gbe.vertices()
['0', '1', '2', 'A', 'B', '5', 'C']

```

TESTS:

```

sage: from sage.numerical.backends.glpk_graph_backend import GLPKGraphBackend
sage: gbe = GLPKGraphBackend()
sage: gbe.add_vertices([None, None, None, '1'])
['0', '2', '3']

```

**cpp()**

Solves the critical path problem of a project network.

OUTPUT:

The length of the critical path of the network

EXAMPLE:

```

sage: from sage.numerical.backends.glpk_graph_backend import GLPKGraphBackend
sage: gbe = GLPKGraphBackend()
sage: gbe.add_vertices([None for i in range(3)])
['0', '1', '2']
sage: gbe.set_vertex_demand('0', 3)
sage: gbe.set_vertex_demand('1', 1)
sage: gbe.set_vertex_demand('2', 4)
sage: a = gbe.add_edge('0', '2')
sage: a = gbe.add_edge('1', '2')
sage: gbe.cpp()
7.0
sage: v = gbe.get_vertex('1')
sage: print 1, v["rhs"], v["es"], v["ls"] # abs tol 1e-6
1 1.0 0.0 2.0

```

**delete\_edge(u, v, params=None)**

Deletes an edge from the graph.

If an edge does not exist it is ignored.

INPUT:

- *u* – The name (as `str`) of the tail vertex of the edge
- *v* – The name (as `str`) of the tail vertex of the edge
- *params* – *params* – An optional `dict` containing the edge parameters (see `meth:add_edge`). If this parameter is not provided, all edges connecting *u* and *v* are deleted. Otherwise only edges with matching parameters are deleted.

**See Also:**

`delete_edges()`

EXAMPLE:

```
sage: from sage.numerical.backends.glpk_graph_backend import GLPKGraphBackend
sage: gbe = GLPKGraphBackend()
sage: edges = [("A", "B", {"low":0.0, "cap":10.0, "cost":5})]
sage: edges.append(("A", "B", {"low":0.0, "cap":15.0, "cost":10}))
sage: edges.append(("B", "C", {"low":0.0, "cap":20.0, "cost":1}))
sage: edges.append(("B", "C", {"low":0.0, "cap":10.0, "cost":20}))
sage: gbe.add_edges(edges)
sage: gbe.delete_edge("A", "B")
sage: gbe.delete_edge("B", "C", {"low":0.0, "cap":10.0, "cost":20})
sage: print gbe.edges()[0][0], gbe.edges()[0][1], gbe.edges()[0][2]['cost']
B C 1.0
```

**delete\_edges(*edges*)**

Deletes edges from the graph.

Non existing edges are ignored.

INPUT:

- edges – An iterable container of edges.

**See Also:**

`delete_edge()`

EXAMPLE:

```
sage: from sage.numerical.backends.glpk_graph_backend import GLPKGraphBackend
sage: gbe = GLPKGraphBackend()
sage: edges = [("A", "B", {"low":0.0, "cap":10.0, "cost":5})]
sage: edges.append(("A", "B", {"low":0.0, "cap":15.0, "cost":10}))
sage: edges.append(("B", "C", {"low":0.0, "cap":20.0, "cost":1}))
sage: edges.append(("B", "C", {"low":0.0, "cap":10.0, "cost":20}))
sage: gbe.add_edges(edges)
sage: gbe.delete_edges(edges[1:])
sage: len(gbe.edges())
1
sage: print gbe.edges()[0][0], gbe.edges()[0][1], gbe.edges()[0][2]['cap']
A B 10.0
```

**delete\_vertex(*vert*)**

Removes a vertex from the graph.

Trying to delete a non existing vertex will raise an exception.

INPUT:

- vert – The name (as str) of the vertex to delete.

EXAMPLE:

```
sage: from sage.numerical.backends.glpk_graph_backend import GLPKGraphBackend
sage: gbe = GLPKGraphBackend()
sage: verts = ["A", "D"]
sage: gbe.add_vertices(verts)
sage: gbe.delete_vertex("A")
sage: gbe.vertices()
['D']
sage: gbe.delete_vertex("A")
Traceback (most recent call last):
...
RuntimeError: Vertex A does not exist.
```

**delete\_vertices** (*verts*)

Removes vertices from the graph.

Trying to delete a non existing vertex will raise an exception.

INPUT:

- *verts* – iterable container containing names (as `str`) of the vertices to delete

EXAMPLE:

```
sage: from sage.numerical.backends.glpk_graph_backend import GLPKGraphBackend
sage: gbe = GLPKGraphBackend()
sage: verts = ["A", "B", "C", "D"]
sage: gbe.add_vertices(verts)
sage: v_d = ["A", "B"]
sage: gbe.delete_vertices(v_d)
sage: gbe.vertices()
['C', 'D']
sage: gbe.delete_vertices(["C", "A"])
Traceback (most recent call last):
...
RuntimeError: Vertex A does not exist.
sage: gbe.vertices()
['C', 'D']
```

**edges** ()

Returns a list of all edges in the graph

OUTPUT:

A list of triples representing the edges of the graph.

EXAMPLE:

```
sage: from sage.numerical.backends.glpk_graph_backend import GLPKGraphBackend
sage: gbe = GLPKGraphBackend()
sage: edges = [("A", "B", {"low":0.0, "cap":10.0, "cost":5})]
sage: edges.append(("B", "C"))
sage: gbe.add_edges(edges)
sage: for ed in gbe.edges():
...     print ed[0], ed[1], ed[2]['cost']
A B 5.0
B C 0.0
```

**get\_edge** (*u*, *v*)

Returns an edge connecting two vertices.

---

**Note:** If multiple edges connect the two vertices only the first edge found is returned.

---

INPUT:

- *u* – Name (as `str`) of the tail vertex
- *v* – Name (as `str`) of the head vertex

OUTPUT:

A triple describing if edge was found or `None` if not. The third value of the triple is a dict containing the following edge parameters:

- *low* – The minimum flow through the edge

- `cap` – The maximum capacity of the edge
- `cost` – The cost of transporting one unit through the edge
- `x` – The actual flow through the edge after solving

EXAMPLE:

```
sage: from sage.numerical.backends.glpk_graph_backend import GLPKGraphBackend
sage: gbe = GLPKGraphBackend()
sage: edges = [("A", "B"), ("A", "C"), ("B", "C")]
sage: gbe.add_edges(edges)
sage: ed = gbe.get_edge("A", "B")
sage: print ed[0], ed[1], ed[2]['x']
A B 0.0
sage: gbe.get_edge("A", "F") is None
True
```

**get\_vertex** (*vertex*)

Returns a specific vertex as a dict Object.

INPUT:

- `vertex` – The vertex label as `str`.

OUTPUT:

The vertex as a dict object or None if the vertex does not exist. The dict contains the values used or created by the different algorithms. The values associated with the keys following keys contain:

- “`rhs`” – The supply / demand value the vertex (mincost alg)
- “`pi`” – The node potential (mincost alg)
- “`cut`” – The cut flag of the vertex (maxflow alg)
- “`es`” – The earliest start of task (cpp alg)
- “`ls`” – The latest start of task (cpp alg)

EXAMPLE:

```
sage: from sage.numerical.backends.glpk_graph_backend import GLPKGraphBackend
sage: gbe = GLPKGraphBackend()
sage: verts = ["A", "B", "C", "D"]
sage: gbe.add_vertices(verts)
sage: sorted(gbe.get_vertex("A").items())
[('cut', 0), ('es', 0.0), ('ls', 0.0), ('pi', 0.0), ('rhs', 0.0)]
sage: gbe.get_vertex("F") is None
True
```

**get\_vertices** (*verts*)

Returns a dictionary of the dictionaries associated to each vertex.

INPUT:

- `verts` – iterable container of vertices

OUTPUT:

A list of pairs (`vertex`, `properties`) where `properties` is a dictionary containing the numerical values associated with a vertex. For more information, see the documentation of [GLPKGraphBackend.get\\_vertex\(\)](#).

EXAMPLE:

```

sage: from sage.numerical.backends.glpk_graph_backend import GLPKGraphBackend
sage: gbe = GLPKGraphBackend()
sage: verts = ['A', 'B']
sage: gbe.add_vertices(verts)
sage: sorted(gbe.get_vertices(verts)['B'].items())
[('cut', 0), ('es', 0.0), ('ls', 0.0), ('pi', 0.0), ('rhs', 0.0)]
sage: gbe.get_vertices(["C", "D"])
{}

```

**maxflow\_ffalg** (*u=None, v=None*)

Finds solution to the maxflow problem with Ford-Fulkerson algorithm.

INPUT:

- *u* – Name (as `str`) of the tail vertex. Default is `None`.
- *v* – Name (as `str`) of the head vertex. Default is `None`.

If *u* or *v* are `None`, the currently stored values for the head or tail vertex are used. This behavior is useful when reading maxflow data from a file. When calling this function with values for *u* and *v*, the head and tail vertex are stored for later use.

OUTPUT:

The solution to the maxflow problem, i.e. the maximum flow.

---

**Note:**

- If the source or sink vertex does not exist, an `IndexError` is raised.
  - If the source and sink are identical, a `ValueError` is raised.
  - This method raises `MIPSolverException` exceptions when the solution can not be computed for any reason (none exists, or the LP solver was not able to find it, etc...)
- 

EXAMPLE:

```

sage: from sage.numerical.backends.glpk_graph_backend import GLPKGraphBackend
sage: gbe = GLPKGraphBackend()
sage: v = gbe.add_vertices([None for i in range(5)])
sage: edges = ((0, 1, 2), (0, 2, 3), (1, 2, 3), (1, 3, 4),
...           (3, 4, 1), (2, 4, 2))
sage: for a in edges:
...     edge = gbe.add_edge(str(a[0]), str(a[1]), {"cap":a[2]})
sage: gbe.maxflow_ffalg('0', '4')
3.0
sage: gbe.maxflow_ffalg()
3.0
sage: gbe.maxflow_ffalg('0', '8')
Traceback (most recent call last):
...
IndexError: Source or sink vertex does not exist

```

**mincost\_okalg** ()

Finds solution to the mincost problem with the out-of-kilter algorithm.

The out-of-kilter algorithm requires all problem data to be integer valued.

OUTPUT:

The solution to the mincost problem, i.e. the total cost, if operation was successful.

---

**Note:** This method raises `MIPSolverException` exceptions when the solution can not be computed for any reason (none exists, or the LP solver was not able to find it, etc...)

---

EXAMPLE:

```
sage: from sage.numerical.backends.glpk_graph_backend import GLPKGraphBackend
sage: gbe = GLPKGraphBackend()
sage: vertices = (35, 50, 40, -45, -20, -30, -30)
sage: vs = gbe.add_vertices([None for i in range(len(vertices))])
sage: v_dict = {}
sage: for i, v in enumerate(vs):
...     v_dict[v] = vertices[i]
sage: gbe.set_vertices_demand(v_dict.items())
sage: cost = ((8, 6, 10, 9), (9, 12, 13, 7), (14, 9, 16, 5))
sage: lcost = range(len(cost))
sage: lcost_0 = range(len(cost[0]))
sage: for i in lcost:
...     for j in lcost_0:
...         gbe.add_edge(str(i), str(j + len(cost)), {"cost":cost[i][j], "cap":100})
sage: gbe.mincost_okalg()
1020.0
sage: for ed in gbe.edges():
...     print ed[0], "->", ed[1], ed[2]["x"]
0 -> 6 0.0
0 -> 5 25.0
0 -> 4 10.0
0 -> 3 0.0
1 -> 6 0.0
1 -> 5 5.0
1 -> 4 0.0
1 -> 3 45.0
2 -> 6 30.0
2 -> 5 0.0
2 -> 4 10.0
2 -> 3 0.0
```

**set\_vertex\_demand** (*vertex*, *demand*)

Sets the demand of the vertex in a mincost flow algorithm.

INPUT:

- *vertex* – Name of the vertex
- *demand* – the numerical value representing demand of the vertex in a mincost flow algorithm (it could be for instance  $-1$  to represent a sink, or  $1$  to represent a source and  $0$  for a neutral vertex). This can either be an `int` or `float` value.

EXAMPLE:

```
sage: from sage.numerical.backends.glpk_graph_backend import GLPKGraphBackend
sage: gbe = GLPKGraphBackend()
sage: vertices = [None for i in range(3)]
sage: gbe.add_vertices(vertices)
['0', '1', '2']
sage: gbe.set_vertex_demand('0', 2)
sage: gbe.get_vertex('0')['rhs']
2.0
sage: gbe.set_vertex_demand('3', 2)
Traceback (most recent call last):
```

```
...
KeyError: 'Vertex 3 does not exist.'
```

**set\_vertices\_demand(pairs)**

Sets the parameters of selected vertices.

INPUT:

- *pairs* – A list of pairs (vertex, demand) associating a demand to each vertex. For more information, see the documentation of `set_vertex_demand()`.

EXAMPLE:

```
sage: from sage.numerical.backends.glpk_graph_backend import GLPKGraphBackend
sage: gbe = GLPKGraphBackend()
sage: vertices = [None for i in range(3)]
sage: gbe.add_vertices(vertices)
['0', '1', '2']
sage: gbe.set_vertices_demand([('0', 2), ('1', 3), ('3', 4)])
sage: sorted(gbe.get_vertex('1').items())
[('cut', 0), ('es', 0.0), ('ls', 0.0), ('pi', 0.0), ('rhs', 3.0)]
```

**vertices()**

Returns the list of all vertices

---

**Note:** Changing elements of the list will not change anything in the the graph.

---



---

**Note:** If a vertex in the graph does not have a name / label it will appear as None in the resulting list.

---

EXAMPLE:

```
sage: from sage.numerical.backends.glpk_graph_backend import GLPKGraphBackend
sage: gbe = GLPKGraphBackend()
sage: verts = ["A", "B", "C"]
sage: gbe.add_vertices(verts)
sage: a = gbe.vertices(); a
['A', 'B', 'C']
sage: a.pop(0)
'A'
sage: gbe.vertices()
['A', 'B', 'C']
```

**write\_ccdata(fname)**

Writes the graph to a text file in DIMACS format.

Writes the data to plain ASCII text file in DIMACS format. A discription of the DIMACS format can be found at <http://dimacs.rutgers.edu/Challenges/>.

INPUT:

- *fname* – full name of the file

OUTPUT:

Zero if the operations was successful otherwise nonzero

EXAMPLE:

```
sage: from sage.numerical.backends.glpk_graph_backend import GLPKGraphBackend
sage: gbe = GLPKGraphBackend()
```

```
sage: a = gbe.add_edge("0", "1")
sage: gbe.write_ccdata(SAGE_TMP+"/graph.dat")
Writing graph to ...
6 lines were written
0
```

**write\_graph** (*fname*)

Writes the graph to a plain text file

INPUT:

- fname* – full name of the file

OUTPUT:

Zero if the operations was successful otherwise nonzero

EXAMPLE:

```
sage: from sage.numerical.backends.glpk_graph_backend import GLPKGraphBackend
sage: gbe = GLPKGraphBackend()
sage: a = gbe.add_edge("0", "1")
sage: gbe.write_graph(SAGE_TMP+"/graph.txt")
Writing graph to ...
2 lines were written
0
```

**write\_maxflow** (*fname*)

Writes the maximum flow problem data to a text file in DIMACS format.

INPUT:

- fname* – Full name of file

OUTPUT:

Zero if successful, otherwise non-zero

EXAMPLE:

```
sage: from sage.numerical.backends.glpk_graph_backend import GLPKGraphBackend
sage: gbe = GLPKGraphBackend()
sage: gbe.add_vertices([None for i in range(2)])
['0', '1']
sage: a = gbe.add_edge('0', '1')
sage: gbe.maxflow_ffalg('0', '1')
0.0
sage: gbe.write_maxflow(SAGE_TMP+"/graph.max")
Writing maximum flow problem data to ...
6 lines were written
0
sage: gbe = GLPKGraphBackend()
sage: gbe.write_maxflow(SAGE_TMP+"/graph.max")
Traceback (most recent call last):
...
IOError: Cannot write empty graph
```

**write\_mincost** (*fname*)

Writes the mincost flow problem data to a text file in DIMACS format

INPUT:

- fname* – Full name of file



OUTPUT:

Zero if successful, otherwise nonzero

EXAMPLE:

```
sage: from sage.numerical.backends.glpk_graph_backend import GLPKGraphBackend
sage: gbe = GLPKGraphBackend()
sage: a = gbe.add_edge("0", "1")
sage: gbe.write_mincost(SAGE_TMP+"/graph.min")
Writing min-cost flow problem data to ...
4 lines were written
0
```

## 6.4 PPL Backend

PPL Backend

AUTHORS:

- Risan (2012-02): initial implementation

```
class sage.numerical.backends.ppl_backend.PPLBackend
Bases: sage.numerical.backends.generic_backend.GenericBackend
```

`x.__init__(...)` initializes `x`; see `help(type(x))` for signature

**add\_col** (*indices, coeffs*)  
Add a column.

INPUT:

- *indices* (list of integers) – this list contains the indices of the constraints in which the variable's coefficient is nonzero
- *coeffs* (list of real values) – associates a coefficient to the variable in each of the constraints in which it appears. Namely, the *i*th entry of *coeffs* corresponds to the coefficient of the variable in the constraint represented by the *i*th entry in *indices*.

---

**Note:** *indices* and *coeffs* are expected to be of the same length.

---

EXAMPLE:

```
sage: from sage.numerical.backends.generic_backend import get_solver
sage: p = get_solver(solver = "PPL")
sage: p.ncols()
0
sage: p.nrows()
0
sage: p.add_linear_constraints(5, 0, None)
sage: p.add_col(range(5), range(5))
sage: p.nrows()
5
```

**add\_linear\_constraint** (*coefficients, lower\_bound, upper\_bound, name=None*)  
Add a linear constraint.

INPUT:

- `coefficients` – an iterable with  $(c, v)$  pairs where  $c$  is a variable index (integer) and  $v$  is a value (real value).
- `lower_bound` – a lower bound, either a real value or `None`
- `upper_bound` – an upper bound, either a real value or `None`
- `name` – an optional name for this row (default: `None`)

EXAMPLE:

```
sage: from sage.numerical.backends.generic_backend import get_solver
sage: p = get_solver(solver = "PPL")
sage: p.add_variables(5)
4
sage: p.add_linear_constraint(zip(range(5), range(5)), 2.0, 2.0)
sage: p.row(0)
([1, 2, 3, 4], [1, 2, 3, 4])
sage: p.row_bounds(0)
(2.0000000000000000, 2.0000000000000000)
sage: p.add_linear_constraint( zip(range(5), range(5)), 1.0, 1.0, name='foo')
sage: p.row_name(-1)
'foo'
```

**add\_linear\_constraints** (*number, lower\_bound, upper\_bound, names=None*)

Add constraints.

INPUT:

- `number` (integer) – the number of constraints to add.
- `lower_bound` – a lower bound, either a real value or `None`
- `upper_bound` – an upper bound, either a real value or `None`
- `names` – an optional list of names (default: `None`)

EXAMPLE:

```
sage: from sage.numerical.backends.generic_backend import get_solver
sage: p = get_solver(solver = "PPL")
sage: p.add_variables(5)
4
sage: p.add_linear_constraints(5, None, 2)
sage: p.row(4)
([], [])
sage: p.row_bounds(4)
(None, 2)
```

**add\_variable** (*lower\_bound=0, upper\_bound=None, binary=False, continuous=True, integer=False, obj=0, name=None*)

Add a variable.

This amounts to adding a new column to the matrix. By default, the variable is both positive and real.

It has not been implemented for selecting the variable type yet.

INPUT:

- `lower_bound` – the lower bound of the variable (default: 0)
- `upper_bound` – the upper bound of the variable (default: `None`)
- `binary` – True if the variable is binary (default: `False`).

- `continuous` – True if the variable is binary (default: True).
- `integer` – True if the variable is binary (default: False).
- `obj` – (optional) coefficient of this variable in the objective function (default: 0)
- `name` – an optional name for the newly added variable (default: None).

OUTPUT: The index of the newly created variable

EXAMPLE:

```
sage: from sage.numerical.backends.generic_backend import get_solver
sage: p = get_solver(solver = "PPL")
sage: p.ncols()
0
sage: p.add_variable()
0
sage: p.ncols()
1
sage: p.add_variable(lower_bound=-2)
1
sage: p.add_variable(name='x', obj=2/3)
2
sage: p.col_name(2)
'x'
sage: p.objective_coefficient(2)
2/3
```

**add\_variables** (*n*, *lower\_bound*=0, *upper\_bound*=None, *binary*=False, *continuous*=True, *integer*=False, *obj*=0, *names*=None)

Add *n* variables.

This amounts to adding new columns to the matrix. By default, the variables are both positive and real.

It has not been implemented for selecting the variable type yet.

INPUT:

- *n* – the number of new variables (must be > 0)
- *lower\_bound* – the lower bound of the variable (default: 0)
- *upper\_bound* – the upper bound of the variable (default: None)
- *binary* – True if the variable is binary (default: False).
- *continuous* – True if the variable is binary (default: True).
- *integer* – True if the variable is binary (default: False).
- *obj* – (optional) coefficient of all variables in the objective function (default: 0)
- *names* – optional list of names (default: None)

OUTPUT: The index of the variable created last.

EXAMPLE:

```
sage: from sage.numerical.backends.generic_backend import get_solver
sage: p = get_solver(solver = "PPL")
sage: p.ncols()
0
sage: p.add_variables(5)
4
sage: p.ncols()
5
```

```
5
sage: p.add_variables(2, lower_bound=-2.0, names=['a','b'])
6
```

**base\_ring()**

**col\_bounds** (*index*)

Return the bounds of a specific variable.

INPUT:

- *index* (integer) – the variable's id.

OUTPUT:

A pair (*lower\_bound*, *upper\_bound*). Each of them can be set to *None* if the variable is not bounded in the corresponding direction, and is a real value otherwise.

EXAMPLE:

```
sage: from sage.numerical.backends.generic_backend import get_solver
sage: p = get_solver(solver = "PPL")
sage: p.add_variable()
0
sage: p.col_bounds(0)
(0, None)
sage: p.variable_upper_bound(0, 5)
sage: p.col_bounds(0)
(0, 5)
```

**col\_name** (*index*)

Return the *index* th col name

INPUT:

- *index* (integer) – the col's id
- *name* (char \*) – its name. When set to *NULL* (default), the method returns the current name.

EXAMPLE:

```
sage: from sage.numerical.backends.generic_backend import get_solver
sage: p = get_solver(solver = "PPL")
sage: p.add_variable(name="I am a variable")
0
sage: p.col_name(0)
'I am a variable'
```

**get\_objective\_value()**

Return the exact value of the objective function.

---

**Note:** Behaviour is undefined unless *solve* has been called before.

---

EXAMPLE:

```
sage: from sage.numerical.backends.generic_backend import get_solver
sage: p = get_solver(solver = "PPL")
sage: p.add_variables(2)
1
sage: p.add_linear_constraint([(0,1), (1,2)], None, 3)
sage: p.set_objective([2, 5])
sage: p.solve()
```

```

0
sage: p.get_objective_value()
15/2
sage: p.get_variable_value(0)
0
sage: p.get_variable_value(1)
3/2

```

**get\_variable\_value** (*variable*)

Return the value of a variable given by the solver.

---

**Note:** Behaviour is undefined unless `solve` has been called before.

---

## EXAMPLE:

```

sage: from sage.numerical.backends.generic_backend import get_solver
sage: p = get_solver(solver = "PPL")
sage: p.add_variables(2)
1
sage: p.add_linear_constraint([(0,1), (1, 2)], None, 3)
sage: p.set_objective([2, 5])
sage: p.solve()
0
sage: p.get_objective_value()
15/2
sage: p.get_variable_value(0)
0
sage: p.get_variable_value(1)
3/2

```

**init\_mip** ()

Converting the matrix form of the MIP Problem to PPL MIP\_Problem.

## EXAMPLE:

```

sage: from sage.numerical.backends.generic_backend import get_solver
sage: p = get_solver(solver = "PPL")
sage: p.base_ring()
Rational Field
sage: type(p.zero())
<type 'sage.rings.rational.Rational'>
sage: p.init_mip()

```

**is\_maximization** ()

Test whether the problem is a maximization

## EXAMPLE:

```

sage: from sage.numerical.backends.generic_backend import get_solver
sage: p = get_solver(solver = "PPL")
sage: p.is_maximization()
True
sage: p.set_sense(-1)
sage: p.is_maximization()
False

```

**is\_variable\_binary** (*index*)

Test whether the given variable is of binary type.

INPUT:

- index (integer) – the variable's id

EXAMPLE:

```
sage: from sage.numerical.backends.generic_backend import get_solver
sage: p = get_solver(solver = "PPL")
sage: p.ncols()
0
sage: p.add_variable()
0
sage: p.is_variable_binary(0)
False
```

**is\_variable\_continuous** (index)

Test whether the given variable is of continuous/real type.

INPUT:

- index (integer) – the variable's id

EXAMPLE:

```
sage: from sage.numerical.backends.generic_backend import get_solver
sage: p = get_solver(solver = "PPL")
sage: p.ncols()
0
sage: p.add_variable()
0
sage: p.is_variable_continuous(0)
True
```

**is\_variable\_integer** (index)

Test whether the given variable is of integer type.

INPUT:

- index (integer) – the variable's id

EXAMPLE:

```
sage: from sage.numerical.backends.generic_backend import get_solver
sage: p = get_solver(solver = "PPL")
sage: p.ncols()
0
sage: p.add_variable()
0
sage: p.is_variable_integer(0)
False
```

**ncols** ()

Return the number of columns/variables.

EXAMPLE:

```
sage: from sage.numerical.backends.generic_backend import get_solver
sage: p = get_solver(solver = "PPL")
sage: p.ncols()
0
sage: p.add_variables(2)
1
sage: p.ncols()
2
```

**nrows()**

Return the number of rows/constraints.

EXAMPLE:

```
sage: from sage.numerical.backends.generic_backend import get_solver
sage: p = get_solver(solver = "PPL")
sage: p.nrows()
0
sage: p.add_linear_constraints(2, 2.0, None)
sage: p.nrows()
2
```

**objective\_coefficient** (*variable, coeff=None*)

Set or get the coefficient of a variable in the objective function

INPUT:

- *variable* (integer) – the variable's id
- *coeff* (integer) – its coefficient

EXAMPLE:

```
sage: from sage.numerical.backends.generic_backend import get_solver
sage: p = get_solver(solver = "PPL")
sage: p.add_variable()
0
sage: p.objective_coefficient(0)
0
sage: p.objective_coefficient(0, 2)
sage: p.objective_coefficient(0)
2
```

**problem\_name** (*name='NULL'*)

Return or define the problem's name

INPUT:

- *name* (char \*) – the problem's name. When set to NULL (default), the method returns the problem's name.

EXAMPLE:

```
sage: from sage.numerical.backends.generic_backend import get_solver
sage: p = get_solver(solver = "PPL")
sage: p.problem_name("There once was a french fry")
sage: print p.problem_name()
There once was a french fry
```

**row** (*i*)

Return a row

INPUT:

- *index* (integer) – the constraint's id.

OUTPUT:

A pair (*indices*, *coeffs*) where *indices* lists the entries whose coefficient is nonzero, and to which *coeffs* associates their coefficient on the model of the `add_linear_constraint` method.

EXAMPLE:

```
sage: from sage.numerical.backends.generic_backend import get_solver
sage: p = get_solver(solver = "PPL")
sage: p.add_variables(5)
4
sage: p.add_linear_constraint(zip(range(5), range(5)), 2, 2)
sage: p.row(0)
([1, 2, 3, 4], [1, 2, 3, 4])
sage: p.row_bounds(0)
(2, 2)
```

**row\_bounds** (*index*)

Return the bounds of a specific constraint.

INPUT:

- index (integer) – the constraint's id.

OUTPUT:

A pair (lower\_bound, upper\_bound). Each of them can be set to None if the constraint is not bounded in the corresponding direction, and is a real value otherwise.

EXAMPLE:

```
sage: from sage.numerical.backends.generic_backend import get_solver
sage: p = get_solver(solver = "PPL")
sage: p.add_variables(5)
4
sage: p.add_linear_constraint(zip(range(5), range(5)), 2, 2)
sage: p.row(0)
([1, 2, 3, 4], [1, 2, 3, 4])
sage: p.row_bounds(0)
(2, 2)
```

**row\_name** (*index*)

Return the index th row name

INPUT:

- index (integer) – the row's id

EXAMPLE:

```
sage: from sage.numerical.backends.generic_backend import get_solver
sage: p = get_solver(solver = "PPL")
sage: p.add_linear_constraints(1, 2, None, names="Empty constraint 1")
sage: p.row_name(0)
'Empty constraint 1'
```

**set\_objective** (*coeff, d=0*)

Set the objective function.

INPUT:

- coeff – a list of real values, whose ith element is the coefficient of the ith variable in the objective function.

EXAMPLE:

```
sage: from sage.numerical.backends.generic_backend import get_solver
sage: p = get_solver(solver = "PPL")
sage: p.add_variables(5)
4
```



```
sage: p.set_objective([1, 1, 2, 1, 3])
sage: map(lambda x :p.objective_coefficient(x), range(5))
[1, 1, 2, 1, 3]
```

**set\_sense** (*sense*)

Set the direction (maximization/minimization).

INPUT:

- sense (integer) :
  - 1 => Maximization
  - 1 => Minimization

EXAMPLE:

```
sage: from sage.numerical.backends.generic_backend import get_solver
sage: p = get_solver(solver = "PPL")
sage: p.is_maximization()
True
sage: p.set_sense(-1)
sage: p.is_maximization()
False
```

**set\_variable\_type** (*variable*, *vtype*)

Set the type of a variable.

EXAMPLE:

```
sage: from sage.numerical.backends.generic_backend import get_solver
sage: p = get_solver(solver = "PPL")
sage: p.add_variables(5)
4
sage: p.set_variable_type(3, -1)
sage: p.set_variable_type(3, -2)
Traceback (most recent call last):
...
Exception: ...
```

**set\_verbosity** (*level*)

Set the log (verbosity) level. Not Implemented.

EXAMPLE:

```
sage: from sage.numerical.backends.generic_backend import get_solver
sage: p = get_solver(solver = "PPL")
sage: p.set_verbosity(0)
```

**solve** ()

Solve the problem.

---

**Note:** This method raises `MIPSolverException` exceptions when the solution can not be computed for any reason (none exists, or the LP solver was not able to find it, etc...)

---

EXAMPLE:

```
sage: from sage.numerical.backends.generic_backend import get_solver
sage: p = get_solver(solver = "PPL")
sage: p.add_linear_constraints(5, 0, None)
```

```
sage: p.add_col(range(5), range(5))
sage: p.solve()
0
sage: p.objective_coefficient(0,1)
sage: p.solve()
Traceback (most recent call last):
...
MIPSolverException: ...
```

**variable\_lower\_bound**(*index*, *value=False*)

Return or define the lower bound on a variable

INPUT:

- *index* (integer) – the variable’s id
- *value* – real value, or None to mean that the variable has not lower bound. When set to None (default), the method returns the current value.

EXAMPLE:

```
sage: from sage.numerical.backends.generic_backend import get_solver
sage: p = get_solver(solver = "PPL")
sage: p.add_variable()
0
sage: p.col_bounds(0)
(0, None)
sage: p.variable_lower_bound(0, 5)
sage: p.col_bounds(0)
(5, None)
sage: p.variable_lower_bound(0, None)
sage: p.col_bounds(0)
(None, None)
```

**variable\_upper\_bound**(*index*, *value=False*)

Return or define the upper bound on a variable

INPUT:

- *index* (integer) – the variable’s id
- *value* – real value, or None to mean that the variable has not upper bound. When set to None (default), the method returns the current value.

EXAMPLE:

```
sage: from sage.numerical.backends.generic_backend import get_solver
sage: p = get_solver(solver = "PPL")
sage: p.add_variable()
0
sage: p.col_bounds(0)
(0, None)
sage: p.variable_upper_bound(0, 5)
sage: p.col_bounds(0)
(0, 5)
sage: p.variable_upper_bound(0, None)
sage: p.col_bounds(0)
(0, None)
```

**zero**()

## 6.5 CVXOPT Backend

CVXOPT Backend

AUTHORS:

- Ingolfur Edvardsson (2014-05) : initial implementation

**class** `sage.numerical.backends.cvxopt_backend.CVXOPTBackend`

Bases: `sage.numerical.backends.generic_backend.GenericBackend`

`x.__init__(...)` initializes `x`; see `help(type(x))` for signature

**add\_col** (*indices, coeffs*)

Add a column.

INPUT:

- *indices* (list of integers) – this list contains the indices of the constraints in which the variable's coefficient is nonzero
- *coeffs* (list of real values) – associates a coefficient to the variable in each of the constraints in which it appears. Namely, the *i*th entry of *coeffs* corresponds to the coefficient of the variable in the constraint represented by the *i*th entry in *indices*.

---

**Note:** *indices* and *coeffs* are expected to be of the same length.

---

EXAMPLE:

```
sage: from sage.numerical.backends.generic_backend import get_solver
sage: p = get_solver(solver = "CVXOPT")
sage: p.ncols()
0
sage: p.nrows()
0
sage: p.add_linear_constraints(5, 0, None)
sage: p.add_col(range(5), range(5))
sage: p.nrows()
5
```

**add\_linear\_constraint** (*coefficients, lower\_bound, upper\_bound, name=None*)

Add a linear constraint.

INPUT:

- *coefficients* an iterable with (*c*, *v*) pairs where *c* is a variable index (integer) and *v* is a value (real value).
- *lower\_bound* - a lower bound, either a real value or `None`
- *upper\_bound* - an upper bound, either a real value or `None`
- *name* - an optional name for this row (default: `None`)

EXAMPLE:

```
sage: from sage.numerical.backends.generic_backend import get_solver
sage: p = get_solver(solver = "CVXOPT")
sage: p.add_variables(5)
4
sage: p.add_linear_constraint(zip(range(5), range(5)), 2.0, 2.0)
sage: p.row(0)
```

```
([1, 2, 3, 4], [1, 2, 3, 4])
sage: p.row_bounds(0)
(2.000000000000000, 2.000000000000000)
sage: p.add_linear_constraint( zip(range(5), range(5)), 1.0, 1.0, name='foo')
sage: p.row_name(-1)
'foo'
```

**add\_linear\_constraints** (*number, lower\_bound, upper\_bound, names=None*)

Add constraints.

INPUT:

- *number* (integer) – the number of constraints to add.
- *lower\_bound* - a lower bound, either a real value or None
- *upper\_bound* - an upper bound, either a real value or None
- *names* - an optional list of names (default: None)

EXAMPLE:

```
sage: from sage.numerical.backends.generic_backend import get_solver
sage: p = get_solver(solver = "CVXOPT")
sage: p.add_variables(5)
4
sage: p.add_linear_constraints(5, None, 2)
sage: p.row(4)
([], [])
sage: p.row_bounds(4)
(None, 2)
```

**add\_variable** (*lower\_bound=0.0, upper\_bound=None, binary=False, continuous=True, integer=False, obj=None, name=None*)

Add a variable.

This amounts to adding a new column to the matrix. By default, the variable is both positive and real.

INPUT:

- *lower\_bound* - the lower bound of the variable (default: 0)
- *upper\_bound* - the upper bound of the variable (default: None)
- *binary* - True if the variable is binary (default: False).
- *continuous* - True if the variable is binary (default: True).
- *integer* - True if the variable is binary (default: False).
- *obj* - (optional) coefficient of this variable in the objective function (default: 0.0)
- *name* - an optional name for the newly added variable (default: None).

OUTPUT: The index of the newly created variable

EXAMPLE:

```
sage: from sage.numerical.backends.generic_backend import get_solver
sage: p = get_solver(solver = "CVXOPT")
sage: p.ncols()
0
sage: p.add_variable()
0
sage: p.ncols()
1
```

```

1
sage: p.add_variable(binary=True)
1
sage: p.add_variable(lower_bound=-2.0, integer=True)
2
sage: p.add_variable(continuous=True, integer=True)      # optional - CVXOPT
Traceback (most recent call last):
...
ValueError: ...
sage: p.add_variable(name='x',obj=1.0)
3
sage: p.col_name(3)
'x'
sage: p.objective_coefficient(3)
1.0000000000000000

```

**add\_variables** (*n*, *lower\_bound=None*, *upper\_bound=None*, *binary=False*, *continuous=True*, *integer=False*, *obj=None*, *names=None*)

Add *n* variables.

This amounts to adding new columns to the matrix. By default, the variables are both positive and real.

INPUT:

- *n* - the number of new variables (must be > 0)
- *lower\_bound* - the lower bound of the variable (default: 0)
- *upper\_bound* - the upper bound of the variable (default: None)
- *binary* - True if the variable is binary (default: False).
- *continuous* - True if the variable is binary (default: True).
- *integer* - True if the variable is binary (default: False).
- *obj* - (optional) coefficient of all variables in the objective function (default: 0.0)
- *names* - optional list of names (default: None)

OUTPUT: The index of the variable created last.

EXAMPLE:

```

sage: from sage.numerical.backends.generic_backend import get_solver
sage: p = get_solver(solver = "CVXOPT")
sage: p.ncols()
0
sage: p.add_variables(5)
4
sage: p.ncols()
5
sage: p.add_variables(2, lower_bound=-2.0, integer=True, names=['a','b'])
6

```

**col\_bounds** (*index*)

Return the bounds of a specific variable.

INPUT:

- *index* (integer) – the variable's id.

OUTPUT:

A pair (lower\_bound, upper\_bound). Each of them can be set to None if the variable is not bounded in the corresponding direction, and is a real value otherwise.

EXAMPLE:

```
sage: from sage.numerical.backends.generic_backend import get_solver
sage: p = get_solver(solver = "CVXOPT")
sage: p.add_variable()
0
sage: p.col_bounds(0)
(0.0, None)
sage: p.variable_upper_bound(0, 5)
sage: p.col_bounds(0)
(0.0, 5)
```

**col\_name** (*index*)

Return the index th col name

INPUT:

- index (integer) – the col's id
- name (char \*) – its name. When set to NULL (default), the method returns the current name.

EXAMPLE:

```
sage: from sage.numerical.backends.generic_backend import get_solver
sage: p = get_solver(solver = "CVXOPT")
sage: p.add_variable(name="I am a variable")
0
sage: p.col_name(0)
'I am a variable'
```

**get\_objective\_value** ()

Return the value of the objective function.

---

**Note:** Behaviour is undefined unless `solve` has been called before.

---

EXAMPLE:

```
sage: from sage.numerical.backends.generic_backend import get_solver
sage: p = get_solver(solver = "cvxopt")
sage: p.add_variables(2)
1
sage: p.add_linear_constraint([(0,1), (1,2)], None, 3)
sage: p.set_objective([2, 5])
sage: p.solve()
0
sage: round(p.get_objective_value(), 4)
7.5
sage: round(p.get_variable_value(0), 4)
0.0
sage: round(p.get_variable_value(1), 4)
1.5
```

**get\_variable\_value** (*variable*)

Return the value of a variable given by the solver.

---

**Note:** Behaviour is undefined unless `solve` has been called before.

---

EXAMPLE:

```
sage: from sage.numerical.backends.generic_backend import get_solver
sage: p = get_solver(solver = "CVXOPT")
sage: p.add_variables(2)
1
sage: p.add_linear_constraint([(0,1), (1, 2)], None, 3)
sage: p.set_objective([2, 5])
sage: p.solve()
0
sage: round(p.get_objective_value(),4)
7.5
sage: round(p.get_variable_value(0),4)
0.0
sage: round(p.get_variable_value(1),4)
1.5
```

**is\_maximization()**

Test whether the problem is a maximization

EXAMPLE:

```
sage: from sage.numerical.backends.generic_backend import get_solver
sage: p = get_solver(solver = "CVXOPT")
sage: p.is_maximization()
True
sage: p.set_sense(-1)
sage: p.is_maximization()
False
```

**is\_variable\_binary** (*index*)

Test whether the given variable is of binary type.

INPUT:

- *index* (integer) – the variable's id

EXAMPLE:

```
sage: from sage.numerical.backends.generic_backend import get_solver
sage: p = get_solver(solver = "CVXOPT")
sage: p.ncols()
0
sage: p.add_variable()
0
sage: p.set_variable_type(0,0) # optional - CVXOPT
sage: p.is_variable_binary(0) # optional - CVXOPT
True
```

**is\_variable\_continuous** (*index*)

Test whether the given variable is of continuous/real type.

INPUT:

- *index* (integer) – the variable's id

EXAMPLE:

```
sage: from sage.numerical.backends.generic_backend import get_solver
sage: p = get_solver(solver = "CVXOPT")
sage: p.ncols()
0
sage: p.add_variable()
```

```
0
sage: p.is_variable_continuous(0)           # optional - CVXOPT
True
sage: p.set_variable_type(0,1)             # optional - CVXOPT
sage: p.is_variable_continuous(0)         # optional - CVXOPT
False
```

**is\_variable\_integer** (*index*)

Test whether the given variable is of integer type.

INPUT:

- index (integer) – the variable's id

EXAMPLE:

```
sage: from sage.numerical.backends.generic_backend import get_solver
sage: p = get_solver(solver = "CVXOPT")    # optional - CVXOPT
sage: p.ncols()                           # optional - CVXOPT
0
sage: p.add_variable()                    # optional - CVXOPT
1
sage: p.set_variable_type(0,1)            # optional - CVXOPT
sage: p.is_variable_integer(0)            # optional - CVXOPT
True
```

**ncols** ()

Return the number of columns/variables.

EXAMPLE:

```
sage: from sage.numerical.backends.generic_backend import get_solver
sage: p = get_solver(solver = "CVXOPT")
sage: p.ncols()
0
sage: p.add_variables(2)
1
sage: p.ncols()
2
```

**nrows** ()

Return the number of rows/constraints.

EXAMPLE:

```
sage: from sage.numerical.backends.generic_backend import get_solver
sage: p = get_solver(solver = "CVXOPT")
sage: p.nrows()
0
sage: p.add_variables(5)
4
sage: p.add_linear_constraints(2, 2.0, None)
sage: p.nrows()
2
```

**objective\_coefficient** (*variable*, *coeff=None*)

Set or get the coefficient of a variable in the objective function

INPUT:

- variable (integer) – the variable's id



- coeff (double) – its coefficient

EXAMPLE:

```
sage: from sage.numerical.backends.generic_backend import get_solver
sage: p = get_solver(solver = "CVXOPT")
sage: p.add_variable()
0
sage: p.objective_coefficient(0)
0.0
sage: p.objective_coefficient(0,2)
sage: p.objective_coefficient(0)
2.0
```

**problem\_name** (name='NULL')

Return or define the problem's name

INPUT:

- name (char \*) – the problem's name. When set to NULL (default), the method returns the problem's name.

EXAMPLE:

```
sage: from sage.numerical.backends.generic_backend import get_solver
sage: p = get_solver(solver = "CVXOPT")
sage: p.problem_name("There once was a french fry")
sage: print p.problem_name()
There once was a french fry
```

**row** (i)

Return a row

INPUT:

- index (integer) – the constraint's id.

OUTPUT:

A pair (indices, coeffs) where indices lists the entries whose coefficient is nonzero, and to which coeffs associates their coefficient on the model of the add\_linear\_constraint method.

EXAMPLE:

```
sage: from sage.numerical.backends.generic_backend import get_solver
sage: p = get_solver(solver = "CVXOPT")
sage: p.add_variables(5)
4
sage: p.add_linear_constraint(zip(range(5), range(5)), 2, 2)
sage: p.row(0)
([1, 2, 3, 4], [1, 2, 3, 4])
sage: p.row_bounds(0)
(2, 2)
```

**row\_bounds** (index)

Return the bounds of a specific constraint.

INPUT:

- index (integer) – the constraint's id.

OUTPUT:

A pair (lower\_bound, upper\_bound). Each of them can be set to None if the constraint is not bounded in the corresponding direction, and is a real value otherwise.

EXAMPLE:

```
sage: from sage.numerical.backends.generic_backend import get_solver
sage: p = get_solver(solver = "CVXOPT")
sage: p.add_variables(5)
4
sage: p.add_linear_constraint(zip(range(5), range(5)), 2, 2)
sage: p.row(0)
([1, 2, 3, 4], [1, 2, 3, 4])
sage: p.row_bounds(0)
(2, 2)
```

**row\_name** (index)

Return the index th row name

INPUT:

- index (integer) – the row's id

EXAMPLE:

```
sage: from sage.numerical.backends.generic_backend import get_solver
sage: p = get_solver(solver = "CVXOPT")
sage: p.add_linear_constraints(1, 2, None, names="Empty constraint 1")
sage: p.row_name(0)
'Empty constraint 1'
```

**set\_objective** (coeff, d=0.0)

Set the objective function.

INPUT:

- coeff – a list of real values, whose ith element is the coefficient of the ith variable in the objective function.
- d (double) – the constant term in the linear function (set to 0 by default)

EXAMPLE:

```
sage: from sage.numerical.backends.generic_backend import get_solver
sage: p = get_solver(solver = "CVXOPT")
sage: p.add_variables(5)
4
sage: p.set_objective([1, 1, 2, 1, 3])
sage: map(lambda x : p.objective_coefficient(x), range(5))
[1, 1, 2, 1, 3]
```

**set\_sense** (sense)

Set the direction (maximization/minimization).

INPUT:

- sense (integer) :
  - +1 => Maximization
  - 1 => Minimization

EXAMPLE:

```

sage: from sage.numerical.backends.generic_backend import get_solver
sage: p = get_solver(solver = "CVXOPT")
sage: p.is_maximization()
True
sage: p.set_sense(-1)
sage: p.is_maximization()
False

```

**set\_variable\_type** (*variable*, *vtype*)

Set the type of a variable.

EXAMPLE:

```

sage: from sage.numerical.backends.generic_backend import get_solver
sage: p = get_solver(solver = "cvxopt")
sage: p.add_variables(5)
4
sage: p.set_variable_type(3, -1)
sage: p.set_variable_type(3, -2)
Traceback (most recent call last):
...
Exception: ...

```

**set\_verbosity** (*level*)

Does not apply for the cvxopt solver

**solve** ()

Solve the problem.

---

**Note:** This method raises `MIPSolverException` exceptions when the solution can not be computed for any reason (none exists, or the LP solver was not able to find it, etc...)

---

EXAMPLE:

```

sage: p = MixedIntegerLinearProgram(solver = "cvxopt", maximization=False)
sage: x=p.new_variable(nonnegative=True)
sage: p.set_objective(-4*x[0] - 5*x[1])
sage: p.add_constraint(2*x[0] + x[1] <= 3)
sage: p.add_constraint(2*x[1] + x[0] <= 3)
sage: round(p.solve(), 2)
-9.0
sage: p = MixedIntegerLinearProgram(solver = "cvxopt", maximization=False)
sage: x=p.new_variable(nonnegative=True)
sage: p.set_objective(x[0] + 2*x[1])
sage: p.add_constraint(-5*x[0] + x[1] <= 7)
sage: p.add_constraint(-5*x[0] + x[1] >= 7)
sage: p.add_constraint(x[0] + x[1] >= 26 )
sage: p.add_constraint( x[0] >= 3)
sage: p.add_constraint( x[1] >= 4)
sage: round(p.solve(), 2)
48.83
sage: p = MixedIntegerLinearProgram(solver = "cvxopt")
sage: x=p.new_variable(nonnegative=True)
sage: p.set_objective(x[0] + x[1] + 3*x[2])
sage: p.solver_parameter("show_progress", True)
sage: p.add_constraint(x[0] + 2*x[1] <= 4)
sage: p.add_constraint(5*x[2] - x[1] <= 8)
sage: round(p.solve(), 2)

```

```
...
      pcost      dcost      gap      pres      dres      k/t
...
8.8
sage: #CVXOPT gives different values for variables compared to the other solvers.
sage: c = MixedIntegerLinearProgram(solver = "cvxopt")
sage: p = MixedIntegerLinearProgram(solver = "ppl")
sage: g = MixedIntegerLinearProgram()
sage: xc=c.new_variable(nonnegative=True)
sage: xp=p.new_variable(nonnegative=True)
sage: xg=g.new_variable(nonnegative=True)
sage: c.set_objective(xc[2])
sage: p.set_objective(xp[2])
sage: g.set_objective(xg[2])
sage: #we create a cube for all three solvers
sage: c.add_constraint(xc[0] <= 100)
sage: c.add_constraint(xc[1] <= 100)
sage: c.add_constraint(xc[2] <= 100)
sage: p.add_constraint(xp[0] <= 100)
sage: p.add_constraint(xp[1] <= 100)
sage: p.add_constraint(xp[2] <= 100)
sage: g.add_constraint(xg[0] <= 100)
sage: g.add_constraint(xg[1] <= 100)
sage: g.add_constraint(xg[2] <= 100)
sage: round(c.solve(),2)
100.0
sage: round(c.get_values(xc[0]),2)
50.0
sage: round(c.get_values(xc[1]),2)
50.0
sage: round(c.get_values(xc[2]),2)
100.0
sage: round(p.solve(),2)
100.0
sage: round(p.get_values(xp[0]),2)
0.0
sage: round(p.get_values(xp[1]),2)
0.0
sage: round(p.get_values(xp[2]),2)
100.0
sage: round(g.solve(),2)
100.0
sage: round(g.get_values(xg[0]),2)
0.0
sage: round(g.get_values(xg[1]),2)
0.0
sage: round(g.get_values(xg[2]),2)
100.0
```

**solver\_parameter** (name, value=None)

Return or define a solver parameter

INPUT:

- name (string) – the parameter
- value – the parameter’s value if it is to be defined, or None (default) to obtain its current value.

---

**Note:** The list of available parameters is available at `solver_parameter()`.

EXAMPLE:

```
sage: from sage.numerical.backends.generic_backend import get_solver
sage: p = get_solver(solver = "CVXOPT")
sage: p.solver_parameter("show_progress")
False
sage: p.solver_parameter("show_progress", True)
sage: p.solver_parameter("show_progress")
True
```

**variable\_lower\_bound**(*index*, *value=None*)

Return or define the lower bound on a variable

INPUT:

- *index* (integer) – the variable's id
- *value* – real value, or None to mean that the variable has not lower bound. When set to None (default), the method returns the current value.

EXAMPLE:

```
sage: from sage.numerical.backends.generic_backend import get_solver

sage: p = get_solver(solver = "CVXOPT")
sage: p.add_variable()
0
sage: p.col_bounds(0)
(0.0, None)
sage: p.variable_lower_bound(0, 5)
sage: p.col_bounds(0)
(5, None)
```

**variable\_upper\_bound**(*index*, *value=None*)

Return or define the upper bound on a variable

INPUT:

- *index* (integer) – the variable's id
- *value* – real value, or None to mean that the variable has not upper bound. When set to None (default), the method returns the current value.

EXAMPLE:

```
sage: from sage.numerical.backends.generic_backend import get_solver

sage: p = get_solver(solver = "CVXOPT")
sage: p.add_variable()
0
sage: p.col_bounds(0)
(0.0, None)
sage: p.variable_upper_bound(0, 5)
sage: p.col_bounds(0)
(0.0, 5)
```

Sage also supports CBC (COIN-OR), CPLEX (ILOG), Gurobi, CVXOPT and PPL. In order to find out how to use them in Sage, please refer to the [Thematic Tutorial on Linear Programming](#).



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