Variable (mathematics)

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In elementary mathematics, a **variable** is an alphabetic character representing a number, the **value** of the variable, which is either arbitrary or not fully specified or unknown. Making algebraic computations with variables as if they were explicit numbers allows one to solve a range of problems in a single computation. A typical example is the quadratic formula, which allows one to solve every quadratic equation by simply substituting the numeric values of the coefficients of the given equation to the variables that represent them.

The concept of **variable** is also fundamental in calculus. Typically, a function $y = f(x)$ involves two variables, y and x, representing respectively the value and the argument of the function. The term "variable" comes from the fact that, when the argument (also called the "variable of the function") *varies*, then the value *varies* accordingly.[1]

In more advanced mathematics, a **variable** is a symbol that denotes a mathematical object, which could be a number, a vector, a matrix, or even a function. In this case, the original property of "variability" of a variable is not kept (except, sometimes, for informal explanations).

Similarly, in computer science, a **variable** is a name (commonly an alphabetic character or a word) representing some value represented in computer memory. In mathematical logic, a **variable** is either a symbol representing an unspecified term of the theory, or a basic object of the theory, which is manipulated without referring to its possible intuitive interpretation.

Contents

Genesis and evolution of the concept

François Viète introduced at the end of 16th century the idea of representing known and unknown numbers by letters, nowadays called variables, and of computing with them as if they were numbers, in order to obtain, at the end, the result by a simple replacement. François Viète's convention was to use consonants for known values and vowels for unknowns.[2]

In 1637, René Descartes "invented the convention of representing unknowns in equations by *x*, *y*, and *z*, and knowns by *a*, *b*, and *c*".[3] Contrarily to Viète's convention, Descartes' one is still commonly in use.

Starting in the 1660s, Isaac Newton and Gottfried Wilhelm Leibniz independently developed the infinitesimal calculus, which essentially consists of studying how an infinitesimal variation of a *variable quantity* induces a corresponding variation of another quantity which is a *function* of the first variable (quantity). Almost a century later Leonhard Euler fixed the terminology of infinitesimal calculus and introduced the notation $y = f(x)$ for a function f, its **variable** x and its value y. Until the end of the 19th century, the word *variable* referred almost exclusively to the arguments and the values of functions.

In the second half of the 19th century, it appeared that the foundation of infinitesimal calculus was not formalized enough to deal with apparent paradoxes such as a continuous function which is nowhere differentiable. To solve this problem, Karl Weierstrass introduced a new formalism consisting of replacing the intuitive notion of limit by a formal definition. The older notion of limit was "when the *variable* x varies and tends toward a, then $f(x)$ tends toward L", without any accurate definition of "tends". Weierstrass replaced this sentence by the formula

$$(\forall\epsilon > 0)(\exists\eta > 0)(\forall x)\ |x - a| < \eta \Rightarrow |L - f(x)| < \epsilon,$$

in which none of the five variables is considered as varying.

This static formulation led to the modern notion of variable which is simply a symbol representing a mathematical object which either is unknown or may be replaced by any element of a given set; for example, the set of real numbers.

Specific kinds of variables

It is common that many variables appear in the same mathematical formula, which play different roles. Some names or qualifiers have been introduced to distinguish them. For example, in the general cubic equation

$$ax^3 + bx^2 + cx + d = 0,$$

there are five variables. Four of them, a, b, c, d represent given numbers, and the last one, x, represents the *unknown* number, which is a solution of the equation. To distinguish them, the variable x is called *a unknown*, and the other variables are called *parameters* or *coefficients*, or sometimes *constants*, although this last terminology is incorrect for an equation and should be reserved for the function defined by the left-hand side of this equation.

In the context of functions, the term *variable* refers commonly to the arguments of the functions. This is typically the case in sentences like "function of a real variable", "x is the variable of the function $f: x \mapsto f(x)$", "f is a function of the variable x" (meaning that the argument of the function is referred to by the variable x).

In the same context, the variables that are independent of x define constant functions and are therefore called *constant*. For example, a *constant of integration* is an arbitrary constant function that is added to a particular antiderivative to obtain the other antiderivatives. Because the strong relationship between polynomials and polynomial function, the term "constant" is often used to denote the coefficients of a polynomial, which are constant functions of the indeterminates.

This use of "constant" as an abbreviation of "constant function" must be distinguished from the normal meaning of the word in mathematics. A **constant**, or **mathematical constant** is a well and unambiguously defined number or other mathematical object, as, for example, the numbers 0, 1, π and the identity element of a group.

Here are other specific names for variables.

- A **unknown** is a variable in which an equation has to be solved for.
- An **indeterminate** is a symbol, commonly called variable, that appears in a polynomial or a formal power series. Formally speaking, an indeterminate is not a variable, but a constant in the polynomial ring of the ring of formal power series. However, because of the strong relationship between polynomials or power series and the functions that they define, many authors consider indeterminates as a special kind of variables.
- A **parameter** is a quantity (usually a number) which is a part of the input of a problem, and remains constant during the whole solution of this problem. For example, in mechanics the mass and the size of a solid body are *parameters* for the study of its movement. It should be noted that in computer science, *parameter* has a different meaning and denotes an argument of a function.
- **Free variables and bound variables**
- A **random variable** is a kind of variable that is used in probability theory and its applications.

It should be emphasized that all these denominations of variables are of semantic nature and that the way of computing with them (syntax) is the same for all.

Dependent and independent variables

In calculus and its application to physics and other sciences, it is rather common to consider a variable, say y, whose possible values depend of the value of another variable, say x. In mathematical terms, the *dependent* variable y represents the value of a function of x. To simplify formulas, it is often useful to use the same symbol for the dependent variable y and the function mapping x onto y. For example, the state of a physical system depends on measurable quantities such as the pressure, the temperature, the spatial position, ..., and all these quantities varies when the system evolves, that is, they are function of the time. In the formulas describing the system, these quantities are represented by variables which are dependent on the time, and thus considered implicitly as functions of the time.

Therefore, in a formula, a **dependent variable** is a variable that is implicitly a function of another (or several other) variables. An **independent variable** is a variable that is not dependent.[4]

The property of a variable to be dependent or independent depends often of the point of view and is not intrinsic. For example, in the notation $f(x, y, z)$, the three variables may be all independent and the notation represents a function of three variables. On the other hand, if y and z depend on x (are *dependent variables*) then the notation represent a function of the single *independent variable* x.[5]

Examples

If one defines a function *f* from the real numbers to the real numbers by

$$f(x) = x^2 + \sin(x + 4)$$

then *x* is a variable standing for the argument of the function being defined, which can be any real number. In the identity

$$\sum_{i=1}^{n} i = \frac{n^2 + n}{2}$$

the variable *i* is a summation variable which designates in turn each of the integers 1, 2, ..., *n* (it is also called **index** because its variation is over a discrete set of values) while *n* is a parameter (it does not vary within the formula).

In the theory of polynomials, a polynomial of degree 2 is generally denoted as $ax^2 + bx + c$, where *a*, *b* and *c* are called coefficients (they are assumed to be fixed, i.e., parameters of the problem considered) while *x* is called a variable. When studying this polynomial for its polynomial function this *x* stands for the function argument. When studying the polynomial as an object in itself, *x* is taken to be an indeterminate, and would often be written with a capital letter instead to indicate this status.

Notation

In mathematics, the variables are generally denoted by a single letter. However, this letter is frequently followed by a subscript, as in x_2, and this subscript may be a number, another variable (x_i), a word or the abbreviation of a word (x_{in} and x_{out}), and even a mathematical expression. Under the influence of computer science, one may encounter in pure mathematics some variable names consisting in several letters and digits.

Following the 17th century French philosopher and mathematician, René Descartes, letters at the beginning of the alphabet, e.g. *a*, *b*, *c* are commonly used for known values and parameters, and letters at the end of the alphabet, e.g. *x*, *y*, *z*, and *t* are commonly used for unknowns and variables of functions.[6] In printed mathematics, the norm is to set variables and constants in an italic typeface.[7]

For example, a general quadratic function is conventionally written as:

$$ax^2 + bx + c\,,$$

where *a*, *b* and *c* are parameters (also called constants, because they are constant functions), while *x* is the variable of the function. A more explicit way to denote this function is

$$x \mapsto ax^2 + bx + c\,,$$

which makes the function-argument status of *x* clear, and thereby implicitly the constant status of *a*, *b* and *c*. Since *c* occurs in a term that is a constant function of *x*, it is called the constant term.[8]:18

Specific branches and applications of mathematics usually have specific naming conventions for variables. Variables with similar roles or meanings are often assigned consecutive letters. For example, the three axes in 3D coordinate space are conventionally called *x*, *y*, and *z*. In physics, the names of variables are largely determined by the physical quantity they describe, but various naming conventions exist. A convention often followed in probability and statistics is to use *X*, *Y*, *Z* for the names of random variables, keeping *x*, *y*, *z* for variables representing corresponding actual values.

There are many other notational usages. Usually, variables that play a similar role are represented by consecutive letters or by the same letter with different subscript. Below are some of the most common usages.

- *a*, *b*, *c*, and *d* (sometimes extended to *e* and *f*) often represent parameters or coefficients.
- a_0, a_1, a_2, ... play a similar role, when otherwise too many different letters would be needed.
- a_i or u_i is often used to denote the *i*-th term of a sequence or the *i*-th coefficient of a series.
- *f* and *g* (sometimes *h*) commonly denote functions.

- i, j, and k (sometimes l or h) are often used to denote varying integers or indices in an indexed family.
- l and w are often used to represent the length and width of a figure.
- l is also used to denote a line. In number theory, l often denotes a prime number not equal to p.
- n usually denotes a fixed integer, such as a count of objects or the degree of an equation.
  - When two integers are needed, for example for the dimensions of a matrix, one uses commonly m and n.
- p often denotes a prime numbers or a probability.
- q often denotes a prime power or a quotient
- r often denotes a remainder.
- x, y and z usually denote the three Cartesian coordinates of a point in Euclidean geometry. By extension, they are used to name the corresponding axes.
- z typically denotes a complex number, or, in statistics, a normal random variable.
- α, β, γ, θ and φ commonly denote angle measures.
- ϵ usually represents an arbitrarily small positive number.
  - ϵ and δ commonly denote two small positives.
- λ is used for eigenvalues.
- σ often denotes a sum, or, in statistics, the standard deviation.

See also

- Free variables and bound variables (Bound variables are also known as dummy variables)
- Variable (programming)
- Mathematical expression
- Physical constant
- Coefficient
- Constant of integration
- Constant term of a polynomial
- Indeterminate (variable)
- Lambda calculus

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