Project #2

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This project is inspired by the case study
"Chuck A. Luck Wagers a Buck: Probabilistic Reasoning and the Gambler's Ruin" by Christopher M. Rump.

Consider the game of *Chuck-a-Luck*. Here is how it's played as told by Mr. Daniel Reisman of Niverville, New York:

"Three dice are rolled in a wire cage. You place a bet on any number from 1 to 6. If any one of the three dice comes up with your number, you win the amount of your bet. (You also get your original stake back.) If more than one die comes up with your number, you win the amount of your bet for each match. For example, if you had a $1 bet on number 5, and each of the three dice came up with 5, you would win $3."

Problem #1 (3 points)

What is the probability that the gambler wins the game?

P(one match) = $\frac{3!}{(3-1)!1!} * (\frac{1}{6})^1 * (\frac{5}{6})^2 = 0.3472$
P(two matches) = $\frac{3!}{(3-2)!2!} * (\frac{1}{6})^2 * (\frac{5}{6})^1 = 0.06944$
P(three matches) = $\frac{3!}{(3-3)!3!} * (\frac{1}{6})^3 * (\frac{5}{6})^0 = 0.04630$

P(wins) = P(one match) + P(two matches) + P(three matches)
= 0.3472 + 0.06944 + 0.004630
= 0.4213

Problem #2 (5 points)

Who has the long-run advantage in the game: the house or the gambler? By how much on the dollar?

The house has the long-run advantage.

Total combinations of three die: 6 * 6 * 6 = 216
Number of zero matches = (5 * 5 * 5) * 3 = 125
Number of one match = (1 * 5 * 5) * 3 = 75
Number of two matches = (1 * 1 * 5) * 3 = 15
Number of three matches = (1 * 1 * 1) * 3 = 1

E[Winnings]= ($3)P(three matches) + ($2)P(two matches) + ($1)P(one match) + (-$1)P(zero matches)
= ($3) $(\frac{1}{216})$ + ($2) $(\frac{15}{216})$ + ($1) $(\frac{75}{216})$ + (-$1) $(\frac{125}{216})$
= -$0.0787

The gambler would lose about 8 cents on the dollar. So, the house would have the advantage and win by about 8 cents on the dollar.

Problem #3 (8 points)

How would you change the amount of the winnings in every round of *Chuck-a-Luck* to make the game fair? Say that doubles pay $$a$ per $1 bet rather than $2 and triples pay $$b$ per $1 bet in stead of $3. What is the condition on a and b?

It is fair if the expected value is 0. So... (b) ($\frac{1}{216}$) + (a) ($\frac{15}{216}$) + (1) ($\frac{75}{216}$) + (-1) ($\frac{125}{216}$) = $0
Simplified: 15a - b = 50
a = $3, b = $5

Problem #4 (10 points)

Get three regular dice or use software to create simulated values of the outcomes of rounds of *Chuck-a-Luck* with the original reward system. Play 100 independent rounds of the game, starting with a new dollar each time. What is the average of the gambler's winnings in these 100 rounds?

We simulated the 100 trials with three die in real life and got an average winning amount of -$0.075.

Problem #5 (2 points)

Take another look at your answers to the above problems. How do they relate to the Strong Law of Large Numbers? If you forgot about this theorem you studied in *Probability*, see Wikipedia.

The law of large numbers says that a large number of trials will tend to average a result that is closer to the expected value. Our answers to problems 2.2 and 2.4 were -0.0787 and -0.075, respectively. This relates to the law because the answer to 2.2 was our expected result, and running a large number of trials in 2.4 gave us a result that was close to the expected value.

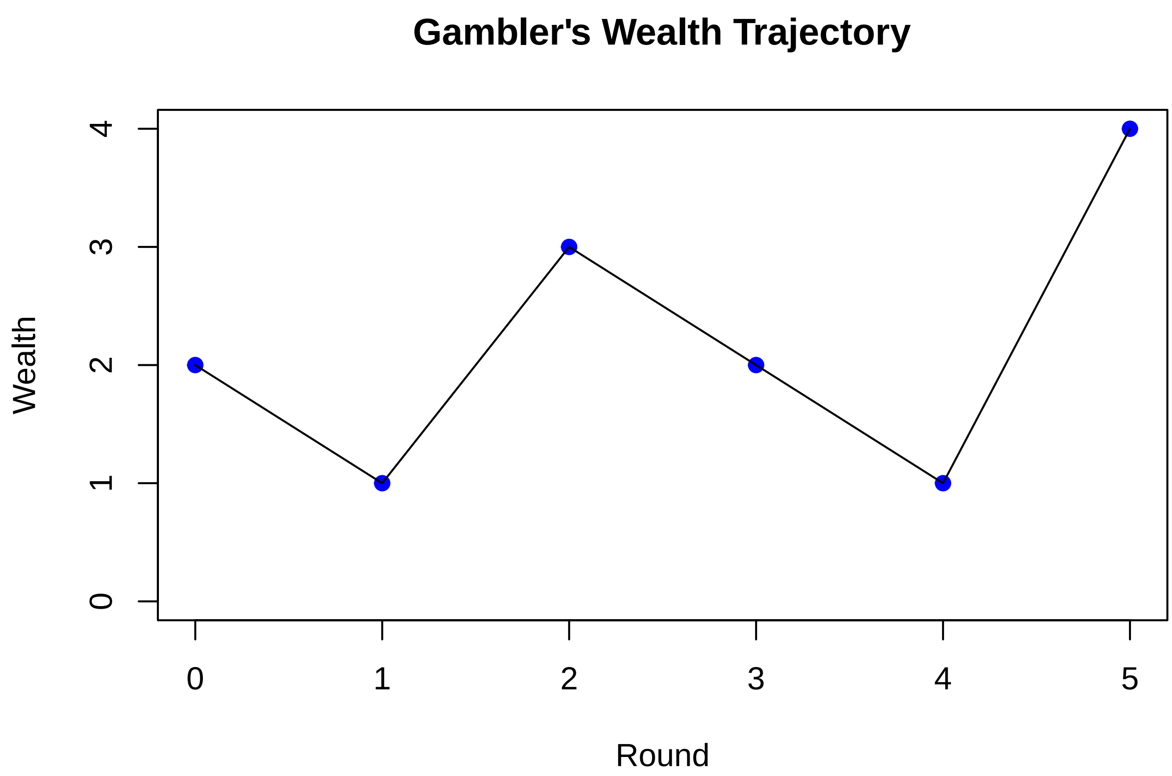
Problem #6 (10 points)

Now we are going to look at a particular gambler who arrives at the casino with $2 in his pocket intending to play *Chuck2-a-Luck*. At the beginning of every round of the game, he invests exactly $1 (if he has the money). He continues to play round after round of the game until he either loses everything, or his wealth reaches at least $4.

Again, you can use real dice or software to play the game. Keep track of the gambler's wealth as the rounds of *Chuck-a-Luck* are repeated until the entire string of rounds is ended by either bankruptcy, or his wealth being at least $4. Display the gambler's wealth trajectory graphically.

We simulated the game in real life and got the wealth trajectory below:

```
project2question6.data <- read.csv("project2.csv")
plot(project2question6.data,
     main="Gambler's Wealth Trajectory",
     xlab="Round",
     ylab="Wealth",
     xlim=c(0,5),
     ylim=c(0,4),
     pch=19,
     col="blue")
lines(project2question6.data)
```



Problem #7 (22 points)

Having completed the above single run, now repeat the same exercise 100 times. More precisely, you can imagine letting 100 gamblers enter the casino with \$2 each and play rounds of *Chuck-a-Luck* as above. This time you do not need to keep track of each gambler's wealth as the game-play progresses. For each gambler you should just record:

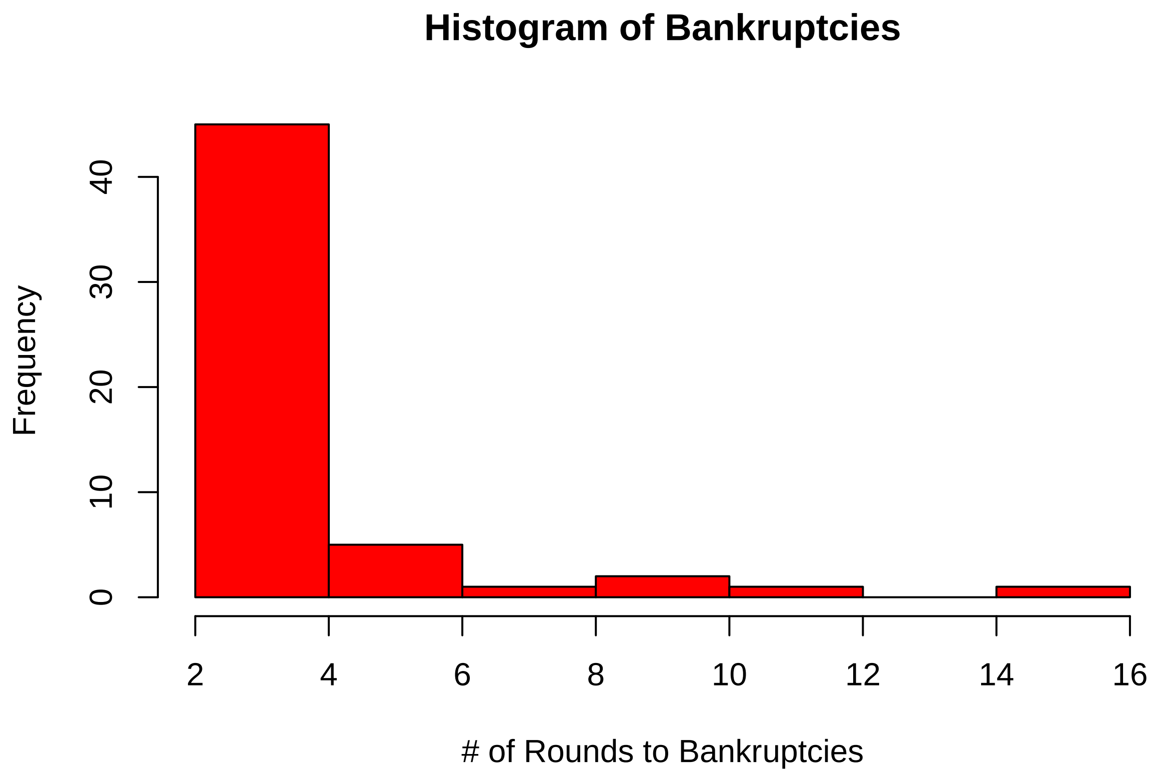
- the **number of rounds of the game** it took them to either go bankrupt or emerge victorious with at least \$4;
- whether the **final result** was bankruptcy or not.

What is the average number of rounds the 100 gamblers played? What is the proportion of gamblers who finished with at least \$4 in their pocket? Draw the histogram of the number of rounds played for the players who went bankrupt only, the histogram of the number of rounds played for the remaining players, and the histogram of the number of rounds played for all players.

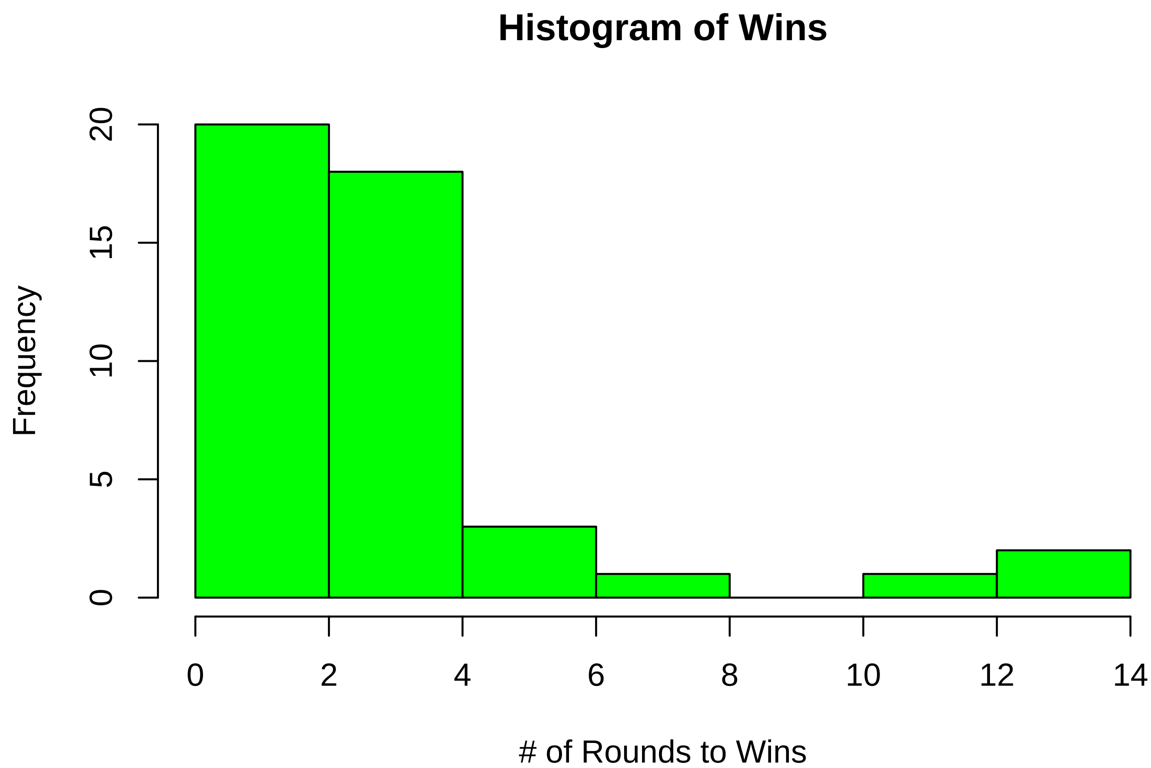
The average number of rounds the 100 gamblers played was mean(c(loss,win)) = 4.2. The proportion of gamblers who finished with at least \$4 in their pocket was

```
initial = 0
gambler = 0
loss = c()
win = c()
```

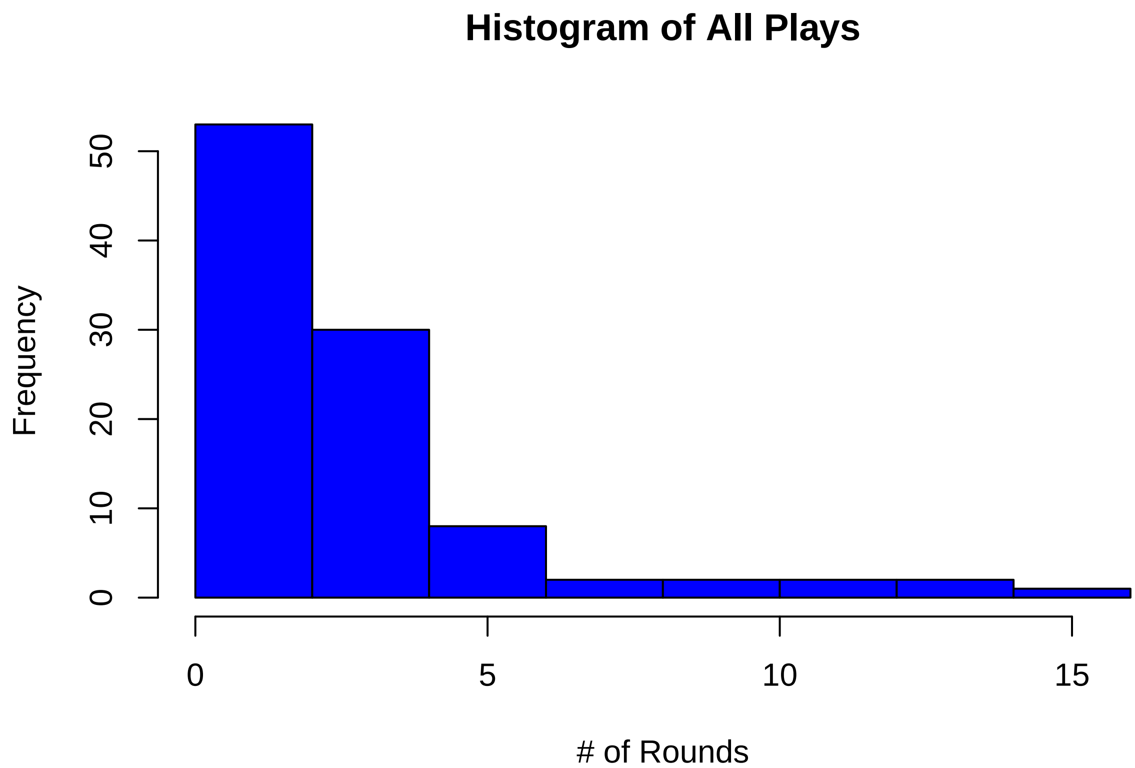
```r
while (initial < 100){
  wealth = 2
  i = 0
  round = 0

  initial = initial + 1

  while (wealth > 0 & wealth < 4){
    roll = rbinom(1, 3, 1/6)
    round = round + 1
    if (roll == 0){
      wealth = wealth - 1
    }
    else{
      wealth = wealth + roll
    }
    if (wealth == 0){
      loss = append(loss, round)
    }
    else{
      if (wealth > 3){
        win = append(win, round)
      }
    }
  }
  if (wealth >= 4){
      gambler = gambler + 1
  }
}

hist(loss,
     main="Histogram of Bankruptcies",
     xlab="# of Rounds to Bankruptcies",
     ylab="Frequency",
     col = "red")
```




```
hist(win,
     main="Histogram of Wins",
     xlab="# of Rounds to Wins",
     ylab="Frequency",
     col = "green")
```




```
hist(c(loss, win),
     main="Histogram of All Plays",
     xlab="# of Rounds",
     ylab="Frequency",
     col = "blue")
```




```
writeLines("The average number of rounds the 100 gamblers played was:")
## The average number of rounds the 100 gamblers played was:
print(mean(c(loss,win)))
## [1] 3.59
writeLines("The proportion of gamblers who finished with at least $4 in their pocket was:")
## The proportion of gamblers who finished with at least $4 in their pocket was:
print(gambler/100)
## [1] 0.45
```