Economic Forecast of GDP and Unemployment Rates

4/24/2022

```
devtools::install_github("ccolonescu/POE5Rdata", force=TRUE)
```

```
## Downloading GitHub repo ccolonescu/POE5Rdata@HEAD

## * checking for file '/private/var/folders/hn/_p299s4n70q31_q8bj97qvrw0000gn/T/RtmpyE78dx/remotes55fc3
## * preparing 'POE5Rdata':
## * checking DESCRIPTION meta-information ... OK
## * checking for LF line-endings in source and make files and shell scripts
## * checking for empty or unneeded directories
## * building 'POE5Rdata_0.1.0.tar.gz'
```

```
library(POE5Rdata)
```

```
##
## Attaching package: 'POE5Rdata'

## The following object is masked from 'package:datasets':
##
##     euro
```

```
library(tseries)
```

```
## Registered S3 method overwritten by 'quantmod':
##   method            from
##   as.zoo.data.frame zoo
```

```
library(forecast)
```

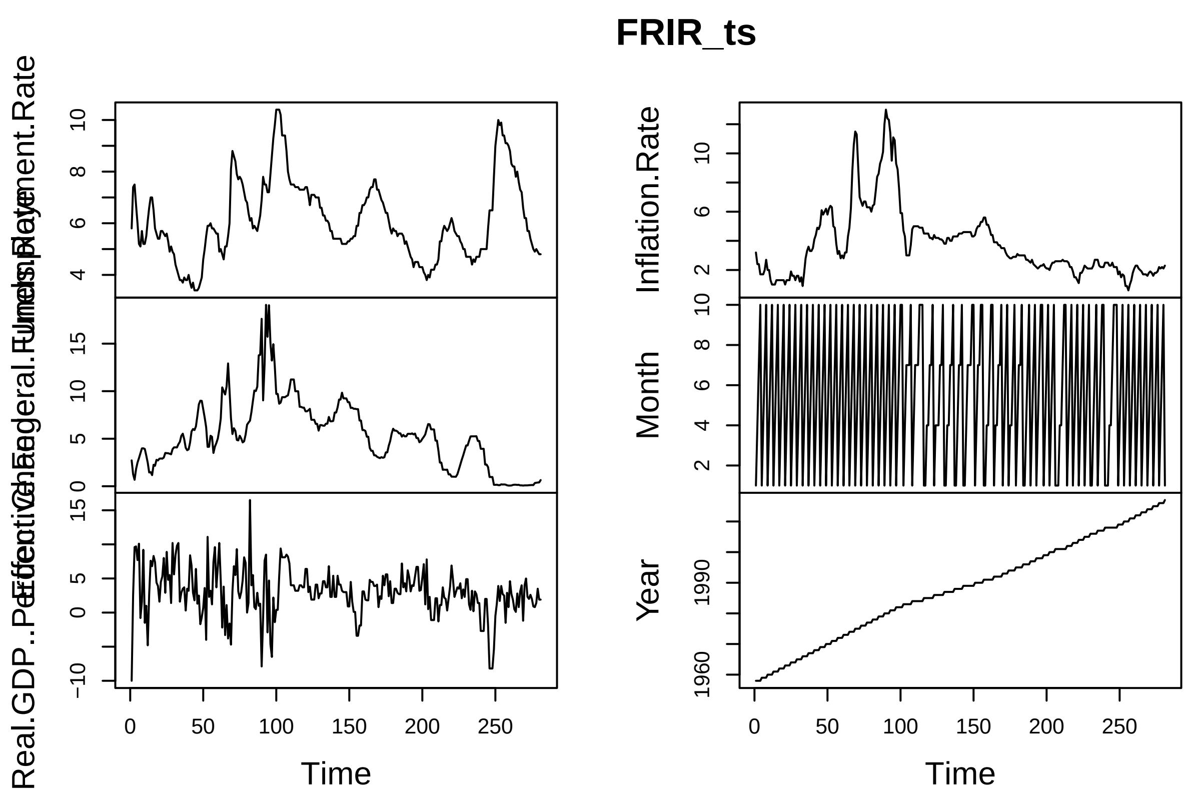
```
##
## Attaching package: 'forecast'

## The following object is masked from 'package:POE5Rdata':
##
##     gold
```

```
FRIR<-read.csv("index.csv")
attach(FRIR)
```

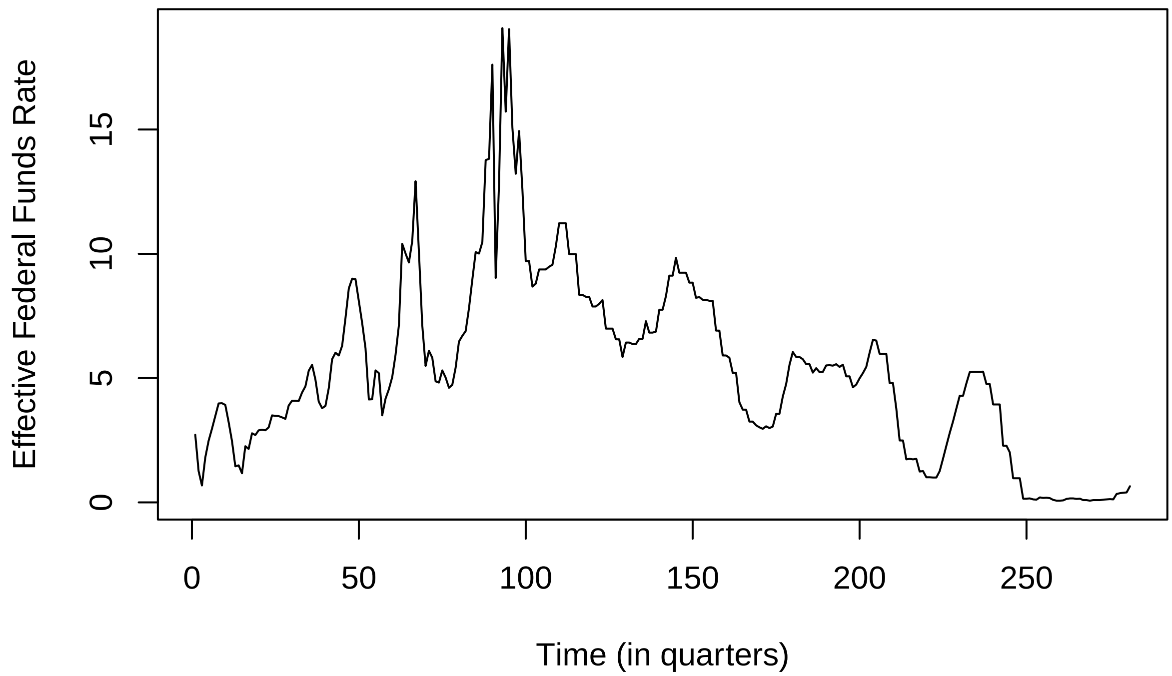
```
#Response Variables: Effective.Federal.Funds.Rate, Unemployment.Rate

#Explanatory Variables: Inflation.Rate, Real.GDP..Percent.Change.

FRIR <- FRIR[, c("Unemployment.Rate", "Effective.Federal.Funds.Rate", "Real.GDP..Percent.Change.", "Infl
FRIR<-FRIR[FRIR$Month %in% c(7, 10, 1, 4),]
FRIR<-FRIR[FRIR$Year >= 1958, ]

library(zoo)
```

```
##
## Attaching package: 'zoo'

## The following objects are masked from 'package:base':
##
##     as.Date, as.Date.numeric
```

```
FRIR.clean<-na.locf(na.locf(FRIR),fromLast=TRUE)

FRIR_ts<-ts(FRIR.clean)
plot(FRIR_ts)
```

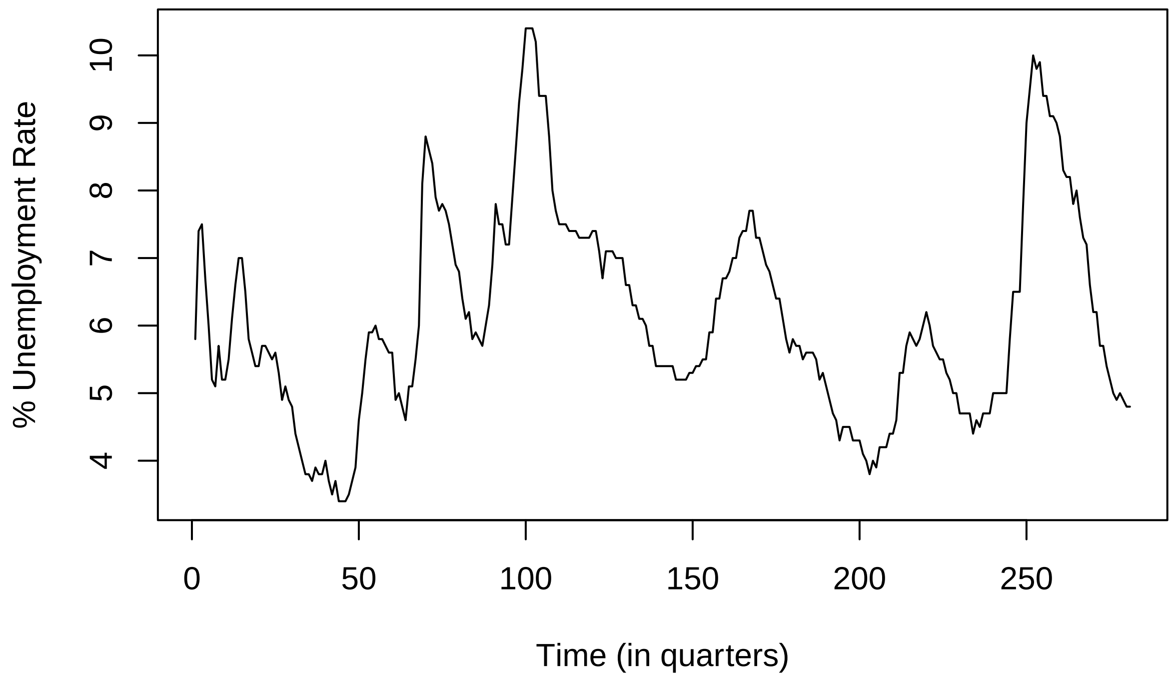


```r
plot(FRIR_ts[,"Effective.Federal.Funds.Rate"], ylab = "Effective Federal Funds Rate", xlab = "Time (in
```



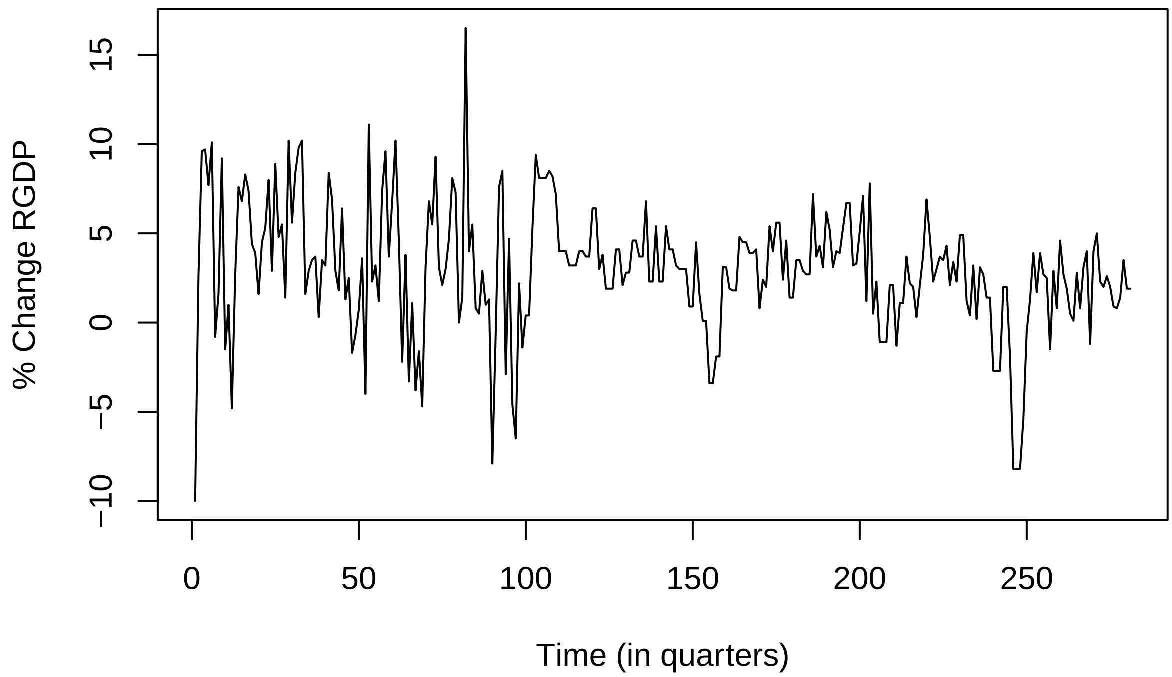
The effective federal funds rate exhibits some stationarity over time, but exhibits a large spike near quarter = 100.

```r
plot(FRIR_ts[,"Unemployment.Rate"], ylab = "% Unemployment Rate", xlab = "Time (in quarters)")
```



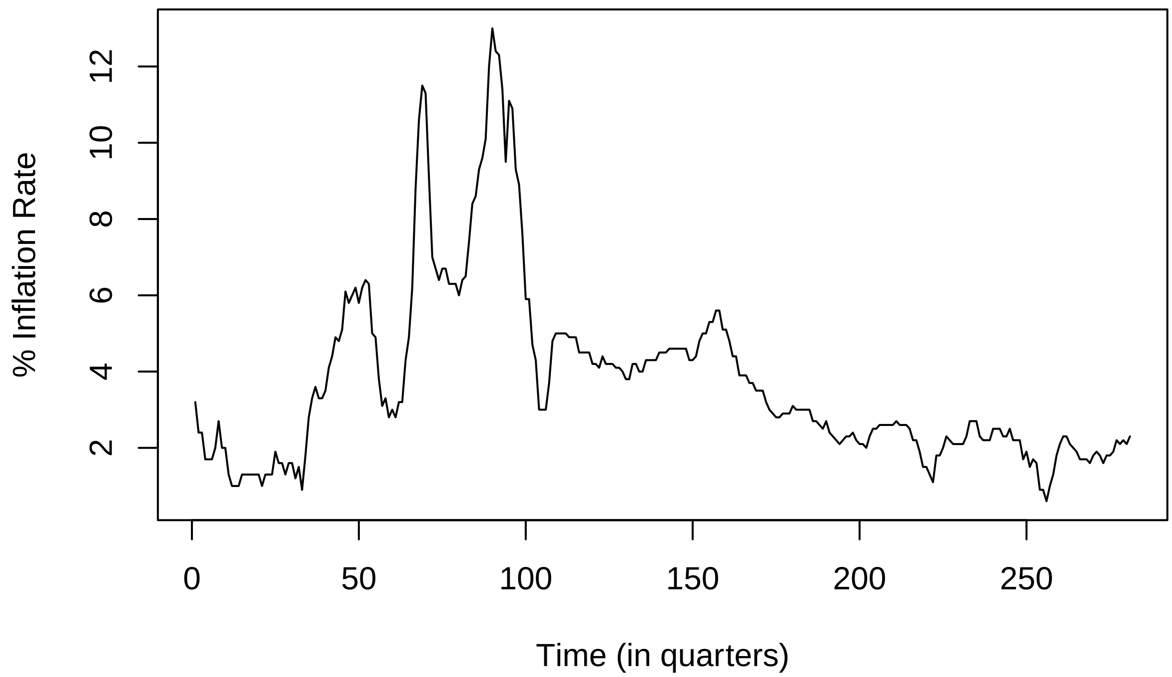
The unemployment rate exhibits more stationarity around the mean of about 6%, with large spikes at quarter = 100 and about quarter = 250.

```
plot(FRIR_ts[,"Real.GDP..Percent.Change."], ylab = "% Change RGDP", xlab = "Time (in quarters)")
```



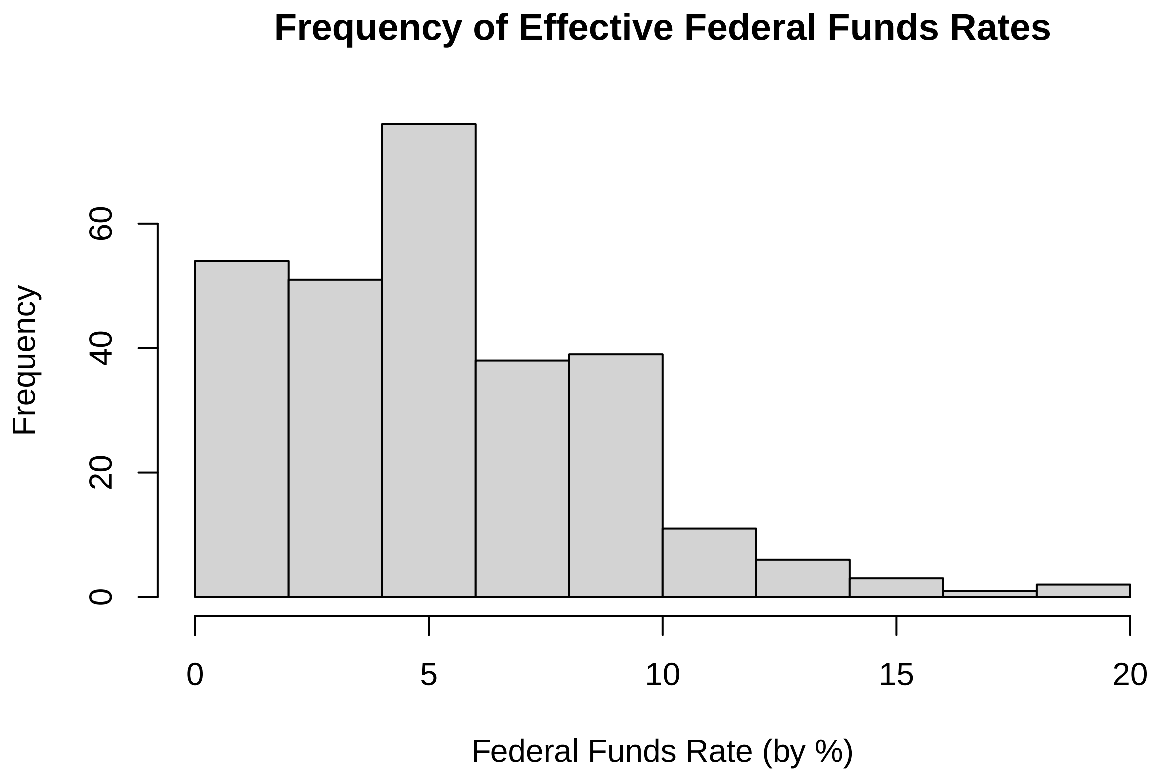
The percent change in RGDP exhibits much more stationarity and less persistence. Such volatile data is understandable given that RGDP is highly responsive to changes in the state of the economy, and measuring the data by percentage change would amplify such responses.

```
plot(FRIR_ts[,"Inflation.Rate"], ylab = "% Inflation Rate", xlab = "Time (in quarters)")
```



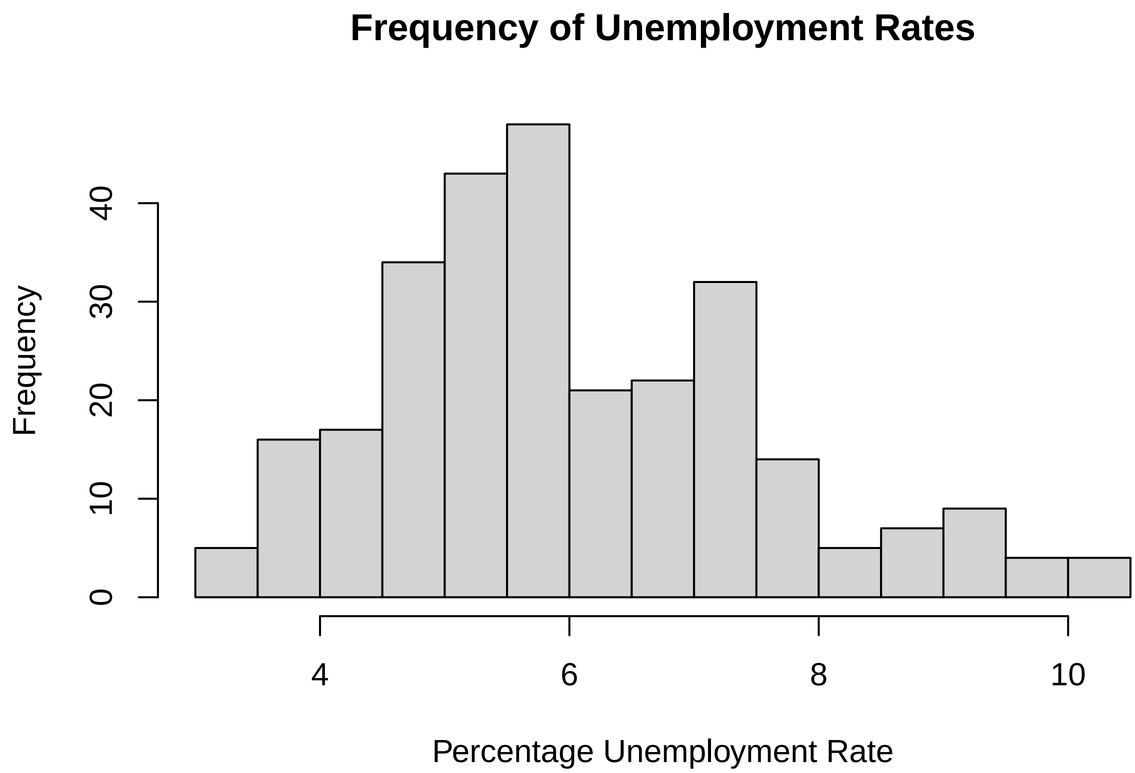
The inflation rate exhibits less mean-reversion and more persistence, reflecting the characteristic that inflation cannot reverse itself as easily as other economic factors can.

```
hist(FRIR_ts[,"Effective.Federal.Funds.Rate"], main="Frequency of Effective Federal Funds Rates", xlab=
```



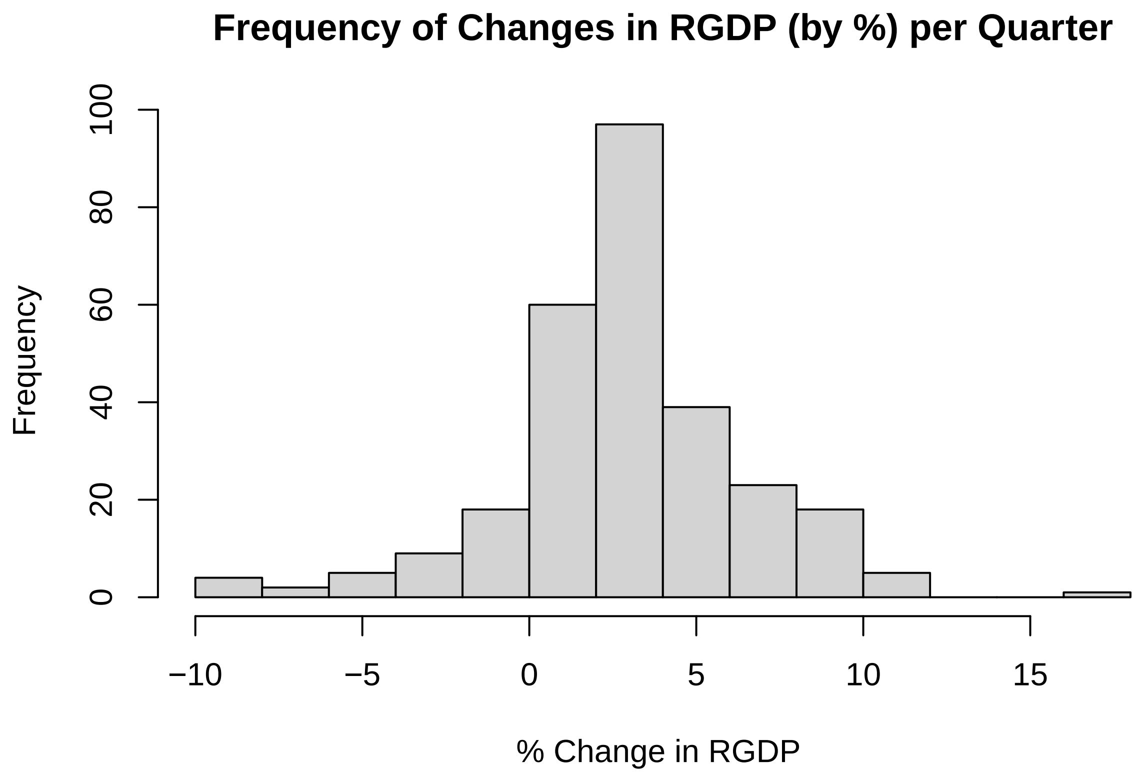
The histogram shows the right- skew of Effective Federal Funds Rates, indicating that the median effective federal funds rate is about 5%. The right skew of the data comes from the 1980's, when the Effective Federal Funds Rates were very high to control inflation.

```
hist(FRIR_ts[,"Unemployment.Rate"], main= "Frequency of Unemployment Rates", xlab="Percentage Unemployme
```



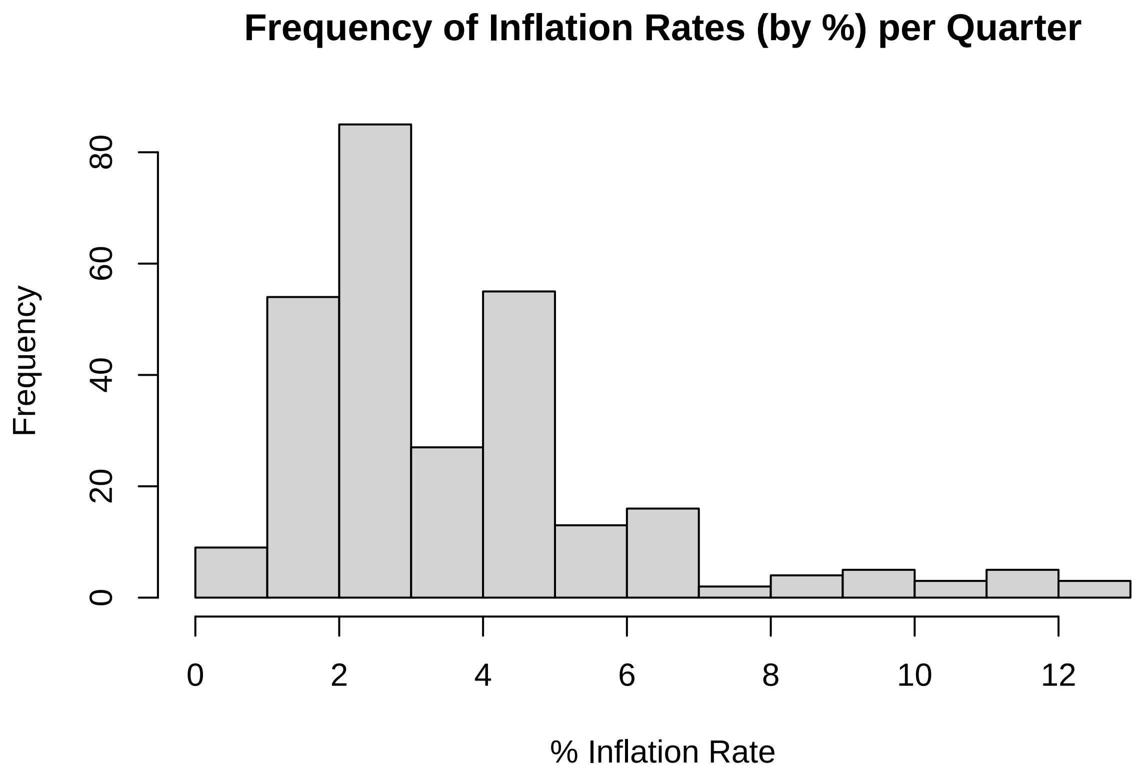
The histogram of unemployment rates is somewhat normal, and indicates that the median unemployment rate for the data is just under 6%. It exhibits somewhat of a right-skew, which is most likely caused by short-lasting recessions where the US experienced high unemployment rates.

```
hist(FRIR_ts[,"Real.GDP..Percent.Change."], main= "Frequency of Changes in RGDP (by %) per Quarter", xla
```



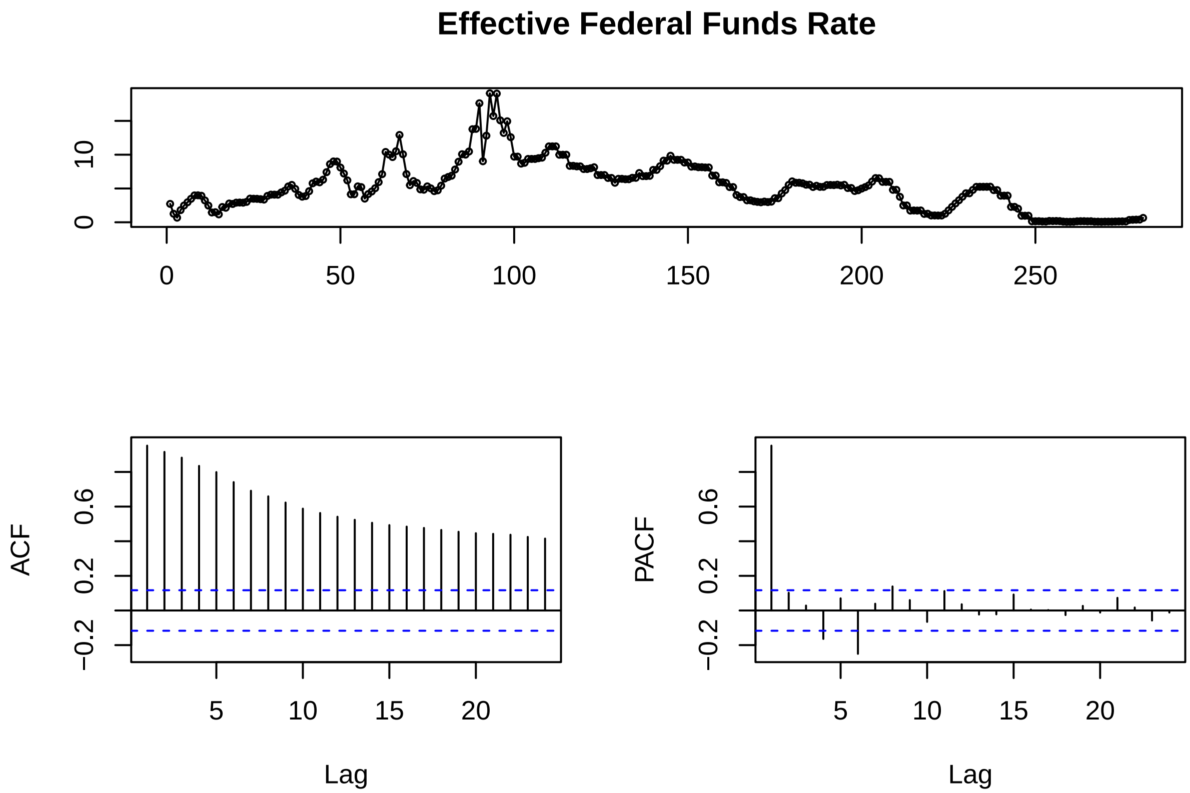
Again, the distribution of the percent changes in Real GDP per quarter follows a roughly normal distribution. The histogram indicates that the median change in GDP was about 3% per quarter. There is a potential outlier at 16 which may need to be removed from the data.

```
hist(FRIR_ts[,"Inflation.Rate"], main= "Frequency of Inflation Rates (by %) per Quarter", xlab="% Infla
```



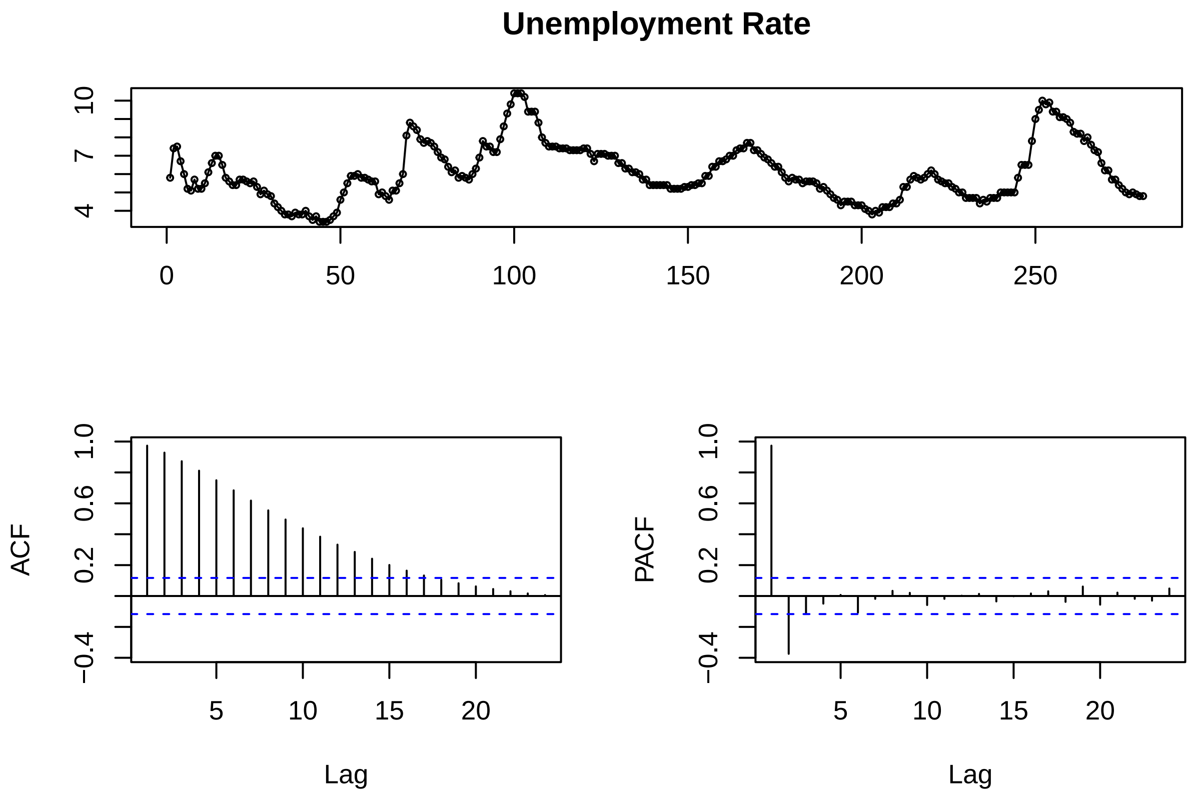
The histogram of inflation rates per quarter is heavily right-skewed and peaks at around 2.5%. Nearly all of the highest rates of inflation come from the 1970's and 80's (sometimes referred to as the "Great Inflation"), when shocks to the price of oil caused runaway inflation and economic downturn

```
tsdisplay(FRIR_ts[,"Effective.Federal.Funds.Rate"],main="Effective Federal Funds Rate")
```



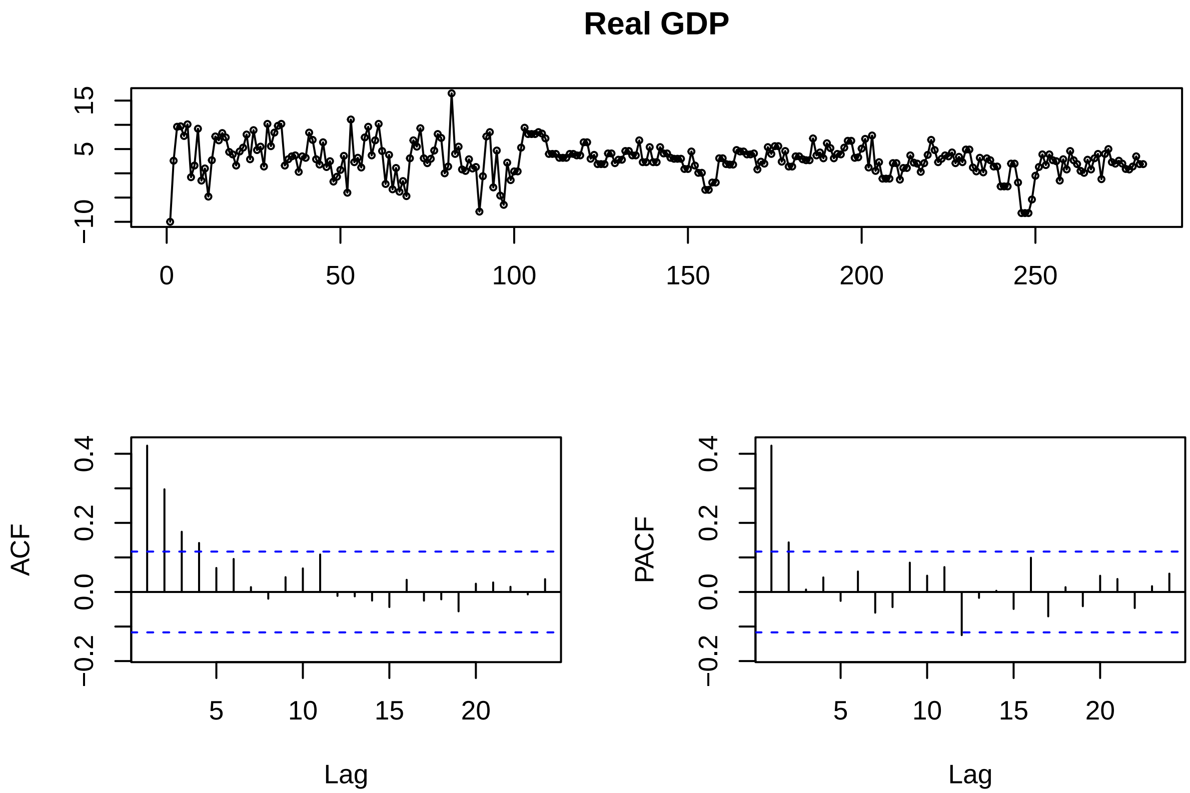
The effective federal funds rate shows some stationarity but somewhat of a steady decline for the last 200 quarters.The ACF shows that there are serial correlations between the effective federal funds rate and the lags of itself. The PACF indicates that the strongest autocorrelations exist 4 and 6 quarters after the effective federal funds rate has changed.

```
tsdisplay(FRIR_ts[,"Unemployment.Rate"], main="Unemployment Rate")
```



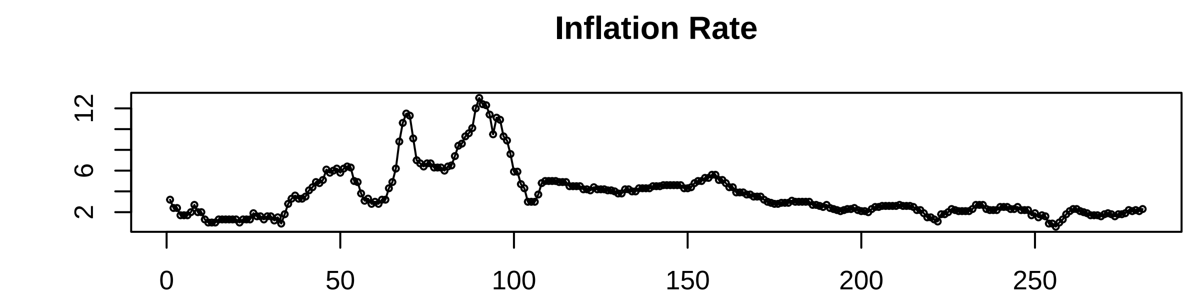
As noted above, the unemployment rate shows some stationarity. The plot is positive for several periods followed by a few negative values, which is an indication of autocorrelation. The ACF graph confirms this, suggesting significant autocorrelation for about 17 lags. The PACF plot highlights the second lag as the lag with the largest effect on the unemployment rate.

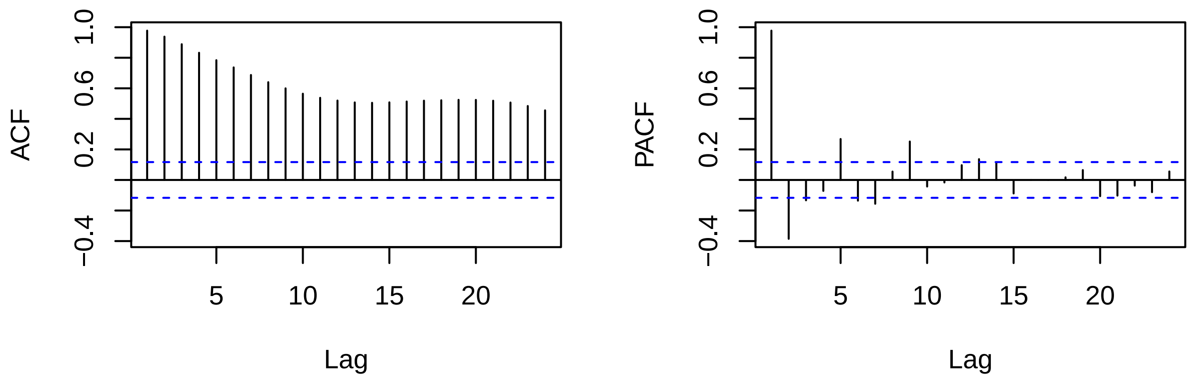
```
tsdisplay(FRIR_ts[,"Real.GDP..Percent.Change."], main="Real GDP")
```



RGDP exhibits the most stationarity of all of the four variables and far fewer serially-corellated lags in the ACF plot. Interestingly, the PACF graph indicates that lags 2, 12, and 16 influence the RGDP at time 0 the most. Fewer significant ACF values and a more volatile plot suggest that RGDP may be more difficult to predict than our other variables.

```
tsdisplay(FRIR_ts[,"Inflation.Rate"], main="Inflation Rate")
```





Much like the effective federal funds rate, the rate of inflation is less mean-reverting and less volatile than the other variables. This characteristic of the data is further emphasized by the ACF graph, which shows significant auto-correlation for at least 25 lags. The PACF suggests that the most influential lags for the inflation rate at T(0) are lags 2, 5, and 9.

```
#AR model #1
library(dynlm)
reg.ar.1a=dynlm(FRIR_ts[,"Unemployment.Rate"]~L(FRIR_ts[,"Unemployment.Rate"], 1:2))
summary(reg.ar.1a)
```

```
##
## Time series regression with "ts" data:
## Start = 3, End = 281
##
## Call:
## dynlm(formula = FRIR_ts[, "Unemployment.Rate"] ~ L(FRIR_ts[,
##     "Unemployment.Rate"], 1:2))
##
## Residuals:
##      Min       1Q   Median       3Q      Max
## -0.78588 -0.17058 -0.02985  0.14558  1.90013
##
## Coefficients:
##                                           Estimate Std. Error t value Pr(>|t|)
## (Intercept)                                0.19936    0.07402   2.693  0.00751
## L(FRIR_ts[, "Unemployment.Rate"], 1:2)1  1.37395    0.05233  26.257  < 2e-16
## L(FRIR_ts[, "Unemployment.Rate"], 1:2)2 -0.40785    0.05239  -7.785 1.41e-13
```

```
##
## (Intercept)                               **
## L(FRIR_ts[, "Unemployment.Rate"], 1:2)1 ***
## L(FRIR_ts[, "Unemployment.Rate"], 1:2)2 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 0.2989 on 276 degrees of freedom
## Multiple R-squared:  0.9624, Adjusted R-squared:  0.9622
## F-statistic:  3536 on 2 and 276 DF,  p-value: < 2.2e-16
```

```
reg.ar.1b=dynlm(FRIR_ts[,"Unemployment.Rate"]~L(FRIR_ts[,"Unemployment.Rate"], 1:3))
summary(reg.ar.1b)
```

```
##
## Time series regression with "ts" data:
## Start = 4, End = 281
##
## Call:
## dynlm(formula = FRIR_ts[, "Unemployment.Rate"] ~ L(FRIR_ts[,
##     "Unemployment.Rate"], 1:3))
##
## Residuals:
##      Min       1Q   Median       3Q      Max
## -0.92622 -0.16577 -0.03571  0.14542  1.86696
##
## Coefficients:
##                                          Estimate Std. Error t value Pr(>|t|)
## (Intercept)                               0.21903    0.07465   2.934  0.00363
## L(FRIR_ts[, "Unemployment.Rate"], 1:3)1  1.36110    0.05981  22.756  < 2e-16
## L(FRIR_ts[, "Unemployment.Rate"], 1:3)2 -0.30829    0.09725  -3.170  0.00170
## L(FRIR_ts[, "Unemployment.Rate"], 1:3)3 -0.08960    0.05749  -1.558  0.12029
##
## (Intercept)                               **
## L(FRIR_ts[, "Unemployment.Rate"], 1:3)1 ***
## L(FRIR_ts[, "Unemployment.Rate"], 1:3)2 **
## L(FRIR_ts[, "Unemployment.Rate"], 1:3)3
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 0.297 on 274 degrees of freedom
## Multiple R-squared:  0.9631, Adjusted R-squared:  0.9627
## F-statistic:  2382 on 3 and 274 DF,  p-value: < 2.2e-16
```

```
#AR model #2
reg.ar.2a=dynlm(FRIR_ts[,"Effective.Federal.Funds.Rate"]~L(FRIR_ts[,"Effective.Federal.Funds.Rate"], 1:4
summary(reg.ar.2a)
```

```
##
## Time series regression with "ts" data:
## Start = 5, End = 281
##
## Call:
```

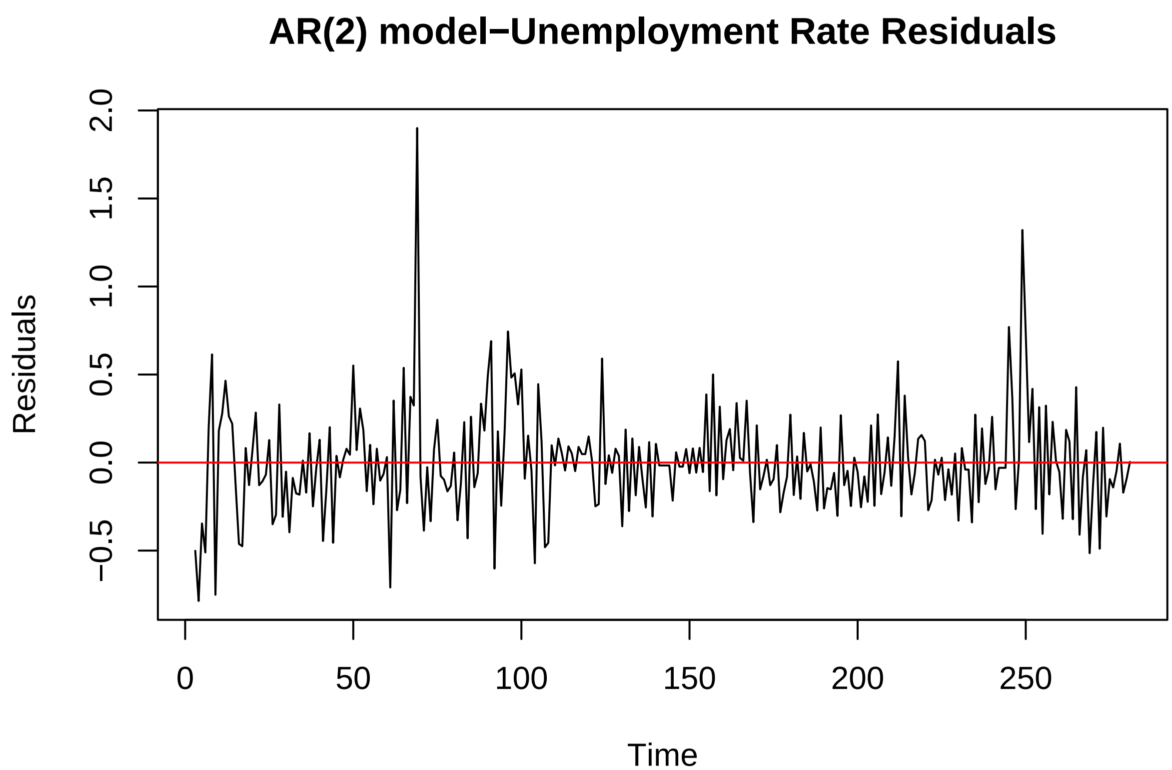
```
## dynlm(formula = FRIR_ts[, "Effective.Federal.Funds.Rate"] ~ L(FRIR_ts[,
##     "Effective.Federal.Funds.Rate"], 1:4))
##
## Residuals:
##     Min      1Q  Median      3Q     Max
## -8.1758 -0.2362 -0.0519  0.3092  6.2895
##
## Coefficients:
##                                                 Estimate Std. Error t value
## (Intercept)                                      0.22594    0.11467   1.970
## L(FRIR_ts[, "Effective.Federal.Funds.Rate"], 1:4)1  0.83219    0.05956  13.971
## L(FRIR_ts[, "Effective.Federal.Funds.Rate"], 1:4)2  0.11469    0.07697   1.490
## L(FRIR_ts[, "Effective.Federal.Funds.Rate"], 1:4)3  0.19122    0.07694   2.485
## L(FRIR_ts[, "Effective.Federal.Funds.Rate"], 1:4)4 -0.18081    0.05940  -3.044
##                                                 Pr(>|t|)
## (Intercept)                                      0.04981 *
## L(FRIR_ts[, "Effective.Federal.Funds.Rate"], 1:4)1  < 2e-16 ***
## L(FRIR_ts[, "Effective.Federal.Funds.Rate"], 1:4)2  0.13735
## L(FRIR_ts[, "Effective.Federal.Funds.Rate"], 1:4)3  0.01355 *
## L(FRIR_ts[, "Effective.Federal.Funds.Rate"], 1:4)4  0.00257 **
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 1.038 on 272 degrees of freedom
## Multiple R-squared:  0.9176, Adjusted R-squared:  0.9164
## F-statistic: 757.3 on 4 and 272 DF,  p-value: < 2.2e-16
```

```
reg.ar.2b=dynlm(FRIR_ts[,"Effective.Federal.Funds.Rate"]~L(FRIR_ts[,"Effective.Federal.Funds.Rate"], 1:6
summary(reg.ar.2b)
```

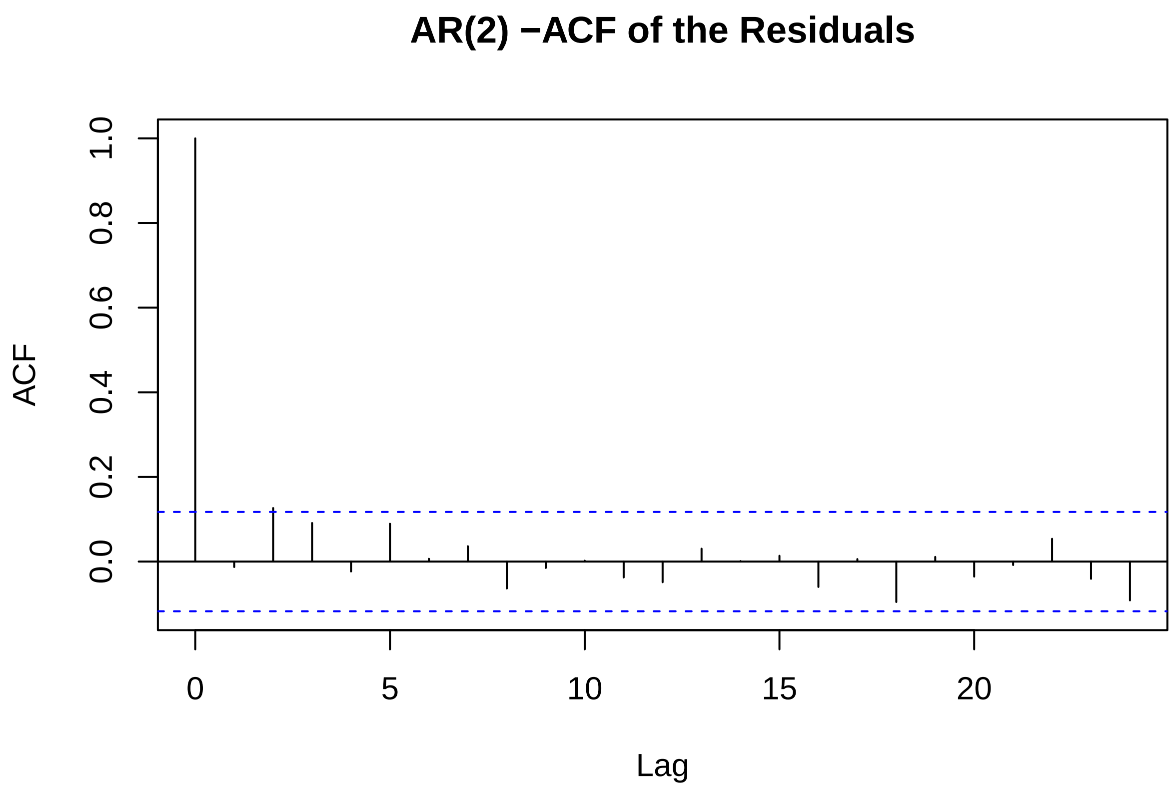
```
##
## Time series regression with "ts" data:
## Start = 7, End = 281
##
## Call:
## dynlm(formula = FRIR_ts[, "Effective.Federal.Funds.Rate"] ~ L(FRIR_ts[,
##     "Effective.Federal.Funds.Rate"], 1:6))
##
## Residuals:
##     Min      1Q  Median      3Q     Max
## -8.2118 -0.2715 -0.0454  0.3280  4.9544
##
## Coefficients:
##                                                 Estimate Std. Error t value
## (Intercept)                                      0.26527    0.11334   2.340
## L(FRIR_ts[, "Effective.Federal.Funds.Rate"], 1:6)1  0.85769    0.05873  14.603
## L(FRIR_ts[, "Effective.Federal.Funds.Rate"], 1:6)2  0.04254    0.07661   0.555
## L(FRIR_ts[, "Effective.Federal.Funds.Rate"], 1:6)3  0.24074    0.07561   3.184
## L(FRIR_ts[, "Effective.Federal.Funds.Rate"], 1:6)4 -0.21031    0.07539  -2.790
## L(FRIR_ts[, "Effective.Federal.Funds.Rate"], 1:6)5  0.29078    0.07639   3.806
## L(FRIR_ts[, "Effective.Federal.Funds.Rate"], 1:6)6 -0.27200    0.05845  -4.653
##                                                 Pr(>|t|)
## (Intercept)                                     0.019997 *
## L(FRIR_ts[, "Effective.Federal.Funds.Rate"], 1:6)1  < 2e-16 ***
```

```
## L(FRIR_ts[, "Effective.Federal.Funds.Rate"], 1:6)2 0.579157
## L(FRIR_ts[, "Effective.Federal.Funds.Rate"], 1:6)3 0.001623 **
## L(FRIR_ts[, "Effective.Federal.Funds.Rate"], 1:6)4 0.005656 **
## L(FRIR_ts[, "Effective.Federal.Funds.Rate"], 1:6)5 0.000175 ***
## L(FRIR_ts[, "Effective.Federal.Funds.Rate"], 1:6)6 5.14e-06 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 1.002 on 268 degrees of freedom
## Multiple R-squared:  0.9241, Adjusted R-squared:  0.9224
## F-statistic: 543.6 on 6 and 268 DF,  p-value: < 2.2e-16
```

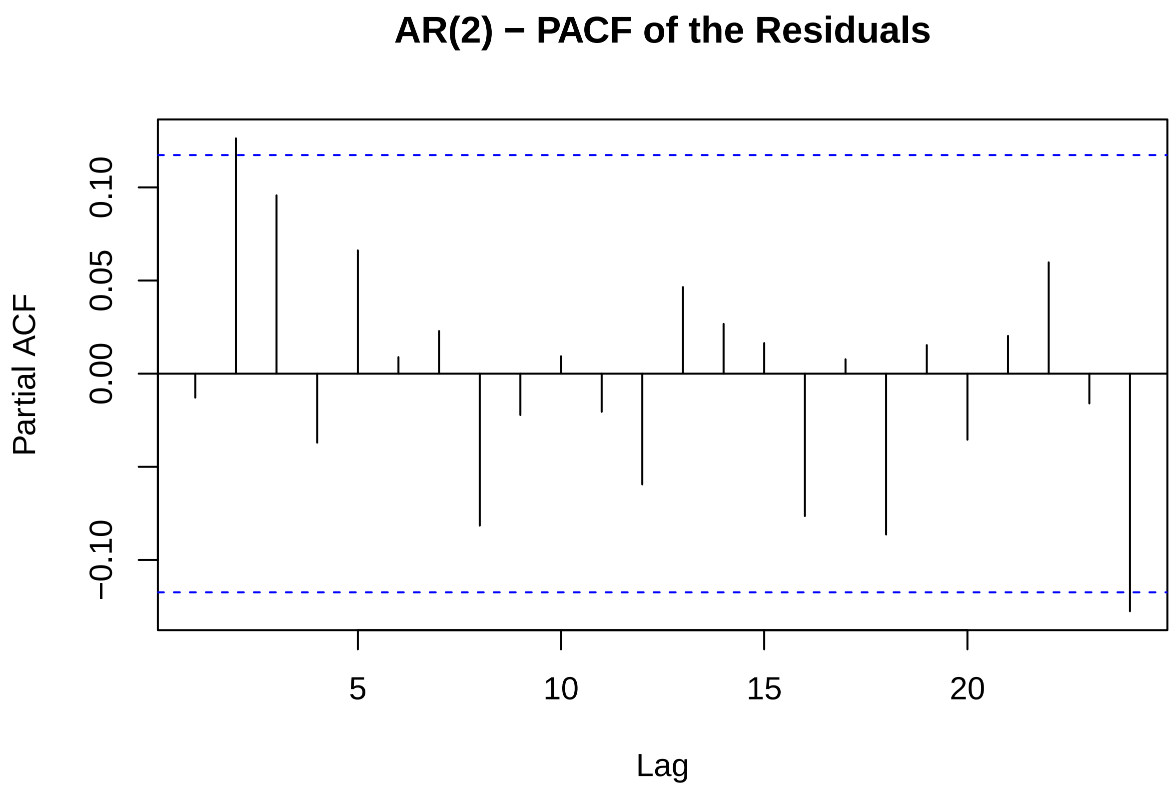
```r
plot(reg.ar.1a$residuals,pch=20,ylab="Residuals", main="AR(2) model-Unemployment Rate Residuals")
abline(h=0,lwd=1,col="red")
```




```r
acf(reg.ar.1a$residuals, main="AR(2) -ACF of the Residuals")
```

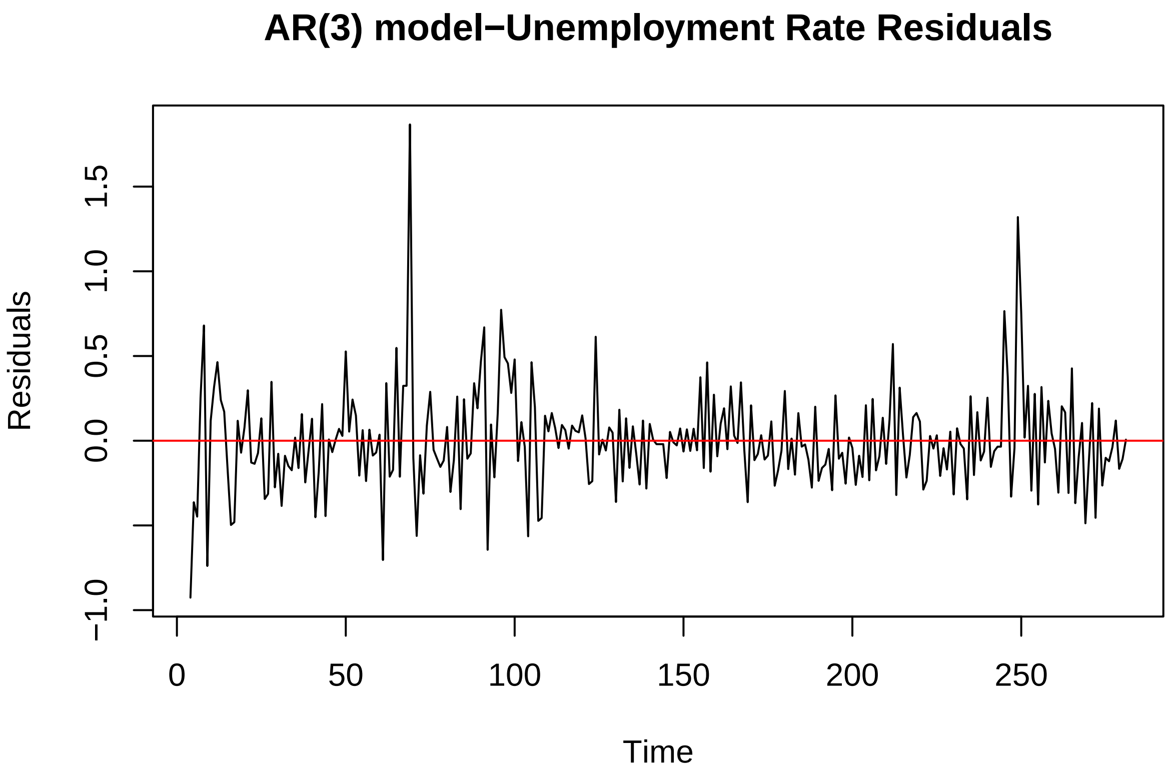



```
pacf(reg.ar.1a$residuals, main="AR(2) - PACF of the Residuals")
```

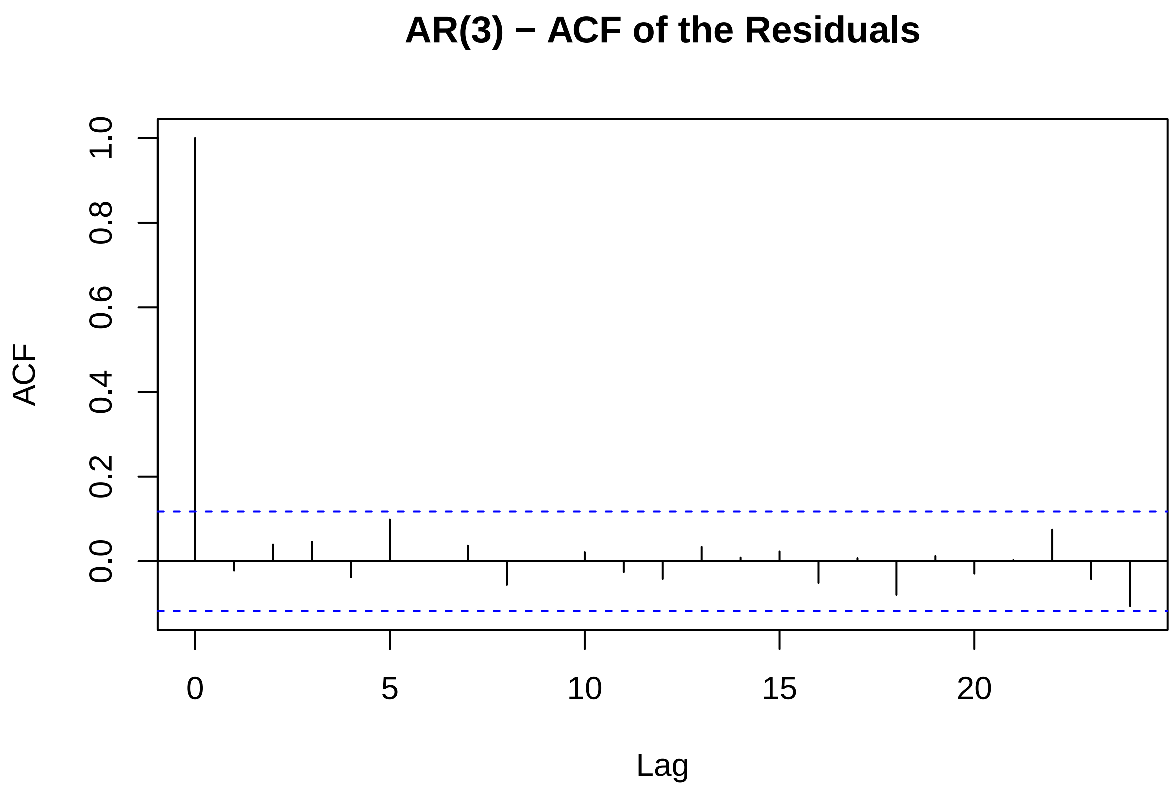


The plot of the residuals shows substantial "white noise" and the ACF and PACF plots confirm this. There are few significant autocorrelations between the data and its residuals and both graphs exhibit no patterns, suggesting that the ar(2) model appropriately captures the dynamics of the data and leaves no patterns among the errors.

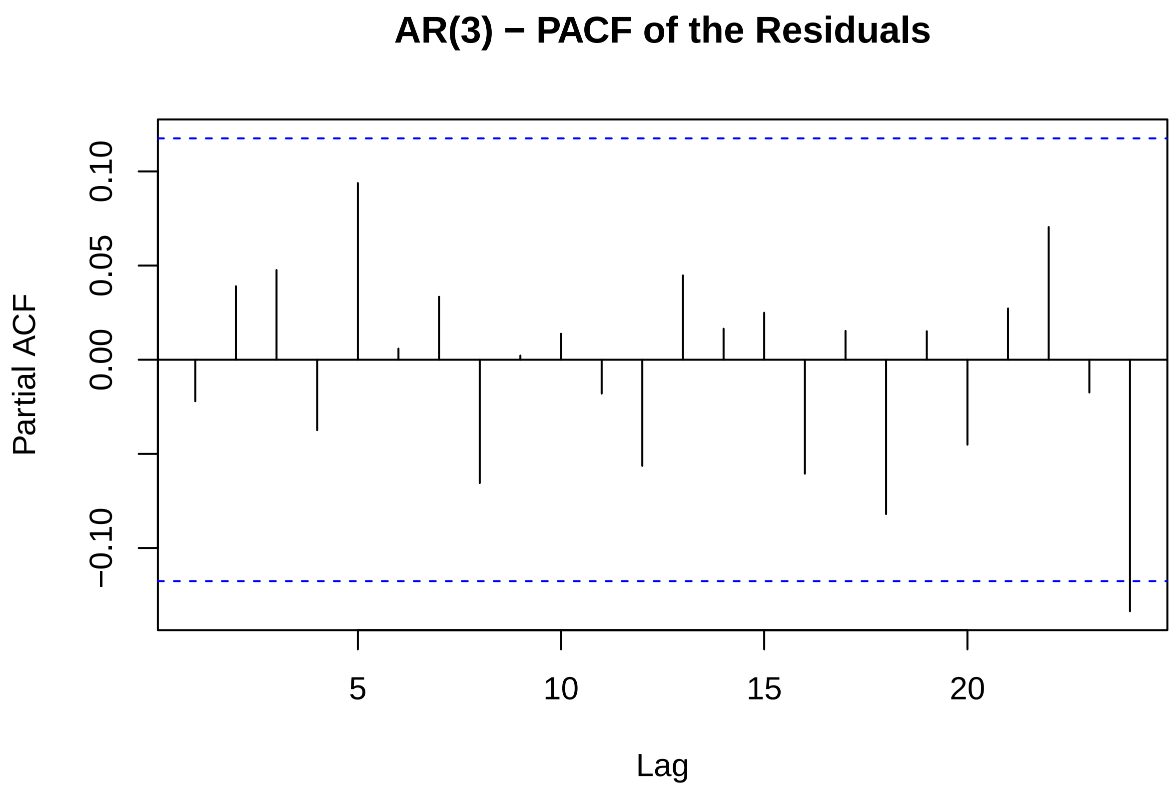
```
plot(reg.ar.1b$residuals,pch=20,ylab="Residuals", main="AR(3) model-Unemployment Rate Residuals")
abline(h=0,lwd=1,col="red")
```




```
acf(reg.ar.1b$residuals, main="AR(3) - ACF of the Residuals")
```

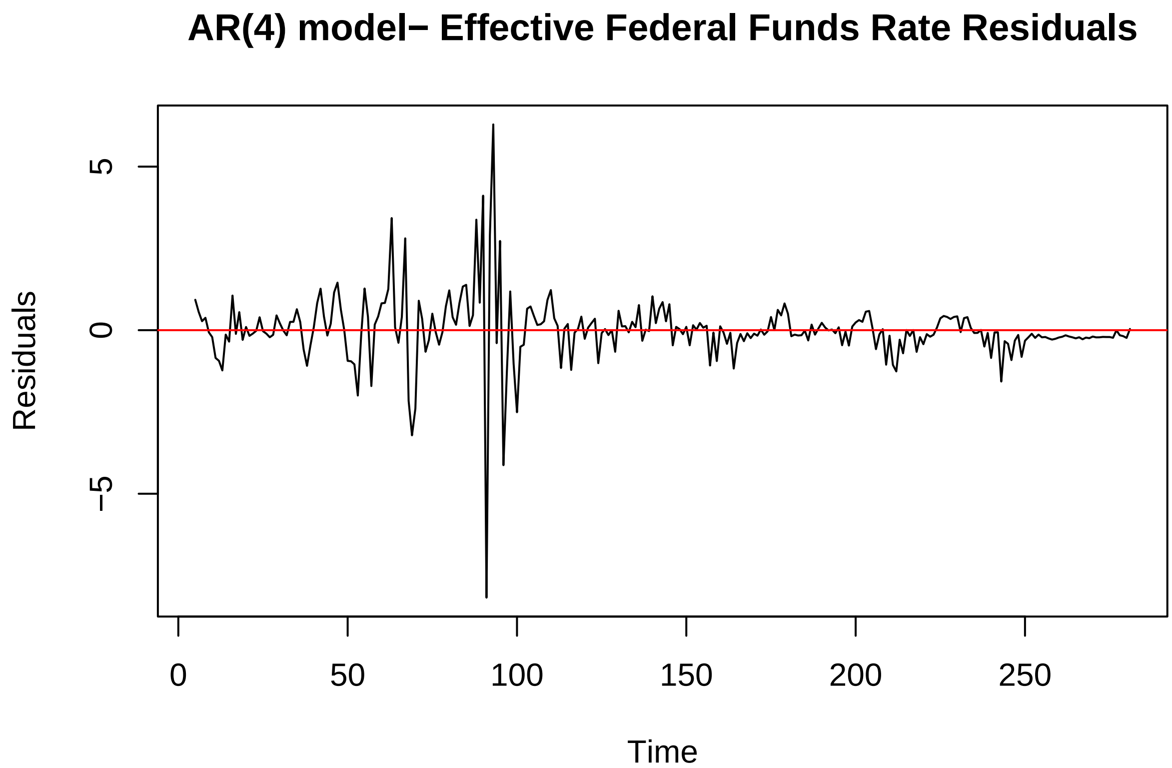



```
pacf(reg.ar.1b$residuals, main="AR(3) - PACF of the Residuals")
```

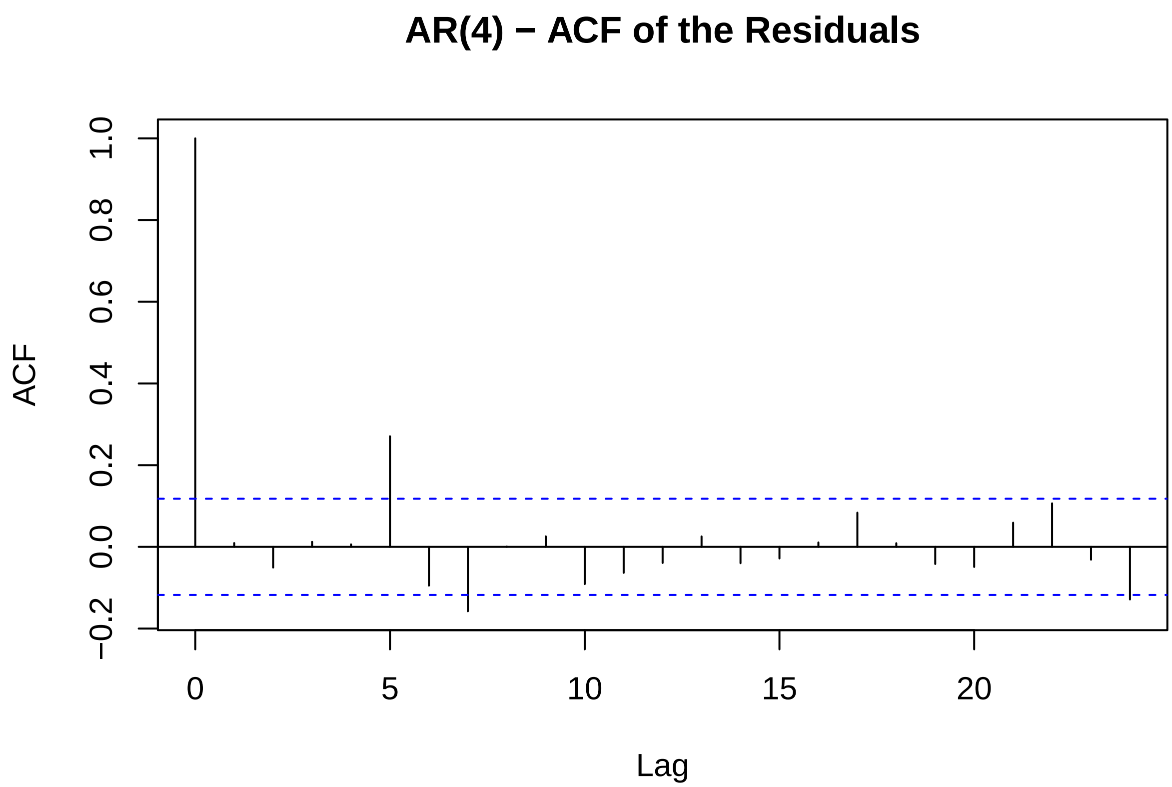


The plot of the unemployment rate residuals for the ar(3) model also shows stationarity and low persistence. There are few significant autocorrelations between the data and its residuals, and the PACF and ACF graphs confirm this. Almost no lags between the data and its residuals are autocorrelated, suggesting that the model is appropriate.

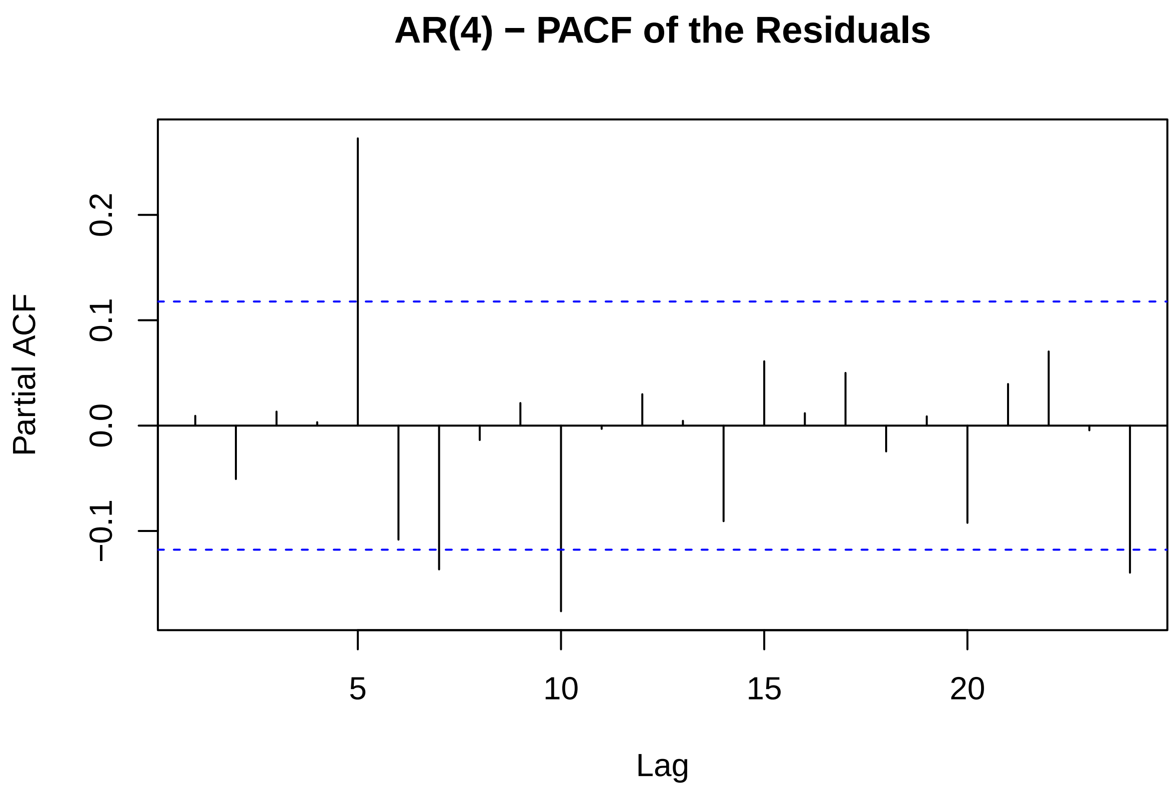
```
plot(reg.ar.2a$residuals,pch=20,ylab="Residuals", main="AR(4) model- Effective Federal Funds Rate Residu
abline(h=0,lwd=1,col="red")
```




```
acf(reg.ar.2a$residuals, main="AR(4) - ACF of the Residuals")
```

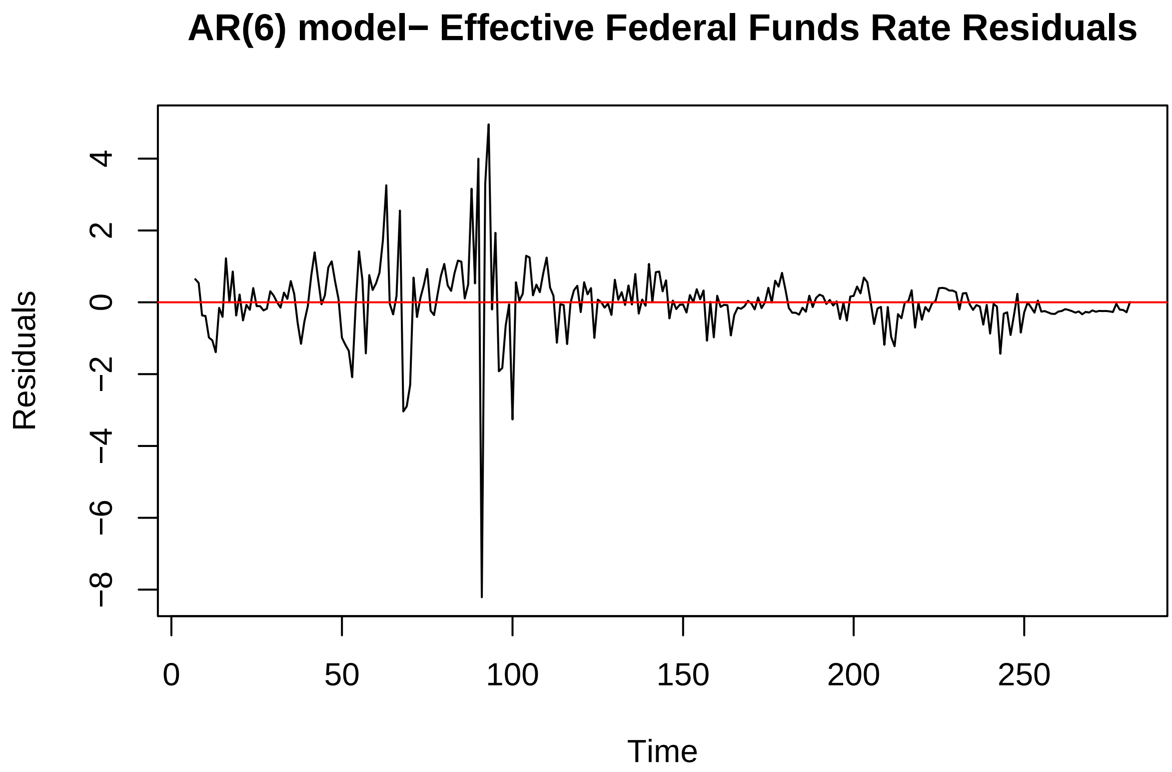



```
pacf(reg.ar.2a$residuals, main="AR(4) - PACF of the Residuals")
```

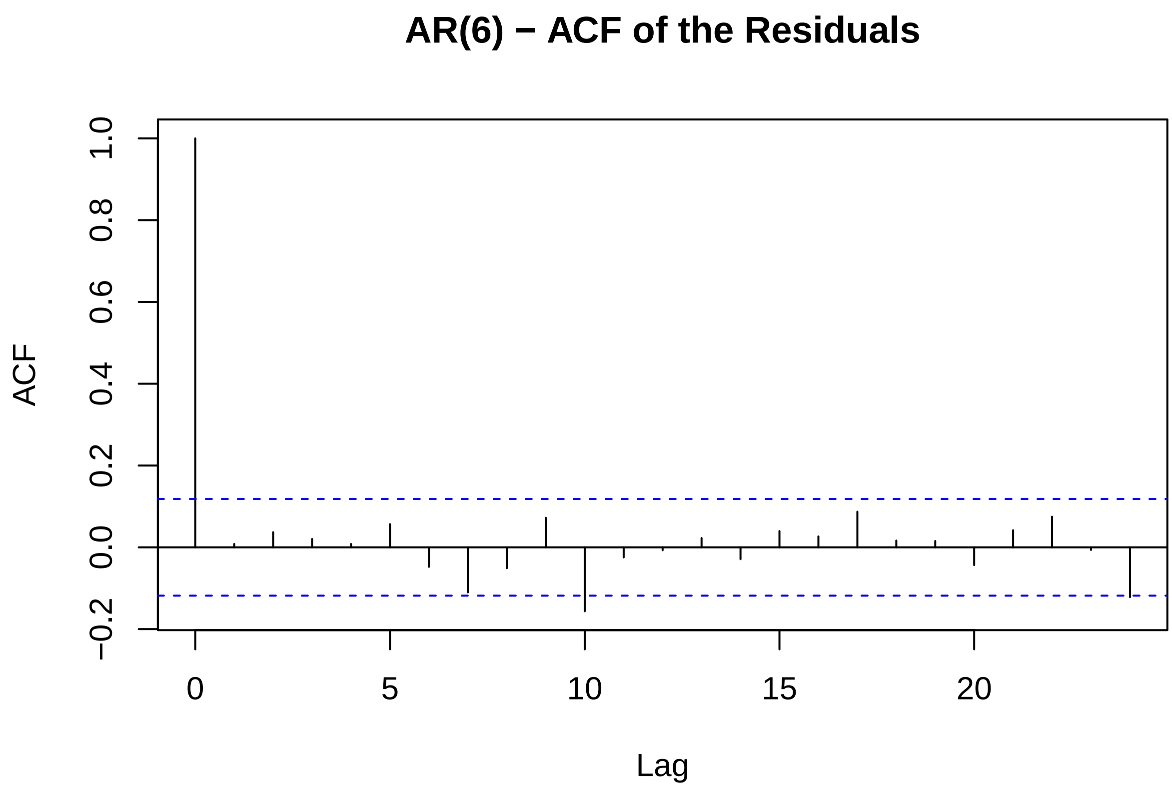


The residuals of the ar(4) model for the effective federal funds rate is decent in that it shows the data is centered around the mean. There is still the spike at Time =~ 90, but this seems to be an anomaly in the original data that would be difficult to capture in a regression model. The ACF and PACF models show no patterns, which is a good indication that the ar(4) model does not fail to capture any dynamics of the data set. However, the PACF of the residuals shows a few significant lags which indicates that the model, while appropriate, may not be the most effective predictor of effective federal funds rates.

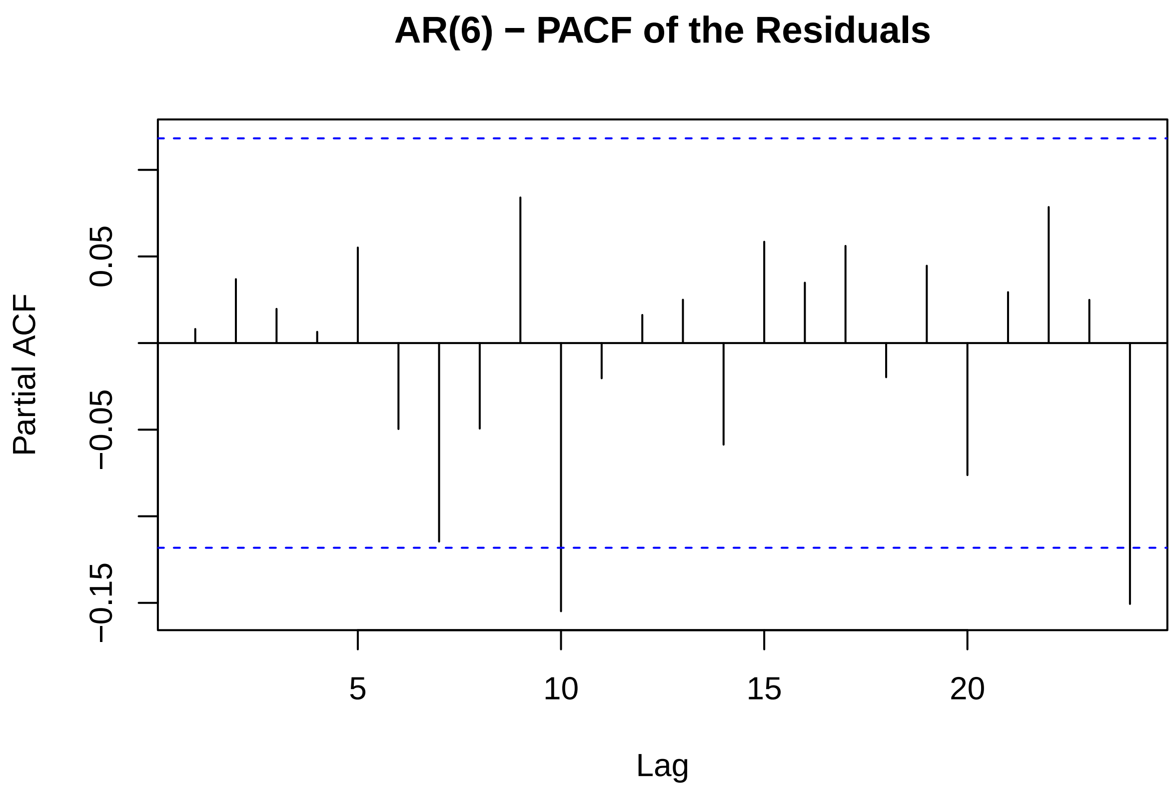
```
plot(reg.ar.2b$residuals,pch=20,ylab="Residuals", main="AR(6) model- Effective Federal Funds Rate Residu
abline(h=0,lwd=1,col="red")
```




```
acf(reg.ar.2b$residuals, main="AR(6) - ACF of the Residuals")
```




```
pacf(reg.ar.2b$residuals, main="AR(6) - PACF of the Residuals")
```



Again, the residuals of the model shows the spike at Time =~90, but all other parts of the plot show less amplitude and are mean reverting. The ACF and the PACF graphs of the residuals still show no patterns and few instances of autocorrelation, which is a good indicator that the AR(6) model does a good job of capturing the dynamics of the data.

```
#1/3 training, 2/3 testing
df = data.frame(FRIR_ts[,"Effective.Federal.Funds.Rate"],FRIR_ts[, "Unemployment.Rate"])
set.seed(1)
row.number <- sample(1:nrow(df), 0.66*nrow(df))
train = df[row.number,]
test = df[-row.number,]
test
```

```
##     FRIR_ts....Effective.Federal.Funds.Rate.. FRIR_ts....Unemployment.Rate..
## 3                                        0.68                            7.5
## 4                                        1.80                            6.7
## 6                                        2.96                            5.2
## 7                                        3.47                            5.1
## 8                                        3.98                            5.7
## 10                                       3.92                            5.2
## 11                                       3.23                            5.5
## 12                                       2.47                            6.1
## 18                                       2.78                            5.6
## 21                                       2.92                            5.7
## 27                                       3.42                            4.9
## 30                                       4.09                            4.8
## 32                                       4.08                            4.2
```

```
## 35                                         5.30                        3.8
## 38                                         4.05                        3.8
## 46                                         7.41                        3.4
## 47                                         8.61                        3.5
## 52                                         6.20                        5.5
## 54                                         4.15                        5.9
## 55                                         5.31                        6.0
## 56                                         5.20                        5.8
## 58                                         4.17                        5.7
## 59                                         4.55                        5.6
## 62                                         7.12                        5.0
## 63                                        10.40                        4.8
## 66                                        10.51                        5.1
## 68                                        10.06                        6.0
## 69                                         7.13                        8.1
## 74                                         4.82                        7.7
## 76                                         5.02                        7.7
## 80                                         6.47                        6.8
## 81                                         6.70                        6.4
## 82                                         6.89                        6.1
## 93                                        19.08                        7.5
## 94                                        15.72                        7.2
## 95                                        19.04                        7.2
## 96                                        15.08                        7.9
## 97                                        13.22                        8.6
## 101                                        9.71                       10.4
## 106                                        9.37                        9.4
## 109                                       10.29                        7.7
## 112                                       11.23                        7.5
## 117                                        8.35                        7.3
## 119                                        8.27                        7.3
## 123                                        8.14                        6.7
## 125                                        6.99                        7.1
## 128                                        6.56                        7.0
## 131                                        6.43                        6.6
## 132                                        6.37                        6.3
## 133                                        6.37                        6.3
## 137                                        6.83                        5.7
## 142                                        8.30                        5.4
## 147                                        9.24                        5.2
## 151                                        8.23                        5.4
## 154                                        8.15                        5.5
## 157                                        6.91                        6.4
## 158                                        6.91                        6.4
## 159                                        5.91                        6.7
## 162                                        5.21                        7.0
## 168                                        3.25                        7.7
## 169                                        3.10                        7.3
## 171                                        2.96                        7.1
## 173                                        2.99                        6.8
## 175                                        3.56                        6.4
## 178                                        4.76                        5.8
## 183                                        5.76                        5.5
## 184                                        5.56                        5.6
```

```
## 188                                        5.24                          5.2
## 194                                        5.45                          4.3
## 195                                        5.54                          4.5
## 196                                        5.07                          4.5
## 199                                        4.74                          4.3
## 200                                        4.99                          4.3
## 205                                        6.51                          3.9
## 210                                        4.80                          4.4
## 212                                        2.49                          5.3
## 216                                        1.73                          5.8
## 220                                        1.01                          6.2
## 222                                        1.00                          5.7
## 227                                        2.79                          5.2
## 229                                        3.78                          5.0
## 232                                        4.79                          4.7
## 233                                        5.24                          4.7
## 237                                        5.26                          4.7
## 240                                        3.94                          5.0
## 243                                        2.28                          5.0
## 244                                        2.28                          5.0
## 245                                        2.01                          5.8
## 247                                        0.97                          6.5
## 254                                        0.20                          9.9
## 255                                        0.18                          9.4
## 258                                        0.10                          9.1
## 259                                        0.07                          9.0
## 260                                        0.07                          8.8
## 261                                        0.08                          8.3
## 275                                        0.13                          5.2
```

```
dim(train)
```

```
## [1] 185   2
```

```
dim(test)
```

```
## [1] 96  2
```

```
ar.train.2 = ar(train,aic=FALSE,order.max=2, method="ols")
ar.test.2 = ar(train,aic=FALSE,order.max=2, method="ols")
ar.train.3 = ar(train,aic=FALSE,order.max=3, method="ols")
ar.test.3 = ar(train,aic=FALSE,order.max=3, method="ols")
ar.train.4 = ar(train,aic=FALSE,order.max=4, method="ols")
ar.test.4 = ar(train,aic=FALSE,order.max=4, method="ols")
ar.train.6 = ar(train,aic=FALSE,order.max=6, method="ols")
ar.test.6 = ar(train,aic=FALSE,order.max=6, method="ols")
```

```
residuals.ar2.train.u = na.locf(na.locf(ar.train.2[["resid"]][,2]),fromLast=TRUE)

MSE.ar2.train.u = mean(residuals.ar2.train.u ^2)
MSE.ar2.train.u
```

```
## [1] 2.41302
```

```r
residuals.ar2.test.u = na.locf(na.locf(ar.test.2[["resid"]][,2]),fromLast=TRUE)

MSE.ar2.test.u = mean(residuals.ar2.test.u ^2)
MSE.ar2.test.u
```

```
## [1] 2.41302
```

```r
residuals.ar3.train.u = na.locf(na.locf(ar.train.3[["resid"]][,2]),fromLast=TRUE)

MSE.ar3.train.u = mean(residuals.ar3.train.u ^2)
MSE.ar3.train.u
```

```
## [1] 2.425426
```

```r
residuals.ar3.test.u = na.locf(na.locf(ar.test.3[["resid"]][,2]),fromLast=TRUE)

MSE.ar3.test.u = mean(residuals.ar3.test.u ^2)
MSE.ar3.test.u
```

```
## [1] 2.425426
```

According to the MSE, AR(2) is a better fit for Unemployment Rate since the MSE for both training and testing are lower than that of the AR(3) model.

```r
residuals.ar4.train.f = na.locf(na.locf(ar.train.4[["resid"]][,1]),fromLast=TRUE)

MSE.ar4.train.f= mean(residuals.ar4.train.f^2)
MSE.ar4.train.f
```

```
## [1] 12.01274
```

```r
residuals.ar4.test.f = na.locf(na.locf(ar.test.4[["resid"]][,1]),fromLast=TRUE)

MSE.ar4.test.f= mean(residuals.ar4.test.f^2)
MSE.ar4.test.f
```

```
## [1] 12.01274
```

```r
residuals.ar6.train.f = na.locf(na.locf(ar.train.6[["resid"]][,1]),fromLast=TRUE)

MSE.ar6.train.f= mean(residuals.ar6.train.f^2)
MSE.ar6.train.f
```

```
## [1] 11.79055
```

```r
residuals.ar6.test.f = na.locf(na.locf(ar.test.6[["resid"]][,1]),fromLast=TRUE)

MSE.ar6.test.f= mean(residuals.ar6.test.f^2)
MSE.ar6.test.f
```

```
## [1] 11.79055
```

According to the MSE, AR(6) is a better fit for Effective Federal Funds Rate since the MSE for both training and testing are lower than that of the AR(4) model.

```r
#AR(2) and AR(3) - Unemployment Rate 1a and 1b
library(dynlm)
reg.ar.1a.compare=dynlm(FRIR_ts[,"Unemployment.Rate"]~L(FRIR_ts[,"Unemployment.Rate"], 1:2), data=train
summary(reg.ar.1a.compare)
```

```
##
## Time series regression with "ts" data:
## Start = 3, End = 281
##
## Call:
## dynlm(formula = FRIR_ts[, "Unemployment.Rate"] ~ L(FRIR_ts[,
##     "Unemployment.Rate"], 1:2), data = train)
##
## Residuals:
##      Min       1Q   Median       3Q      Max
## -0.78588 -0.17058 -0.02985  0.14558  1.90013
##
## Coefficients:
##                                          Estimate Std. Error t value Pr(>|t|)
## (Intercept)                               0.19936    0.07402   2.693  0.00751
## L(FRIR_ts[, "Unemployment.Rate"], 1:2)1  1.37395    0.05233  26.257  < 2e-16
## L(FRIR_ts[, "Unemployment.Rate"], 1:2)2 -0.40785    0.05239  -7.785 1.41e-13
##
## (Intercept)                             **
## L(FRIR_ts[, "Unemployment.Rate"], 1:2)1 ***
## L(FRIR_ts[, "Unemployment.Rate"], 1:2)2 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 0.2989 on 276 degrees of freedom
## Multiple R-squared:  0.9624, Adjusted R-squared:  0.9622
## F-statistic:  3536 on 2 and 276 DF,  p-value: < 2.2e-16
```

```r
reg.ar.1b.compare=dynlm(FRIR_ts[,"Unemployment.Rate"]~L(FRIR_ts[,"Unemployment.Rate"], 1:3), data=train
summary(reg.ar.1b.compare)
```

```
##
## Time series regression with "ts" data:
## Start = 4, End = 281
##
## Call:
## dynlm(formula = FRIR_ts[, "Unemployment.Rate"] ~ L(FRIR_ts[,
##     "Unemployment.Rate"], 1:3), data = train)
##
## Residuals:
##      Min       1Q   Median       3Q      Max
## -0.92622 -0.16577 -0.03571  0.14542  1.86696
```

```
##
## Coefficients:
##                                        Estimate Std. Error t value Pr(>|t|)
## (Intercept)                             0.21903    0.07465   2.934  0.00363
## L(FRIR_ts[, "Unemployment.Rate"], 1:3)1  1.36110    0.05981  22.756  < 2e-16
## L(FRIR_ts[, "Unemployment.Rate"], 1:3)2 -0.30829    0.09725  -3.170  0.00170
## L(FRIR_ts[, "Unemployment.Rate"], 1:3)3 -0.08960    0.05749  -1.558  0.12029
##
## (Intercept)                             **
## L(FRIR_ts[, "Unemployment.Rate"], 1:3)1 ***
## L(FRIR_ts[, "Unemployment.Rate"], 1:3)2 **
## L(FRIR_ts[, "Unemployment.Rate"], 1:3)3
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 0.297 on 274 degrees of freedom
## Multiple R-squared:  0.9631, Adjusted R-squared:  0.9627
## F-statistic:  2382 on 3 and 274 DF,  p-value: < 2.2e-16
```

```
#AR(4) and AR(6) model Effective Federal Funds Rate 2a and 2b
reg.ar.2a.compare=dynlm(FRIR_ts[,"Effective.Federal.Funds.Rate"]~L(FRIR_ts[,"Effective.Federal.Funds.Ra
summary(reg.ar.2a.compare)
```

```
##
## Time series regression with "ts" data:
## Start = 5, End = 281
##
## Call:
## dynlm(formula = FRIR_ts[, "Effective.Federal.Funds.Rate"] ~ L(FRIR_ts[,
##     "Effective.Federal.Funds.Rate"], 1:4), data = train)
##
## Residuals:
##     Min      1Q  Median      3Q     Max
## -8.1758 -0.2362 -0.0519  0.3092  6.2895
##
## Coefficients:
##                                                   Estimate Std. Error t value
## (Intercept)                                        0.22594    0.11467   1.970
## L(FRIR_ts[, "Effective.Federal.Funds.Rate"], 1:4)1  0.83219    0.05956  13.971
## L(FRIR_ts[, "Effective.Federal.Funds.Rate"], 1:4)2  0.11469    0.07697   1.490
## L(FRIR_ts[, "Effective.Federal.Funds.Rate"], 1:4)3  0.19122    0.07694   2.485
## L(FRIR_ts[, "Effective.Federal.Funds.Rate"], 1:4)4 -0.18081    0.05940  -3.044
##                                                   Pr(>|t|)
## (Intercept)                                        0.04981 *
## L(FRIR_ts[, "Effective.Federal.Funds.Rate"], 1:4)1  < 2e-16 ***
## L(FRIR_ts[, "Effective.Federal.Funds.Rate"], 1:4)2  0.13735
## L(FRIR_ts[, "Effective.Federal.Funds.Rate"], 1:4)3  0.01355 *
## L(FRIR_ts[, "Effective.Federal.Funds.Rate"], 1:4)4  0.00257 **
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 1.038 on 272 degrees of freedom
## Multiple R-squared:  0.9176, Adjusted R-squared:  0.9164
## F-statistic: 757.3 on 4 and 272 DF,  p-value: < 2.2e-16
```

```r
reg.ar.2b.compare=dynlm(FIR_ts[,"Effective.Federal.Funds.Rate"]~L(FRIR_ts[,"Effective.Federal.Funds.Ra
summary(reg.ar.2b.compare)
```

```
##
## Time series regression with "ts" data:
## Start = 7, End = 281
##
## Call:
## dynlm(formula = FRIR_ts[, "Effective.Federal.Funds.Rate"] ~ L(FRIR_ts[,
##     "Effective.Federal.Funds.Rate"], 1:6), data = train)
##
## Residuals:
##     Min      1Q  Median      3Q     Max
## -8.2118 -0.2715 -0.0454  0.3280  4.9544
##
## Coefficients:
##                                                   Estimate Std. Error t value
## (Intercept)                                        0.26527    0.11334   2.340
## L(FRIR_ts[, "Effective.Federal.Funds.Rate"], 1:6)1  0.85769    0.05873  14.603
## L(FRIR_ts[, "Effective.Federal.Funds.Rate"], 1:6)2  0.04254    0.07661   0.555
## L(FRIR_ts[, "Effective.Federal.Funds.Rate"], 1:6)3  0.24074    0.07561   3.184
## L(FRIR_ts[, "Effective.Federal.Funds.Rate"], 1:6)4 -0.21031    0.07539  -2.790
## L(FRIR_ts[, "Effective.Federal.Funds.Rate"], 1:6)5  0.29078    0.07639   3.806
## L(FRIR_ts[, "Effective.Federal.Funds.Rate"], 1:6)6 -0.27200    0.05845  -4.653
##                                                   Pr(>|t|)
## (Intercept)                                       0.019997 *
## L(FRIR_ts[, "Effective.Federal.Funds.Rate"], 1:6)1  < 2e-16 ***
## L(FRIR_ts[, "Effective.Federal.Funds.Rate"], 1:6)2 0.579157
## L(FRIR_ts[, "Effective.Federal.Funds.Rate"], 1:6)3 0.001623 **
## L(FRIR_ts[, "Effective.Federal.Funds.Rate"], 1:6)4 0.005656 **
## L(FRIR_ts[, "Effective.Federal.Funds.Rate"], 1:6)5 0.000175 ***
## L(FRIR_ts[, "Effective.Federal.Funds.Rate"], 1:6)6 5.14e-06 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 1.002 on 268 degrees of freedom
## Multiple R-squared:  0.9241, Adjusted R-squared:  0.9224
## F-statistic: 543.6 on 6 and 268 DF,  p-value: < 2.2e-16
```

```r
#AIC and BIC for each model
AIC(reg.ar.1a.compare)
```

```
## [1] 122.8099
```

```r
AIC(reg.ar.1b.compare)
```

```
## [1] 119.8427
```

```r
AIC(reg.ar.2a.compare)
```

```
## [1] 813.8688
```

```
AIC(reg.ar.2b.compare)
```

```
## [1] 790.4967
```

```
BIC(reg.ar.1a.compare)
```

```
## [1] 137.3347
```

```
BIC(reg.ar.1b.compare)
```

```
## [1] 137.9808
```

```
BIC(reg.ar.2a.compare)
```

```
## [1] 835.6129
```

```
BIC(reg.ar.2b.compare)
```

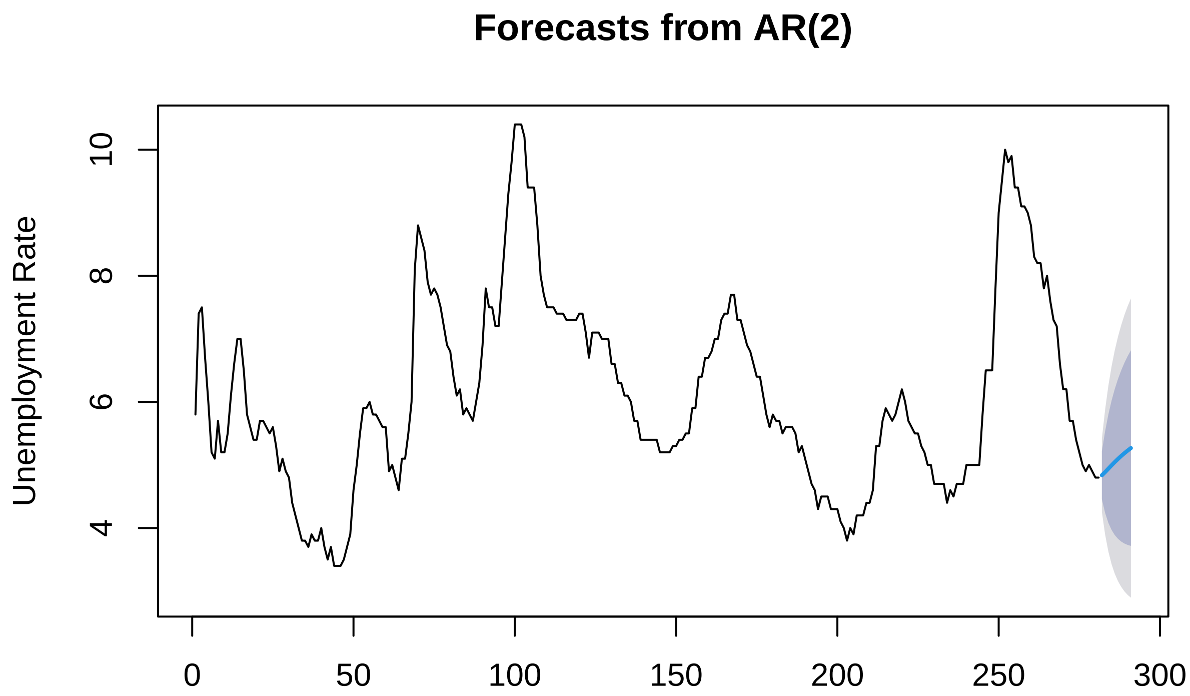
```
## [1] 819.4309
```

Continuing to support the idea that AR(2) for Unemployment Rate and AR(6) for Effective Federal Funds Rate are the best models, we can look to the AIC and BIC. The lowest BIC for Unemployment Rate is given by the AR(2) model, and the lowest BIC for Effective Federal Funds Rate is given by the AR(6) model.

```
y1<- FRIR_ts[, "Unemployment.Rate"]
ar1a = ar(y1, aic = FALSE, order.max = 2, method="ols")
ar1b = ar(y1, aic = FALSE, order.max = 3, method="ols")

y2<- FRIR_ts[,"Effective.Federal.Funds.Rate"]
ar2a = ar(y2, aic = FALSE, order.max = 4, method="ols")
ar2b = ar(y2, aic = FALSE, order.max = 6, method="ols")

forecast(ar1a, 10)
```

```
##     Point Forecast    Lo 80    Hi 80    Lo 95    Hi 95
## 282       4.836628 4.455689 5.217568 4.254032 5.419225
## 283       4.886954 4.239611 5.534297 3.896928 5.876980
## 284       4.941160 4.082753 5.799567 3.628339 6.253981
## 285       4.995111 3.969592 6.020629 3.426715 6.563506
## 286       5.047128 3.887664 6.206593 3.273881 6.820376
## 287       5.096594 3.828125 6.365064 3.156637 7.036551
## 288       5.143342 3.784891 6.501793 3.065770 7.220914
## 289       5.187397 3.753761 6.621033 2.994840 7.379954
## 290       5.228859 3.731776 6.725943 2.939268 7.518451
## 291       5.267859 3.716804 6.818915 2.895725 7.639994
```

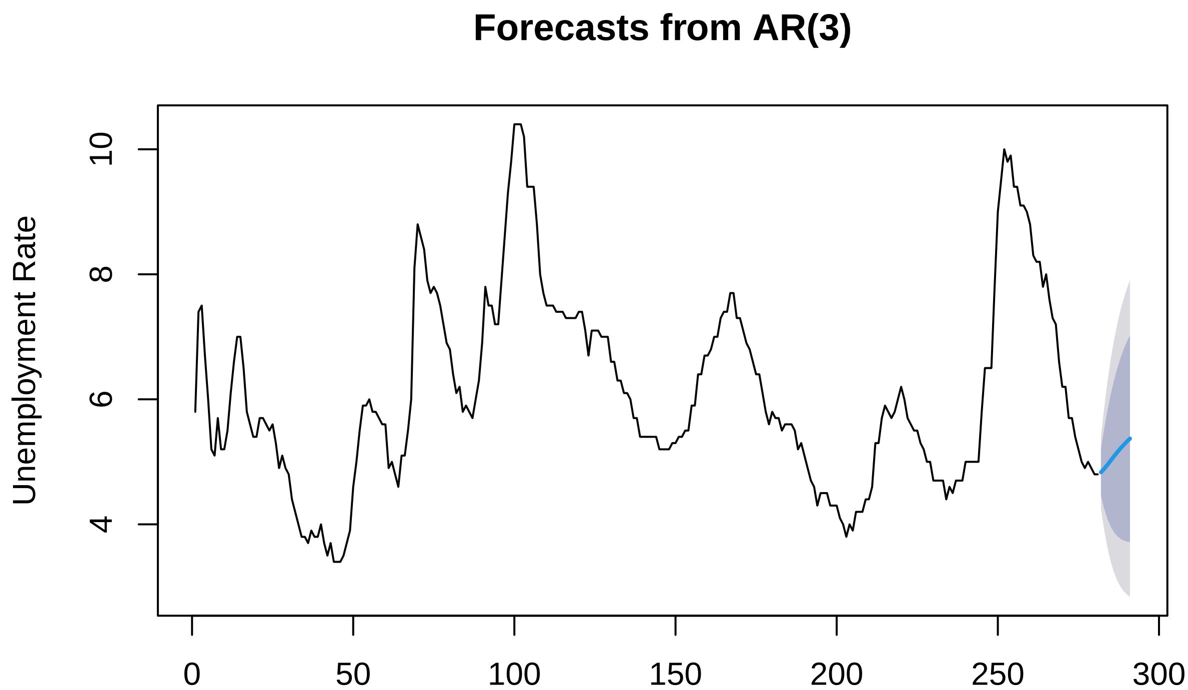
```
plot(forecast(ar1a, 10), ylab = "Unemployment Rate")
```




```
forecast(ar1b, 10)
```

```
##     Point Forecast    Lo 80    Hi 80    Lo 95    Hi 95
## 282       4.833450 4.455619 5.211281 4.255608 5.411293
## 283       4.887940 4.249798 5.526081 3.911986 5.863893
## 284       4.951793 4.087109 5.816476 3.629374 6.274211
## 285       5.018907 3.965429 6.072385 3.407751 6.630062
## 286       5.085688 3.876719 6.294657 3.236730 6.934647
## 287       5.150172 3.813787 6.486557 3.106348 7.193996
## 288       5.211339 3.770593 6.652086 3.007907 7.414772
## 289       5.268731 3.742359 6.795103 2.934346 7.603115
## 290       5.322210 3.725394 6.919027 2.880090 7.764331
## 291       5.371828 3.716874 7.026782 2.840794 7.902862
```

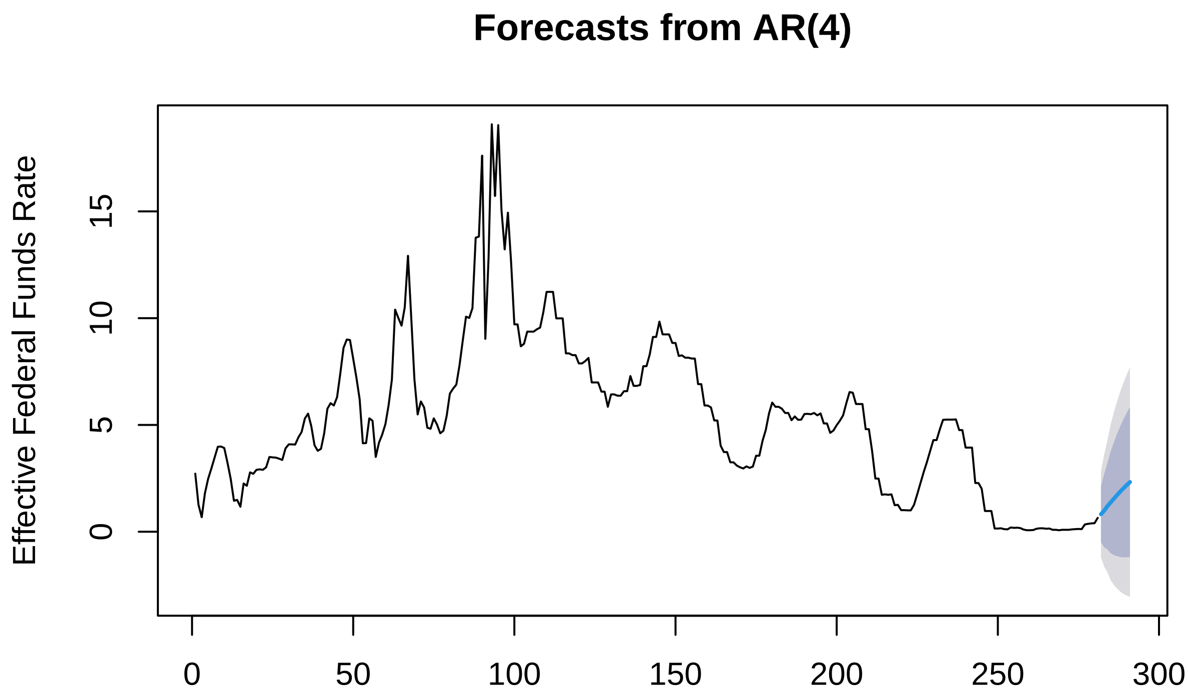
```
plot(forecast(ar1b, 10), ylab = "Unemployment Rate")
```




```r
forecast(ar2a, 10)
```

```
##      Point Forecast        Lo 80    Hi 80     Lo 95    Hi 95
## 282       0.8204178 -0.4981546 2.138990 -1.196165 2.837000
## 283       0.9892056 -0.7262239 2.704635 -1.634317 3.612729
## 284       1.1952101 -0.8236042 3.214024 -1.892300 4.282720
## 285       1.3733873 -1.0083505 3.755125 -2.269166 5.015941
## 286       1.5467528 -1.0959871 4.189493 -2.494969 5.588475
## 287       1.7203338 -1.1433716 4.584039 -2.659326 6.099993
## 288       1.8814920 -1.1856707 4.948655 -2.809329 6.572313
## 289       2.0364483 -1.1994022 5.272299 -2.912358 6.985255
## 290       2.1857298 -1.1975787 5.569038 -2.988594 7.360054
## 291       2.3271632 -1.1872690 5.841595 -3.047697 7.702023
```

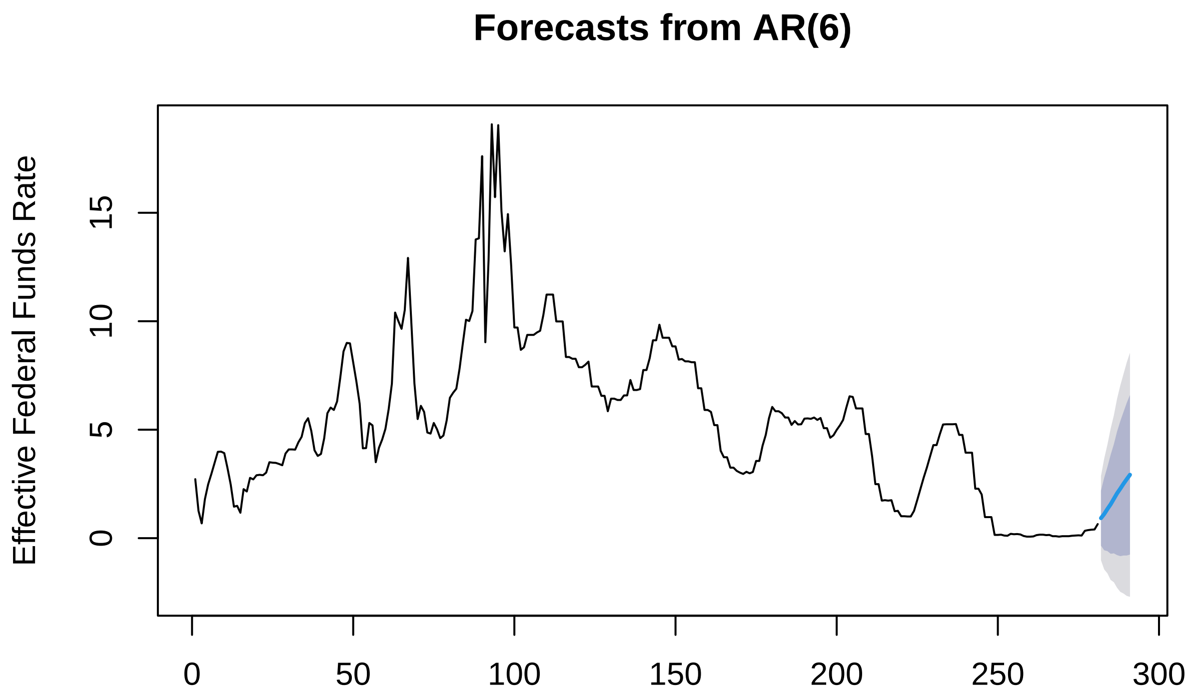
```r
plot(forecast(ar2a, 10), ylab = "Effective Federal Funds Rate")
```




```r
forecast(ar2b, 10)
```

```
##     Point Forecast       Lo 80    Hi 80     Lo 95    Hi 95
## 282      0.9220813 -0.3457513 2.189914 -1.016901 2.861064
## 283      1.1131623 -0.5571214 2.783446 -1.441316 3.667641
## 284      1.3443628 -0.5955335 3.284259 -1.622453 4.311178
## 285      1.5611818 -0.7186410 3.841005 -1.925506 5.047870
## 286      1.8157348 -0.7002956 4.331765 -2.032202 5.663671
## 287      2.0698755 -0.7813506 4.921102 -2.290699 6.430449
## 288      2.2838089 -0.8285375 5.396155 -2.476114 7.043732
## 289      2.5090460 -0.7996410 5.817733 -2.551154 7.569246
## 290      2.7191383 -0.7987636 6.237040 -2.661028 8.099305
## 291      2.9220131 -0.7564240 6.600450 -2.703671 8.547697
```

```r
plot(forecast(ar2b, 10), ylab = "Effective Federal Funds Rate")
```




```
library(ARDL)
```

```
## To cite ARDL in publications use:
##
## Kleanthis Natsiopoulos and Nickolaos Tzeremes (2021). ARDL: ARDL, ECM and Bounds-Test for Cointegrati
```

```
auto_ardl(Unemployment.Rate~Real.GDP..Percent.Change., data = FRIR_ts, max_order= 5, selection = "AIC")
```

```
## Warning: The `x` argument of `as_tibble.matrix()` must have unique column names if `.name_repair` is
## Using compatibility `.name_repair`.
## This warning is displayed once every 8 hours.
## Call `lifecycle::last_lifecycle_warnings()` to see where this warning was generated.
```

```
## $best_model
##
## Time series regression with "ts" data:
## Start = 5, End = 281
##
## Call:
## dynlm::dynlm(formula = full_formula, data = data, start = start,
##     end = end)
##
## Coefficients:
##                      (Intercept)          L(Unemployment.Rate, 1)
```

```
##                        0.3682675                           1.1099054
##          L(Unemployment.Rate, 2)           L(Unemployment.Rate, 3)
##                       -0.0905647                          -0.0474826
##        Real.GDP..Percent.Change.  L(Real.GDP..Percent.Change., 1)
##                       -0.0276477                          -0.0162098
## L(Real.GDP..Percent.Change., 2)  L(Real.GDP..Percent.Change., 3)
##                       -0.0144974                          -0.0089431
## L(Real.GDP..Percent.Change., 4)
##                       -0.0002863
##
##
## $best_order
## [1] 3 4
##
## $top_orders
##    Unemployment.Rate Real.GDP..Percent.Change.      AIC
## 1                  3                         4 39.95313
## 2                  4                         3 39.95615
## 3                  4                         2 40.38873
## 4                  2                         2 40.90676
## 5                  2                         3 41.26449
## 6                  3                         5 41.27452
## 7                  4                         4 41.95311
## 8                  1                         4 42.68547
## 9                  3                         3 43.19658
## 10                 1                         5 43.37019
## 11                 5                         5 45.08063
## 12                 1                         3 45.40306
## 13                 1                         2 46.94692
## 14                 1                         1 83.47256
```

```
auto_ardl(Unemployment.Rate~Inflation.Rate, data = FRIR_ts, max_order= 5, selection = "AIC")
```

```
## $best_model
##
## Time series regression with "ts" data:
## Start = 5, End = 281
##
## Call:
## dynlm::dynlm(formula = full_formula, data = data, start = start,
##     end = end)
##
## Coefficients:
##             (Intercept)  L(Unemployment.Rate, 1)  L(Unemployment.Rate, 2)
##                0.129969                 1.298896                -0.169131
## L(Unemployment.Rate, 3)           Inflation.Rate     L(Inflation.Rate, 1)
##               -0.167889                 0.046892                 0.041957
##    L(Inflation.Rate, 2)     L(Inflation.Rate, 3)     L(Inflation.Rate, 4)
##               -0.005461                -0.159325                 0.102595
##
##
## $best_order
## [1] 3 4
##
```

```
## $top_orders
##    Unemployment.Rate Inflation.Rate       AIC
## 1                  3              4  93.41317
## 2                  4              5  93.47028
## 3                  3              5  93.88133
## 4                  4              4  94.48246
## 5                  5              5  95.44971
## 6                  2              4  99.48033
## 7                  2              5 100.36566
## 8                  3              3 107.72866
## 9                  2              3 108.19997
## 10                 2              2 108.50282
## 11                 1              5 142.13430
## 12                 1              4 143.43379
## 13                 1              3 146.35835
## 14                 1              2 149.16422
## 15                 1              1 175.77111
```

```
auto_ardl(Effective.Federal.Funds.Rate~Real.GDP..Percent.Change., data = FRIR_ts, max_order= 5, selecti
```

```
## $best_model
##
## Time series regression with "ts" data:
## Start = 6, End = 281
##
## Call:
## dynlm::dynlm(formula = full_formula, data = data, start = start,
##     end = end)
##
## Coefficients:
##                         (Intercept)  L(Effective.Federal.Funds.Rate, 1)
##                           -0.211644                            0.752474
## L(Effective.Federal.Funds.Rate, 2)  L(Effective.Federal.Funds.Rate, 3)
##                            0.171868                            0.247031
## L(Effective.Federal.Funds.Rate, 4)           Real.GDP..Percent.Change.
##                           -0.207355                            0.008349
##    L(Real.GDP..Percent.Change., 1)     L(Real.GDP..Percent.Change., 2)
##                            0.083999                            0.029489
##    L(Real.GDP..Percent.Change., 3)     L(Real.GDP..Percent.Change., 4)
##                            0.009267                           -0.029609
##    L(Real.GDP..Percent.Change., 5)
##                            0.032802
##
##
## $best_order
## [1] 4 5
##
## $top_orders
##    Effective.Federal.Funds.Rate Real.GDP..Percent.Change.      AIC
## 1                             4                         5 784.1505
## 2                             5                         5 785.2623
## 3                             4                         4 787.1358
## 4                             3                         5 793.6182
## 5                             2                         5 794.2684
```

```
## 6                                3                          4 796.9039
## 7                                3                          3 797.0327
## 8                                2                          4 797.3418
## 9                                2                          3 797.4538
## 10                               2                          2 798.8979
## 11                               1                          5 804.3302
## 12                               1                          4 807.8754
## 13                               1                          3 808.1266
## 14                               1                          2 809.5294
## 15                               1                          1 810.0502
```

```r
auto_ardl(Effective.Federal.Funds.Rate~Inflation.Rate, data = FRIR_ts, max_order= 5, selection = "AIC")
```

```
## $best_model
##
## Time series regression with "ts" data:
## Start = 6, End = 281
##
## Call:
## dynlm::dynlm(formula = full_formula, data = data, start = start,
##     end = end)
##
## Coefficients:
##                        (Intercept)  L(Effective.Federal.Funds.Rate, 1)
##                            0.11376                             0.81775
## L(Effective.Federal.Funds.Rate, 2)                      Inflation.Rate
##                            0.11242                             0.82321
##                  L(Inflation.Rate, 1)                L(Inflation.Rate, 2)
##                           -1.09611                            -0.09824
##                  L(Inflation.Rate, 3)                L(Inflation.Rate, 4)
##                            0.81254                            -0.26150
##                  L(Inflation.Rate, 5)
##                           -0.11272
##
##
## $best_order
## [1] 2 5
##
## $top_orders
##    Effective.Federal.Funds.Rate Inflation.Rate      AIC
## 1                             2              5 778.1967
## 2                             2              4 779.2919
## 3                             3              5 779.5861
## 4                             1              5 779.5876
## 5                             1              4 779.8974
## 6                             3              4 780.6612
## 7                             4              5 780.8561
## 8                             4              4 782.1308
## 9                             5              5 782.8381
## 10                            2              3 789.9230
## 11                            3              3 789.9669
## 12                            1              3 791.1161
## 13                            1              2 797.2431
## 14                            2              2 797.4332
```

```
## 15                                1               1 809.9694
```

```r
ardl.1a<- ardl(Unemployment.Rate~Real.GDP..Percent.Change., data =FRIR_ts, order=c(3,4))
summary(ardl.1a)
```

```
##
## Time series regression with "ts" data:
## Start = 5, End = 281
##
## Call:
## dynlm::dynlm(formula = full_formula, data = data, start = start,
##     end = end)
##
## Residuals:
##      Min       1Q   Median       3Q      Max
## -0.71351 -0.15921 -0.00167  0.12707  1.61049
##
## Coefficients:
##                                  Estimate Std. Error t value Pr(>|t|)
## (Intercept)                     0.3682675  0.0730063   5.044 8.40e-07 ***
## L(Unemployment.Rate, 1)         1.1099054  0.0604328  18.366  < 2e-16 ***
## L(Unemployment.Rate, 2)        -0.0905647  0.0902876  -1.003  0.31673
## L(Unemployment.Rate, 3)        -0.0474826  0.0601069  -0.790  0.43024
## Real.GDP..Percent.Change.      -0.0276477  0.0051522  -5.366 1.74e-07 ***
## L(Real.GDP..Percent.Change., 1) -0.0162098  0.0056321  -2.878  0.00432 **
## L(Real.GDP..Percent.Change., 2) -0.0144974  0.0058006  -2.499  0.01304 *
## L(Real.GDP..Percent.Change., 3) -0.0089431  0.0055978  -1.598  0.11131
## L(Real.GDP..Percent.Change., 4) -0.0002863  0.0051000  -0.056  0.95528
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 0.255 on 268 degrees of freedom
## Multiple R-squared:  0.9733, Adjusted R-squared:  0.9726
## F-statistic:  1223 on 8 and 268 DF,  p-value: < 2.2e-16
```

```r
ardl.1b<- ardl(Unemployment.Rate~Inflation.Rate, data =FRIR_ts, order=c(3,4))
summary(ardl.1b)
```

```
##
## Time series regression with "ts" data:
## Start = 5, End = 281
##
## Call:
## dynlm::dynlm(formula = full_formula, data = data, start = start,
##     end = end)
##
## Residuals:
##      Min       1Q   Median       3Q      Max
## -0.69779 -0.15238 -0.02228  0.13153  1.51226
##
## Coefficients:
##                         Estimate Std. Error t value Pr(>|t|)
## (Intercept)             0.129969   0.076352   1.702  0.08987 .
```

```
## L(Unemployment.Rate, 1)  1.298896   0.059221  21.933  < 2e-16 ***
## L(Unemployment.Rate, 2) -0.169131   0.096272  -1.757  0.08009 .
## L(Unemployment.Rate, 3) -0.167889   0.059657  -2.814  0.00525 **
## Inflation.Rate           0.046892   0.038697   1.212  0.22667
## L(Inflation.Rate, 1)     0.041957   0.061592   0.681  0.49633
## L(Inflation.Rate, 2)    -0.005461   0.061644  -0.089  0.92948
## L(Inflation.Rate, 3)    -0.159325   0.060736  -2.623  0.00921 **
## L(Inflation.Rate, 4)     0.102595   0.038325   2.677  0.00789 **
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 0.2809 on 268 degrees of freedom
## Multiple R-squared:  0.9677, Adjusted R-squared:  0.9667
## F-statistic:  1003 on 8 and 268 DF,  p-value: < 2.2e-16
```

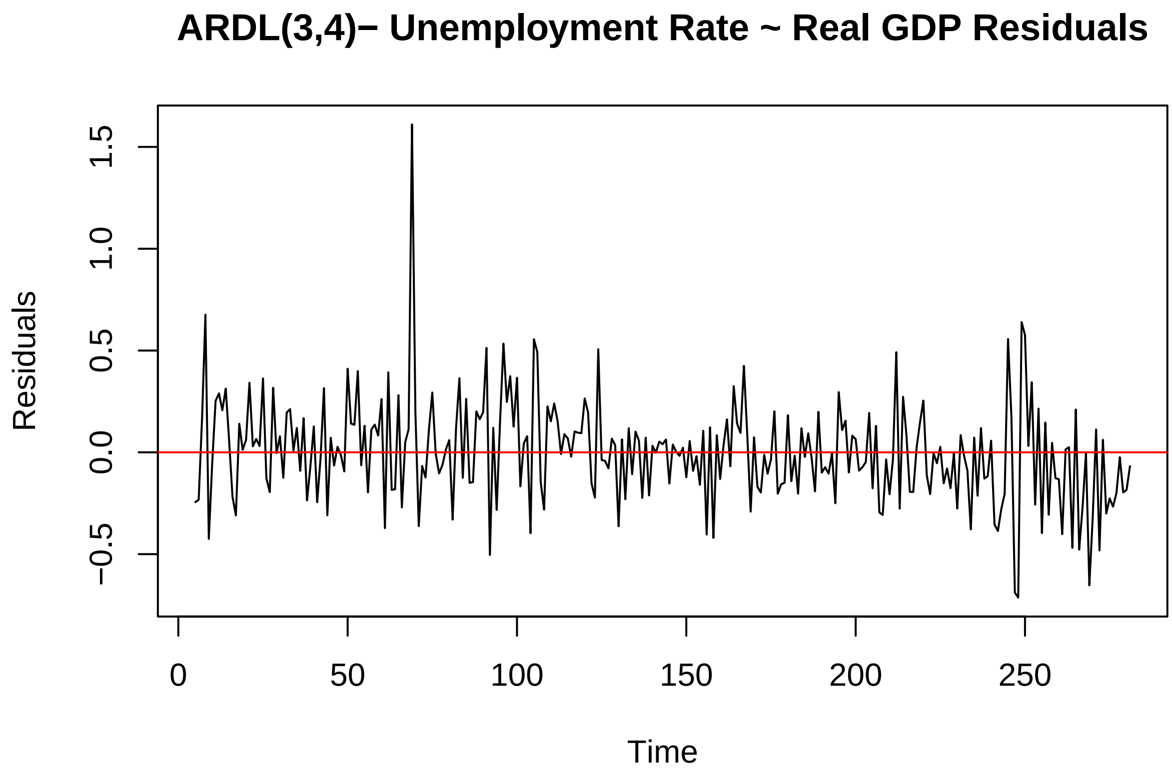
```
ardl.2a<- ardl(Effective.Federal.Funds.Rate~ Real.GDP..Percent.Change., data =FRIR_ts, order=c(4,5))
summary(ardl.2a)
```

```
##
## Time series regression with "ts" data:
## Start = 6, End = 281
##
## Call:
## dynlm::dynlm(formula = full_formula, data = data, start = start,
##     end = end)
##
## Residuals:
##     Min      1Q  Median      3Q     Max
## -6.9248 -0.3291 -0.0646  0.3082  6.0031
##
## Coefficients:
##                                    Estimate Std. Error t value Pr(>|t|)
## (Intercept)                       -0.211644   0.138489  -1.528 0.127647
## L(Effective.Federal.Funds.Rate, 1)  0.752474   0.061413  12.253  < 2e-16 ***
## L(Effective.Federal.Funds.Rate, 2)  0.171868   0.075180   2.286 0.023038 *
## L(Effective.Federal.Funds.Rate, 3)  0.247031   0.075512   3.271 0.001212 **
## L(Effective.Federal.Funds.Rate, 4) -0.207355   0.061842  -3.353 0.000916 ***
## Real.GDP..Percent.Change.           0.008349   0.020125   0.415 0.678593
## L(Real.GDP..Percent.Change., 1)     0.083999   0.021171   3.968 9.35e-05 ***
## L(Real.GDP..Percent.Change., 2)     0.029489   0.021798   1.353 0.177262
## L(Real.GDP..Percent.Change., 3)     0.009267   0.021721   0.427 0.669984
## L(Real.GDP..Percent.Change., 4)    -0.029609   0.021124  -1.402 0.162181
## L(Real.GDP..Percent.Change., 5)     0.032802   0.018610   1.763 0.079115 .
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 0.9787 on 265 degrees of freedom
## Multiple R-squared:  0.9285, Adjusted R-squared:  0.9258
## F-statistic: 344.2 on 10 and 265 DF,  p-value: < 2.2e-16
```

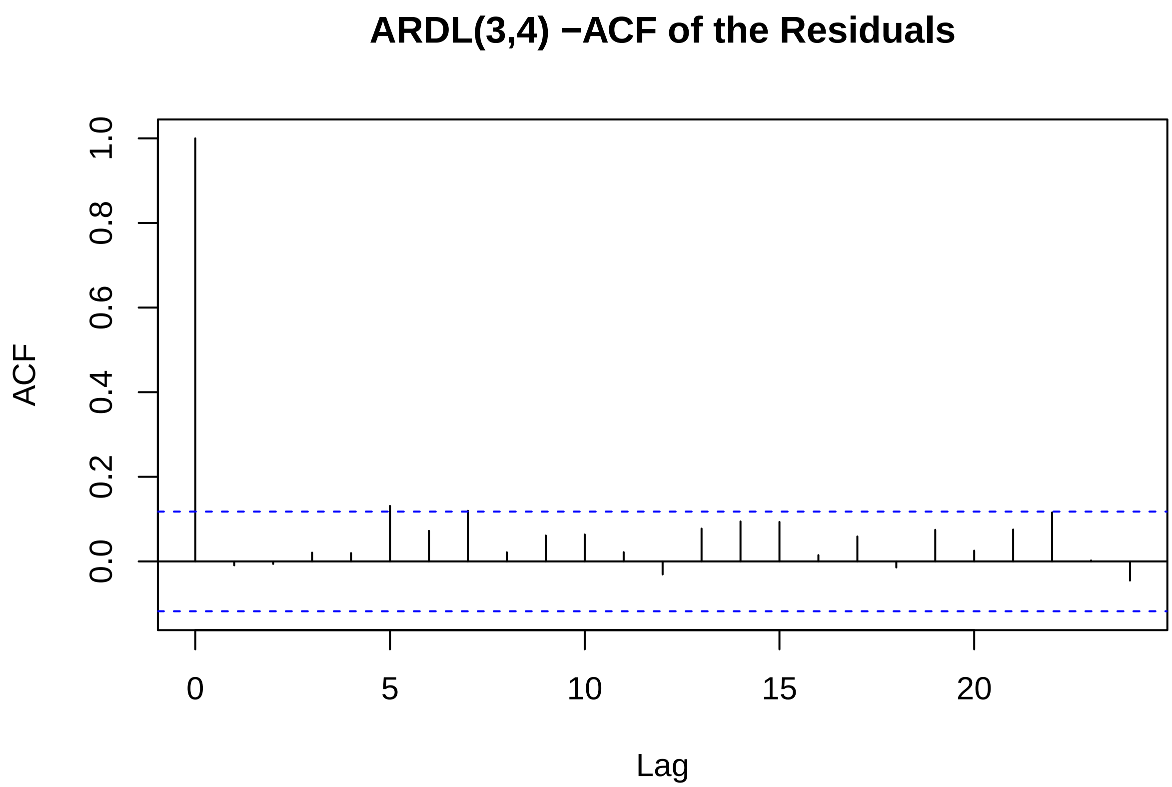
```
ardl.2b<- ardl(Effective.Federal.Funds.Rate~Inflation.Rate, data =FRIR_ts, order=c(2,5))
summary(ardl.2b)
```

```
##
## Time series regression with "ts" data:
## Start = 6, End = 281
##
## Call:
## dynlm::dynlm(formula = full_formula, data = data, start = start,
##     end = end)
##
## Residuals:
##     Min      1Q  Median      3Q     Max
## -6.4654 -0.3180  0.0074  0.3074  6.5049
##
## Coefficients:
##                                    Estimate Std. Error t value Pr(>|t|)
## (Intercept)                         0.11376    0.11508   0.988 0.323812
## L(Effective.Federal.Funds.Rate, 1)  0.81775    0.06099  13.408  < 2e-16 ***
## L(Effective.Federal.Funds.Rate, 2)  0.11242    0.06188   1.817 0.070377 .
## Inflation.Rate                      0.82321    0.13641   6.035 5.29e-09 ***
## L(Inflation.Rate, 1)               -1.09611    0.22172  -4.944 1.36e-06 ***
## L(Inflation.Rate, 2)               -0.09824    0.22378  -0.439 0.661017
## L(Inflation.Rate, 3)                0.81254    0.21273   3.820 0.000166 ***
## L(Inflation.Rate, 4)               -0.26150    0.22354  -1.170 0.243117
## L(Inflation.Rate, 5)               -0.11272    0.13754  -0.820 0.413204
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 0.9716 on 267 degrees of freedom
## Multiple R-squared:  0.929,  Adjusted R-squared:  0.9269
## F-statistic: 436.8 on 8 and 267 DF,  p-value: < 2.2e-16
```

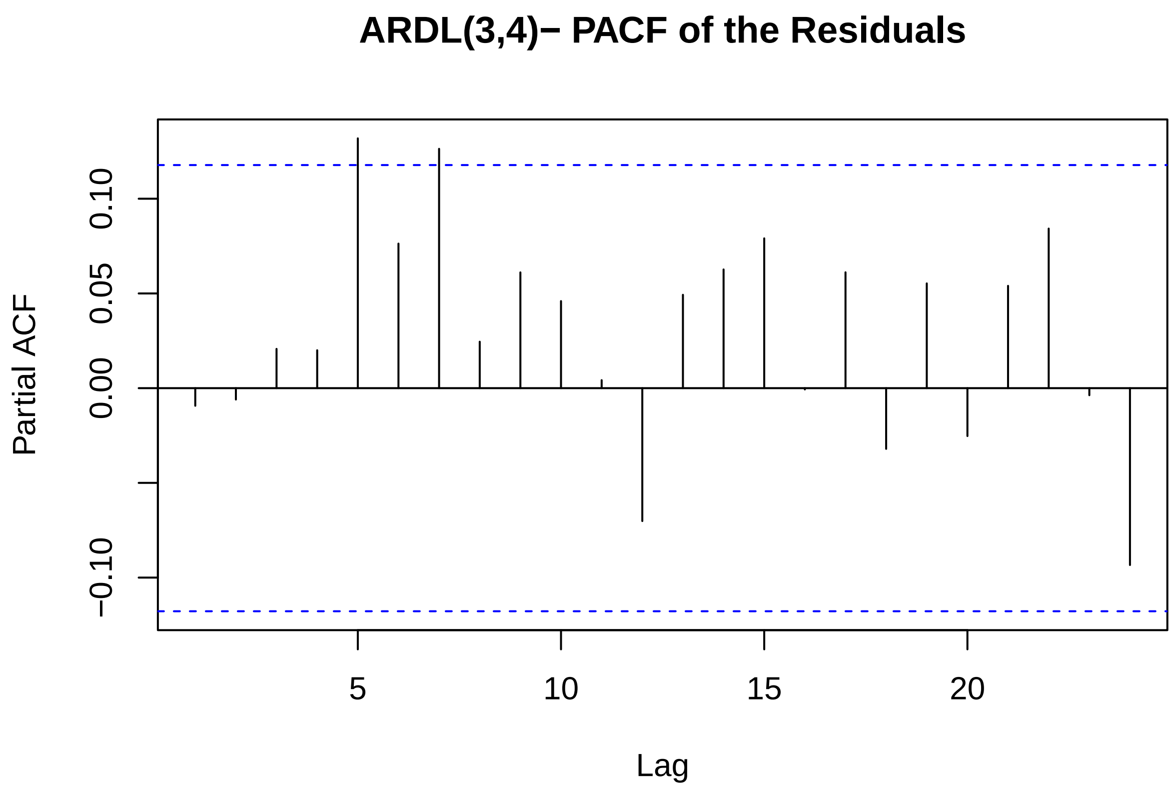
```
plot(ardl.1a$residuals,pch=20,ylab="Residuals", main="ARDL(3,4)- Unemployment Rate ~ Real GDP Residuals
abline(h=0,lwd=1,col="red")
```




```
acf(ardl.1a$residuals, main="ARDL(3,4) -ACF of the Residuals")
```

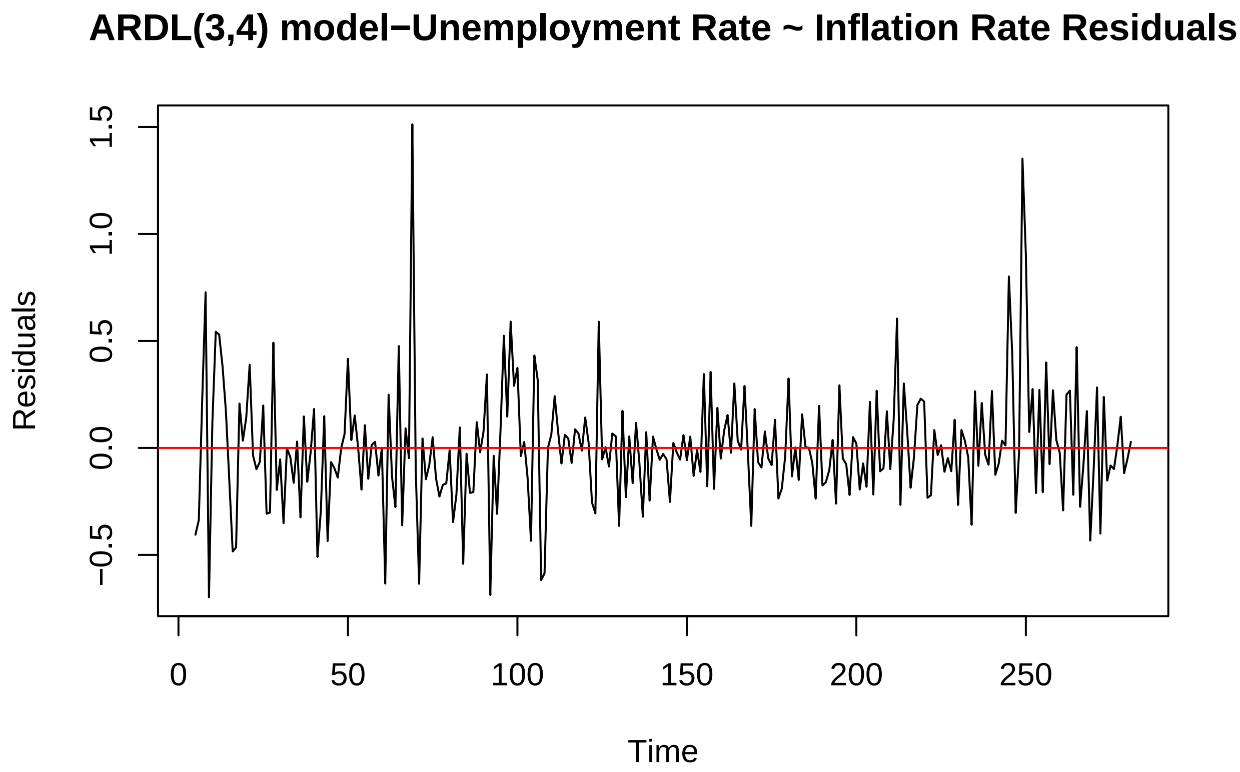



```
pacf(ardl.1a$residuals, main="ARDL(3,4)- PACF of the Residuals")
```

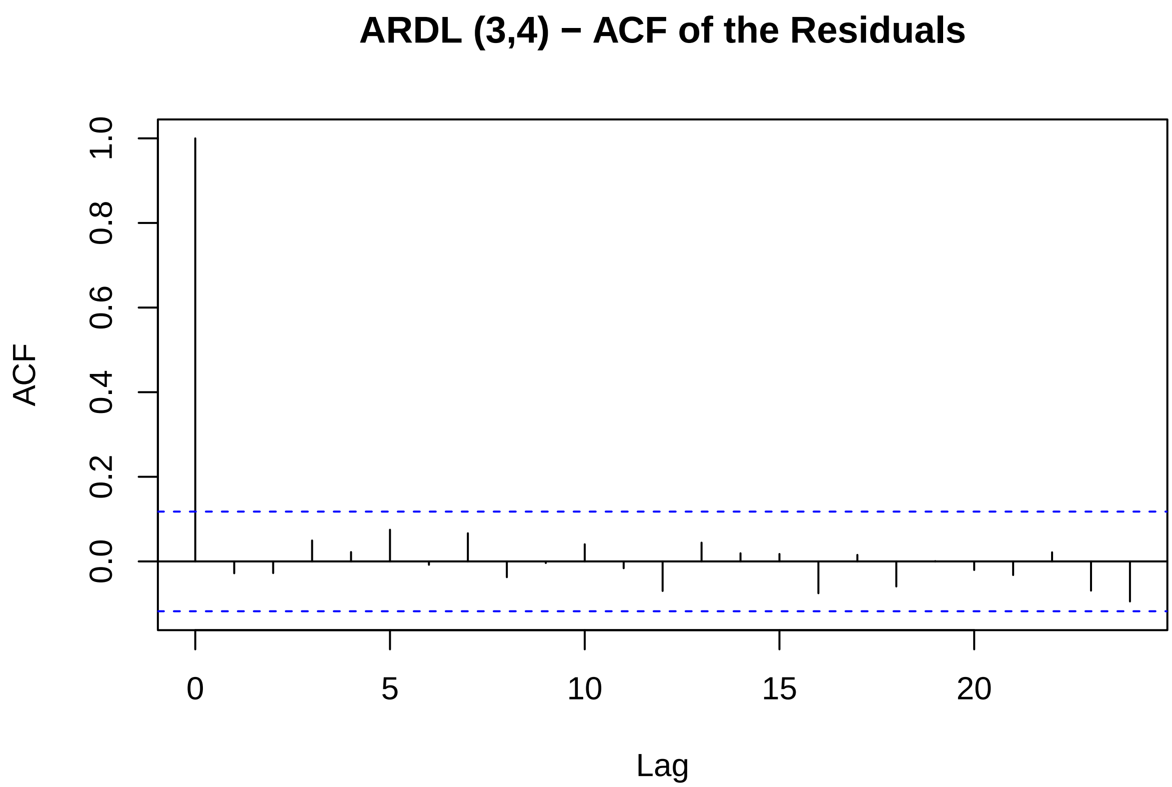


The ARDL Residuals plot (1a) for Unemployment rate shows high stationarity, although it does display a spike at around time 70. The data is mean reverting and persistent. The ACF shows that there are no high spikes which is a good indicator that serial correlation isn't present and therefore that this is an appropriate model.The PCF displays a slight spike at lag 5 which could indicate serial correlation but is not high enough to where we would disregard the model.

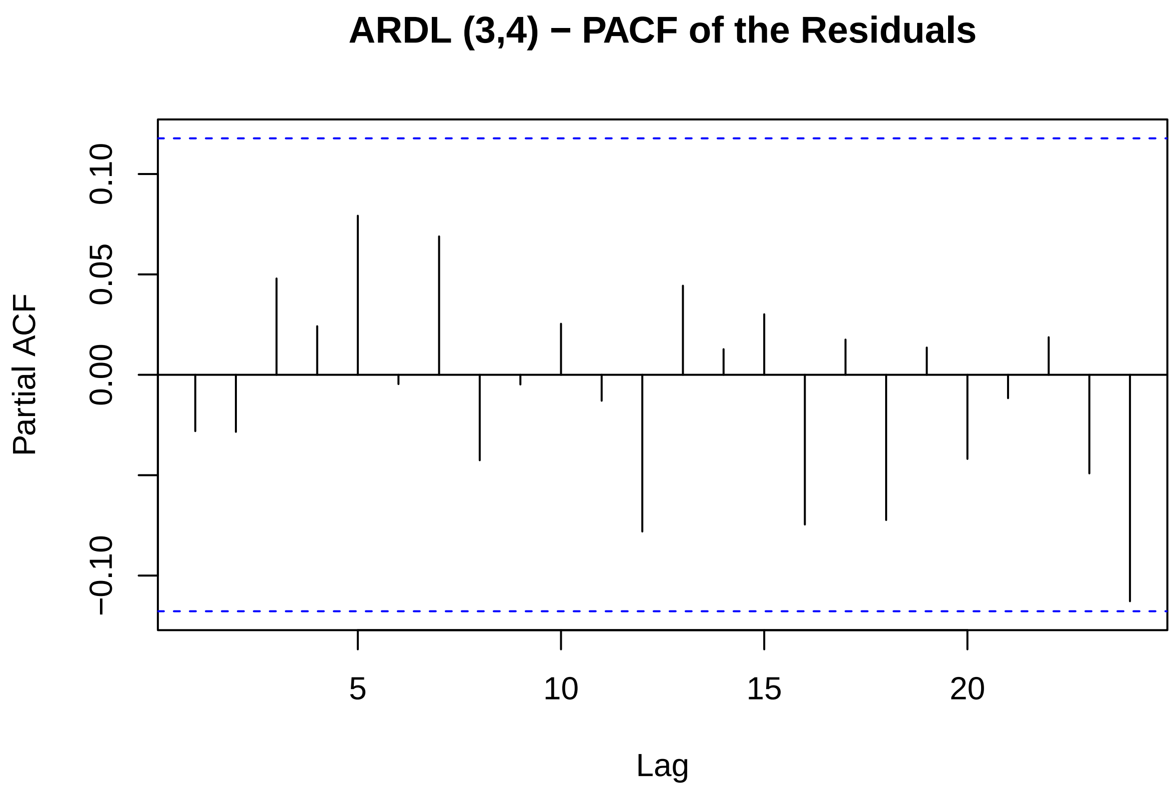
```
plot(ardl.1b$residuals,pch=20,ylab="Residuals", main="ARDL(3,4) model-Unemployment Rate ~ Inflation Rat
abline(h=0,lwd=1,col="red")
```




```
acf(ardl.1b$residuals, main="ARDL (3,4) - ACF of the Residuals")
```

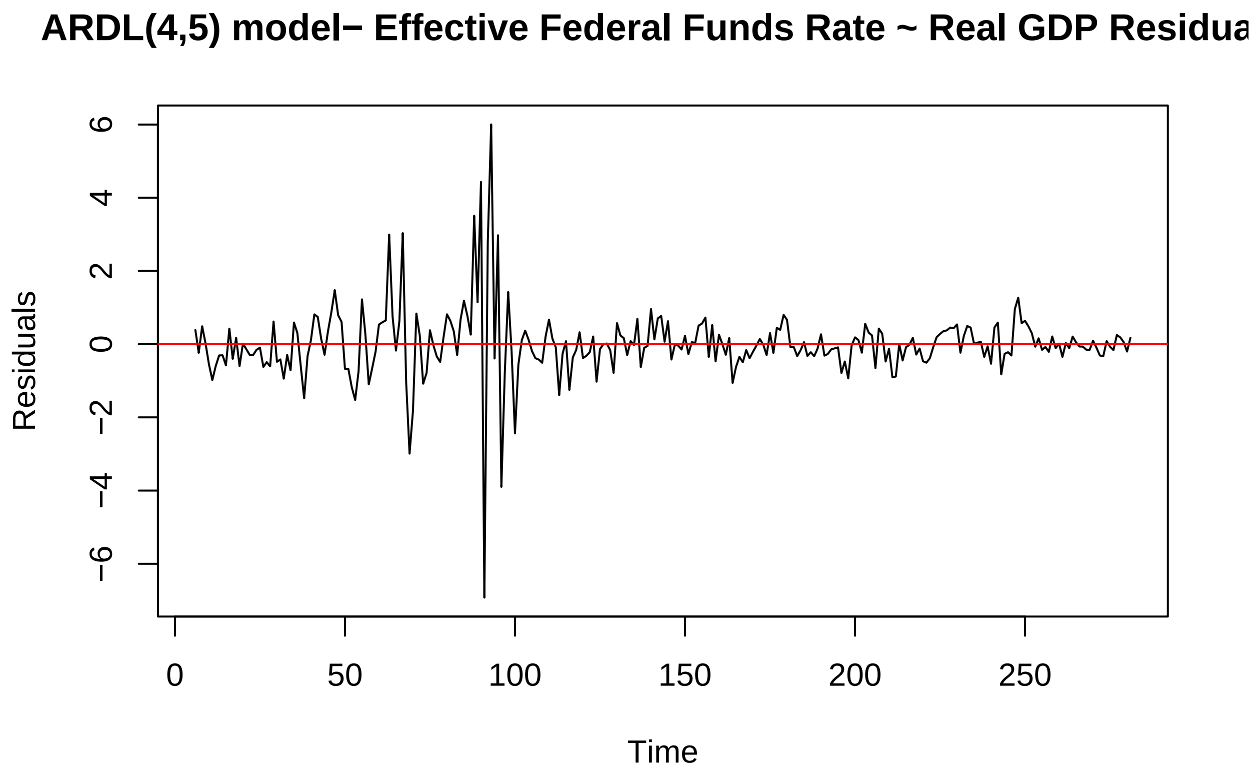



```
pacf(ardl.1b$residuals, main="ARDL (3,4) - PACF of the Residuals")
```

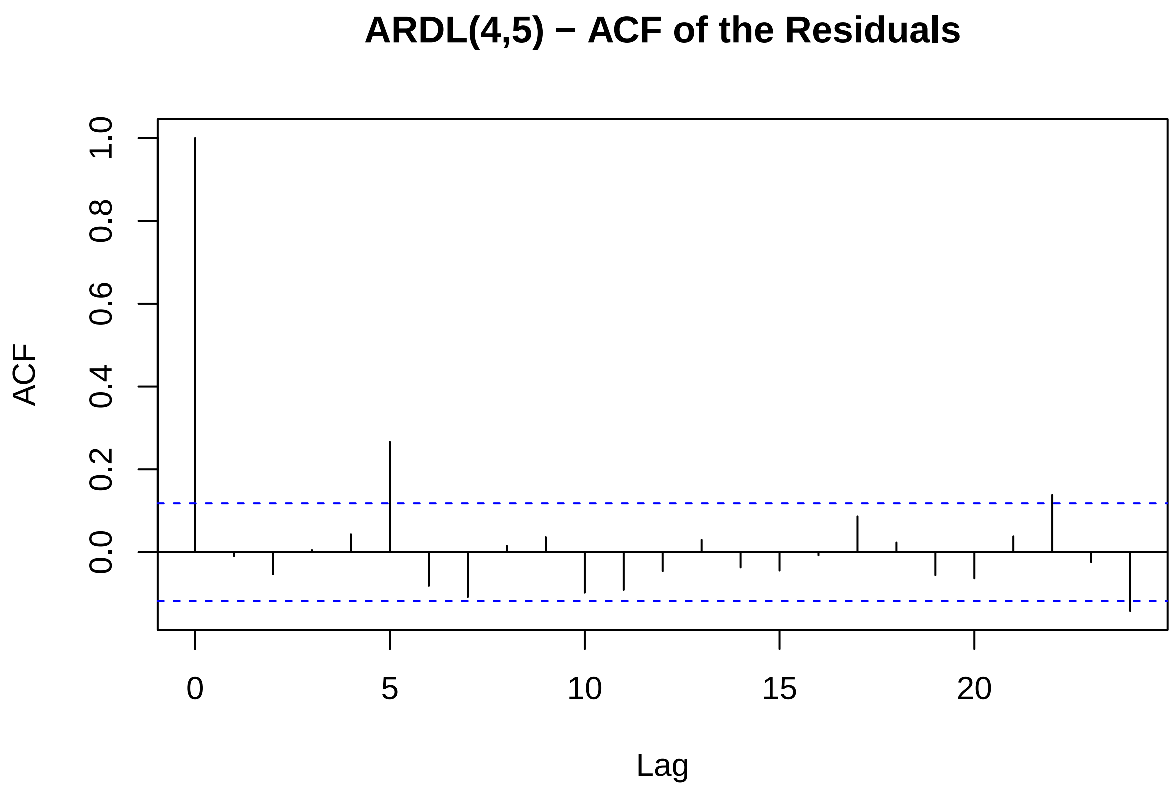


The ARDL Residuals plot (1b) for Unemployment rate shows a good amount of stationarity and persistance but has more variance than the (1a) residuals. There are two significant spikes again around time 70 and at time 250. When looking at the ACF there are slight spikes at lag 1 and 2 which could indicate a level of serial correlation at those lags but are not significantly high enough where we would disregard the model as inappropriate. However, this could indicate that another model could be a better model of the data. The PCF displays no significant spikes at any lags which is good as that means there is no serial correlation.

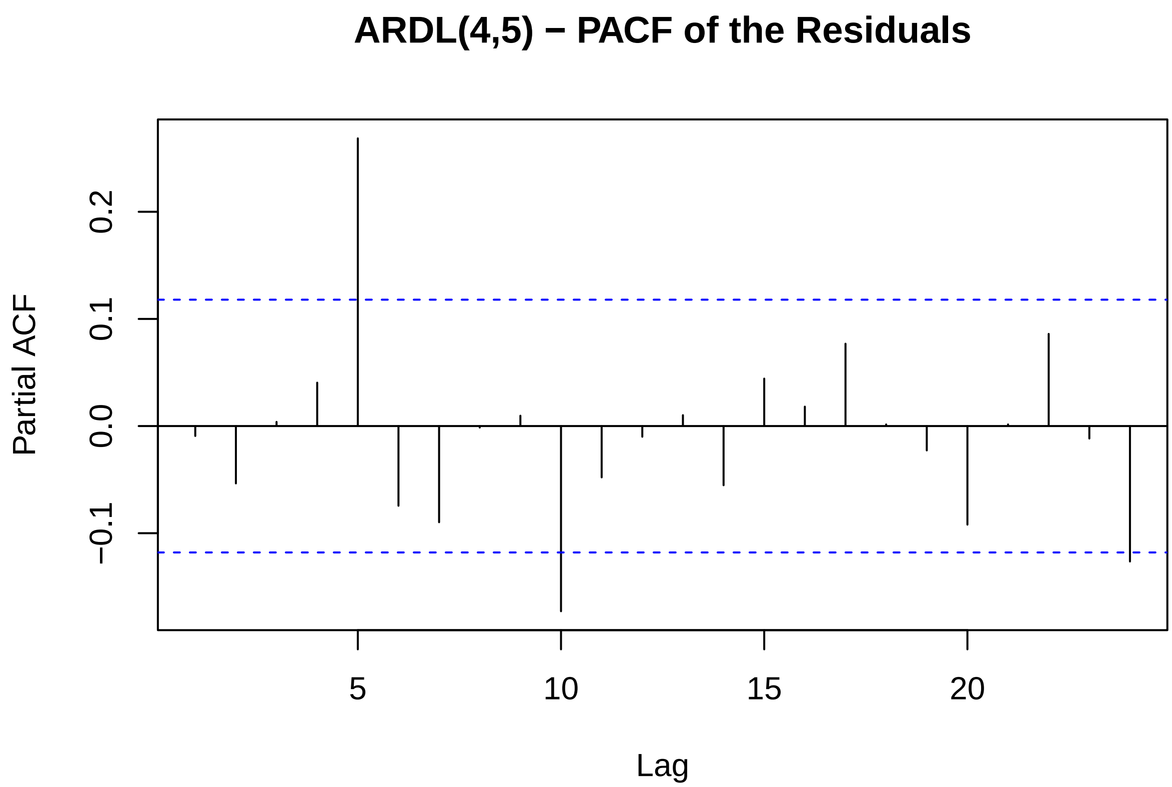
```
plot(ardl.2a$residuals,pch=20,ylab="Residuals", main="ARDL(4,5) model- Effective Federal Funds Rate ~ R
abline(h=0,lwd=1,col="red")
```




```
acf(ardl.2a$residuals, main="ARDL(4,5) - ACF of the Residuals")
```

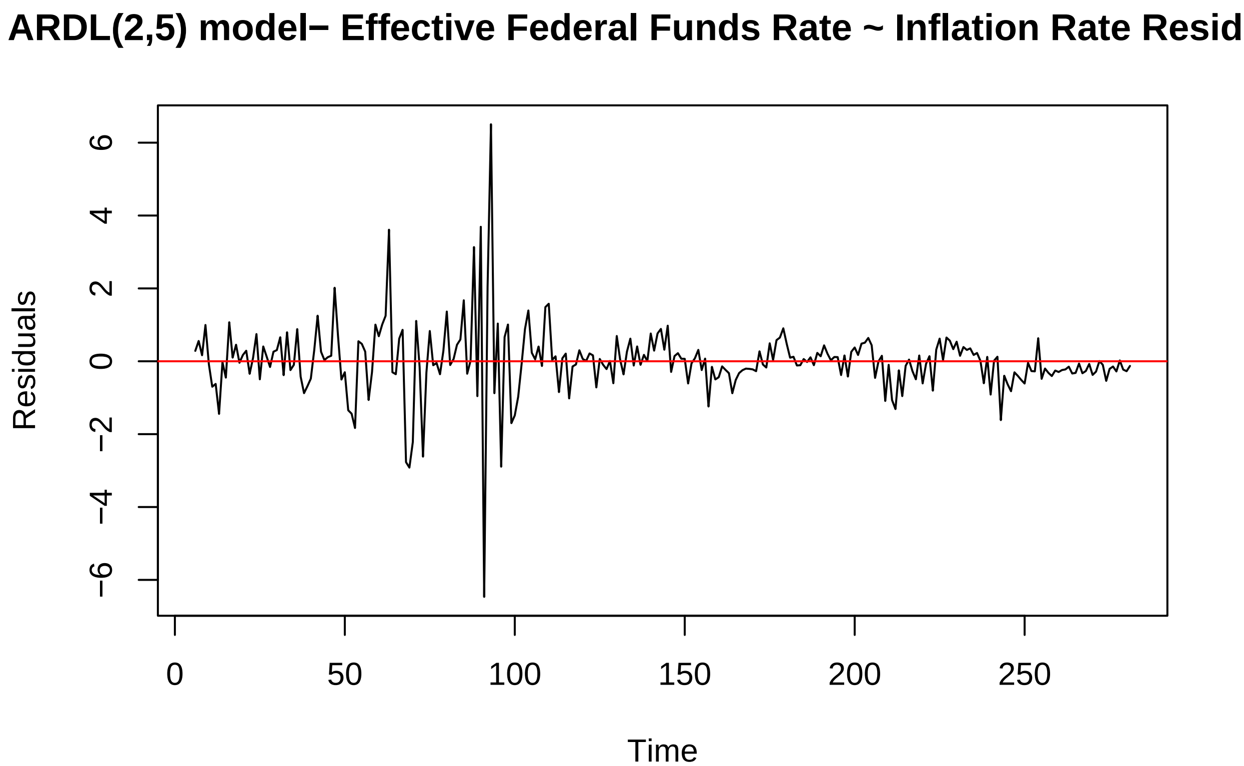



```
pacf(ardl.2a$residuals, main="ARDL(4,5) - PACF of the Residuals")
```

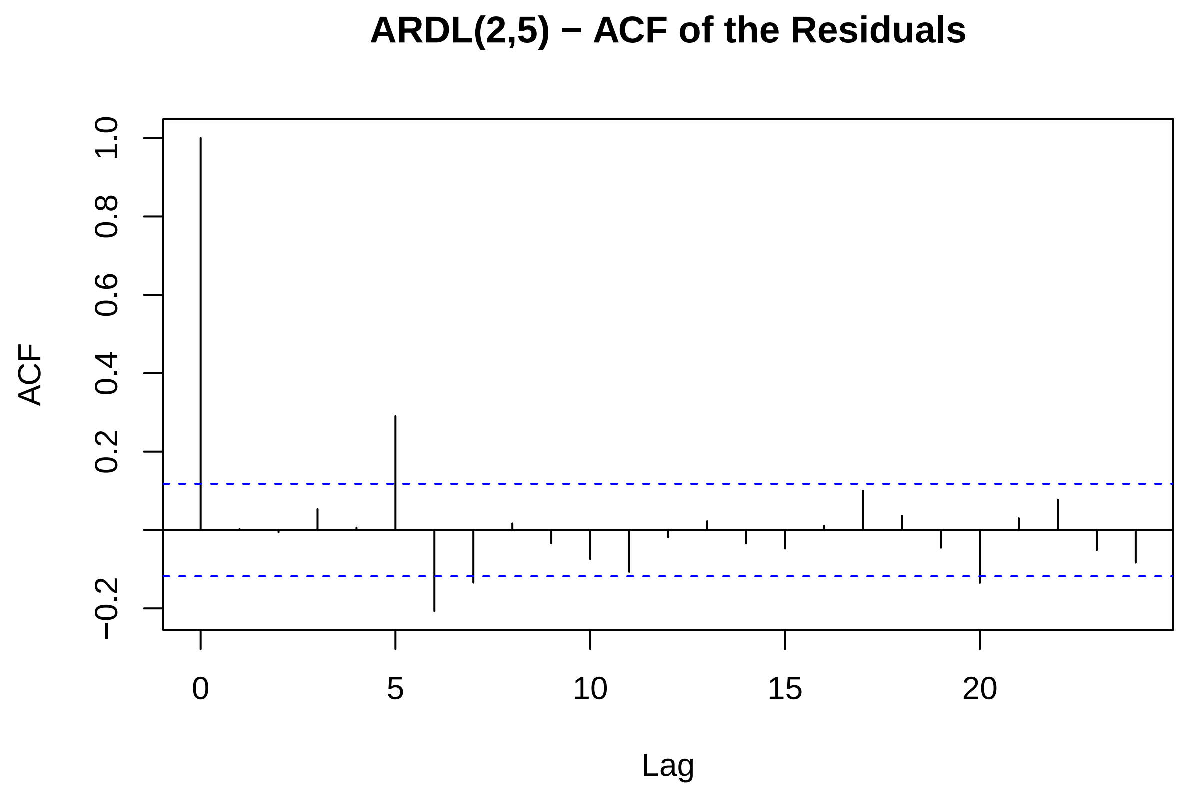


The ARDL Residuals plot (2a) for Effective Federal Funds Rate is extremely stationary, persistant, with constant variance, and mean reverting, in exception to a very significant spike around time 90 and and earlier spike around time 70. The ACF shows a slight spike at lag 3 and lag 5 which indicates that there possibly is serial correlation at those lags but it is not high enough to cause us to disregard the model. However, this could indicate that a different model could be better. The PCF also shows the same slight spikes at lag 3 and lag 5.

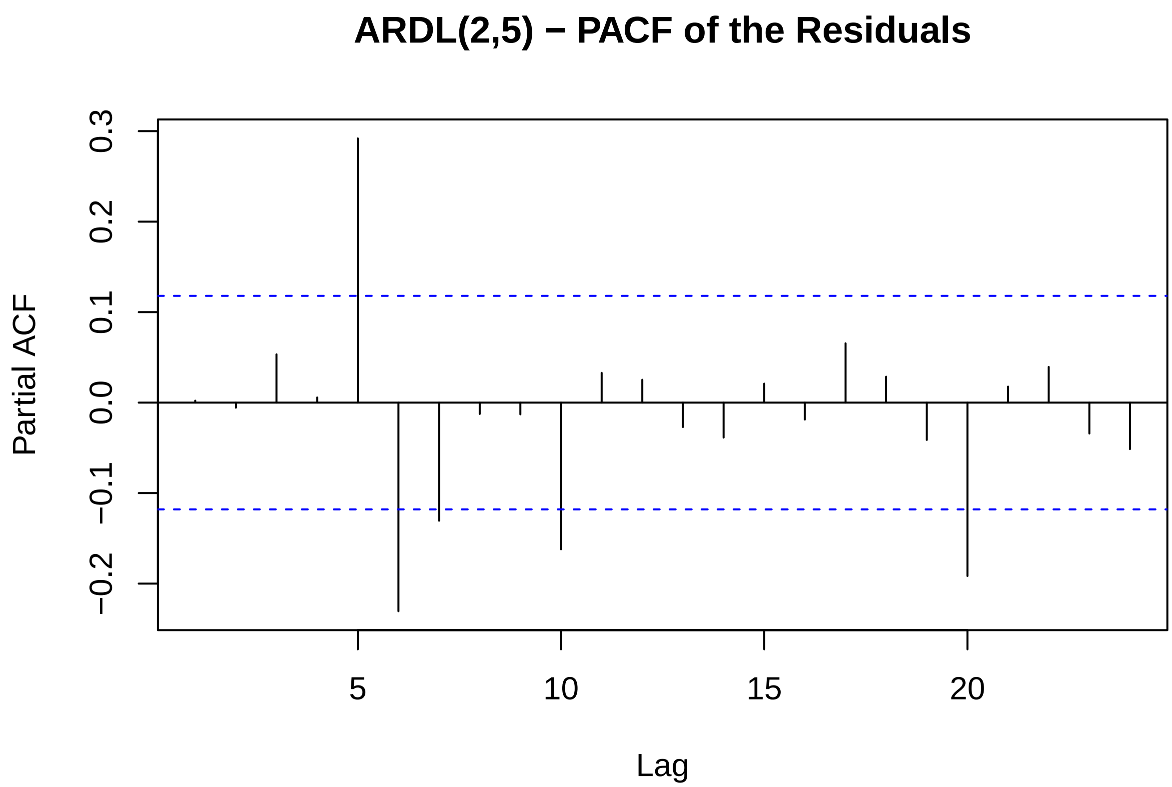
```
plot(ardl.2b$residuals,pch=20,ylab="Residuals", main="ARDL(2,5) model- Effective Federal Funds Rate ~ In
abline(h=0,lwd=1,col="red")
```




```
acf(ardl.2b$residuals, main="ARDL(2,5) - ACF of the Residuals")
```




```
pacf(ardl.2b$residuals, main="ARDL(2,5) - PACF of the Residuals")
```



The ARDL Residuals plot (2b) for Effective Federal Funds Rate is almost identical to the residuals plot for (2a). Again there is a spike around time 70 and time 90. The ACF also is identical to the ACF of (2a) with a slight spike at lag 5 again indicating that there could be some serial correlation at that lag but the level on the ACF is not high enough for us to completely disregard the model. The PCF also displays that slight spike at lag 5.

It is interesting to note that there is a spike around time 70 in each residual plot for all four ARDL models. This could indicate that there was a shock to the economy or some other significant event that has caused this spike.

```
df<-data.frame(FRIR_ts)
row.number<-1:188
train=df[row.number,]
test=df[-row.number,]

#ARDL(3,4) - Unemployment Rate ~ Real GDP 1a
library(ARDL)
library(dLagM)
```

```
## Loading required package: nardl

##
## Attaching package: 'dLagM'

## The following object is masked from 'package:forecast':
##
##     forecast
```

```r
ardl.1a.compare=ardl(Unemployment.Rate~Real.GDP..Percent.Change., order=c(3,4), data=train)
summary(ardl.1a.compare)
```

```
##
## Time series regression with "ts" data:
## Start = 5, End = 188
##
## Call:
## dynlm::dynlm(formula = full_formula, data = data, start = start,
##     end = end)
##
## Residuals:
##      Min       1Q   Median       3Q      Max
## -0.53721 -0.14312  0.00013  0.10495  1.50460
##
## Coefficients:
##                                  Estimate Std. Error t value Pr(>|t|)
## (Intercept)                      0.417813   0.094256   4.433 1.64e-05 ***
## L(Unemployment.Rate, 1)          1.030321   0.074717  13.790  < 2e-16 ***
## L(Unemployment.Rate, 2)         -0.039087   0.107495  -0.364 0.716586
## L(Unemployment.Rate, 3)         -0.010358   0.075214  -0.138 0.890627
## Real.GDP..Percent.Change.       -0.037080   0.005578  -6.647 3.65e-10 ***
## L(Real.GDP..Percent.Change., 1) -0.023987   0.006325  -3.793 0.000205 ***
## L(Real.GDP..Percent.Change., 2) -0.019261   0.006687  -2.881 0.004465 **
## L(Real.GDP..Percent.Change., 3) -0.010356   0.006173  -1.678 0.095192 .
## L(Real.GDP..Percent.Change., 4)  0.003331   0.005474   0.608 0.543676
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 0.2422 on 175 degrees of freedom
## Multiple R-squared:  0.9736, Adjusted R-squared:  0.9724
## F-statistic: 806.7 on 8 and 175 DF,  p-value: < 2.2e-16
```

```r
train.ur.ardl.1<-ardl(Unemployment.Rate~Real.GDP..Percent.Change.,data=train ,order=c(3,4))
MSE.train.ur.ardl.1= mean(train.ur.ardl.1[["residuals"]]^2)
MSE.train.ur.ardl.1
```

```
## [1] 0.05580404
```

```r
test.ur.ardl.1<-ardl(Unemployment.Rate~Real.GDP..Percent.Change.,data=test ,order=c(3,4))
MSE.test.ur.ardl.1 =mean(test.ur.ardl.1[["residuals"]]^2)
MSE.test.ur.ardl.1
```

```
## [1] 0.05206973
```

```r
#ARDL(3,4) model-Unemployment Rate ~ Inflation Rate 1b
ardl.1b.compare=ardl(Unemployment.Rate~Inflation.Rate, order=c(3,4), data=train)
summary(ardl.1b.compare)
```

```
##
## Time series regression with "ts" data:
```

```
## Start = 5, End = 188
##
## Call:
## dynlm::dynlm(formula = full_formula, data = data, start = start,
##     end = end)
##
## Residuals:
##      Min       1Q   Median       3Q      Max
## -0.70784 -0.15415 -0.00405  0.11125  1.50303
##
## Coefficients:
##                          Estimate Std. Error t value Pr(>|t|)
## (Intercept)               0.12850    0.10574   1.215  0.22592
## L(Unemployment.Rate, 1)  1.27064    0.07356  17.273  < 2e-16 ***
## L(Unemployment.Rate, 2) -0.20328    0.11607  -1.751  0.08162 .
## L(Unemployment.Rate, 3) -0.11810    0.07311  -1.615  0.10803
## Inflation.Rate            0.04932    0.04093   1.205  0.22980
## L(Inflation.Rate, 1)      0.03831    0.06502   0.589  0.55645
## L(Inflation.Rate, 2)      0.01068    0.06509   0.164  0.86986
## L(Inflation.Rate, 3)     -0.16639    0.06374  -2.610  0.00983 **
## L(Inflation.Rate, 4)      0.10805    0.04089   2.643  0.00897 **
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 0.2802 on 175 degrees of freedom
## Multiple R-squared:  0.9647, Adjusted R-squared:  0.963
## F-statistic: 597.2 on 8 and 175 DF,  p-value: < 2.2e-16
```

```r
train.ur.ardl.2<-ardl(Unemployment.Rate~Inflation.Rate,data=train ,order=c(3,4))
MSE.train.ur.ardl.2 = mean(train.ur.ardl.2[["residuals"]]^2)
MSE.train.ur.ardl.2
```

```
## [1] 0.07468998
```

```r
test.ur.ardl.2<-ardl(Unemployment.Rate~Inflation.Rate,data=test ,order=c(3,4))
MSE.test.ur.ardl.2=mean(test.ur.ardl.2[["residuals"]]^2)
MSE.test.ur.ardl.2
```

```
## [1] 0.07260941
```

According to the MSE, the model of ARDL(3,4) for Unemployment.Rate ~ Real.GDP..Percent.Change. is a better fit as the MSE for both training and testing is lower than that of ARDL (3,4) for Unemployment.Rate ~ Inflation.Rate

```r
#ARDL(4,5) model- Effective Federal Funds Rate ~ Real GDP
ardl.2a.compare=ardl(Effective.Federal.Funds.Rate~ Real.GDP..Percent.Change., order=c(4,5), data=train)
summary(ardl.2a.compare)
```

```
##
## Time series regression with "ts" data:
## Start = 6, End = 188
##
```

```
## Call:
## dynlm::dynlm(formula = full_formula, data = data, start = start,
##     end = end)
##
## Residuals:
##     Min      1Q  Median      3Q     Max
## -6.6499 -0.4545 -0.0536  0.4134  5.8367
##
## Coefficients:
##                                    Estimate Std. Error t value Pr(>|t|)
## (Intercept)                        -0.49919    0.34394  -1.451 0.148498
## L(Effective.Federal.Funds.Rate, 1)  0.73190    0.07732   9.466  < 2e-16 ***
## L(Effective.Federal.Funds.Rate, 2)  0.19172    0.09267   2.069 0.040062 *
## L(Effective.Federal.Funds.Rate, 3)  0.26274    0.09326   2.817 0.005408 **
## L(Effective.Federal.Funds.Rate, 4) -0.20145    0.07977  -2.525 0.012460 *
## Real.GDP..Percent.Change.           0.01431    0.02845   0.503 0.615607
## L(Real.GDP..Percent.Change., 1)     0.10668    0.02885   3.698 0.000292 ***
## L(Real.GDP..Percent.Change., 2)     0.04319    0.02959   1.460 0.146225
## L(Real.GDP..Percent.Change., 3)     0.01315    0.02983   0.441 0.659900
## L(Real.GDP..Percent.Change., 4)    -0.03601    0.02857  -1.261 0.209124
## L(Real.GDP..Percent.Change., 5)     0.03708    0.02499   1.484 0.139671
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 1.176 on 172 degrees of freedom
## Multiple R-squared:  0.8774, Adjusted R-squared:  0.8703
## F-statistic: 123.1 on 10 and 172 DF,  p-value: < 2.2e-16
```

```
train.effr.ardl.1<-ardl(Effective.Federal.Funds.Rate~Real.GDP..Percent.Change.,data=train ,order=c(4,5))
MSE.train.effr.ardl.1 =mean(train.effr.ardl.1[["residuals"]]^2)
MSE.train.effr.ardl.1
```

```
## [1] 1.299018
```

```
test.effr.ardl.1<-ardl(Effective.Federal.Funds.Rate~Real.GDP..Percent.Change.,data=test ,order=c(4,5))
MSE.test.effr.ardl.1=mean(test.ur.ardl.1[["residuals"]]^2)
MSE.test.effr.ardl.1
```

```
## [1] 0.05206973
```

```
#ARDL(2,5) model- Effective Federal Funds Rate ~ Inflation Rate
ardl.2b.compare=ardl(Effective.Federal.Funds.Rate~Inflation.Rate, order=c(2,5), data=train)
summary(ardl.2b.compare)
```

```
##
## Time series regression with "ts" data:
## Start = 6, End = 188
##
## Call:
## dynlm::dynlm(formula = full_formula, data = data, start = start,
##     end = end)
##
```

```
## Residuals:
##     Min      1Q  Median      3Q     Max 
## -6.1131 -0.4235 -0.0210  0.3528  6.5588 
## 
## Coefficients:
##                                     Estimate Std. Error t value Pr(>|t|)    
## (Intercept)                          0.40697    0.20443   1.991  0.04808 *  
## L(Effective.Federal.Funds.Rate, 1)   0.77338    0.07562  10.227  < 2e-16 ***
## L(Effective.Federal.Funds.Rate, 2)   0.10781    0.07632   1.413  0.15958    
## Inflation.Rate                       0.86459    0.16783   5.152 6.95e-07 ***
## L(Inflation.Rate, 1)                -1.12319    0.27565  -4.075 6.99e-05 ***
## L(Inflation.Rate, 2)                -0.12675    0.27984  -0.453  0.65116    
## L(Inflation.Rate, 3)                 0.87702    0.26519   3.307  0.00115 ** 
## L(Inflation.Rate, 4)                -0.30380    0.27829  -1.092  0.27650    
## L(Inflation.Rate, 5)                -0.09815    0.16958  -0.579  0.56346    
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
## 
## Residual standard error: 1.147 on 174 degrees of freedom
## Multiple R-squared:  0.8819, Adjusted R-squared:  0.8765 
## F-statistic: 162.5 on 8 and 174 DF,  p-value: < 2.2e-16
```

```
train.effr.ardl.2<-ardl(Effective.Federal.Funds.Rate~Inflation.Rate,data=train ,order=c(2,5))
MSE.train.effr.ardl.2 =mean(train.effr.ardl.2[["residuals"]]^2)
MSE.train.effr.ardl.2
```

```
## [1] 1.251344
```

```
test.effr.ardl.2<-ardl(Effective.Federal.Funds.Rate~Inflation.Rate,data=test ,order=c(2,5))
MSE.test.effr.ardl.2=mean(test.ur.ardl.2[["residuals"]]^2)
MSE.test.effr.ardl.2
```

```
## [1] 0.07260941
```

According to the MSE, the model of ARDL(2,5) for Effective.Federal.Funds.Rate ~ Inflation.Rate is a better fit as the MSE for both training and testing is lower than that of ARDL(4,5) for Effective.Federal.Funds.Rate ~ Real.GDP..Percent.Change.

```
AIC(ardl.1a.compare)
```

```
## [1] 11.16212
```

```
AIC(ardl.1b.compare)
```

```
## [1] 64.79805
```

```
AIC(ardl.2a.compare)
```

```
## [1] 591.2058
```

```
AIC(ardl.2b.compare)
```

```
## [1] 580.3635
```

```
BIC(ardl.1a.compare)
```

```
## [1] 43.31148
```

```
BIC(ardl.1b.compare)
```

```
## [1] 96.94741
```

```
BIC(ardl.2a.compare)
```

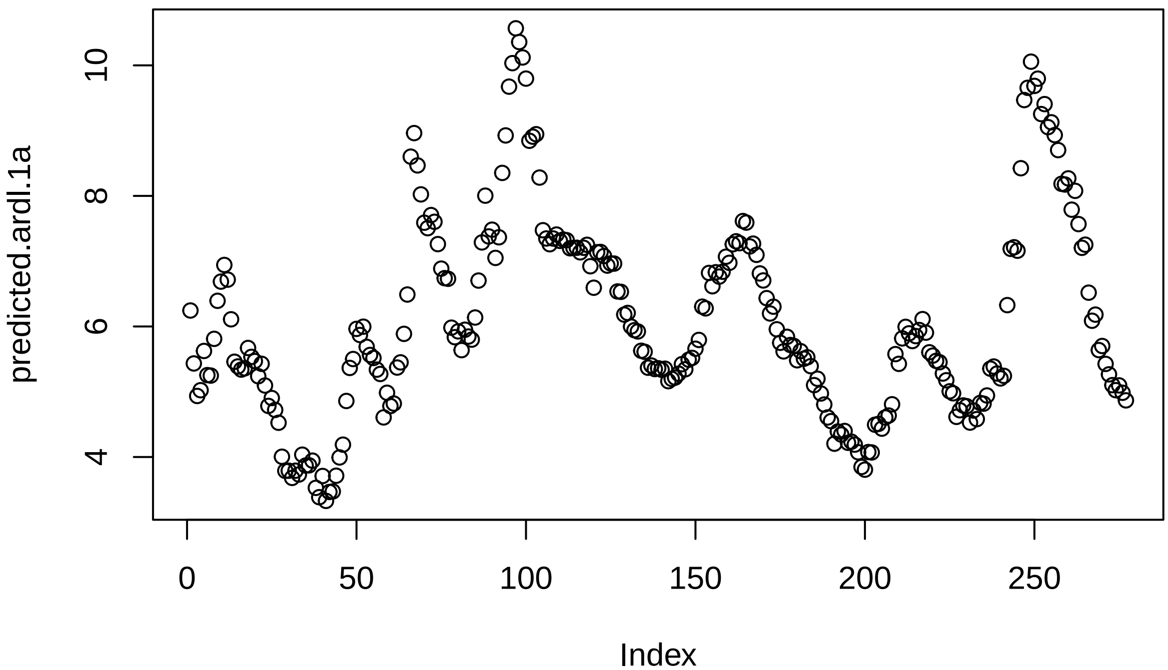
```
## [1] 629.7197
```

```
BIC(ardl.2b.compare)
```

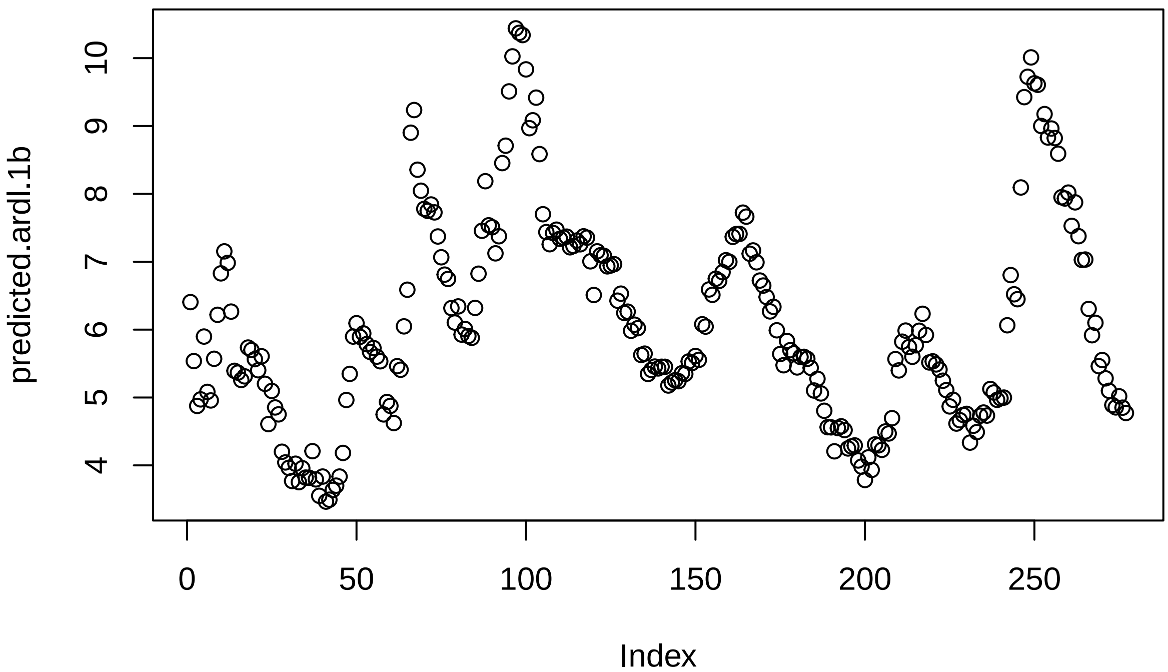
```
## [1] 612.4584
```

Continuing to support the idea that ARDL(3,4) for Unemployment.Rate ~ Real.GDP..Percent.Change. and ARDL(2,5) for Effective.Federal.Funds.Rate ~ Inflation.Rate are the best models, we can look to the AIC and BIC. The lowest BIC for Unemployment.Rate models is given by the Unemployment.Rate ~ Real.GDP..Percent.Change model, and the lowest BIC for Effective.Federal.Funds.Rate models is given by the Effective.Federal.Funds.Rate ~ Inflation.Rate model, supporting our conclusion further.

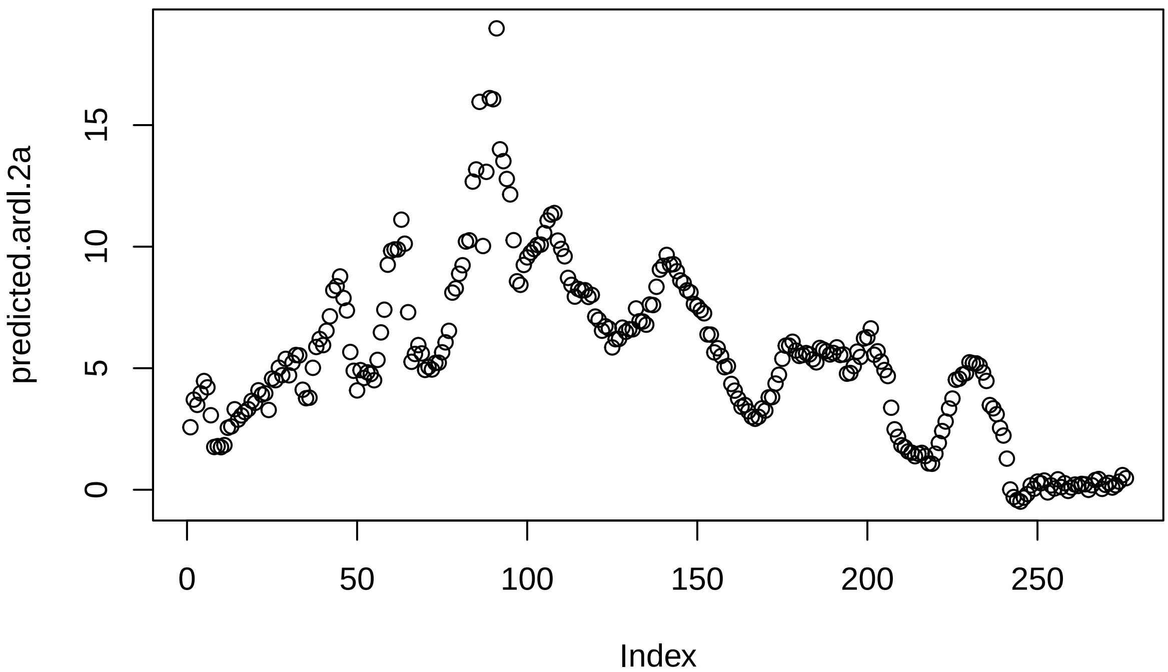
```
predicted.ardl.1a<- predict(ardl.1a, n.ahead=10)
plot(predicted.ardl.1a)
```



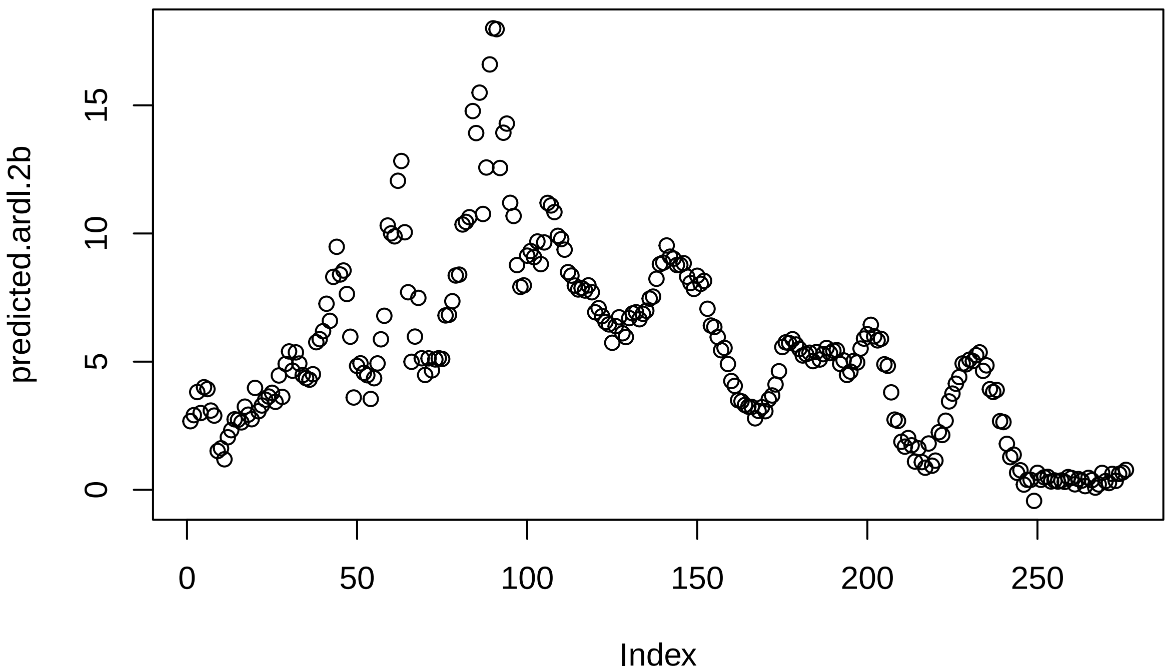

```
predicted.ardl.1b<- predict(ardl.1b, n.ahead=10)
plot(predicted.ardl.1b)
```




```
predicted.ardl.2a<- predict(ardl.2a, n.ahead=10)
plot(predicted.ardl.2a)
```




```
predicted.ardl.2b<- predict(ardl.2b, n.ahead=10)
plot(predicted.ardl.2b)
```




```
y=cbind(FRIR_ts[,"Effective.Federal.Funds.Rate"],FRIR_ts[, "Unemployment.Rate"])
y_tot=data.frame(y)

library(vars)
```

```
## Loading required package: MASS

## Loading required package: strucchange

## Loading required package: sandwich

## Loading required package: urca

## Loading required package: lmtest
```

```
VARselect(y_tot, lag.max=12)
```

```
## $selection
## AIC(n)  HQ(n)  SC(n) FPE(n)
##      6      6      2      6
##
## $criteria
##                  1            2            3            4            5            6
```

```
## AIC(n) -2.2873622 -2.53883530 -2.56657732 -2.58641132 -2.56429002 -2.69057836
## HQ(n)  -2.2551620 -2.48516825 -2.49144346 -2.48981064 -2.44622253 -2.55104404
## SC(n)  -2.2071827 -2.40520290 -2.37949197 -2.34587302 -2.27029876 -2.34313413
## FPE(n)  0.1015341  0.07895899  0.07679976  0.07529351  0.07698084  0.06785192
##                 7           8           9          10          11          12
## AIC(n) -2.67036114 -2.66684464 -2.6517659 -2.63258722 -2.63664407 -2.62546602
## HQ(n)  -2.50936000 -2.48437668 -2.4478311 -2.40718563 -2.38977566 -2.35713080
## SC(n)  -2.26946396 -2.21249450 -2.1439628 -2.07133117 -2.02193506 -1.95730405
## FPE(n)  0.06924326  0.06949453  0.0705598  0.07193801  0.07166112  0.07248395
```

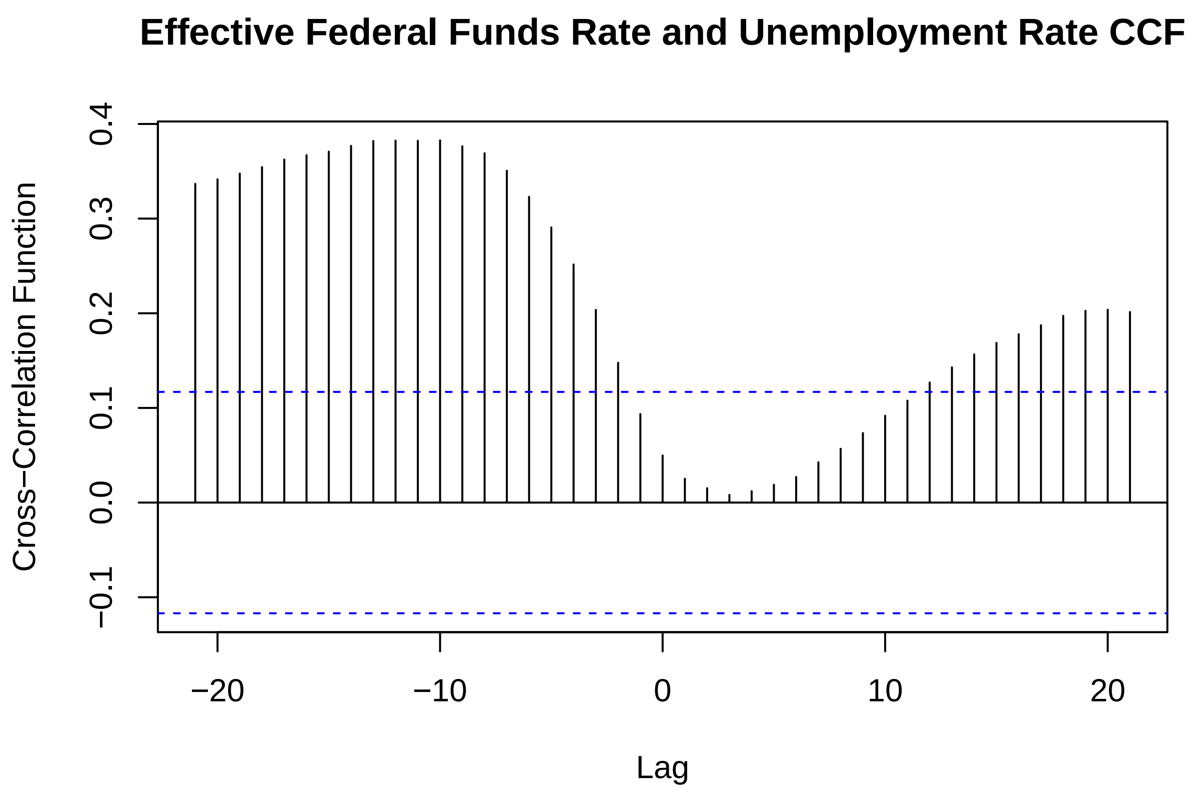
```r
VAR_model=VAR(y_tot, p=6)
summary(VAR_model)
```

```
##
## VAR Estimation Results:
## =========================
## Endogenous variables: FRIR_ts....Effective.Federal.Funds.Rate.., FRIR_ts....Unemployment.Rate..
## Deterministic variables: const
## Sample size: 275
## Log Likelihood: -388.19
## Roots of the characteristic polynomial:
## 0.9667 0.8754 0.857 0.857 0.8441 0.8441 0.7947 0.7947 0.6517 0.6517 0.5937 0.5937
## Call:
## VAR(y = y_tot, p = 6)
##
##
## Estimation results for equation FRIR_ts....Effective.Federal.Funds.Rate..:
## ==========================================================================
## FRIR_ts....Effective.Federal.Funds.Rate.. = FRIR_ts....Effective.Federal.Funds.Rate...l1 + FRIR_ts..
##
##                                              Estimate Std. Error t value
## FRIR_ts....Effective.Federal.Funds.Rate...l1  0.74022    0.06151  12.035
## FRIR_ts....Unemployment.Rate...l1            -1.06530    0.21503  -4.954
## FRIR_ts....Effective.Federal.Funds.Rate...l2  0.08048    0.07628   1.055
## FRIR_ts....Unemployment.Rate...l2             0.99925    0.33834   2.953
## FRIR_ts....Effective.Federal.Funds.Rate...l3  0.28895    0.07608   3.798
## FRIR_ts....Unemployment.Rate...l3            -0.26835    0.34366  -0.781
## FRIR_ts....Effective.Federal.Funds.Rate...l4 -0.09389    0.07549  -1.244
## FRIR_ts....Unemployment.Rate...l4             0.46384    0.34189   1.357
## FRIR_ts....Effective.Federal.Funds.Rate...l5  0.30093    0.07508   4.008
## FRIR_ts....Unemployment.Rate...l5             0.16284    0.33100   0.492
## FRIR_ts....Effective.Federal.Funds.Rate...l6 -0.34662    0.06270  -5.528
## FRIR_ts....Unemployment.Rate...l6            -0.31046    0.19866  -1.563
## const                                         0.25907    0.26446   0.980
##                                              Pr(>|t|)
## FRIR_ts....Effective.Federal.Funds.Rate...l1  < 2e-16 ***
## FRIR_ts....Unemployment.Rate...l1            1.30e-06 ***
## FRIR_ts....Effective.Federal.Funds.Rate...l2 0.292394
## FRIR_ts....Unemployment.Rate...l2            0.003429 **
## FRIR_ts....Effective.Federal.Funds.Rate...l3 0.000181 ***
## FRIR_ts....Unemployment.Rate...l3            0.435604
## FRIR_ts....Effective.Federal.Funds.Rate...l4 0.214681
## FRIR_ts....Unemployment.Rate...l4            0.176049
## FRIR_ts....Effective.Federal.Funds.Rate...l5 7.99e-05 ***
```

```
## FRIR_ts....Unemployment.Rate...l5           0.623154
## FRIR_ts....Effective.Federal.Funds.Rate...l6 7.82e-08 ***
## FRIR_ts....Unemployment.Rate...l6           0.119315
## const                                       0.328185
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
##
## Residual standard error: 0.95 on 262 degrees of freedom
## Multiple R-Squared: 0.9333,  Adjusted R-squared: 0.9302
## F-statistic: 305.5 on 12 and 262 DF,  p-value: < 2.2e-16
##
##
## Estimation results for equation FRIR_ts....Unemployment.Rate..:
## ============================================================
## FRIR_ts....Unemployment.Rate.. = FRIR_ts....Effective.Federal.Funds.Rate...l1 + FRIR_ts....Unemployme
##
##                                               Estimate Std. Error t value
## FRIR_ts....Effective.Federal.Funds.Rate...l1  0.007638   0.018336   0.417
## FRIR_ts....Unemployment.Rate...l1             1.269341   0.064104  19.801
## FRIR_ts....Effective.Federal.Funds.Rate...l2  0.026221   0.022740   1.153
## FRIR_ts....Unemployment.Rate...l2            -0.168316   0.100863  -1.669
## FRIR_ts....Effective.Federal.Funds.Rate...l3  0.022120   0.022680   0.975
## FRIR_ts....Unemployment.Rate...l3            -0.037686   0.102450  -0.368
## FRIR_ts....Effective.Federal.Funds.Rate...l4 -0.007347   0.022504  -0.326
## FRIR_ts....Unemployment.Rate...l4            -0.103995   0.101922  -1.020
## FRIR_ts....Effective.Federal.Funds.Rate...l5 -0.019049   0.022384  -0.851
## FRIR_ts....Unemployment.Rate...l5             0.140089   0.098676   1.420
## FRIR_ts....Effective.Federal.Funds.Rate...l6 -0.017066   0.018692  -0.913
## FRIR_ts....Unemployment.Rate...l6            -0.136084   0.059223  -2.298
## const                                         0.158738   0.078840   2.013
##                                              Pr(>|t|)
## FRIR_ts....Effective.Federal.Funds.Rate...l1   0.6773
## FRIR_ts....Unemployment.Rate...l1              <2e-16 ***
## FRIR_ts....Effective.Federal.Funds.Rate...l2   0.2499
## FRIR_ts....Unemployment.Rate...l2              0.0964 .
## FRIR_ts....Effective.Federal.Funds.Rate...l3   0.3303
## FRIR_ts....Unemployment.Rate...l3              0.7133
## FRIR_ts....Effective.Federal.Funds.Rate...l4   0.7443
## FRIR_ts....Unemployment.Rate...l4              0.3085
## FRIR_ts....Effective.Federal.Funds.Rate...l5   0.3955
## FRIR_ts....Unemployment.Rate...l5              0.1569
## FRIR_ts....Effective.Federal.Funds.Rate...l6   0.3621
## FRIR_ts....Unemployment.Rate...l6              0.0224 *
## const                                          0.0451 *
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
##
## Residual standard error: 0.2832 on 262 degrees of freedom
## Multiple R-Squared: 0.9678,  Adjusted R-squared: 0.9664
## F-statistic: 656.8 on 12 and 262 DF,  p-value: < 2.2e-16
##
##
```

```
##
## Covariance matrix of residuals:
##                                          FRIR_ts....Effective.Federal.Funds.Rate..
## FRIR_ts....Effective.Federal.Funds.Rate..                                   0.90253
## FRIR_ts....Unemployment.Rate..                                             -0.09395
##                                          FRIR_ts....Unemployment.Rate..
## FRIR_ts....Effective.Federal.Funds.Rate..                      -0.09395
## FRIR_ts....Unemployment.Rate..                                  0.08021
##
## Correlation matrix of residuals:
##                                          FRIR_ts....Effective.Federal.Funds.Rate..
## FRIR_ts....Effective.Federal.Funds.Rate..                                    1.0000
## FRIR_ts....Unemployment.Rate..                                              -0.3492
##                                          FRIR_ts....Unemployment.Rate..
## FRIR_ts....Effective.Federal.Funds.Rate..                       -0.3492
## FRIR_ts....Unemployment.Rate..                                   1.0000
```

```
ccf(FRIR_ts[,"Effective.Federal.Funds.Rate"],FRIR_ts[, "Unemployment.Rate"], ylab="Cross-Correlation Fu
```



The Cross-Correlation function suggests that unemployment rates are maximally correlated with the effective federal funds rates lagged by 5-20 quarters. This is a rather large interval but intuitively makes sense given that effective federal funds rates should not immediately affect unemployment, but should create observable effects between a year and 5 years.

```
grangertest((FRIR_ts[,"Unemployment.Rate"] ~ FRIR_ts[,"Effective.Federal.Funds.Rate"]), order = 6)
```

```
## Granger causality test
```

```
##
## Model 1: FRIR_ts[, "Unemployment.Rate"] ~ Lags(FRIR_ts[, "Unemployment.Rate"], 1:6) + Lags(FRIR_ts[,
## Model 2: FRIR_ts[, "Unemployment.Rate"] ~ Lags(FRIR_ts[, "Unemployment.Rate"], 1:6)
##   Res.Df Df      F   Pr(>F)
## 1    262
## 2    268 -6 3.0948 0.006045 **
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

```
grangertest((FRIR_ts[,"Effective.Federal.Funds.Rate"] ~ FRIR_ts[,"Unemployment.Rate"]), order = 6)
```

```
## Granger causality test
##
## Model 1: FRIR_ts[, "Effective.Federal.Funds.Rate"] ~ Lags(FRIR_ts[, "Effective.Federal.Funds.Rate"],
## Model 2: FRIR_ts[, "Effective.Federal.Funds.Rate"] ~ Lags(FRIR_ts[, "Effective.Federal.Funds.Rate"],
##   Res.Df Df      F    Pr(>F)
## 1    262
## 2    268 -6 6.0349 6.232e-06 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

The p-value for the Granger-Causality test of unemployment rate on the effective federal funds rate is significant at the .001 level, which means that the effective federal funds rate Granger-causes unemployment rate. ## The p-value for the Granger causality test of effective federal funds rate on unemployment rate is significant at the 0.00 level, meaning we can reject H0 and conclude that the unemployment rate Granger-causes the effective federal funds rate as well.

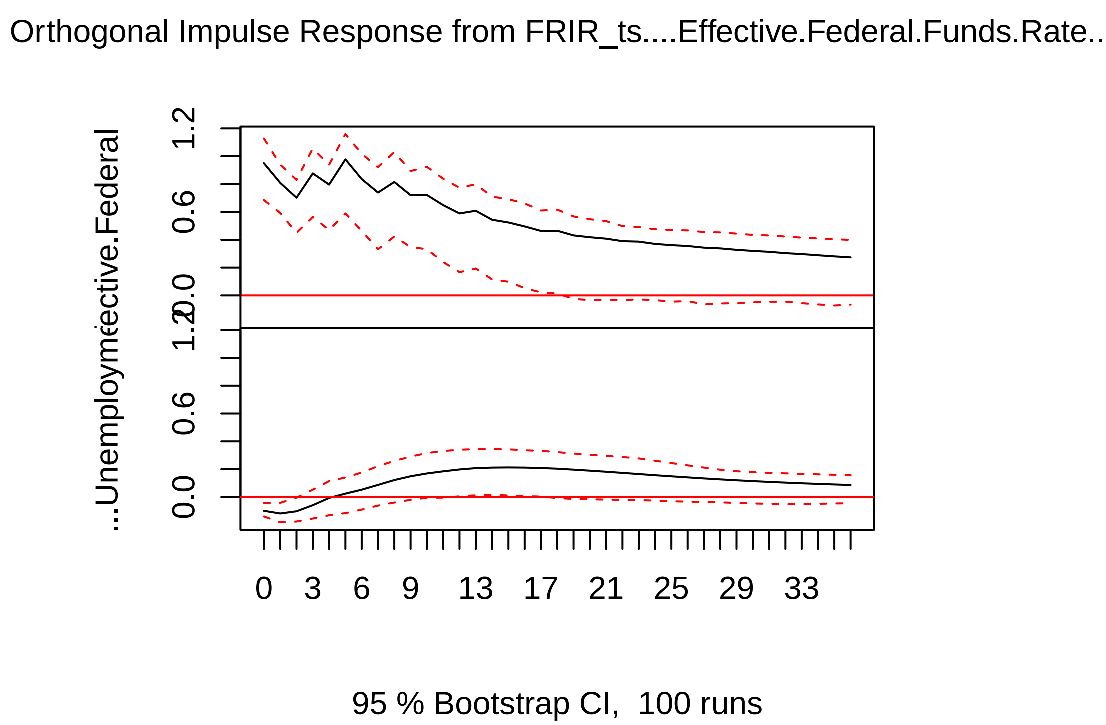
```
irf(VAR_model)
```

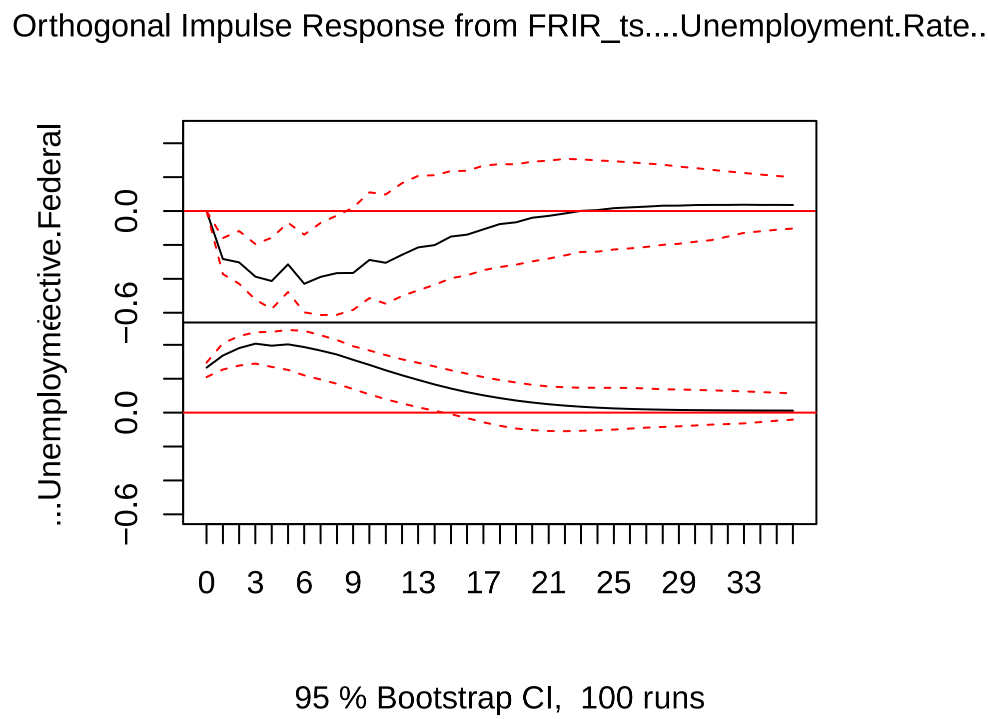
```
##
## Impulse response coefficients
## $FRIR_ts....Effective.Federal.Funds.Rate..
##       FRIR_ts....Effective.Federal.Funds.Rate.. FRIR_ts....Unemployment.Rate..
##  [1,]                                 0.9500168                   -0.098890670
##  [2,]                                 0.8085695                   -0.118269260
##  [3,]                                 0.7021498                   -0.102392644
##  [4,]                                 0.8767568                   -0.058758559
##  [5,]                                 0.7960811                   -0.006595483
##  [6,]                                 0.9775163                    0.024386554
##  [7,]                                 0.8362240                    0.052777381
##  [8,]                                 0.7390940                    0.087009736
##  [9,]                                 0.8152179                    0.121692791
## [10,]                                 0.7200458                    0.149166412
## [11,]                                 0.7217112                    0.169280370
##
## $FRIR_ts....Unemployment.Rate..
##       FRIR_ts....Effective.Federal.Funds.Rate.. FRIR_ts....Unemployment.Rate..
##  [1,]                                 0.0000000                      0.2653850
##  [2,]                                -0.2827148                      0.3368639
##  [3,]                                -0.3029474                      0.3807671
##  [4,]                                -0.3872357                      0.4068954
##  [5,]                                -0.4129942                      0.3949509
```

```
##  [6,]                          -0.3147229                    0.4027037
##  [7,]                          -0.4291799                    0.3866484
##  [8,]                          -0.3884827                    0.3660819
##  [9,]                          -0.3662371                    0.3429378
## [10,]                          -0.3653773                    0.3115040
## [11,]                          -0.2885406                    0.2815541
##
##
## Lower Band, CI= 0.95
## $FRIR_ts....Effective.Federal.Funds.Rate..
##       FRIR_ts....Effective.Federal.Funds.Rate.. FRIR_ts....Unemployment.Rate..
##  [1,]                           0.7438186                 -0.152900333
##  [2,]                           0.5811508                 -0.175325699
##  [3,]                           0.4549374                 -0.179498166
##  [4,]                           0.5968698                 -0.137655961
##  [5,]                           0.5068778                 -0.114980624
##  [6,]                           0.6542861                 -0.106051740
##  [7,]                           0.5165130                 -0.083970847
##  [8,]                           0.3701983                 -0.064321736
##  [9,]                           0.4248392                 -0.034133434
## [10,]                           0.3224746                 -0.007120399
## [11,]                           0.3136222                  0.018428020
##
## $FRIR_ts....Unemployment.Rate..
##       FRIR_ts....Effective.Federal.Funds.Rate.. FRIR_ts....Unemployment.Rate..
##  [1,]                           0.0000000                   0.21901986
##  [2,]                          -0.3817788                   0.27492387
##  [3,]                          -0.3940527                   0.29416767
##  [4,]                          -0.5217539                   0.30034336
##  [5,]                          -0.5349486                   0.27472712
##  [6,]                          -0.4398461                   0.27240206
##  [7,]                          -0.6071607                   0.24635905
##  [8,]                          -0.5690459                   0.21210413
##  [9,]                          -0.5591765                   0.17603221
## [10,]                          -0.5853562                   0.13402292
## [11,]                          -0.5169282                   0.09875794
##
##
## Upper Band, CI= 0.95
## $FRIR_ts....Effective.Federal.Funds.Rate..
##       FRIR_ts....Effective.Federal.Funds.Rate.. FRIR_ts....Unemployment.Rate..
##  [1,]                           1.1941179                  -0.05031494
##  [2,]                           0.9984117                  -0.05595714
##  [3,]                           0.9015743                  -0.02909236
##  [4,]                           1.1493141                   0.03175627
##  [5,]                           1.0306034                   0.09683502
##  [6,]                           1.2378217                   0.13574661
##  [7,]                           1.0982742                   0.15713742
##  [8,]                           0.9993570                   0.19552372
##  [9,]                           1.0527889                   0.24716542
## [10,]                           0.9924827                   0.27436189
## [11,]                           0.9857516                   0.29040822
##
## $FRIR_ts....Unemployment.Rate..
```

```
##        FRIR_ts....Effective.Federal.Funds.Rate.. FRIR_ts....Unemployment.Rate..
##   [1,]                                0.00000000                      0.2990668
##   [2,]                               -0.14223757                      0.3726506
##   [3,]                               -0.10865509                      0.4300241
##   [4,]                               -0.20691861                      0.4565801
##   [5,]                               -0.19869018                      0.4480704
##   [6,]                               -0.07105609                      0.4718469
##   [7,]                               -0.14345494                      0.4590124
##   [8,]                               -0.07829776                      0.4463767
##   [9,]                               -0.04721466                      0.4386158
## [10,]                               -0.03512629                      0.4035042
## [11,]                                0.06692395                      0.3726743
```

```
plot(irf(VAR_model, n.ahead=36))
```

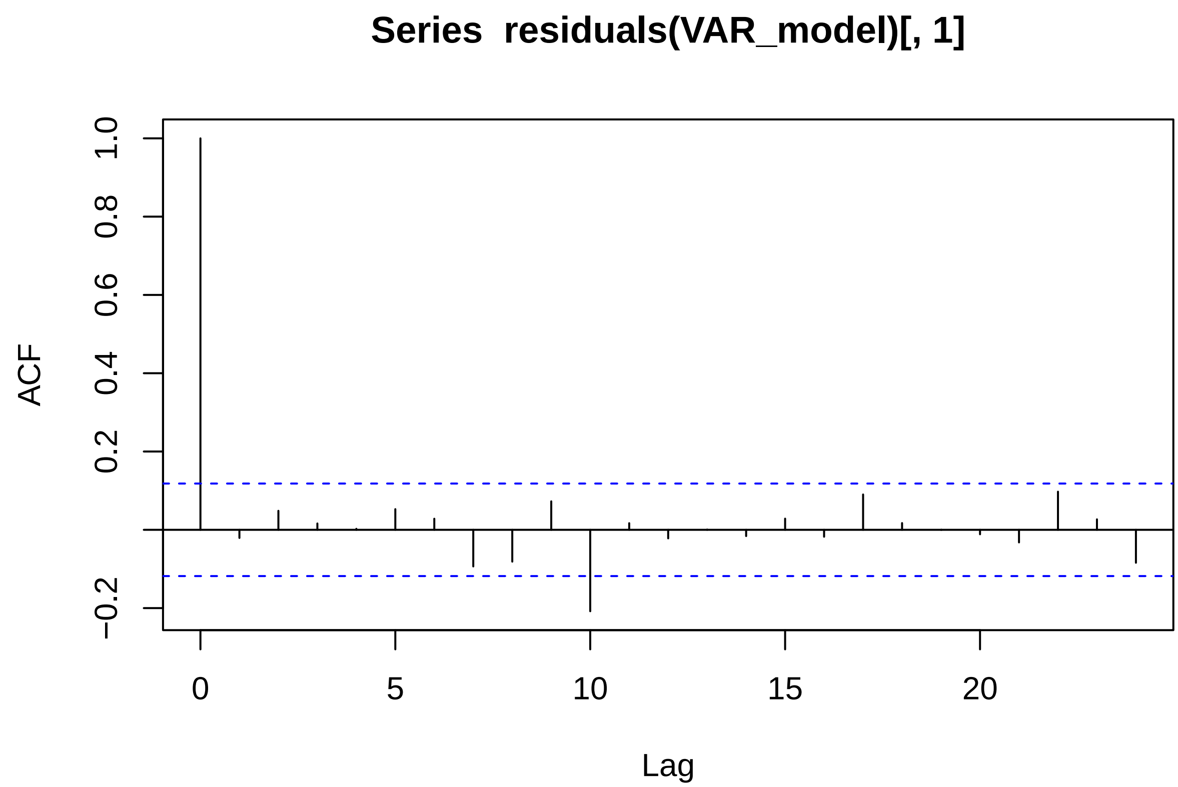




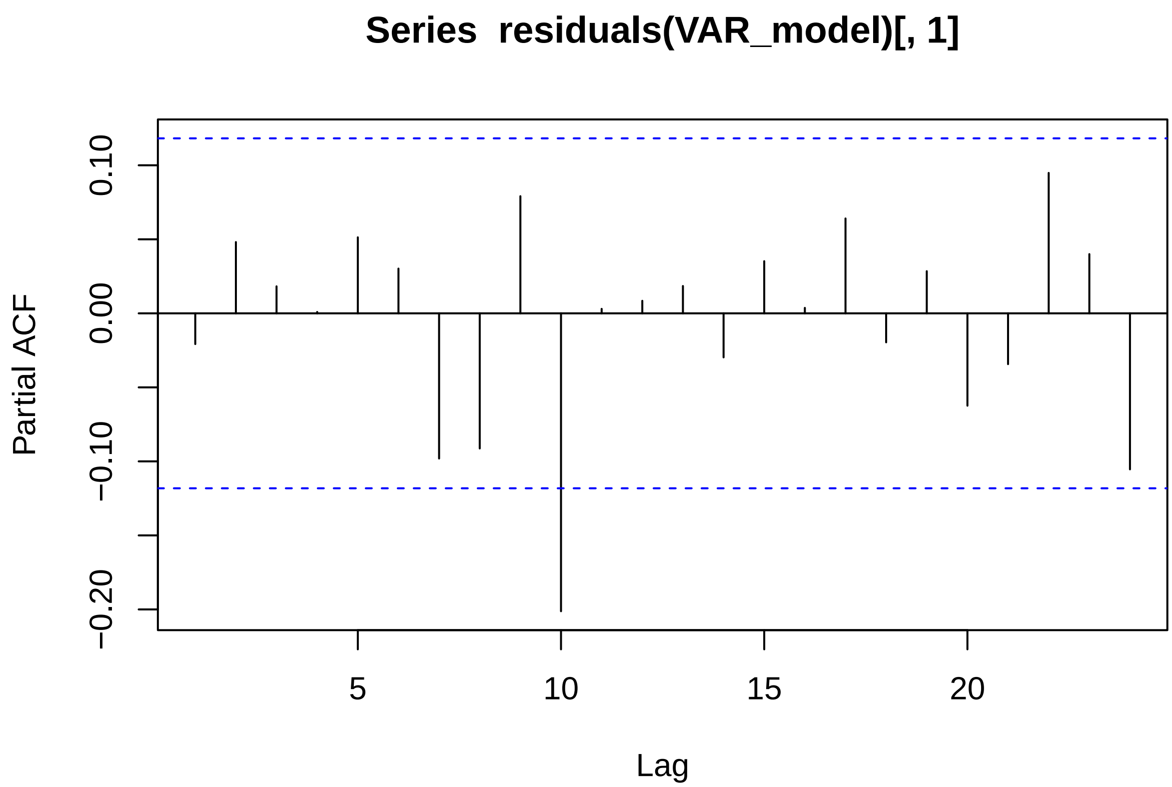
The own-variable impulse response function for the effective federal funds rate is significant until the 16th lag, indicating that shocks to the effective federal funds rate have a large positive effect on the effective federal funds rate of subsequent quarters but then decays at about 4 years. The cross-variable impulse response function indicates that the shocks to the effective federal funds rate has a much less significant effect on the unemployment rates of subsequent quarters. The effects are significant between 10 and 21 quarters.

The own-variable impulse response function for the unemployment rate is significant until the 12th lag, indicating that shocks to the unemployment rate have large negative effects on subsequent unemployment rates up until a year after the shock. The cross-variable impulse response function indicates that shocks to the unemployment rate positively affects the effective federal funds rate, before causing it to decay again betweeen 1 month and 1 year.

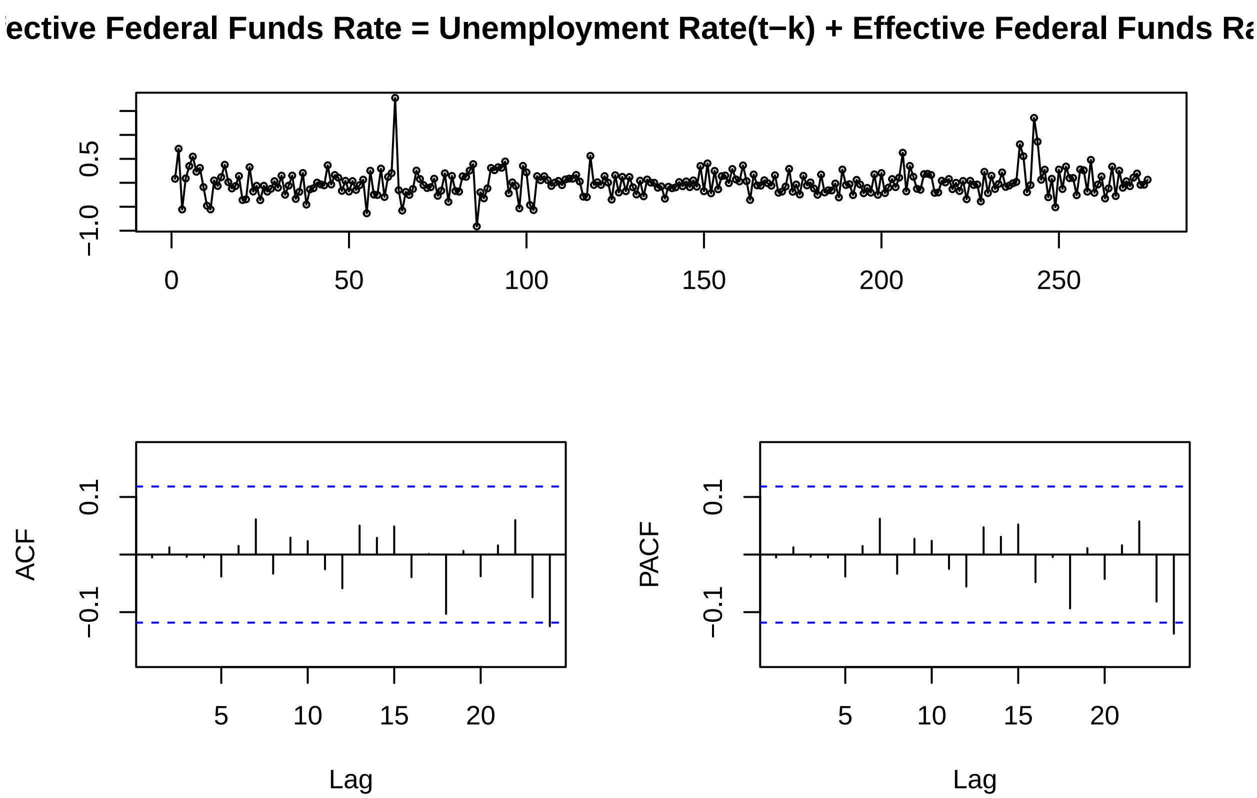
```
acf(residuals(VAR_model)[,1])
```




```
pacf(residuals(VAR_model)[,1])
```




```
tsdisplay(residuals(VAR_model)[,2],main ="Effective Federal Funds Rate = Unemployment Rate(t-k) + Effec
```



The fitted values show stationarity and low persistence, indicating that the VAR model for effective federal funds rate is robust. The ACF and the PACF of the residuals show no patterns and few lags with serial auto-correlation, indicating that the residuals exhibit stationarity and that the VAR model effectively captures the dynamics of the data.

```
df = data.frame(FRIR_ts[,"Effective.Federal.Funds.Rate"],FRIR_ts[, "Unemployment.Rate"])
set.seed(1)
row.number <- sample(1:nrow(df), 0.66*nrow(df))
train = df[row.number,]
test = df[-row.number,]
test
```

```
##     FRIR_ts....Effective.Federal.Funds.Rate.. FRIR_ts....Unemployment.Rate..
## 3                                        0.68                            7.5
## 4                                        1.80                            6.7
## 6                                        2.96                            5.2
## 7                                        3.47                            5.1
## 8                                        3.98                            5.7
## 10                                       3.92                            5.2
## 11                                       3.23                            5.5
## 12                                       2.47                            6.1
## 18                                       2.78                            5.6
## 21                                       2.92                            5.7
## 27                                       3.42                            4.9
## 30                                       4.09                            4.8
## 32                                       4.08                            4.2
## 35                                       5.30                            3.8
```

```
## 38                                       4.05                         3.8
## 46                                       7.41                         3.4
## 47                                       8.61                         3.5
## 52                                       6.20                         5.5
## 54                                       4.15                         5.9
## 55                                       5.31                         6.0
## 56                                       5.20                         5.8
## 58                                       4.17                         5.7
## 59                                       4.55                         5.6
## 62                                       7.12                         5.0
## 63                                      10.40                         4.8
## 66                                      10.51                         5.1
## 68                                      10.06                         6.0
## 69                                       7.13                         8.1
## 74                                       4.82                         7.7
## 76                                       5.02                         7.7
## 80                                       6.47                         6.8
## 81                                       6.70                         6.4
## 82                                       6.89                         6.1
## 93                                      19.08                         7.5
## 94                                      15.72                         7.2
## 95                                      19.04                         7.2
## 96                                      15.08                         7.9
## 97                                      13.22                         8.6
## 101                                      9.71                        10.4
## 106                                      9.37                         9.4
## 109                                     10.29                         7.7
## 112                                     11.23                         7.5
## 117                                      8.35                         7.3
## 119                                      8.27                         7.3
## 123                                      8.14                         6.7
## 125                                      6.99                         7.1
## 128                                      6.56                         7.0
## 131                                      6.43                         6.6
## 132                                      6.37                         6.3
## 133                                      6.37                         6.3
## 137                                      6.83                         5.7
## 142                                      8.30                         5.4
## 147                                      9.24                         5.2
## 151                                      8.23                         5.4
## 154                                      8.15                         5.5
## 157                                      6.91                         6.4
## 158                                      6.91                         6.4
## 159                                      5.91                         6.7
## 162                                      5.21                         7.0
## 168                                      3.25                         7.7
## 169                                      3.10                         7.3
## 171                                      2.96                         7.1
## 173                                      2.99                         6.8
## 175                                      3.56                         6.4
## 178                                      4.76                         5.8
## 183                                      5.76                         5.5
## 184                                      5.56                         5.6
## 188                                      5.24                         5.2
```

```
## 194                                          5.45                              4.3
## 195                                          5.54                              4.5
## 196                                          5.07                              4.5
## 199                                          4.74                              4.3
## 200                                          4.99                              4.3
## 205                                          6.51                              3.9
## 210                                          4.80                              4.4
## 212                                          2.49                              5.3
## 216                                          1.73                              5.8
## 220                                          1.01                              6.2
## 222                                          1.00                              5.7
## 227                                          2.79                              5.2
## 229                                          3.78                              5.0
## 232                                          4.79                              4.7
## 233                                          5.24                              4.7
## 237                                          5.26                              4.7
## 240                                          3.94                              5.0
## 243                                          2.28                              5.0
## 244                                          2.28                              5.0
## 245                                          2.01                              5.8
## 247                                          0.97                              6.5
## 254                                          0.20                              9.9
## 255                                          0.18                              9.4
## 258                                          0.10                              9.1
## 259                                          0.07                              9.0
## 260                                          0.07                              8.8
## 261                                          0.08                              8.3
## 275                                          0.13                              5.2
```

```
dim(train)
```

```
## [1] 185   2
```

```
dim(test)
```

```
## [1] 96  2
```

```
var.train = VAR(train, p=6)
var.test = VAR(test, p =6)
predictor.train =predict(var.train,train)
predictor.test =predict(var.test,test)

MSE.train.fedfunds = mean(predictor.train[["model"]][["varresult"]][["FRIR_ts....Effective.Federal.Funds
MSE.test.fedfunds = mean(predictor.test[["model"]][["varresult"]][["FRIR_ts....Effective.Federal.Funds.
MSE.train.fedfunds
```

```
## [1] 11.79055
```

```
MSE.test.fedfunds
```

```
## [1] 2.606139
```

```r
MSE.train.rate = mean(predictor.train[["model"]][["varresult"]][["FRIR_ts....Unemployment.Rate.."]][["re
MSE.test.rate = mean(predictor.test[["model"]][["varresult"]][["FRIR_ts....Unemployment.Rate.."]][["resi
MSE.train.rate
```

```
## [1] 2.341177
```

```r
MSE.test.rate
```

```
## [1] 0.4249512
```

```r
AIC.var<-AIC(VAR_model$varresult$FRIR_ts....Effective.Federal.Funds.Rate..,VAR_model$varresult$FRIR_ts.
AIC.var
```

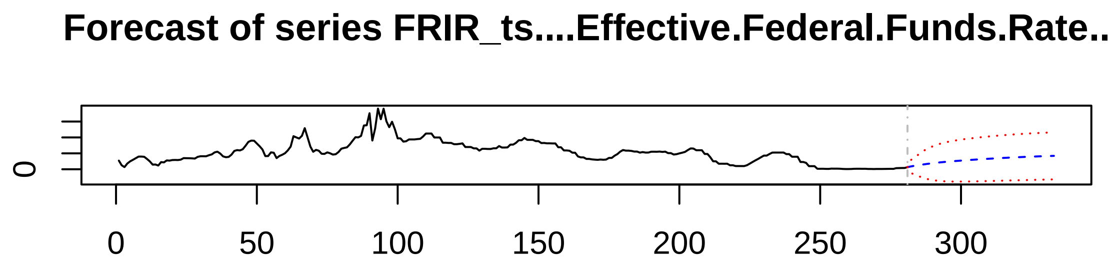
```
##                                                                  df      AIC
## VAR_model$varresult$FRIR_ts....Effective.Federal.Funds.Rate.. 14 766.8973
## VAR_model$varresult$FRIR_ts....Unemployment.Rate..            14 101.2394
```

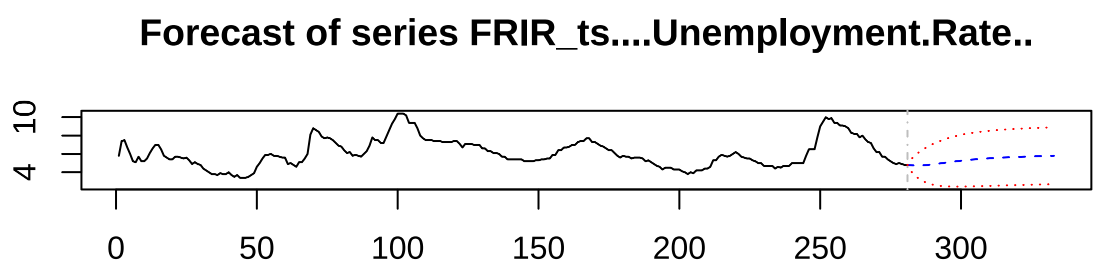
```r
BIC.var<-BIC(VAR_model$varresult$FRIR_ts....Effective.Federal.Funds.Rate..,VAR_model$varresult$FRIR_ts.
BIC.var
```

```
##                                                                  df      BIC
## VAR_model$varresult$FRIR_ts....Effective.Federal.Funds.Rate.. 14 817.5321
## VAR_model$varresult$FRIR_ts....Unemployment.Rate..            14 151.8742
```

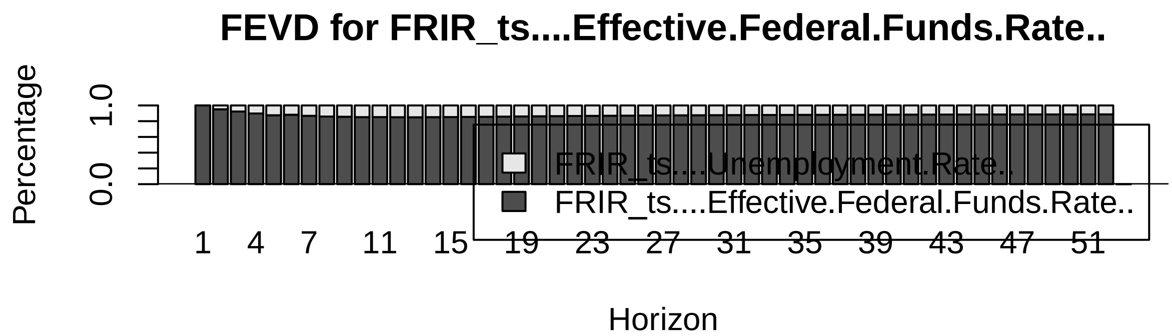
Based on MSE, AIC, and BIC results, we can conclude that Unemployment.Rate ~ Effective.Federal.Funds.Rate is a better model as Unemployment.Rate has lower MSE for both training and testing, and lower AIC and BIC.

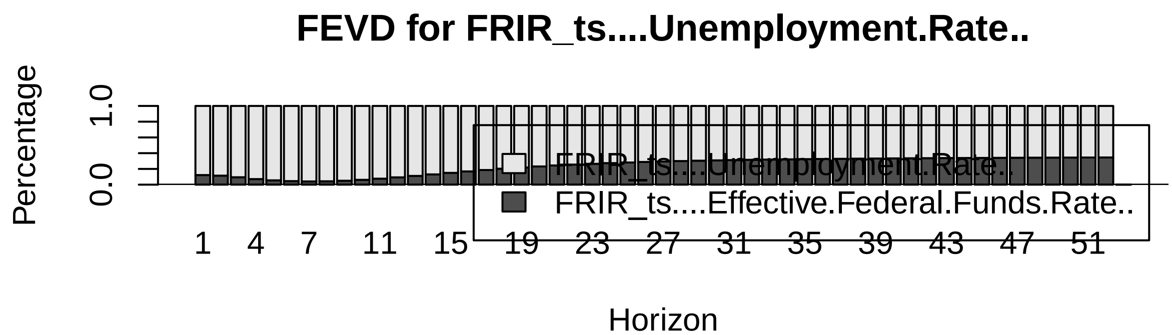
```r
var.predict=predict(object=VAR_model,n.ahead=52)
plot(var.predict)
```






```
#Forecast Error Variance Decomposition
plot(fevd(VAR_model,n.ahead=52))
```





The FEVD plot displays the contributions for each individual economic shock as the portion of the total area of each individual rectangle displayed on the graphs below. Approximately 80% of the variation in the effective federal funds rate is from shocks to the effective federal funds rate itself, while the remaining 20% is from the unemployment rate.

Summary - We constructed three types of models (AR, ARDL, and VAR) to estimate the relatiohnsip between Effective Federal Funds Rate and Unemployment Rate. According to our AR models, we found that and AR(2) model is best for Unemployment Rate, while an AR(6) model is best for Effective Federal Funds Rate. When including explanatory variables Percent CHange in Real GDP and Inflation Rate, we estimated ARDL models and determined that models of ARDL(3,4) for Unemployment Rate ~ Percent Change in Real GDP and ARDL (2,5) for Effective Federal Funds Rate ~ Inflation rate where the best fits. Further, when looking at VAR models, we concluded that Unemployment Rate is useful in predicting the Effective Federal Funds Rate.