Grammatical Ambiguity and Precedence

Concepts of Programming Languages
Lecture 12

CAS CS 320

Outline

Discuss ambiguity in grammar

Look at ways of avoiding ambiguity

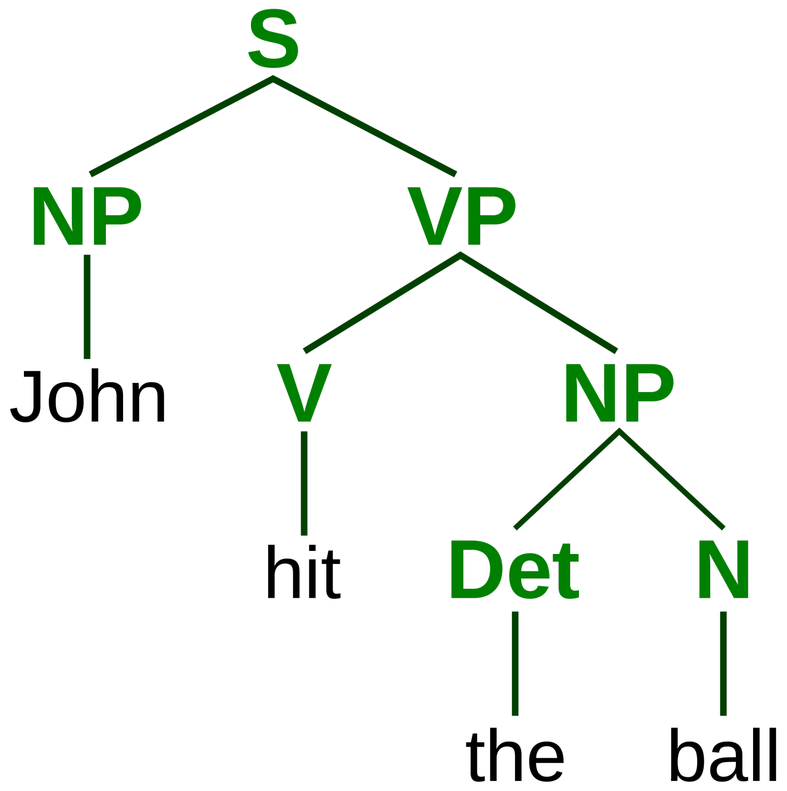
Analyze the relationship between operator fixity, precedence, and ambiguity

Recap

Recall: What is Grammar?

John hit the ball

⇓

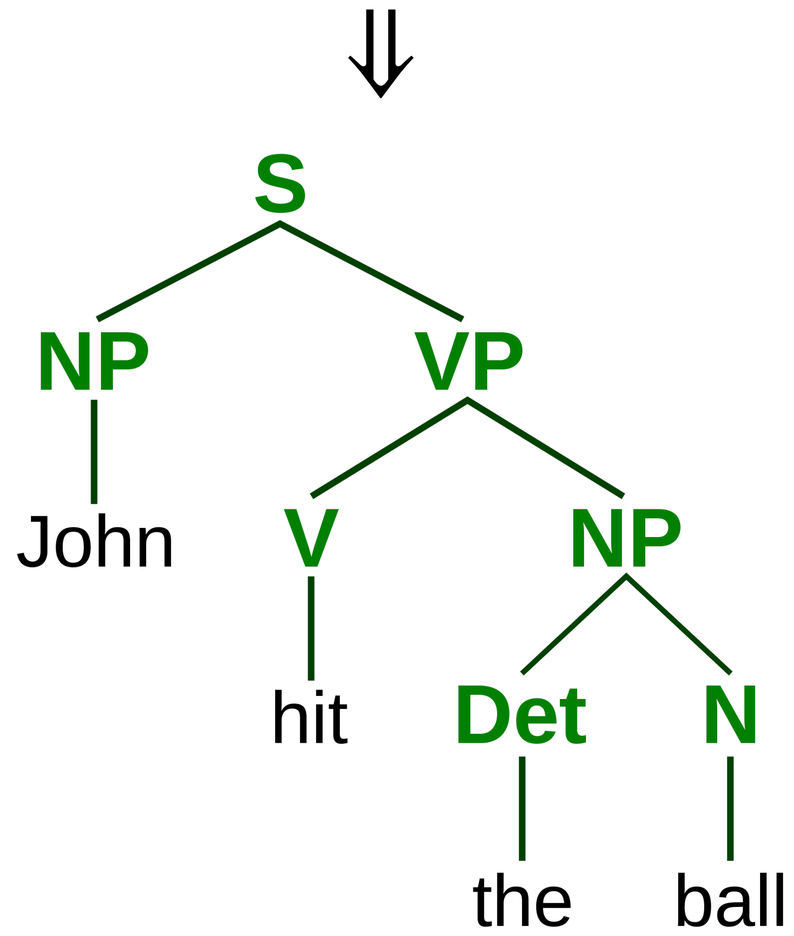


https://commons.wikimedia.org/wiki/File:ParseTree.svg

Recall: What is Grammar?

Grammar refers to the rules which govern what statements are well-formed

John hit the ball



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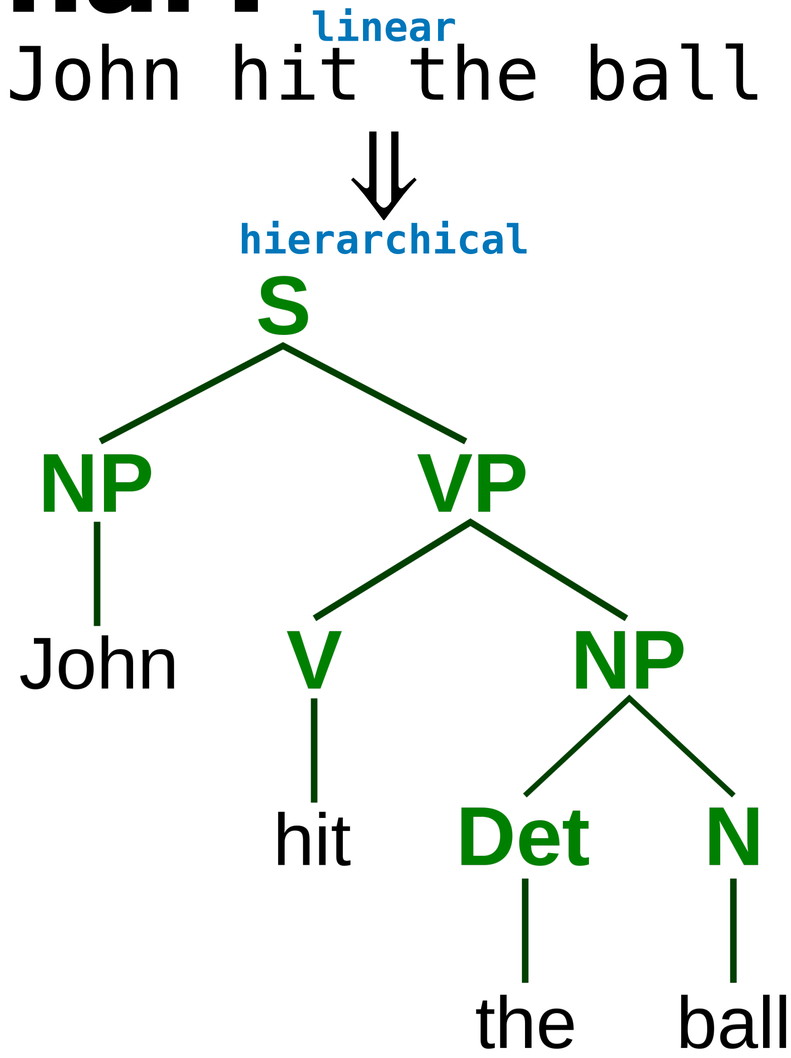
Grammar gives **linear** statements (in natural language or code) their **hierarchical** structure



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Recall: Grammar vs. Semantics

I taught my car in the refrigerator. 

vs.

My the car taught I refrigerator. 

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Grammar is not (typically) interested in meaning, just structure

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(As we will see, it is useful to separate these two concerns)

Grammars for Programming Languages

Grammars for Programming Languages

Formal grammars for PL
tell us which programs
are well-formed

Grammars for Programming Languages

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# let f x = x + 1;;
val f : int -> int = <fun>
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Grammars for Programming Languages

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Exception: Failure "hd".
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(In OCaml, well-formed programs are the ones we can type-check)

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Exception: Failure "hd".
# let x = ;;
Line 1, characters 8-10:
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            ^^
Error: Syntax error
```

Recall: Production Rules

```
<non-term> ::= sent-form1 | sent-form2 | ...
```

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A **(BNF) production rule** describes what we can replace a non-terminal symbol with in a derivation

Recall: Production Rules

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```

A **(BNF) production rule** describes what we can replace a non-terminal symbol with in a derivation

The "|" means: we can replace it with one or the other sentential forms on either side of the "|"

Recall: BNF Grammar

```
<sentence>    ::= <noun-phrase> <verb-phrase>
<verb-phrase> ::= <verb> <prep-phrase>
                | <verb>
<prep-phrase> ::= <prep> <noun-phrase>
<noun-phrase> ::= <article> <noun>
<article>     ::= the
<noun>        ::= cow
                | moon
<verb>        ::= jumped
<prep>        ::= over
```

Recall: BNF Grammar

A **BNF grammar** is defined by a collection of production rules and a **starting (nonterminal) symbol**

```
<sentence>    ::= <noun-phrase> <verb-phrase>
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Note. We don't specify the symbols of a grammar, they are implicit in the rules

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Recall: BNF Grammar

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Note. We don't specify the start symbol, it's the left nonterminal symbol in the **first rule**

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<verb-phrase> ::= <verb> <prep-phrase>
                | <verb>
<prep-phrase> ::= <prep> <noun-phrase>
<noun-phrase> ::= <article> <noun>
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```

Example

```
<expr>  ::= <op1> <expr>
          | <op2> <expr> <expr>
          | <var>
<op1>   ::= not
<op2>   ::= and | or
<var>   ::= x | y | z
```

Example

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tokens (terminal symbols)

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abstractions (non-terminal symbols)

tokens (terminal symbols)

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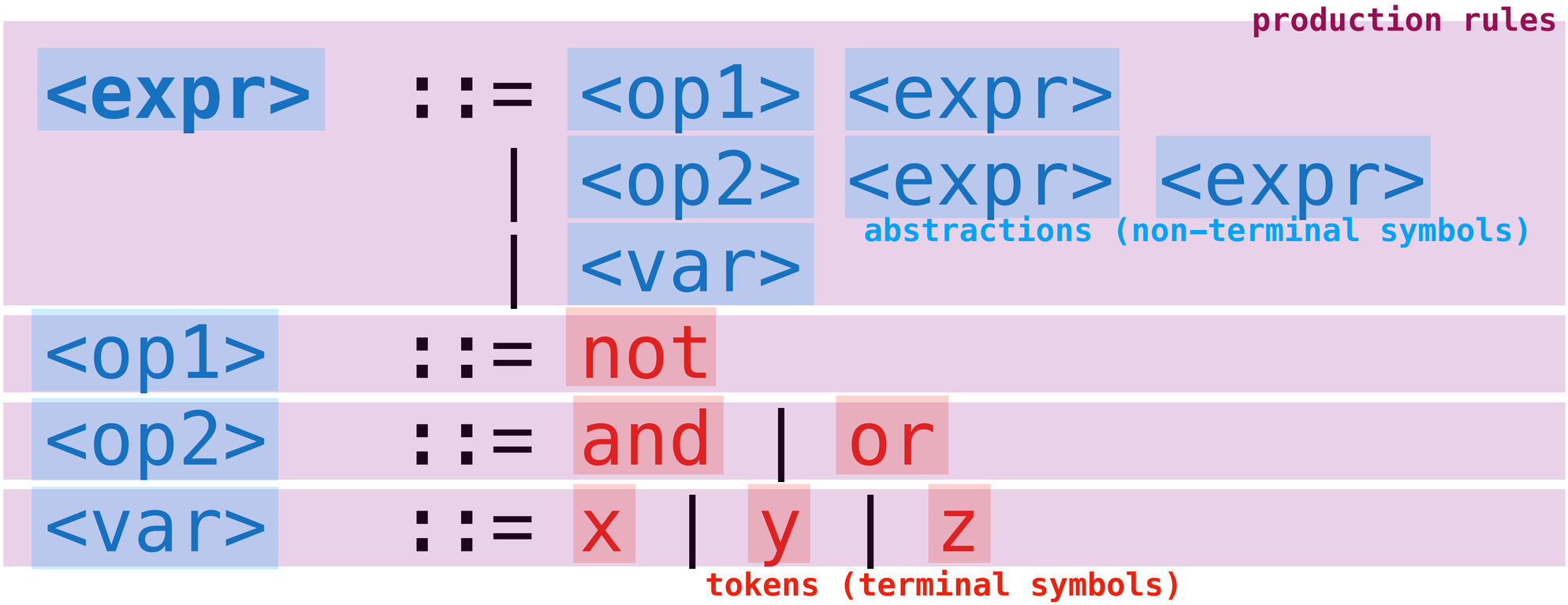
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Recall: Derivations and Parse Trees

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```
<expr>
<op2> <expr> <expr>
and <expr> <expr>
and <op1> <expr> <expr>
and not <expr> <expr>
and not <var> <expr>
and not x <expr>
and not x <var>
and not x y
```

Derivations and Parse Trees

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Derivations and Parse Trees

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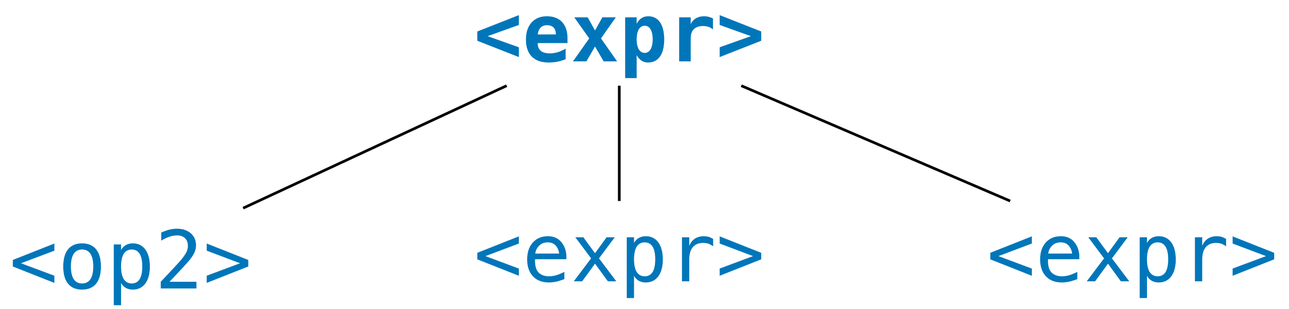

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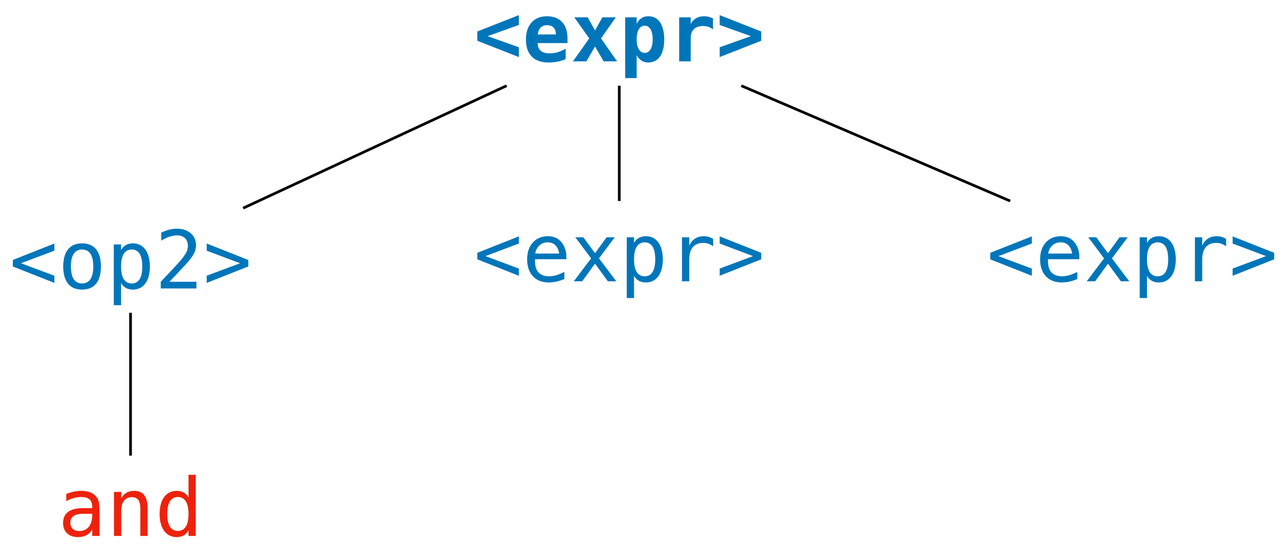

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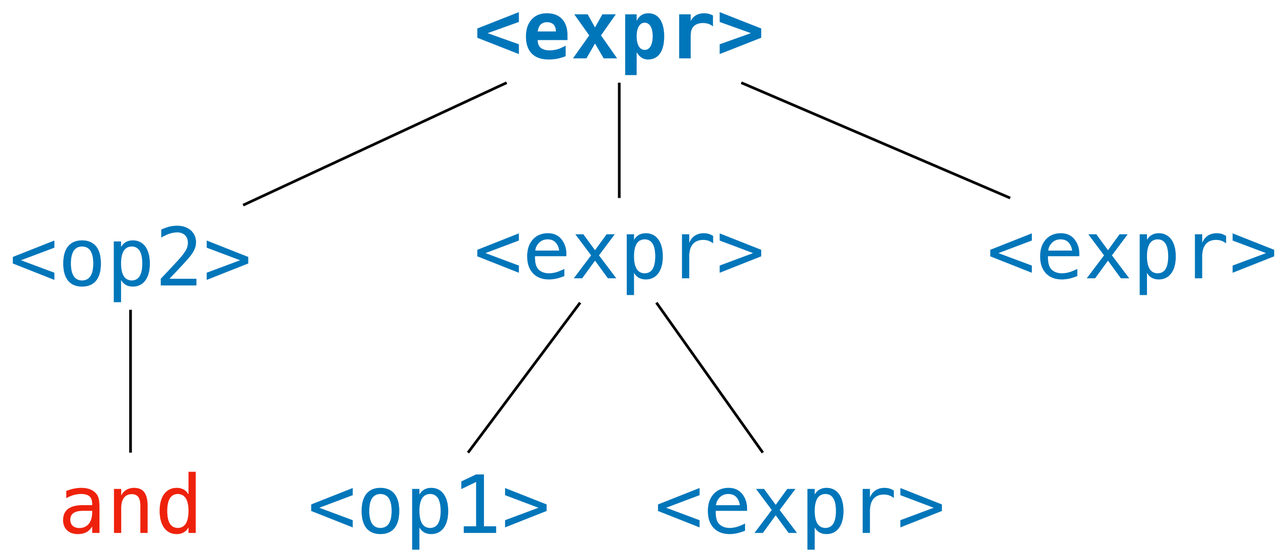

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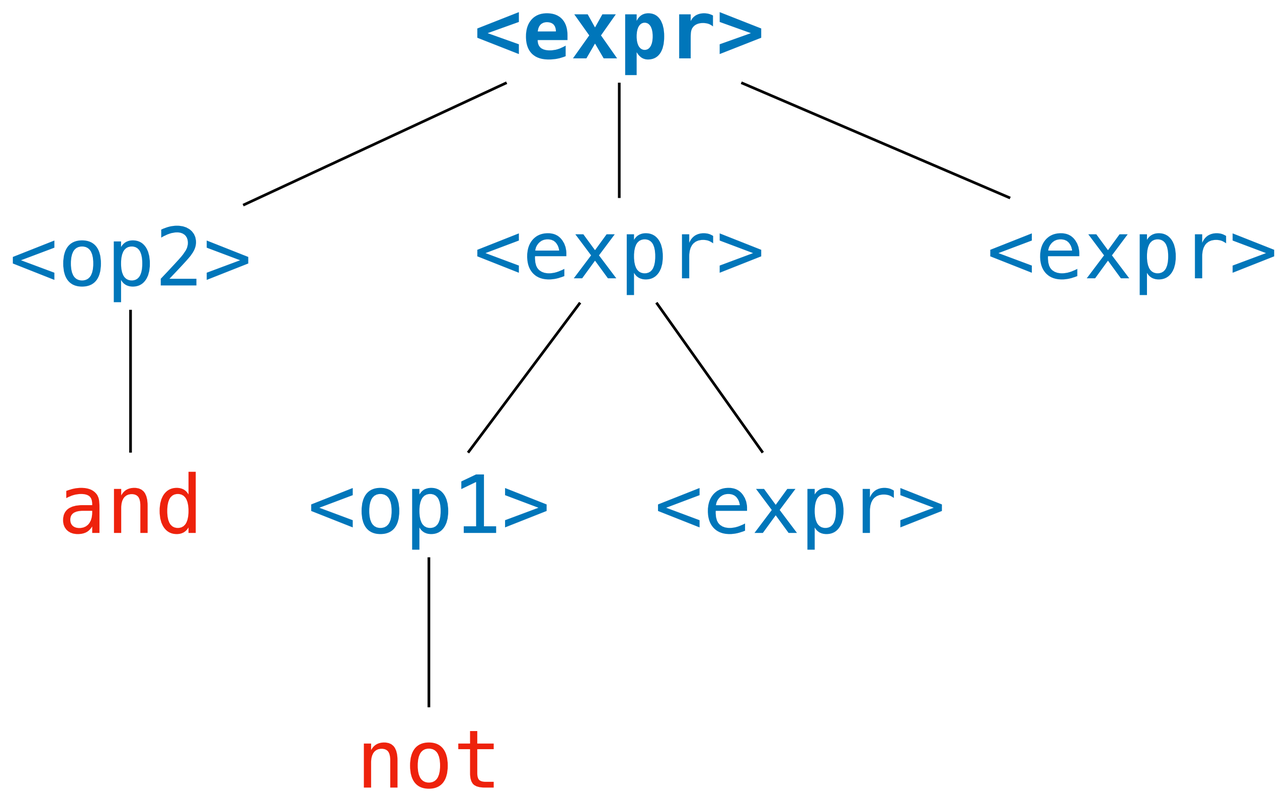

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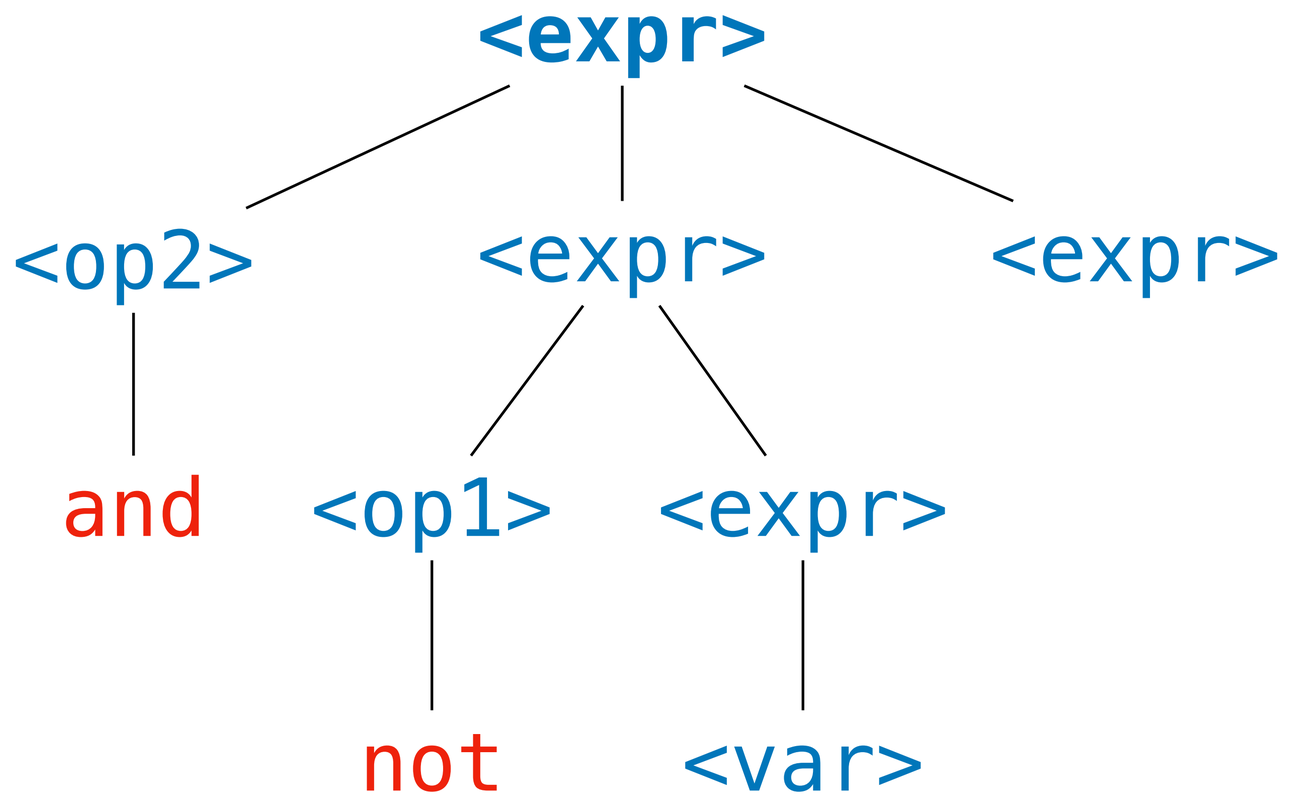

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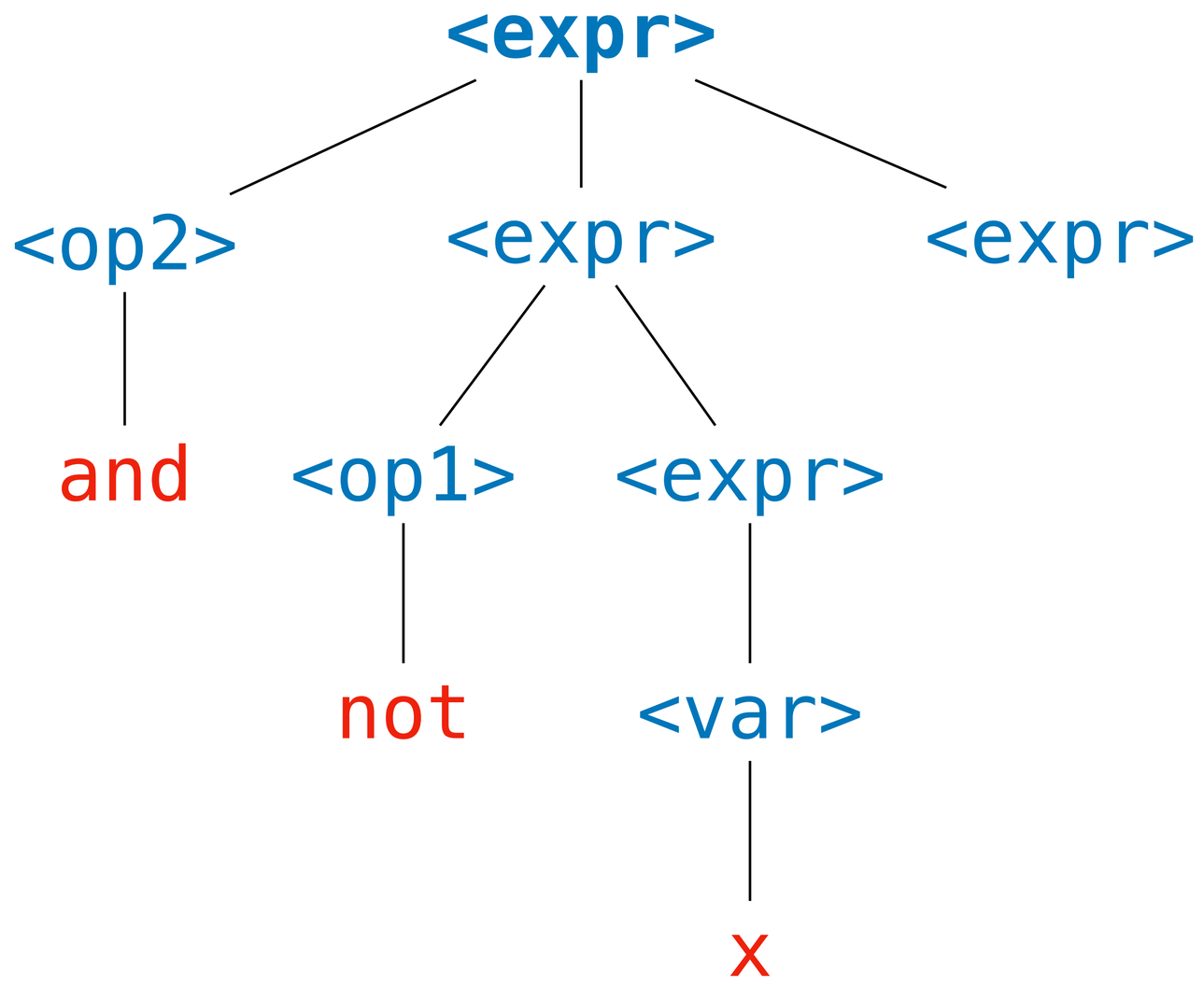

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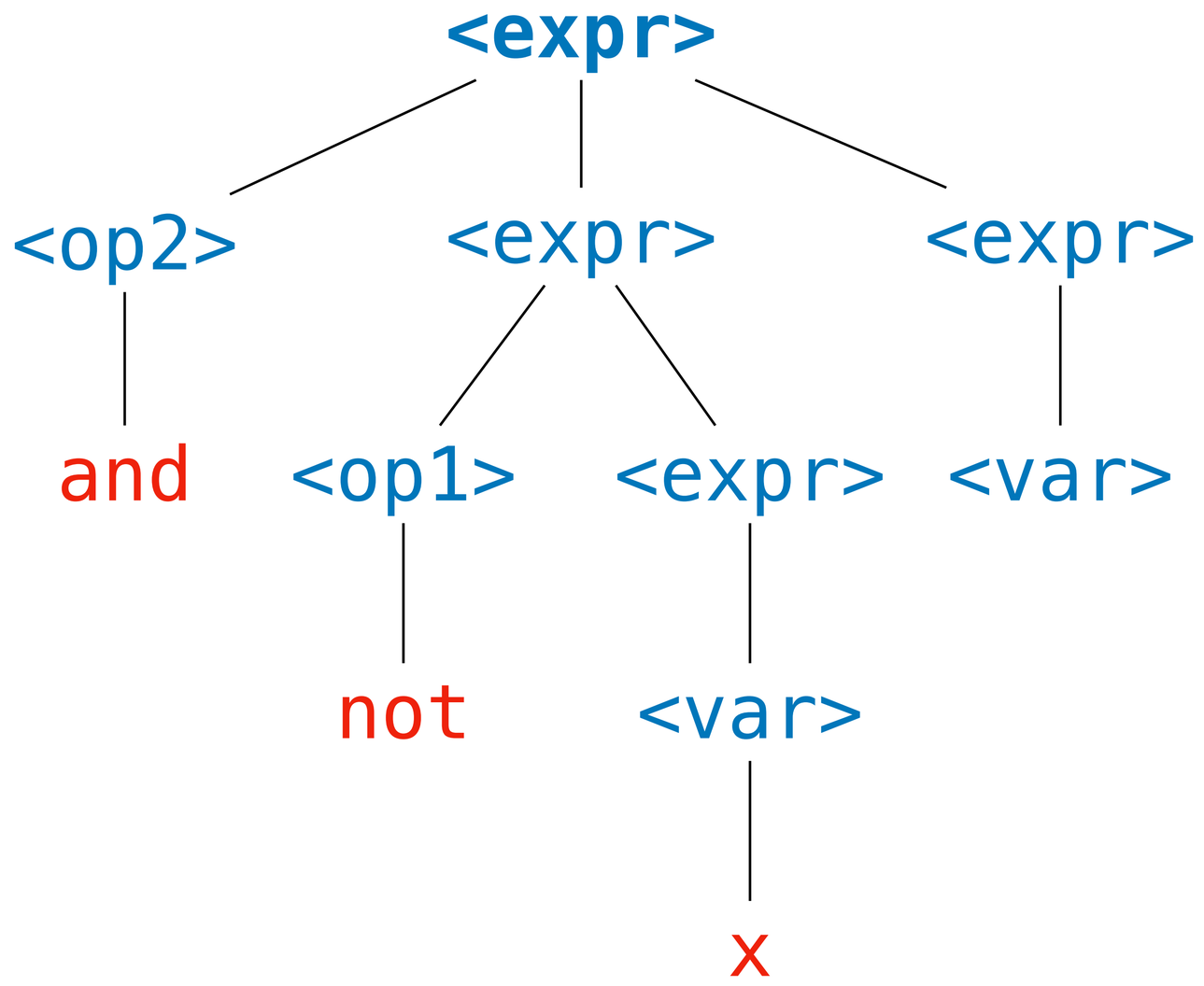

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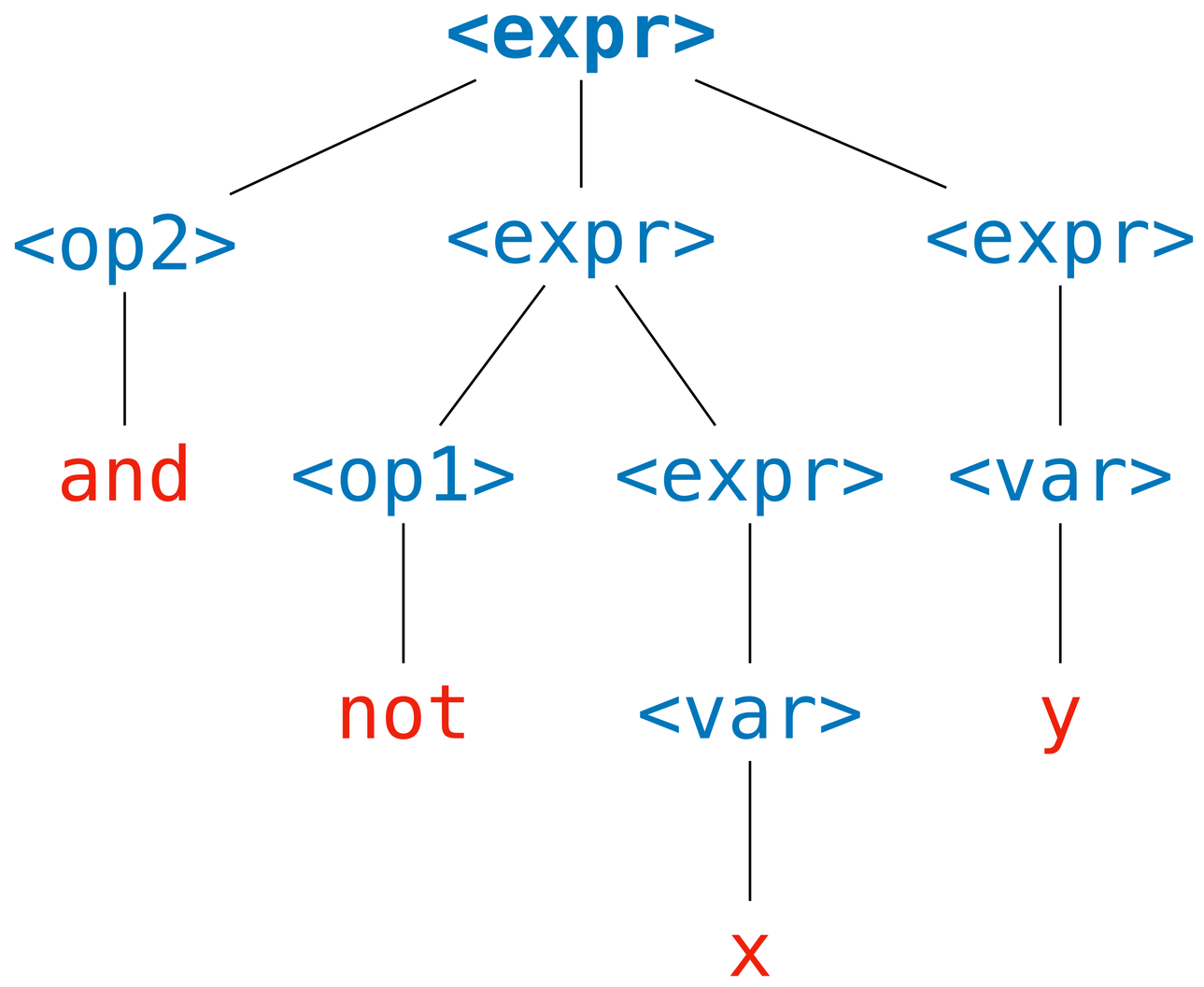

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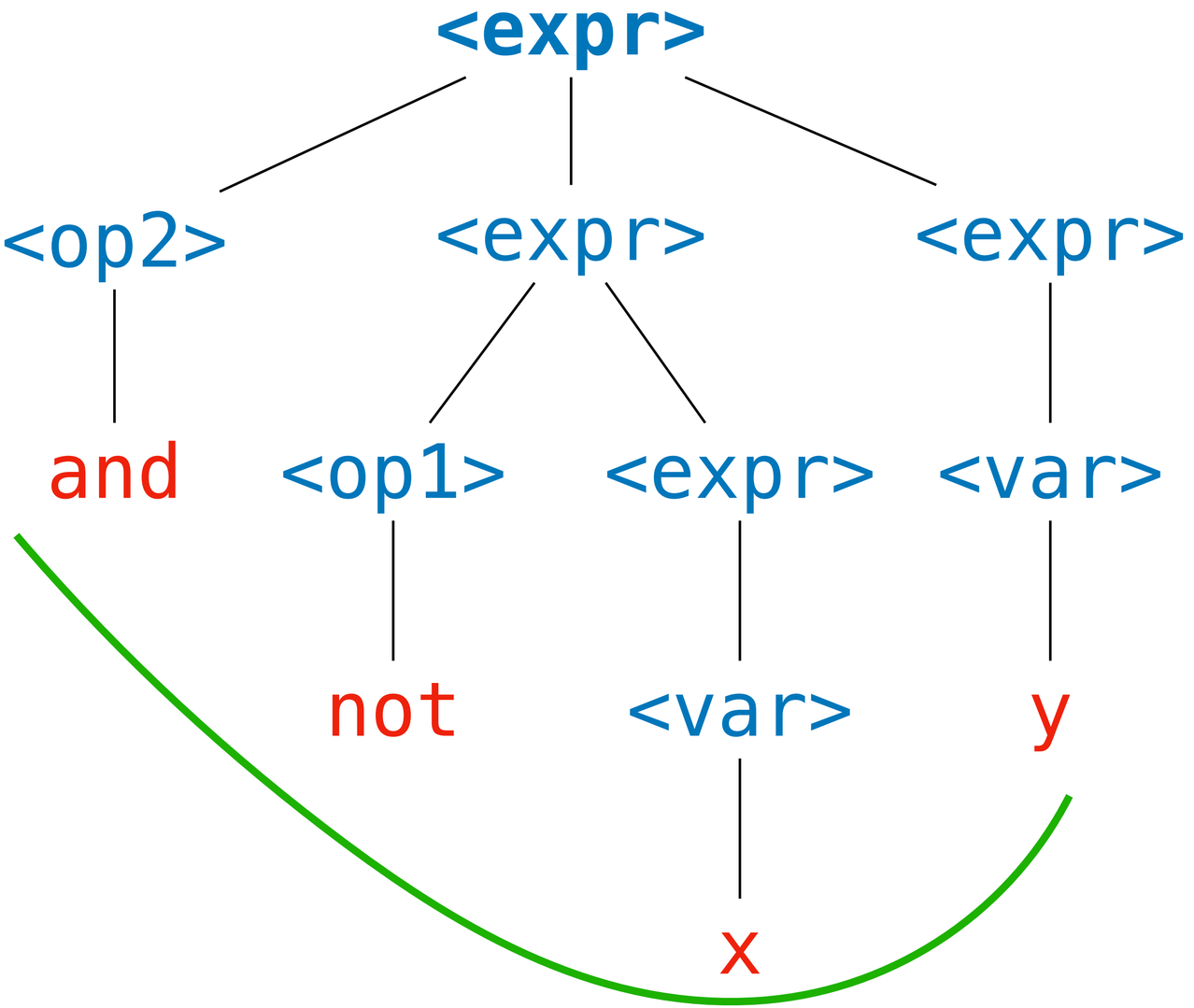

Terminology

```
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          | <var>
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<op2>   ::= and | or
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```

```
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and not x y
```

A sentence is **recognized** by a grammar if there is a derivation of that sentence in the grammar

For example, the above grammar recognizes **and not x y** because of the given derivation

Practice Problem

```
<s> ::= A <a> | A <b>
<a> ::= A B
<b> ::= B <b> | B <s>
```

Is the following sentence recognized by the above grammar?

A B B A A B

Answer

<s> ::= A <a> | A <b>
<a> ::= A B
<b> ::= B <b> | B <s>

<s>

A <b>

A B <b>

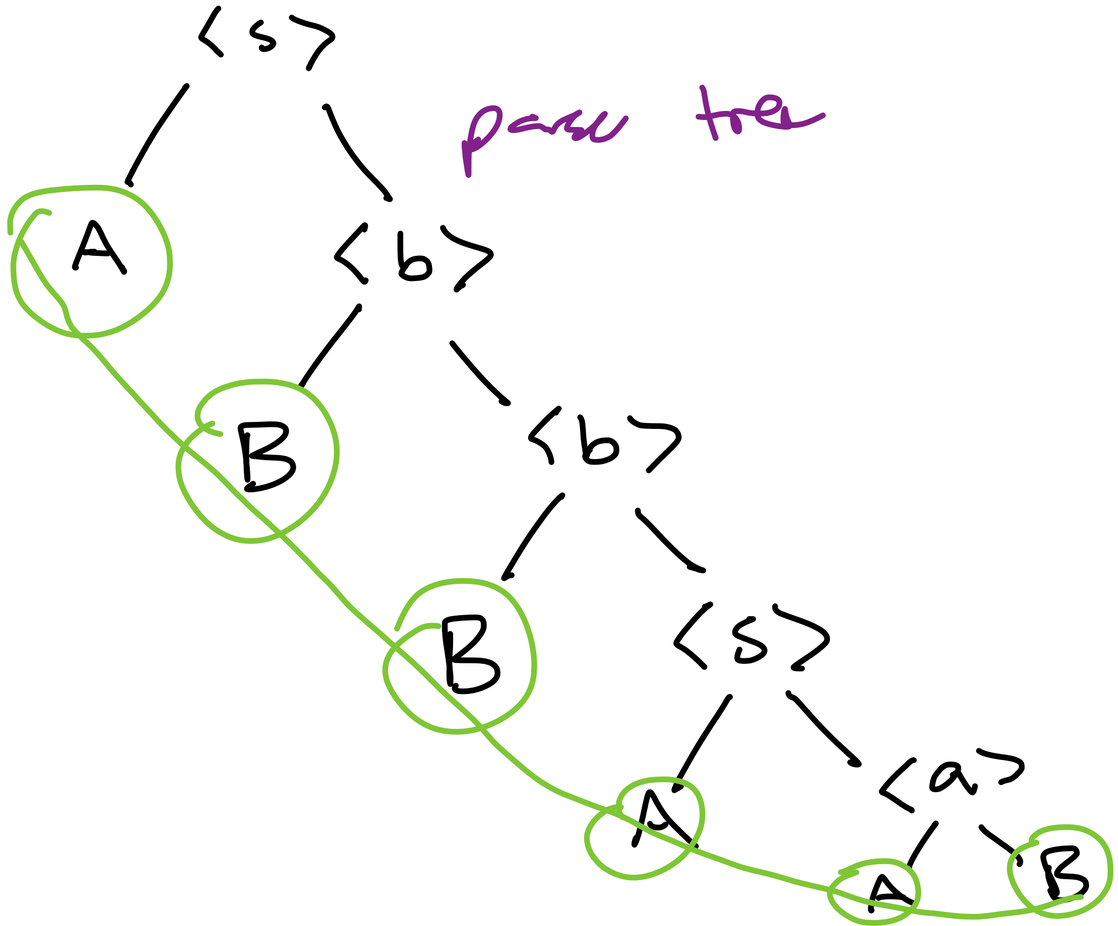
A B B <s>

A B B A <a>

A B B A A B

derivation

A B B A A B



parse tree

Ambiguity

Ambiguity in Natural Language

The duck is ready for dinner.

John saw the man on the mountain with a telescope.

He said on Tuesday there would be an exam.

(examples taken from around the internet)

Ambiguity in Natural Language

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Natural language has ambiguities that can confuse the meaning of a sentence

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Natural language has ambiguities that can confuse the meaning of a sentence

We have informal tactics for avoiding these pitfalls

(examples taken from around the internet)

Ambiguity in Natural Language

*The duck is ready **to eat** dinner.*

*John saw the man on the mountain **using** a telescope.*

*He said the exam would **be held** on Tuesday.*

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Ambiguity in Natural Language

*The duck is ready **to eat** dinner.*

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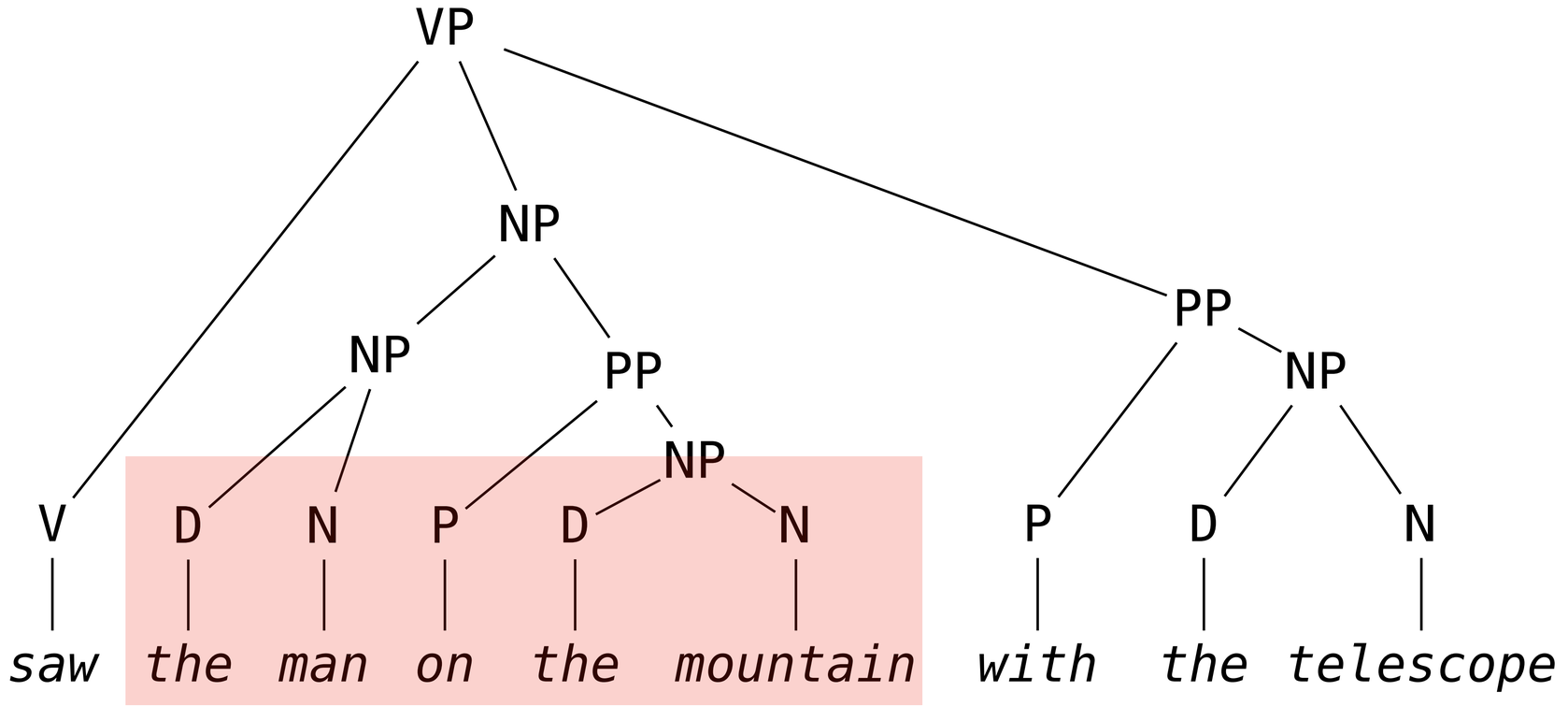
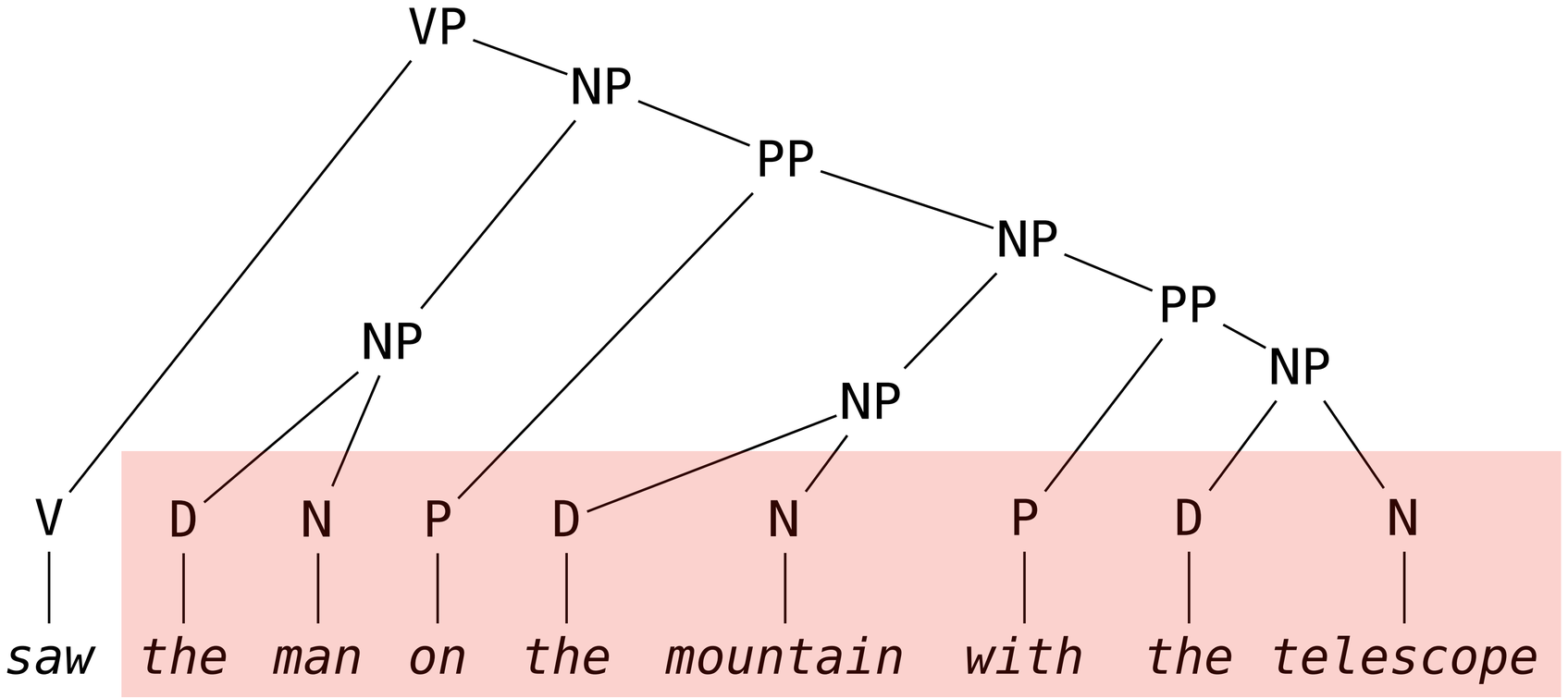
Aside: Ambiguity and Linearity



Ambiguity is caused by writing down hierarchical structures in a linear fashion

Aside: Ambiguity and Linearity



There is no ambiguity in the grammatical parse tree of this statement

The hierarchical structure changes the meaning of the sentence

Ambiguity in Formal Grammar

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```
<expr> ::= <expr> <op> <expr>
         | <var>
<op>   ::= +
<var>  ::= x | y | z
```

Ambiguity in Formal Grammar

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```
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<var>  ::= x | y | z
```

x + y + z can be derived as

Ambiguity in Formal Grammar

Definition. A BNF grammar is **ambiguous** if there is a sentence with multiple parse trees/ leftmost derivations.

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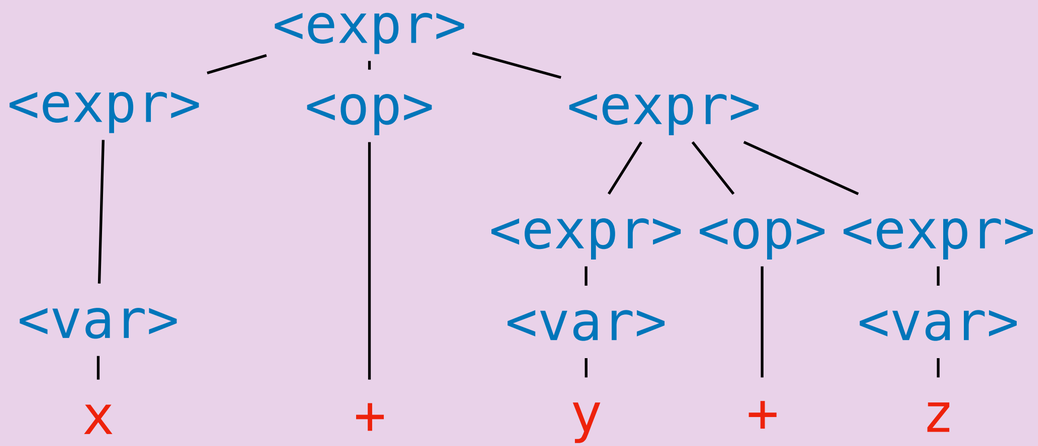

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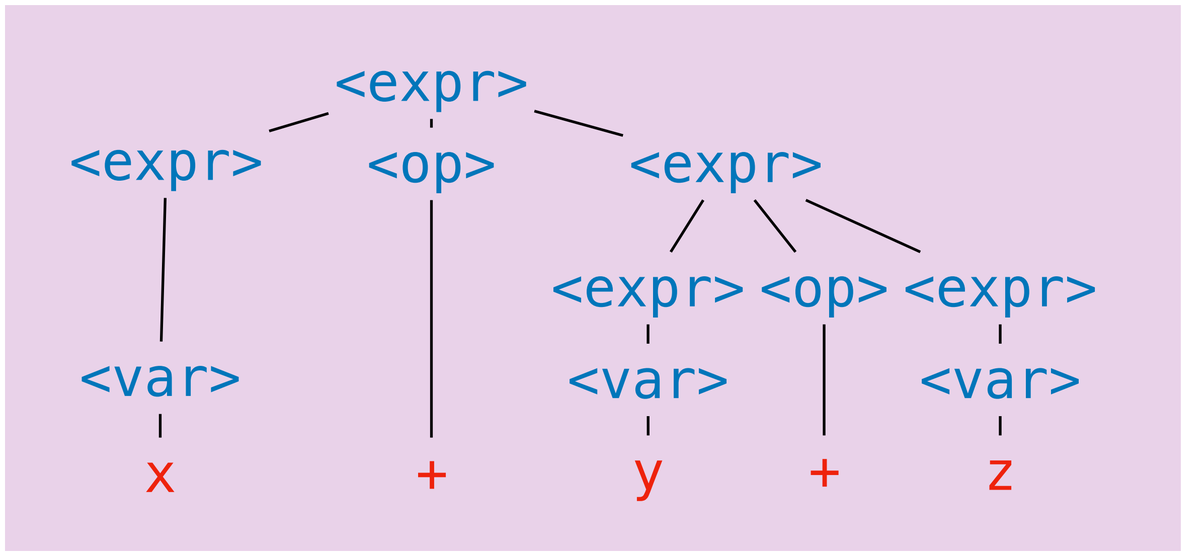

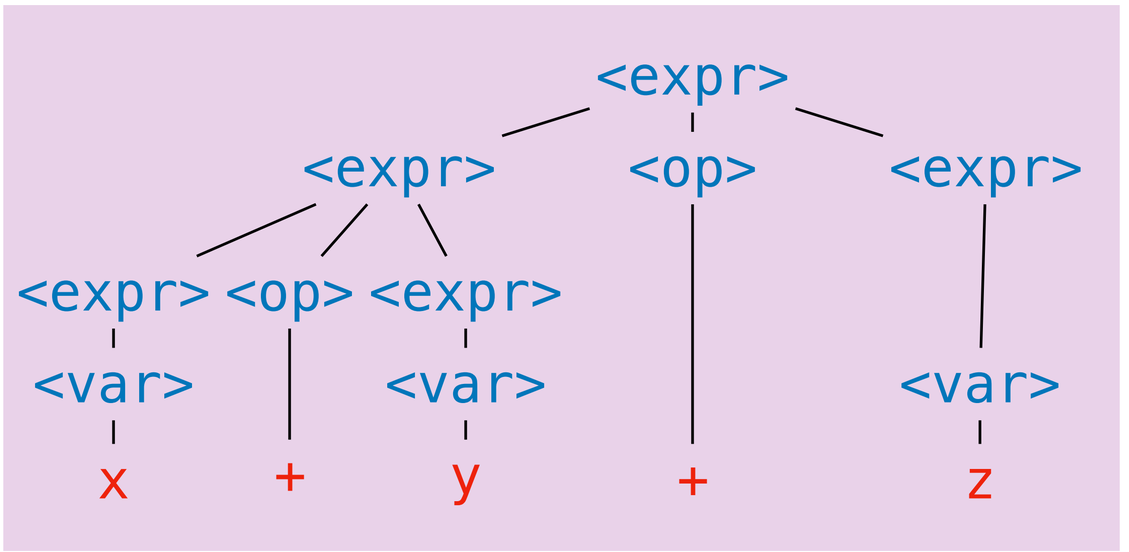

Again, why do we care?

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false && destory_everything () || false
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Again, why do we care?

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false && destory_everything () || false
```

Note that **1 + 1 + 1** is not ambiguous with respect to its *meaning* (it's value is **3** according to the standard definition of +)

But we make a promise to the user of a language that we won't make any unspoken assumptions about what they meant when they wrote down their program

Practice Problem

```
<expr> ::= x
         | if <expr> then <expr>
         | if <expr> then <expr> else <expr>
```

Show that the above grammar is ambiguous

Answer

```
<expr> ::= x
         | if <expr> then <expr>
         | if <expr> then <expr> else <expr>
```

if x then if x then x else x

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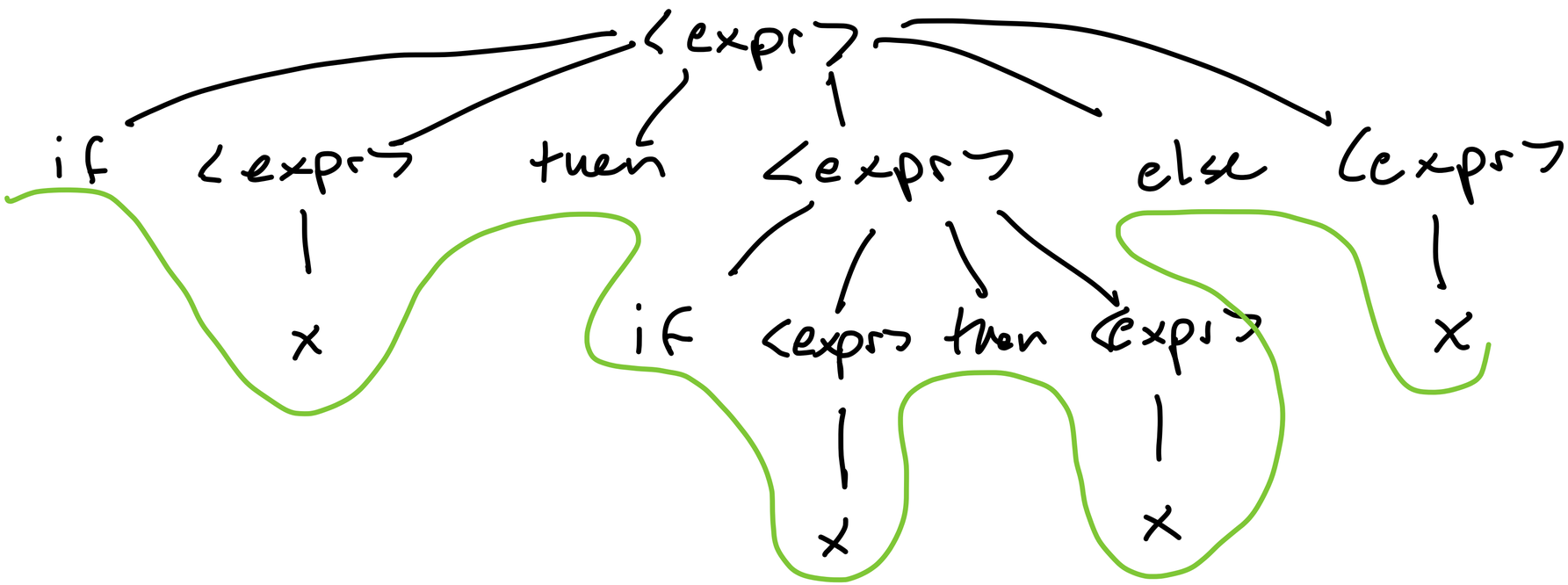

What can we do about ambiguity?

Aside: Ambiguity and Computability

```
let is_ambiguous(g : grammar) : bool = ???
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Not just hard, but **literally impossible**

That's not to say we can't determine that *particular* grammars are ambiguous

Fixity

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```
prefix        f x , (- x)

postfix       a! (get from ref)

infix         a * b, a + b, a mod b

mixfix        if b then x else y
```

Polish Notation

```
- / + 2 * 1 - 2 3
```

is equivalent to

```
- (2 + (1 * (- 2) / 3)
```

To avoid ambiguity, we can make all operators prefix (or postfix) operators. *We don't even need parentheses*

(This how early calculators worked)

Example

```
<expr> ::= <bool>
         | <var>
         | ifthen <expr> <expr>
         | ifthenelse <expr> <expr> <expr>
<bool>::= tru | fls
<var> ::= x | y | z
```

No more ambiguity. But programs written like this are notoriously difficult to read...

Lots of Parentheses

```
<expr>  ::= (<op1> <expr>)
          | (<expr> <op2> <expr>)
          | <var>
<op1>   ::= not
<op2>   ::= and | or
<var>   ::= x | y | z
```

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If we want infix operators, we *could* add parentheses around all operators

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<op1>   ::= not
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<var>   ::= x | y | z
```

```
(((not x) and (not (not y)) or z)

((((x or y) or z) or x) or y)

(not ((not x) and (not y)) or
      (x and z))

(x and y)
```

If we want infix operators, we *could* add parentheses around all operators

But we run into a similar issue: *Too many parentheses are difficult to read*

Can we get away without (or with fewer) parentheses?

Aside: The Cult of Parentheses

```
(defun fib (n)
  "Return the nth Fibonacci number."
  (if (< n 2)
      n
      (+ (fib (- n 1))
         (fib (- n 2)))))
```

Two Ingredients (or Flavors of Ambiguity)

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Associativity:

How should arguments be grouped in an expression like **1 + 2 + 3 + 4**?

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Associativity:

How should arguments be grouped in an expression like **1 + 2 + 3 + 4**?

Precedence:

How should arguments be grouped in an expression like **1 + 2 * 3 + 4**?

Associativity

The associativity of an infix operator refers to how its arguments are grouped in the absence of parentheses:

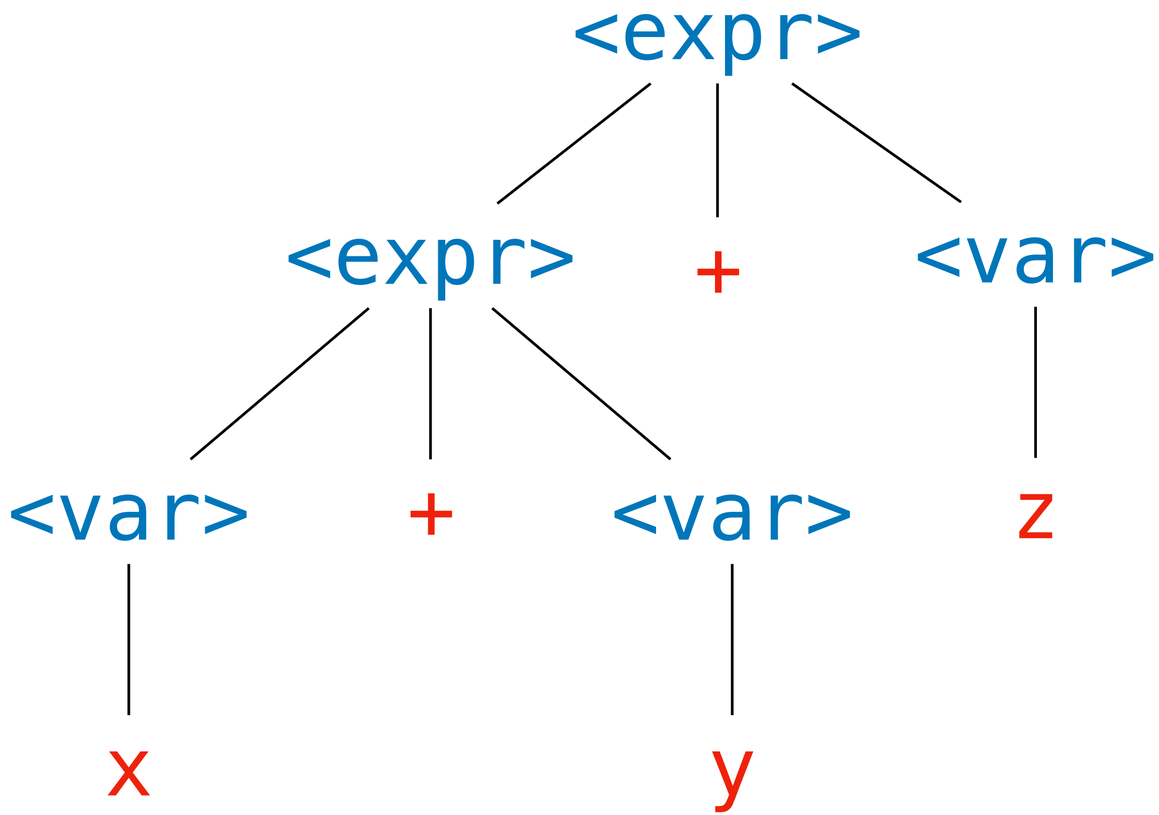
```
left associative      1 +  2 +  3 ⇒ (1 +  2) +  3

right associative     a -> b -> c ⇒  a -> (b -> c)
```

Associativity

```
<expr> ::= <expr> + <expr>
         | <var>
<var>  ::= x | y | z
```

x + y + z ⇒



"*add the sum of x and y to z*"

How do we enforce that we get a tree of this shape?

The Culprit

```
<expr> ::= <expr> + <expr>
         | <var>
<var>  ::= x | y | z
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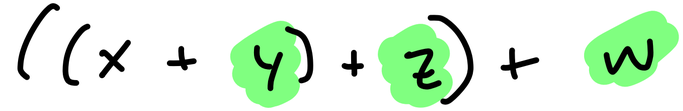
Any time we have a rule like this, we should be suspicious...

<expr> + <expr> ⇒ <expr> + <expr> + <expr>

Which <expr> did we replace?

The Solution: Breaking Symmetry

```
<expr> ::= <expr> + <var>
         | <var>
<var>  ::= x | y | z
```

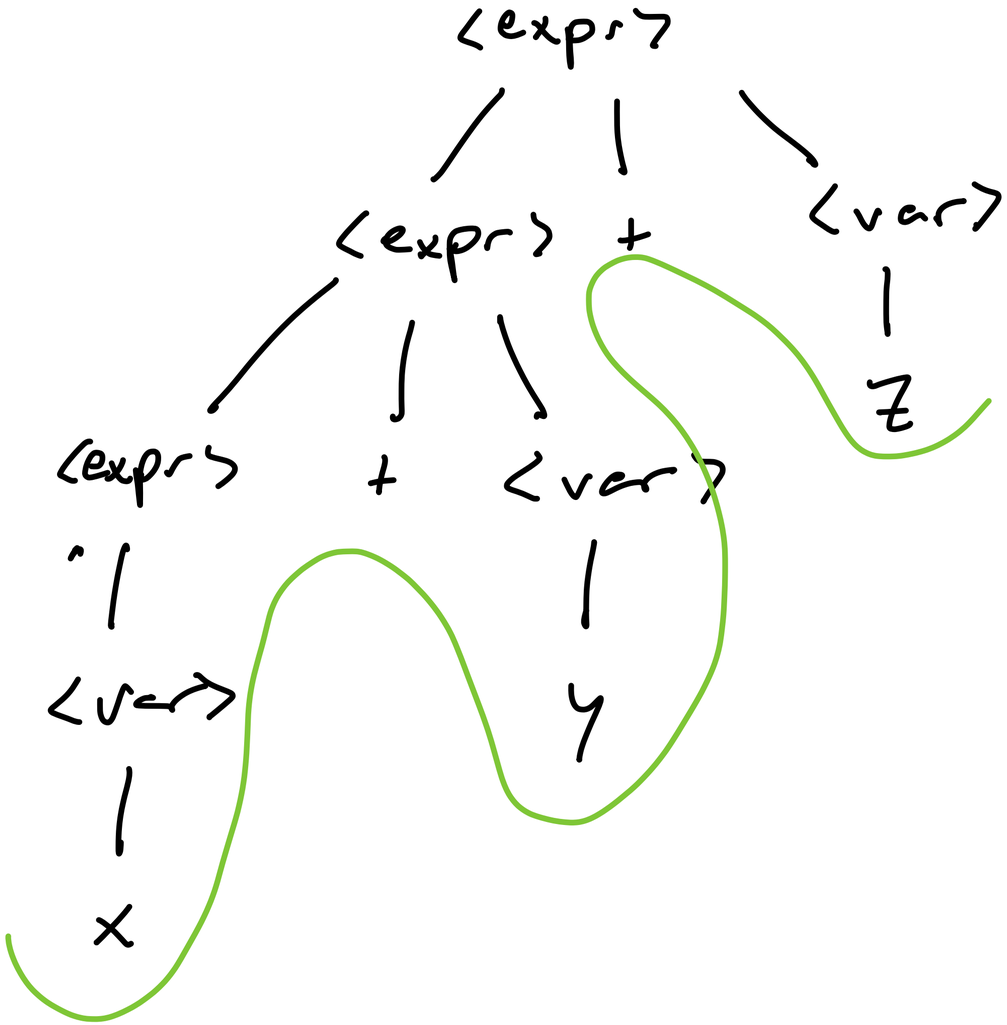



```
<expr>
<expr> + <var>
<expr> + z
<expr> + <var> + z
<expr> + y + z
<var> + y + z
x + y + z
```

We make sure that one of the arguments must be "simpler"

By enforcing that the second argument is a <var>, we will get the left-associative parse tree

Example Parse Tree

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<expr> ::= <expr> + <var>
         | <var>
<var>  ::= x | y | z
```

```
<expr>
<expr> + <var>
<expr> + z
<expr> + <var> + z
<expr> + y + z
<var> + y + z
x + y + z
```

And Right Associativity

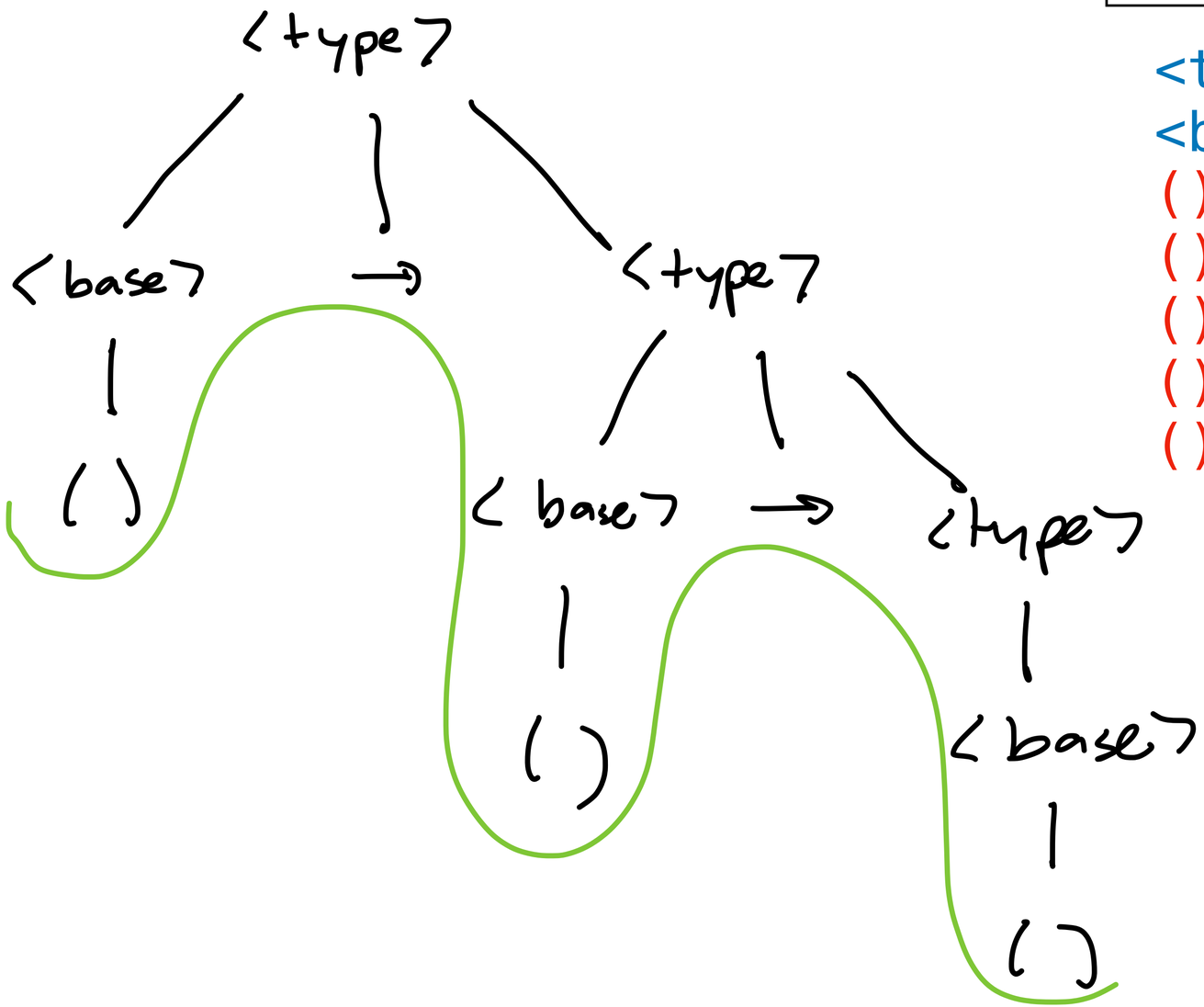
```
<type> ::= <base> -> <type>
         | <base>
<base> ::= ()
```

```
<type>
<base> -> <type>
() -> <type>
() -> <base> -> <type>
() -> () -> <type>
() -> () -> <base>
() -> () -> ()
```

For right associativity, we break symmetry by "factoring out" the *left* argument.

Example Parse Tree

```
<type> ::= <base> -> <type>
         | <base>
<base> ::= ()
```

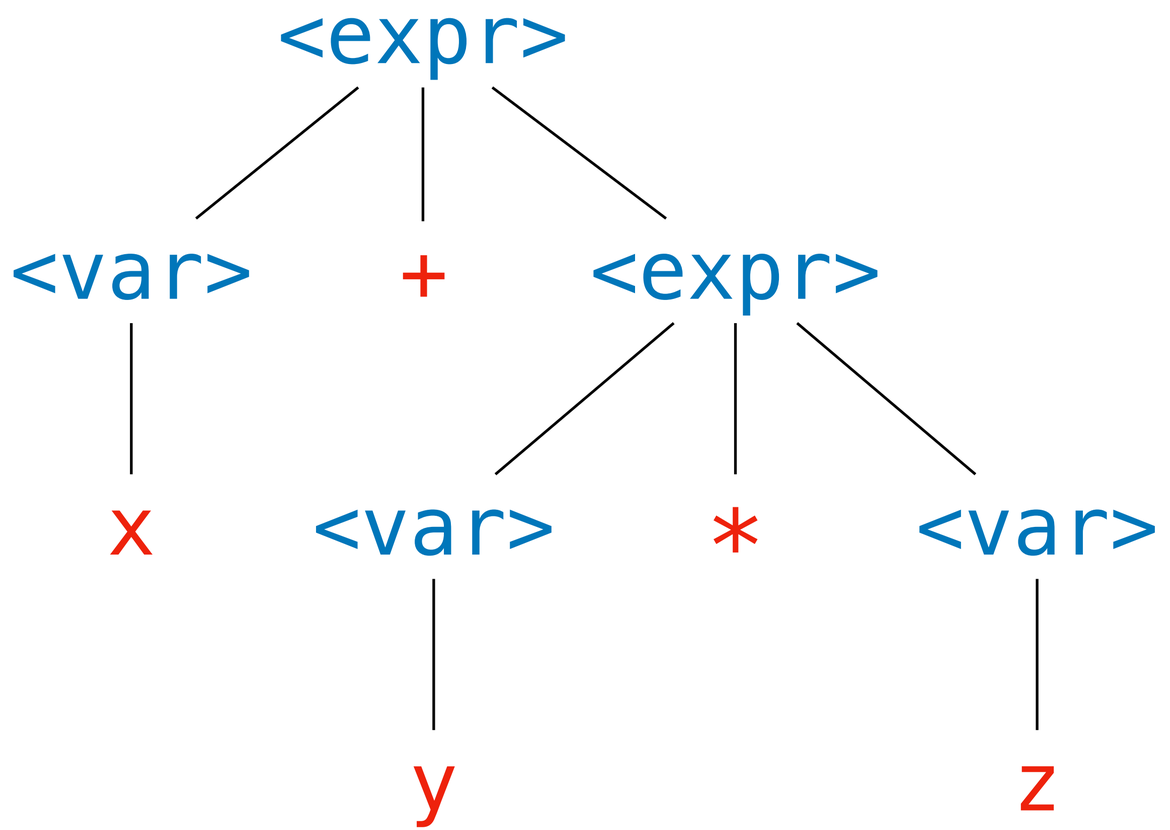



```
<type>
<base> -> <type>
() -> <type>
() -> <base> -> <type>
() -> () -> <type>
() -> () -> <base>
() -> () -> ()
```

Multiple Operators

```
<expr> ::= <expr> <op> <var>
         | <var>
<op>   ::= + | *
<var>  ::= x | y | z
```

x + y * z ⇒



"*add x to the product of y and z*"

Question. *What if we have multiple operators? Which one should "bind tighter"?*

Precedence

$$2 + 3 \times 6 = 2 + (3 \times 6) = 20$$

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The precedence of an operator refers to order in which an operator should be considered, relative to other operators

Example. **PEMDAS (paren, exp, mul, div, add, sub)**

Higher precedence means it "binds tighter"

Dealing with Precedence within the Grammar

```
<expr> ::= <expr> + <term>
         | <term>
<term> ::= <term> * <var>
         | <var>
<var>  ::= x | y | z
```

We factor out the * part of the <expr> rule

Note that we handle *lower* precedence terms first, since terms *deeper* in the parse tree are evaluated first

Practice Problem

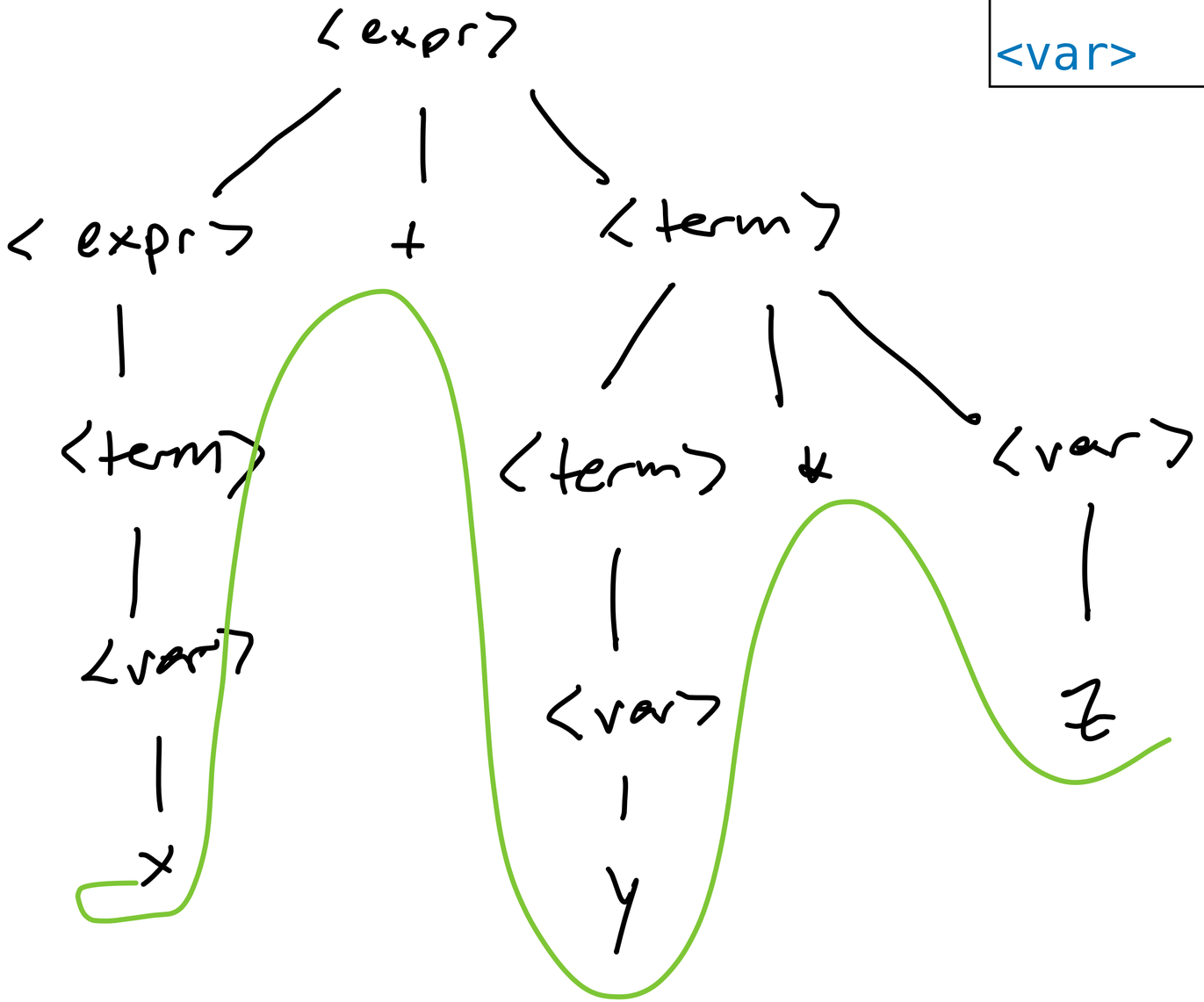
```
<expr> ::= <expr> + <term>
         | <term>
<term> ::= <term> * <var>
         | <var>
<var>  ::= x | y | z
```

Write down the parse tree for ***x + y * z***

Answer

```
<expr> ::= <expr> + <term>
         | <term>
<term> ::= <term> * <var>
         | <var>
<var>  ::= x | y | z
```

x + y * z



A Note on Associativity and Precedence

```
%token PLUS
%token MINUS
%token TIMES
%token DIVIDE

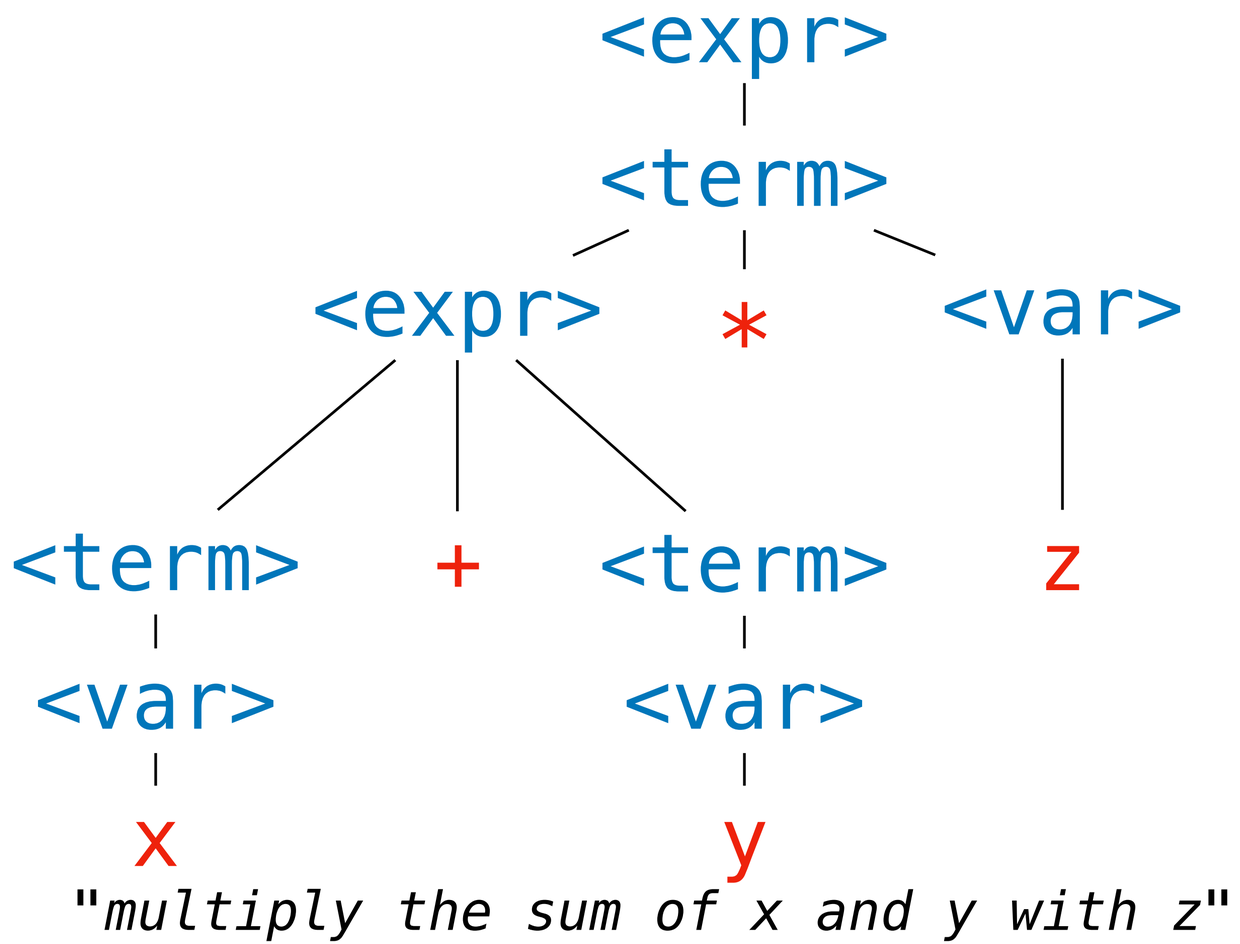
%left PLUS, MINUS
%left TIMES, DIVIDE
```

In most situations, we actually won't deal with associativity and precedence in this way

Using a parser generator we'll often be able to *specify* the precedence of an operator

The Issue of Parentheses Returns

```
<expr> ::= <expr> + <term>
         | <term>
<term> ::= <term> * <var>
         | <var>
<var>  ::= x | y | z
```



"*multiply the sum of x and y with z*"

Question. *Can we derive this parse tree?*

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<expr> ::= <expr> + <term>
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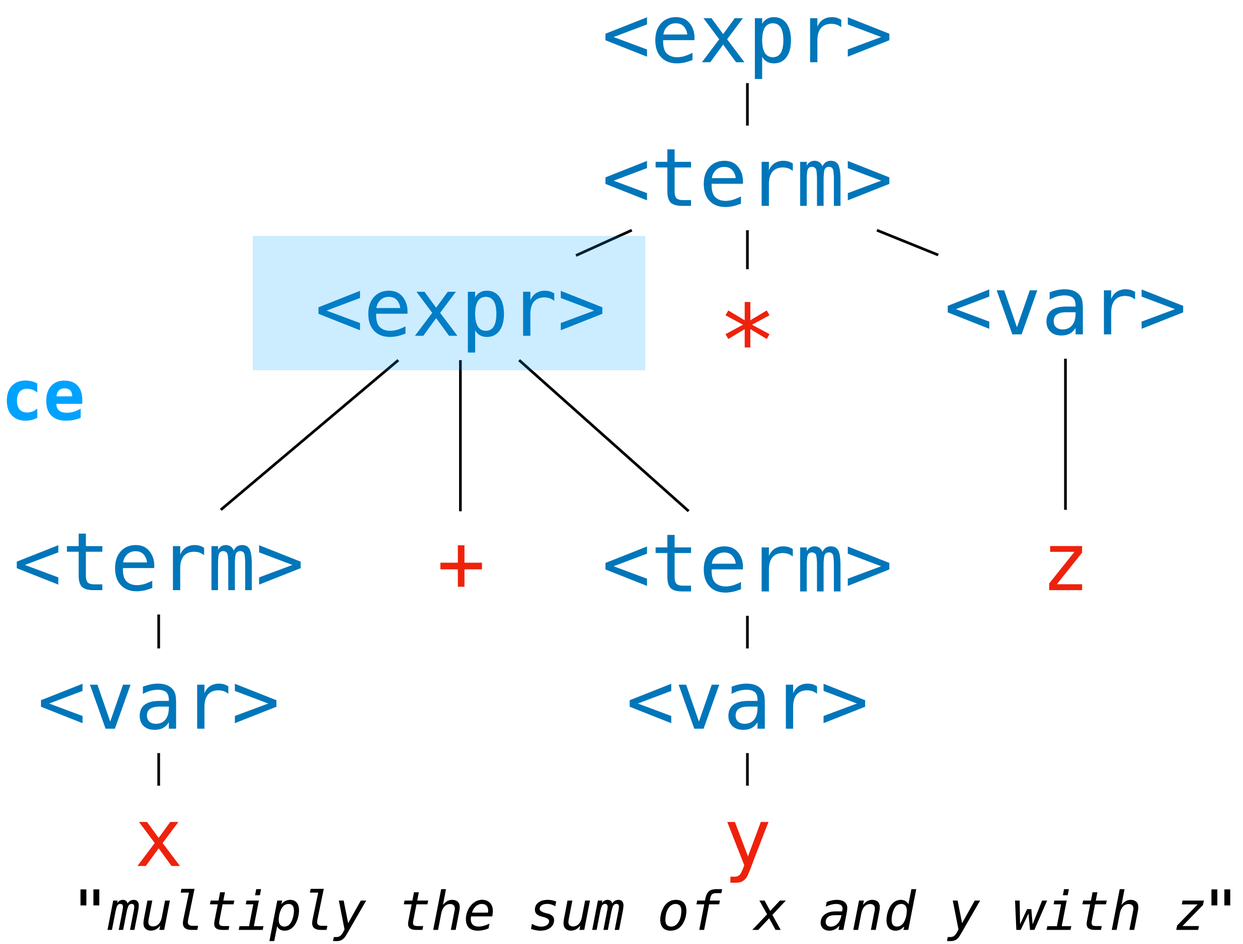
```

No, we need to introduce parentheses again.



"*multiply the sum of x and y with z*"

Question. *Can we derive this parse tree?*

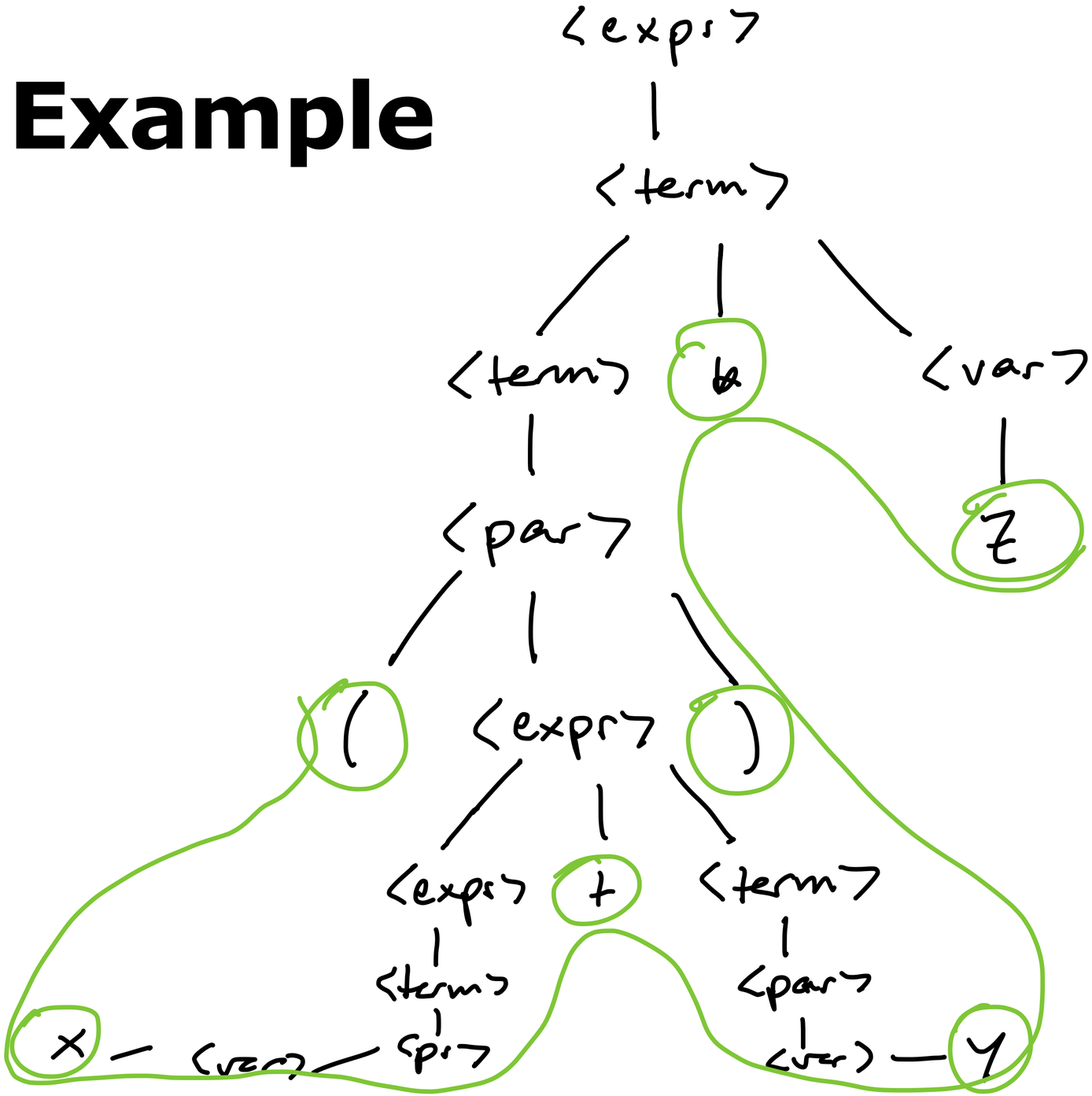
Dealing with Parentheses

```
<expr> ::= <expr> + <term>
         | <term>
<term> ::= <term> * <var>
         | <pars>
<pars> ::= <var> | ( <expr> )
<var>  ::= x | y | z
```

We further factor out the part of the rule for parentheses. Note that any expression can appear in the parentheses

(This is a circular, or mutually recursive, production rule)

Example

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<expr> ::= <expr> + <term>
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         | <pars>
<pars> ::= <var> | ( <expr> )
<var>  ::= x | y | z
```

(x + y) * z

Other Considerations

There's a lot left to make a working grammar:

» actual values (e.g., **(1 + 23) * 4**)

» variable names (e.g, **valid_var + 33**)

» multiple operations with the same precedence
(e.g. **1 + 3 - 2**)

» multiple operations with different associativity (?)

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There's a lot left to make a working grammar:

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» variable names (e.g, **valid_var + 33**)

» multiple operations with the same precedence
(e.g. **1 + 3 - 2**)

» multiple operations with different associativity (?)

This is what we will be doing when we build our interpreters

Summary

To avoid ambiguity, we make choices beforehand about the fixity, associativity and precedence

Determining ambiguity can be tricky, but usually possible for simple grammars

We make the grammars of programming languages unambiguous so that we don't make unspoken assumptions about the users input