Ex 5 Find a Θ-evaluation for the sum

$S_n = 1\sqrt{1} + 2\sqrt{2} + \ldots + n\sqrt{n}$ $\qquad \sum n\sqrt{n}$

AKA find f such that $S = \Theta(f(n))$

show work for upper + lower bound ① $f(n)$ is inc

$$S_n \geq \int_0^n (n\sqrt{n})\,dx \qquad S_n \geq \int_0^n f(x)\,dx$$

$$= \int_0^n n^{3/2}\,dx = \qquad S_n \leq \int_1^{n+1} f(x)\,dx$$

$$= \left(\frac{2n^{5/2}}{5}\right]_0^n = \frac{2n^{5/2}}{5} \leq S_n$$

$$S_n \leq \int_1^{n+1} (n\sqrt{n}) = \frac{2n^{5/2}}{5}\Big]_1^{n+1} = \left[\frac{2(n+1)^{5/2}}{5} - \frac{2(1)^{5/2}}{5}\right] \geq S_n$$