Data Structures and Algorithms

COSC 336 Assignment 2

Instructions.

1. Submit by the date and time indicated on Blackboard.
2. This is a team assignment. Work in teams of 2-3 students. Submit on Blackboard one assignment per team, with the names of all students making the team.
3. For editing your homework. I recommend that you use Latex and Overleaf, see the template files posted on the Blackboard: assignment-template.tex and assignment-template.pdf.
4. If a problem has more questions, write down your answers in the same order as the order of questions. In principle, this should help you.

Exercise 1.

a Find a Θ evaluation for the function $(4n+1)8^{\log(n^2)}$. (Hint: $8^{\log(n^2)}$ can be written in a simpler way.)

b Give an example of two functions $t_1(n)$ and $t_2(n)$ that satisfy the relations: $t_1(n) = \Theta(n^2)$, $t_2(n) = \Theta(n^2)$ and $t_1(n) - t_2(n) = o(n^2)$.

c Give an example of a function $t_1(n)$ such that $t_1(n) = \Theta(t_1(2n))$.

d Give an example of a function $t_2(n)$ such that $t_2(n) = o(t_2(2n))$.

(Note: For (b), (c), (d), seek your examples among polynomials, logarithms, exponentials, factorial.)

Exercise 2. Fill the table from Exercise 3-2, page 61 (3-rd edition) in the textbook (also attached below), except row c, as asked in the exercise. For example the entry on the first cell in the top row is "yes" because $\log^k n = O(n^\epsilon)$. (Note: in row c all the entries are "no", because $n^{\sin n}$ oscillates.)

3-2 *Relative asymptotic growths*

Indicate, for each pair of expressions (A, B) in the table below, whether A is O, o, Ω, ω, or Θ of B. Assume that $k \geq 1$, $\epsilon > 0$, and $c > 1$ are constants. Your answer should be in the form of the table with "yes" or "no" written in each box.

	A	B	O	o	Ω	ω	Θ
a.	$\lg^k n$	n^ϵ					
b.	n^k	c^n					
c.	$\sqrt{n}$	$n^{\sin n}$					
d.	2^n	$2^{n/2}$					
e.	$n^{\lg c}$	$c^{\lg n}$					
f.	$\lg(n!)$	$\lg(n^n)$					

Exercise 3. For each of the following program fragments give a $\Theta(\cdot)$ estimation of the running time as a function of n.

(a)
```
sum = 0;
for (int i = 0; i< n * n; i++) {
      for(int j =0;  j < n/2; j++)
        sum++;
}
```

(b)
```
 sum = 0;
 for (int i = 0; i< n; i++) {
sum++;}

for(int j = 0;  j < n/2; j++){
      sum++;}
```

(c)
```
 sum = 0;
 for (int i = 0; i< n * n; i++) {
      for(int j = 0;  j < n * n; j++)
           sum++
}
```

(d)
```
 sum = 0;
 for (int i = 1; i< n; i = 2*i)
           sum++
```

(e)
```
 sum = 0;
 for (int i = 0; i< n; i++) {
      for(int j = 1;  j < n * n; j = 2*j)
           sum++
}
```

Exercise 4. (a) Compute the sum $S_1 = 500 + 501 + 502 + 503 + \ldots + 999$ (the sum of all integers from 500 to 999). Do not use a program.

(b) Compute the sum $S_2 = 1 + 3 + 5 + \ldots + 999$ (the sum of all odd integers from 1 to 999). Do not use a program.

(c) A group of 30 persons need to form a committee of 3 persons. How many such committees are possible?

(d) Let C_n be the number of committees of 4 persons selected from a group of n persons. Is the estimation $C_n = o(n^3)$ correct? Justify your answer. (Hint: use the formula that gives the number of committees as a function of n.)

Exercise 5. Find a $\Theta(\cdot)$ evaluation for the sum

$$S = 1\sqrt{1} + 2\sqrt{2} + \ldots + n\sqrt{n}.$$

In other words, find a function f such that $S = \Theta(f(n))$.

Show the work for both the upper bound and the lower bound. You can use the technique with integrals, or the method with bounding the terms of the sum.