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COSC 237
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Assignment 2

Instructions.

1. Submit by the date and time indicated on Blackboard.
2. This is a team assignment. Work in teams of 2-3 students. Submit on Blackboard one assignment per team, with the names of all students making the team.
3. For editing your homework. I recommend that you use Latex and Overleaf, see the template files posted on the Blackboard: assignment-template.tex and assignment-template.pdf.
4. If a problem has more questions, write down your answers in the same order as the order of questions. In principle, this should help you.

Exercise 1.

a Find a Θ evaluation for the function $(4n+1)8^{\log(n^2)}$. (Hint: $8^{\log(n^2)}$ can be written in a simpler way.)

Step 1: $(4n+1) = n$

Step 2: $8^{\log(n^2)} = (2^3)^{2\log(n)} = 2^{6\log(n)} = 2^{\log(n^6)} = n^6$

Step 3: $n * n^6 = n^7$

$(4n+1)8^{\log(n^2)} = \Theta(n^7)$

b Give an example of two functions $t_1(n)$ and $t_2(n)$ that satisfy the relations: $t_1(n) = \Theta(n^2)$, $t_2(n) = \Theta(n^2)$ and $t_1(n) - t_2(n) = o(n^2)$.

$t_1(n) = 3n^2 + n$

$t_2(n) = 3n^2$

$(3n^2 + n) - (3n^2) = n$

$n = o(n^2)$

c Give an example of a function $t_1(n)$ such that $t_1(n) = \Theta(t_1(2n))$.

$t_1(n) = \log(n)$

$t_1(2n) = \log(2n) = \log(n)$

$\log(n) = \Theta(\log(2n))$

d Give an example of a function $t_2(n)$ such that $t_2(n) = o(t_2(2n))$.

$t_2(n) = n!$

$t_2(2n) = 2n!$

$n! = o(2n!)$

(Note: For (b), (c), (d), seek your examples among polynomials, logarithms, exponentials, factorial.)

Exercise 2. Fill the table from Exercise 3-2, page 61 (3-rd edition) in the textbook (also attached below), except row c, as asked in the exercise. For example the entry on the first cell in the top row is "yes" because $\log^k n = O(n^\epsilon)$. (Note: in row c all the entries are "no", because $n^{\sin n}$ oscillates.)

3-2 Relative asymptotic growths

Indicate, for each pair of expressions (A, B) in the table below, whether A is O, o, Ω, ω, or Θ of B. Assume that $k \geq 1$, $\epsilon > 0$, and $c > 1$ are constants. Your answer should be in the form of the table with "yes" or "no" written in each box.

	A	B	O	o	Ω	ω	Θ
a.	$\lg^k n$	n^ϵ	Yes	Yes	No	No	No
b.	n^k	c^n	Yes	Yes	No	No	No
c.	$\sqrt{n}$	$n^{\sin n}$					
d.	2^n	$2^{n/2}$	No	No	Yes	Yes	No
e.	$n^{\lg c}$	$c^{\lg n}$	Yes	No	Yes	No	Yes
f.	$\lg(n!)$	$\lg(n^n)$	Yes	No	Yes	No	Yes

Exercise 3. For each of the following program fragments give a $\Theta(\cdot)$ estimation of the running time as a function of n.

(a)
```
sum = 0;
for (int i = 0; i< n * n; i++) {
      for(int j =0;  j < n/2; j++)
        sum++;
}
```

$i(n) = n^2 * j(n)$

$j(n) = n/2$

$i(n) = n^2 * n/2$

$= \Theta(n^3)$

(b)
```
sum = 0;
 for (int i = 0; i< n; i++) {
sum++;}

for(int j = 0;  j < n/2; j++){
      sum++;}
```

$i(n) = n + j(n)$

$j(n) = n$

$i(n) = 2n$

$= \Theta(n)$

(c)
```
sum = 0;
 for (int i = 0; i< n * n; i++) {
      for(int j = 0;  j < n * n; j++)
            sum++
}
```

$i(n) = n^2 * j(n)$

$j(n) = n^2$

$i(n) = n^2 * n^2$

$= \Theta(n^4)$

(d)
```
sum = 0;
 for (int i = 1; i< n; i = 2*i)
            sum++
```

$= \Theta(2^n)$

(e)
```
sum = 0;
 for (int i = 0; i< n; i++) {
      for(int j = 1;  j < n * n; j = 2*j)
            sum++
}
```

$= \Theta(2^{2n})$

Exercise 4.

(a) Compute the sum $S_1 = 500 + 501 + 502 + 503 + \ldots + 999$ (the sum of all integers from 500 to 999). Do not use a program.

$n(n+1)/2 = 999(1000)/2 = 499,500$
$n(n+1)/2 = 499(500)/2 = 124,750$
$499,500 - 124,750 = 374,750$

374,750

(b) Compute the sum $S_2 = 1 + 3 + 5 + \ldots + 999$ (the sum of all odd integers from 1 to 999). Do not use a program.

$na + d(n-1)/2 = n(1) + 2(n-1)/2 = n + 2n - 2/2$
$n + 2n - 2/2 = 500 + 2(500) - 2/2 = 124,998$

124,998

(c) A group of 30 persons need to form a committee of 3 persons. How many such committees are possible?

$\binom{30}{3} = 30 * 29 * 28/1 * 2 * 3$

$30 * 29 * 28/1 * 2 * 3 = 24360/6 = 4,060$

4,060

(d) Let C_n be the number of committees of 4 persons selected from a group of n persons. Is the estimation $C_n = o(n^3)$ correct? Justify your answer. (Hint: use the formula that gives the number of committees as a function of n.)

$\binom{n}{4} = n(n-1)(n-2)(n-3)/1 * 2 * 3 * 4$

No $\mathrm{C}_n = \Theta(n^4)$ so, $C_n > (n^3)$ and the estimation should be $\omega(n^3)$ instead.

Exercise 5. Find a $\Theta(\cdot)$ evaluation for the sum

$$S = 1\sqrt{1} + 2\sqrt{2} + \ldots + n\sqrt{n}.$$

In other words, find a function f such that $S = \Theta(f(n))$.

Show the work for both the upper bound and the lower bound. You can use the technique with integrals, or the method with bounding the terms of the sum.

Ex 5 Find a Θ evaluation for the sum

$S_n = 1\sqrt{1} + 2\sqrt{2} + \dots + n\sqrt{n}$ $\qquad \sum n\sqrt{n}$

AKA find f such that $S = \Theta(f(n))$

show work for upper + lower bound $\qquad$ ① $f(n)$ is inc

$S_n \geq \int_0^n (n\sqrt{n})\,dx$ $\qquad S_n \geq \int_0^n f(x)\,dx$

$= \int_0^n n^{3/2}\,dx =$ $\qquad S_n \leq \int_1^{n+1} f(x)\,dx$

$= \left(\frac{2n^{5/2}}{5}\right]_0^n = \frac{2n^{5/2}}{5} \leq S_n$

$S_n \leq \int_1^{n+1} (n\sqrt{n}) = \frac{2n^{5/2}}{5}\Big]_1^{n+1} = \left[\frac{2(n+1)^{5/2}}{5} - \frac{2(1)^{5/2}}{5}\right] \geq S_n$

Lower bound:

$(2/5)n^{5/2} <= S$
$S = \Theta(n^{5/2})$

Upper bound:

$(2/5)(n+1)^{5/2} >= S$
$(n+1)^{5/2} >= S$
$S = \Theta((n+1)^{5/2})$