qs 1

$T(n) = 3T(n-1) + 12n$ given $T(0) = 5$

find, $T(2) = ?$

Ans

$T(2) = 3T(2-1) + 12(2)$

$= 3T(1) + 24 \quad — 1$

$T(1) = 3T(1-1) + 12(1)$

$= 3T(0) + 12$

$= 3 \times 5 + 12 \quad \{$ as $T(0) = 5$

$= 15 + 12$

$= 27$

substituting vaue of $T(1)$ in eqn 1

$T(2) = 3 \times 27 + 12(2)$

$= 81 + 24$

$= \boxed{105}$ (Ans)

ans 2.a)

$$T(n) = T(n-1) + c$$

Here we don't have a base case, so let's assume
$T(0) = a$ (some constant)

$T(n) = T(n-1) + c$
substituting for $T(n-1)$ we get

$$T(n) = T(n-1-1) + c + c$$
$$= T(n-2) + 2c$$

continuing this way, we get
$$T(n) = T(n-k) + kc$$

Continuing this way we get

$$T(n) = T(n-K) + KC$$

Continuing until the base case (ie: $K=n$)

$$T(n) = T(0) + nC$$
$$= a + nC$$ { as assumed

So the solution to the recurrence relation

$T(n) = T(n-1) + C$ is,

$$T(n) = a + nC$$ (Ans)

i.e. $O(n)$

ans 2.b>

$$T(n) = 2T\left(\frac{n}{2}\right) + n \quad \text{— } (1)$$

substituting n with $n/2$ we get

$$\Rightarrow T\left(\frac{n}{2}\right) = 2T\left(\frac{n}{4}\right) + \frac{n}{2} \quad \text{— } (2)$$

substituting n with $n/4$ we get

$$\Rightarrow T\left(\frac{n}{4}\right) = 2T\left(\frac{n}{8}\right) + \frac{n}{4} \quad \text{— } (3)$$

using (2) in (1)

$$T(n) = 2\left[2T\left(\frac{n}{4}\right) + \frac{n}{2}\right] + n$$

$$= 2 \cdot 2T\left(\frac{n}{4}\right) + n + n$$

$$= 2^2 T\left(\frac{n}{2^2}\right) + 2n \quad \text{—} \quad (4)$$

using (3) in (4)

$$T(n) = 2^2\left[2T\left(\frac{n}{8}\right) + \frac{n}{4}\right] + 2n$$

$$= 2^3 T\left(\frac{n}{2^3}\right) + 3n \quad \text{—} \quad (5)$$

From (4) & (5) we can figure out a pattern

$$2^2 T\left(\frac{n}{2^2}\right) + 2n$$

$$2^3 T\left(\frac{n}{2^3}\right) + 3n$$

$$2^4 T\left(\frac{n}{2^4}\right) + 4n$$

for K times, $2^K T\left(\frac{n}{2^K}\right) + Kn$ — (6)

Here the base case is $T(n) = 1$ if $n = 1$

ie: $T(1) = 1$

so $\frac{n}{2^K} = 1$ — (7)

$\Rightarrow n = 2^K$ — (8)

$\Rightarrow \log_2 n = K \log_2 2$ { taking both sides for $\log_2$

$\Rightarrow \log_2 n = K$

Now working with eq^n (6)

$2^K T(1) + Kn$

$$= 2^{k} \cdot 1 + n \cdot \log_2 n$$

$$= n + n \log_2 n$$

ie: $O(n \log_2 n)$ (Ans)

qs 2.c)

$$T(n) = 2T\left(\frac{n}{2}\right) + c$$

as this is similar to the previous question but for the "c" term.

So the answer would be $O(n)$ (Ans)

qs 2.d)

$$T(n) = T\left(\frac{n}{2}\right) + c \quad (1)$$

substituting n with $\frac{n}{2}$

$$T(n/2) = T(n/4) + c \quad \text{--- (2)}$$

Substituting n with $n/4$

$$T(n/4) = T(n/8) + c \quad \text{--- (3)}$$

using (2) in (1)

$$\Rightarrow T(n) = T(n/4) + c = T\left(\frac{n}{2^2}\right) + c$$

using (3) in (2)

$$\Rightarrow T(n) = T\left(\frac{n}{8}\right) + c = T\left(\frac{n}{2^3}\right) + c$$

doing it 'K' times

$\vdots$

$$T\left(\frac{n}{2^k}\right) + c$$

the base case is $\frac{n}{2^k} = 1 \Rightarrow n = 2^k$

$\Rightarrow \log_2 n = k \log_2 2$

$\Rightarrow \log_2 n = k$

Now $T(1) + c$

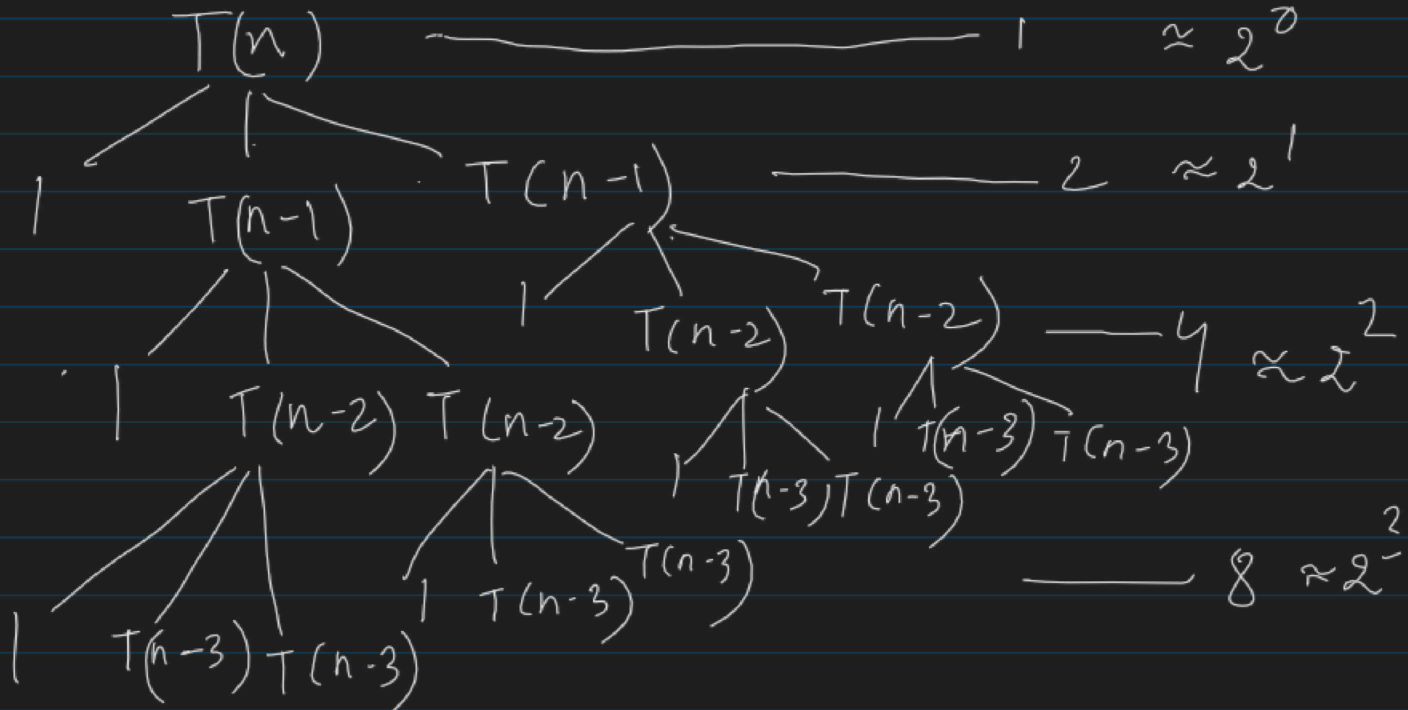
ie: $O(1)$ (Ans)

qs 3. N>

$T(n) = 2T(n-1) + 1$ { solving it using recursive tree

re writing the question

$$T(n) = \begin{cases} 1 & n = 0 \\ 2T(n-1) + 1 & n > 0 \end{cases} \quad \text{— (1)}$$



$T(0) \qquad T(0) \qquad 2^K$

adding for all the steps

$1 + 2^1 + 2^2 + 2^3 + \ldots 2^K = 2^{K+1} - 1$ — (2)

GP series

$a + ar + ar^2 + ar^3 + \ldots ar^K = \frac{a(r^{K+1} - 1)}{r - 1}$

to reach base case

$n - K = 0$

$\Rightarrow n = K$

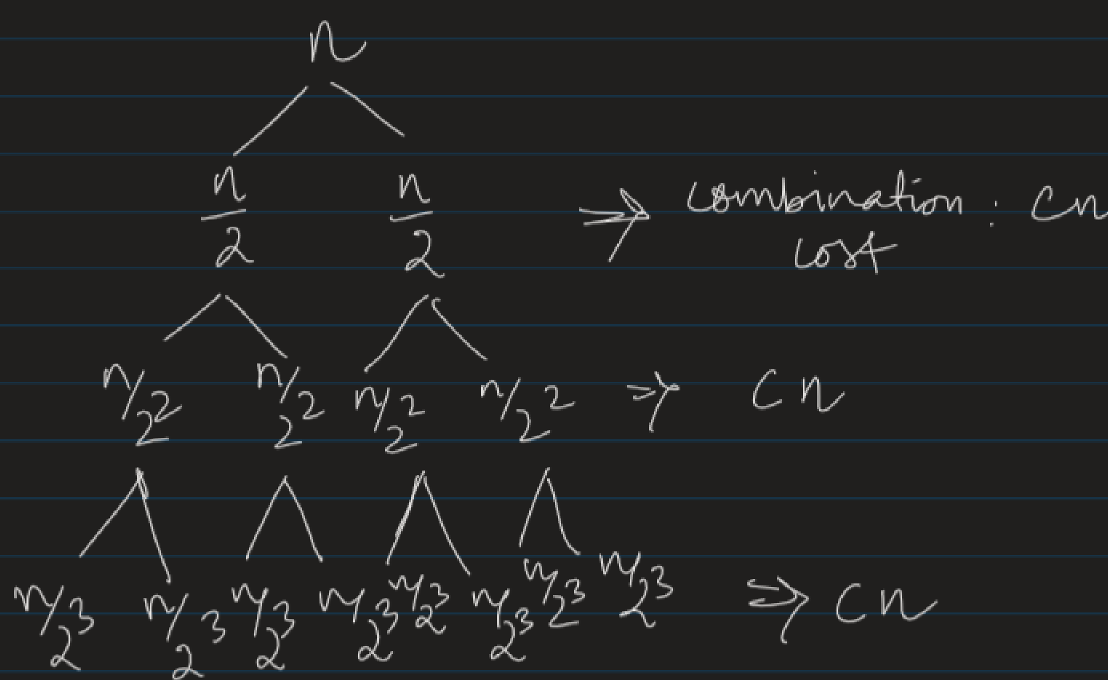
using the above in eqn (2)

$$= 2^{n+1} - 1$$

$$\boxed{\text{ie: } O(2^n) \text{ (Ans)}}$$

qs 3.b)

$$T(n) = \begin{cases} 1 & \text{if } n = 1 \\ 2T(n/2) + n & \text{if } n > 1 \end{cases}$$



$$\frac{n}{2^k} \quad \frac{n}{2^k} \quad \frac{n}{2^k} \quad \Rightarrow cn$$

for the base case $\frac{n}{2^k} = 1 \Rightarrow n = 2^k$

$\Rightarrow \log_2 n = k$

So combining all the costs

$$k \cdot c \cdot n$$

$$= \log_2 n \cdot c \cdot n$$

$$\approx n \log_2 n$$

ie: $O(n \log n)$ (Ans)