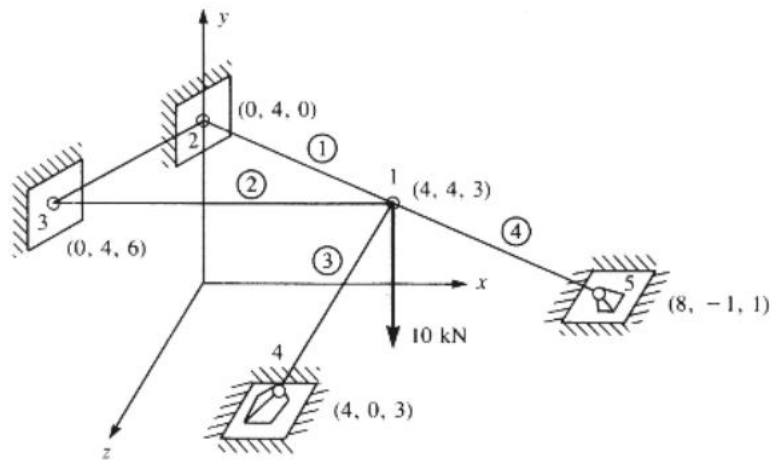


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## Problem Statement



Use Mathcad, Matlab, etc to Logan Problem 3-40.

## Notes:

1. All elements are push-pull only.
2. Nodes 2,3,4,5 constrain the assembly in all directions.
3. What looks like an element drawn between nodes 2 and 3 is probably an error in the drawing.

```
clc
clear all
```

## Connectivity table, node coordinate matrices, and problem setup

```
connect = [ 2 1;
           3 1;
           4 1;
           5 1]; %connectivity table defining 2d trusses

node_coordinates = [ 4 4 3;
                    0 4 0;
                    0 4 6;
                    4 0 3;
                    8 -1 1]; %coordinate table of node locations

F = [0 -10000 0]'; %left DOFs 4-15 out since those are unknown reactions

DOF = 3; %degrees of freedom per node
E = 2.1e11; %modulus of elasticity (Southern Pine)
A = 10e-4; %cross-sectional area of a truss
```

## Calculation of element stiffness matrix and population of global stiffness matrix

```
num_nodes = size(node_coordinates, 1);
k_global = zeros(num_nodes*DOF); %initialization of empty global stiffness matrix
T = zeros(size(connect, 1), 6);
DOF_ids = zeros(6, size(connect, 1));

for i = 1:size(connect, 1)
    element_node1 = connect(i, 1); %transforms global node into node numbers relative to element
    element_node2 = connect(i, 2);
```

```

%pick x coordinates of truss out of node coordinate matrix
x1 = node_coordinates(element_node1, 1);
x2 = node_coordinates(element_node2, 1);

%pick y coordinates of truss out of node coordinate matrix
y1 = node_coordinates(element_node1, 2);
y2 = node_coordinates(element_node2, 2);

%pick z coordinates of truss out of node coordinate matrix
z1 = node_coordinates(element_node1, 3);
z2 = node_coordinates(element_node2, 3);

truss_length = sqrt((x2-x1)^2 + (y2-y1)^2 + (z2-z1)^2); %truss length
Cx = (x2-x1)/truss_length; %cosine of truss defining angle
Cy = (y2-y1)/truss_length; %cosine of truss defining angle
Cz = (z2-z1)/truss_length; %cosine of truss defining angle

%DOF of nodes associated with truss
DOFaddress = [(element_node1*3)-2 (element_node1*3)-1 (element_node1*3) ...
              (element_node2*3)-2 (element_node2*3)-1 (element_node2*3)];

rotation = [Cx Cy Cz 0 0 0;
            0 0 0 Cx Cy Cz]; %rotation matrix for stress calculation later

k_local = [Cx 0;
           Cy 0;
           Cz 0;
           0 Cx;
           0 Cy;
           0 Cz];
k_local = (A*E/truss_length)*k_local*[1 -1;-1 1]*rotation; %local stiffness matrix

T(i,:) = E/truss_length*[-Cx -Cy -Cz Cx Cy Cz]; %rotation matrix for solving for stresses later
DOF_ids(:,i) = DOFaddress'; %ids of DOFs associated with this truss

for i = 1:6 %adds local stiffness matrix to global stiffness matrix
    for j = 1:6
        globalIn1 = DOFaddress(i);
        globalIn2 = DOFaddress(j);
        k_global(globalIn1, globalIn2) = k_global(globalIn1, globalIn2)+k_local(i, j);
    end
end
end
k_global

```

k\_global =

1.0e+07 \*

Columns 1 through 7

6.4891	-1.3913	-0.5565	-2.6880	0	-2.0160	-2.6880
-1.3913	6.9892	0.6957	0	0	0	0
-0.5565	0.6957	3.3023	-2.0160	0	-1.5120	2.0160
-2.6880	0	-2.0160	2.6880	0	2.0160	0
0	0	0	0	0	0	0
-2.0160	0	-1.5120	2.0160	0	1.5120	0
-2.6880	0	2.0160	0	0	0	2.6880
0	0	0	0	0	0	0
2.0160	0	-1.5120	0	0	0	-2.0160
0	0	0	0	0	0	0
0	-5.2500	0	0	0	0	0
0	0	0	0	0	0	0
-1.1131	1.3913	0.5565	0	0	0	0
1.3913	-1.7392	-0.6957	0	0	0	0
0.5565	-0.6957	-0.2783	0	0	0	0

Columns 8 through 14

0	2.0160	0	0	0	-1.1131	1.3913
0	0	0	-5.2500	0	1.3913	-1.7392
0	-1.5120	0	0	0	0.5565	-0.6957
0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	-2.0160	0	0	0	0	0
0	0	0	0	0	0	0
0	1.5120	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	5.2500	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	1.1131	-1.3913
0	0	0	0	0	-1.3913	1.7392
0	0	0	0	0	-0.5565	0.6957

Column 15

0.5565

```

-0.6957
-0.2783
 0
 0
 0
 0
 0
 0
 0
 0
 0
-0.5565
0.6957
0.2783

```

### Reducing global stiffness matrix (did it manually since the partition is obvious)

```
k_reduced = k_global(1:3, 1:3)
```

```

k_reduced =

1.0e+07 *

    6.4891   -1.3913   -0.5565
   -1.3913    6.9892    0.6957
   -0.5565    0.6957    3.3023

```

### Solving simultaneous equations for unconstrained displacements

```

U = (inv(k_reduced)*F);
U = [U;0;0;0;0;0;0;0;0;0;0];

```

### Post-processing

#### Calculate stress in each element [Pa]

```

%displacements associated with each truss in matrix form
truss_displacements = zeros(4, 6);

%populate it according the the nodes each truss is connected to
for i=1:size(DOF_ids, 1)
    for j=1:size(DOF_ids, 2)
        truss_displacements(i, j) = U(DOF_ids(i,j));
    end
end

stress = diag(T*truss_displacements) %stress is in the diagonal entries cause linear algebra

```

```

stress =

1.0e+06 *

   -0.3387
   -1.6933
   -7.9681
   -2.7261

```

#### Calculate strain in each element

```
strain = stress/E
```

```

strain =

1.0e-04 *

   -0.0161
   -0.0806
   -0.3794
   -0.1298

```