Problem 4.24

Use Mathcad, Matlab, ect to solve Logan Problems. Find the rections, nodal displacement and plot the interpolated displacement for the beams

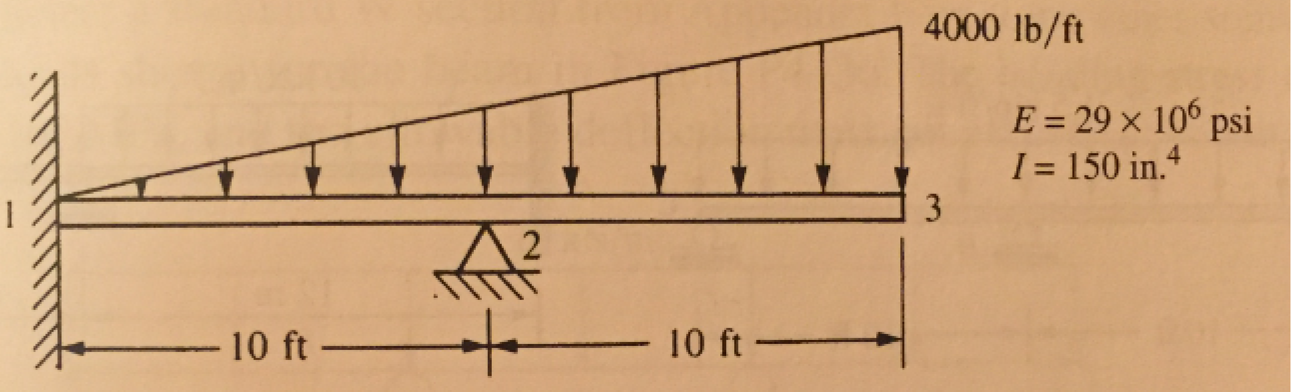


Figure P4–24

In [1]:
```
# Import necessary packages

from matplotlib import pyplot as plt
import numpy as np
import sympy as sp
sp.init_printing()
%matplotlib inline
```

In [2]:
```
# Initialize problem parameters

E = 29e6
I = 150
L = 10*12
w = 4000/12

global_stiffness_matrix = E*I/sp.Pow(L, 3)*sp.Matrix([[12, 6*L, -12, 6*L, 0, 0
],\
                                                      [6*L, 4*sp.Pow(L, 2), -6*L, 2*sp.Pow(L, 2
), 0,0],
                                                      [-12, -6*L, 24, 0, -12, 6*L],
                                                      [6*L, 2*sp.Pow(L, 2), 0, 8*sp.Pow(L, 2),
-6*L, 2*sp.Pow(L, 2)],
                                                      [0, 0, -12, -6*L, 12, -6*L],
                                                      [0, 0, 6*L, 2*sp.Pow(L, 2), -6*L, 4*sp.Po
w(L, 2)]])
global_stiffness_matrix
```

Out[2]:

$$\begin{bmatrix} 30208.3333333333 & 1812500.0 & -30208.3333333333 & 1812500.0 & \\ 1812500.0 & 145000000.0 & -1812500.0 & 72500000.0 & \\ -30208.3333333333 & -1812500.0 & 60416.6666666667 & 0 & -3020 \\ 1812500.0 & 72500000.0 & 0 & 290000000.0 & - \\ 0 & 0 & -30208.3333333333 & -1812500.0 & 3020 \\ 0 & 0 & 1812500.0 & 72500000.0 & - \end{bmatrix}$$

In [3]:
```
# Reduce stiffness matrix according to what is constrained

reduced_stiffness_matrix = global_stiffness_matrix[3:, 3:]
reduced_stiffness_matrix
```

Out[3]:

$$\begin{bmatrix} 290000000.0 & -1812500.0 & 72500000.0 \\ -1812500.0 & 30208.3333333333 & -1812500.0 \\ 72500000.0 & -1812500.0 & 145000000.0 \end{bmatrix}$$

In [4]:
```
# Generate equivalent node forces for distributed loads

F_reduced_equivalent_nodal_forces = sp.Matrix([[-w*sp.Pow(L, 2)/30],\
                                               [-17*w*L/40],
                                               [w*sp.Pow(L,2)/15]])
F_reduced_equivalent_nodal_forces
```

Out[4]:

$$\begin{bmatrix} -160000.0 \\ -17000.0 \\ 320000.0 \end{bmatrix}$$

In [5]:
```
# Calculate displacements that are not constrained

non_constrained_displacements = reduced_stiffness_matrix.LUsolve(F_reduced_equivalent_nodal_forces)
non_constrained_displacements
```

Out[5]:

$$\begin{bmatrix} -0.0129655172413793 \\ -3.27724137931034 \\ -0.0322758620689655 \end{bmatrix}$$

In [6]:
```
# Calculate actual reaction forces

global_displacements = sp.Matrix([0, 0, 0, non_constrained_displacements])
effective_forces = global_stiffness_matrix*global_displacements
equivalent_nodal_forces = sp.Matrix([-3*w*L/40,\
                                    -w*sp.Pow(L, 2)/60,
                                    -w*L/2,
                                    F_reduced_equivalent_nodal_forces])
F = effective_forces - equivalent_nodal_forces
F
```

Out[6]:

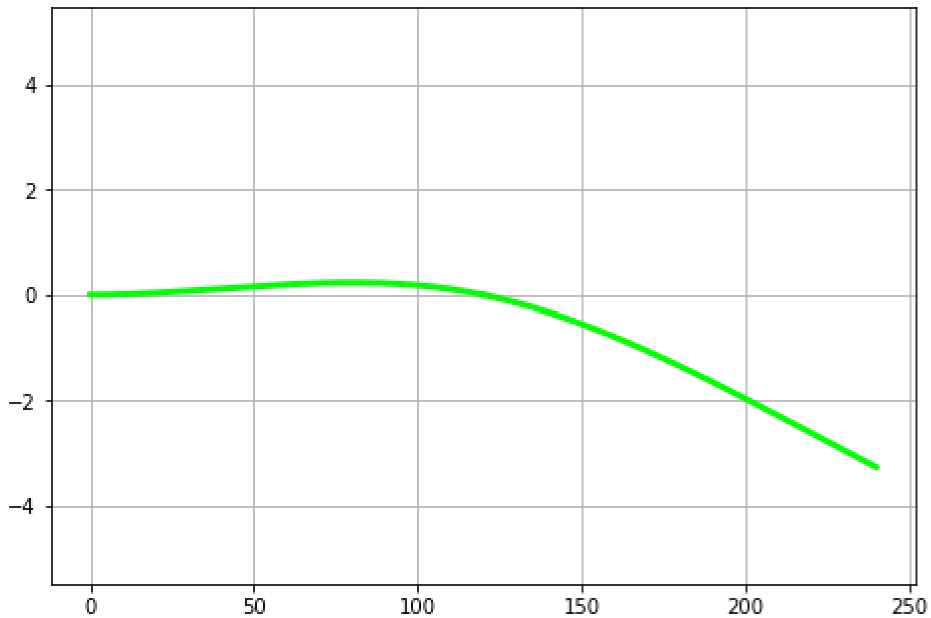
$$\begin{bmatrix} -20500.0 \\ -859999.999999999 \\ 60500.0 \\ 0 \\ -3.63797880709171 \cdot 10^{-12} \\ 0 \end{bmatrix}$$

In [7]:
```
# Interpolate from nodal displacements according to shape functions

def calculate_interpolated_displacement(x_data, length, v_1, phi_1, v_2, phi_2):
    N1 = (1/(L**3))*(2*x_data**3 - 3*x_data**2*L + L**3)
    N2 = (1/(L**3))*(L*x_data**3 - 2*L**2*x_data**2 + x_data*L**3)
    N3 = (1/(L**3))*(-2*x_data**3 + 3*L*x_data**2)
    N4 = (1/(L**3))*(L*x_data**3 - x_data**2*L**2)
    return N1*v_1 + N2*phi_1 + N3*v_2 + N4*phi_2
```

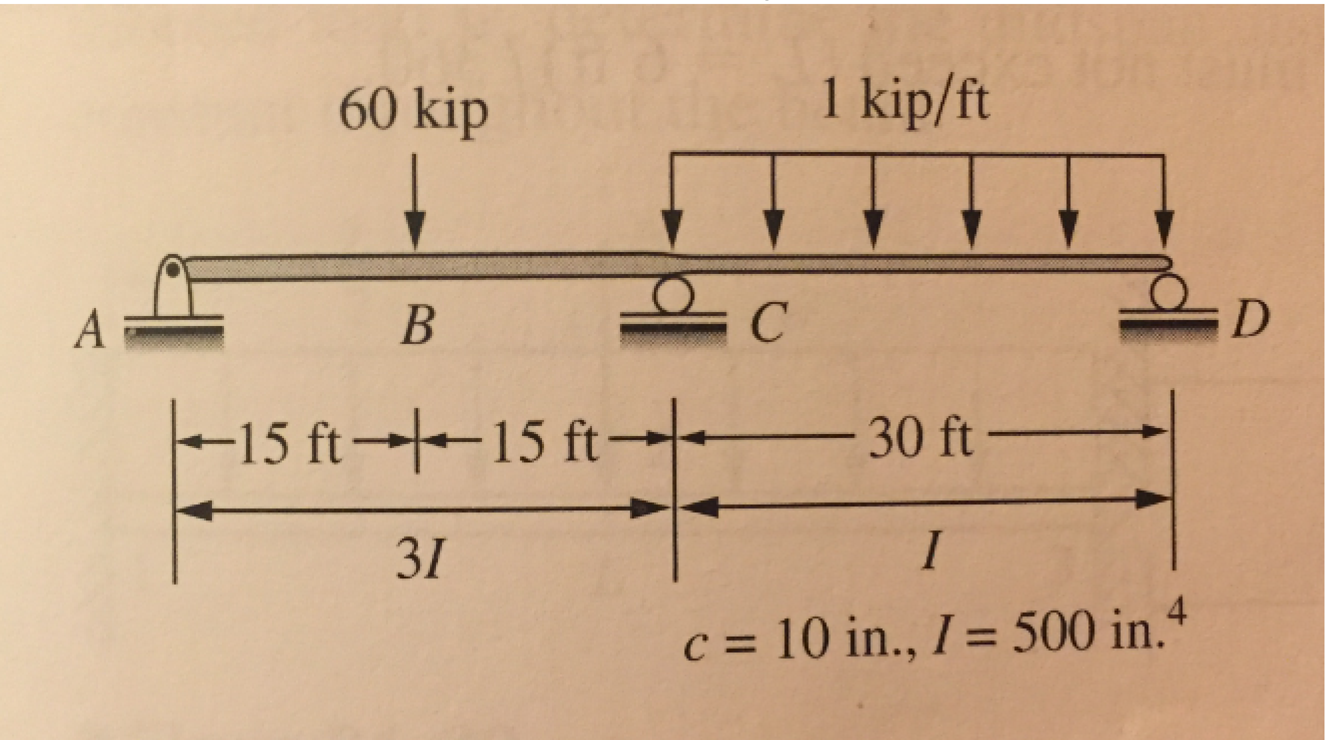
In [8]:
```
# Plot interpolated displacements

fig = plt.figure()
ax = fig.add_axes([0, 0, 1, 1])
x = np.arange(0, L, 0.1)
beam_element_1_displacement = calculate_interpolated_displacement(x, L, 0, 0,
0, non_constrained_displacements[0])
beam_element_2_displacement = calculate_interpolated_displacement(x, L, 0, non
_constrained_displacements[0], non_constrained_displacements[1], non_constrain
ed_displacements[2])
linewidth = 3
ax.plot(x, beam_element_1_displacement, color='lime', linewidth=linewidth)
ax.plot(x+L, beam_element_2_displacement, color='lime', linewidth=linewidth)
plt.ylim(-5.5, 5.5)
ax.grid(True)
```



Problem 4.29

Use Mathcad, Matlab, ect to solve Logan Problem 4.29. Find the rections, nodal displacement and plot the interpolated displacement for the beam. Also plot the stress along the beam and confirm with CREO



■ Figure P4–29

In [9]:

```
# Initialize problem parameters

E, I, L, P, w = sp.symbols('E, I, L, P, w')
E = 30e6
I = 500
L = 30*12
P = 60e3
w = 1000/12

k_1 = ((E*3*I)/sp.Pow(L, 3))*sp.Matrix([[12, 6*L, -12, 6*L],\
                                        [6*L, 4*sp.Pow(L, 2), -6*L, 2*sp.Pow(L
, 2)],
                                        [-12, -6*L, 12, -6*L],
                                        [6*L, 2*sp.Pow(L, 2), -6*L, 4*sp.Pow(L
, 2)]])
k_2 = ((E*I)/sp.Pow(L, 3))*sp.Matrix([[12, 6*L, -12, 6*L],\
                                      [6*L, 4*sp.Pow(L, 2), -6*L, 2*sp.Pow(L,
2)],
                                      [-12, -6*L, 12, -6*L],
                                      [6*L, 2*sp.Pow(L, 2), -6*L, 4*sp.Pow(L,
2)]])
```

In [10]:

```
# Generate global stiffness matrix for this problem

global_stiffness_matrix = sp.zeros(6, 6)
global_stiffness_matrix[0:4, 0:4] = global_stiffness_matrix[0:4, 0:4] + k_1
global_stiffness_matrix[2:6, 2:6] = global_stiffness_matrix[2:6, 2:6] + k_2
global_stiffness_matrix
```

Out[10]:

$$\begin{bmatrix} 11574.0740740741 & 2083333.33333333 & -11574.0740740741 & 2083333.333 \\ 2083333.33333333 & 500000000.0 & -2083333.33333333 & 25000000 \\ -11574.0740740741 & -2083333.33333333 & 15432.0987654321 & -1388888.88 \\ 2083333.33333333 & 250000000.0 & -1388888.88888889 & 666666666.6 \\ 0 & 0 & -3858.02469135802 & -694444.444 \\ 0 & 0 & 694444.444444444 & 83333333.33 \end{bmatrix}$$

In [11]:
```
# construct reduced stiffness matrix (manually cause I'm running out of time)

k_reduced = sp.zeros(3, 3)
k_reduced[0, 0] = global_stiffness_matrix[1, 1]
k_reduced[0, 1] = global_stiffness_matrix[1, 3]
k_reduced[0, 2] = global_stiffness_matrix[1, 5]
k_reduced[1, 0] = global_stiffness_matrix[3, 1]
k_reduced[1, 1] = global_stiffness_matrix[3, 3]
k_reduced[1, 2] = global_stiffness_matrix[3, 5]
k_reduced[2, 0] = global_stiffness_matrix[5, 1]
k_reduced[2, 1] = global_stiffness_matrix[5, 3]
k_reduced[2, 2] = global_stiffness_matrix[5, 5]
k_reduced
```

Out[11]:

$$\begin{bmatrix} 500000000.0 & 250000000.0 & 0 \\ 250000000.0 & 666666666.666667 & 83333333.3333333 \\ 0 & 83333333.3333333 & 166666666.666667 \end{bmatrix}$$

In [12]:
```
# Generate equivalent node forces for distributed loads

F_reduced_equivalent_nodal_forces = sp.Matrix([-P*L/8, -w*sp.Pow(L, 2)/12 + P*
L/8, w*sp.Pow(L, 2)/12])
F_reduced_equivalent_nodal_forces
```

Out[12]:

$$\begin{bmatrix} -2700000.0 \\ 1800000.0 \\ 900000.0 \end{bmatrix}$$

In [13]:
```
# Calculate displacements that are not constrained

non_constrained_displacements = k_reduced.LUsolve(F_reduced_equivalent_nodal_f
orces)
non_constrained_displacements
```

Out[13]:

$$\begin{bmatrix} -0.0081 \\ 0.0054 \\ 0.0027 \end{bmatrix}$$

In [14]:
```
# Calculate all displacements

global_displacements = sp.Matrix([0,\
                                  non_constrained_displacements[0],
                                  0,
                                  non_constrained_displacements[1],
                                  0,
                                  non_constrained_displacements[2]])
effective_forces = global_stiffness_matrix*global_displacements
equivalent_nodal_forces = sp.Matrix([-P/2,
                                     -P*L/8,
                                     -P/2 - w*L/2,
                                     -w*sp.Pow(L, 2)/12 + P*L/8,
                                     -w*L/2,
                                     w*sp.Pow(L, 2)/12])
global_displacements
```

Out[14]:

$$\begin{bmatrix} 0 \\ -0.0081 \\ 0 \\ 0.0054 \\ 0 \\ 0.0027 \end{bmatrix}$$

In [15]:
```
# Calculate actual reaction forces

F = effective_forces - equivalent_nodal_forces
F
```

Out[15]:

$$\begin{bmatrix} 24375.0 \\ -4.65661287307739 \cdot 10^{-10} \\ 56250.0 \\ -2.3283064365387 \cdot 10^{-10} \\ 9375.0 \\ 0 \end{bmatrix}$$

$R_a = 24.4\,kip \qquad R_b = 56.3\,kip \qquad R_c = 9.38\,kip$

In [19]:
```
# Plot interpolated deflections

fig = plt.figure()
ax = fig.add_axes([0, 0, 1, 1])
x = np.arange(0, L, 0.1)
beam_element_1_displacement = calculate_interpolated_displacement(x, L, 0, non
_constrained_displacements[0], 0, non_constrained_displacements[1])
beam_element_2_displacement = calculate_interpolated_displacement(x, L, 0, non
_constrained_displacements[1], 0, non_constrained_displacements[2])
max_displacement = np.amax(np.abs(beam_element_1_displacement))
max_test = np.amax(np.abs(beam_element_1_displacement))
if max_test > max_displacement:
    max_displacement = max_test
print("max displacement is {} in.".format(max_test))
linewidth = 3
ax.plot(x, beam_element_1_displacement, color='lime', linewidth=linewidth)
ax.plot(x+L, beam_element_2_displacement, color='lime', linewidth=linewidth)
ax.set_title('Deflection Curve')
plt.ylim(-3, 3)
ax.grid(True)
```

```
max displacement is 0.613458658854167 in.
```