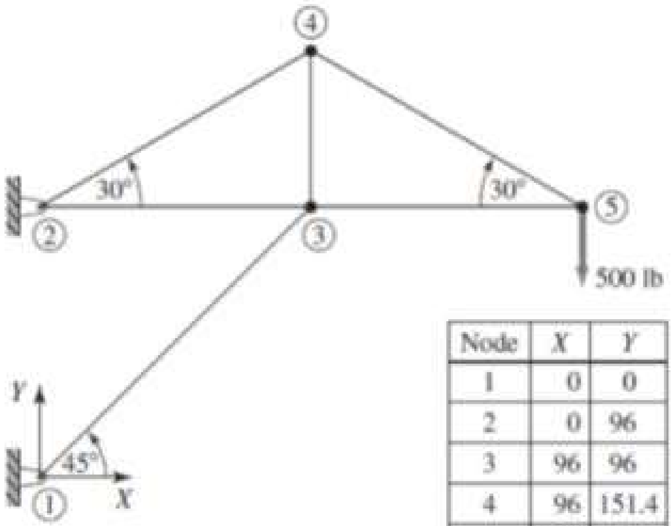
Contents

- Problem Statement
- Connectivity table, node coordinate matricies, and problem setup
- Calculation of element stiffness matrix and population of global stiffness matrix
- Reducing global stiffness matrix
- Solving simultaneous equations for unconstrained displacements
- Post-processing
- Calculate force vector (including reactions)
- Calculate stress in each element
- Calculate strain in each element
- Plot original and deformed structure

Problem Statement



Node	X	Y
1	0	0
2	0	96
3	96	96
4	96	151.4
5	192	96

(inches)

The cantilever truss shown here was constructed by a builder to support a winch and cable system to lift and lower construction materials. The truss members are 1.75x3.5in southern yellow pine, E=2x106 psi.

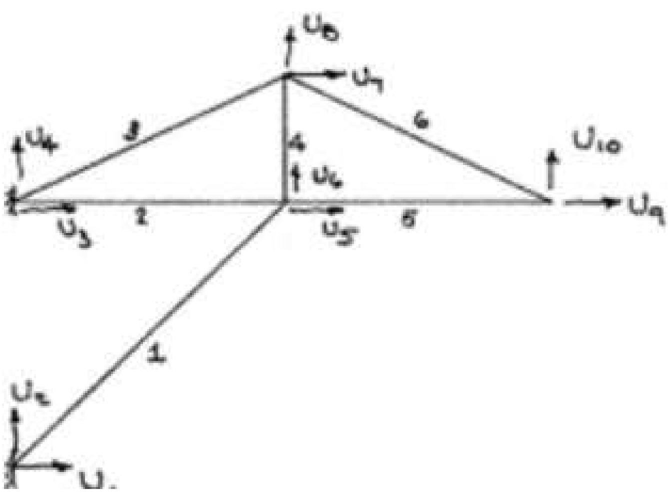
Part 1

Analyze the truss using Mathcad, Matlab, etc. calculate:

A. The global displacement components of all unconstrained nodes.
B. The axial stress and strain in each member.
C. Reaction forces at the constrained nodes.

Please use the element numbering system shown below.

Part 2



```
clc
clear all
```

Connectivity table, node coordinate matricies, and problem setup

```
connect = [1 3;
           2 3;
           2 4;
```

```
            3 4;
            3 5;
            4 5]; %connectivity table defining 2d trusses

node_coordinates = [0 0;
           0 96;
           96 96;
           96 151.4
           192 96]; %coordinate table of node locations

stationary_DOF = [1 2 3 4]; %vector of constrained DOF numbers
F = [0 0 0 0 0 -500]'; %left DOFs 1-4 out since those are unknown reactions
scale = 150; %displacement scale so we can plot the results later

DOF = 2; %degrees of freedom per node
E = 2e6; %modulus of elasticity (Southern Pine)
A = 1.75*3.5; %cross-sectional area of a truss
```

Calculation of element stiffness matrix and population of global stiffness matrix

```
num_nodes = size(node_coordinates, 1);
k_global = zeros(num_nodes*DOF);%initialization of empty global stiffness matrix
T = zeros(size(connect, 1), 4);
DOF_ids = zeros(4, size(connect, 1));

for i = 1:size(connect, 1)
    element_node1 = connect(i, 1); %transforms global node into node numbers relative to elem
ent
    element_node2 = connect(i, 2);

    %pick x coordinates of truss out of node coordinate matrix
    x1 = node_coordinates(element_node1, 1);
    x2 = node_coordinates(element_node2, 1);

    %pick y coordinates of truss out of node coordinate matrix
    y1 = node_coordinates(element_node1, 2);
    y2 = node_coordinates(element_node2, 2);

    truss_length = sqrt((x2-x1)^2 + (y2-y1)^2); %truss length
    C = (x2-x1)/truss_length; %cosine of truss defining angle
    S = (y2-y1)/truss_length; %sine of truss defining angle

    %DOF of nodes associated with truss
    DOFaddress = [(element_node1*2)-1 (element_node1*2) (element_node2*2)-1 (element_node2*2)
];

    k_local = (A*E/truss_length)*[C^2 C*S -C^2 -C*S;
                                   C*S S^2 -C*S -S^2;
                                  -C^2 -C*S C^2 C*S;
                                  -C*S -S^2 C*S S^2]; %local stiffness matrix


    T(i,:) = E/truss_length*[-C -S C S]; %rotation matrix for solving for stresses later
    DOF_ids(:,i) = DOFaddress'; %ids of DOFs associated with this truss

    for i = 1:4 %adds local stiffness matrix to global stiffness matrix
```

```
        for j = 1:4
            globalIn1 = DOFaddress(i);
            globalIn2 = DOFaddress(j);
            k_global(globalIn1, globalIn2) = k_global(globalIn1, globalIn2)+k_local(i, j);
        end
    end

end
```

Reducing global stiffness matrix

```
dim = num_nodes*2-length(stationary_DOF);
k_reduced = zeros(dim); %initialized reduced stiffness matrix
DOFfree = ones(1, num_nodes*2); %initialize mask indicating free DOFs

%populate mask as appropriate
for i = 1:length(stationary_DOF)
    DOFfree(stationary_DOF) = 0;
end

a = 1;
b = 1;
%reduce global stiffness matrix according to mask
for i = 1:length(DOFfree)
    for j = 1:length(DOFfree)

        if (DOFfree(i) && DOFfree(j))
            k_reduced(a,b)=k_global(i,j);
            if (a == b)
                U_label(a) = i;
            end
            b = b+1;
            if (b > dim)
               b = 1;
               a = a+1;
            end
        end

    end
end
U_label = U_label';
k_reduced
```

```
k_reduced =

   1.0e+05 *

    3.0032    0.4511         0         0   -1.2760         0
    0.4511    2.6623         0   -2.2112         0         0
         0         0    1.6582         0   -0.8291    0.4785
         0   -2.2112         0    2.7634    0.4785   -0.2761
   -1.2760         0   -0.8291    0.4785    2.1051   -0.4785
         0         0    0.4785   -0.2761   -0.4785    0.2761
```

Solving simultaneous equations for unconstrained displacements

```
U = (inv(k_reduced)*F);
U = [0; 0; 0; 0; U]
```

```
U =

         0
         0
         0
         0
    0.0010
   -0.0232
    0.0265
   -0.0277
   -0.0057
   -0.1016
```

Post-processing

Calculate force vector (including reactions)

```
force_vector = k_global*U
```

```
force_vector =

   1.0e+03 *

    1.0000
    1.0000
   -1.0000
   -0.5000
    0.0000
   -0.0000
   -0.0000
   -0.0000
    0.0000
   -0.5000
```

Calculate stress in each element

```
%displacements associated with each truss in matrix form
truss_displacements = zeros(4, 6);

%populate it according the the nodes each truss is connected to
for i=1:size(DOF_ids, 1)
    for j=1:size(DOF_ids, 2)
```

```
        truss_displacements(i, j) = U(DOF_ids(i,j));
    end
end

stress = diag(T*truss_displacements)
```

```
stress =

 -230.8920
   21.8080
  163.3219
 -163.2653
 -141.4573
  163.3219
```

Calculate strain in each element

```
strain = stress/E
```

```
strain =

   1.0e-03 *

   -0.1154
    0.0109
    0.0817
   -0.0816
   -0.0707
    0.0817
```

Plot original and deformed structure

```
U = U(length(stationary_DOF)+1:end) * scale;
newNodeCo = node_coordinates;

%increment original node coordinates by displacements
for i = 1:length(U_label)
    if (~mod(U_label(i),2))
        node = U_label(i)/2;
        newNodeCo(node,2) = newNodeCo(node,2) + U(i);
    else
        node = ceil(U_label(i)/2);
        newNodeCo(node,1) = newNodeCo(node,1) + U(i);
    end
end
dotSize = 25;

%plot structures
pbaspect([1 1 1])
```

```
hold on
xlim([-10, 200])
ylim([-10, 200])
grid()

%plot nodes just for fun
scatter(node_coordinates(:,1),node_coordinates(:,2),dotSize,[1 0 0],'filled')
scatter(newNodeCo(:,1),newNodeCo(:,2),dotSize,[.49 .99 0],'filled')

%plot each truss
for i = 1:size(connect, 1)
   element_node1 = connect(i, 1);
   element_node2 = connect(i, 2);
   X = [node_coordinates(element_node1,1), node_coordinates(element_node2,1)];
   Y = [node_coordinates(element_node1,2), node_coordinates(element_node2,2)];
   line(X,Y,'Color', 'red')
   X = [newNodeCo(element_node1,1), newNodeCo(element_node2,1)];
   Y = [newNodeCo(element_node1,2), newNodeCo(element_node2,2)];
   line(X,Y,'Color', 'green')
end
quiver(node_coordinates(end, 1), node_coordinates(end, 2), 0, -30, 'Color', 'k', 'MaxHeadSize
', 0.75)
legend('original structure', 'deformed structure')
```