MF 772 Group 08 Project Report
Dynamic Credit Migration-Based Trading Framework

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Abstract

We have developed a dynamic daily credit scoring framework utilizing the Kealhofer-Merton-Vasicek (KMV) and Hidden Markov Models (HMM). The KMV model calculates the distance to default and default probabilities, which are used as inputs for the HMM. The HMM determines transition and emission probabilities across predefined credit rating states. Following this, a credit score is derived by combining the normalized states with a financial health score obtained from key financial ratios. The final credit score guides a long/short trading strategy in CDS and equities, leveraging market and credit conditions effectively.

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Chapter 1

Introduction

In this report, we introduce a novel approach to credit rating transition analysis by integrating the KMV model and Hidden Markov Models (HMM). Traditional credit ratings are static and lack the flexibility to capture real-time changes in a firm's financial health. By employing a dynamic framework, we aim to provide a daily credit score that reflects a firm's risk profile more accurately.

The KMV model provides a structural approach to estimating credit risk by calculating the distance to default and the probability of default. This model is further enhanced by HMM, which models credit rating transitions as a probabilistic state-switching process. Inputs such as financial ratios are used to derive a financial health score, enabling the system to reflect underlying economic changes effectively.

The dynamic credit rating model is then applied in a trading strategy involving credit default swaps (CDS) and equities, enabling long/short trades based on predicted regime-switching probabilities.

1.1 Data Sources

To develop and validate the model, we utilize the following data sources:

- **Bloomberg:** Historical CDS Spreads, Financial Ratios, Company financials.
- **FRED (Federal Reserve Economic Data):** 10-Year Treasury Yield.
- **U.S. Bureau of Labor Statistics:** Inflation rates.
- **Yahoo Finance:** Equity prices for companies and the volatility index (VIX).

Chapter 2

KMV Model

2.1 Introduction

The KMV model is a structural credit risk model that estimates the probability of default (PD) for a firm by assessing the value of its assets relative to its liabilities. It builds upon the Merton model by incorporating real-world data and practical assumptions, making it suitable for estimating creditworthiness dynamically.

In this project, we apply the KMV model to historical market data for 15 companies. For each company, we have time series data for the firm's equity value, short-term debt (STD), long-term debt (LTD), and other relevant financial indicators. We use this data to estimate the firm's asset value and asset volatility, and subsequently compute the distance to default and probability of default.

2.2 Assumptions

The KMV model is based on the following assumptions:

- A firm defaults when the value of its assets falls below a threshold, known as the default point (D).
- The asset value follows a geometric Brownian motion:

$$dV = \mu V \, dt + \sigma V \, dW \tag{2.1}$$

  where V is the firm's asset value, μ is the expected return on assets, σ is the asset volatility, and W is a Wiener process.
- Equity value represents a call option on the firm's assets, and the Black-Scholes formula is used for valuation.

2.3 Market Value of Assets and Asset Volatility Estimation

To implement the KMV model, we need the market value of the firm's assets (V) and its asset volatility (σ). We start with observed market values of equity (E) and the firm's capital structure (STD and LTD). The relationship between equity and assets in the

Merton framework is given by the Black-Scholes option pricing formula, treating equity as a call option on the assets:

$$E = V\Phi(d_1) - De^{-rT}\Phi(d_2)$$

where:

$$d_1 = \frac{\ln\left(\frac{V}{D}\right) + \left(r + \frac{\sigma^2}{2}\right)T}{\sigma\sqrt{T}} \quad \text{and} \quad d_2 = d_1 - \sigma\sqrt{T}$$

Here:

- E: Market value of equity
- D: Default point, often $D = \text{STD} + 0.5 \times \text{LTD}$
- V: Market value of assets (unknown)
- σ: Asset volatility (unknown)
- r: Risk-free rate
- T: Time horizon
- $\Phi(\cdot)$: Cumulative standard normal distribution

To find V and σ from observed E, D, and known parameters, we solve the system consisting of the equity equation and an additional condition relating equity volatility (σ_E) to asset volatility (σ):

$$\sigma_E = \frac{\partial E}{\partial V}\sigma = \Phi(d_1)\sigma\frac{V}{E}$$

This provides a second equation to solve for V and σ. In practice, σ_E can be observed from equity market data, and we treat V and σ as unknowns.

The estimation involves an iterative procedure:

1. Start with an initial guess for V and σ based on the firm's observed equity and leverage.
2. Compute E and σ_E from the guessed V and σ.
3. Compare the computed E and σ_E to the observed equity values and equity volatility from market data.
4. Update V and σ (e.g., using a numerical root-finding method like Newton-Raphson) until the computed values match the observed values within a specified tolerance.

This iterative process converges to a set of values for V and σ that are consistent with the observed market data. Performing this procedure for each of the 15 companies provides a time series of V and σ for each firm, enabling dynamic tracking of creditworthiness.

2.4 Key Formulas

2.4.1 Distance to Default (DD)

The distance to default is defined as:

$$DD = \frac{\ln\left(\frac{V}{D}\right) + \left(\mu - \frac{\sigma^2}{2}\right)T}{\sigma\sqrt{T}} \tag{2.2}$$

2.4.2 Probability of Default (PD)

The probability of default is derived from the standard normal distribution:

$$PD = \Phi(-DD) \tag{2.3}$$

2.5 Methodology

The KMV model is implemented in the following steps:

1. **Data Collection:** Obtain historical market data for 15 companies, including market value of equity, STD, LTD, and other financial indicators.

2. **Asset Value and Volatility Estimation:** Use the iterative procedure described in Section 2.3 to solve for V and σ.

3. **Default Point Calculation:** The default point (D) is calculated as STD + 0.5 $\times$ LTD.

4. **Distance to Default:** Compute the distance to default (DD) using the formula provided.

5. **Probability of Default:** Compute the probability of default (PD) using the cumulative normal distribution.

6. **Incorporation into HMM Model:** The DD and PD values are used as inputs for the Hidden Markov Model (HMM) to determine state transitions in credit quality.

2.6 Results

The results of our KMV model are shown below in comparison with the Bloomberg 1-year distance to default (DD). The Bloomberg model incorporates certain qualitative adjustments in their distance-to-default calculations, such as additional financial ratios like the Interest Coverage Ratio (ICR). In contrast, our approach remains purely statistical, relying solely on market and balance sheet data without incorporating additional factors. The trendline of our calculated distance to default follows closely with the Bloomberg distance to default, as illustrated in the figures below.

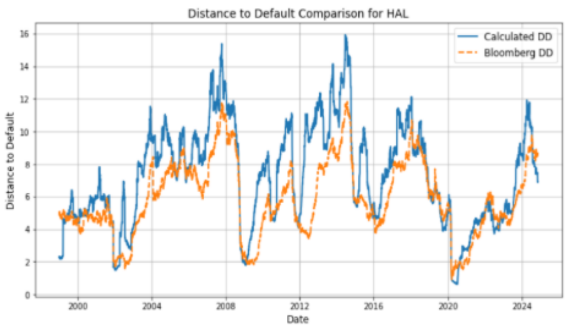

Figure 2.1: Distance to Default Comparison for HAL

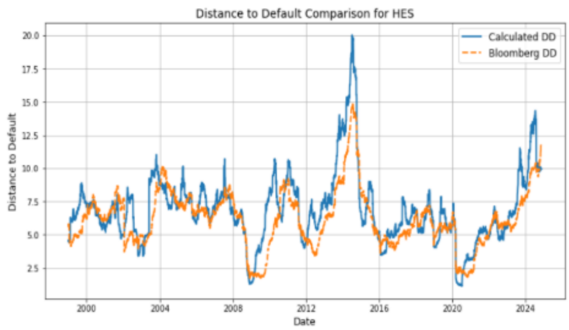

Figure 2.2: Distance to Default Comparison for HES

Chapter 3

Hidden Markov Model

3.1 Introduction

The Hidden Markov Model (HMM) is a statistical framework that models sequences of observations generated by underlying, unobservable states. In this project, the HMM dynamically estimates credit rating transitions for 15 companies using historical financial and market data. The hidden states correspond to credit ratings, while the observed variables include CDS spreads, inflation rates, treasury yields, and metrics derived from the KMV model.

3.2 Model Components

The HMM consists of the following key components:

- **Hidden States:** Represent 21 discrete credit rating categories.
- **Transition Probabilities:** Define the likelihood of transitioning from one credit state to another:

  $$P(S_t = j|S_{t-1} = i) = a_{ij}, \quad \sum_j a_{ij} = 1 \tag{3.1}$$

  where a_{ij} is the probability of transitioning from state i to state j, and the row sum of the transition matrix A must equal 1.

- **Emission Probabilities:** Specify the likelihood of observing data given a state:

  $$P(O_t|S_t = j) = b_j(O_t) \tag{3.2}$$

  where $b_j(O_t)$ is the emission probability of observation O_t given state j.

- **Initial State Probabilities:** The probability distribution over initial states:

  $$\pi_i = P(S_1 = i), \quad \sum_i \pi_i = 1 \tag{3.3}$$

3.3 Key Formulas

3.3.1 Transition Matrix

The transition matrix A captures probabilities of moving between states:

$$A = \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1N} \\ a_{21} & a_{22} & \cdots & a_{2N} \\ \vdots & \vdots & \ddots & \vdots \\ a_{N1} & a_{N2} & \cdots & a_{NN} \end{bmatrix}$$

where N is the number of hidden states, and each row represents the transition probabilities from a given state.

3.3.2 Emission Probabilities

Emission probabilities are modeled using Gaussian distributions:

$$b_j(O_t) = \frac{1}{\sqrt{2\pi\sigma_j^2}} \exp\left(-\frac{(O_t - \mu_j)^2}{2\sigma_j^2}\right) \tag{3.4}$$

where μ_j and σ_j are the mean and standard deviation of the observation in state j.

3.3.3 Observation Sequence Likelihood

The probability of an observation sequence $O = \{O_1, O_2, \ldots, O_T\}$ given model parameters $\lambda = (A, B, \pi)$:

$$P(O|\lambda) = \sum_S P(O, S|\lambda) \tag{3.5}$$

where S is the sequence of hidden states.

3.3.4 Viterbi Algorithm for State Prediction

The most probable sequence of hidden states S^*:

$$S^* = \arg\max_S P(S|O, \lambda) \tag{3.6}$$

3.4 Methodology

The implementation of the HMM involves three main steps:

1. **Data Preparation:** Observed variables (e.g., CDS spreads, inflation rates) are standardized, and missing values are handled using interpolation or historical averages.

2. **Model Training:** The Baum-Welch algorithm iteratively optimizes transition probabilities a_{ij} and emission probabilities $b_j(O_t)$. Transition probabilities are constrained to align with historical rating boundaries.

3. **State Prediction:** The Viterbi algorithm identifies the most probable sequence of credit states for each company, yielding a dynamic credit health trajectory.

3.5 Outputs and Applications

The HMM produces the following outputs:

- **Transition Matrix:** Captures the probabilities of moving between different credit states.
- **Emission Probabilities:** Provide insights into how observed data aligns with each hidden state.
- **Inferred States:** A sequence of most probable credit rating states for each company.

These outputs enable dynamic analysis of creditworthiness over time. The resulting credit state predictions and transition probabilities will be used to calculate credit scores in subsequent stages, informing the trading strategy.

3.6 Results

The evolution of predicted hidden credit states for multiple firms is illustrated in Figure 3.2. Each horizontal line represents one company, offset for clarity, with each state number reflecting a level of credit quality inferred by the HMM.

	Ticker	Min_State	Max_State
0	CPB	13.0	19.0
3	BWA	11.0	14.0
4	EMR	16.0	20.0
5	KEY	13.0	17.0
6	LMT	12.0	16.0
7	JNPR	13.0	13.0
8	HES	11.0	14.0
9	WELL	11.0	14.0
10	TSN	9.0	15.0
11	PARA	9.0	15.0
12	HAL	13.0	18.0
13	D	13.0	14.0
14	BAC	13.0	20.0
15	TXT	12.0	16.0
16	GILD	14.0	15.0
19	STX	10.0	13.0

Figure 3.1: Historical Minimum and maximum credit ratings.

The bounds shown in Figure 3.1 ensure that the model's predictions remain realistic, grounded in historically observed rating limits.

Historical constraints are provided by the min-max rating bounds, which maintain plausible state ranges and prevent unrealistic predictions. Market sensitivity is also demonstrated, as economic stress or periods of market volatility correspond to more frequent state changes, capturing how factors like CDS spreads, inflation, treasury yields, and distance-to-default influence perceived credit risk.

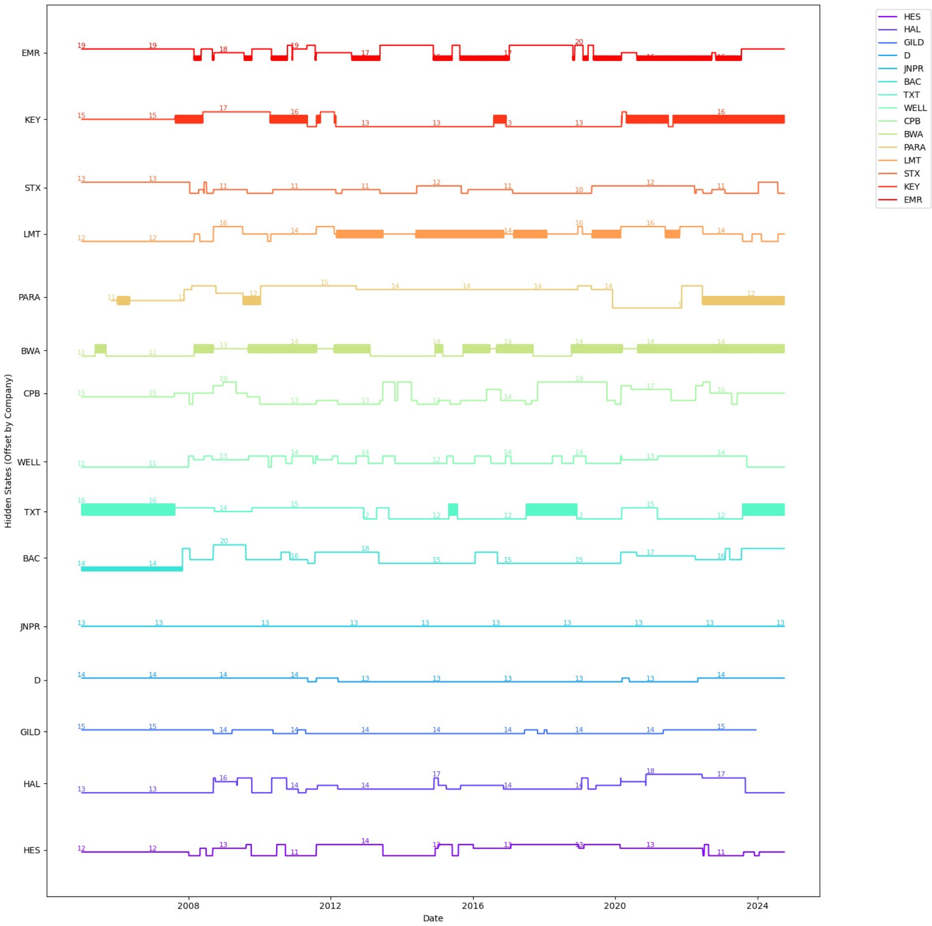

Figure 3.2: Predicted hidden credit states for multiple firms from 2008 to 2024.

Chapter 4

Scoring Framework

4.1 Financial Health Score

The Financial Health Score (FHS) is a quantitative measure of a company's overall financial stability, derived from key financial ratios. It serves as an input to the credit scoring framework, integrating various aspects of financial health into a single metric. Principal Component Analysis (PCA) is used to determine the weights for each factor.

4.1.1 Inputs

The following financial ratios are used as inputs:

- Interest Coverage Ratio: Measures a company's ability to pay interest expenses.
- Total Debt-to-Total Asset Ratio: Indicates leverage relative to total assets.
- Total Debt-to-Total Equity Ratio: Reflects financial leverage.
- Current Market Capitalization: Represents market valuation as a proxy for company size.

4.1.2 PCA to Derive Weights

The financial ratios are first standardized using Min-Max normalization:

$$x' = \frac{x - x_{\min}}{x_{\max} - x_{\min}} \tag{4.1}$$

where x' is the normalized value, x is the original value, and $x_{\min}$ and $x_{\max}$ are the minimum and maximum values of x.

PCA is applied to calculate factor weights:

$$w_i = \text{Eigenvector corresponding to the largest eigenvalue of the covariance matrix} \tag{4.2}$$

where w_i is the weight assigned to the i-th factor.

4.1.3 Financial Health Score Formula

The Financial Health Score is calculated as a weighted average of the standardized factors:

$$\text{FHS} = \sum_{i=1}^{n} w_i \cdot x_i' \tag{4.3}$$

where:

- w_i: Weight of the i-th factor.
- x_i': Normalized value of the i-th factor.

4.2 Credit Scoring

The credit scoring framework combines the Financial Health Score and the normalized outputs from the Hidden Markov Model (HMM). The Technique for Order Preference by Similarity to Ideal Solution (TOPSIS) is used to derive a comprehensive credit score.

4.2.1 Inputs

- **Financial Health Score (FHS):** Derived from financial ratios.
- **Normalized Hidden States:** Represents credit ratings as a continuous scale between 0 and 1, based on HMM outputs.

4.2.2 TOPSIS Methodology

The TOPSIS method identifies the positive ideal solution (PIS) and the negative ideal solution (NIS) based on the given criteria.

Normalization of Criteria

$$r_{ij} = \frac{x_{ij}}{\sqrt{\sum_{i=1}^{m} x_{ij}^2}} \tag{4.4}$$

where:

- r_{ij}: Normalized value of the i-th company for the j-th criterion.
- x_{ij}: Original value of the i-th company for the j-th criterion.

Weighted Normalized Matrix

$$v_{ij} = w_j \cdot r_{ij} \tag{4.5}$$

where:

- w_j: Weight of the j-th criterion (e.g., FHS, normalized hidden states).

Positive and Negative Ideal Solutions

$$v^{+} = \left(\max_{i} v_{ij}\right), \quad v^{-} = \left(\min_{i} v_{ij}\right) \tag{4.6}$$

where v^{+} and v^{-} represent the PIS and NIS, respectively.

Separation Measures

$$S_i^{+} = \sqrt{\sum_{j=1}^{n} \left(v_{ij} - v_j^{+}\right)^2}, \quad S_i^{-} = \sqrt{\sum_{j=1}^{n} \left(v_{ij} - v_j^{-}\right)^2} \tag{4.7}$$

where:

- S_i^{+}: Separation from the positive ideal solution.
- S_i^{-}: Separation from the negative ideal solution.

Relative Closeness to the Ideal Solution

$$C_i = \frac{S_i^{-}}{S_i^{+} + S_i^{-}} \tag{4.8}$$

where C_i is the credit score for the i-th company, ranging between 0 and 1.

4.2.3 Final Credit Score

The final credit score is scaled to a range of 0–800:

$$\text{Credit Score} = 800 \cdot C_i \tag{4.9}$$

Chapter 5

Hedged Trading Strategy Development

This chapter presents the development of a hedged trading strategy combining Credit Default Swaps (CDS) and equities. The objective is to leverage the complementary nature of these instruments for robust, risk-adjusted returns. Equities are chosen over bonds due to their higher liquidity, volatility, and faster response to credit events. While CDS reflect direct credit risk, equities incorporate broader market sentiment and financial health, creating a well-rounded approach.

The strategy employs two key inputs:

- **Credit Scores:** Derived from the HMM, reflecting company-specific creditworthiness.
- **CBOE Volatility Index (VIX):** Capturing systemic market volatility.

The output is a Long/Short signal for CDS and an opposite equity position for hedging. Signals use a 45-day lookback and are executed over a 30-day holding period, with monthly rebalancing. Signals are normalized for comparability, and various weight combinations were tested. The optimal weights of 0.5 for Credit Scores and -0.5 for VIX balanced idiosyncratic and systemic risks effectively.

5.1 Results

Three portfolio construction methods were tested:

1. **Equally Weighted Portfolio:** Sharpe Ratio = 0.9.
2. **Market Cap-Weighted Portfolio:** Sharpe Ratio = 0.6.
3. **Mean-Variance Optimized (MVO) Portfolio:** Sharpe Ratio = 1.2.

The chosen parameters (0.5/-0.5 weights, 45-day lookback, 30-day holding) consistently delivered the highest risk-adjusted returns. The MVO portfolio outperformed, demonstrating that combining CDS and equities with informed weighting schemes can enhance performance.

5.2 Key Insights

- **Comprehensive Risk Coverage:** Credit Scores for firm-specific risk, VIX for market-wide volatility.
- **Balanced Signal:** Credit Scores offer stable, long-term insights; VIX reacts quickly to shocks.

5.3 Conclusion

This strategy's effectiveness lies in integrating CDS and equities with Credit Scores and VIX. By balancing idiosyncratic and systemic risks, it achieves superior performance, especially in the MVO portfolio with a Sharpe Ratio of 1.2. This underscores the potential of data-driven frameworks in optimizing returns while managing risk.

Figure 5.1: Cumulative Return of the CDS Strategy Over the Sample Period

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