



# ADITYA DEGREE COLLEGES

ANDHRA PRADESH

IV SEMESTER - PREFINAL EXAMINATIONS

II B.Sc - MATHEMATICS

Max. Marks : 75

Time : 3 Hours

Date:

## SECTION - A

I. Answer any FIVE questions from the following

5X5= 25M

1. State and prove Sandwich theorem
2. Test for convergence  $\sum_{n=1}^{\infty} \left( \sqrt[3]{n^3 + 1} - n \right)$
3. Test for convergent  $\sum \frac{x^n}{x^n + a^n} (x > 0, a > 0)$
4. Examine the continuity of  $f(x) = |x| + |x-1|$  at  $x=0, 1$
5. Prove that  $\tan x > x > \sin x \forall x \in \left( 0, \frac{\pi}{2} \right)$
6. Verify Cauchy's mean value theorem for  $f(x) = \sqrt{x}$  and  $g(x) = 1/\sqrt{x}$  in  $[a, b]$  where  $0 < a < b$
7. If  $f(x) = x^2$  on  $[0, 1]$  &  $P = \left\{ 0, \frac{1}{3}, \frac{2}{3}, 1 \right\}$  find  $U(P, f)$  &  $L(P, f)$ .
8. Show that every monotonic on  $[a, b]$  then  $f$  is integrable on  $[a, b]$

## SECTION - B

II. Answer ALL Questions

5X10=50M

9. a) State and prove Cauchy's general principle for convergence.

(OR)

- b) State and prove monotone converge theorem

10. a) State and prove Ratio test.

(OR)

- b) State and prove leibnitz's Test and prove that  $1 - \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{3}} - \frac{1}{\sqrt{4}} + \dots + \frac{(-1)^{n-1}}{\sqrt{n}} + \dots$  converges

11. a) State and prove absolute maximum minimum theorem.

(OR)

(i) Every continuous function on  $[a, b]$  is uniformly continuous on  $[a, b]$

(ii) Discuss the types of discontinuities

12. a) State and prove Rolle's theorem.

(OR)

b) (i) Show that  $f(x) = x^2 \cos(1/x)$ ,  $x \neq 0$ ;  $f(x) = 0, x = 0$  is derivable everywhere but the derivative is not continuous at 0

(ii) Show that

for  $0 < u < v$ .

13. a) (i) If  $f \in R[a, b]$  and  $\phi$  is a primitive of  $f$ , then  $\int_a^b f(x) dx = \phi(b) - \phi(a)$

(ii) Prove that  $\frac{1}{\pi} \leq \int_0^1 \frac{\sin \pi x}{1+x^2} dx \leq \frac{2}{\pi}$ .

(OR)

b) State and prove a necessary and sufficient condition for R-integrable on  $[a, b]$ .

$$\frac{\phi - u}{1 + v^2} < \tan^{-1} v - \tan^{-1} u < \frac{v - u}{1 + u^2}$$