Aditya Group of Degree Colleges

ADITYA DEGREE COLLEGES

* ANDHRA PRADESH *

PRE-FINAL - EXAMINATIONS

III B.SC :: *MATHEMATICS*

SPECIAL FUNCTIONS

Date: 24.03.2020

Max. Marks : 75 M
Time: 3hrs

SECTION-A

I. Answer any FIVE of the following questions: **5 x 5 = 25 M**

1. Prove that $H_n^{II}(x) = 4n(n-1)H_{n-2}(x)$
2. Find the first five Hermite polynomials.
3. Prove that $xL_n^{II}(x) + (1-x)L_n^1(x) + nL_n(x) = 0$
4. Show that $L_n(x) = \dfrac{e^x}{n!}\dfrac{d^n}{dx^n}\left(x^n e^{-x}\right)$
5. Prove that
6. Show that $P_n(-x) = (-1)^n P_n(x)$
7. Prove that (i) $J_0^1 = -J_1$ (ii) $J_2 - J_0 = 2J_0^{11}$
8. Evaluate $\int_0^1 x^4(1-x)^2\,dx$

$$(2n+1)P_n = P_{n+1}^1 - P_{n-1}^1$$

SECTION-B

II. Answer the following questions: **5 x 10 = 50 M**

9. a) Prove that $e^{2tx-t^2} = \sum_{n=0}^{\infty} \dfrac{t^n}{n!} H_n(x)$

(Or)

b) Prove that $H_n(x) = 2^n \left\{ \exp\left(\dfrac{-1}{4}\dfrac{d^2}{dx^2} \right) x^n \right\}$

10. a) Prove that $\dfrac{1}{1-t} e^{\frac{-tx}{1-t}} = \sum_{n=0}^{\infty} t^n L_n(x)$

(Or)

b) Prove that $\int_0^{\infty} e^{-x} L_n(x) L_m(x)\,dx = \begin{cases} 0 & if \;\; m \neq n \\ 1 & if \;\; m = n \end{cases}$

11. a) Prove that $(2n+1)xP_n(x)=(n+1)P_{n+1}(x)+nP_{n-1}(x)$

(Or)

b) Show that $p_n(x)$ is the coefficient of h^n in the expansion in ascending powers of $(1-2xh+h^2)^{\frac{-1}{2}}$

12. a) Prove that $x\,J_n^1(x)=nJ_n(x)-x\,J_{n+1}(x)$

(Or)

b) Show that (i) $J_{-n}(x)=(-1)^n J_n(x)$ when n is a +ve integer

(ii) $J_n(-x)=(-1)^n J_n(x)$ when n is a -ve integer

13.a) Evaluate (i) $\int_0^\infty x^2 e^{-x^4}dx$ (ii) $\int_0^2 \frac{x^2}{\sqrt{2-x}}dx$

(Or)

b) Show that $\beta(m,n)=\frac{\Gamma(m).\Gamma(n)}{\Gamma(m+n)}$