ADITYA DEGREE COLLEGES

ANDHRA PRADESH

II SEMESTER - PREFINAL EXAMINATIONS

I B.Sc - MATHEMATICS

Date:

Max. Marks : 75
Time : 3 Hours

SECTION - A

I. Answer any FIVE questions from the following **5X5= 25M**

1. A variable plane is at a constant distance 'p' from the origin and meets the axes in A, B, C. Show that the locus of the centroid of the tetrahedron OABC is $x^{-2}+y^{-2}+z^{-2}=16p^{-2}$

2. Find the equation of the plane through the intersection of the planes $x+y+z=1$, $2x+3y+4z=5$ and perpendicular to the plane $x-y+z=0$.

3. Find the image of the line $\frac{x-1}{2}=\frac{y-2}{2}=\frac{z-3}{4}$ in the plane $x+y+z=1$.

4. Find the equations of the line intersecting the lines $2x+y-1=0=x-2y+3z$; and is parallel to the line $\frac{x-1}{1}=\frac{y-2}{2}=\frac{z-3}{3}$.

5. A sphere of constant radius k passes through the origin and intersects the axes in A, B, C. Prove that the centroid of the ΔABC lies on the sphere $9(x^2+y^2+z^2)=4k^2$.

6. Find the equations of the spheres passing through the circle $x^2+y^2=4$, $z=0$ and is intersected by the plane $x+2y+2z=0$ in a circle of radius 3.

7. Find the enveloping cone of the sphere $x^2+y^2+z^2+2x-2y=2$, with its vertex at (1, 1, 1).

8. Show that if a right circular cone has set of three mutually perpendicular generators, its semi vertical angle must be $\tan^{-1}\sqrt{2}$

SECTION - B

II. Answer ALL Questions **5X10=50M**

9. a) Find the bisecting plane of the acute angle between the planes $3x-2y-6z+2=0$, $-2x+y-2z-2=0$.

(OR)

b) Prove that the equation $2x^2-6y^2-12z^2+18yz+2zx+xy=0$ represents a pair of planes and find the angle between them.

10. a) Prove that the lines $\frac{x-1}{2}=\frac{y-2}{3}=\frac{z-3}{4}$, $\frac{x-2}{3}=\frac{y-3}{4}=\frac{z-4}{5}$ are coplanar. Also find their point of intersection and the plane containing the lines.

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(OR)

b) Find the S.D between the lines $\frac{x-3}{3}=\frac{y-8}{-1}=\frac{z-3}{1}$, $\frac{x+3}{-3}=\frac{y+7}{2}=\frac{z-6}{4}$. Find also the equations and the points in which the S.D meets the given lines.

11. a) Show that the four points (-8, 5, 2), (-5, 2, 2), (-7, 6, 6), (-4, 3, 6) are concyclic.

(OR)

b) Show that the two circles $x^2+y^2+z^2-y+2z=0$, $x-y+z=2$, $x^2+y^2+z^2+x-3y+z-5=0$, $2x-y+4z-1=0$ lie on the same sphere and find its equation.

12. a) Find the limiting pointsof the coaxal system defined by spheres $x^2+y^2+z^2+4x-2y+2z+6=0$ and $x^2+y^2+z^2+2x-4y-2z+6=0$.

(OR)

b) (i) Prove that if the angle between the lines of Intersection of the plane $x+y+z=0$ and the cone $ayz+bzx+cxy=0$ is $\frac{\pi}{2}$ then .

(ii) If $ax^2+by^2+cz^2+2ux+2vy+2wz+d=0$ represents a cone prove that $\frac{u^2}{a}+\frac{v^2}{b}+\frac{w^2}{c}=d$ $a+b+c=0$

13. a) Prove that the equation $\sqrt{fx}\pm\sqrt{gy}\pm\sqrt{hz}=0$ represents a cone that touches the coordinate planes and find its reciprocal cone.

(OR)

b) (i) Find the equation to the cone which passes through the three coordinate axes as well as the three lines $\frac{1}{2}x=y=-z, x=\frac{1}{3}y=\frac{1}{5}z$ *and* $\frac{1}{8}x=-\frac{1}{11}y=\frac{1}{5}z$.

(ii) Find the equation of the cone whose vertex is (1,1,0) and whose guiding curve is y=0, $x^2+z^2=4$

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