Question 1 – Linear Programming and variants [6 + 6 + 4 + 3 + 3 + 3 + 3 + 4 + 2 + 2 +

3 + 3 + 3 + 3 + 3 + 3 + 3 + 3 = 60 marks]

We use resources to make products. Consider 6 such resources and 5 such products. The various resources that we use might include (e.g.) linen, elastic, plastic, foam, etc. The various products that we make might include (e.g.) soap, sanitiser, washable masks, disposable masks, filters, shields, other personal protection equipment (PPE), etc. (Alternatively, the products might possibly be various graphical processing units - or GPUs - and the various resources might possiblybe Solder, Copper wire, Plastic, Aluminium, Bearings, Die size.) We show below the profit of each product, the number of each resource required to make each product, and the total availability of each resource. Unless we are explicitly told that a variable is integer-valued (or otherwise discrete-valued or binary, etc), it will probably be safer not to make such an assumption and rather instead allow the variable to be continuous-valued. (Note that sub-questions such as part 1m will occur later.) If unsure, clearly state and justify any assumptions. Please state such continuous values to at least three decimal places.

	Product 1	*Product 2*	*Product 3*	*Product 4*	*Product 5*	
Profit of Product	$510	$300	$510	$270	$810	
						Resource Availability
Resource 1	2	10	2	3	6	2487
Resource 2	6	3	6	3	10	3030
Resource 3	2	3	10	6	2	5217
Resource 4	7	6	5	4	3	4000
Resource 5	5	6	3	10	2	4999
Resource 6	10	3	5	3	4	2769

We wish to produce products - given constraints - so as to optimise our objective function.

Bearing in mind the introductory material above, the questions follow below:

1a) Formulate a Linear Programming (an LP) model for this problem.

1b) Create a Microsoft Excel spreadsheet model for this problem. Store the model in your Excel workbook (6 marks)

1c) Solve the problem - using Microsoft Excel Solver. Generate the Sensitivity report for the problem (4 marks)

1d) What is the optimal production plan (X 1 , X 2 , X 3 , X 4 , X 5) and the associated profit? Refer to your answers to any of a), b) and/or c) above as appropriate. (3 marks)

1e) Which constraints, if any, are binding? Refer to your answers to any of the above parts as appropriate, and explain your reasoning. (3 marks)

1f) The people running the company are now offered the opportunity of an exchange of goods. The offer is for the company to receive 1 of Resource 2, 10 of Resource 4 and 100 of Resource 5 but for the company to have to relinquish (or surrender, or give away. or pay for these resources with) 10 of Resource 1, 5 of Resource 3 and 3 of Resource 6. Should the company accept this offer?

Clearly explain with clear calculations (to at least 3 decimal places) how much money the company would gain or lose by agreeing to such an exchange, making it clear whether this would result in a gain or a loss.

Let us return to the original problem above (prior to the company being made an offer) from part d.

A proposal is put forward to produce a new product called Product 6. Product 6 would have a profit of $155 and would require the following resources: 2 of Resource 2, 4 of Resource 4 and 5 of Resource 5.

1g) Would we expect Product 6 to be produced - i.e., if we are to produce products to optimise our objective function, would we produce any copies of this new product? If we would expect Product 6 to be produced, then how much less profitable could Product 6 be and still be produced? If we would not expect Product 6 to be produced, then how much more profitable would Product 6 need to be in order to be produced?

Let us again return to the original problem above from part d, where the profitability of the various products was (510, 300, 510, 270, 810). Various employees at the company have considered making changes which would affect the profitability of various products. One change would result in (512, 301, 511, 269, 811).

1h) Explaining your reasoning, when compared to your original answer using (510, 300, 510, 270, 810), would your optimal amount to be produced of each of Product1, ..., Product5 change? Explain clearly why or why not. And, if the amounts produced would change, explain clearly with any necessary or relevant calculations what they would change to.

1i) A second change, if it really could be carried out in practice, would double the values to become (1020, 600, 1020, 540, 1620). Explaining your reasoning, when compared to your original answer using (510, 300, 510, 270, 810), would your optimal amount to be produced of each of Product1, ..., Product5 change? Explain clearly why or why not. And, if the amounts produced would change, explain clearly with any necessary or relevant calculations what they would change to.

1j) A third change, which would probably not be a good idea, would halve the values to become (255, 150, 255, 135, 405). Nonetheless, if such a change were to take place then, explaining your reasoning, when compared to your original answer using (510, 300, 510, 270, 810), would your optimal amount to be produced of each of Product1, ..., Product5 change? Explain clearly why or why not. And, if the amounts produced would change, explain clearly with any necessary or relevant calculations what they would change to.

1k) Returning to the original problem and solution from part d, suppose we now introduce the requirement that Product1, Product2 and Product 5 must be produced in equal amounts. Compared to the original feasible region (from part d), does adding this new requirement make the feasible region larger, stay the same, smaller, or something else? Clearly explain your answer.

1l part 1) Continuing from part k with this newly introduced requirement that Product1, Product2 and Product 5 must be produced in equal amounts, what is the optimal amount to be produced of each of Product1, Product2, Product3, Product4, Product5?

11 part 2) What is the resultant profit (stated to at least 3 decimal places)? For both part 1 and part 2 of 1l, (in keeping with **note 4** ,) clearly show all working. Returning to the original problem from part d, suppose we now introduce the additional requirement that Product1, Product2, Product 3, Product 4 and Product 5 must be produced in integer amounts.

1m part 1) What is the optimal amount to be produced of each of Product1, Product2, Product3, Product4, Product5?

1m part 2) What is the optimal value of the objective function?

1n) Continuing on from part m above, assume the same unit profits as before but now with fixed start-up costs as given below.

Product	Product 1	Product 2	Product 3	Product 4	Product 5
Unit Profit	\$510	\$300	\$510	\$270	\$810
Fixed-cost (Start-up cost)	2000	4000	8000	16000	1000

1n part 1) What is the optimal amount to be produced of each of Product1, Product2, Product3, Product4, Product5?

1n part 2) What is the optimal value of the objective function?

Return to part n above.

Suppose we now impose the additional constraint that, if Product3 is produced, then there must be a minimum of 225 and a maximum of 325 of Product3 produced.

1o part 1) What is the optimal amount to be produced of each of Product1, Product2, Product3, Product4, Product5?

1o part 2) What is the optimal value of the objective function?

Return to part n above.

Suppose we now change the additional constraint from part 1o (immediately above) to be that, if Product3 is produced, then there must be a minimum of 300 and a maximum of 450 Product 3 produced, and (also) the amount produced of Product3 must also be a multiple of 50.

1p part 1) What is the optimal amount to be produced of each of Product1, Product2, Product3, Product4, Product5?

1p part 2) What is the optimal value of the objective function?

Return to part n above.

Now suppose that we introduce the requirement that, if Product 2 is produced, then the amount of Product 2 produced must be one of 102, 103, 105, 107, 111 and we also introduce the further additional requirement that, if Product 4 is produced, then the amount produced of Product4 must be one of 320, 330, 350, 370, 410.

1q part 1) What is the optimal amount to be produced of each of Product1, Product2, Product3, Product4, Product5?

1q part 2) What is the optimal value of the objective function?

Remove these most recent additional constraints and again return to part n above. Suppose we now add the requirements that Product1 and Product2 are produced in equal abundance, Product4 and Product5 are produced in equal abundance, if Product 3 is produced then neither product 1 nor product 2 is produced, if Product 3 is produced then at least 10 and at most 100 of Product 5 are produced.

1r part 1) What is the optimal amount to be produced of each of Product1, Product2, Product3, Product4, Product5?

1r part 2) What is the optimal value of the objective function?

Question 2 – Transhipment and networks [2 + 3 + 6 + 4 + 3 + 3 + 3 + 3 + 3 + 2 =

32 marks]

Suppose we have a product (possibly masks, possibly shields, possibly containers of hand sanitiser) that we wish to move from two locations (let’s call them node 1 and node 2, both with a supply of 75) to two other locations (let’s call them node 7 and node 8, with demands of 80 and 70 respectively).

We initially assume the transportation costs along edges in the network to be as follows:

From	To	Unit Cost ($)
1	3	50
1	4	80
2	3	70
2	4	40
3	5	70
3	6	50
4	5	40
4	6	80
5	7	80
5	8	40
6	7	60
6	8	70

Students are expected and required to address question 2 in terms of linear programming (LP) and - if required - the closest possible variants.

2a) State the variables, and use these variables to state the objective function that we wish to optimise. (We assume that the cost is something that we wish to minimise.)

2b) How many variables are there? Informally in terms of the network, being as specific as you can, what do the variables correspond to?

2c) Solve the problem of the flow along edges giving the minimum cost. Show the amounts of flow along the edges. State the value of the objective function. State the number of edges with non-zero flow (and, for ease of reference, call this *e 2c*).

2d) Assuming that the number of edges with non-zero flow is less than e 2c (equivalently, less than or equal to *e 2c - 1*), again solve the problem of the flow along edges giving the minimum cost. Show the amounts of flow along the edges. State the value of the objective function. State the number of edges with non-zero flow.

2e) If the problem is to have a solution of finite cost (any possible solution at all) in which goods get from the source/supply/starting points to the demand/sink destination points, what is the smallest number of edges that can have non-zero flow for such a solution to occur? *Hint* : One way of doing this is to introduce a very large penalty for each edge with non-zero flow.In that case (if we require that only this smallest possible number of edges be used), what is the minimum such cost? (If you followed the *hint* immediately above, then make sure to remove the newly introduced large penalty when giving your answer.)

2f) Return to the problem from parts a, b and c above.

Due to maintenance problems along the edge between node 4 and node 5, the unit cost of using this edge is $40/unit up to 30 units, then $60/unit thereafter. Show how to solve this problem. In keeping with **note 4** , solve this problem.

2g) Following on from part f above, due to further maintenance problems along the edge between node 4 and node 5, the unit cost of using this edge is $40/unit up to 30 units, then $60/unit up to 55 units (i.e., we could have 30 units @ $40/unit and 25 units @ $60/unit, as 30 + 25 = 55), then $110/unit thereafter.

2h) We modify the original problem from parts a, b and c to be a shortest path problem. The edge costs (from parts a, b and c) should now be assumed to be the length of the edge. What is the shortest path from node 2 to node 8, and what is the length of the path? Show how to solve this problem. In keeping with **note 4** , solve this problem.

2i part 1) Following on from part h, how would you modify your answer if we require that the path from node 2 to node 8 has to go through node 5?

2i part 2) Following on from part h, how would you modify your answer if we require that the path from node 2 to node 8 has to go through node 6?

Following on from the themes of part h and part i above, we now ask an open question worth bonus marks. (The motivation might be that someone has to collect face masks and shields on their way to a destination, but the order in which they collect them doesn't matter.)

2j) Suppose we have a start node (call it A), and a destination node (call it D) and two intermediate nodes (call them B and C respectively) that we have to go through. Suppose also that we are allowed to go A to B to C to D and we are also allowed to go A to C to B to D, and that this is not known or specified in advance. How might we set this up as a linear programming (LP) problem? We do not require a complete solution for 2j immediately above but wish you to explain in detail how you would set this up.

Question 3 – Economic Order Quantity (EOQ) [10 marks]

Suppose that we have an ordering problem with variable costs. We have a deterministic annual demand of 1000. The cost of placing an order (of any positive non-zero amount) is $21 for an order. The holding cost of storing items is 25% (or 1/4) per annum of the cost of the goods. A s many goods as required can be held in inventory indefinitely and not be thrown away. The cost of each good is $4.00 up to 794 units ordered. If we order from 795 up to 1099, we get a 5% discount and the cost of each good is $3.80. If we order from 1100 up to 1859, we get an 8% discount and the cost of each good is $3.68. If we order 1860 or more, we get a 15% discount and the cost of each good is $3.40. What is the optimal order quantity and the optimal total annual cost?