Question 1a

Let a, b, c, d, e be the number of products for each product P1, P2, P3, P4, P5 respectively.

Optimization Function: Maximize Profits
$510a + 300b + 510c + 270d + 810e$

Constraints:
$2a + 10b + 2c + 3d + 6e <= 2487$
$6a + 3b + 6c + 3d + 10e <= 3030$
$2a + 3b + 10c + 6d + 2e <= 5217$
$7a + 6b + 5c + 4d + 3e <= 4000$
$5a + 6b + 3c + 10d + 2e <= 4999$
$10a + 3b + 5c + 3d + 4e <= 2769$
$a>=0$
$b>=0$
$c>=0$
$d>=0$
$e>=0$

Question 1d

The final value under variable cells heading from the sensitivity report (Qu1c Sensitivity Rep) suggests that the maximum profit can be obtained from the combination of the number of products a=3.999, b=83, c=277, d=365 and e=0. The optimised profit value obtained from the solver is $266,760 (Working shown in LotsOfProducts 1b).

From Qu1c Sensitivity Rep, we can deduct that as part of optimal solution, the number of product 5 production doesn't contribute to optimal solution as long as the profit for product 5 is increased up to $858.339 or decreased to infinity from its current value $810.

Question 1e

As per the Qu1e Answer Rep in spreadsheet, except for resource 4, all other resources are binding. Also, using Qu1b Sensitivity Rep in spreadsheet, we can see that all other resources except for resource 4 have been used up to their maximum limit of availability. Even though the number for resource 4 used has a room until its limit, increasing the number for resource 4 serves no purpose in the optimal solution in this case.

Question 1f

Below table is created using Qu1c Sensitivity Rep (allowable increase and decrease under constraints table)

Resource	Current Available Value	Exchange Value	New Available Value	Allowable Increased Available Value	Allowable Decreased Available Value
R1	2487.000	-10	2477.000	2592.302	2487.000
R2	3030.000	1	3031.000	3030.000	3009.994
R3	5217.000	-5	5212.000	5250.541	5217.000
R4	4000.000	10	4010.000	1E+30	3371.000
R5	4999.000	100	5099.000	4999.000	4824.846
R6	2769.000	-3	2766.000	1E+30	2769.000

Based on the above table, we see:

1. For R1, the new available value is not within the allowable increase and decrease range.
2. For R2, the new available value is not within the allowable increase and decrease range.
3. For R3, the new available value is not within the allowable increase and decrease range.
4. For R4, the new available value is **within** the allowable increase and decrease range.
5. For R5, the new available value is not within the allowable increase and decrease range.
6. For R6, the new available value is not within the allowable increase and decrease range.

As we can see that except for R4, new available value for all other resources fall outside the allowable range. Therefore, these changes will impact the optimal solution.

Upon solving the problem with new constraints, the maximum profit we get using solver is $266,759.032 and the resultant difference is **loss** of **$0.968**. So, strictly speaking, the company shouldn't accept this offer as it ends up in loss. The working is shown in LotsOfProducts 1f.

Question 1g

According to spreadsheet tab 'LotsOfProducts 1g', we see that 0 quantity of Product 6 was produced as we obtained the optimum profit value of same as the previous optimum profit value of $266,760. Using 'Qu1g Sensitivity Rep', we see that the allowable increase is 13.869 while allowable decrease is 1E+30. Hence, for product 6 to be produced, it should be able to make a profit of more than **$168.869** (Current Profit + Allowable Increase).

Question 1h

Product	Original Profit Value	Allowable Increase	Allowable Decrease	New Profit Value	Absolute Change Value	Increase/ Decrease	% Change
1	510.000	1.333	27.692	512.000	2	Increase	150.000%
2	300.000	22.500	38.143	301.000	1	Increase	4.444%
3	510.000	8.780	1.488	511.000	1	Increase	11.389%
4	270.000	48.396	5.000	269.000	1	Decrease	20.000%
5	810.000	48.339	1E+30	811.000	1	Increase	2.069%
Sum of % Change							**187.902%**

Above table is created based on Qu1c Sensitivity Rep. The overall change accounts to 187.902%. In particular, the new profit value for product 1 has gone beyond the break even point and hence, this change gives an opportunity to produce more of this product. However, this is violating the 100% rule, the overall optimal solution will change. Upon solving this question with updated profit values, the quantity for product 1 changes from 3.99 to 4 and the optimal profit value is $266,763 (There's a gain of $3). Working is shown in LosOfProducts 1h.

Question 1i

Here, keeping everything else the same, all the profit values are simply doubled. In such cases, the objective function value (optimised profit) will get doubled but the solution (quantity of the products produced) remains the same as the level curve passes through the same solution point/s. The new optimised profit value is $533,520. Working is shown in LotsOfProducts 1i.

Question 1j

Similar to the previous question, keeping everything else the same, all the profit values are simply halved. In such cases, the objective function value (optimised profit) will get halved but the solution (quantity of the products produced) remains the same as the level curve passes through the same solution point/s. The new optimised profit value is $133,380. Working is shown in LotsOfProducts 1j.

Question 1k

Adding a new constraint results into two things – original solution satisfies the constraint or it does not. Adding the new requirement/constraint makes the problem harder to satisfy, so we can't get a better optimal solution than we got earlier. Hence, in this case, adding a new requirement has made the feasible region smaller which can be clearly seen in LotsOfProducts 1k. The new optimal profit value is $263,686.841 whereas original profit value was $266,760.000. The new optimal profit value has dropped down by $3,073.159 resulting into smaller feasibility region.

Question 1l part 1

The optimal number of products to be produced as per added constraint from question 1k are as below (Working is shown in LotsOfProducts 1l):

Products	Product 1	Product 2	Product 3	Product 4	Product 5
# Of Products	9.685	9.685	271.384	405.894	9.685

Question 1l part 2

The resultant optimal profit value is $263,686.841. Working is shown in LotsOfProducts 1l.

Question 1m part 1

The optimal number of products to be produced as per added integer constraint are as below (Working is shown in LotsOfProducts 1m):

Products	Product 1	Product 2	Product 3	Product 4	Product 5
# Of Products	4.000	83.000	277.000	365.000	0.000

Question 1m part 2

The resultant optimal profit value is $266,760.000. Working is shown in LotsOfProducts 1m.

Question 1n part1

The optimal number of products to be produced as per added fixed cost are as below (Working is shown in LotsOfProducts 1n):

Products	Product 1	Product 2	Product 3	Product 4	Product 5
# Of Products	0	164	423	0	0

Question 1n part 2

The resultant optimal profit value is $252,930. Working is shown in LotsOfProducts 1n.

Question 1o part 1

The optimal number of products to be produced as per added constraints on the number of products of type product 3 are as below (Working is shown in LotsOfProducts 1o):

Products	Product 1	Product 2	Product 3	Product 4	Product 5
# Of Products	177	116	0	0	161

The solver doesn't consider that P3 should be produced as it has found an optimal profit value without producing P3.

Question 1o part 2

The resultant optimal profit value is $248,480. Working is shown in LotsOfProducts 1o.

Question 1p part 1

The optimal number of products to be produced as per added constraint on the number of products of type product 3 are as below (Working is shown in LotsOfProducts 1p):

Products	Product 1	Product 2	Product 3	Product 4	Product 5
# Of Products	177	116	0	0	162

Question 1p part 2

The resultant optimal profit value is $249,290.000 (Working is shown in LotsOfProducts 1p).

Question 1q part 1

The optimal number of products to be produced as per added constraint on the number of products of type product 2 and 4 are as below (Working is shown in LotsOfProducts 1q):

Products	Product 1	Product 2	Product 3	Product 4	Product 5
# Of Products	0	0	505	0	0

Except for production of P3, production of other products doesn't result into optimal profit value.

Question 1q part 2

The resultant optimal profit value is $249,550.000 (Working is shown in LotsOfProducts 1q).

Question 1r part 1

The optimal number of products to be produced as per mentioned constraints are as below (Working is shown in LotsOfProducts 1r):

Products	Product 1	Product 2	Product 3	Product 4	Product 5
# Of Products	0	0	479	12	12

Question 1r part 2

The resultant optimal profit value is $232,250.000 (Working is shown in LotsOfProducts 1r).

Question 2a and 2b

There are 12 variables:

Variable	Description
P13	Amount shipped from node 1 to node 3.
P14	Amount shipped from node 1 to node 4.
P23	Amount shipped from node 2 to node 3.
P24	Amount shipped from node 2 to node 4.
P35	Amount shipped from node 3 to node 5.
P36	Amount shipped from node 3 to node 6.
P45	Amount shipped from node 4 to node 5.
P46	Amount shipped from node 4 to node 6.
P57	Amount shipped from node 5 to node 7.
P58	Amount shipped from node 5 to node 8.
P67	Amount shipped from node 6 to node 7.
P68	Amount shipped from node 6 to node 8.

Objective function (Minimize the cost):
50P13 + 80P14 + 70P23 + 40P24 + 70P35 + 50P36 + 40P45 + 80P46 + 80P57 + 40P58 + 60P67 + 70P68

Question 2c

Below is the flow in between nodes:

Ship	From	To
75	1	3
0	1	4
0	2	3
75	2	4
0	3	5
75	3	6
75	4	5
0	4	6
5	5	7
70	5	8
75	6	7
0	6	8

The optimal transportation cost is $21,200.000. The number of edges with non-zero flow is 7. Working is shown in T&N 2c.

Question 2f

Considering the maintenance issues, I added a new row with the shipment value, nodes and cost associated with it. After this, I added a constraint for first route 4-5 that shipment value must be less than or equal to 30. There is no need to add a constraint of the new row for route 4-5 because its value can be 0 or anything greater than 0. It will be taken care by solver based on the previous constraints added. Working is shown in T&N 2f.

Question 2g

Considering the maintenance issues, I removed the rows associated with route 4-5 and added 3 new rows with the shipment value, nodes (4-5) and cost associated with it. After this, I added a constraint for first route 4-5 that shipment value must be less than or equal to 30 and other constraint for the second 4-5 route that shipment value must be less than or equal to 25. There is no need to add a constraint of the last row for route 4-5 because its value can be 0 or anything greater than 0. It will be taken care by solver based on the previous constraints added. Working is shown in T&N 2g.

Question 2h

I updated the table values such that I removed all the paths except for the ones that connect from node 2 to node 8. Since, we are finding shortest path, the supply/demand at any given node will be 0, -1 or 1. Since supply is equal to demand in this case, we equate the net flow to the supply/demand (-1 from node 2 and +1 at node 8 cancel out). Using the solver and setting the constraints, we get the shortest path that is nothing but the sum product of the selection values (either 0 or 1) and cost associated with that route i.e. distance in this case. The shortest path as per solver is 120. Path followed is 2-4-5-8. Working is shown in T&N 2h.

Question 2i part 1

Keeping the previous setting same, we add a new constraint such that the sum of the selection value into the node 5 from node 3 and node 4 is equal to 1. The selection value in this problem has to be either 0 or 1. This ensures that node 5 is taken. Working is shown in T&N 2i part 1. However, the solution to this and previous sub part are same.

Question 2i part 2

Similar to the previous question, we add a new constraint such that the sum of the selection value into the node 6 from node 3 and node 4 is equal to 1. The selection value in this problem has to be either 0 or 1. This ensures that node 6 is taken. Working is shown in T&N 2i part 2.

Question 2j

Based on the question, we can move from A-B-C-D which is uni-directional. There is another route A-C-B-D which makes us go from A to C first, travel back to B and then to D from B. In case, we don't know about this in advance, we can draw down each route first and then combine both. By doing so, we get the following:
From A, we can go to B and C
From B, we can go to C and D
From C, we can go to B and D
D is the destination so there's nothing from D.

Now that we have the directions and relations between the node A, B, C and D, we can solve this as a LP problem.

Question 3

Below is the optimal order quantity that should be ordered for each lot of order size.

Order Size	1-794	795-1099	1100-1859	1860-∞
Order Quantity, Q*	1.000	795.000	1100.000	1860.000

If we order 106.904 each time, total cost = $2500.500
If we order 795 each time, total cost = $4,204
If we order 1100 each time, total cost = $4,205
If we order 1860 each time, total cost = $4,202

Therefore, recommendation is to order: Q=1860, Cost = $4,202

Upon manual trails on excel by changing the order quantity for order size 1-794, I found that no matter what the quantity is (as long as it is within the order size range), for this order-size, the cost associated is always more compared with the costs for other order sizes. I also noticed that as we approach to 206, the cost falls down to $4204. 97 which is close to costs for other order sizes. The solver picked the minimum order size as the optimal order quantity respective order sizes. Working is shown in EOQ q3.