Abstract

Risk Management and Risk Modelling are becoming critical day by day for most of the businesses esp. those in Banking and finance. Sub-prime crisis, 2008 has made adaptation of risk measuring tools faster in the finance world. In this study, we analyze and compare the results of various methods used in forecasting Value at Risk (VaR) with three market indices. Using around ten years of the daily return data from on S&P 500, Nifty 50 and S&P/TSX composite index, we find the Value at Risk estimated by various models and strategies at offset of global equities market steep downfall during March,2020 due to pandemic.

After the analysis, it is found that most approaches perform inadequately, although several models gave acceptable VaR figures considering what happened later in the month of March,2020. The accuracy of VaR estimation depends on the distribution of the data and hence the methods used. VaR is certainly a better risk monitoring measure that historical volatility, etc. The predictive performance of several recently advanced and some new VaR models has been examined. The majority of these suffer from excessive VaR violations, implying an underestimation of market risk.

Introduction

The stock market crashes on follows a illogical cycle of around 8-10 years. It happened in 1987, 1996(Asian crisis) to 2008 big financial crises with the bankruptcy of Lehman Brothers and now 2020 Covid pandemic correction. This has attracted a great deal of attention among investors, practitioners, and researchers. The recent worldwide pandemic crisis characterized by the substantial increase in market volatility, and the big drops in market indices has further generated discussions on market risk and margin setting for financial institutions. As a result, value at risk (VaR) has become the standard measure of market risk in risk management. Its usefulness and weaknesses are widely discussed.

Risk Monitoring has become a vital component for finance world. In the context of risk measurement, we distinguish between:

- Risk measure, which is the operation that assigns a value to a risk, and
- Risk metric, which is the attribute of risk that is being measured.

A measure is an operationally defined procedure for assigning values. An attribute is that which is being measured—the object of the measurement. Like, volatility and credit exposure are attributes of bond risk that might be measured. Volatility and credit exposure are risk metrics. According to Holton (2004), risk has two components:

- Exposure
- Uncertainty

Credit exposure is a risk metric that only quantifies exposure. Market risk is exposure to the uncertain market value of a portfolio. Risk metrics that quantify uncertainty—either alone or in combination with exposure—are usually probabilistic. Many summarize risk with a parameter of some probability distribution.

Imagine an investment manager indicates that, based on the composition of its portfolio and on current market conditions, there is a 95% probability it will either make a profit or otherwise not lose more than USD 1.3MM over the next trading day. This is an example of a value-at-risk (VaR) measurement. For a given time period and probability, a value-at-risk measure purports to indicate an amount of money such that there is that probability of the portfolio not losing more than that amount of money over that time period. Stated another way, value-at-risk purports to indicate a quantile of the probability distribution for a portfolio's loss over a specified time period. To specify a value-at-risk metric, we must identify three things:

1. The period over which a possible loss will be calculated—1 day, 3 weeks, 1 month, etc. This is called the value-at-risk horizon. In our analysis, the value-at-risk horizon is one trading day and 15 trading day for some methods.
2. A quantile of that possible loss. In the example, the portfolio's value-at-risk is expressed as a .95 quantile of loss. In our analysis, we have used 99% quantile of loss.
3. The currency in which the possible loss is denominated. This is called the base currency. In our analysis, the base currency is USD, INR and CAD for different indices. We have assumed 100 units of base currency invested in each index on which we find VaR.

Applications of Value-at-Risk:

a. Risk Reporting and Oversight
b. Bank Regulatory Capital Requirements
c. Economic Capital Calculations
d. Corporate Disclosures
e. Risk Budgeting

The Value-at-Risk (VaR) concept has emerged as the most prominent measure of downside market risk. It places an upper bound on losses in the sense that these will exceed the VaR threshold with only a small target probability, 1-p, typically chosen between 1% and 5%.

Let, $\Delta V(l)$ be the change in value of underlying assets of financial position from time t to t+l. L(l) be the respective loss function. L(l) is a positive or negative function of $\Delta V(l)$ depending on the position being short or long. Denote the cumulative distribution function (CDF) of L(l) by $F_l(x)$. We define the VaR of a financial position over the time horizon l with tail probability p as:

$$\boxed{p = \Pr[L(l) \geq VaR] = 1 - \Pr[L(l) < VaR]}$$

The probability that the position holder would encounter a loss greater than or equal to VaR over the time horizon l is p. Alternatively, VaR can be interpreted as follows. With probability (1 − p), the potential loss encountered by the holder of the financial position over the time horizon l is less than VaR. VaR is concerned with the upper tail behavior of the loss CDF.

For a long financial position, loss occurs when the returns are negative. Therefore, we have used negative returns in data analysis for a long financial position. Since log returns correspond approximately to percentage changes in value of a financial asset, we use log returns r_t in data analysis. The VaR calculated from the upper quantile of the distribution of r_{t+1} given information available at time t is therefore in percentage. The base currency amount of VaR is then the cash value of the financial position times the VaR of the log return series. That is, VaR = Value × VaR (of loss function).

VaR does not fully describe the upper tail behavior of the loss function. In practice, two assets may have the same VaR yet encounter different losses when the VaR is exceeded. Furthermore, the VaR does not satisfy the sub-additivity property, which states that a risk measure for two portfolios after they have been merged should be no greater than the sum of their risk measures before they were merged. Therefore, care must be exercised in using VaR to measure risk. Regardless of some of its criticisms, regulatory requirements are heavily geared toward VaR. In the light of this practical relevance of the VaR concept, the need for reliable VaR estimation and prediction strategies arises. The purpose of this analysis is to compare alternative approaches for univariate VaR prediction.

For implementing univariate VaR-based measures, one seeks a precise quantile estimate relatively far out in the left tail of the return distribution for some specified future date. Existing approaches for obtaining such an estimate may be classified as follows: historical simulation simply utilizes empirical quantiles based on the available (possibly prefiltered) past data; fully parametric models describe the entire distribution of returns, including possible volatility dynamics; extreme value theory parametrically models only the tails of the return distribution; and, finally, quantile regression directly models a specific quantile rather than the whole return distribution. Following are some methods used to calculate VaR in this analysis:

Models

RISKMETRICS:

A watershed in the history of value-at-risk (VaR) was the publication of J.P. Morgan's RiskMetrics technical document. RiskMetrics assumes that the continuously compounded daily return of a portfolio follows a conditional normal distribution. The loss function is symmetrically distributed with $\mu = 0$ and σ^2 follows an IGARCH without drift.

IGARCH:

$$x_t | F_{1:t-1} \sim N(0, \sigma_t^2)$$

$$a_t \sim (0, \sigma_t^2)$$

$$\sigma_t^2 = (1-\alpha)\, a_{t-1}^2 + \alpha\, \sigma_{t-1}^2 \,,\ 0 < \alpha < 1$$

VaR:

$$VaR_{1-p}(1) = z_{1-p}\, \sigma_{t+1}$$

$$VaR_{1-p}(k) = z_{1-p}\, \sigma_{t+1} \sqrt{k}$$

ECONOMETRIC MODELLING:

Since VaR is based on volatility modelling of loss function, various econometric models can be used to estimate volatility and ultimately Value-at-Risk. Although various models have been tested and many are under study, in our analysis, we have used GARCH(1,1) with normal and standard t-distribution. Econometric Modelling makes multi-period forecast of VaR difficult to calculate.
If x_t is the loss function, general ARMA(p,q)-GARCH(m,s) can be given as:

$$x_t = \phi_0 + \sum_{i=1}^{p} \phi_i\, x_{t-i} + a_t - \sum_{j=1}^{q} \theta_j\, a_{t-j}$$

$$a_t = \epsilon_t\, \sigma_t \,,\ \epsilon_t \sim D(0,1)$$

$$\sigma_t^2 = \alpha_0 + \sum_{i=1}^{m} \alpha_i\, a_{t-1}^2 + a_t - \sum_{j=1}^{s} \beta_j\, \sigma_{t-j}^2$$

VaR:

For Normal Distribution:

$$VaR_{1-p}(1) = x_t(1) + z_{1-p}\, \sigma_t(1)$$

For Student t-Distribution:

$$\text{VaR}_{1-p}(1) = x_t(1) + t_v^*(1-p)\,\sigma_t(1)$$

where, $t_v^*(1-p)$ is the $(1-p)$th quantile of a standardized Student-t distribution with v degrees of freedom.

EMPIRICAL QUANTILES:

It is a non-parametric approach for Value-at-Risk calculations. Here, no assumption about distribution of loss function data is required except for the assumption that returns in prediction period is same as that in sample period. one can use the empirical quantile of the return r_t to calculate VaR. The order statistics of the sample are these values arranged in increasing order. We use the notation:

$$r(1) \leq r(2) \leq \cdots \leq r(n)$$

to denote the arrangement and refer to r(i) as the i^{th} order statistic of the sample. In particular, r(1) is the sample minimum and r(n) the sample maximum. The advantages of using the empirical quantile method to VaR calculation include (a) simplicity and (b) using no specific distributional assumption. However, the approach has several drawbacks. First, it assumes that the distribution of the return r_t remains unchanged from the sample period to the prediction period. Given that VaR is concerned mainly with tail probability, this assumption implies that the predicted loss cannot be greater than that of the historical loss. It is not so in practice. Second, when the tail probability p is small, the empirical quantile is not an efficient estimate of the theoretical quantile. Third, the direct quantile estimation fails to consider the effect of explanatory variables that are relevant to the portfolio under study. In real application, VaR obtained by the empirical quantile can serve as a lower bound for the actual VaR.

EXTREME VALUE THEORY METHOD:

All the methods discussed above uses whole of sample distribution. This works fine for first and second moments that is, near the central region about mean. In contrast, the financial data usually have prominent fat tails. When the event happens in tail region(although rarely), it creates catastrophic situations. Hence, It is more important to focus on tail regions to assess extreme loss cases. This is what EVT does.

Assume that the returns r_t are serially independent with a common cumulative distribution function F(x) and that the range of the return rt is [l, u]. In practice, the CDF F(x) of r_t is unknown and, hence, Fn,n(x) of r(n) is unknown. However, as n increases to infinity, Fn,n(x) becomes degenerated—namely, $F_{n,n}(x) \to 0$ if $x < u$ and $F_{n,n}(x) \to 1$ if $x \geq u$ as n goes to infinity.

$$F_*(x) = \begin{cases} \exp[-(1+\xi x)^{-1/\xi}] & \text{if } \xi \neq 0, \\ \exp[-\exp(-x)] & \text{if } \xi = 0, \end{cases}$$

The sequence $\{\beta_n\}$ is a location series and $\{\alpha_n\}$ is a series of scaling factors. The parameter ξ is referred to as the shape parameter that governs the tail behavior of the limiting distribution. The parameter $\alpha = 1/\xi$ is called the tail index of the distribution.

The tail behavior of CDF F(x) of x_t determines the limiting distribution $F_*(x)$ of the normalized maximum. The $\{\beta_n\}$ series and $\{\alpha_n\}$ series depends on the CDF. The tail index ξ does not depend on time interval of x_t .Since, we can not estimate all three parameters , we tend to divide the sample into sub-samples and then apply EVT to each sample.

$$\boxed{\underbrace{T}_{\text{total no. of observations}} = \underbrace{n}_{\text{size of subsample}} \times \underbrace{g}_{\text{no. of subsamples}}}$$

We have taken n =21 for analysis and used maximum likelihood estimates.

VaR:

$$\boxed{\text{VaR} = \left\{ \begin{array}{ll} \beta_n - \frac{\alpha_n}{\xi_n}\left\{1 - \left[-n\ \ln(1-p)\right]^{-\xi_n}\right\} & \text{if } \xi_n \neq 0 \\ \beta_n - \alpha_n \ln[-n\ \ln(1-p)] & \text{if } \xi_n = 0, \end{array} \right\}}$$

Multi-Period VaR:

$$\boxed{\text{VaR}(\ell) = \ell^{\xi}\text{VaR}}$$

The approach to VaR calculation using the extreme value theory encounters some difficulties. First, the choice of subperiod length n is not clearly defined. Second, the approach is unconditional and, hence, does not take into consideration effects of other explanatory variables.

PEAK OVER THRESHOLD:

To overcome these difficulties of traditional EVT, a modern approach to extreme value theory has been proposed in the statistical literature. Instead of focusing on the extremes (maximum or minimum), the new approach focuses on exceedances of the measurement over some high threshold and the times at which the exceedances occur. Thus, this new approach is also referred to as peaks over thresholds (POT).

This approach requires specification of threshold(η). Different choices of the threshold η lead to different estimates of the shape parameter k (and hence the tail index 1/ξ). In the literature, some researchers believe that the choice of η is a statistical problem as well as a financial one, and it cannot be determined based purely on statistical theory. The new approach considers the conditional distribution of $x = r_t - \eta$ given $r_t \leq \eta$ for a long position. Occurrence of the event $\{rt \leq \eta\}$ follows a point process.

$$F_*(x) = \begin{cases} \exp[-(1+\xi x)^{-1/\xi}] & \text{if } \xi \neq 0 \\ \\ \exp[-\exp(-x)] & \text{if } \xi = 0 \end{cases}$$

GENERALIZED PARETO DISTRIBUTION:

If we can use $\psi(\eta) = \alpha + \xi(\eta - \beta)$, then we can generate probability distribution with cumulative distribution function as:

$$G_{\xi,\psi(\eta)} = \begin{cases} 1 - [1 + \frac{\xi y}{\psi(\eta)}]^{-1/\xi} & \text{if } \xi \neq 0 \\ \\ 1 - \exp[\frac{-y}{\psi(\eta)}] & \text{if } \xi = 0 \end{cases}$$

This is called Generalized Pareto Distribution. Here, N_η is the number of returns that exceed η and r_{ti} are the values of the corresponding returns.

VaR:

$$\mathrm{VaR}_{1-p} = \eta - \frac{\psi(\eta)}{\xi}\{1 - [\frac{T}{N_\eta}p]^{-\xi}\}$$

Data

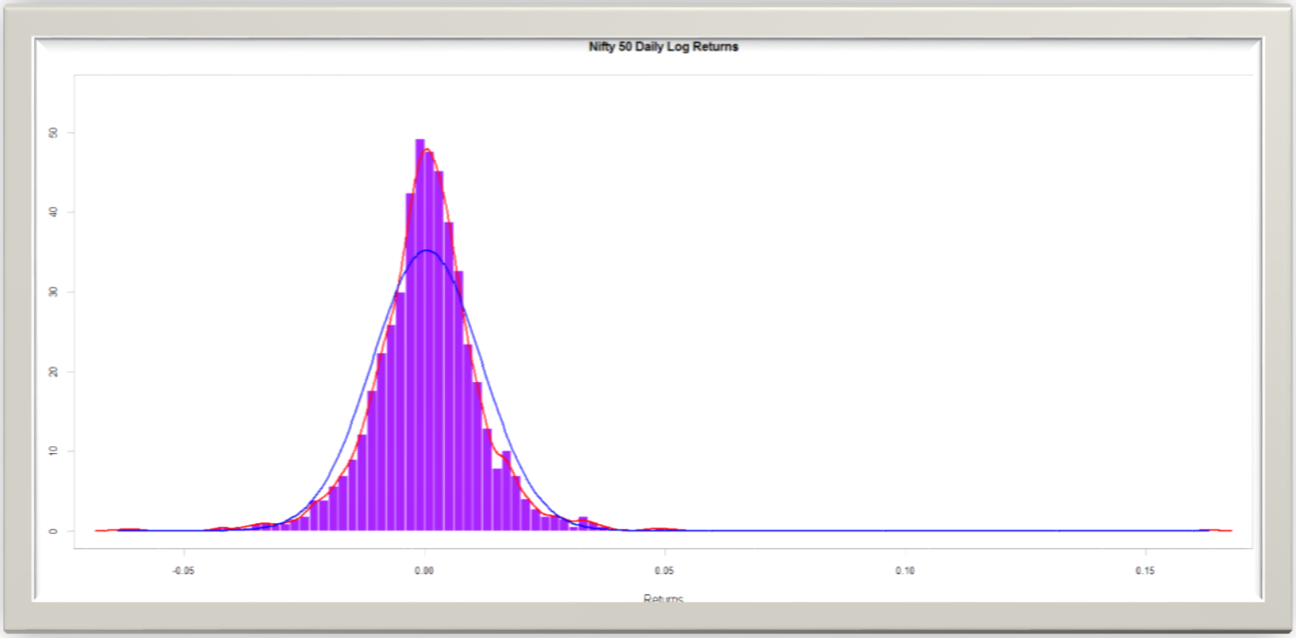
We have used the most popular equity market indices from USA, India, Canada namely S&P 500, Nifty 50, S&P/TSX respectively. Adjusted closing indices values for each trading day has been used to calculate daily log returns. The NA value has been dealt appropriately. The duration of data is from 3rd January 2009 to 1st March 2020. The data has been fetched using "quantmod" package in R which relays it from yahoo finance. VaR calculation has been done for 1% probability.

Table 1: Important basic statistics of log daily returns for S&P 500, Nifty 50, S&P/TSX. The time period is from 3rd January 2009 to 1st March 2020.

	S&P 500	Nifty 50	S&P/TSX
nobs	2806	2720	2798
NAs	0	0	0
Minimum	-0.068958	-0.063802	-0.055104
Maximum	0.068366	0.163343	0.051784
1. Quartile	-0.00348	-0.005374	-0.003866
3. Quartile	0.005304	0.006211	0.004596
Mean	0.000413	0.00047	2.00E-04
Median	0.000666	0.000467	0.000689
Sum	1.158551	1.277772	0.560445
SE Mean	0.000194	0.000217	0.000165
LCL Mean	3.20E-05	4.40E-05	-0.000122
UCL Mean	0.000794	0.000896	0.000523
Variance	0.000106	0.000128	7.60E-05
Stdev	0.010303	0.011327	0.008704
Skewness	-0.387206	0.973826	-0.387304
Kurtosis	5.081281	17.61096	3.73896

We have calculated compounded daily log returns with around 2800 observations for each index. From table 1, we can make following observations:

a. The mean for all log daily returns is almost zero. This can be further verified by t-test.
b. The median, which is a robust estimator, is also around zero for all three indices.
c. The returns data for S&P 500 & S&P/TSX looks slightly asymmetrical with negative skewness. While Nifty 50 has significantly positively skewed distribution.
d. As expected for financial data, all indices show fat tails as excess kurtosis is greater than 3. This feature is significant more in Nifty 50. So, all indices returns are leptokurtic in nature.


Nifty 50 Daily Log Returns
0
10
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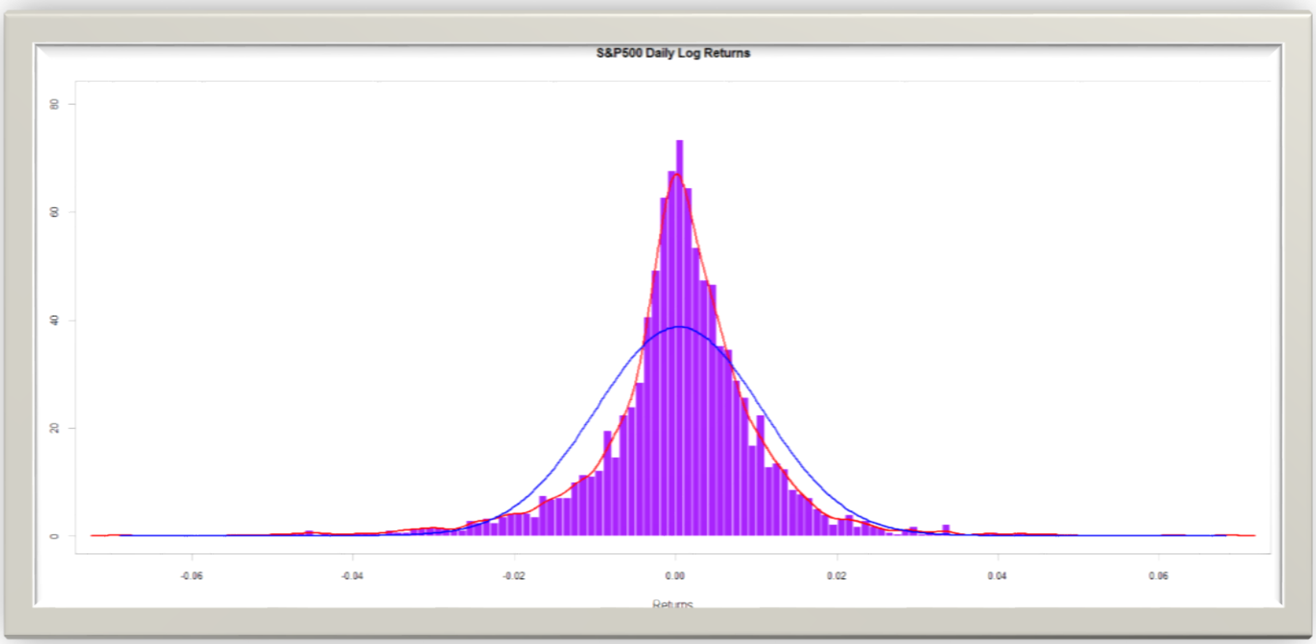
30
40
50
-0.05
0.00
0.05
0.10
0.15
Returns


S&P500 Daily Log Returns
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80
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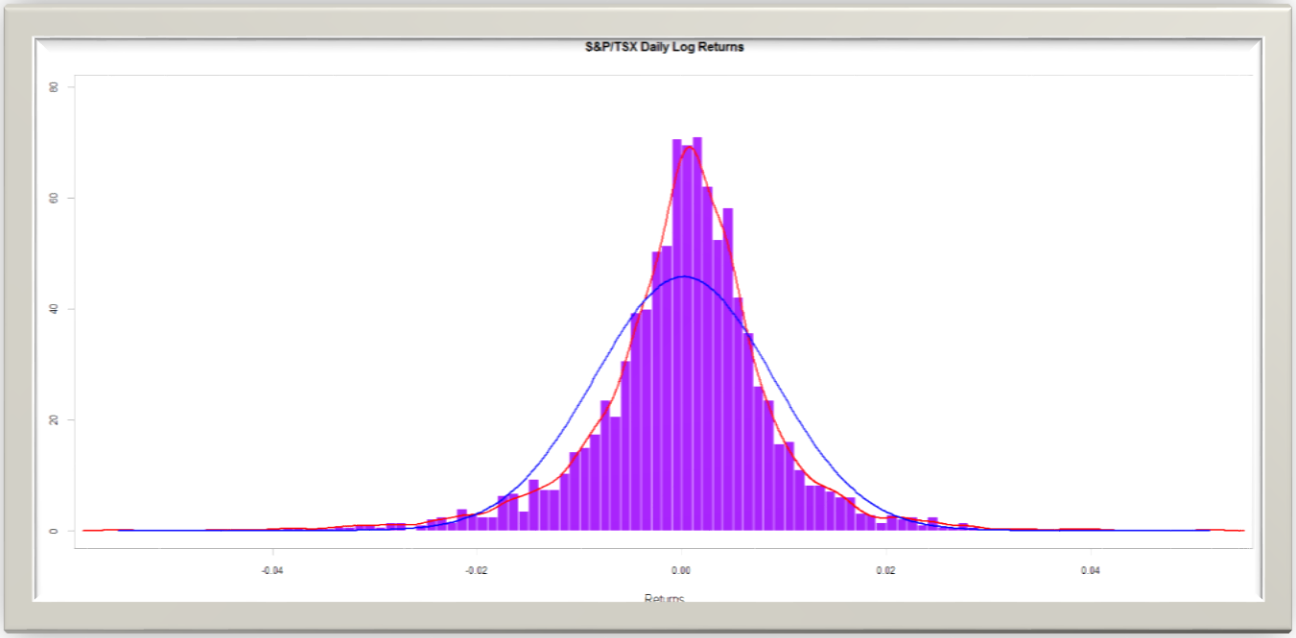
-0.04
-0.02
0.00
0.02
0.04
0.06
Returns



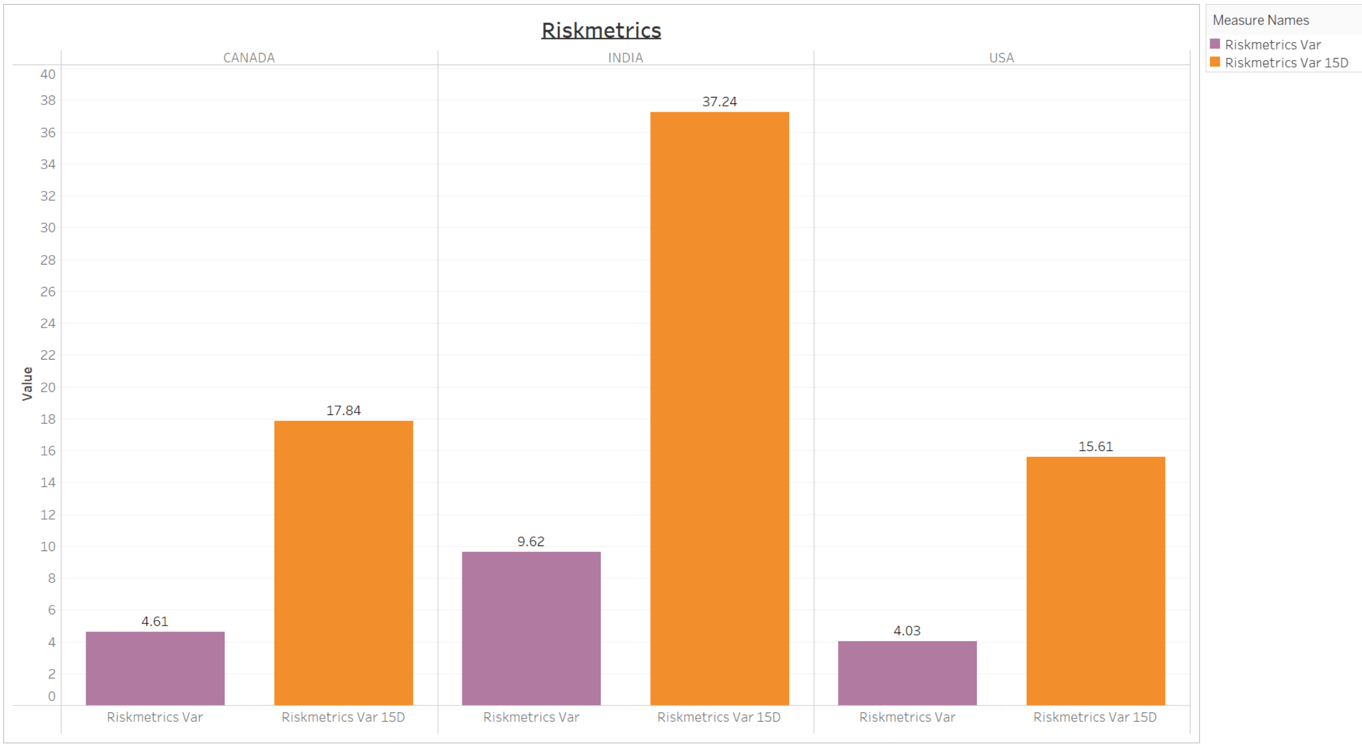
The above graphs clearly visualize the basic statistics discussed before. None of the return's distribution is close to normal distribution. We have considered a long position in all three indices with initial value of 100 base currency invested in each one. So,

VaR of each position = VaR from model * 100

Observations & Results

RISKMETRICS:

Using the RiskMetrics method developed by JP Morgan, we computed the VaR for next one trading day as well as for next 15 days. The results are shown in the below chart 1.



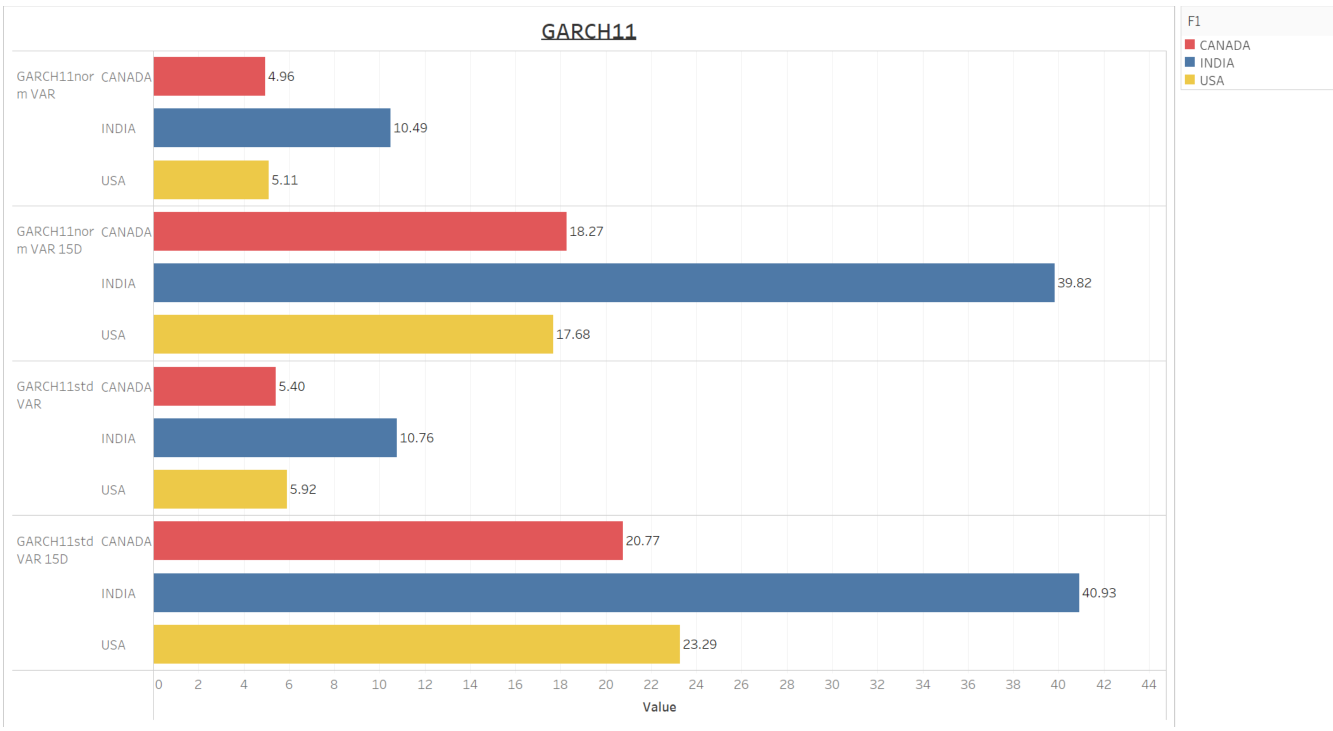
The observations are:

- The next one-day VaR value for Indian market was almost double of Canadian & American markets.
- The 15-days VaR value for Indian market was again very high at 37.28 units. For Canadian & American indices 15-days VaR was 17.84 & 16.81 units. This makes sense now as all the major indices fell sharply in the month of March 2020 and Indian market saw the maximum drawdown in terms of percentage.
- Although, Riskmetric method considers normal distribution of data for calculation of VaR , which is actually not the case here, still we can say that this method is giving some relevant results.

ECONOMETRIC MODELS (GARCH[1,1]):

We used two Econometric models

1. GARCH(1,1) with Normal Distribution
2. GARCH(1,1) with Student t-Distribution

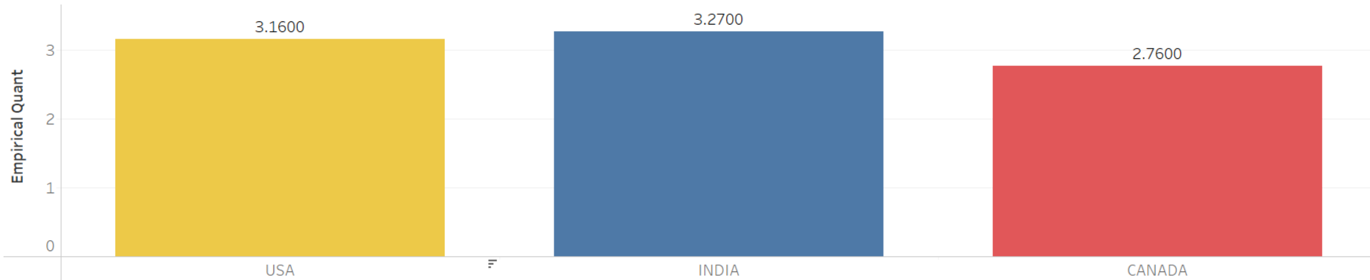


The observations are:

- The next one-day VaR value for Indian market was again almost double of Canadian & American markets for both GARCH model (normal & student t-distribution)
- The 15-days VaR value from normal distribution for Indian market was as high as 39.82 units. For Canadian & American indices with same distribution ,15-days VaR was around 18 units.
- Although, GARCH model with t-distribution is almost in sync with GARCH with normal distribution. As we know t-distribution fits better for financial data, so GARCH t-distribution should be a better estimate.

EMPIRICAL QUANTILES:

Using the Empirical Quantiles method, we obtained following VaR values for next one day:

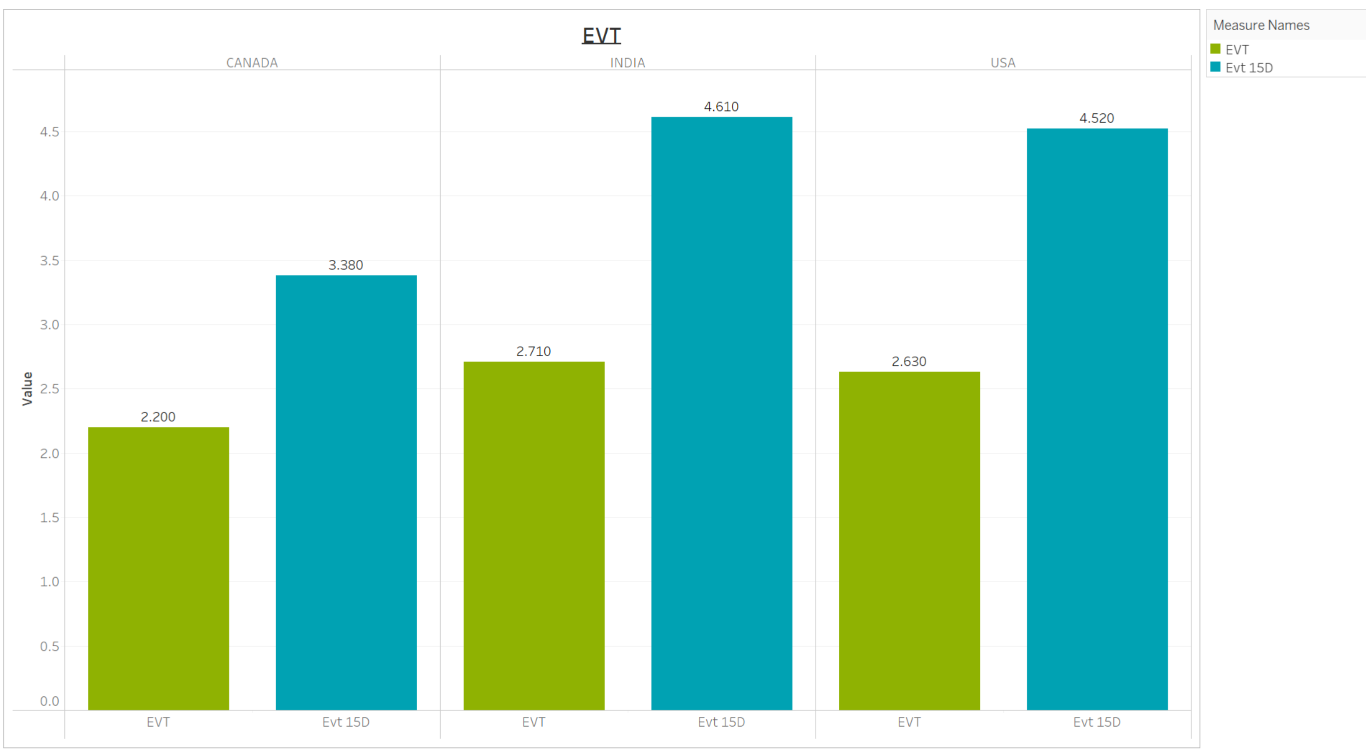


The observations are:

- Overall VaR values for every country is significantly lower than all the models before.
- The next one-day VaR value for Indian & American market was around 3.2 units.
- Canadian TSX had the lowest VaR value at 2.76 units.

EXTREME VALUE THEORY (EVT):

The EVT method gave following VaR values for next one and 15 days:



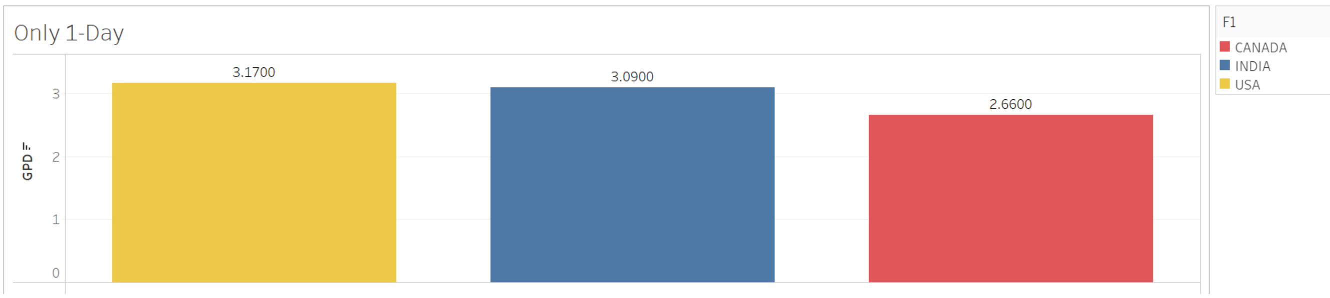
The observations are:

- Overall VaR values for every country is significantly lower than all the models before. Even below the empirical quantiles method.

- The next one-day and 15 days VaR value for Indian & American market was around 2.7 & 4.6 units respectively.
- Canadian TSX had the lowest VaR value at 2.2 units for one day ahead.

Generalized Pareto Distribution (GPD):

The GPD method gave following VaR values for next one day:

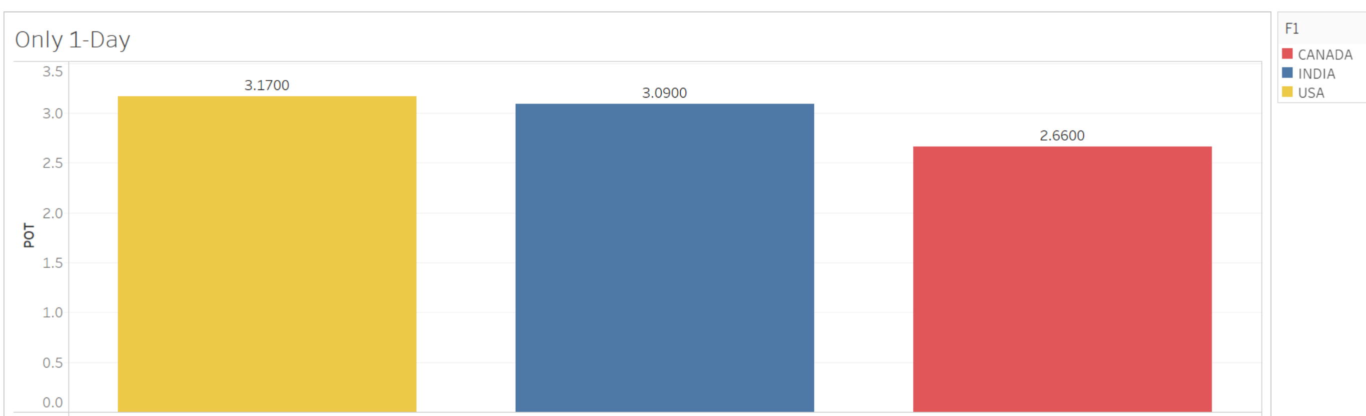


The observations are:

- GPD method gave VaR values for next one day as 3.17, 3.09 and 2.66 for USA, Canada & India respectively.

Peak Over Threshold Method (POT):

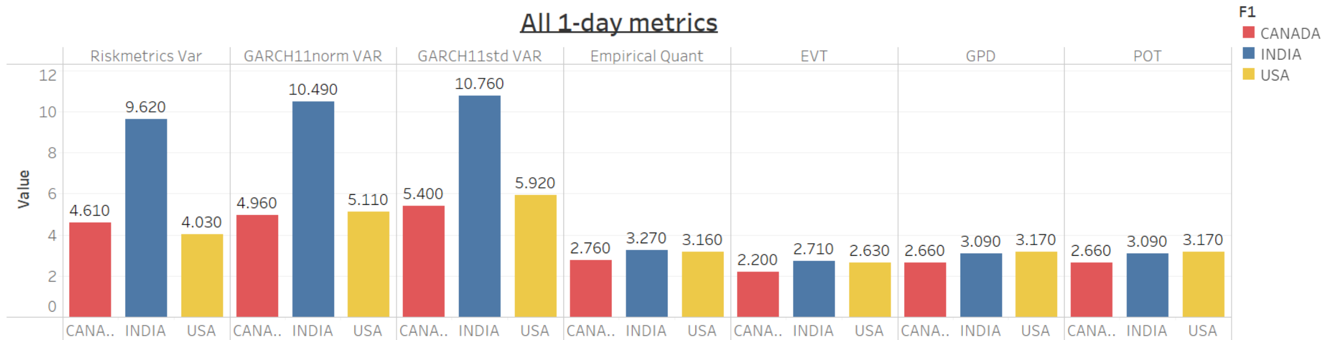
The POT method gave following VaR values for next one day:

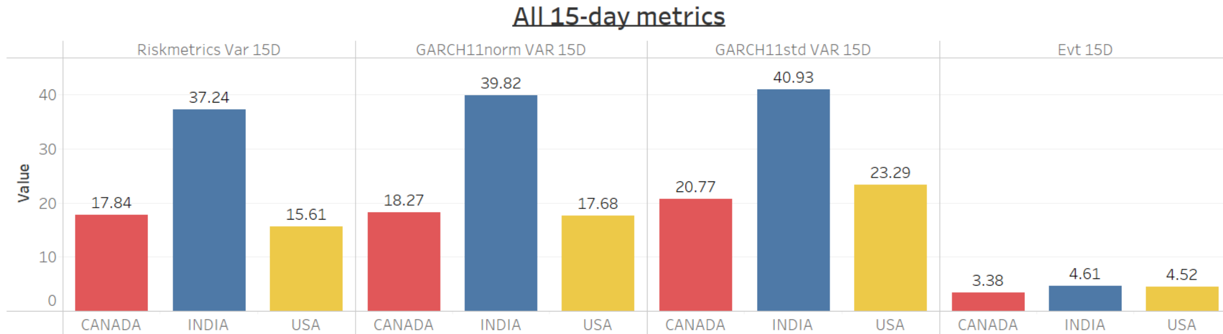


The observations are:

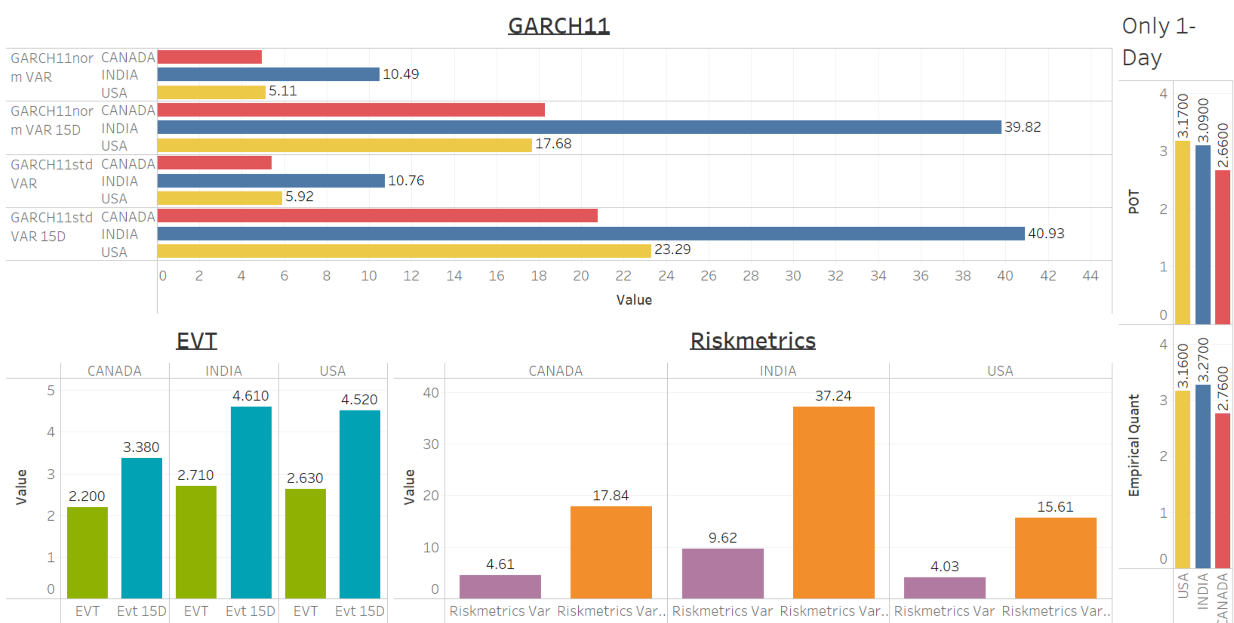
- The values are identical with GPD method. This might be because of some error.
- The next one-day VaR value for Indian & American market was around 3.09 & 3.17 units respectively.
- Canadian TSX had the lowest VaR value at 2.66 units for one day ahead.

Overall Observations & Results





The one day VaR out of all models used are giving maximum value for Indian market. The average VaR of all there indices is greater from RiskMetrics and econometric models. For 15 day VaR, GARCH(1,1) with student-t distribution models gives the maximum VaR values all indices. The VaR from EVT ,GPD, POT are significantly smaller.



	RISKMETRICS_VAR	RISKMETRICS_VAR_15D	GARCH11norm_VAR	GARCH11norm_VAR_15D	GARCH11std_VAR	GARCH11std_VAR_15D	EmpiricalQuant	EVT	EVT_15D	GPD	POT
USA	4.03	15.61	5.11	17.68	5.92	23.29	3.16	2.63	4.52	3.17	3.17
INDIA	9.62	37.24	10.49	39.82	10.76	40.93	3.27	2.71	4.61	3.09	3.09
CANADA	4.61	17.84	4.96	18.27	5.4	20.77	2.76	2.2	3.38	2.66	2.66

Conclusion

In the analysis, we were trying to calculate one day and 15 day VaR for three indices using various methods from RiskMetrics to Extreme Value Theory. The aim of this analysis was to see which methods were signaling a huge drawdown in March 2020 as due to onset of pandemic , there was a sharp fall in the global markets. Emerging markets such as India went through more than 40% correction while developed markets like USA and Canada fell almost 20-30% from February 2020 levels. If a fund manager were relying on the Value-at-Risk measure for monitoring the risk in his 100 units portfolio in each of three markets on 1st March 2020, what would each of the methods used in VaR calculations were signaling about riskiness of his portfolio.

From the results we observed that every method gave different estimates for Value at Risk for one day and 15 day estimates. The Indian market index ,Nifty 50 fell the most post 1st March 2020 and this has been reflected in estimation of VaR by almost all methods. The GARCH(1,1) with Student t-distribution gave most accurate estimates about what to come in future. There are substantial differences among different approaches. This is not surprising because there exists substantial uncertainty in estimating tail behavior of a statistical distribution. Since there is no true VaR available to compare the accuracy of different approaches, we recommend that one applies several methods to gain insight into the range of VaR.

In practice, VaR prediction is hampered by the fact that financial returns exhibit ‘‘nonstandard’’ statistical properties. Specifically, they are not independently and identically distributed (iid) and, moreover, they are not normally distributed. This is reflected by three widely reported stylized facts: (i) volatility clustering, indicated by high autocorrelation of absolute and squared returns; (ii) substantial kurtosis, that is, the density of the unconditional return distribution is more peaked around the center and possesses much fatter tails than the normal density; and (iii) mild skewness of the returns, possibly of a time-varying nature. As a consequence, ‘‘standard’’ methods, based on the assumption of iid-ness and normality, tend not to suffice, which has led to various alternative strategies for VaR prediction.

Appendix: R Code

```
library('quantmod')
library('fBasics')
library('PerformanceAnalytics')
tick = c("^GSPC","^NSEI","^GSPTSE")
getSymbols(tick, from = "2009-01-03", to ='2020-03-01')
us_ret = diff(log(as.numeric(GSPC$GSPC.Adjusted[!is.na(GSPC$GSPC.Adjusted)])))
ind_ret = diff(log(as.numeric(NSEI$NSEI.Adjusted[!is.na(NSEI$NSEI.Adjusted)])))
can_ret = diff(log(as.numeric(GSPTSE$GSPTSE.Adjusted[!is.na(GSPTSE$GSPTSE.Adjusted)])))
nret = data.frame()
nret = cbind(-us_ret,-ind_ret,-can_ret)
colnames(nret) = c("USA","INDIA","CANADA")
us_b = data.frame(basicStats(us_ret))
ind_b = data.frame(basicStats(ind_ret))
can_b = data.frame(basicStats(can_ret))
basic_stats = data.frame(us_b)
basic_stats = cbind(basic_stats,ind_b$ind_ret,can_b$can_ret)
names(basic_stats) = c("S&P 500","Nifty 50","S&P/TSX")
write.csv(basic_stats, "C:\\Users\\Lighthouse\\Desktop\\basicstats.csv", row.names=TRUE)
chart.Histogram(us_ret,methods = c("add.density","add.normal"),
        colorset = c("purple","red","blue"),main = "S&P500 Daily Log Returns")
chart.Histogram(ind_ret,methods = c("add.density","add.normal"),
        colorset = c("purple","red","blue"),main = "Nifty 50 Daily Log Returns")
chart.Histogram(can_ret,methods = c("add.density","add.normal"),
        colorset = c("purple","red","blue"),main = "S&P/TSX Daily Log Returns")

source("RMfit.R")
v= c()
v_15 = c()
for (i in 1:3){
  m1 =RMfit(nret[,i])
  vpred=sqrt(m1$par[1]*(m1$volatility[2806])^2+(1-m1$par[1])*(nret[2806,i])^2)

  v[i] = vpred*qnorm(0.99)  #VaR at p=0.05    using  Vpred=0.007133031
    # 15-day holding period
  v_15[i] = sqrt(15)*vpred*qnorm(0.99) #note that qnorm(0.95)=1.645
}
result = data.frame()
result = cbind(v,v_15)
rownames(result)  = c("USA","INDIA","CANADA")
colnames(result)  = c("RISKMETRICS_VAR","RISKMETRICS_VAR_15D")
```

```
### Econometric modeling- GARCH11-norm #########
require(fGarch)
source('RMeasure.R')
v = c()
v_15 = c()
for (i in 1:3){

  m2=garchFit(~garch(1,1),data=nret[,i],trace=F)
  pm2=predict(m2,15)
  sigma=pm2[1,3]
  mu=pm2[1,1]
  a = data.frame(RMeasure(mu,sigma))
  v[i] = a[2,2]
  v1=sqrt(sum(pm2$standardDeviation^2))
  b = data.frame(RMeasure(pm2$meanForecast[1]*15,v1))
  v_15[i] = b[2,2]

}
result = as.data.frame(cbind(result,v,v_15))
colnames(result)[3] = c("GARCH11norm_VAR")
colnames(result)[4] = c("GARCH11norm_VAR_15D")

### Econometric modeling- GARCH11-std #########
v = c()
v_15 = c()
for (i in 1:3){

  m3=garchFit(~garch(1,1),data=nret[,i],trace=F,cond.dist = "std")
  pm3=predict(m3,15)

  df=m3@fit$par[5]
  sigma=pm3[1,3]
  mu=pm3[1,1]

  c = data.frame(RMeasure(mu,sigma,cond.dist="std",df=df))
  v[i] = c[2,2]
  v1=sqrt(sum(pm3$standardDeviation^2))
  d = data.frame(RMeasure(pm3$meanForecast[15],v1,cond.dist = "std",df=df))
  v_15[i] = d[2,2]

}
result = as.data.frame(cbind(result,v,v_15))
colnames(result)[5] = c("GARCH11std_VAR")
colnames(result)[6] = c("GARCH11std_VAR_15D")

### Empirical quantiles #########
```

```
v = c()
v_15 = c()
for (i in 1:3){
   m4=quantile(nret[,i],c(0.99))
   v[i] = m4[1]
}
result = as.data.frame(cbind(result,v))
colnames(result)[7] = c("EmpiricalQuant")


### Extreme value theory #########
require(evir)
source("evtVaR.R")
v = c()
v_15 = c()
for (i in 1:3){

  m5=gev(nret[,i],block=21)

  e = as.data.frame(evtVaR(m5$par.ests[1],m5$par.ests[2],m5$par.ests[3],21,0.01))
  v[i] = e[1,1]
  v_15[i] = (15^m5$par.ests[1])*v[i]


}
result = as.data.frame(cbind(result,v,v_15))
colnames(result)[8] = c("EVT")
colnames(result)[9] = c("EVT_15D")

### Generalized Pareto distribution #########
v = c()
v_15 = c()
for (i in 1:3){

  m6=gpd(nret[,i],0.01)  # Threshhold=0.01
  f = as.data.frame(riskmeasures(m6,c(0.99)))
  v[i] = f[1,2]
}
result = as.data.frame(cbind(result,v))
colnames(result)[10] = c("GPD")

### POT: Peaks over Threshold #########

v = c()
v_15 = c()
```

```
for (i in 1:3){

  m7=pot(nret[,i],threshold=0.01)  # Threshold =0.01

  g = as.data.frame(riskmeasures(m7,c(0.99)))
  v[i] = g[1,2]
}
result = as.data.frame(cbind(result,v))
colnames(result)[11] = c("POT")

result2 =  round(result * 100,2)
write.csv(result2, "C:\\Users\\Lighthouse\\Desktop\\result.csv", row.names=TRUE)
```

References

1. Analysis of Financial Time Series by Ruey S. Tsay.
2. https://www.value-at-risk.net/
3. "Value-at-Risk Prediction: A Comparison of Alternative Strategies"
4. MQIM-6602 Lecture Notes