AIAA Propulsion and Energy Forum
August 24-28, 2020, VIRTUAL EVENT
AIAA Propulsion and Energy 2020 Forum

10.2514/6.2020-3545

Optimal Weight Power System Design and Synthesis for More Electric Aircraft

Angel A. Recalde[1*‡], Serhiy Bozhko[2*], Jason Atkin[3†]

[*] *School of Electrical and Electronic Engineering, University of Nottingham, NG7 2TU, UK*

[†] *School of Computer Science, University of Nottingham, NG8 1BB, United Kingdom*

[‡] *Escuela Superior Politecnica del Litoral ESPOL, EC 090112, Ecuador*

The synthesis of a power distribution architecture for More Electric Aircraft requires weight optimization in order to reduce energy consumption. The weight of an aircraft power distribution system depends on various factors such as the functional and safety requirements, as well as component selection and location. Functional and safety requirements can be translated into a set of connectivity and reliability constraints to produce an architecture that represents an abstract topology of the power system. However, component selection and location aims to produce a solution that is closer to a final implementation. Then, this paper presents an optimization based design formulation that synthesizes a power system architecture considering component selection and location in order to shorten the gap between the topology and the physical implementation. Given the complexity in producing and solving such formulation, linear transformations are performed to enable the use powerful commercial solvers and reach a minimum weight electrical distribution system. Therefore, the design is presented as a Mixed Integer Linear Programming problem and a case study is used to exemplify the synthesis of a power distribution architecture that is optimal.

I. Nomenclature

Sets

$\mathscr{g}$	=	set of available generators
$\mathcal{E}$	=	set of available distribution paths (generator – load connections)
$\mathcal{N}$	=	set of available distribution components
$\mathcal{A}$	=	set of available connections between distribution components
K	=	set of selected distribution paths (generator – load connections), $K \subseteq \mathcal{E}$

Generator Selection and Generator-Load Pairing

$\mathcal{G} = (\mathscr{g}, \mathcal{E})$	=	graph containing set of generators $\mathscr{g}$ and distribution paths $\mathcal{E}$
w_s, P_s^G	=	weight, and power rating for generator s
$L_l, r_{\mathrm{TARGET},\,l}$	=	load demand and reliability target for load l
g_s	=	Boolean selection for generator s
y_{sl}	=	Boolean selection for distribution path that connects generator s with load l
r_{sl}	=	reliability of distribution system or path that connects generator s with load l
u_{sh}, w_h	=	weight linearization Boolean variable and coefficient

Power Distribution Design

$\mathcal{G} = (\mathcal{N}, \mathcal{A})$	=	graph containing set of components $\mathcal{N}$ and connections $\mathcal{A}$
k, r_{sl}	=	distribution path ($k \in K$, $k \in \mathcal{E}$), reliability target for distribution path k

[1] PEMC Group, MSCA Researcher, angel.recalde@nottingham.ac.uk
[2] PEMC Group, Institute for Aerospace Technology, serhiy.bozhko@nottingham.ac.uk
[3] COL Lab, School of Computer Science, jason.atkin@nottingham.ac.uk

$P_{ij,k}, P_{i,k}$	=	power flow on connection i, j and component i for distribution path k
w_{ij}, w_i	=	fixed weight for connection i, j; fixed weight for component i
$w_{ij}^{\text{kW}}, w_i^{\text{kW}}$	=	variable weight for connection i, j; variable weight for component i
x_{ij}, x_i	=	Boolean selection for connection i, j and component i
r_{ij}, r_i	=	reliability of connection $i, j,$ reliability of component i
d_k^{LOAD}, d_k^{GEN}	=	load demand and power generation of the distribution path k
$z_{ij,k}, z_{i,k}$	=	Boolean variables to associate connection i, j, component i with distribution path k
m_i, n_i	=	Abscissa and ordinate position of component i
$c_{ij,o}^{\text{AWG}}$	=	Boolean variable to select a unique standard wire gauge if connection i, j is selected
$I_o^{\text{AWG}}, r_o^{\text{AWG}}$	=	ampacity and resistance per-unit-of-length of standard gauge type o
w_o^{AWG}	=	weight per-unit-of-length of standard gauge type o
q_w	=	Boolean variable to select alternative paths for distribution path k
$V_{p.u.}^{\text{DROP}}, V_{ij}^{\text{OP}}$	=	permissible voltage drop (in p.u.), and operational voltage for connection i, j

II. Introduction

The More Electric Aircraft (MEA) power distribution system consists of a set of interconnected components that performs power transmission and conversion to supply loads at the required voltage level and format (AC or DC, high, medium, and low voltage, etc.) [1], [2]. Recently, DC distribution systems have captured a lot of interest for MEA applications due to the reduced number of conductors [3], inexistence of reactive power [4], and higher dynamic stability [5] if compared to the AC counterpart. In addition, among several wiring configurations studied for MEA power system architectures, DC systems provide maximum power transfer and maximum weight to power ratio for the same voltage range [3]. Several DC distribution topologies have been proposed in MEA to provide adequate number of power conversion levels, high reliability, and high-power density [1], [5]–[7]. Hence, the main objective in determining a power system architecture for MEA is minimizing the amount of weight, i.e. reduce the system's payload, while guaranteeing safety and power transfer availability.

Safety specifications can be translated into a set of reliability constraints that ensures the power system comply with a certain reliability level (reliability metric) by assembling sufficient number of components and distribution paths [8]–[10]. Several approaches based on reliability-based network optimization problems [11] have included reliability constraints in a Mixed Integer Linear Programming (MILP) problem that synthesizes an optimum power distribution topology [10], [12]–[14]. These formulations can be solved either with iterative techniques, or sequential algorithms. In the former, the number of reliability constraints increase depending on the reliability requirements [12], so that the connectivity complexity is increased only if required by specification, otherwise, simpler topologies could be feasible candidates. In the latter, starting at an abstract representation (platform) of the system considering functional requirements, the design walks through a certain number of refinement steps until the platform is very close to a physical implementation that complies to the initial specifications [15]. Although these design frameworks have been used to synthesize an optimal power system architecture that complies with a set of requirements, further steps must be performed in order to produce a final implementation. This paper pretends to close this gap by exploring weight minimization via component selection and location while complying with a set of reliability constraints. The design exercise is formulated as a MILP to experience the advantage of reaching optimality in polynomial time (depending on the number of variables) and using powerful MILP solvers available commercially. The rest of the paper is outlined as follows. Section III develops the design formulation to synthesize a power distribution architecture considering connectivity, reliability, component selection, and location constraints. Then, Section IV presents a linear transformation of the design problem. Section V exemplifies the use of the proposed formulation in the case study. Finally, Section VI comes to the conclusion of the paper.

III. Design Formulation

The power system design must consider the functional and safety specifications from the initial design stage. In the case of a MEA power system, these specifications are based primarily on the electrical loads' power and energy requirements, the airworthiness standards, and the aircraft state-of-the-art design practices. Besides, the design outcome is inherently related to the type of aircraft, its engine capabilities, and the flight purpose or length (military, cargo, commercial short-/ long-haul, etc.). Nevertheless, the specifications and requirements can be translated into a set of connectivity and reliability constraints which are satisfied by the optimum obtained through the design framework. A set of constraints on the components' selection and location are included in an attempt to get closer to a physical power system implementation whose weight is minimum.

The MEA power distribution system comprises a set of interconnected power sources (generators, batteries, etc.) and distribution devices that performs generation, transmission, conversion, and distribution to supply power to critical and non-critical loads. For the rest of the paper, the power sources are assumed to be generators driven by the aircraft' jet engines. Given that the power conversion weight is driven by a power density value (kW/kg) [16], [17], its total weight is proportional to the amount of power to convert. Consequently, the weight of the power distribution system is independent from the generator-load pairing arrangement. Following a PBD methodology [18], the MEA power system can be synthesized in two sequential steps [15], [19]: generator selection and generator-load pairing (GS&GLP), and power distribution design (PDD). The former selects a number of power sources (generator selection) and determines which loads are supplied by those power sources at any time (generator-load pairs or distribution paths). Then, the latter synthesizes a power distribution topology whose number of components and their corresponding sizes depend on the previous generator-load pair arrangement or selected distribution paths [15], [20].

A. Generator Selection and Generator-Load Pairing step

The design exercise requires to supply a pre-specified set of critical loads $l \subseteq L$ with reliability requirements. In the GS&GLP, the functional requirement produces an abstract representation of the power system where source points deliver power to load points. Let a set of $\mathcal{g}$ generators and a set of $\mathcal{E}$ distribution paths (generator-load connections) be represented by a template (graph) $\mathcal{G} = (\mathcal{g}, \mathcal{E})$. Each generator s has a set of parameters, i.e. weight w_s, reliability r_s, and power rating P_s^G. The GS&GLP step attempts to select subsets $\mathcal{s} \subseteq \mathcal{g}$ and $K \subseteq \mathcal{E}$ of generators and distribution paths respectively that minimizes generation weight w, supply each of the critical loads l (from the set L), and satisfies the connectivity and reliability constraints. Let a Boolean g_s determine the selection ($g_s = 1$) or rejection ($g_s = 0$) of a generator s of the set $\mathcal{g}$. The minimization of the generation's total weight can be written as:

$$\underset{w,g,y}{\text{minimize}} \left(\sum_s w_s g_s \right) \tag{1}$$

In (1), w_s is the weight of generator s and the product $w_s g_s$ equals w_s if $g_s = 1$ (otherwise, generator s is not selected and its weight contribution is 0). The generator weight w_s is assumed to be a function of the power rating P_s^G [16]. The weight of the distribution system is not included in (1) because in the abstraction of the generation platform, the function of the distribution system is to allocate load power to generator power according to the generator's capacity and the loads' reliability requirements. Later in the PDD step, a set of functional requirements are defined and the distribution paths are constructed to allow power transmission. Consistently, a set of connectivity and reliability constraints always ensure sufficient generation and distribution capacity to supply critical loads.

In GS&GLP, the connectivity constraints allow load l to be connected to a generator s. Each of the selected generators s must have a power rating P_s^G greater than the minimum power rating P_{MIN}^G but less than the maximum power rating available P_{MAX}^G. A set H of h commercial power rating values P_h^G can be used to select a specific P_s^G for generator s. Let a Boolean u_{sh} select a power rating value $h \in H$ for generator s. Each generator has a unique power rating if selected, then,

$$\sum_h u_{sh} \leq g_s \qquad \forall s \tag{2}$$

Let the Boolean y_{sl} determine the selection ($y_{sl} = 1$) or rejection ($y_{sl} = 0$) of the distribution path sl connecting generator s to the load l. A load l is connected to a generator s if that generator has been selected, then,

$$y_{sl} \leq \sum_h u_{sh} \qquad \forall l, \forall s \tag{3}$$

Each load is connected at least to one generator, then,

$$\sum_s y_{sl} \geq 1 \qquad \forall l \tag{4}$$

The power rating P_h^G of the generator s is greater than or equal to the total load connected to it, then,

$$\sum_l L_l y_{sl} \leq \sum_h P_h^G u_{sh} \qquad \forall s \tag{5}$$

For each load l, the reliability is considered as the availability of power supply on the load's terminals, i.e. there is a reliability target $r_{\text{TARGET},\,l}$ that needs to be satisfied. Let r_s be the reliability of the generator s, and r_{sl} the reliability of each distribution path y_{sl} (generator-load connection). When a load l is connected to a generator s, the reliability on the load terminals is $r_{sl}r_s$, which is the reliability of a series-system. When a load l is connected to more than one generator, the reliability of a parallel series-system can be applied. Then, the reliability of a load l connected to multiple generators must be greater than or equal to $r_{\text{TARGET},\,l}$, then

$$1-\prod_{s}(1-y_{sl}r_{sl}r_s) \geq r_{\text{TARGET},\,l} \qquad \forall l \qquad (6)$$

The product $y_{sl}r_{sl}$ in (6) allows to set r_{sl} to 0 if the distribution path y_{sl} is not selected ($y_{sl}=0$). As mentioned before, the outcome of the GS&GLP step is a group of generators $\mathscr{s} \subseteq \mathscr{g}$ with their corresponding ratings P_h^G and a group of distribution paths (generator-load connections) $K \subseteq \mathcal{E}$ with their corresponding reliabilities r_{sl}. The reliability constraint in (6) contains the distribution path's reliability variable r_{sl} which is used as the reliability target of the distribution path in the PDD step.

B. Power Distribution Design step

The PDD step aims to synthesize a power distribution architecture that implements the distribution paths arrangement (generator-load connections) found in the GS&GLP step. The functional abstraction of the distribution platform considers appropriate power conversion/transmission between devices and topology reconfiguration. Consider a set $\mathcal{N}$ of power distribution components and a set $\mathcal{A}$ of feasible connections. Let these two sets be represented by a template (graph) $\mathcal{G}=\{\mathcal{N},\mathcal{A}\}$. Each component i has a group of design parameters, i.e. weight w_i, reliability r_i, and power capacity P_i. Similarly, for each connection ij the group of parameters include weight w_{ij}, reliability r_{ij}, and power capacity P_{ij}. The PDD attempts to select a subset of components $n \subseteq \mathcal{N}$ and a subset of connections $a \subseteq \mathcal{A}$ that minimizes power distribution weight, while satisfying a group of connectivity and reliability constraints. The power distribution weight consists of a fixed weight (installation payload) and a variable weight that depends on the amount of power flow converted/transferred.

Let a Boolean x_i select a component i from the set $\mathcal{N}$ (all selected components will form the subset n). Also, let component i have a fixed weight w_i, a variable weight w_i^{KW} that represents the ratio between a unit of weight and a unit of power converted/transferred, and a power flow P_i. Similarly, let a Boolean x_{ij} select a connection (from the set $\mathcal{A}$) between component i and j (all selected connections will form the subset a). Let each connection ij have a fixed weight w_{ij}, a variable weight w_{ij}^{KW} that represents the ratio between a unit of weight and a unit of power transferred, and a power flow P_{ij}. The parameters w_i^{KW} and w_{ij}^{KW} (in kg/kW) are the inverse of the power density (in kW/kg) for components and connections respectively. Then, the total power distribution's weight (fixed weight plus variable weight) can be written in terms of the selection variables x_i, x_{ij}, and the power flows P_i, P_{ij},

$$\underset{P,x,v}{\text{minimize}}\left[\left(\sum_{(i,j)\in\mathcal{A}} w_{ij}x_{ij}+\sum_{i\in\mathcal{N}} w_i x_i\right)+\left(\sum_{(i,j)\in\mathcal{A}} w_{ij}^{\text{kW}}P_{ij}+\sum_{i\in\mathcal{N}} w_i^{\text{kW}}P_i\right)\right] \qquad (7)$$

The first term in (7) is the summation of all fixed weights, and the second term is the summation of all variables weights (which depends on the amount of power flow converted/transferred). Given that the PDD step implements the solution of the GS&GLP (constructing the selected distribution paths y_{sl}), there is a number of K distribution paths that are built with selected components x_i and connections x_{ij}. Each distribution path k ($k \in K$, k exists if and only if $y_{sl}=1$) connects the generator s and the load l of its corresponding path y_{sl}. It is possible that two or more distribution paths k could share the same component i and connection ij (depending on the reliability constraints also). Also, all the component's power flows can be expressed in terms of the connections' power flows P_{ij}. Let a connection ij have a power flow $P_{ij,k}$ for every distribution path k such that $P_{ij}=\sum_{k\in K}P_{ij,k}$ (if connection ij serves only one distribution path k, then only one $P_{ij,k}\neq 0$, and all other $P_{ij,k}=0$), and let a component i have a power flow $P_{i,k}$ for every distribution path k such that $P_i=\sum_{k\in K}P_{i,k}$ (if component i serves only one distribution path k, then only one $P_{i,k}\neq 0$, and all other $P_{i,k}=0$). Note that $P_{i,k}$ can be expressed in terms of the incoming (or outgoing) connections' power flows, i.e. $P_{i,k}=\sum_{\mathscr{j}}P_{\mathscr{j}i,k}$ such that the flow through component i is the summation of all incoming (or outgoing) flows coming from other components $\mathscr{j}$. Because $P_{i,k}=\sum_{\mathscr{j}}P_{\mathscr{j}i,k}$, then the flow through component i is $P_{i,}=\sum_{k\in K}\sum_{\mathscr{j}}P_{\mathscr{j}i,k}$. For any component i, it is assumed that summation of all incoming flows is equal to the summation of all outgoing flows, i.e. $\sum_{j} P_{ji,k} = \sum_{j} P_{ij,k}$, in other words, internal losses are negligible (efficiency of 1.0), otherwise, internal losses must be modelled. Now, the power distribution's weight in (7) can be written as,

$$\underset{P,x}{\text{minimize}} \left[\left(\sum_{(i,j)\in\mathcal{A}} w_{ij} x_{ij} + \sum_{i\in\mathcal{N}} w_i x_i \right) + \left(\sum_{(i,j)\in\mathcal{A}} \left(w_{ij}^{\text{kW}} \sum_{k\in K} P_{ij,k} \right) + \sum_{i\in\mathcal{N}} \left(w_i^{\text{kW}} \sum_{k\in K} \sum_{j} P_{ji,k} \right) \right) \right] \tag{8}$$

Similarly to (7), the first term in (8) is the summation of all fixed weights, and the second term is the summation of all variables weights (which depends on the amount of power flow converted/transferred). In (8), all power flows are expressed in terms of the connections' power flows $P_{ij,k}$. The functional requirements are enforced with a set of connectivity constraints and the reliability requirements enforce sufficient number of components and connections to maintain the critical load l of the distribution path k (for which $y_{sl} = 1$) connected to the generator s.

The first connectivity constraint is the power flow balance on every component $i \in \mathcal{N}$, i.e. nodal equation. Within each distribution path k, generator s supplies a power of d_k^{GEN} to the load l that consumes a power of d_k^{LOAD}. In general, each component in path k is a node that can host a generator, a load, a conversion device, or a bus (step node). Thus, for the generator node, $d_k^{GEN} \neq 0$ and $d_k^{LOAD} = 0$, and for the load node, $d_k^{GEN} = 0$ and $d_k^{LOAD} \neq 0$. For the rest of cases, $d_k^{LOAD} = d_k^{GEN} = 0$. Then, for every component $i \in \mathcal{N}$ of the distribution path $k \in K$,

$$\sum_{i|(j,i)\in\mathcal{A}} P_{ji,k} - \sum_{i|(i,j)\in\mathcal{A}} P_{ij,k} + d_k^{LOAD} - d_k^{GEN} = 0 \qquad \forall i \in \mathcal{N}, \forall k \in K \tag{9}$$

The first term in (9) represents the incoming flow, while the second term is the outgoing power flow. In the case of converters, (9) could include a loss function in terms of total power converted. By convention, incoming flow and load demand is positive, while outgoing flow and generation power is negative. For every component i that is selected, some of its incoming (and outgoing) connections can be selected, then,

$$x_{ij} \leq x_i \qquad \forall (i,j) \in \mathcal{A}, \forall i \in \mathcal{N} \tag{10}$$

Power flow is allowed only on selected connections and components (otherwise, it is 0). Given that distribution path k delivers power demand d_k^{LOAD} from a generator supplying d_k^{GEN}, the flow of the distribution path k is limited to d_k^{LOAD}. For simplicity notation, let d_k be equal to d_k^{LOAD}, then,

$$P_{ij,k} \leq d_k x_{ij} \qquad \forall (i,j) \in \mathcal{A}, \forall k \in K \tag{11}$$

$$\sum_{j\in\mathcal{N}} P_{ji,k} \leq d_k x_i \qquad \forall i \in \mathcal{N}, \forall k \in K \tag{12}$$

Constraints (9)-(12) allow connectivity between power distribution components and connections. Now, the reliability constraints enforce that a sufficient number of components and connections are selected in order to ensure that the distribution path k supplies its critical load from the corresponding generator. Recall that the template $\mathcal{G} = \{\mathcal{N}, \mathcal{A}\}$ contains all permissible interconnections between components (from generators to loads) so that the critical load receives power from an adequate power conversion level or bus. A distribution path k is, on its simplest structure, a series system that connects a generator with a critical load. Then, the reliability constraint of a distribution path k (series system) can be written as,

$$\left(\prod_{j|(i,j)\in k} x_{ij} r_{ij} \right) \left(\prod_{i\in k} x_i r_i \right) \geq r_{sl} \qquad \forall k \in K \tag{13}$$

where r_i is the reliability of component i, r_{ij} is the reliability of connection ij, and r_{sl} is the reliability target (equal to the reliability of the distribution path k of the GS&GLP step). In this case, (13) implements the distribution path k as a series system, which fails if any of its components fail. In order to avoid such behavior, network design approaches that can be used to build a failure resistant system which is amenable for reconfiguration. These approaches are known as resilient designs [21] and they synthesize a number of w alternate paths in the case that any component or connection of the distribution path k (series system) fails. Such reliability constraint can be written as,

$$1 - \prod_{w} \left(1 - \left(\prod_{j|(i,j)\in k} x_{ij} r_{ij} \right) \left(\prod_{i\in k} x_i r_i \right) \right) \geq r_{sl} \qquad \forall k \in K \tag{14}$$

The left-hand side of (14) is the reliability of a parallel-series system. Depending on the reliability target r_{sl} of the path k, (13) could not synthesize a distribution path but (14) can surely combine several alternate paths such that the reliability target r_{sl} is achieved. Up to this point, a minimum-weight topology for a power distribution system can be synthesized using (1)-(14). In order to get closer to a final implementation, additional constraints on the components' selection and location are included. These constraints allow to determine the impact of location and sizing on the overall system's construction.

C. Location constraints for power distribution design

Let the 2- dimensional space availability on a MEA airplane be represented with two straight lines, one (horizontal) spanning from the left- to the right-wing tips (wing line), and the other (vertical) crossing from the cockpit to the APU at the back of the plane (fuselage line). The components can be located in any position along each one of these two lines. If selected, a component i is located in a unique position (m, n), then,

$$m_i^{\mathrm{MIN}} x_i \le m_i \le m_i^{\mathrm{MAX}} x_i \qquad \forall i \in \mathcal{N} \qquad (15)$$

$$n_i^{\mathrm{MIN}} x_i \le n_i \le n_i^{\mathrm{MAX}} x_i \qquad \forall i \in \mathcal{N} \qquad (16)$$

where m_i, n_i are the abscissa and ordinate location variables for component i; $m_i^{\mathrm{MIN}}, m_i^{\mathrm{MAX}}$ are the permissible minimum and maximum abscissa location, and $n_i^{\mathrm{MIN}}, n_i^{\mathrm{MAX}}$ are the minimum and maximum ordinate location. If $x_i = 0$ in (15)-(16) (component i is not selected), the location variables m_i, n_i are automatically set to 0. The connection's length l_{ij} connecting two components i and j (i.e. the length of connection i, j) can be determined by,

$$l_{ij} = |m_i - m_j| + |n_i - n_j| \qquad \forall (i,j) \in \mathcal{A} \qquad (17)$$

In (17), the length is measured as the sum of the absolute difference of the components' locations (1-Norm). There is no need to use Euclidean distance (2-Norm) because the components are only located along the two lines (the wing line, and the fuselage line). To ensure that each component has a unique location (no two components are placed on the same position), the constraint for unique location is,

$$l_{ij} \ge \epsilon x_{ij} \qquad \forall (i,j) \in \mathcal{A} \qquad (18)$$

where ϵ is a tolerance gap (distance gap) to force either $|m_i - m_j| \ge \epsilon$ or $|n_i - n_j| \ge \epsilon$ when connection x_{ij} is selected (located either along the wing or the fuselage lines only), hence $m_i \ne m_j$ or $n_i \ne n_j$. The selection of components is completed by including the location constraints (15)-(18). In addition, these constraints require additional conditions on cable sizing (connection) due to the reason that location affects length, voltage drop, and ampacity.

D. Constraints for cabling sizing

It is assumed that a component's location does not influence its size, i.e. the component is selected only if it satisfies the connectivity and reliability constraints. However, the connection's size is certainly influenced by the location of the components that it connects. There are two main characteristics that change in a connection depending on its length: 1) ampacity (gauge type), and 2) voltage drop. The selection of the connection's ampacity depends on the power flow transferred between any components ij, while the voltage drop depends on the maximum permissible voltage drop and connection's length. Consider a set $\mathcal{O}$ of standard wire gauges that are available (commercial cables). Let a Boolean $c_{ij,o}^{\mathrm{AWG}}$ select a standard wire gauge o ($o \in \mathcal{O}$), such that $c_{ij,o}^{\mathrm{AWG}} = 1$ for the selected wire gauge o. Then, each connection x_{ij} that has been selected must have a unique standard wire gauge, so,

$$\sum_{o \in \mathcal{O}} c_{ij,o}^{\mathrm{AWG}} \le x_{ij} \qquad \forall (i,j) \in \mathcal{A} \qquad (19)$$

Each standard wire gauge o has an ampacity I_o^{AWG}, a resistance per-unit-of-length r_o^{AWG}, a weight per-unit-of-length w_o^{AWG}. Note that w_o^{AWG} is different from w_{ij}^{kW} (inverse of the power density or power-to-weight ratio). Let the operating voltage of the connection ij be V_{ij}^{OP} (this voltage has been pre-defined according to the power conversion levels considered in the template $\mathcal{G} = \{\mathcal{N}, \mathcal{A}\}$). The ampacity constraint ensures that the current flowing through connection ij is less than or equal to the ampacity I_o^{AWG} corresponding to the wire gauge o, then,

$$\frac{P_{ij}}{V_{ij}^{\mathrm{OP}}} \le \sum_{o \in \mathcal{O}} I_o^{\mathrm{AWG}} c_{ij,o}^{\mathrm{AWG}} \qquad \forall (i,j) \in \mathcal{A} \qquad (20)$$

where $P_{ij}/V_{ij}^{\mathrm{OP}}$ represent the current flowing through connection ij. Now, let the maximum permissible voltage drop be $V_{p.u.}^{\mathrm{DROP}}$ (in p.u.). The voltage drop constraint ensures that the voltage drop across the connection is less than or equal to the maximum permissible voltage drop, then,

$$\frac{P_{ij}}{V_{ij}^{\mathrm{OP}}} r_o^{\mathrm{AWG}} l_{ij} \leq V_{p.u.}^{\mathrm{DROP}} V_{ij}^{\mathrm{OP}} + V_{ij}^{\mathrm{OP}}\left(1 - c_{ij,o}^{\mathrm{AWG}}\right) \qquad \forall (i,j) \in \mathcal{A}, \forall o \in \mathcal{O} \tag{21}$$

where (21) is written using the Big-M method [22]. When $c_{ij,o}^{\mathrm{AWG}} = 1$ (i.e. a wire gauge is chosen for a selected connection), constraint (21) enforces the actual voltage drop to be less than or equal to $V_{p.u.}^{\mathrm{DROP}} V_{ij}^{\mathrm{OP}}$; if $c_o^{\mathrm{AWG}} = 0$, the actual voltage drop is arbitrarily set. In summary, constraints (15)-(21) are introduced as component location and cabling selection constraints and these constraints aim to determine the cabling size of the power distribution system. If these constraints were not considered in the design, the topology's weight would still need to be adjusted after power system architecture synthesis, possibly at later refinement steps after PDD. In the following section, the linearization of constraints (6), (13), (14), and (21) will be discussed.

IV. Linear Transformations for Non-linear constraints

The reliability constraints (6), (13), and (14) are non-linear due to the products between Boolean variables. In the GS&GLP step, the reliability contribution of any generator or distribution path is 0 when it is not selected. Similarly, the reliability contribution of components and connections is 0 when they are not selected in the PDD step. In the case of the multiplications in (6), (13), and (14), logarithm functions can linearize the product series, then,

$$\sum_s y_{sl} \ln(1 - r_{sl} r_s) \leq \ln\left(1 - r_{\mathrm{TARGET},\, l}\right) \qquad \forall l \tag{22}$$

where the Boolean y_{sl} has been taken outside the logarithm because $\ln(1 - y_{sl} r_{sl} r_s) = 0$ when $y_{sl} = 0$, then if y_{sl} is multiplied by $\ln(1 - r_{sl} r_s)$ the result is the same for $y_{sl} = 0$ (same results are obtained for $y_{sl} = 1$). For (13),

$$\sum_{j|(i,j)\in k} z_{ij,k} \ln\left(r_{ij}\right) + \sum_{i\in k} z_{i,k} \ln(r_i) \geq \ln(r_{sl}) \qquad \forall k \in K \tag{23}$$

where $z_{ij,k}$ and $z_{i,k}$ are Boolean variables that determine if a connection x_{ij} and component x_i are part of the distribution path k (series system). Linearization of (14) (parallel-series system) requires the selection of a specific number of alternate paths. Let a Boolean q_w be set ($q_w = 1$) if an alternate path w is constructed. Also, let the series system of the inner product have a reliability r_q and let this series system be linearized in the same way as presented in the left-hand side of (23). Then, the linearization of (14) can be written as,

$$\sum_w q_w \ln\left(1 - r_q\right) \leq \ln(1 - r_{sl}) \qquad \forall\, k \in K \tag{24}$$

$$\sum_{j|(i,j)\in\mathcal{A}} z_{ij,k} \ln\left(r_{ij}\right) + \sum_{i\in\mathcal{N}} z_{i,k} \ln(r_i) = \ln\left(r_q\right) \qquad \forall k \in K \tag{25}$$

In the case of the linearization of absolute values in constraints, the reader can refer to [23] to transform (17). Finally, the left hand-side of (21) presents a non-linear product of two continuous variables, P_{ij} (power flow) and l_{ij} (connection's length that depends on the location of components). One alternative is to use two-dimensional piecewise linear functions [24], [25], or McCormick envelopes [26], [27]. The main idea is to produce tight constraints in a form of grid such that a value could be located according to the selection of specific bounds (lower and upper). In any case, the product $P_{ij} l_{ij}$ is replaced by a third variable δ_{ij} which is constrained accordingly, then,

$$\left(V_{ij}^{\mathrm{OP}}\right)^{-1} r_o^{\mathrm{AWG}} \delta_{ij} \leq V_{p.u.}^{\mathrm{DROP}} V_{ij}^{\mathrm{OP}} + V_{ij}^{\mathrm{OP}}\left(1 - c_{ij,o}^{\mathrm{AWG}}\right) \qquad \forall (i,j) \in \mathcal{A}, \forall o \in \mathcal{O} \tag{26}$$

V. Case Study

A case study is presented to exemplify the use of the design formulation detailed in Section III for a short-haul small MEA application. Two different MEA power distribution systems are synthesized to satisfy the requirements of the demands listed in Table 1. In both cases, total load is 100kW. L_2 and L_3 are critical in case 1, while all loads are critical in case 2. The permissible voltage drop across any connection is 2.5% of the nominal HV or LV DC value.

Table 1 Load power requirements for the MEA power systems

	L_1	L_2	L_3	L_4
Case 1 L_l [kW]	10	40	40	10
Case 2 L_l [kW]	50	25	25	-

The reliability target for all the loads is 1.0×10^{-3}, which is the probability of not being supplied with electrical power, i.e. $1 - r_{\text{TARGET},l}$. Loads L_2, L_3 are LV DC, and L_1, L_4 are HV DC loads. The design templates $\mathcal{G} = \{\mathcal{g}, \mathcal{E}\}$ for the GS&GLP and $\mathcal{G} = \{\mathcal{N}, \mathcal{A}\}$ for the PDD steps are shown in Fig. 1. Due to the reason that there are LV DC and HV DC loads, the system is assumed to have two voltage levels (high- and low-voltage DC), which require power conversion devices between HV and LV DC levels. Hence, there are three types of power distribution components: source matrix contactor (HV box) on HV level, HV/LV DC converter, and LV bus (LV DC level). In the LV DC side, LV DC buses distribute the power to the LV DC loads. In the HV DC side, the HV box distributes the generation power which is assumed to be rectified (e.g. AC generator with rectifier unit). Power conversion is performed by HV/LV DC converters (e.g. dual active bridge or DAB) which can be fed from several generators via the HV box and their LV DC output can be paralleled to other converters to supply the same load bus.

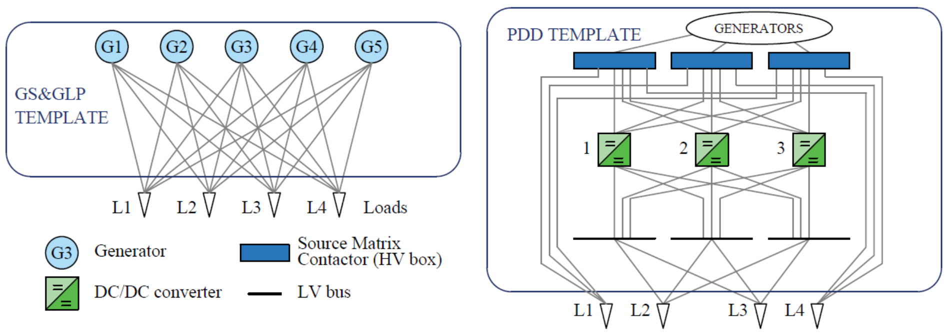

Fig. 1 Templates for the GS&GLP step (left), and PDD step (right)

The HV box is a fast switching device that receives power from multiple generators, and routes it either to the HV/LV DC converters, or to the HV DC loads directly. The HV box switches generation power in such a way that there is no risk of cross-connecting generators. The GS&GLP and PDD optimization problems are in Table 2. The components' parameters of the abstraction platforms for GS&GLP and PDD are listed in Table 3. The components' power capacities are expressed as a range $[\cdot]$ between minimum and maximum (P_s^G for the generators). For the generators' weight w_s, a function in terms of the power rating $w_s(P_s^G)$ is used, while for the distribution platform, there are fixed weight values (w_i, w_{ij}) and values for the inverse of the power density $(w_i^{\text{kW}}, w_{ij}^{\text{kW}})$ $[(\text{kW/kg})^{-1}]$. Finally, the components' reliabilities are presented as the probability of not supplying power, i.e. $1 - r_s$ in the case of GS&GLP, and $1 - r_i$ or $1 - r_{ij}$ for the PDD. In the GS&GLP, the abstraction of the distribution path requires a model in which the only parameter is the reliability r_{sl}, which is related to the allocation of the load's power to a specific generator (i.e. the probability that the distribution path is able to deliver power from generator s to load l). On the other hand, the location of the generators and loads is fixed. The two LV DC loads are located in the wings (one on the left, and the other on the right wing), one LV DC load is located in the airplane's cockpit, and the HV DC load is located at the back of the aircraft. The generators are located next to the engines (left and right), and there is also one generator in the tail of the aircraft (e.g. driven by the auxiliary power unit). The rest of the power distribution components are free to be located along the wing or the fuselage lines only.

Table 2 Optimization formulations for the synthesis of a MEA architecture

GS&GLP step	PDD step
min Eq. (1)	**min** Eq. (8)
subject to Eqs. (2) – (5), (22)	**subject to** Eqs. (9) – (12), (23) or (24) – (25), (15) – (21)

The selected components and connections from the GS&GLP and PDD assessments produce an optimal power system that has minimum weight. The case study is solved using a Windows High Spec PC Intel Xeon 64-bit 3.60GHz running CPLEX Studio IDE 12.9.0 [28]. The optimal topologies and location for cases 1 and 2 are shown in Fig. 2.

Table 3 Components' parameters of the abstraction platforms for GS&GLP and PDD

	GS&GLP		PDD			
	Generators	Distribution path	HV box	Converters	LV DC bus	Connections
P [kW]	[25, 150]	-	[0, 500]	[0, 800]	[0, 500]	[0, 1000]
w [kg]	$12.2 + 0.1428P_s^G$	-	1.8^* 0.1^{**}	8.5^* 0.3^{**}	1.8^* 0.1^{**}	0.20^* 0.05^{**}
$1 - r$	1.0×10^{-5}	$[1.0 \times 10^{-1}, 3.0 \times 10^{-4}]$	5.0×10^{-6}	1.0×10^{-4}	5.0×10^{-6}	2.0×10^{-6}

* Fixed weight, ** variable weight in terms of the inverse of the power density

It can be seen from Fig. 2 that topologies for cases 1 and 2 have different number of components and these components are located differently in the aircraft (except the generators and loads). The results reflects that the initial requirements, i.e. load demand, load criticality, and type of supply (HV or LV DC), produces different optimal architectures. In this case, both power systems were synthesized to reach minimum total weight.

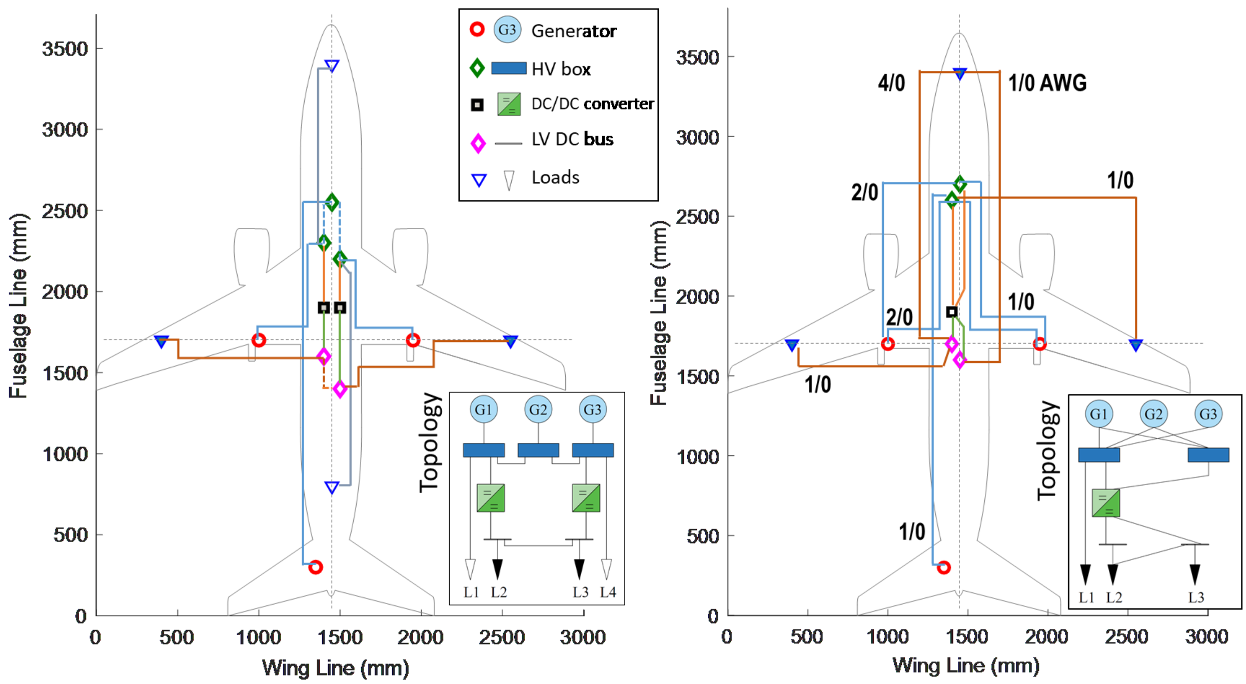

Fig. 2 Optimal architecture for MEA power system, case 1 (left), case 2

VI. Conclusion

An optimization-based formulation for the design and synthesis of a MEA power system architecture that has minimum weight has been proposed. Functional and safety requirements have been translated to a set of connectivity constraints and reliability constraints. In an attempt to close the gap between architecture and physical implementation, a set of location and cabling sizing constraints were introduced because system's weight depends directly on these parameters. Given the non-linearity of some of the reliability, location, and cabling sizing constraints, linearization is introduced as an enabler to obtain global optimum via commercial Mixed Integer Linear Programming solver. The results have shown that the initial load requirements influences heavily on the topology synthesized.

Acknowledgments

This work is funded by the INNOVATIVE doctoral programme. The INNOVATIVE programme is partially funded by the Marie-Curie Initial Training Networks (ITN) action (project number 665468), and partially by the Institute for Aerospace Technology (IAT) at the University of Nottingham.

References

[1] G. Buticchi, S. Bozhko, M. Liserre, P. Wheeler, and K. Al-Haddad, "On-board microgrids for the more electric aircraft - Technology review," *IEEE Trans. Ind. Electron.*, vol. 66, no. 7, pp. 5588–5599, 2019.

[2] B. Karanayil, M. Ciobotaru, and V. Agelidis, "Power Flow Management of Isolated Multiport Converter for More Electric Aircraft," *IEEE Trans. Power Electron.*, vol. 8993, no. c, pp. 1–1, 2016.

[3] I. Christou, A. Nelms, I. Cotton, and M. Husband, "Choice of optimal voltage for more electric aircraft wiring systems," *IET Electr. Syst. Transp.*, vol. 1, no. 1, pp. 24–30, 2011.

[4] X. Roboam, B. Sareni, and A. De Andrade, "More Electricity in the Air," *Industrial Electronics Magazine, IEEE*, vol. 6, no. 4, pp. 6–17, Dec-2012.

[5] J. Chen, C. Wang, and J. Chen, "Investigation on the selection of electric power system architecture for future more electric aircraft," *IEEE Trans. Transp. Electrif.*, vol. 4, no. 2, pp. 563–576, 2018.

[6] A. Tardy *et al.*, "Towards More Optimization for Aircraft Energy Conversion Systems," in *MEA 2015 – More Electric Aircraft –*, 2015, no. February, p. 16.

[7] J. Brombach, A. Lucken, B. Nya, M. Johannsen, and D. Schulz, "Comparison of Different Electrical HVDC- Architectures for Aircraft Application," in *Electrical Systems for Aircraft, Railway and Ship Propulsion (ESARS), 2012*, 2012, pp. 1–6.

[8] L. Jun and X. Huibin, "Reliability Analysis of Aircraft Equipment Based on FMECA Method," in *2012 International Conference on Solid State Devices and Materials Science*, 2012, vol. 25, pp. 1816–1822.

[9] R. D. Telford, S. J. Galloway, and G. M. Burt, "Evaluating the reliability & availability of more-electric aircraft power systems," in *Proceedings of the Universities Power Engineering Conference*, 2012.

[10] A. A. Recalde, J. A. Atkin, and S. V. Bozhko, "Optimal Design and Synthesis of MEA Power System Architectures considering Reliability Specifications," *IEEE Trans. Transp. Electrif.*, pp. 1–1, 2020.

[11] H. Ounis, B. Sareni, X. Roboam, and A. De Andrade, "Multi-level integrated optimal design for power systems of more electric aircraft," in *Mathematics and Computers in Simulation*, 2016, vol. 130, pp. 223–235.

[12] P. Nuzzo *et al.*, "A contract-based methodology for aircraft electric power system design," *IEEE Access*, vol. 2, pp. 1–25, 2014.

[13] J. Finn, P. Nuzzo, and A. Sangiovanni-Vincentelli, "A mixed discrete-continuous optimization scheme for Cyber-Physical System architecture exploration," in *2015 IEEE/ACM International Conference on Computer-Aided Design (ICCAD)*, 2015, pp. 216–223.

[14] A. A. Recalde, S. Bozhko, and J. Atkin, "Design of More Electric Aircraft DC Power Distribution Architectures considering Reliability Performance," in *AIAA Propulsion and Energy 2019 Forum*, 2019.

[15] A. Pinto, S. Becz, and H. Reeve, "Correct-by-Construction Design of Aircraft Electric Power Systems," in *10th AIAA Aviation Technology, Integration, and Operations (ATIO) Conference*, 2010, pp. 1–11.

[16] J.-K. Mueller *et al.*, "Design Considerations for the Electrical Power Supply of Future Civil Aircraft with Active High-Lift Systems," *Energies*, vol. 11, no. 2, p. 179, 2018.

[17] N. A. of Sciences Engineering and Medicine and C. on Propulsion, *Commercial Aircraft Propulsion and Energy Systems Research*. Washington, D.C.: National Academies Press, 2016.

[18] A. Sangiovanni-Vincentelli, "Quo vadis, SLD? Reasoning about the trends and challenges of system level design," *Proc. IEEE*, vol. 95, no. 3, pp. 467–506, 2007.

[19] S. Becz, A. Pinto, L. E. Zeidner, R. Khire, A. Banaszuk, and H. M. Reeve, "Design system for managing complexity in aerospace systems," in *10th AIAA ATIO/ISSMO Conference*, 2010, pp. 1–7.

[20] P. Nuzzo, J. Finn, M. Mozumdar, and A. Sangiovanni-vincentelli, "Platform-Based Design Methodology and Modeling for Aircraft Electric Power Systems," in *Proceedings of Green Energy and Systems Conference*, 2013, pp. 1–7.

[21] R. L. Y. Chen and C. A. Phillips, "K-Edge failure resilient network design," *Electron. Notes Discret. Math.*, vol. 41, pp. 375–382, 2013.

[22] E. Burke *et al.*, *Search Methodologies- Introductory Tutorials in Optimization and Decision Support Techniques*, 1st ed. UK: Springer Science & Bussiness Media, 2005.

[23] S. Boyd, B. S. P., and L. Vandenberghe, *Convex Optimization*, 1st ed. Cambridge University Press, 2004.

[24] C. D'Ambrosio, A. Lodi, and S. Martello, "Piecewise linear approximation of functions of two variables in MILP models," *Oper. Res. Lett.*, vol. 38, no. 1, pp. 39–46, Jan. 2010.

[25] R. Misener and C. A. Floudas, "Piecewise-Linear Approximations of Multidimensional Functions," *J. Optim. Theory Appl.*, vol. 145, no. 1, pp. 120–147, Apr. 2010.

[26] P. M. Castro, "Tightening piecewise McCormick relaxations for bilinear problems," *Comput. Chem. Eng.*, vol. 72, pp. 300–311, Jan. 2015.

[27] M. L. Bergamini, P. Aguirre, and I. Grossmann, "Logic-based outer approximation for globally optimal synthesis of process networks," *Comput. Chem. Eng.*, vol. 29, no. 9, pp. 1914–1933, Aug. 2005.

[28] "IBM ILOG CPLEX Optimization Studio." [Online]. Available: https://www.ibm.com/products/ilog-cplex-optimization-studio.