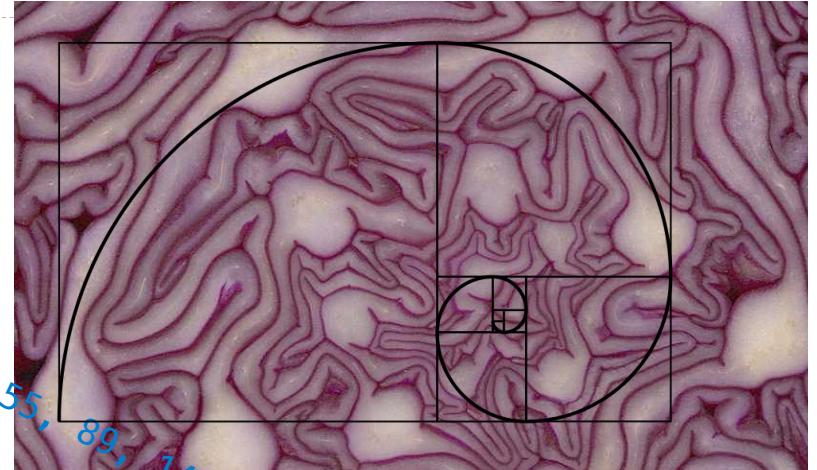
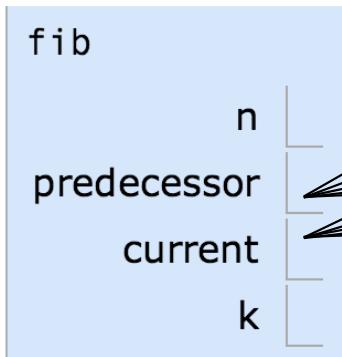


61A Lecture 4

Announcements

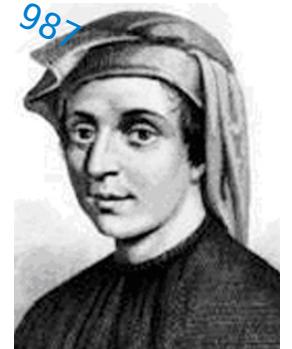
Iteration Example

The Fibonacci Sequence



```
def fib(n):
    """Compute the nth Fibonacci number, for N >= 1."""
    pred, curr = 0, 1 # Zeroth and first Fibonacci numbers
    k = 1             # curr is the kth Fibonacci number
    while k < n:
        pred, curr = curr, pred + curr
        k = k + 1
    return curr
```

The next Fibonacci number is the sum of the current one and its predecessor



Discussion Question 1



$$n^2$$



$$(n + 1)^2$$



$$2 \cdot (n + 1)$$



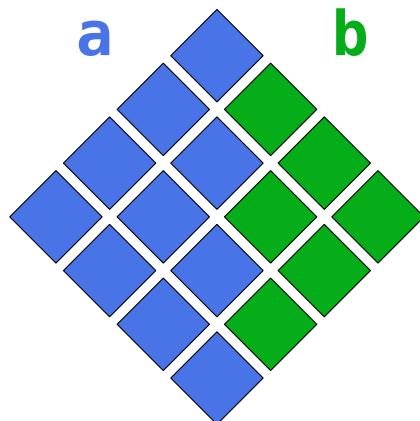
$$n^2 + 1$$



$$n \cdot (n + 1)$$

What does pyramid compute?

```
def pyramid(n):
    a, b, total = 0, n, 0
    while b:
        a, b = a+1, b-1
        total = total + a + b
    return total
```



I'm still here



Designing Functions

Characteristics of Functions

```
def square(x):
    """Return X * X."""
```

```
def fib(n):
    """Compute the nth Fibonacci number, for N >= 1."""
```

A function's domain is the set of all inputs it might possibly take as arguments.

x is a real number

n is an integer greater than or equal to 1

A function's range is the set of output values it might possibly return.

*returns a non-negative
real number*

returns a Fibonacci number

A pure function's behavior is the relationship it creates between input and output.

*return value is the
square of the input*

return value is the nth Fibonacci number

A Guide to Designing Function

Give each function exactly one job.



not



Don't repeat yourself (DRY). Implement a process just once, but execute it many times.

Define functions generally.

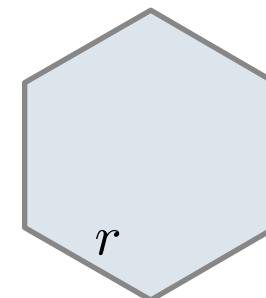
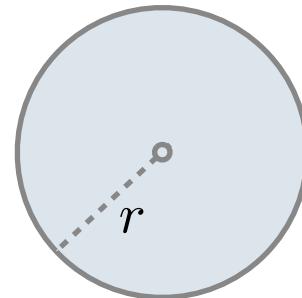
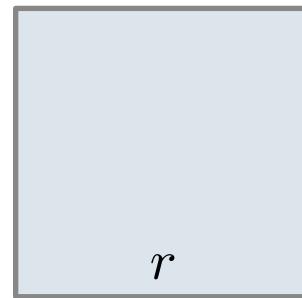


Generalization

Generalizing Patterns with Arguments

Regular geometric shapes relate length and area.

Shape:



Area:

$$\boxed{1} \cdot r^2$$

$$\boxed{\pi} \cdot r^2$$

$$\boxed{\frac{3\sqrt{3}}{2}} \cdot r^2$$

Finding common structure allows for shared implementation

(Demo)

Higher-Order Functions

Generalizing Over Computational Processes

The common structure among functions may be a computational process, rather than a number.

$$\sum_{k=1}^5 k = 1 + 2 + 3 + 4 + 5 = 15$$

$$\sum_{k=1}^5 k^3 = 1^3 + 2^3 + 3^3 + 4^3 + 5^3 = 225$$

$$\sum_{k=1}^5 \frac{8}{(4k-3) \cdot (4k-1)} = \frac{8}{3} + \frac{8}{35} + \frac{8}{99} + \frac{8}{195} + \frac{8}{323} = 3.04$$

(Demo)

Summation Example

```
def cube(k):
    return pow(k, 3)
```

Function of a single argument
(not called "term")

```
def summation(n, term):
    """Sum the first n terms of a sequence.
```

A formal parameter that will
be bound to a function

```
>>> summation(5, cube)
225
"""
total, k = 0, 1
while k <= n:
    total, k = total + term(k), k + 1
return total
```

The cube function is passed
as an argument value

$0 + 1 + 8 + 27 + 64 + 125$

The function bound to term
gets called here

Functions as Return Values

(Demo)

Locally Defined Functions

Functions defined within other function bodies are bound to names in a local frame

```
A function that  
returns a function  
  
def make_adder(n):  
    """Return a function that takes one argument k and returns k + n.  
  
    >>> add_three = make_adder(3)  
    >>> add_three(4)  
    7  
    """  
def adder(k):  
    return k + n  
return adder
```

The name `add_three` is bound to a function

A def statement within another def statement

Can refer to names in the enclosing function

Call Expressions as Operator Expressions

