A Project Report

On

HOPFIELD NEURAL NETWORK

BY

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It is an immense pleasure for us (Rishi & Sasanka) to work on **“HOPFIELD NEURAL NETWORKS”** as part of our 4^{rd} Year project.

We are grateful to work under the guidance of **Professor G. Rama Murthy**, Faculty of Computer Science Department, Mahindra University, without whom the project would not have been successful.

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Certificate

This is to certify that the project report entitled "**HOPFIELD NEURAL NETWORK**" submitted by Mr. S RISHI SHRINIVAS (HT No. 18XJ1A0543),
Mr. AKILI VENKATA SASANKA SAI (HT No. 18XJ1A0205), in partial fulfillment of the requirements of the course PR 301, Project Course, embodies the work done by him under my supervision and guidance.

G. Rama Murthy

Mahindra University, Hyderabad.

Date:

ABSTRACT

Decision making process is one of the most important aspects in the field of Computer Science. Artificial Neural Networks is the area that widely deals with decision making and hopes to replicate human brain. Transforming how the brain works, or at least how we think the brain works, on to simple terms leads us to understand what a neuron/node is, how it is connected to other neurons and how the whole collection of neurons/network work. We overly simplify this process by making the inputs bipolar/binary and so it is corresponding outputs.

We started off from a type of neuron called McCulloch-Pitts-Neuron. This neuron has a set of inputs, set of weights associated with each input, a certain threshold and an output associated for each neuron. The geometrical structure of this network will depend upon the number of neurons in the network. Every structure is a unit structure with origin at centre. A basic structure will be a square for two neurons, then for three neurons it is a cube and as the number of neurons increase, the network's geometry will be of a hyper cube. The vertices will be bipolar value, so the state of the network at any given instant will be one of the corners of the hyper cube.

The Hopfield model is a collection of simple nodes called McCulloch-Pitts-Neuron. Each of these neurons have multiple bipolar inputs and single bipolar output. Also, there is a threshold value that determines whether the neuron needs to be fired or not. There are synaptic weights associated with each edge connecting a node, which is symmetric. The summation of these weights multiplied with the bipolar outputs of each neuron is computed. This value is compared with the threshold and if it exceeds the threshold only then the current neuron’s state is +1, else it is -1.

The Hopfield neural network with multiple layers is introduced in this project. It is observed that such an model also works as an associative memory. In Hopfield Association Memory(HAM), the synaptic weight matrix is assumed to be a symmetric matrix with non negative diagonal elements. This project explores the Hopfield Network whose synaptic weight matrix is a block symmetric matrix, unlike the traditional symmetric matrix.

Interestingly this modified network also results and performs similar to the traditional Hopfield network. The advantages of this approach led us to incrementally expand the network and check for its convergence. One of it's applications can be understanding memory recall.

Further, Dynamics of Complex Recurrent Hopfield Neural Network (CRHNN) whose synaptic weight matrix is non-hermition is investigated. The concepts of left side and right side state updation is introduced and used to demonstrate that CRHNN exhibits “Orientation Selectivity” with regard to memory states reached.

CONTENTS

INTRODUCTION

The main task we are trying to perform is to replicate a human brain. The main challenge is to translate human thinking into a machine code. How well we can make the computer understand and identify objects depends on the approach we decide to tackle the problem. There are several approaches to train the network, appropriate one is choosing to increase efficiency.

This project explores a new way of approaching the Hopfield neural network. In the proposed Artificial Neural Network, connections between neurons in the same layers are Symmetric. But the synaptic weight matrix, in general(between neurons in different layers) is only block symmetric and need not be a symmetric matrix.

The Hopfield Network updates in SERIAL and FULLY PARALLEL modes of operations. Since the expanded model has multiple networks clubbed layer wise, we introduce the notion of LAYER WISE SERIAL updation and LAYER WISE FULLY PARALLEL updation.
The model proposed works as an associative memory. We can combine multiple Hopfield networks with same or different weight matrices. The network structure is in such a way that the connection between neurons of different layers need not be symmetric, but only the connection between neurons of the same matrix must be symmetric. This is an interesting result. This can be viewed as individual Hopfield networks that follows symmetric nature but the different layers not necessarily be symmetric. Since the the individual networks update parallely, one network can reach its stable state before the other. This individual set of stable states can correspond to the segments of memory recall.

Further, orientation selectivity (with regard to same initial condition as row vector and as column vector) of stable/anti-stable states reached in a Complex Recurrent Hopfield Neural Network (CRHNN) is investigated. Using a cascade/parallel connection of CRHNN's the concept of joint memory states is introduced.

Artificial Neural Network has varieties of applications like fault detection, pattern matching, sound, and facial recognitions and more. We can feed the data required and when a new similar data is entered, starting from a very vague point the network converges at the accurate data. For instance, a sound is fed during training and while testing, we start from a very noisy sound and finally converge at the original sound. Similarly, we can also calculate how good the new inputs match with the given initial inputs, thereby reducing the percentage of match between the inputs. Arduous tasks of fetching the right data becomes easy when approached using artificial Neural networks.
Personalized predictions can be made when this is implemented. For example, with the history of people's previous searches as data, the network can give personalized recommendations.
Furthermore, this could be used in the fields of security. We can initially train the device with our voice or face and set it as a device lock. Only when the patterns match the device could later start for the use.

PROBLEM DEFINITION

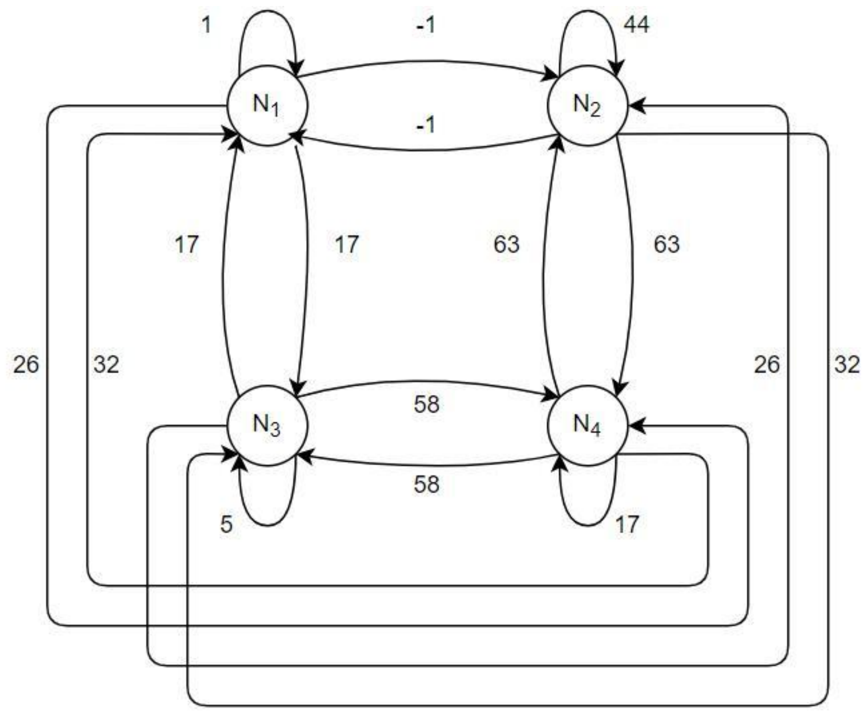
Hopfield Neural Network (HNN) model was subjected to intense research efforts leading to Bi-Directional Associative Memory, Auto/Hetero associative memories. This research paper proposes an Artificial Neural Network model of Associative memory where neurons are arranged in multiple layers and the connections between the neurons are constrained.

The model proposed is motivated by :

i. Homogeneous Associative Memories : At all layers the synaptic weight matrices are same within layers and across layer.

ii. Heterogeneous Associative Memories : These are layered ANN's which are not homogeneous. In such ANN's the connections between neurons in different layers are captured by asymmetric matrices.

Representation of the proposed network:



The above representation is of the proposed Multi-Layer Hopfield Network where in one layer nodes N1 and N2 are placed while the nodes N3 and N4 are from a different layer.

In the above representation the key point to be noted is the connection weight from node N1 to N4 is different from that of node N4 to N1. Similarly, the connection weight from node N2 to N3 is different from the connection weight from node N3 to N2.

Also, behaviour of a CRNN to both left and right side state updation for the same initial vector is discussed.

BACKGROUND WORK

The Computational and Mathematical aspects of the project is described in detail in this section:

We take an Artificial Neural Network (ANN) where the neurons are placed in multiple layers. The neurons follows the McCulloch-Pitts model. Thus, each neuron is in state +1 or -1 . Let us conside{r such a ANN with 'M' layers and 'N' neurons in each layer. Neurons are connected to ea}ch other by links with associated synaptic weights. In our model, the synaptic weight matrix is a block symmetric matrix. Thus, the closed ANN starts in an initial state vector lying on the symmetric, unit hypercube, namely 'MN' dimensional unit symmetrichypercube. The state is updated as follows:

$$V_i(m+1) = Sign\{\sum_{j=1}^{MN} W_{ij} V_j(m) - t_i\} \quad (1)$$

(Where t_i is the i'th neuron's threshold value.)

The modes of operation of the ANN is as follows:

i. Serial Mode: For any given instance, above state updating takes place strictly at one neuron

ii. Fully Parallel Mode: For any given instance, the above state updating takes place at all the "MN" neurons. i.e,

$$\overline{V}(n+1) = Sign\{\overline{W}.\overline{V}(n) - \overline{T}\} \quad (3)$$

iii. Other Parallel Modes of Operation: At any given instance, the states are updated as mentioned above, at more than one node but strictly less than all the nodes.

-> The Synaptic weight matrix $\overline{W}$ is block Symmetric I.e.

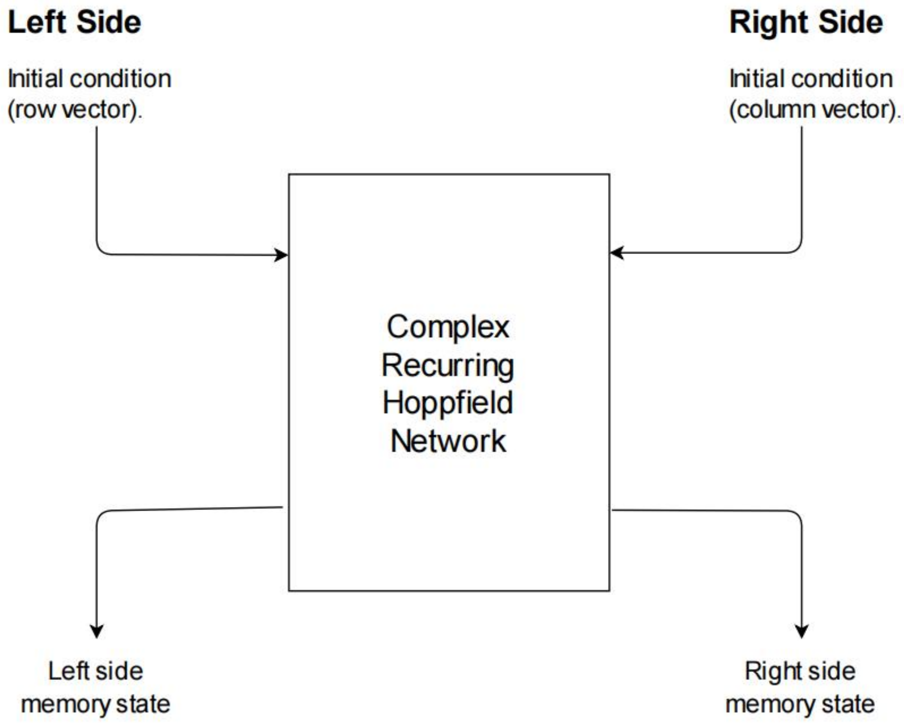
$$\overline{W} = \begin{bmatrix} \overline{W_{11}} & \overline{W_{12}} & .. & \overline{W_{1M}} \\ \overline{W_{21}} & \overline{W_{22}} & .. & \overline{W_{2M}} \\ . & . & .. & . \\ . & . & .. & . \\ \overline{W_{M1}} & \overline{W_{M2}} & .. & \overline{W_{MM}} \end{bmatrix} \quad where\ \overline{W_{ij}} = \overline{W_{ji}}\ and\ \overline{W_{ii}} = \overline{W_{ii}}^T \quad (4)$$

Also, W_{ii} provides synaptic weights between neurons in the i'th layer and W_{ij} provides Synaptic weights between neurons in the i'th layer and j'th layer. In summary, Multilayer Hopfield neural network (based on a graph with associated synaptic weight matrix) is a homogeneous nonlinear dynamical system, evolving in time, with State Space being the symmetric, unit hypercube.

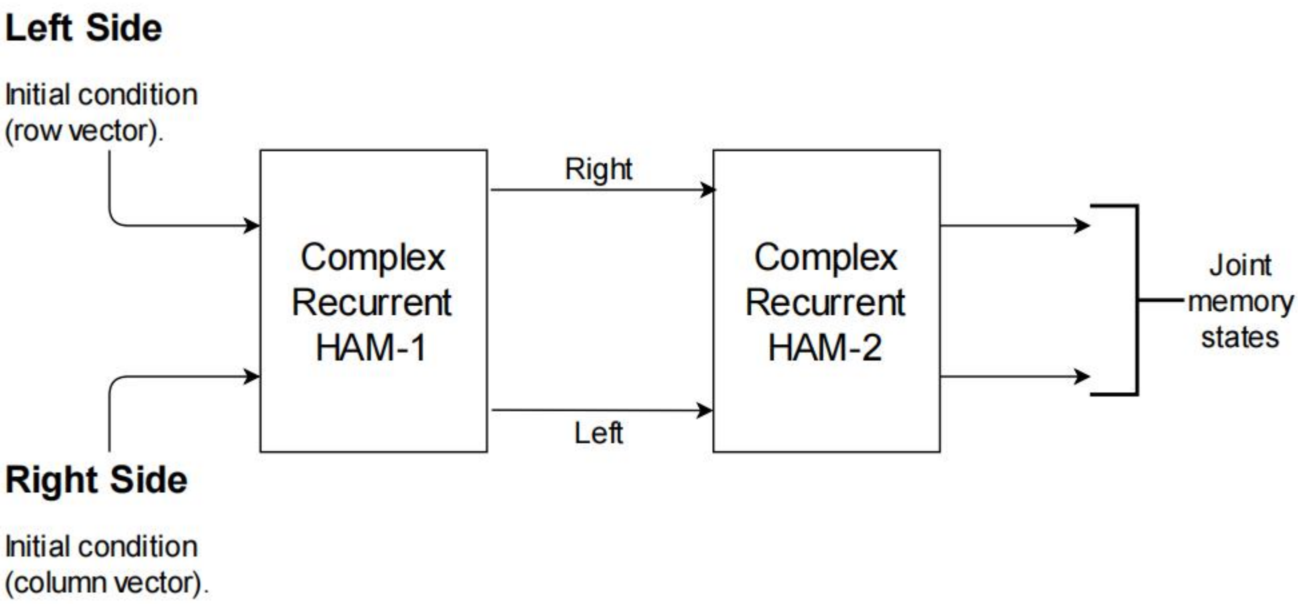
Furthermore, in the proposed complex recurrent neural network, inputs, synaptic weights, outputs are complex numbers (e.g a+jb where a,b are real numbers) i.e. Given a neuron with inputs: *{x'is}*, synaptic weights *{w'is}* (*{x'is}, {w'is}* are complex numbers), the output, y is given by

$$y = CSign\{\sum_{i=1}^{N} x_i w_i - T\}$$

Where $CSign(a + jb) = Sign(a) + jSign(b)$



Orientation selectivity in complex recurrent Hopfield network



Cascade connection of two complex recurrent Hopfield associative memories(HAM's)

IMPLEMENTATION

Consider several Hopfield Associative memories which are based on symmetric synaptic weight matrices in isolation. These are connected incrementally using a Block Symmetric synaptic weight network. In this sense, our model of ANN enables emulating biological neural network which is incrementally evolved.

It is well known that Hopfield Neural Network operates in serial, fully parallel modes of operation. In the case of Multi-layer Hopfield like neural network, we introduce the following mode of operation :-

i. Layer-wise Serial Updation :- The state of the ANN is updated in such a way that only the states of neurons in any one layer is updated at any given time(state of neurons in all other layers remains unchanged).

ii. Layer-wise Fully Parallel Mode of Operation :- At any given time, the state of neurons in all the layers of the Hopfield like neural network is updated simultaneously.

The aim is also to study the dynamics of such CVNN in serial and fully parallel modes.These modes of operation are like in Hopfield Neural Network with initial state of the network is taken as row vector for the left side updation and as column vector for the right side updation.

Left Side Updation :

$$\overline{u}(n+1) = CSign\{\overline{u}(n)\overline{W} - \overline{T}\}$$

Right Side Updation :

$$\overline{v}(n+1) = CSign\{\overline{W}\,\overline{v}(n) - \overline{T}\}$$

NUMERICAL RESULTS

Table 1. Serial Mode of Operation

Synaptic weight matrix	Initial State	Final state
$\begin{bmatrix} 15 & -120 & 57 & -119 \\ -120 & 91 & -25 & 3 \\ 57 & -119 & 7 & -80 \\ -25 & 3 & -80 & 17 \end{bmatrix}$	$[-1\ 1\ 1\ -1]$	$[1\ -1\ 1\ -1]$
$\begin{bmatrix} 1 & -2 & 3 & 7 & 8 & 9 \\ -2 & 4 & -5 & -10 & 11 & 12 \\ 3 & -5 & 6 & -13 & 14 & 15 \\ 7 & 8 & 9 & 16 & 17 & -18 \\ -10 & 11 & 12 & 17 & 19 & 20 \\ -13 & 14 & 15 & -18 & 20 & 21 \end{bmatrix}$	$[-1\ 1\ -1\ 1\ -1\ 1]$	$[-1\ -1\ -1\ -1\ -1\ -1]$

Table 2. Fully Parallel Mode of Operation

Synaptic weight matrix	Initial State	Final state(s)
$\begin{bmatrix} 1 & -1 & 17 & 26 \\ -1 & 44 & 32 & 63 \\ 17 & 26 & 5 & 58 \\ 32 & 63 & 58 & 17 \end{bmatrix}$	$[1\ -1\ -1\ 1]$	Cycle of length 2 between : $[1\ -1\ 1\ -1]$ $[-1\ -1\ -1\ 1]$
$\begin{bmatrix} 29 & -36 & 49 & 62 & 54 & -97 \\ -36 & 63 & -82 & 44 & 3 & 77 \\ 49 & -82 & 91 & -115 & 4 & 6 \\ 62 & 54 & -97 & 49 & 51 & -12 \\ 44 & 3 & 77 & 51 & 17 & -13 \\ -115 & 4 & 6 & -12 & -13 & 111 \end{bmatrix}$	$[-1\ -1\ 1\ 1\ 1\ 1]$	Cycle of length 2 between $[1\ 1\ 1\ -1\ 1\ 1]$ $[-1\ -1\ 1\ 1\ 1\ 1]$

COMPLEX RECURRENT HOPFIELD NEURAL NETWORK (CRHNN)

Example 1:

2X2 Matrix

Entered matrix is:

$$\begin{bmatrix} 17-12j & 18-11j \\ -16+13j & -12-19j \end{bmatrix}$$

The initial state is

$$\begin{bmatrix} -1+j \\ 1+j \end{bmatrix}$$

In fully parallel mode

Left side state updation:

$$\begin{bmatrix} -1+j \\ 1-j \end{bmatrix} \rightarrow \begin{bmatrix} -1+j \\ -1+j \end{bmatrix} \rightarrow \begin{bmatrix} -1+j \\ 1+j \end{bmatrix}$$

Right side state updation:

$$\begin{bmatrix} 1+j \\ 1-j \end{bmatrix} \rightarrow \begin{bmatrix} 1-j \\ -1-j \end{bmatrix} \rightarrow \begin{bmatrix} -1-j \\ -1+j \end{bmatrix} \rightarrow \begin{bmatrix} -1+j \\ 1+j \end{bmatrix}$$

Example 2:

2X2 Matrix
Entered matrix is:

$$\begin{bmatrix} 12-19j & 11+15j \\ -18+13j & 16+14j \end{bmatrix}$$

The initial state is $\begin{bmatrix} -1-j \\ 1+j \end{bmatrix}$

In serial mode
Left side state updation:

$$\begin{bmatrix} -1+j \\ 1+j \end{bmatrix} \to \begin{bmatrix} -1+j \\ -1+j \end{bmatrix} \to \begin{bmatrix} 1+j \\ -1+j \end{bmatrix} \to \begin{bmatrix} 1+j \\ -1+j \end{bmatrix} \to$$

$$\to \begin{bmatrix} 1-j \\ -1+j \end{bmatrix} \to \begin{bmatrix} 1-j \\ -1+j \end{bmatrix} \to \begin{bmatrix} -1-j \\ -1+j \end{bmatrix} \to$$

$$\to \begin{bmatrix} -1-j \\ -1-j \end{bmatrix} \to \begin{bmatrix} 1+j \\ -1-j \end{bmatrix} \to \begin{bmatrix} 1+j \\ -1-j \end{bmatrix} \to$$

$$\to \begin{bmatrix} 1-j \\ -1-j \end{bmatrix} \to \begin{bmatrix} 1-j \\ 1-j \end{bmatrix} \to \begin{bmatrix} -1+j \\ 1-j \end{bmatrix} \to$$

$$\to \begin{bmatrix} -1+j \\ 1-j \end{bmatrix} \to \begin{bmatrix} 1+j \\ 1-j \end{bmatrix} \to \begin{bmatrix} 1+j \\ 1+j \end{bmatrix} \to$$

$$\to \begin{bmatrix} 1-j \\ 1+j \end{bmatrix} \to \begin{bmatrix} 1-j \\ 1+j \end{bmatrix} \to \begin{bmatrix} -1-j \\ 1+j \end{bmatrix} \to$$

$$\to \begin{bmatrix} -1-j \\ 1+j \end{bmatrix} \to \begin{bmatrix} -1+j \\ -1+j \end{bmatrix} \to \begin{bmatrix} -1+j \\ 1+j \end{bmatrix} \to$$

$$\to \begin{bmatrix} 1+j \\ 1+j \end{bmatrix} \to \begin{bmatrix} 1+j \\ 1-j \end{bmatrix}$$

i.e A cycle of length **20** is reached

Right side state updation:

$$\begin{bmatrix} -1+j \\ 1+j \end{bmatrix} \to \begin{bmatrix} -1+j \\ 1-j \end{bmatrix} \to \begin{bmatrix} 1+j \\ 1-j \end{bmatrix} \to \begin{bmatrix} 1+j \\ -1-j \end{bmatrix} \to$$

$$\to \begin{bmatrix} 1-j \\ -1-j \end{bmatrix} \to \begin{bmatrix} 1-j \\ -1+j \end{bmatrix} \to \begin{bmatrix} -1-j \\ -1+j \end{bmatrix} \to \begin{bmatrix} -1-j \\ 1+j \end{bmatrix}$$

i.e A cycle of length **8** is reached

Cascade connection of two complex recurrent Hopfield associative memories(HAM's):

2X2 matrix

Entered matrices are:

1) $\begin{bmatrix} 1-9j & 2-8j \\ 5+6j & -4-7j \end{bmatrix}$

2) $\begin{bmatrix} -1+7j & -5+10j \\ 3-4j & 5+9j \end{bmatrix}$

The initial state is

$$\begin{bmatrix} 1-j \\ -1+j \end{bmatrix}$$

In fully parallel mode

Left side state updation:

$\begin{bmatrix} -1+j \\ 1+j \end{bmatrix} \rightarrow \begin{bmatrix} -1-j \\ -1+j \end{bmatrix}$

i.e A **anti-stable** state is reached

$\begin{bmatrix} 1-j \\ -1-j \end{bmatrix} \rightarrow \begin{bmatrix} 1+j \\ 1-j \end{bmatrix}$

i.e A cycle of length of **4** is reached

Right side updation

$\begin{bmatrix} -1-j \\ -1+j \end{bmatrix} \rightarrow \begin{bmatrix} -1+j \\ 1+j \end{bmatrix} \rightarrow \begin{bmatrix} 1+j \\ 1-j \end{bmatrix} \rightarrow \begin{bmatrix} 1-j \\ -1-j \end{bmatrix}$

i.e A cycle of length of **4** is reached

CONCLUSIONS

When the multi-layer HNN reaches a stable state, the layer-wise states could correspond to ”segments” of memory state.

In the dynamics of Multi-layer HNN, it can so happen that the state vector reaches a layer wise stable state much earlier than the stable state of entire neural network(i.e, for all layers). Such a situation resembles biological memory with regard to memory state retrieval incrementally. Also, the sequence of state updation in various layers emulates the biological memory state recall.

The modes of state updation of multi-layer HNN involves parallel and distributed memory recall mechanisms. Particularly, layer-wise serial updation corresponds to partial parallel modes of operation(in the case of HAM).

Also the incremental expansion of the network can be done with the same Hopfield Networks. In such an architecture the diagonal blocks correspond to HAMs associated with neurons in various individual layers(local HAM’s). Each such individual layer based HAM has associated stable states starting in any initial condition.

They are compared with stable states of ANN with synaptic weight matrix which is a Blocked Symmetric matrix(fully symmetric) and the initial condition vector is obtained by concatenating the individual initial condition vectors. It is realized that the following inference holds.
The final stable state reached was the concatenations of the final stable state vectors obtained individually.

Orientation Selectivity is exhibited by a Complex Recurrent Hopfield Neural Network. Such dynamic behaviour can be capitalized to store larger number of memory States that are correlated. It is reasoned that a network of CRANN’s can be effectively utilized to store joint memory States.

REFERENCES

- Hebbs rule - Hopfield network - Wikipedia
- Hopfield Network serial mode example- A Hopfield Net example
- Recurrent Hopfield network - Hopfield network Research
- Multi-Layer Hopfield like Neural Network
- Dynamics of a Recurrent HopfieldNeural Network : Orientation Selectivity