**Topics: Normal distribution, Functions of Random Variables**

1. The time required for servicing transmissions is normally distributed with *μ* = 45 minutes and *σ* = 8 minutes. The service manager plans to have work begin on the transmission of a customer’s car 10 minutes after the car is dropped off and the customer is told that the car will be ready within 1 hour from drop-off. What is the probability that the service manager cannot meet his commitment?
2. 0.3875
3. 0.2676
4. 0.5
5. 0.6987
6. The current age (in years) of 400 clerical employees at an insurance claims processing center is normally distributed with mean *μ* = 38 and Standard deviation *σ* =6. For each statement below, please specify True/False. If false, briefly explain why.
7. More employees at the processing center are older than 44 than between 38 and 44.
8. A training program for employees under the age of 30 at the center would be expected to attract about 36 employees.
9. If *X1* ~ *N*(μ, σ2) and *X*2 ~ *N*(μ, σ2) are *iid* normal random variables, then what is the difference between 2 *X*1 and *X*1 + *X*2? Discuss both their distributions and parameters.

ANS:

**Distribution of 2X1:**If X1 ~ N(μ, σ^2), then 2X1 will follow a normal distribution.

The mean of 2X1 will be 2μ because the mean of a constant times a random variable is the constant times the mean of the random variable.

The variance of 2X1 will be 4σ^2 because the variance of a constant times a random variable is the constant squared times the variance of the random variable.

So, 2X1 ~ N(2μ, 4σ^2).

**Distribution of X1 + X2:**If X1 and X2 are i.i.d., their sum will follow a normal distribution.

The mean of X1 + X2 will be μ + μ = 2μ because the mean of the sum of two random variables is the sum of their means.The variance of X1 + X2 will be σ^2 + σ^2 = 2σ^2 because the variance of the sum of two independent random variables is the sum of their variances.So, X1 + X2 ~ N(2μ, 2σ^2).In summary, both 2X1 and X1 + X2 follow normal distributions, but they have different parameters:

2X1 ~ N(2μ, 4σ^2), with a mean of 2μ and a variance of 4σ^2.

X1 + X2 ~ N(2μ, 2σ^2), with a mean of 2μ and a variance of 2σ^2.

The key difference is in their variances. The variance of 2X1 is twice as large as the variance of X1 + X2. This means that 2X1 is more spread out (has higher variability) compared to X1 + X2 when X1 and X2 are i.i.d. normal random variables.

1. Let X ~ N(100, 202). Find two values, *a* and *b*, symmetric about the mean, such that the probability of the random variable taking a value between them is 0.99.
2. 90.5, 105.9
3. 80.2, 119.8
4. 22, 78
5. 48.5, 151.5
6. 90.1, 109.9
7. Consider a company that has two different divisions. The annual profits from the two divisions are independent and have distributions Profit1 ~ N(5, 32) and Profit2 ~ N(7, 42) respectively. Both the profits are in $ Million. Answer the following questions about the total profit of the company in Rupees. Assume that $1 = Rs. 45
8. Specify a Rupee range (centered on the mean) such that it contains 95% probability for the annual profit of the company.
9. Specify the 5th percentile of profit (in Rupees) for the company
10. Which of the two divisions has a larger probability of making a loss in a given year?