Solution to Problem 1

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Environment:

Matlab 2014 b

Additional package used from MIT Strategic Engineering: http://strategic.mit.edu/

Checked several toolboxes for graph theory, including grTheory toolbox and MatlabBGL library, find the best one here at

Matlab Tools for Network Analysis (2006-2011)

http://strategic.mit.edu/downloads.php?page=matlab_networks

It is very clear and simple, functions all based on adjacency matrix with detailed description, all you need to do is downloading the package, then unzip it and set the path in Matlab.

1. Overview of problem 1

From the conception of number of connected components, we know it equals the multiplicity of 0 as an eigenvalue of the Laplacian matrix of the graph.

The Laplacian matrix ***L*** is defined as ***L=D-A***, where ***D*** is the degree matrix and ***A*** is the adjacency matrix, to compute the eigenvalues of a matrix isn't a difficult thing nowadays, though the mathematical proof requires some knowledge, there might be other ways of finding the number of connected components, for me, the conception of matrix is much easier to understand and implement in Matlab.

2. Data structure and operations

The sample data gives the number of nodes and information of edges, we need to use this to build an adjacency matrix in Matlab , the idea is simple: first construct an N x N matrix A with all zeros, if there is an edge from Node i to j, we set the value A(i,j) to be 1,

```
M=dlmread('n10.txt')%Import Delimited Numeric Data

N=M(1,1); % number of nodes
Edge=M(2:end,:);%  Create Edge table
NumOfEdge=length(Edge(:,1)) % Number of edge

B=Edge+1; % Since the indices start from 0

%Create Adjacency matrix A from Edge table B
```

```
A=zeros(N);
for i=1:NumOfEdge
    ii=B(i,1);
    jj=B(i,2);
    A(ii,jj)=1;
end
```

2.1 Number of connected components

This doesn't guarantee a symmetric matrix as we have to treat the graph as undirected, plus the functions we need require undirected graph as input, so convert A to AA as an undirected graph

Knowing the functions below are all from the package 'Matlab Tools for Network Analysis', function `num_conn_comp` is dependent of several functions in the package for computing the Laplacian and its eigenvalues

```
%convert it to the symmetric matrix for undirected graph
 AA=adj2simple(A+A');
 % plot
 % Draw a circular graph with links and nodes in order of degree
draw_circ_graph(AA)
 % Calculate the number of connected components
 s=num_conn_comp(AA)
```

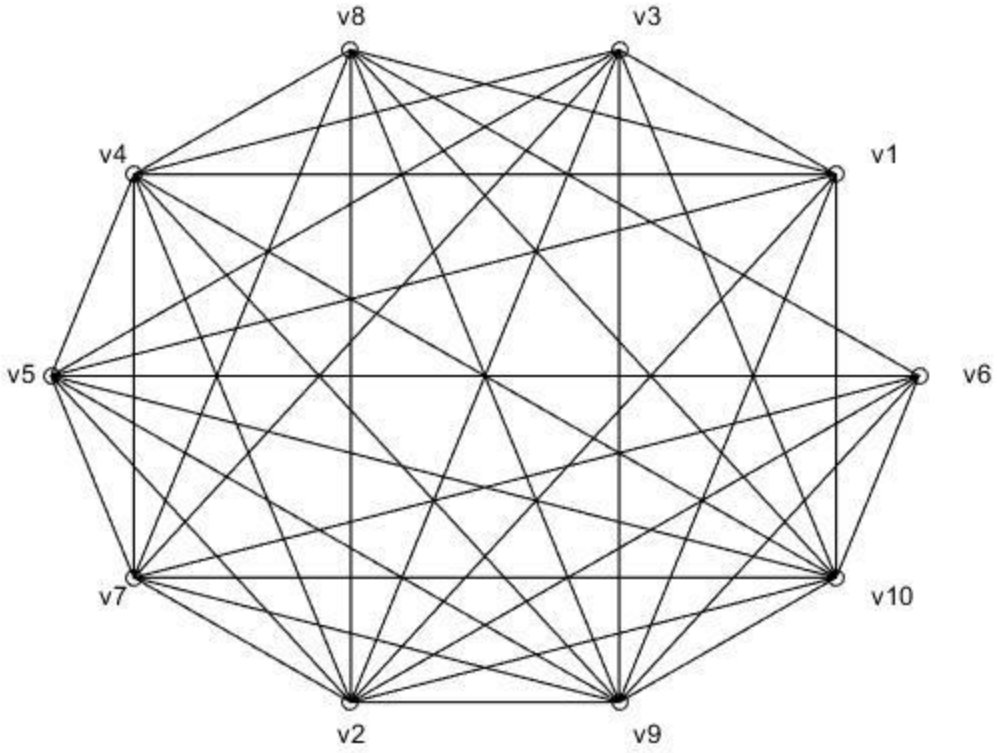
2.2 Histogram of the distribution of degrees of nodes

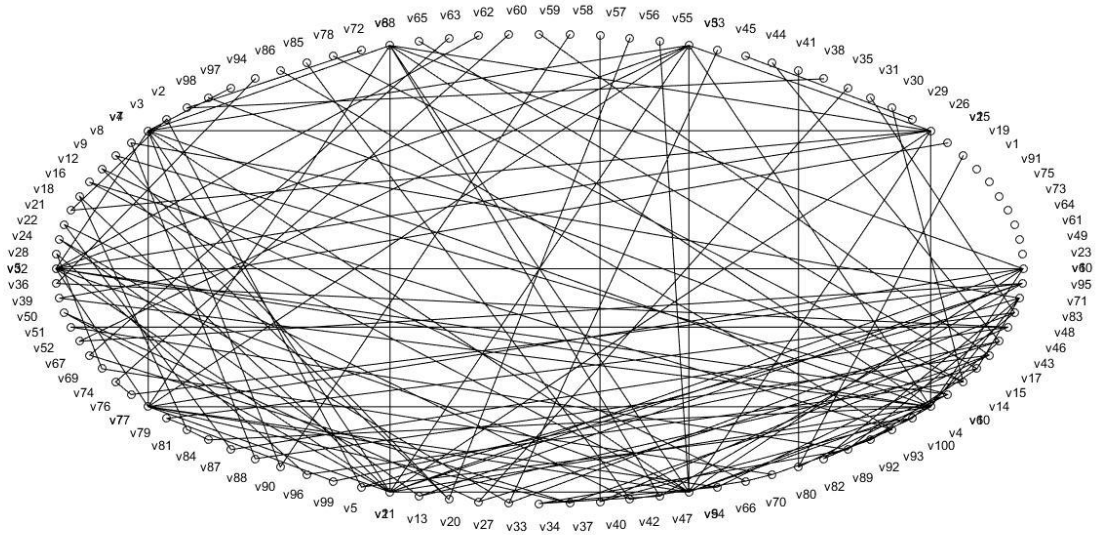
The degree of each node is simply the sum of each row, thus it gives a 1 x N vector deg

```
% Degree of each node
 deg = sum(AA);
 % Histogram
 h = histogram(deg)
```

3. Result

The plot of N10 and N100





For N1000, it is nearly impossible to figure out the individual node, for N10000, my laptop didn't work

The number of connected components is given as s:

N10: s=1

N100: s=12,

N1000: s=17,

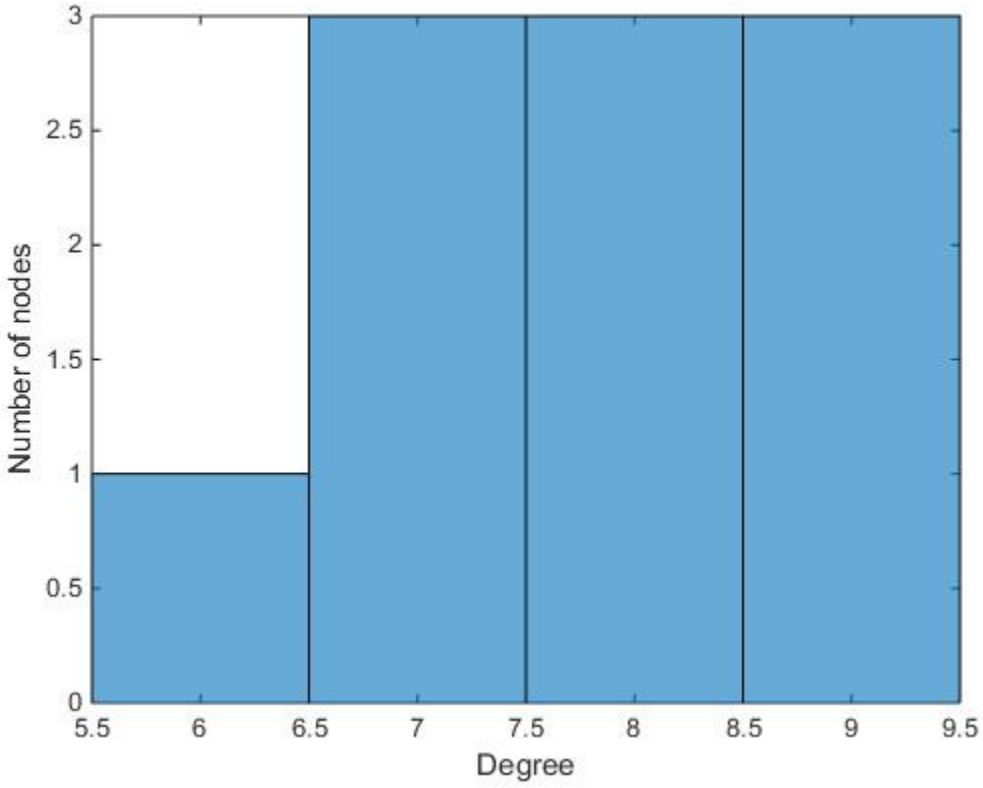
N10000: s=12.

1) Histogram for N10

Data: [7 9 7 8 8 6 8 7 9 9]

Values: [1 3 3 3]

NumBins: 4

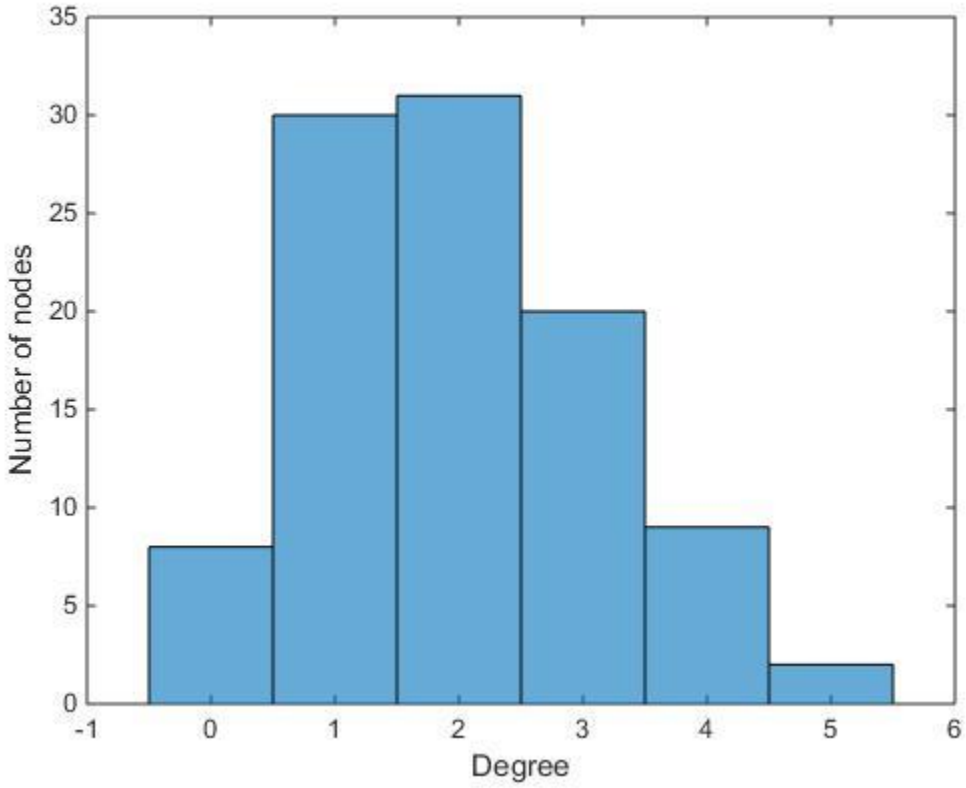


2) Histogram for N100

Data: [1x100 double]

Values: [8 30 31 20 9 2]

NumBins: 6

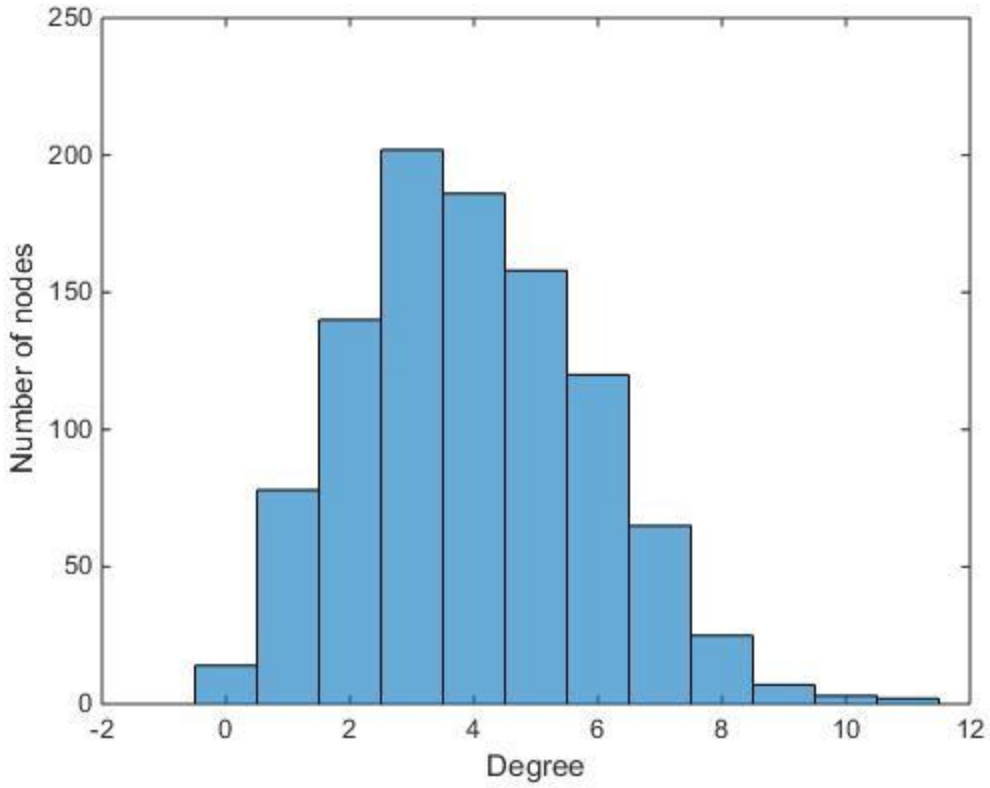


3) Histogram for N1000

Data: [1x1000 double]

Values: [14 78 140 202 186 158 120 65 25 7 3 2]

NumBins: 12

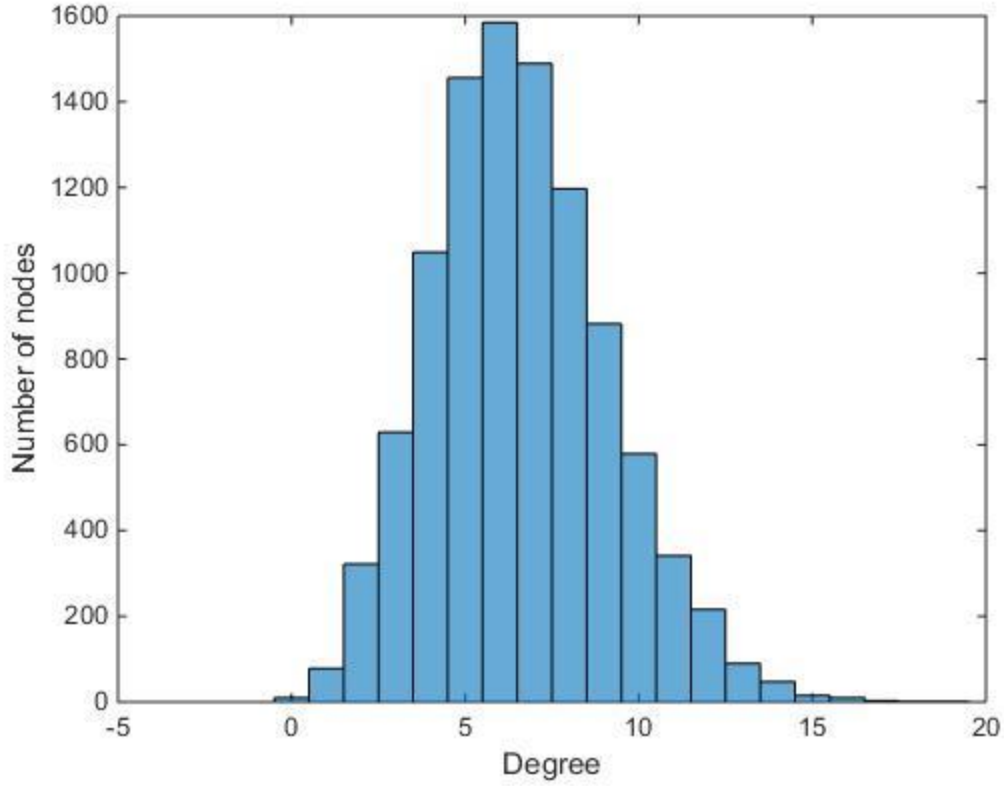


4) Histogram for N10000

Data: [1x10000 double]

Values: [10 78 322 629 1049 1456 1585 1489 1197 882 579 341 216 90 47 15 10 3 1 1]

NumBins: 20


1600
1400
1200
1000
800
600
400
200
0
Number of nodes
-5
0
5
10
15
20
Degree