彩虹散射的半经典模型研究

赵莹杉

同济大学物理科学与工程学院

April 12, 2022

Outline

1 研究背景

- 彩虹散射现象
- 半经典视角下的核虹

2 理论模型

3 结果分析

4 总结与展望

5 附录

彩虹散射现象

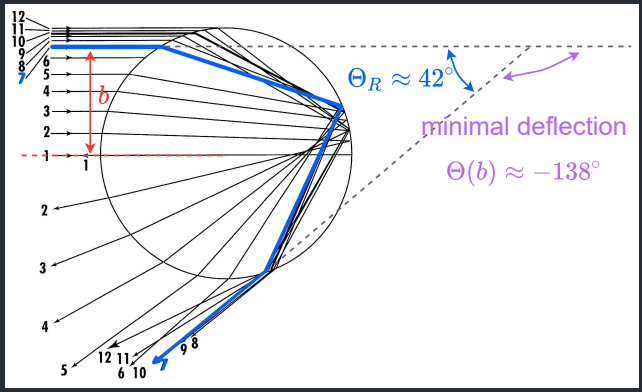

Figure 1: 大气初级彩虹形成示意图 [1]

$$\frac{\mathrm{d}\Theta_R}{\mathrm{d}b} = 0, \quad \frac{\mathrm{d}\sigma}{\mathrm{d}\Omega} = \sum_b \frac{b}{\sin\Theta(b)|\mathrm{d}\Theta(b)/\mathrm{d}b|} \to \infty$$

[1] J. Adam, Physics Reports **356**, 229 (2002)

半经典视角下的核虹

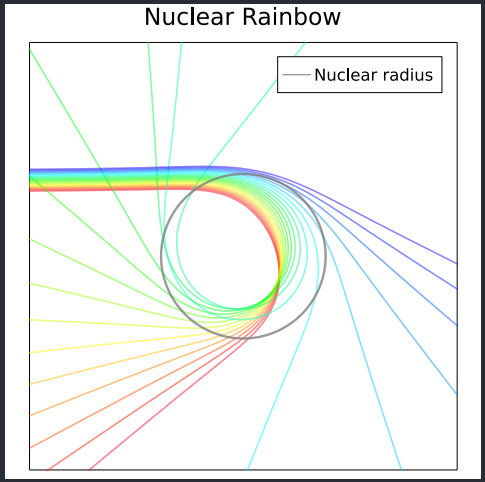

Figure 2: 原子核弹性散射经典轨迹示意图

弹核的德布罗意波长

$$\lambda_B(r) = h/\sqrt{2\mu[E - V(r)]}$$

核势作用范围内“折射率”[2]

$$n(r) = \sqrt{1 - V(r)/E} > 1$$

[2] D. T. Khoa et al., **34**, R111 (2007)

Outline

1 研究背景

2 理论模型

- 光学势与分波法
- 散射波的近远端分解

3 结果分析

4 总结与展望

5 附录

光学势与分波法

Woods-Saxon 形式的光学势

$$V(r) = -\frac{V}{1+\exp(\frac{r-R_V}{a_V})} - i\frac{W}{1+\exp(\frac{r-R_W}{a_W})}$$

Internal Solution

$$u'' = \left[\frac{l(l+1)}{\rho^2} + U_C(\rho) + U_N(\rho) - 1\right] u$$

渐进解为初始条件，数值求解

$$\rho^{l+1} \xleftarrow{\rho\to 0} u(\rho) \xrightarrow{\text{numerical solution}}$$

Boundary

匹配对象：
$u,\ u'$

待求参数：
矩阵元 $\mathbf{S}_l$

External Solution

$$u'' = \left[\frac{l(l+1)}{\rho^2} + \frac{2\eta}{\rho} - 1\right] u$$

Coulomb Hankel functions
Linear combination [3]

$$u_l^{\text{ext}}(\rho) = \frac{i}{2}[H_l^-(\rho) - \mathbf{S}_l H_l^+(\rho)]$$

[3] I. J. Thompson and F. M. Nunes, *Nuclear reactions for astrophysics: principles, calculation and applications of low-energy reactions*, (Cambridge University Press, 2009)

光学势与分波法

散射波函数 → 总散射截面

$$\psi_{\mathsf{sc}} = \psi_C^+ + \psi_N \;\rightarrow\; \sigma_{\mathsf{total}}(\theta) = \left|f_C(\theta) + f_N(\theta)\right|^2$$

点库伦势散射 → 库伦散射振幅 [4]

$$\psi_C^+(r,\theta) = \frac{1}{kr}\sum_l^{\infty}(2l+1)i^l P_l(\cos\theta)F_l(kr) \;\rightarrow\; f_C(\theta) = -\frac{\eta}{2k\sin^2(\theta/2)}\exp\left[-i\eta\ln\left(\sin^2(\theta/2)\right) + 2i\sigma_0(\eta)\right]$$

核势影响的散射 → 额外附加的核散射振幅

$$\psi_N(r,\theta) = \frac{1}{kr}\sum_l(2l+1)i^l P_l(\cos\theta)\left[u_l(kr) - F_l(kr)\right] \;\rightarrow\; f_N(\theta) = \frac{1}{2ik}\sum_l(2l+1)P_l(\cos\theta)(\mathbf{S}_l^N - 1)e^{2i\sigma_l(\eta)}$$

[4] 相关解析解请参考[4]　库伦相移 $\sigma_l(\eta) = \arg\Gamma(1+l+i\eta)$

散射波的近远端分解

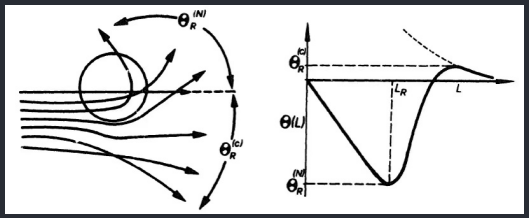

Figure 3: 经典力学散射轨迹[5]

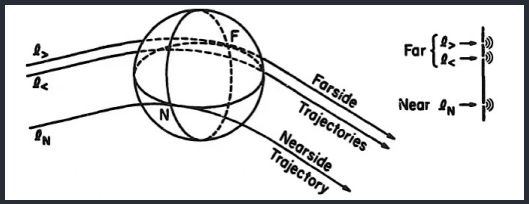

Figure 4: 类三狭缝干涉现象[6]

- 使用 R.C.Fuller 的方法[5]：

$$\tilde{Q}_l^{\pm}(\cos\theta) = \frac{1}{2}P_l(\cos\theta) \mp \frac{i}{\pi}Q_l(\cos\theta)$$

$$P_l(\cos\theta) = \tilde{Q}_l^{+}(\cos\theta) + \tilde{Q}_l^{-}(\cos\theta)$$

近远端散射振幅

$$f_{\mathbf{F/N}}(\theta) = \sum_l (2l+1)\tilde{Q}_l^{\pm}(\cos\theta) \cdot \frac{1}{2ik}(\mathbf{S}_l - 1)$$

[5] R. C. Fuller, Phys. Rev. C **12**, 1561 (1975)

Outline

1 研究背景

2 理论模型

3 结果分析

- 散射截面及散射波函数可视化
- 核势散射及核虹现象分析

4 总结与展望

5 附录

散射截面及散射波函数可视化

System	$E_{\mathrm{lab}}(\mathrm{MeV})$	$V(\mathrm{MeV})$	$r_V(\mathrm{fm})$	$a_V(\mathrm{fm})$	$W(\mathrm{MeV})$	$r_W(\mathrm{fm})$	$a_W(\mathrm{fm})$
$^{16}\mathrm{O}+{}^{12}\mathrm{C}$	132.0	282.2	0.586	0.978	13.86	1.183	0.656

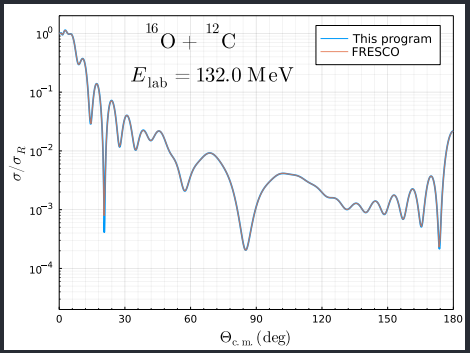

Figure 5: $^{16}\mathrm{O}+{}^{12}\mathrm{C}$ 计算结果

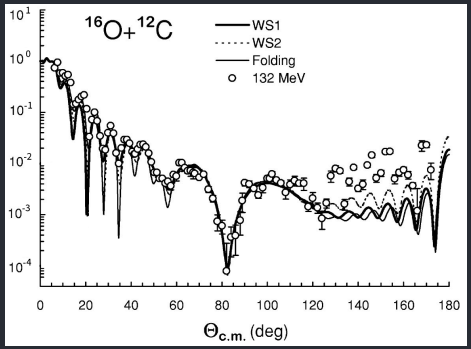

Figure 6: Ogloblin 实验结果[6]

[6] A. A. Ogloblin et al., Phys. Rev. C **62**, 044601 (2000)

散射截面及散射波函数可视化

System	$E_{\mathrm{lab}}(\mathrm{MeV})$	$V(\mathrm{MeV})$	$r_V(\mathrm{fm})$	$a_V(\mathrm{fm})$	$W(\mathrm{MeV})$	$r_W(\mathrm{fm})$	$a_W(\mathrm{fm})$
$\alpha + {}^{90}\mathrm{Zr}$	79.5	141.2	1.225	0.821	18.49	1.575	0.565

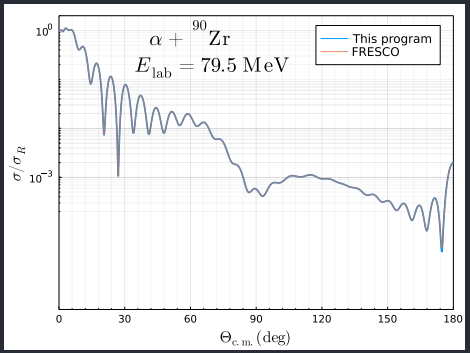

Figure 7: $\alpha + {}^{90}\mathrm{Zr}$ 计算结果

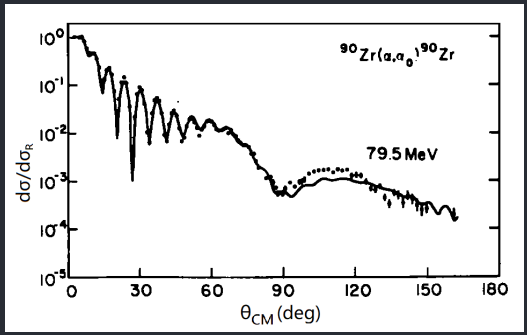

Figure 8: Put 和 Paans 实验结果[7]

[7] L. Put and A. Paans, Nuclear Physics A **291**, 93 (1977)

散射截面及散射波函数可视化

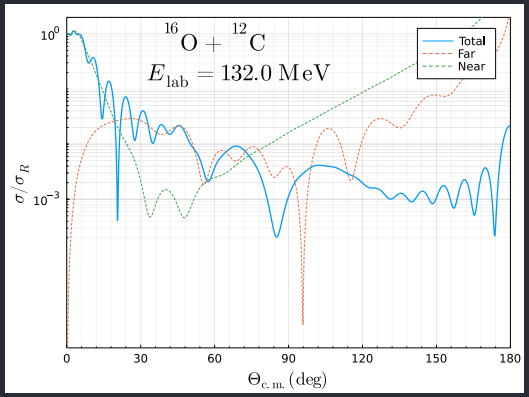

Figure 9: 将散射截面分解为近端部分与远端部分，两者叠加干涉产生艾里振荡图样

- 前角区：夫琅禾费衍射
- 后角区：远端散射主导
- 初级、次级艾里斑

$\Longrightarrow$

- OP 实部足够深
- OP 虚部足够弱

散射截面及散射波函数可视化

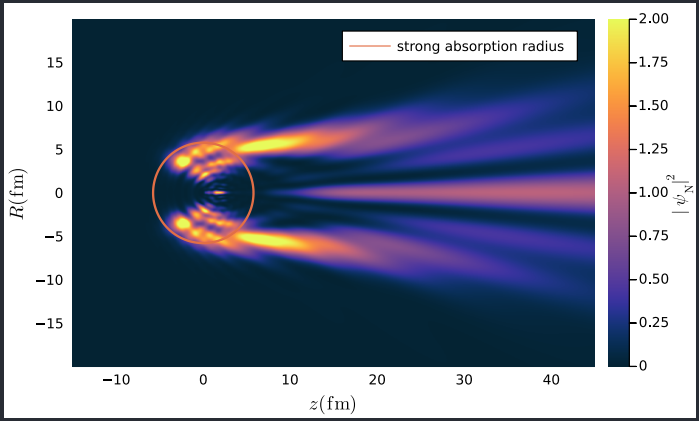

Figure 10: 核势影响的散射波函数 ψ_N 在沿入射 z 轴的二维截面上的可视化，颜色代表散射波的振幅强度 $|\psi_N|^2$。原点处为系统质心，圆圈为强吸收半径 $R_{1/2}$[10]。

$$R_{1/2} = \{\eta + [\eta^2 + L(L+1)]\}/k, \quad \text{where } \mathrm{Re}(\mathbf{S}_L) = 1/2$$

核势散射及核虹现象分析

考虑纯核势散射。上图为实验应测得的散射截面，下图为任意位置的“散射截面”随距离变化的温度图。

任意距离 r，散射角 θ 处 $\sigma(r,\theta)$

$$\left| r \sum_l (2l+1) i^l P_l(\cos\theta) \left[u_l(kr)/kr - j_l(kr) \right] \right|^2$$

无穷远处测得的散射截面

$$\sigma(\theta) = \left| \frac{1}{2ik} \sum_l \left[(2l+1) P_l(\cos\theta) (\mathbf{S}_l - 1) \right] \right|^2$$

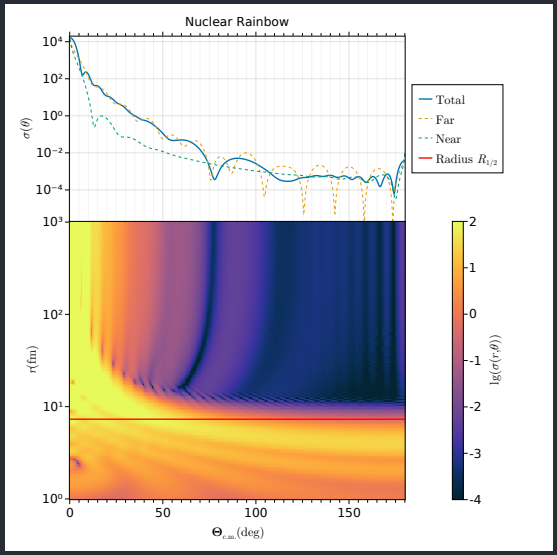

Figure 11: 核虹现象

核势散射及核虹现象分析

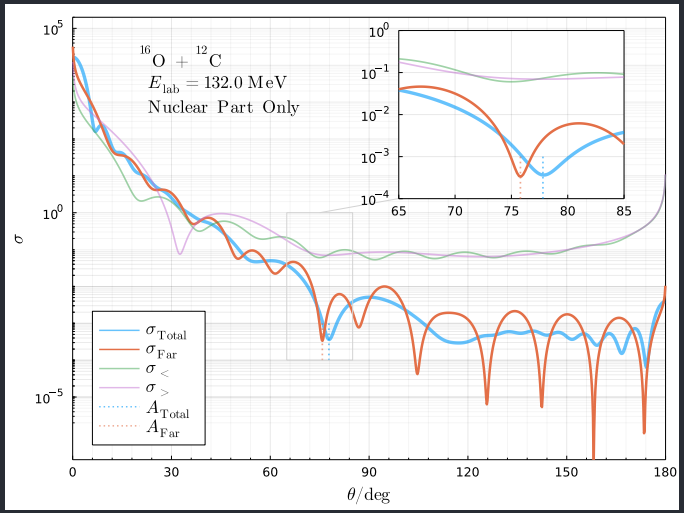

Figure 12: 将纯核势散射的远端散射截面按角动量大小进一步分解为$l_<$与$l_>$两部分。

偏转函数

在半经典 WKB 近似下的偏转函数可以表示成[2]：

$$\Theta(l) = 2\frac{\mathrm{d}\delta_l}{\mathrm{d}l}$$

对半经典偏转函数进行插值并根据极值位置确定进一步分解的分界线 l_c.

干涉现象

$\sigma_<$ 和 $\sigma_>$ 幅值相近且变化平缓，二者干涉相消形成尖锐的远端截面极小值。

Outline

1 研究背景

2 理论模型

3 结果分析

4 总结与展望

5 附录

总结与展望

1. 讨论了一些情况下散射截面随着角度出现振荡的特殊现象-虹现象
2. 进行了半经典模型的定性分析与量子模型分波法的定量计算
3. 通过散射波函数可视化研究了核势散射的近场特征
4. 晕核？致密物质性质？

Outline

1 研究背景

2 理论模型

3 结果分析

4 总结与展望

5 附录

- 参考文献

参考文献

[1] J. Adam, Physics Reports **356**, 229 (2002).

[2] D. T. Khoa, W. von Oertzen, H. G. Bohlen, and S. Ohkubo, **34**, R111 (2007).

[3] I. J. Thompson and F. M. Nunes, *Nuclear reactions for astrophysics: principles, calculation and applications of low-energy reactions*, (Cambridge University Press, 2009).

[4] B. H. Bransden and C. J. Joachain, *Physics of atoms and molecules*, 2nd ed (Prentice Hall, Harlow, England ; New York, 2003).

[5] G. Satchler, Nuclear Physics A **409**, 3 (1983).

[6] K. McVoy and M. Brandan, Nuclear Physics A **542**, 295 (1992).

[7] R. C. Fuller, Phys. Rev. C **12**, 1561 (1975).

[8] A. A. Ogloblin, Y. A. Glukhov, W. H. Trzaska, A. S. Dem'yanova, S. A. Goncharov, R. Julin, S. V. Klebnikov, M. Mutterer, M. V. Rozhkov, V. P. Rudakov, G. P. Tiorin, D. T. Khoa, and G. R. Satchler, Phys. Rev. C **62**, 044601 (2000).

[9] L. Put and A. Paans, Nuclear Physics A **291**, 93 (1977).

[10] C. J. Batty, E. Friedman, H. J. Gils, and H. Rebel, "Experimental methods for studying nuclear density distributions", in *Advances in nuclear physics*, edited by J. W. Negele and E. Vogt (Springer US, Boston, MA, 1989), pp. 1–188.