Neural Network Prediction and Weight Update

Given Values

- Inputs: x1 = 2, x2 = 3

- True Outputs: y1 = 4, y2 = 5

- Weight: w = 1

- Bias: b = 1

Step 1: Make Predictions

The prediction function for a single input is given by:

$$\hat{y} = w * x + b$$

Predictions

1. For x1 = 2:

$$\hat{y}_1 = w * x1 + b = 1 * 2 + 1 = 2 + 1 = 3$$

2. For x2 = 3:

$$\hat{y}_2 = w * x2 + b = 1 * 3 + 1 = 3 + 1 = 4$$

Step 2: Calculate the Loss

Using Mean Squared Error (MSE) as our loss function:

$$\text{Loss} = (1/2) * \text{sum}((\hat{y}_i - y_i)^2)$$

Calculating the Loss

1. For the first prediction:

$$(\hat{y}_1 - y1)^2 = (3 - 4)^2 = (-1)^2 = 1$$

2. For the second prediction:

$$(\hat{y}_2 - y2)^2 = (4 - 5)^2 = (-1)^2 = 1$$

Total loss:

Loss = (1/2) * (1 + 1) = (1/2) * 2 = 1

Step 3: Calculate the Gradients (Partial Derivatives)

1. Gradient with respect to w

Using the formula:

(dLoss/dw) = sum((y_hat_i - y_i) * x_i)

Calculating:

1. For i = 1:

(y_hat_1 - y1) * x1 = (3 - 4) * 2 = -1 * 2 = -2

2. For i = 2:

(y_hat_2 - y2) * x2 = (4 - 5) * 3 = -1 * 3 = -3

Sum of gradients:

(dLoss/dw) = -2 + (-3) = -5

2. Gradient with respect to b

Using the formula:

(dLoss/db) = sum((y_hat_i - y_i))

Calculating:

1. For i = 1:

y_hat_1 - y1 = 3 - 4 = -1

2. For i = 2:

y_hat_2 - y2 = 4 - 5 = -1

Sum of gradients:

(dLoss/db) = -1 + (-1) = -2

Step 4: Update the Weights and Bias

Assuming a learning rate eta = 0.1.

1. Update w

Using the update rule:

$$w = w - \eta * (dLoss/dw)$$

$$w = 1 - 0.1 * (-5) = 1 + 0.5 = 1.5$$

2. Update b

Using the update rule:

$$b = b - \eta * (dLoss/db)$$

$$b = 1 - 0.1 * (-2) = 1 + 0.2 = 1.2$$

Final Results

- Predictions:
  - $\hat{y}_1 = 3$
  - $\hat{y}_2 = 4$

- Loss:

  $Loss = 1$

- Updated Weights and Bias:
  - $w = 1.5$
  - $b = 1.2$

Now you can make predictions again using the updated values of w and b.