Travelling Salesman Problem

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Definition

- Find the shortest possible route that visits each city exactly once and returns to the origin city-> Hamiltonian cycle
- Posed such computational complexity that any programmable efforts to solve such problems would grow super polynomial with the problem size.

Problem Statement

- If there are n cities and cost of traveling from any city to any other city is given.
- Then we have to obtain the cheapest round-trip such that each city is visited exactly ones returning to starting city, completes the tour.
- Typically travelling salesman problem is represent by weighted graph.

Travelling Salesman Problem

You are given-

- A set of some cities.
- Distance between every pair of cities

Travelling Salesman Problem states-

- A salesman has to visit every city exactly once.
- He has to come back to the city from where he starts his journey.
- What is the shortest possible route that the salesman must follow to complete his tour?

	1	2	3	4	5
1	—	10	8	9	7
2	10	—	10	5	6
3	8	10	—	8	9
4	9	5	8	—	6
5	7	6	9	6	—

Step - 0 :-

Sum of least numbers in rows :-

$7 + 5 + 8 + 5 + 6 = 31$

Sum of least numbers in columns :-

$7 + 5 + 8 + 5 + 6 = 31$

Step - 1 :-

Sum of X_{12} :-

$10 + 5 + 8 + 6 + 6 = 35$

Sum of X_{13} :-

$8 + 5 + 8 + 5 + 6 = 32$

Sum of X_{14} :-

$9 + 6 + 8 + 5 + 6 = 34$

Sum of X_{15} :-

$7 + 5 + 8 + 5 + 6 = 31$

Step -2 :- Sum of X_{21} in X_{15} :-

	2	3	4
3	10	—	8
4	5	8	—
5	6	9	6

∴ 2-1-5

5-2 is not possible because it will make a subtou

⇒

	2	3	4
3	10	—	8
4	5	8	—
5	—	9	6

$7 + 10 + 8 + 5 + 6 = 36$

Sum of X_{23} in X_{15} :-

	1	2	4
3	8	10	8
4	9	5	—
5	7	6	6

	1	2	4
3	8	—	8
4	9	5	—
5	7	6	6

$7 + 10 + 8 + 5 + 6 = 36$

Sum of X_{24} in X_{15} :-

	1	2	3
3	8	10	—
4	9	5	8
5	7	6	6

	1	2	3
3	8	10	—
4	9	—	8
5	7	6	9

$7 + 5 + 8 + 8 + 6 = 34$

Sum of X_{21} in X_{13} :-

	2	4	5
3	10	8	9
4	5	—	6
5	6	6	—

$8 + 10 + 8 + 5 + 6 = 37$

2 - 1 - 3

3 - 2 is not possible because it will make a subtour.

Sum of X_{24} in X_{13} :-

	1	2	5
3	8	10	9
4	9	—5	6
5	7	6	—

	1	2	5
3	8	10	9
4	9	—	6
5	7	6	—

$8 + 5 + 8 + 6 + 6 = 33$

Sum of X_{25} in X_{13} :-

	1	2	4
3	8	10	8
4	9	5	—
5	7	—	6

$8 + 6 + 8 + 5 + 6 = 33$

Step - 4 :-

Sum of X_{32} in X_{24} :-

$1 - 3 - 2 - 4 - 5 - 1$

$8 + 10 + 5 + 6 + 7 = 36$

$1 - 3 = 8$
$3 - 2 = 10$
$2 - 4 = 5$
$4 - 5 = 6$
$5 - 1 = 7$
36

Sum of X_{35} in X_{24} :-

$1-3-5-2-4-1 = 37$

$1\ 8 + 9 + 6 + 5 + 9 = 37$

$1-3 = 8$
$3-5 = 9$
$5-2 = 6$
$2-4 = 5$
$4-1 = 9$
37

Sum of X_{32} in X_{25} :-

$1-3-2-5-4-1$
$8+10+6+6+9 = 39$

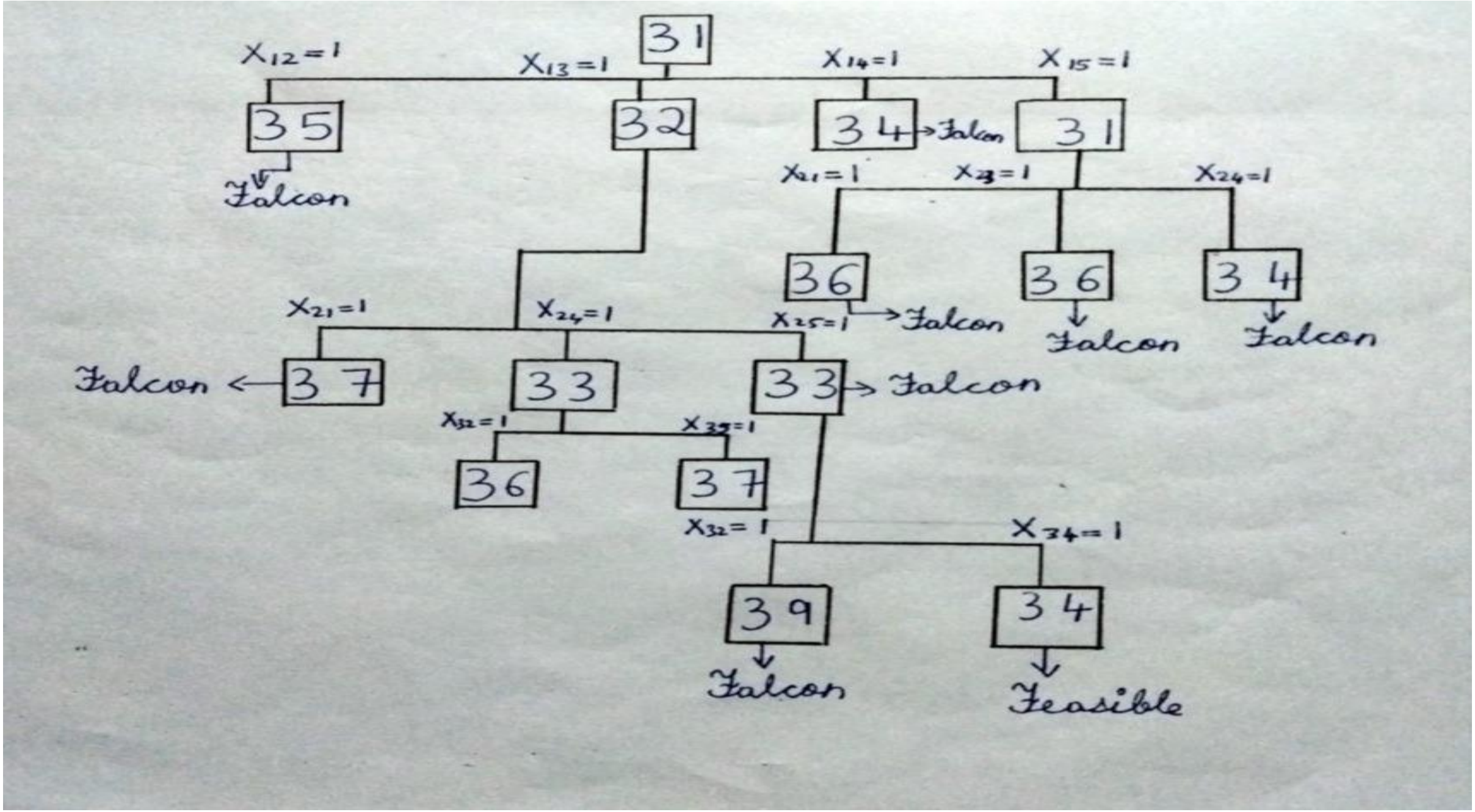
$1-3 = 8$
$3-2 = 10$
$2-5 = 6$
$5-4 = 6$
$4-1 = 9$
39

Sum of X_{34} in X_{25} :-

$1-3-4-2-5-1$
$8+8+5+6+7 = 34$

$\Rightarrow \boxed{Z = 34}$

$1-3 = 8$
$3-4 = 8$
$4-2 = 5$
$2-5 = 6$
$5-1 = 7$
34


31
$X_{12}=1$
$X_{13}=1$
$X_{14}=1$
$X_{15}=1$
35
32
34
Falcon
31
Falcon
$X_{21}=1$
$X_{23}=1$
$X_{24}=1$
36
36
34
$X_{25}=1$
Falcon
Falcon
Falcon
$X_{21}=1$
$X_{24}=1$
Falcon
37
33
33
Falcon
$X_{32}=1$
$X_{33}=1$
36
37
$X_{32}=1$
$X_{34}=1$
39
34
Falcon
Feasible

Pseudo-Code

```
Home_City = Visited = Current_city
While(!visited All City)
        Node=Find Shortest distance(Current_node)
        Add Node (Node)
        Current_node=Node
Result=All node+ Home_City[0][last_Visit_Node]
return Final_result= Result
```

Applications in TSP

- Can be used in:

(i) Transportation: school bus routes, service calls, delivering meals.

(ii) Manufacturing: an industrial robot that drills holes in printed circuit boards.

(iii) VLSI (microchip) layout communication: planning new telecommunication networks.

Summary

- Complexity of this algorithm is $o(n^2)$ which is much better than brute force algorithm.
- It is Fast, Easy and Efficient.