3 Jan 2022.

EXPONENTIAL SMOOTHING METHOD

Exponential smoothing weighs past data from previous time periods with exponentially decreasing importance in the forecast so that the most recent data carries more weight in the moving average.

Simple Exponential Smoothing:

The **forecast** is made up of the actual value for the present time period Xt multiplied by a value between 0 and 1 (the exponential smoothing constant) referred to as α (not the same as used for a Type I error) plus the product of the present time period forecast Ft and (1 – α). The formula is stated algebraically as follows:

X_t

$$F_{t+1} = \alpha X_t + (1-\alpha) F_t = F_t + \alpha (X_t - F_t)$$

where F_{t+1} = Forecast for the next time period $(t + 1)$;
F_t = forecast for the present time period (t);
α = a weight called exponentially smoothing constant $(0 \le \alpha \le 1)$;
X_t = actual value for the present time period (t).

If exponential smoothing has been used over a period of time, the forecast for F_t will have been obtained by

$$F_t = \alpha X_{t-1} + (1-\alpha) F_{t-1}.$$

$$F_{t+1} = \alpha X_t + (1-\alpha) F_t$$

$$= \alpha X_t + F_t - \alpha F_t = F_t + \alpha (X_t - F_t)$$

$$\Rightarrow F_t = \alpha X_{t-1} + (1-\alpha) F_{t-1}$$

PROBLEM:

A firm uses simple exponential smoothing with α = 0.1 to forecast demand. The forecast for the week of February 1 was 500 units whereas actual demand turned out to be 450 units.
(a) Forecast the demand for the week of February 8.
(b) Assume the actual demand during the week of February 8 turned out to be 505 units. Forecast the demand for the week of February 15. Continue forecasting through March 15, assuming that subsequent demands were actually 516, 488, 467, 554, and 510 units.

(a)

Given $F_{t-1} = 500$, $D_{t-1} = 450$, and $\alpha = 0.1$

(a) $F_t = F_{t-1} + \alpha(D_{t-1} - F_{t-1}) = 500 + 0.1(450 - 500) = 495$ units.

(b) Forecast of demand for the week of February 15 is

Forecast of Demand

Week	*Demand* D_{t-1}	*Old Forecast* F_{t-1}	*Forecast Error* $(D_{t-1} - F_{t-1})$	*Correction* $\alpha(D_{t-1} - F_{t-1})$	*New Forecast* (F_t) $F_{t-1} + \alpha(D_{t-1} - F_{t-1})$
Feb. 1	450	500	−50	−5	495
8	505	495	10	1	496
15	516	496	20	2	498
22	488	498	−10	−1	497
Mar. 1	467	497	−30	−3	494
8	554	494	60	6	500
15	510	500	10	1	501

If no previous forecast value is known, the old forecast starting point may be estimated or taken to be an average of some preceding periods.

Exponential Trend Model

The characteristic property of this law is that the rate of growth, that is, the rate of change of *y* with respect to *x* is proportional to the values of the function. The following function has this property.

$$y=ab^x,\ a > 0$$

The letter *b* is a fixed constant, usually either 10 or *e*, where *a* is a constant to be determined from the data.
In order to find out the values of constants *a* and *b* in the exponential function, the two normal equations to be solved are

$$\log y = \log(ab^x)$$
$$= \log a + \log b^x$$
$$= \log a + x \log b$$

$$\log y = \log a + x \log b.$$

$$\Sigma \log y = n \log a + \log b\ \Sigma x$$
$$\Sigma x \log y = \log a\ \Sigma x + \log b\ \Sigma x^2.$$

When the data are coded so that $\Sigma x = 0$, the two normal equations become

$$\Sigma \log y = n \log a \quad \text{or} \quad \log a = \frac{1}{n}\Sigma \log y$$
$$\Sigma x \log y = \log b\ \Sigma x^2 \quad \text{or} \quad \log b = \frac{\Sigma x \log y}{\Sigma x^2}.$$

PROBLEM

The sales (Rs. in million) of a company for the years 1993 to ~~1999~~ 2001 are as follows:

Year :	1997	1998	1999	2000	2001
Sales :	1.6	4.5	13.8	40.2	125.0

Find the exponential trend for the given data and estimate the sales for 2004.

The computational time can be reduced by coding the data. For this consider $u = x - 3$.

u6

Year	*Time Period* x	$u = x - 3$	u^2	*Sales* y	$\log y$	$u \log y$
1997	1	−2	4	1.60	0.2041	−0.4082
1998	2	−1	1	4.50	0.6532	−0.6532
1999	3	0	0	13.80	1.1390	0
2000	4	1	1	40.20	1.6042	1.6042
2001	5	2	4	125.00	2.0969	4.1938
		3	10	$\Sigma \log y$	5.6983	4.7366

2002

2003

2004 →

$$\log a = \frac{1}{n} \Sigma \log y = \frac{1}{5} (5.6983) = 1.1397$$

$$\log b = \frac{\Sigma u \log y}{\Sigma u^2} = \frac{4.7366}{10} = 0.4737.$$

Therefore, $\log y = \log a + (x + 3) \log b = 1.1397 + 0.4737x.$

For sales during 2004, $x = 3$ and we obtain

$$\log y = 1.1397 + 0.4737\,(3) = 2.5608$$

or $$y = \text{antilog}\,(2.5608) = 363.80.$$

Try :-

find the logarithmic straight line to the foll. data.

years	prod.
2005	64
6	70
7	75
8	82
9	88
10	95