Working Procedure [One-way classification CRD]

1. H_0 : There is no significant difference between the treatments.

2. H_1 : There is significant difference between the treatments.

Step 1 : Find N i.e., the number of observations

Step 2 : Find T i.e., the total value of all observations

Step 3 : Find $\frac{T^2}{N}$ i.e., the correction factor

Step 4 : Calculate the total sum of squares.

$$TSS = \sum X_1^2 + \sum X_2^2 + \dots - \frac{T^2}{N}$$

Step 5 : Calculate the column sum of squares

$$SSC = \frac{(\Sigma X_1)^2}{N_1} + \frac{(\Sigma X_2)^2}{N_1} + \frac{(\Sigma X_3)^2}{N_1} + \dots - \frac{T^2}{N}$$

Here, N_1 is the number of elements in each column.

$$SSE = TSS - SSC$$

Step 6 : Prepare the ANOVA table to calculate F-ratio.

Step 7 : Find the table value.

Step 8 : Conclusion :

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Example 2.2.4

The following table shows the lives in hours of four brands of electric lamps.

Brand A :	1610	1610	1650	1680	1700	1720	1800	
B :	1580	1640	1640	1700	1750			
C :	1460	1550	1600	1620	1640	1660	1740	1820
D :	1510	1520	1530	1570	1600	1680		

Perform an analysis of variance test the homogneity of the mean lives of the four brands of Lamps. **[A.U. A/M. 2008] [A.U N/D 2011] [A.U Tvli M/J 2011] [A.U A/M 2015 R-13]**

Solution :

H_0 : There is no significant difference between the four brands.

H_1 : There is a significant difference between the four brands.

Subtract 1600 and then divided by 10, we get

X_1 A	X_2 B	X_3 C	X_4 D	Total	X_1^2	X_2^2	X_3^3	X_4^2
1	–2	–14	–9	–24	1	4	196	81
1	4	–5	–8	–8	1	16	25	64
5	4	0	–7	2	25	16	0	49
8	10	2	–3	17	64	100	4	9
10	15	4	0	29	100	225	16	0
12	–	6	8	26	144	–	36	64
20	–	14	–	34	400	–	196	–
–	–	22	–	22	–	–	484	–
57	**31**	**29**	**–19**	**98**	**735**	**361**	**957**	**267**

Step 1 : N = 26

Step 2 : T = 98

Step 3 : C.F $= \dfrac{T^2}{N} = \dfrac{9604}{26} = 369.39$

Step 4 : TSS $= \Sigma X_1^2 + \Sigma X_2^2 + \Sigma X_3^2 + \Sigma X_4^2 - \dfrac{T^2}{N}$

$= 735 + 361 + 957 + 267 - 369.39$

$= 1950.61$

Step 5 : SSC $= \dfrac{(\Sigma X_1)^2}{N_1} + \dfrac{(\Sigma X_2)^2}{N_1} + \dfrac{(\Sigma X_3)^2}{N_1} + \dfrac{(\Sigma X_4)^2}{N_1} - \dfrac{T^2}{N}$

$N_1 \rightarrow$ Number of elements in their respective columns.

$= \dfrac{(57)^2}{7} + \dfrac{(31)^2}{5} + \dfrac{(29)^2}{8} + \dfrac{(-19)^2}{6} - 369.39$

$= \dfrac{3249}{7} + \dfrac{961}{5} + \dfrac{841}{8} + \dfrac{361}{6} - 369.39$

$= 464.14 + 192.2 + 105.13 + 60.17 - 369.39 = 452.25$

SSE = TSS – SSC

$= 1950.61 - 452.25 = 1498.36$

Step 6 : ANOVA

Source of Variation	Sum of squares	d.f.	Mean square	Variance Ratio	Table value 5% level
Between columns	SSC = 452.25	C – 1 = 4 – 1 = 3	$MSC = \dfrac{SSC}{C-1} = \dfrac{452.25}{3} = 150.75$	$F_C = \dfrac{MSC}{MSE} = \dfrac{150.75}{68.11} = 2.21 > 1$	F_C (3,22) = 3.05
Error	SSE = 1498.36	N – C = 26 – 4 = 22	$MSE = \dfrac{SSE}{N-C} = \dfrac{1498.36}{22} = 68.11$	Since $\dfrac{MSE}{MSC} < 1$	

Step 7 : Conclusion : Cal F_c < Table F_c

So, we accept H_0

Example 2.2.5

The accompanying data resulted from an experiment comparing the degree of soiling for fabric co-polymerized with the three different mixtures of methacrcylic acid. Analysis is the given classification.

Mixture 1	.56	1.12	.90	1.07	.94
Mixture 2	.72	.69	.87	.78	.91
Mixture 3	.62	1.08	1.07	.99	.93

[A.U A/M 2017 R-13]

Solution :

1. H_0 : The true average degree of soiling is identical for three mixtures.
2. H_1 : The true average degree of soiling is not identical for three mixtures.

X_1	X_2	X_3	Total	X_1^2	X_2^2	X_3^2
0.56	0.72	0.62	1.9	0.3136	0.5184	0.3844
1.12	0.69	1.08	2.89	1.2544	0.4761	1.1664
0.90	0.87	1.07	2.84	0.8100	0.7569	1.1449
1.07	0.78	0.99	2.84	1.1449	0.6084	0.9801
0.94	0.91	0.93	2.78	0.8836	0.8281	0.8649
4.59	3.97	4.69	13.25	4.4065	3.1879	4.5407

Step 1 : N = 15

Step 2 : T = 13.25

Step 3 : $\frac{T^2}{N} = \frac{175.5625}{15} = 11.7042$

Step 4 : TSS $= \Sigma X_1^2 + \Sigma X_2^2 + \Sigma X_3^2 - \frac{T^2}{N}$

$= 4.4065 + 3.1879 + 4.5407 - 11.7042 = 0.4309$

Step 5 : SSC $= \frac{(\Sigma X_1)^2}{N_1} + \frac{(\Sigma X_2)^2}{N_1} + \frac{(\Sigma X_3)^2}{N_1} - \frac{T^2}{N}$

[N_1 = Number of elements in each column]

$= \frac{(4.59)^2}{5} + \frac{(3.97)^2}{5} + \frac{(4.69)^2}{5} - 11.7042$

$= 0.0608$

SSE = TSS − SSC

$= 0.4309 - 0.0608$

$= 0.3701$

Step 6 : ANOVA

Source of Variation	Sum of Squares	Degrees of freedom	Means square	Variance ratio	Table value at 5% level
Treatment	SSC = 0.0608	C − 1 = 3 − 1 = 2	$MSC = \frac{SSC}{C-1} = \frac{0.0608}{2} = 0.0304$	$F_c = \frac{MSE}{MSC} = \frac{0.3084}{0.0304} = 10.144$	$F_c(12, 2) = 19.41$
Error	SSE = 0.3701	N − C = 15 − 3 = 12	$MSE = \frac{SSE}{N-C} = \frac{0.3701}{12} = 0.3084$		
Total	TSS = 0.4309	14			

Step 7 : Conclusion : Cal F_C < Tab F_C. So, we accept H_0.

errors.

Arrange calculation of sum of squares.

Step 1 : Find N. **Step 2 :** Find T. **Step 3 :** Find $\frac{T^2}{N}$

Step 4 : Find TSS **Step 5 :** Find SSC. **Step 6 :** Find SSR

Step 7 : SSE = TSS − SSC − SSR

Prepare the ANOVA Table

Step 8 : Find Table F_c and F_R

Step 9 : Conclusion

Example 2.3.2

Perform two-way ANOVA for the given below : [A.U. N/D 2005]

	Treatment			
Plots of land	A	B	C	D
I	38	40	41	39
II	45	42	49	36
III	40	38	42	42

Use coding method, subtracting 40 from the given numbers.

Solution : Subtract 40 from all the numbers. By doing so, F ratio is unaffected and reduces the numbers to smaller numbers.

	A	B	C	D
I	−2	0	1	−1
II	5	2	9	−4
III	0	−2	2	2

	X_1	X_2	X_3	X_4	Total	X_1^2	X_2^2	X_3^2	X_4^2
Y_1	−2	0	1	−1	−2	4	0	1	1
Y_2	5	2	9	−4	12	25	4	81	16
Y_3	0	−2	2	2	2	0	4	4	4
Total	3	0	12	−3	12	29	8	86	21

1. H_0 : There is no significant difference between column means as well as row means.

2. H_1 : There is significant difference between column means or the row means.

Step 1 : $N = 3 + 3 + 3 + 3 = 12$

Step 2 : $T = 3 + 0 + 12 - 3 = 12$

Step 3 : $\dfrac{T^2}{N} = \dfrac{12^2}{12} = 12$

Step 4 : $TSS = \Sigma X_1^2 + \Sigma X_2^2 + \Sigma X_3^2 + \Sigma X_4^2 - \dfrac{T^2}{N}$

$= 29 + 8 + 86 + 21 - 12 = 132$

Step 5 : $SSC = \frac{(\Sigma X_1)^2}{N_1} + \frac{(\Sigma X_2)^2}{N_1} + \frac{(\Sigma X_3)^2}{N_1} + \frac{(\Sigma X_4)^2}{N_1} - \frac{T^2}{N}$

[N_1 = Number of elements in each column]

$= \frac{9}{3} + \frac{0}{3} + \frac{144}{3} + \frac{9}{3} - 12$

$= 3 + 0 + 48 + 3 - 12 = 42$

Step 6 : $SSR = \frac{(\Sigma Y_1)^2}{N_2} + \frac{(\Sigma Y_2)^2}{N_2} + \frac{(\Sigma Y_3)^2}{N_2} - \frac{T^2}{N}$

[N_2 = Number of elements in each row]

$= \frac{4}{4} + \frac{144}{4} + \frac{4}{4} - 12 = 26.0$

$SSE = TSS - SSC - SSR = 132 - 42 - 26 = 64$

Step 7 : ANOVA Table

Source of Variation	SS	DF	MSS	VR	Table value at 5% level
Between columns	SSC = 42	$c - 1 = 4 - 1 = 3$	$MSC = \frac{SSC}{c-1} = \frac{42}{3} = 14$	$F_C = \frac{MSC}{MSE} = \frac{14}{10.67} = 1.31$	$F_C(3, 6) = 4.76$
Between rows	SSR = 26	$r - 1 = 3 - 1 = 2$	$MSR = \frac{SSR}{r-1} = \frac{26}{2} = 13$	$F_R = \frac{MSR}{MSE} = \frac{13}{10.67} = 1.22$	$F_R(2,6) = 5.14$
Error	SSE = 64	$N - c - r + 1 = 6$	$MSE = \frac{SSE}{N-c-r+1} = \frac{64}{6} = 10.67$		
Total	TSS = 132	11			

Step 8 : Conclusion :

Cal. $F_C <$ Table F_C. So, we accept H_0

Cal. $F_R <$ Table F_R. So, we accept H_0

Hence there is no significant difference between column means as well as row means.

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Example 2.3.3

Analyse the following RBD and find your conclusion. [A.U N/D 2013]

		Treatments			
		T_1	T_2	T_3	T_4
	B_1	12	14	20	22
	B_2	17	27	19	15
Blocks	B_3	15	14	17	12
	B_4	18	16	22	12
	B_5	19	15	20	14

Solution :

1. H_0 : There is no significant difference between blocks and treatments.
2. H_1 : There is significant difference between blocks and treatments.

Subtract 15 from each number.

	X_1	X_2	X_3	X_4	Total	X_1^2	X_2^2	X_3^2	X_4^2
Y_1	−3	−1	5	7	8	9	1	25	49
Y_2	2	12	4	0	18	4	144	16	0
Y_3	0	−1	2	−3	−2	0	1	4	9
Y_4	3	1	7	−3	8	9	1	49	9
Y_5	4	0	5	−1	8	16	0	25	1
Total	6	11	23	0	40	38	147	119	68

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Step 1 : $N = 20$

Step 2 : $T = 40$

Step 3 : $\frac{T^2}{N} = \frac{(40)^2}{20} = 80$

Step 4 : $TSS = \Sigma X_1^2 + \Sigma X_2^2 + \Sigma X_3^2 + \Sigma X_4^2 - \frac{T^2}{N}$

$= 38 + 147 + 119 + 68 - 80$

$= 292$

Step 5 : $SSC = \frac{(\Sigma X_1)^2}{N_1} + \frac{(\Sigma X_2)^2}{N_1} + \frac{(\Sigma X_3)^2}{N_1} + \frac{(\Sigma X_4)^2}{N_1} - \frac{T^2}{N}$

[$N_1 \rightarrow$ number of elements in each column]

$= \frac{(6)^2}{5} + \frac{(11)^2}{5} + \frac{(23)^2}{5} + 0 - 80$

$= 57.2$

Step 6 : $SSR = \frac{(\Sigma Y_1)^2}{N_2} + \frac{(\Sigma Y_2)^2}{N_2} + \frac{(\Sigma Y_3)^2}{N_2} + \frac{(\Sigma Y_4)^2}{N_2} + \frac{(\Sigma Y_5)^2}{N_2} - \frac{T^2}{N}$

[$N_2 \rightarrow$ number of elements in each row]

$= \frac{8^2}{4} + \frac{18^2}{4} + \frac{(-2)^2}{4} + \frac{8^2}{4} + \frac{8^2}{4} - 80$

$= 50$

$SSE = TSS - SSC - SSR$

$= 292 - 57.2 - 50$

$= 184.8$

Step 7 : ANOVA Table

Source of Variation	Sum of squares	Degrees of freedom	Mean sum of squares	Variance ratio	Table value at 5% level
Column treatment	SSC = 57.2	$c-1$ $= 4-1=3$	MSC $= \frac{SSC}{c-1}$ $= \frac{57.2}{3}$ $= 19.1$	$F_C = \frac{MSC}{MSE} =$ 1.24	$F_C(3, 12)$ $= 3.49$
Between Rows (block)	SSR = 50	$r-1$ $=5-1=4$	MSR $= \frac{SSR}{r-1}$ $= \frac{50}{4} = 12.5$	$F_R = \frac{MSE}{MSR}$ $= 1.23$	$F_R(12,4)$ $=5.91$
Ramainder or Error	SSE = 184.8	$N-c-r+1$ $=20-4-5+1$ $= 12$	MSE $= \frac{SSE}{12}$ $= \frac{184.8}{12}$ $= 15.4$		

Step 8 : Conclusion : Cal $F_C <$ Table F_C, so accept H_0

Cal $F_R <$ Table F_R, so accept H_0

5. Working Rule

The analysis is done in a way similar to two-way classification. The different sums of squares are obtained as follows :

Step 1 : Find N.

Step 2 : Find T.

Step 3 : Find $\frac{T^2}{N}$

Step 4 : Find TSS

Step 5 : Find SSC

Step 6 : Find SSR, Find SSK

Step 7 : ANOVA table

Step 8 : Conclusion

Example 2.4.1

The following is a Latin square of a design, when 4 varieties of seeds are being tested. Set up the analysis of variance table and state your conclusion. You may carry out suitable change of origin and scale.

A	105	B	95	C	125	D	115
C	115	D	125	A	105	B	105
D	115	C	95	B	105	A	115
B	95	A	135	D	95	C	115

[A.U. M/J 2013, N/D 2013] [A.U A/M 2017 R-8]

Solution : Subtract 100 and then divided by 5, we get

A	1	B	−1	C	5	D	3
C	3	D	5	A	1	B	1
D	3	C	−1	B	1	A	3
B	−1	A	7	D	−1	C	3

	X_1	X_2	X_3	X_4	Total	X_1^2	X_2^2	X_3^2	X_4^2
Y_1	1	−1	5	3	8	1	1	25	9
Y_2	3	5	1	1	10	9	25	1	1
Y_3	3	−1	1	3	6	9	1	1	9
Y_4	−1	7	−1	3	8	1	49	1	9
Total	6	10	6	10	32	20	76	28	28

H_0 : There is no significant difference between rows, columns and treatments.

H_1 : There is significant difference between rows, columns and treatments.

Step 1 : $N = 16$

Step 2 : $T = 32$

Step 3 : $\dfrac{T^2}{N} = \dfrac{(32)^2}{16} = 64$

Step 4 : TSS $= \Sigma X_1^2 + \Sigma X_2^2 + \Sigma X_3^2 + \Sigma X_4^2 - \frac{T^2}{N}$

$= 20 + 76 + 28 + 28 - 64 = 88$

Step 5 : SSC $= \frac{(\Sigma X_1)^2}{N_1} + \frac{(\Sigma X_2)^2}{N_1} + \frac{(\Sigma X_3)^2}{N_1} + \frac{(\Sigma X_4)^2}{N_1} - \frac{T^2}{N}$

[N_1 = Number of elements in each column]

$= \frac{6^2}{4} + \frac{10^2}{4} + \frac{6^2}{4} + \frac{10^2}{4} - 64$

$= 9 + 25 + 9 + 25 - 64 = 4$

Step 6 : To find SSR $= \frac{(\Sigma Y_1)^2}{N_2} + \frac{(\Sigma Y_2)^2}{N_2} + \frac{(\Sigma Y_3)^2}{N_2} + \frac{(\Sigma Y_4)^2}{N_2} - \frac{T^2}{N}$

[N_2 = Number of elements in each row]

$= \frac{(8)^2}{4} + \frac{(10)^2}{4} + \frac{(6)^2}{4} + \frac{(8)^2}{4} - 64$

$= 16 + 25 + 9 + 16 - 64 = 2$

To find SSK :

Arrange the elements in the order of treatment.

A	1	1	3	7	12
B	−1	1	1	−1	0
C	5	3	−1	3	10
D	3	5	3	−1	10

SSK $= \frac{(12)^2}{4} + \frac{0^2}{4} + \frac{(10)^2}{4} + \frac{(10)^2}{4} - \frac{T^2}{N}$

$= 36 + 0 + 25 + 25 - 64 = 22$

SSE = TSS − SSC − SSR − SSK

$= 88 - 4 - 2 - 22 = 60$

Step 7 : ANOVA Table

Source of Variation	SS	DF	MSS	Variance Ratio	Table value at 5% level
Between Rows	SSR = 2	$K-1=3$	$MSR = \frac{SSR}{K-1} = 0.67$	$F_R = \frac{MSE}{MSR} = \frac{10}{0.67} = 14.9$	$F_R(6,3) = 8.94$
Between Columns	SSC = 4	$K-1=3$	$MSC = \frac{SSC}{K-1} = 1.33$	$F_C = \frac{MSE}{MSC} = \frac{10}{1.33} = 7.52$	$F_C(6,3) = 8.94$
Between Treatments	SSK = 22	$K-1=3$	$MSK = \frac{SSK}{K-1} = 7.33$	$F_T = \frac{MSE}{MSK} = \frac{10}{7.33} = 1.36$	$F_T(6,3) = 8.94$
Error	SSE = 60	$(K-1)(K-2) = 6$	$MSE = \frac{SSE}{(K-1)(K-2)} = 10$		
	88	15			

Step 8 : Conclusion : Here,

Cal F_C < Table F_C

Cal F_R > Table F_R

Cal F_T < Table F_T

Example 2.4.2

A variable trial was conducted on wheat with 4 varieties in a Latin Square Design. The plan of the experiment and the per plot yield are given below :

C	25	B	23	A	20	D	20
A	19	D	19	C	21	B	18
B	19	A	14	D	17	C	20
D	17	C	20	B	21	A	15

Analyse data and interpret the result. **[A.U M/J 2012]**
[A.U N/D 2016 R-13] [A.U A/M 2017 R-13]

Solution : Subtract 20 from all the items.

Y_n	X_1	X_2	X_3	X_4	Total	X_1^2	X_2^2	X_3^2	X_n^2
Y_1	5	3	0	0	8	25	9	0	0
Y_2	−1	−1	1	−2	−3	1	1	1	4
Y_3	−1	−6	−3	0	−10	1	36	9	0
Y_4	−3	0	1	−5	−7	9	0	1	25
	0	−4	−1	−7	−12	36	46	11	29

H_0 : There is no significant difference between rows, columns and treatments.

H_1 : There is significant difference between rows, columns and treatments.

Step 1 : $N = 16$

Step 2 : $T = -12$

Step 3 : $\frac{T^2}{N} = \frac{144}{16} = 9$

Step 4 : $TSS = \Sigma X_1^2 + \Sigma X_2^2 + \Sigma X_3^2 + \Sigma X_4^2 - \frac{T^2}{N}$

$= 36 + 46 + 11 + 29 - 9 = 113$

Step 5 : $SSC = \frac{(\Sigma X_1)^2}{N_1} + \frac{(\Sigma X_2)^2}{N_1} + \frac{(\Sigma X_3)^2}{N_1} + \frac{(\Sigma X_4)^2}{N_1} - \frac{T^2}{N}$

[N_1 = Number of elements in each column]

$= \frac{0^2}{4} + \frac{16}{4} + \frac{1}{4} + \frac{49}{4} - 9 = 4 + 0.25 + 12.25 - 9 = 7.5$

Step 6 : $SSR = \frac{(\Sigma Y_1)^2}{N_2} + \frac{(\Sigma Y_2)^2}{N_2} + \frac{(\Sigma Y_3)^2}{N_2} + \frac{(\Sigma Y_4)^2}{N_2} - \frac{T^2}{N}$

[N_2 = Number of elements in each row]

$= \frac{8^2}{4} + \frac{(-3)^2}{4} + \frac{(-10)^2}{4} + \frac{(-7)^2}{4} - 9$

$= 16 + 2.25 + 25 + 12.25 - 9 = 46.5$

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To find SSK :

					T
A	0	−1	−6	−5	−12
B	3	−2	−1	1	1
C	5	1	0	0	6
D	0	−1	−3	−3	−7

$$SSK = \frac{(-12)^2}{4} + \frac{(1)^2}{4} + \frac{(6)^2}{4} + \frac{(-7)^2}{4} - \frac{T^2}{N}$$

$$= 36 + 0.25 + 9 + 12.25 - 9 = 48.5$$

$$SSE = TSS - SSC - SSR - SSK$$

$$= 113 - 7.5 - 46.5 - 48.5 = 10.5$$

Step 7 : ANOVA Table

Source of Variation	SS	DF	MSS	Variance Ratio	Table value at 5% level
Between Row	SSR=46.5	K − 1=3	$MSR = \frac{SSR}{K-1} = 15.5$	$F_R = \frac{MSR}{MSE} = 8.86$	$F_R(3,6) = 4.76$
Between Column	SSC=7.5	K − 1=3	$MSC = \frac{SSC}{K-1} = 2.5$	$F_C = \frac{MSC}{MSE} = 1.43$	$F_C(3,6) = 4.76$
Between Treatment	SSK=48.5	K − 1=3	$MSK = \frac{SSK}{K-1} = 16.17$	$F_T = \frac{MSK}{MSE} = 9.24$	$F_T(3,6) = 4.76$
Error	SSE = 10.5	(K−1)(K−2) = 6	$MSE = \frac{SSE}{(K-1)(K-2)} = 1.75$		
Total	TSS = 113				

Step 8 : Conclusion : Cal F_R > table F_R

Cal F_C < table F_C

Cal F_T > table F_T

There is significant difference between treatments and rows.

But, there is no significant differene between columns.