LATIN SQUARE DESIGN

- ANALYSIS OF VARIANCE FOR THREE FACTORS OF CLASSIFICATIONS
- Let N=n^2 variate values $\{x_{ij}\}$ representing the yield of paddy, be classified according to three factors. Let the rows, columns and letters stand for the three factors, say soil fertility, seed quality and treatment respectively.
- Our null hypothesis is that the rows, columns and treatment are homogeneous .

ANOVA FOR THREE FACTORS OF CLASSIFICATION

S.V.	S.S.	d.f.	M.S.	F
Between rows	Q_1	$n-1$	$Q_1/(n-1) = M_1$	$\left(\frac{M_1}{M_4}\right)^{\pm 1}$
Between columns	Q_2	$n-1$	$Q_2/(n-1) = M_2$	$\left(\frac{M_2}{M_4}\right)^{\pm 1}$
Between letters	Q_3	$n-1$	$Q_3/(n-1) = M_3$	$\left(\frac{M_3}{M_4}\right)^{\pm 1}$
Residual	Q_4	$(n-1)(n-2)$	$Q_4/(n-1)(n-2) = M_4$	—
Total	Q	n^2-1	—	—

1. $Q = \sum\sum x_{ij}^2 - \frac{T^2}{n^2}$, *where* $T = \sum\sum x_{ij}$

2. $Q_1 = \frac{1}{n}\sum T_i^2 - \frac{T^2}{n^2}$, *where* $T_i = \sum_{j=1}^{n} x_{ij}$

3. $Q_2 = \frac{1}{n}\sum T_j^2 - \frac{T^2}{n^2}$, *where* $T_j = \sum_{i=1}^{n} x_{ij}$

4. $Q_3 = \frac{1}{n}\sum T_k^2 - \frac{T^2}{n^2}$, *where* T_k *is the sum of all* x_{ij}*'s receiving the* k^{th} *treatment.*

5. $Q_4 = Q - Q_1 - Q_2 - Q_3$.

Also $T = \sum_i T_i = \sum_j T_j = \sum_k T_k$

PROBLEM

Example 9 *The following data resulted from an experiment to compare three burners B_1, B_2 and B_3. A Latin square design was used as the tests were made on 3 engines and were spread over 3 days.*

	Engine 1	Engine 2	Engine 3
Day 1	$B_1 - 16$	$B_2 - 17$	$B_3 - 20$
Day 2	$B_2 - 16$	$B_3 - 21$	$B_1 - 15$
Day 3	$B_3 - 15$	$B_1 - 12$	$B_2 - 13$

Test the hypothesis that there is no difference between the burners.

Solution We subtract 16 from the given values and work out with new values of x_{ij}.

	E_1	E_2	E_3	T_i	$\frac{T_i^2}{n}$	$\sum_j x_{ij}^2$
D_1	$0(B_1)$	$1(B_2)$	$4(B_3)$	5	8.33	17
D_2	$0(B_2)$	$5(B_3)$	$-1(B_1)$	4	5.33	26
D_3	$-1(B_3)$	$-4(B_1)$	$-3(B_2)$	-8	21.33	26
T_j	-1	2	0	$T = 1$	$\sum T_i^2 / n = 35$	69
T_j^2 / n	0.33	1.33	0	$\sum T_i^2 / n = 1.66$		
$\sum_i x_{ij}^2$	1	42	26	69		

Rearranging the data values according to the burners, we have

Burner		x_k		T_k	T_k^2 / n
B_1	0	-1	-4	-5	8.33
B_2	1	0	-3	-2	1.33
B_3	4	5	-1	8	21.33
	Total		$T = 1$	$\sum \frac{T_k^2}{n} = 31$	

$$Q = \sum\sum x_{ij}^2 - \frac{T^2}{N} = 69 - \frac{1}{9} = 68.89$$

$$Q_1 = \frac{1}{n}\sum T_i^2 - \frac{T^2}{N} = 35 - \frac{1}{9} = 34.89$$

$$Q_2 = \frac{1}{n}\sum T_j^2 - \frac{T^2}{N} = 1.67 - \frac{1}{9} = 1.56$$

$$Q_3 = \frac{1}{n}\sum T_k^2 - \frac{T^2}{N} = 31 - \frac{1}{9} = 30.89$$

$$Q_4 = Q - Q_1 - Q_2 - Q_3 = 1.55$$

ANOVA table

S.V.	S.S.	d.f.	M.S.	F_0
Between rows (days)	$Q_1 = 34.89$	$n-1=2$	17.445	$\frac{17.445}{0.775} = 22.51$
Between Cols. (engines)	$Q_2 = 1.56$	$n-1=2$	0.780	$\frac{0.780}{0.775} = 1.01$
Between letters (burners)	$Q_3 = 30.89$	$n-1=2$	15.445	$\frac{15.445}{0.775} = 19.93$
Residual	$Q_4 = 1.55$	$(n-1)(n-2) = 2$	0.775	–
Total	$Q = 68.89$	$n^2-1=8$	–	–

From the F-tables, $F_{5\%}\ (v_1 = 2, v_2 = 2) = 19.00$

Since $F_0 (= 19.93) > F_{5\%}\ (= 19.00)$ for the burners, there is significant difference between the burners.

Incidentally, since $F_0 > F_{5\%}$ for the rows, the difference between the days is significant and since $F_0 < F_{5\%}$ for the columns, the difference between the engine is not significant.