

PAIRED T-TEST

29/11/2021

- (i) The sample sizes are equal; i.e., $n_1 = n_2 = n$ (say), and
- (ii) The sample observations (x_1, x_2, \dots, x_n) and (y_1, y_2, \dots, y_n) are not completely independent but they are dependent in pairs.

Let $d_i = X_i - Y_i$ ($i = 1, 2, \dots, n$) denote the difference in the observations for the i th unit.

$H_0: \mu_1 = \mu_2$ i.e. the increments are just by chance

$H_1: \mu_1 \neq \mu_2$ ($\mu_1 > \mu_2$ (or) $\mu_1 < \mu_2$)

test statistic

$$t_0 = \left| \frac{\bar{d}}{S/\sqrt{n}} \right|$$

$$\text{where } \bar{d} = \frac{\sum d}{n} \text{ and } S^2 = \frac{1}{n-1} \sum (d - \bar{d})^2 = \frac{1}{n-1} \left[\sum d^2 - \frac{(\sum d)^2}{n} \right]$$

Expected value:

$$t_e = \left| \frac{\bar{d}}{S/\sqrt{n}} \right| \text{ follows } \underline{\underline{t\text{-distribution with } n-1 \text{ d.f}}}$$

Problems :-

1.

To test the desirability of a certain modification in typists desks, 9 typists were given two tests of as nearly as possible the same nature, one on the desk in use and the other on the new type. The following difference in the number of words typed per minute were recorded:

Typists	A	B	C	D	E	F	G	H	I
Increase in number of words	2	4	0	3	-1	4	-3	2	5

Do the data indicate the modification in desk promotes speed in typing?

$$\alpha = 0.05$$

$$H_0: \mu_1 = \mu_2$$

$$H_1: \mu_1 < \mu_2 \quad (\text{left tail})$$

(one tail)

Typist	d	d ²
A	2	4
B	4	16
C	0	0
D	3	9
E	-1	1
F	4	16
G	-3	9
H	2	4
I	5	10
	<hr/> 16	<hr/> 84

$$n = 9$$

$$\bar{d} = \frac{\sum d}{n} = \frac{16}{9} = 1.778$$

$$S = \sqrt{\frac{1}{n-1} \left[\sum d^2 - \frac{(\sum d)^2}{n} \right]}$$

$$= \sqrt{\frac{1}{8} \left[84 - \frac{(16)^2}{9} \right]}$$

$$= 2.62$$

$$t_{cal} = \left| \frac{\bar{d}}{s/\sqrt{n}} \right| = \left| \frac{1.718}{2.62/\sqrt{9}} \right| = \underline{\underline{2.02}}$$

$$t_{table} \text{ (one tail + 5\% LOS)} = 1.86$$

$$df = n-1 = 9-1 = 8$$

$$t = 2.02 > 1.86 \text{ (table)}$$

H_0 is rejected -

paired \rightarrow data \rightarrow two pairs of data are given -

$d \rightarrow$ diff in pairs

large samples:-

Z test \rightarrow single mean
 \rightarrow diff. of mean
 \rightarrow single proportion
 \rightarrow diff. of proportion

Small samples:-

t - test \rightarrow single sample.
 \rightarrow diff. of means of two samples.
 \rightarrow paired t-test

F - test

chi square test \rightarrow independence of attributes.
 \rightarrow goodness of fit.

One Way ANOVA

ANOVA \rightarrow Analysis of Variance.