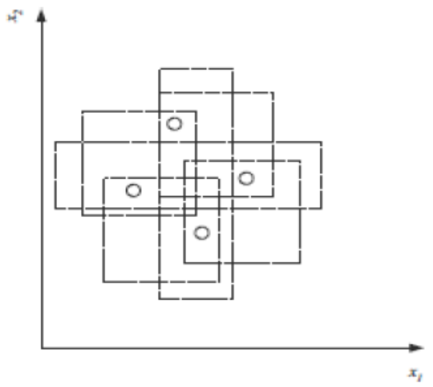


Fig: An axis-aligned rectangle can shattered four points. Only rectangle covering two points are shown.

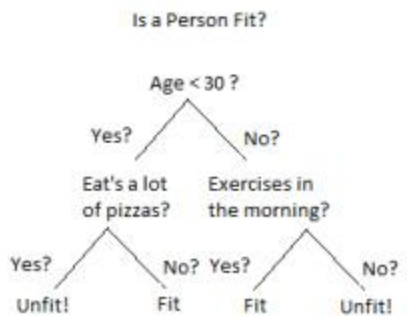
VC dimension may seem pessimistic. It tells us that using a rectangle as our hypothesis class, we can learn only datasets containing four points and not more.

Unit II
Supervised and Unsupervised Learning

Topics: Decision Trees: ID3, Classification and Regression Trees, Regression: Linear Regression, Multiple Linear Regression, Logistic Regression, Neural Networks: Introduction, Perception, Multilayer Perception, Support Vector Machines: Linear and Non-Linear, Kernel Functions, K Nearest Neighbors. Introduction to clustering, K-means clustering, K-Mode Clustering.

2.1. Decision Tree

Introduction Decision Trees are a type of Supervised Machine Learning (that is you explain what the input is and what the corresponding output is in the training data) where the data is continuously split according to a certain parameter. The tree can be explained by two entities, namely **decision nodes and leaves**. The leaves are the decisions or the final outcomes. And the decision nodes are where the data is split.



An example of a decision tree can be explained using above binary tree. Let’s say you want to predict whether a person is fit given their information like age, eating habit, and physical activity, etc. The decision nodes here are questions like ‘What’s the age?’, ‘Does he exercise?’, and ‘Does he eat a lot of pizzas’? And the leaves, which are outcomes like either ‘fit’, or ‘unfit’. In this case this was a binary classification problem (a yes no type problem). There are two main types of Decision Trees:

1. **Classification trees** (Yes/No types)

What we have seen above is an example of classification tree, where the outcome was a variable like ‘fit’ or ‘unfit’. Here the decision variable is **Categorical**.

2. **Regression trees** (Continuous data types)

Here the decision or the outcome variable is **Continuous**, e.g. a number like 123. **Working** Now that we know what a Decision Tree is, we’ll see how it works internally. There are many algorithms out there which construct Decision Trees, but one of the best is called as **ID3 Algorithm**. ID3 Stands for **Iterative Dichotomiser 3**. Before discussing the ID3 algorithm, we’ll go through few definitions. **Entropy** Entropy, also called as Shannon Entropy is denoted by H(S) for a finite set S, is the measure of the amount of uncertainty or randomness in data.

$$H(S) = \sum_{x \in X} p(x) \log_2 \frac{1}{p(x)}$$

Intuitively, it tells us about the predictability of a certain event. Example, consider a coin toss whose probability of heads is 0.5 and probability of tails is 0.5. Here the entropy is the highest possible, since there’s no way of determining what the outcome might be. Alternatively, consider a coin which has heads on both the sides, the entropy of such an event can be predicted perfectly since we know beforehand that it’ll always be heads. In other words, this event has **no randomness** hence it’s entropy is zero. In particular, lower values imply less uncertainty while higher values imply high uncertainty. **Information Gain** Information gain is also called as Kullback-Leibler divergence denoted by IG(S,A) for a set S is the effective change in entropy after deciding on a particular attribute A. It measures the relative change in entropy with respect to the independent variables

$$IG(S,A) = H(S) - H(S,A)$$

Alternatively,

$$IG(S,A) = H(S) - \sum_{i=0}^{n} P(x) * H(x)$$

where IG(S, A) is the information gain by applying feature A. H(S) is the Entropy of the entire set, while the second term calculates the Entropy after applying the feature A, where P(x) is the probability of event x. Let’s understand this with the help of an example Consider a piece of data collected over the course of 14 days where the features are Outlook, Temperature, Humidity, Wind and the outcome variable is whether Golf was played on the day. Now, our job is to build a predictive model which takes in above 4 parameters and predicts whether Golf will be played on the day. We’ll build a decision tree to do that using **ID3 algorithm.**

Day	**Outlook**	**Temperature**	**Humidity**	**Wind**	**Play Golf**
D1	Sunny	Hot	High	Weak	No
D2	Sunny	Hot	High	Strong	No
D3	Overcast	Hot	High	Weak	Yes

D4	Rain	Mild	High	Weak	Yes
D5	Rain	Cool	Normal	Weak	Yes
D6	Rain	Cool	Normal	Strong	No
D7	Overcast	Cool	Normal	Strong	Yes
D8	Sunny	Mild	High	Weak	No
D9	Sunny	Cool	Normal	Weak	Yes
D10	Rain	Mild	Normal	Weak	Yes
D11	Sunny	Mild	Normal	Strong	Yes
D12	Overcast	Mild	High	Strong	Yes
D13	Overcast	Hot	Normal	Weak	Yes
D14	Rain	Mild	High	Strong	No

2.1.1 ID3

ID3 Algorithm will perform following tasks recursively

1. Create root node for the tree
2. If all examples are positive, return leaf node 'positive'
3. Else if all examples are negative, return leaf node 'negative'
4. Calculate the entropy of current state H(S)
5. For each attribute, calculate the entropy with respect to the attribute 'x' denoted by H(S, x)
6. Select the attribute which has maximum value of IG(S, x)
7. Remove the attribute that offers highest IG from the set of attributes
8. Repeat until we run out of all attributes, or the decision tree has all leaf nodes.

Now we'll go ahead and grow the decision tree. The initial step is to calculate H(S), the Entropy of the current state. In the above example, we can see in total there are 5 No's and 9 Yes's.

Yes	No	Total
9	5	14

$$Entropy(S) = \sum_{x \in X} p(x) \log_2 \frac{1}{p(x)}$$

$$Entropy(S) = -\left(\frac{9}{14}\right)\log_2\left(\frac{9}{14}\right) - \left(\frac{5}{14}\right)\log_2\left(\frac{5}{14}\right)$$

$$= 0.940$$

where 'x' are the possible values for an attribute. Here, attribute 'Wind' takes two possible values in the sample data, hence x = {Weak, Strong} we'll have to calculate:

1. $H(S_{weak})$
2. $H(S_{strong})$
3. $P(S_{weak})$
4. $P(S_{strong})$
5. $H(S) = 0.94$ which we had already calculated in the previous example

Amongst all the 14 examples we have **8 places where the wind is weak and 6 where the wind is Strong**.

Wind = Weak	Wind = Strong	Total
8	6	14

$$P(S_{weak}) = \frac{Number\ of\ Weak}{Total}$$
$$= \frac{8}{14}$$
$$P(S_{strong}) = \frac{Number\ of\ Strong}{Total}$$
$$= \frac{6}{14}$$

Now out of the 8 Weak examples, 6 of them were ‘Yes’ for Play Golf and 2 of them were ‘No’ for ‘Play Golf’. So, we have,

$$Entropy(S_{weak}) = -\left(\frac{6}{8}\right)\log_2\left(\frac{6}{8}\right) - \left(\frac{2}{8}\right)\log_2\left(\frac{2}{8}\right)$$
$$= 0.811$$

Similarly, out of 6 Strong examples, we have **3 examples where the outcome was ‘Yes’ for Play Golf and 3 where we had ‘No’ for Play Golf**.

$$Entropy(S_{strong}) = -\left(\frac{3}{6}\right)\log_2\left(\frac{3}{6}\right) - \left(\frac{3}{6}\right)\log_2\left(\frac{3}{6}\right)$$
$$= 1.000$$

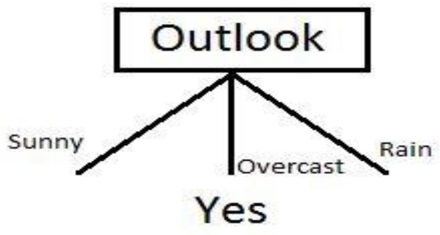
Remember, here half items belong to one class while other half belong to other. Hence we have perfect randomness. Now we have all the pieces required to calculate the Information Gain,

$$IG(S, Wind) = H(S) - \sum_{i=0}^{n} P(x) * H(x)$$
$$IG(S, Wind) = H(S) - P(S_{weak}) * H(S_{weak}) - P(S_{strong}) * H(S_{strong})$$
$$= 0.940 - \left(\frac{8}{14}\right)(0.811) - \left(\frac{6}{14}\right)(1.00)$$
$$= 0.048$$

Which tells us the Information Gain by considering ‘Wind’ as the feature and give us information gain of **0.048**. Now we must similarly calculate the Information Gain for all the features.

$$IG(S, Outlook) = 0.246$$
$$IG(S, Temperature) = 0.029$$
$$IG(S, Humidity) = 0.151$$
$$IG(S, Wind) = 0.048 \text{ (Previous example)}$$

We can clearly see that IG(S, Outlook) has the highest information gain of 0.246, **hence we chose Outlook attribute as the root node**. At this point, the decision tree looks like.



Here we observe that whenever the outlook is Overcast, Play Golf is always 'Yes', it's no coincidence by any chance, the simple tree resulted because of **the highest information gain is given by the attribute Outlook**. Now how do we proceed from this point? We can simply apply **recursion**, you might want to look at the algorithm steps described earlier. Now that we've used Outlook, we've got three of them remaining Humidity, Temperature, and Wind. And, we had three possible values of Outlook: Sunny, Overcast, Rain. Where the Overcast node already ended up having leaf node 'Yes', so we're left with two subtrees to compute: Sunny and Rain.

Next step would be computing$H(S_{sunny})$.

Table where the value of Outlook is Sunny looks like:

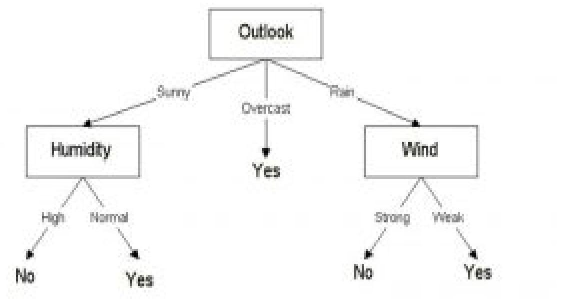
Temperature	Humidity	Wind	Play Golf
Hot	High	Weak	No
Hot	High	Strong	No
Mild	High	Weak	No
Cool	Normal	Weak	Yes
Mild	Normal	Strong	Yes

$$H(S_{sunny}) = \left(\frac{3}{5}\right)\log_2\left(\frac{3}{5}\right) - \left(\frac{2}{5}\right)\log_2\left(\frac{2}{5}\right) = 0.96$$

As we can see the **highest Information Gain is given by Humidity**. Proceeding in the same way with

$$S_{rain}$$

will give us Wind as the one with highest information gain. The final Decision Tree looks something like this. The final Decision Tree looks something like this.



2.1.2. Classification and Regression Trees

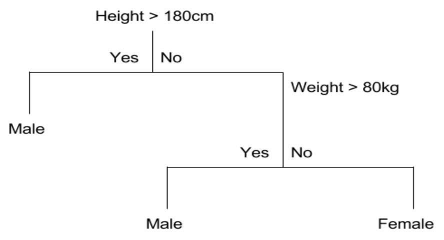
2.1.2.1. Classification Trees

A classification tree is an algorithm where the target variable is fixed or categorical. The algorithm is then used to identify the "class" within which a target variable would most likely fall.

An example of a classification-type problem would be determining who will or will not subscribe to a digital platform; or who will or will not graduate from high school.

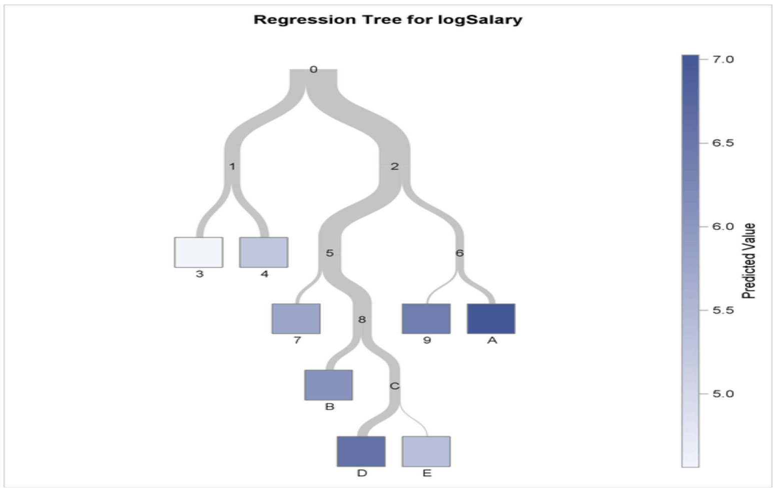
These are examples of simple binary classifications where the categorical dependent variable can assume only one of two, mutually exclusive values. In other cases, you might have to predict among a number of different variables. For instance, you may have to predict which type of smartphone a consumer may decide to purchase.

In such cases, there are multiple values for the categorical dependent variable. Here's what a classic classification tree looks like



2.1.2.2. Regression Trees

A regression tree refers to an algorithm where the target variable is and the algorithm is used to predict it's value. As an example of a regression type problem, you may want to predict the selling prices of a residential house, which is a continuous dependent variable.
This will depend on both continuous factors like square footage as well as categorical factors like the style of home, area in which the property is located and so on.



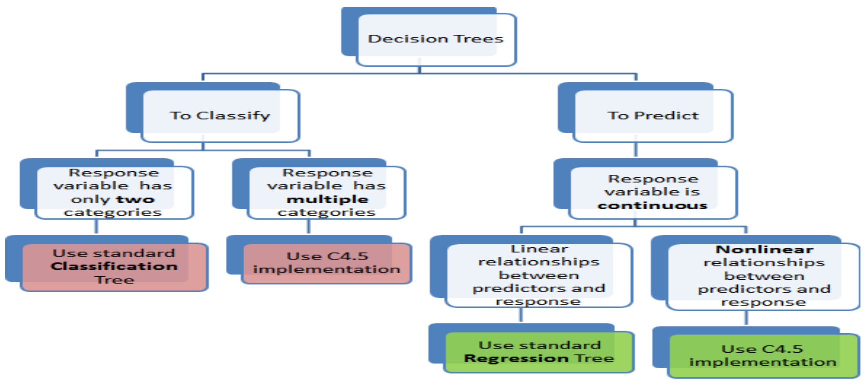
When to use Classification and Regression Trees

Classification trees are used when the dataset needs to be split into classes which belong to the response variable. In many cases, the classes Yes or No.
In other words, they are just two and mutually exclusive. In some cases, there may be more than two classes in which case a variant of the classification tree algorithm is used.
Regression trees, on the other hand, are used when the response variable is continuous. For instance, if the response variable is something like the price of a property or the temperature of the day, a regression tree is used.
In other words, regression trees are used for prediction-type problems while classification trees are used for classification-type problems.

How Classification and Regression Trees Work

A classification tree splits the dataset based on the homogeneity of data. Say, for instance, there are two variables; income and age; which determine whether or not a consumer will buy a particular kind of phone.
If the training data shows that 95% of people who are older than 30 bought the phone, the data gets split there and age becomes a top node in the tree. This split makes the data "95% pure". Measures of impurity like entropy or Gini index are used to quantify the homogeneity of the data when it comes to classification trees.

In a regression tree, a regression model is fit to the target variable using each of the independent variables. After this, the data is split at several points for each independent variable.
At each such point, the error between the predicted values and actual values is squared to get “A Sum of Squared Errors” (SSE). The SSE is compared across the variables and the variable or point which has the lowest SSE is chosen as the split point. This process is continued recursively.


Decision Trees
To Classify
To Predict
Response variable has only two categories
Response variable has multiple categories
Response variable is continuous
Use standard Classification Tree
Use C4.5 implementation
Linear relationships between predictors and response
Nonlinear relationships between predictors and response
Use standard Regression Tree
Use C4.5 implementation

Advantages of Classification and Regression Trees
The purpose of the analysis conducted by any classification or regression tree is to create a set of if-else conditions that allow for the accurate prediction or classification of a case.
(i) The Results are Simplistic
The interpretation of results summarized in classification or regression trees is usually fairly simple. The simplicity of results helps in the following ways.

- It allows for the rapid classification of new observations. That’s because it is much simpler to evaluate just one or two logical conditions than to compute scores using complex nonlinear equations for each group.
- It can often result in a simpler model which explains why the observations are either classified or predicted in a certain way. For instance, business problems are much easier to explain with if-then statements than with complex nonlinear equations.

(ii) Classification and Regression Trees are Nonparametric & Nonlinear
The results from classification and regression trees can be summarized in simplistic if-then conditions. This negates the need for the following implicit assumptions.

- The predictor variables and the dependent variable are linear.
- The predictor variables and the dependent variable follow some specific nonlinear link function.
- The predictor variables and the dependent variable are monotonic.

Since there is no need for such implicit assumptions, classification and regression tree methods are well suited to data mining. This is because there is very little knowledge or assumptions that can be made beforehand about how the different variables are related.
As a result, classification and regression trees can actually reveal relationships between these variables that would not have been possible using other techniques.
(iii) Classification and Regression Trees Implicitly Perform Feature Selection
Feature selection or variable screening is an important part of analytics. When we use decision trees, the top few nodes on which the tree is split are the most important variables within the set. As a result, feature selection gets performed automatically and we don’t need to do it again.
Limitations of Classification and Regression Trees

Classification and regression tree tutorials, as well as classification and regression tree ppts, exist in abundance. This is a testament to the popularity of these decision trees and how frequently they are used. However, these decision trees are not without their disadvantages.
There are many classification and regression trees examples where the use of a decision tree has not led to the optimal result. Here are some of the limitations of classification and regression trees.

(i) Overfitting

Overfitting occurs when the tree takes into account a lot of noise that exists in the data and comes up with an inaccurate result.

(ii) High variance

In this case, a small variance in the data can lead to a very high variance in the prediction, thereby affecting the stability of the outcome.

(iii) Low bias

A decision tree that is very complex usually has a low bias. This makes it very difficult for the model to incorporate any new data.

What is a CART in Machine Learning?

A Classification and Regression Tree (CART) is a predictive algorithm used in machine learning. It explains how a target variable's values can be predicted based on other values.
It is a decision tree where each fork is a split in a predictor variable and each node at the end has a prediction for the target variable.
The CART algorithm is an important decision tree algorithm that lies at the foundation of machine learning. Moreover, it is also the basis for other powerful machine learning algorithms like bagged decision trees, random forest and boosted decision trees.

Summing up

The Classification and regression tree (CART) methodology is one of the oldest and most fundamental algorithms. It is used to predict outcomes based on certain predictor variables.
They are excellent for data mining tasks because they require very little data pre-processing. Decision tree models are easy to understand and implement which gives them a strong advantage when compared to other analytical models.

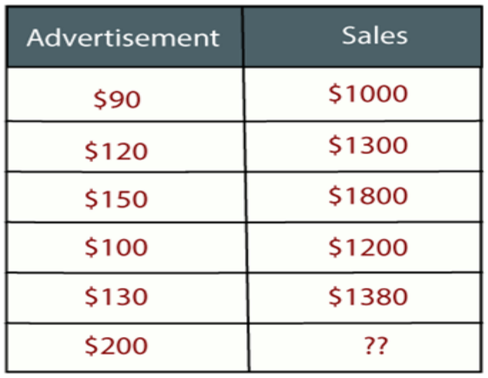
2.2. Regression

Regression Analysis in Machine learning

Regression analysis is a statistical method to model the relationship between a dependent (target) and independent (predictor) variables with one or more independent variables. More specifically, Regression analysis helps us to understand how the value of the dependent variable is changing corresponding to an independent variable when other independent variables are held fixed. It predicts continuous/real values such as **temperature, age, salary, price,** etc.

We can understand the concept of regression analysis using the below example:

Example: Suppose there is a marketing company A, who does various advertisement every year and get sales on that. The below list shows the advertisement made by the company in the last 5 years and the corresponding sales:



Advertisement	Sales
$90	$1000
$120	$1300
$150	$1800
$100	$1200
$130	$1380
$200	??

Now, the company wants to do the advertisement of $200 in the year 2019 **and wants to know the prediction about the sales for this year**. So to solve such type of prediction problems in machine learning, we need regression analysis.
Regression is a supervised learning technique which helps in finding the correlation between variables and enables us to predict the continuous output variable based on the one or more predictor variables. It is mainly used for **prediction, forecasting, time series modeling, and determining the causal-effect relationship between variables**.
In Regression, we plot a graph between the variables which best fits the given datapoints, using this plot, the machine learning model can make predictions about the data. In simple words, ***"Regression shows a line or curve that passes through all the datapoints on target-predictor graph in such a way that the vertical distance between the datapoints and the regression line is minimum."*** The distance between datapoints and line tells whether a model has captured a strong relationship or not.

Some examples of regression can be as:

- Prediction of rain using temperature and other factors
- Determining Market trends
- Prediction of road accidents due to rash driving.

Terminologies Related to the Regression Analysis:

- **Dependent Variable:** The main factor in Regression analysis which we want to predict or understand is called the dependent variable. It is also called **target variable**.
- **Independent Variable:** The factors which affect the dependent variables or which are used to predict the values of the dependent variables are called independent variable, also called as a **predictor**.
- **Outliers:** Outlier is an observation which contains either very low value or very high value in comparison to other observed values. An outlier may hamper the result, so it should be avoided.
- **Multicollinearity:** If the independent variables are highly correlated with each other than other variables, then such condition is called Multicollinearity. It should not be present in the dataset, because it creates problem while ranking the most affecting variable.
- **Underfitting and Overfitting:** If our algorithm works well with the training dataset but not well with test dataset, then such problem is called **Overfitting**. And if our algorithm does not perform well even with training dataset, then such problem is called **underfitting**.

Why do we use Regression Analysis?

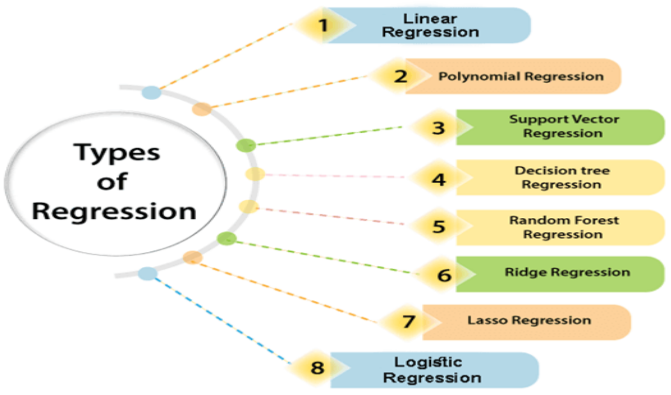
As mentioned above, Regression analysis helps in the prediction of a continuous variable. There are various scenarios in the real world where we need some future predictions such as weather condition, sales prediction, marketing trends, etc., for such case we need some technology which can make predictions more accurately. So for such case we need Regression analysis which is a statistical method and used in machine learning and data science. Below are some other reasons for using Regression analysis:

- Regression estimates the relationship between the target and the independent variable.
- It is used to find the trends in data.
- It helps to predict real/continuous values.
- By performing the regression, we can confidently determine the **most important factor, the least important factor, and how each factor is affecting the other factors**.

Types of Regression

There are various types of regressions which are used in data science and machine learning. Each type has its own importance on different scenarios, but at the core, all the regression methods analyze the effect of the independent variable on dependent variables. Here we are discussing some important types of regression which are given below:

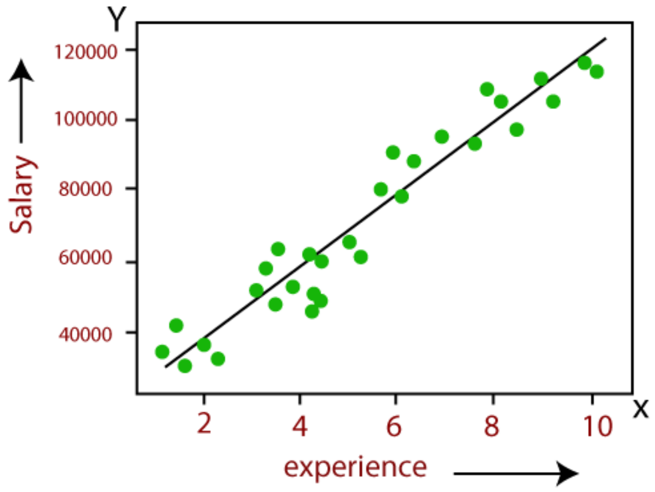
- **Linear Regression**
- **Logistic Regression**
- **Polynomial Regression**
- **Support Vector Regression**
- **Decision Tree Regression**
- **Random Forest Regression**
- **Ridge Regression**
- **Lasso Regression**



2.2.1. Linear Regression:

- Linear regression is a statistical regression method which is used for predictive analysis.
- It is one of the very simple and easy algorithms which works on regression and shows the relationship between the continuous variables.
- It is used for solving the regression problem in machine learning.
- Linear regression shows the linear relationship between the independent variable (X-axis) and the dependent variable (Y-axis), hence called linear regression.

- If there is only one input variable (x), then such linear regression is called **simple linear regression**. And if there is more than one input variable, then such linear regression is called **multiple linear regression**.
- The relationship between variables in the linear regression model can be explained using the below image. Here we are predicting the salary of an employee on the basis of **the year of experience**.



Below is the mathematical equation for Linear regression:

Y= aX+b

Here, Y = dependent variables (target variables),
X= Independent variables (predictor variables),
a and b are the linear coefficients

Some popular applications of linear regression are:

- Analyzing trends and sales estimates
- Salary forecasting
- Real estate prediction
- Arriving at ETAs in traffic.

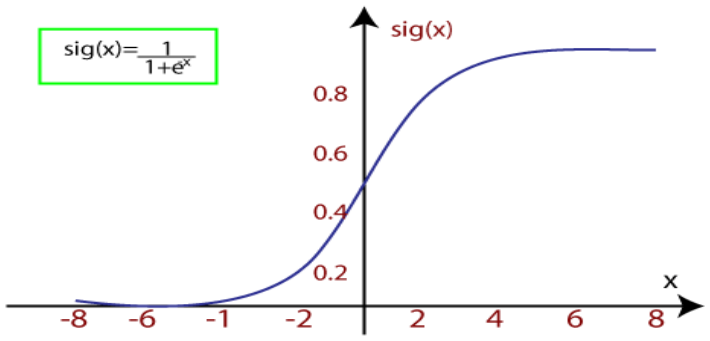
2.2.2. Logistic Regression:

- Logistic regression is another supervised learning algorithm which is used to solve the classification problems. In **classification problems**, we have dependent variables in a binary or discrete format such as 0 or 1.
- Logistic regression algorithm works with the categorical variable such as 0 or 1, Yes or No, True or False, Spam or not spam, etc.
- It is a predictive analysis algorithm which works on the concept of probability.
- Logistic regression is a type of regression, but it is different from the linear regression algorithm in the term how they are used.
- Logistic regression uses **sigmoid function** or logistic function which is a complex cost function. This sigmoid function is used to model the data in logistic regression. The function can be represented as:

$$f(x)=\frac{1}{1+e^{-x}}$$

- f(x)= Output between the 0 and 1 value.
- x= input to the function
- e= base of natural logarithm.

  When we provide the input values (data) to the function, it gives the S-curve as follows:



- It uses the concept of threshold levels, values above the threshold level are rounded up to 1, and values below the threshold level are rounded up to 0.

There are three types of logistic regression:

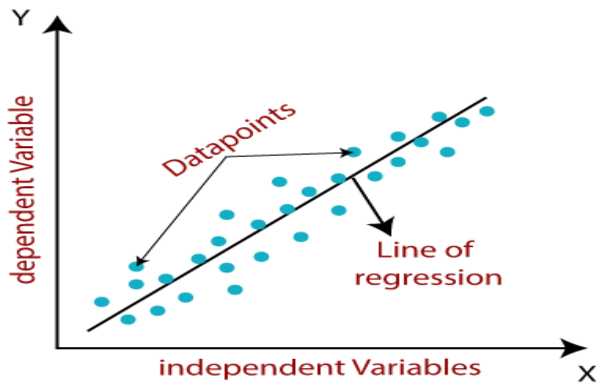
- **Binary(0/1, pass/fail)**
- **Multi(cats, dogs, lions)**
- **Ordinal(low, medium, high)**

Linear Regression in Machine Learning

Linear regression is one of the easiest and most popular Machine Learning algorithms. It is a statistical method that is used for predictive analysis. Linear regression makes predictions for continuous/real or numeric variables such as **sales, salary, age, product price,** etc.

Linear regression algorithm shows a linear relationship between a dependent (y) and one or more independent (y) variables, hence called as linear regression. Since linear regression shows the linear relationship, which means it finds how the value of the dependent variable is changing according to the value of the independent variable.

The linear regression model provides a sloped straight line representing the relationship between the variables. Consider the below image:



Mathematically, we can represent a linear regression as:

$$y= a_0+a_1x+ \varepsilon$$

Here,
Y= Dependent Variable (Target Variable)
X= Independent Variable (predictor Variable)
a0= intercept of the line (Gives an additional degree of freedom)
a1 = Linear regression coefficient (scale factor to each input value).
ε = random error
The values for x and y variables are training datasets for Linear Regression model representation.

Types of Linear Regression

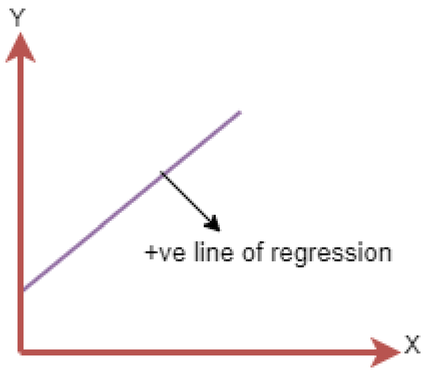
Linear regression can be further divided into two types of the algorithm:

- **Simple Linear Regression:**
  If a single independent variable is used to predict the value of a numerical dependent variable, then such a Linear Regression algorithm is called Simple Linear Regression.

- **Multiple Linear regression:**
  If more than one independent variable is used to predict the value of a numerical dependent variable, then such a Linear Regression algorithm is called Multiple Linear Regression.

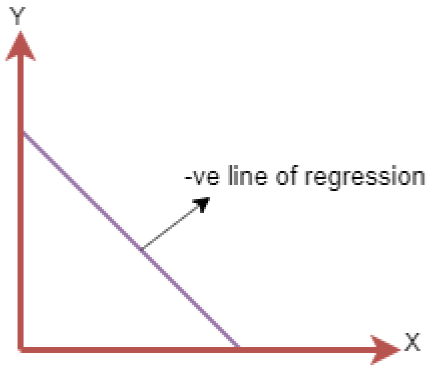
Linear Regression Line:
A linear line showing the relationship between the dependent and independent variables is called a **regression line**.
A regression line can show two types of relationship:

- **Positive Linear Relationship:**
  If the dependent variable increases on the Y-axis and independent variable increases on X-axis, then such a relationship is termed as a Positive linear relationship.



The line equation will be: $\mathbf{Y= a_0+a_1x}$

- **Negative Linear Relationship:**
  If the dependent variable decreases on the Y-axis and independent variable increases on the X-axis, then such a relationship is called a negative linear relationship.



The line of equation will be: $\mathbf{Y= -a_0+a_1x}$

Finding the best fit line:

When working with linear regression, our main goal is to find the best fit line that means the error between predicted values and actual values should be minimized. The best fit line will have the least error.

The different values for weights or the coefficient of lines (a_0, a_1) gives a different line of regression, so we need to calculate the best values for a_0 and a_1 to find the best fit line, so to calculate this we use cost function.

Cost function-

- The different values for weights or coefficient of lines (a_0, a_1) gives the different line of regression, and the cost function is used to estimate the values of the coefficient for the best fit line.

- Cost function optimizes the regression coefficients or weights. It measures how a linear regression model is performing.

- We can use the cost function to find the accuracy of the **mapping function**, which maps the input variable to the output variable. This mapping function is also known as **Hypothesis function**.

For Linear Regression, we use the **Mean Squared Error (MSE)** cost function, which is the average of squared error occurred between the predicted values and actual values. It can be written as:

For the above linear equation, MSE can be calculated as:

$$\text{MSE}= 1\frac{1}{N}\sum_{i=1}^{n}(y_i - (a_1x_i + a_0))^2$$

Where,

N=Total number of observation
Yi = Actual value
$(a1x_i+a_0)$= Predicted value.

Residuals: The distance between the actual value and predicted values is called residual. If the observed points are far from the regression line, then the residual will be high, and so cost function will high. If the scatter points are close to the regression line, then the residual will be small and hence the cost function.

Gradient Descent:

- Gradient descent is used to minimize the MSE by calculating the gradient of the cost function.
- A regression model uses gradient descent to update the coefficients of the line by reducing the cost function.
- It is done by a random selection of values of coefficient and then iteratively update the values to reach the minimum cost function.

Model Performance:

The Goodness of fit determines how the line of regression fits the set of observations. The process of finding the best model out of various models is called **optimization**. It can be achieved by below method:

1. R-squared method:

- R-squared is a statistical method that determines the goodness of fit.
- It measures the strength of the relationship between the dependent and independent variables on a scale of 0-100%.
- The high value of R-square determines the less difference between the predicted values and actual values and hence represents a good model.
- It is also called a **coefficient of determination,** or **coefficient of multiple determination** for multiple regression.
- It can be calculated from the below formula:

$$\text{R-squared}=\frac{\text{Explained variation}}{\text{Total Variation}}$$

Assumptions of Linear Regression

Below are some important assumptions of Linear Regression. These are some formal checks while building a Linear Regression model, which ensures to get the best possible result from the given dataset.

- **Linear relationship between the features and target:**
  Linear regression assumes the linear relationship between the dependent and independent variables.
- **Small or no multicollinearity between the features:**
  Multicollinearity means high-correlation between the independent variables. Due to multicollinearity, it may difficult to find the true relationship between the predictors and target variables. Or we can say, it is

difficult to determine which predictor variable is affecting the target variable and which is not. So, the model assumes either little or no multicollinearity between the features or independent variables.

- **Homoscedasticity Assumption:**
  Homoscedasticity is a situation when the error term is the same for all the values of independent variables. With homoscedasticity, there should be no clear pattern distribution of data in the scatter plot.

- **Normal distribution of error terms:**
  Linear regression assumes that the error term should follow the normal distribution pattern. If error terms are not normally distributed, then confidence intervals will become either too wide or too narrow, which may cause difficulties in finding coefficients.
  It can be checked using the **q-q plot**. If the plot shows a straight line without any deviation, which means the error is normally distributed.

- **No autocorrelations:**
  The linear regression model assumes no autocorrelation in error terms. If there will be any correlation in the error term, then it will drastically reduce the accuracy of the model. Autocorrelation usually occurs if there is a dependency between residual errors.

Simple Linear Regression in Machine Learning

Simple Linear Regression is a type of Regression algorithms that models the relationship between a dependent variable and a single independent variable. The relationship shown by a Simple Linear Regression model is linear or a sloped straight line, hence it is called Simple Linear Regression.

The key point in Simple Linear Regression is that the ***dependent variable must be a continuous/real value***. However, the independent variable can be measured on continuous or categorical values.

Simple Linear regression algorithm has mainly two objectives:

- **Model the relationship between the two variables.** Such as the relationship between Income and expenditure, experience and Salary, etc.

- **Forecasting new observations.** Such as Weather forecasting according to temperature, Revenue of a company according to the investments in a year, etc.

Simple Linear Regression Model:

The Simple Linear Regression model can be represented using the below equation:

$$y= a_0+a_1x+ \varepsilon$$

Where,

a0= It is the intercept of the Regression line (can be obtained putting x=0)
a1= It is the slope of the regression line, which tells whether the line is increasing or decreasing.
ε = The error term. (For a good model it will be negligible)

2.2.3. Multiple Linear Regressions

In the previous topic, we have learned about Simple Linear Regression, where a single Independent/Predictor(X) variable is used to model the response variable (Y). But there may be various cases in which the response variable is affected by more than one predictor variable; for such cases, the Multiple Linear Regression algorithm is used.

Moreover, Multiple Linear Regression is an extension of Simple Linear regression as it takes more than one predictor variable to predict the response variable.

We can define it as:
"*Multiple Linear Regression is one of the important regression algorithms which models the linear relationship between a single dependent continuous variable and more than one independent variable.*"

Example:
Prediction of CO_2 emission based on engine size and number of cylinders in a car.

Some key points about MLR:

- For MLR, the dependent or target variable(Y) must be the continuous/real, but the predictor or independent variable may be of continuous or categorical form.

- Each feature variable must model the linear relationship with the dependent variable.

- MLR tries to fit a regression line through a multidimensional space of data-points.

MLR equation:
In Multiple Linear Regression, the target variable(Y) is a linear combination of multiple predictor variables x_1, x_2, x_3, ...,x_n. Since it is an enhancement of Simple Linear Regression, so the same is applied for the multiple linear regression equation, the equation becomes:
Y= b<sub>0</sub>+b<sub>1</sub>x<sub>1</sub>+ b<sub>2</sub>x<sub>2</sub>+ b<sub>3</sub>x<sub>3</sub>+...... bnxn .... (a)
Where,
Y= Output/Response variable
b_0, b_1, b_2, b_3 , b_n....= Coefficients of the model.
x_1, x_2, x_3, x_4,...= Various Independent/feature variable
Assumptions for Multiple Linear Regression:

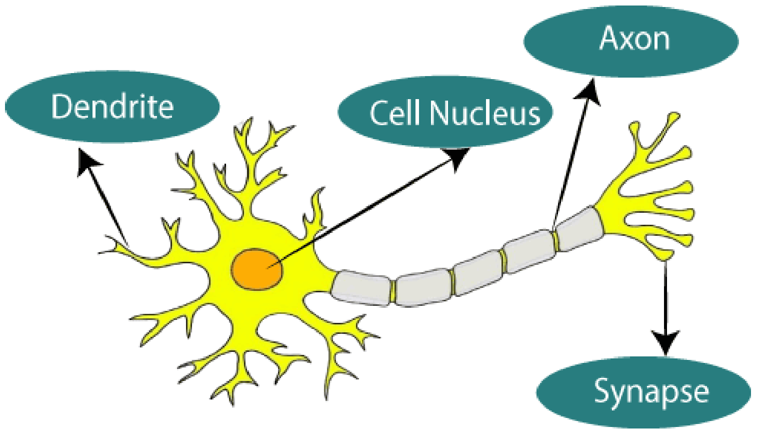
- A linear relationship should exist between the Target and predictor variables.

- The regression residuals must be normally distributed.

- MLR assumes little or no multicollinearity (correlation between the independent variable) in data.

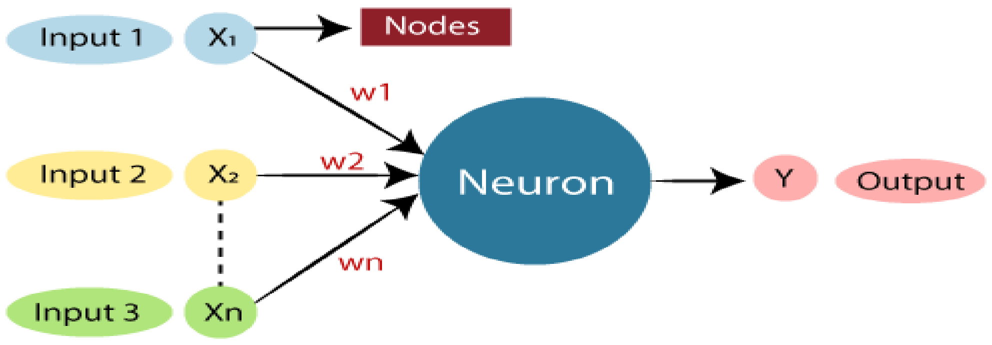
2.3. Neural Networks (ANN - Artificial Neural Network)

2.3.1. Introduction
The term "Artificial Neural Network" is derived from Biological neural networks that develop the structure of a human brain. Similar to the human brain that has neurons interconnected to one another, artificial neural networks also have neurons that are interconnected to one another in various layers of the networks. These neurons are known as nodes.



The given figure illustrates the typical diagram of Biological Neural Network.

The typical Artificial Neural Network looks something like the given figure.



Dendrites from Biological Neural Network represent inputs in Artificial Neural Networks, cell nucleus represents Nodes, synapse represents Weights, and Axon represents Output.

Relationship between Biological neural network and artificial neural network:

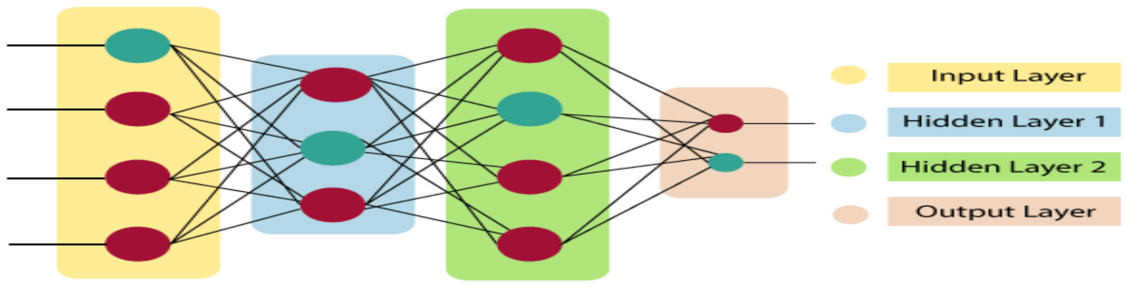
Biological Neural Network	**Artificial Neural Network**
Dendrites	Inputs
Cell nucleus	Nodes
Synapse	Weights
Axon	Output

An **Artificial Neural Network** in the field of **Artificial intelligence** where it attempts to mimic the network of neurons makes up a human brain so that computers will have an option to understand things and make decisions in a human-like manner. The artificial neural network is designed by programming computers to behave simply like interconnected brain cells.

There are around 1000 billion neurons in the human brain. Each neuron has an association point somewhere in the range of 1,000 and 100,000. In the human brain, data is stored in such a manner as to be distributed, and we can extract more than one piece of this data when necessary from our memory parallelly. We can say that the human brain is made up of incredibly amazing parallel processors.

We can understand the artificial neural network with an example, consider an example of a digital logic gate that takes an input and gives an output. "OR" gate, which takes two inputs. If one or both the inputs are "On," then we get "On" in output. If both the inputs are "Off," then we get "Off" in output. Here the output depends upon input. Our brain does not perform the same task. The outputs to inputs relationship keep changing because of the neurons in our brain, which are "learning."

The architecture of an artificial neural network:



Input Layer:

As the name suggests, it accepts inputs in several different formats provided by the programmer.

Hidden Layer:

The hidden layer presents in-between input and output layers. It performs all the calculations to find hidden features and patterns.

Output Layer:

The input goes through a series of transformations using the hidden layer, which finally results in output that is conveyed using this layer.

The artificial neural network takes input and computes the weighted sum of the inputs and includes a bias. This computation is represented in the form of a transfer function.

$$\sum_{i=1}^{n} Wi * Xi + b$$

It determines weighted total is passed as an input to an activation function to produce the output. Activation functions choose whether a node should fire or not. Only those who are fired make it to the output layer. There are distinctive activation functions available that can be applied upon the sort of task we are performing.

Advantages of Artificial Neural Network (ANN)

Parallel processing capability:

Artificial neural networks have a numerical value that can perform more than one task simultaneously.

Storing data on the entire network:

Data that is used in traditional programming is stored on the whole network, not on a database. The disappearance of a couple of pieces of data in one place doesn't prevent the network from working.

Capability to work with incomplete knowledge:

After ANN training, the information may produce output even with inadequate data. The loss of performance here relies upon the significance of missing data.

Having a memory distribution:

For ANN is to be able to adapt, it is important to determine the examples and to encourage the network according to the desired output by demonstrating these examples to the network. The succession of the network is directly proportional to the chosen instances, and if the event can't appear to the network in all its aspects, it can produce false output.

Having fault tolerance:

Extortion of one or more cells of ANN does not prohibit it from generating output, and this feature makes the network fault-tolerance.

Disadvantages of Artificial Neural Network:

Assurance of proper network structure:

There is no particular guideline for determining the structure of artificial neural networks. The appropriate network structure is accomplished through experience, trial, and error.

Unrecognized behavior of the network:

It is the most significant issue of ANN. When ANN produces a testing solution, it does not provide insight concerning why and how. It decreases trust in the network.

Hardware dependence:

Artificial neural networks need processors with parallel processing power, as per their structure. Therefore, the realization of the equipment is dependent.

Difficulty of showing the issue to the network:

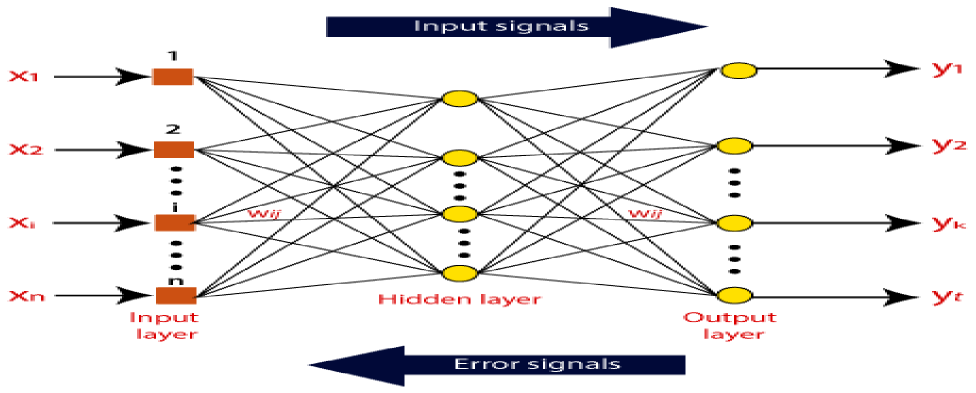
ANNs can work with numerical data. Problems must be converted into numerical values before being introduced to ANN. The presentation mechanism to be resolved here will directly impact the performance of the network. It relies on the user's abilities.

The duration of the network is unknown:

The network is reduced to a specific value of the error, and this value does not give us optimum results.
"*Science artificial neural networks that have steeped into the world in the mid-20th century are exponentially developing. In the present time, we have investigated the pros of artificial neural networks and the issues encountered in the course of their utilization. It should not be overlooked that the cons of ANN networks, which are a flourishing science branch, are eliminated individually, and their pros are increasing day by day. It means that artificial neural networks will turn into an irreplaceable part of our lives progressively important.*"

How do artificial neural networks work?

Artificial Neural Network can be best represented as a weighted directed graph, where the artificial neurons form the nodes. The association between the neurons outputs and neuron inputs can be viewed as the directed edges with weights. The Artificial Neural Network receives the input signal from the external source in the form of a pattern and image in the form of a vector. These inputs are then mathematically assigned by the notations x(n) for every n number of inputs.



Afterward, each of the input is multiplied by its corresponding weights (these weights are the details utilized by the artificial neural networks to solve a specific problem). In general terms, these weights normally represent the strength of the interconnection between neurons inside the artificial neural network. All the weighted inputs are summarized inside the computing unit.

If the weighted sum is equal to zero, then bias is added to make the output non-zero or something else to scale up to the system's response. Bias has the same input, and weight equals to 1. Here the total of weighted inputs can be in the range of 0 to positive infinity. Here, to keep the response in the limits of the desired value, a certain maximum value is benchmarked, and the total of weighted inputs is passed through the activation function.

The activation function refers to the set of transfer functions used to achieve the desired output. There is a different kind of the activation function, but primarily either linear or non-linear sets of functions. Some of the commonly used sets of activation functions are the Binary, linear, and Tan hyperbolic sigmoidal activation functions. Let us take a look at each of them in details:

Binary:

In binary activation function, the output is either a one or a 0. Here, to accomplish this, there is a threshold value set up. If the net weighted input of neurons is more than 1, then the final output of the activation function is returned as one or else the output is returned as 0.

Sigmoidal Hyperbolic:

The Sigmoidal Hyperbola function is generally seen as an "**S**" shaped curve. Here the tan hyperbolic function is used to approximate output from the actual net input. The function is defined as:

F(x) = (1/1 + exp(-????x))

Where ???? is considered the Steepness parameter.

Types of Artificial Neural Network:

There are various types of Artificial Neural Networks (ANN) depending upon the human brain neuron and network functions, an artificial neural network similarly performs tasks. The majority of the artificial neural networks will have some similarities with a more complex biological partner and are very effective at their expected tasks. For example, segmentation or classification.

Feedback ANN:

In this type of ANN, the output returns into the network to accomplish the best-evolved results internally. As per the **University of Massachusetts**, Lowell Centre for Atmospheric Research. The feedback networks feed information back into itself and are well suited to solve optimization issues. The Internal system error corrections utilize feedback ANNs.

Feed-Forward ANN:

A feed-forward network is a basic neural network comprising of an input layer, an output layer, and at least one layer of a neuron. Through assessment of its output by reviewing its input, the intensity of the network can be noticed based on group behavior of the associated neurons, and the output is decided. The primary advantage of this network is that it figures out how to evaluate and recognize input patterns.

Prerequisite

No specific expertise is needed as a prerequisite before starting this tutorial.

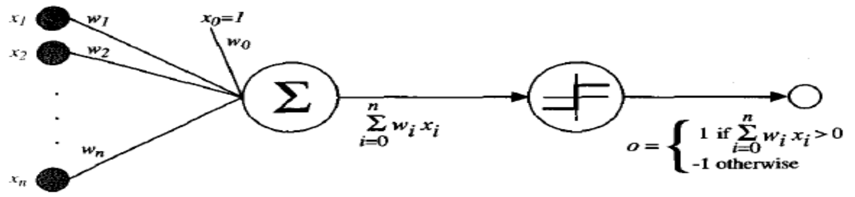
Audience

Our Artificial Neural Network Tutorial is developed for beginners as well as professionals, to help them understand the basic concept of ANNs.

2.3.2. PERCEPTRONS

One type of ANN system is based on a unit called a perceptron, illustrated in below Figure: A perceptron takes a vector of real-valued inputs, calculates a linear combination of these inputs, then outputs a 1 if the result is greater than some threshold and -1 otherwise. More precisely, given inputs ***x₁*** through ***xₙ*** the output ***o(x₁,*** . . . , ***xₙ)*** computed by the perceptron is

$$o(x_1, \ldots, x_n) = \begin{cases} 1 \text{ if } w_0 + w_1 x_1 + w_2 x_2 + \cdots + w_n x_n > 0 \\ -1 \text{ otherwise} \end{cases}$$


x_1
x_2
x_n
w_1
w_2
w_n
$x_0=1$
w_0
Σ
$\sum_{i=0}^{n} w_i x_i$
$o = \begin{cases} 1 \text{ if } \sum_{i=0}^{n} w_i x_i > 0 \\ -1 \text{ otherwise} \end{cases}$

where each w_i is a real-valued constant, or weight, that determines the contribution of input x_i to the perceptron output. Notice the quantity $(-w_0)$ is a threshold that the weighted combination of inputs $w_1 x_1 + \ldots + w_n x_n$ must surpass in order for the perceptron to output a 1.

To simplify notation, we imagine an additional constant input $x_0 = 1$, allowing us to write the above inequality as $\sum_{i=0}^{n} w_i x_i > 0$, or in vector form as $\vec{w} \cdot \vec{x} > 0$. For brevity, we will sometimes write the perceptron function as

$$o(\vec{x}) = sgn(\vec{w} \cdot \vec{x})$$

where

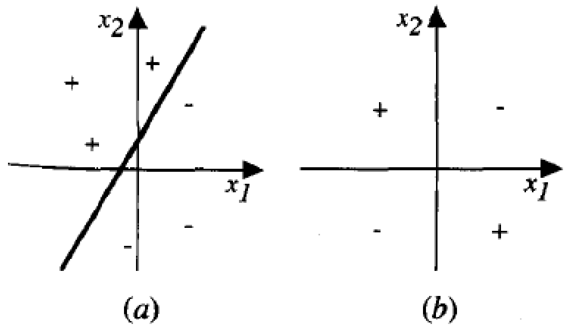
$$sgn(y) = \begin{cases} 1 \text{ if } y > 0 \\ -1 \text{ otherwise} \end{cases}$$

Learning a perceptron involves choosing values for the weights $w_0, \ldots, w_n$ Therefore, the space H of candidate hypotheses considered in perceptron learning is the set of all possible real-valued weight vectors.

$$H = \{\vec{w} \mid \vec{w} \in \Re^{(n+1)}\}$$

Representational Power of Perceptrons:

We can view the perceptron as representing a hyperplane decision surface in the n--dimensional space of instances (i.e., points). The perceptron outputs a 1 for instances lying on one side of the hyperplane and outputs a -1 for instances lying on the other side, as illustrated in Figure below The equation for this decision hyperplane is $\vec{w}.\vec{x} = 0$. Of course, some sets of positive and negative examples cannot be separated by any hyperplane. Those that can be separated are called linearly separable sets of examples.



The decision surface represented by a two-input perceptron. *(a)* A set of training examples and the decision surface of a perceptron that classifies them correctly. *(b)* A set of training examples that is not linearly separable (i.e., that cannot be correctly classified by any straight line). x_l and x_2 are the Perceptron inputs. Positive examples are indicated by "+", negative by "-". The inputs are fed to multiple units, and the outputs of these units are then input to a second, final stage. One way is to represent the Boolean function in disjunctive normal form (i.e., as the disjunction (OR) of a set of conjunctions (ANDs) of the inputs and their negations). Note that the input to an AND perceptron can be negated simply by changing the sign of the corresponding input weight. Because networks of threshold units can represent a rich variety of functions and because single units alone cannot, we will generally be interested in learning multilayer networks of threshold units.

The Perceptron Training Rule

Although we are interested in learning networks of many interconnected units, let us begin by understanding how to learn the weights for a single perceptron. Here the precise learning problem is to determine a weight vector that causes the perceptron to produce the correct ± 1 output for each of the given training examples.

Several algorithms are known to solve this learning problem. Here we consider two: the perceptron rule and the delta rule. These two algorithms are guaranteed to converge to somewhat different acceptable hypotheses, under somewhat different conditions. They are important to ANNs because they provide the basis for learning networks of many units.

One way to learn an acceptable weight vector is to begin with random weights, then iteratively apply the perceptron to each training example, modifying the perceptron weights whenever it misclassifies an example. This process is repeated, iterating through the training examples as many times as needed until
the perceptron classifies all training examples correctly. Weights are modified at each step according to the perceptron training rule, which revises the weight $\boldsymbol{w_i}$ associated with input $\mathbf{x}_i$ according to the rule

$$w_i \leftarrow w_i + \Delta w_i$$

where

$$\Delta w_i = \eta(t - o)x_i$$

Here ***t*** is the target output for the current training example, ***o*** is the output generated by the

perceptron, and η is a positive constant called the ***learning rate.*** The role of the learning rate is to moderate the degree to which weights are changed at each step. It is usually set to some small value (e.g., 0.1) and is sometimes made to decay as the number of weight-tuning iterations increases.
Why should this update rule converge toward successful weight values? To get an intuitive feel, consider some specific cases. Suppose the training example is correctly classified already by the perceptron. In this case, ***(t - o)*** is zero, making Δw_i zero, so that no weights are updated. Suppose the perceptron outputs a -1, when the target output is +1. To make the perceptron output a+1 instead of -1 in this case, the weights must be altered to increase the value of $\vec{w}.\vec{x}$ For example, if $x_i>0$, then increasing w_i will bring the perceptron closer to correctly classifying this example. Notice the training rule will increase ***w,*** in this case, because ***(t - o),*** η, and x_i are all positive. For example, if $x_i = .8$, $\eta = 0.1$, $t = 1$, and $o = -1$, then the weight update will be $\Delta w_i = \eta\ (t - o)x_i = 0.1(1 - (-1))0.8 = 0.16$. On the other hand, if ***t*** = ***-1*** and ***o*** = 1, then weights associated with positive x_i will be decreased rather than increased.

In fact, the above learning procedure can be proven to converge within a finite number of applications of the perceptron training rule to a weight vector that correctly classifies all training examples, ***provided the training examples are linearly separable*** and provided a sufficiently small η is used. If the data are not linearly separable, convergence is not assured.

Gradient Descent and the Delta Rule

Although the perceptron rule finds a successful weight vector when the training examples are linearly separable, it can fail to converge if the examples are not linearly separable. **A** second training rule, called the ***delta rule,*** is designed to overcome this difficulty. If the training examples are not linearly separable, the delta rule converges toward a best-fit approximation to the target concept. The key idea behind the delta rule is to use ***gradient descent*** to search the hypothesis space of possible weight vectors to find the weights that best fit the training examples. This rule is important because gradient descent provides the basis for the BACKPROPAGATION algorithm, which can learn networks with many interconnected units. It is also important because gradient descent can serve as the basis for learning algorithms that must search through hypothesis spaces containing many different types of continuously parameterized hypotheses.
The delta training rule is best understood by considering the task of training an ***unthresholded*** perceptron; that is, a ***linear unit*** for which the output ***o*** is given by

$$o(\vec{x}) = \vec{w} \cdot \vec{x}$$

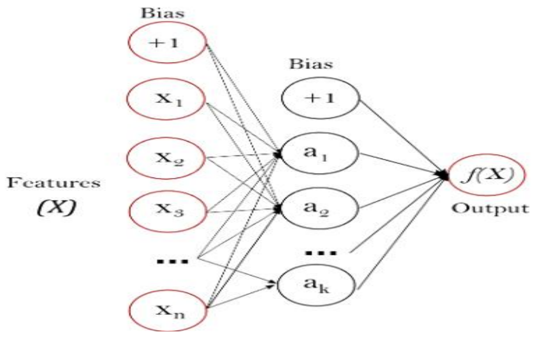
Thus, a linear unit corresponds to the first stage of a perceptron, without the threshold.
In order to derive a weight learning rule for linear units, let us begin by specifying a measure for the ***training error*** of a hypothesis (weight vector), relative to the training examples. Although there are many ways to define this error, one common measure that will turn out to be especially convenient is

$$E(\vec{w}) \equiv \frac{1}{2} \sum_{d \in D} (t_d - o_d)^2$$

where D is the set of training examples, ***td*** is the target output for training example *d,* and o_d is the output of the linear unit for training example d. By this definition, $E(\vec{w})$ is simply half the squared difference between the target output $\boldsymbol{t_d}$ and the hear unit output $\boldsymbol{o_d}$**,** summed over all training examples. Here we characterize ***E*** as a function of $(\vec{w})$**,** because the linear unit output ***o*** depends on this weight vector. Of course ***E*** also depends on the particular **set** of training examples, but we assume these are fixed during training, so we do not bother to write E as an explicit function of these. In particular, there we show that under certain conditions the hypothesis that minimizes E is also the most probable hypothesis in H given the training data.

2.3.2. Multi-layer Perceptron

Multi-layer Perceptron (MLP) is a supervised learning algorithm that learns a function $f(\cdot):R^m \rightarrow R^o$ by training on a dataset, where m is the number of dimensions for input and o is the number of dimensions for output. Given a set of features $X=x^1,x^2,...,x^m$ and a target y, it can learn a non-linear function approximator for either classification or regression. It is different from logistic regression, in that between the input and the output layer, there can be one or more non-linear layers, called hidden layers. Figure shows a one hidden layer MLP with scalar output.



The leftmost layer, known as the input layer, consists of a set of neurons $\{x_i|x_1,x_2,...,x_m\}$ representing the input features. Each neuron in the hidden layer transforms the values from the previous layer with a weighted linear summation $w_1x_1+w_2x_2+...+w_mx_m$, followed by a non-linear activation function $g(\cdot):R\rightarrow R$ - like the hyperbolic tan function. The output layer receives the values from the last hidden layer and transforms them into output values.

The module contains the public attributes coefs_ and intercepts_. coefs_ is a list of weight matrices, where weight matrix at index i represents the weights between layer i and layer i+1. intercepts_ is a list of bias vectors, where the vector at index i represents the bias values added to layer i+1.

The advantages of Multi-layer Perceptron are:

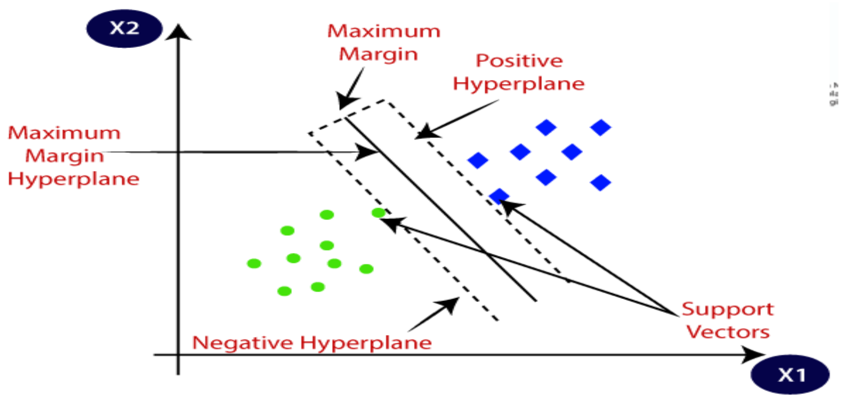
- Capability to learn non-linear models.
- Capability to learn models in real-time (on-line learning) using partial_fit.

The disadvantages of Multi-layer Perceptron (MLP) include:

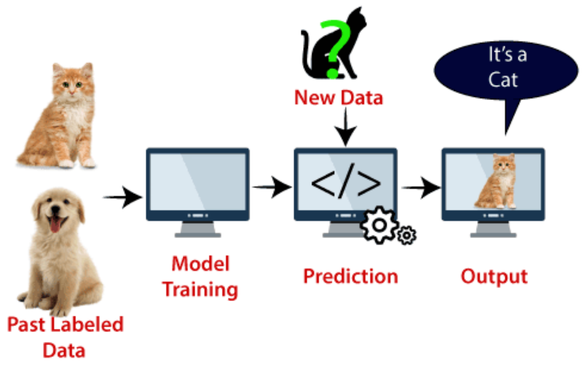
- MLP with hidden layers have a non-convex loss function where there exists more than one local minimum. Therefore different random weight initializations can lead to different validation accuracy.
- MLP requires tuning a number of hyperparameters such as the number of hidden neurons, layers, and iterations.
- MLP is sensitive to feature scaling.

2.4. Support Vector Machines

Support Vector Machine or SVM is one of the most popular Supervised Learning algorithms, which is used for Classification as well as Regression problems. However, primarily, it is used for Classification problems in Machine Learning. The goal of the SVM algorithm is to create the best line or decision boundary that can segregate n-dimensional space into classes so that we can easily put the new data point in the correct category in the future. This best decision boundary is called a hyperplane.
SVM chooses the extreme points/vectors that help in creating the hyperplane. These extreme cases are called as support vectors, and hence algorithm is termed as Support Vector Machine. Consider the below diagram in which there are two different categories that are classified using a decision boundary or hyperplane:



Example: SVM can be understood with the example that we have used in the KNN classifier. Suppose we see a strange cat that also has some features of dogs, so if we want a model that can accurately identify whether it is a cat or dog, so such a model can be created by using the SVM algorithm. We will first train our model with lots of images of cats and dogs so that it can learn about different features of cats and dogs, and then we test it with this strange creature. So as support vector creates a decision boundary between these two data (cat and dog) and choose extreme cases (support vectors), it will see the extreme case of cat and dog. On the basis of the support vectors, it will classify it as a cat. Consider the below diagram:



SVM algorithm can be used for **Face detection, image classification, text categorization,** etc.
Types of SVM
SVM can be of two types:

- **Linear SVM:** Linear SVM is used for linearly separable data, which means if a dataset can be classified into two classes by using a single straight line, then such data is termed as linearly separable data, and classifier is used called as Linear SVM classifier.

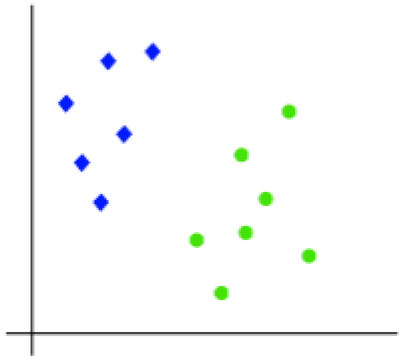
- **Non-linear SVM:** Non-Linear SVM is used for non-linearly separated data, which means if a dataset cannot be classified by using a straight line, then such data is termed as non-linear data and classifier used is called as Non-linear SVM classifier.

***Hyperplane and Support Vectors in the SVM algorithm*:**

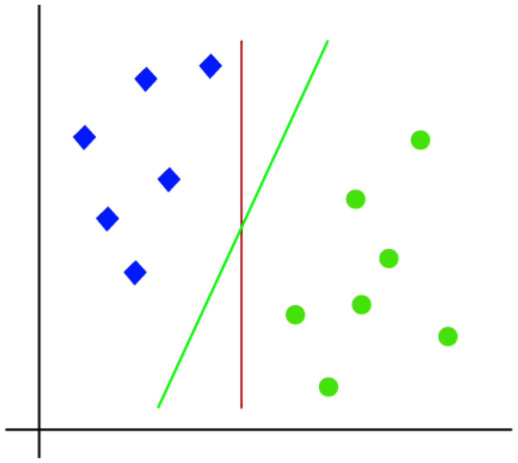
Hyperplane: There can be multiple lines/decision boundaries to segregate the classes in n-dimensional space, but we need to find out the best decision boundary that helps to classify the data points. This best boundary is known as the hyperplane of SVM.
The dimensions of the hyperplane depend on the features present in the dataset, which means if there are 2 features (as shown in image), then hyperplane will be a straight line. And if there are 3 features, then hyperplane will be a 2-dimension plane.
We always create a hyperplane that has a maximum margin, which means the maximum distance between the data points.

Support Vectors:
The data points or vectors that are the closest to the hyperplane and which affect the position of the hyperplane are termed as Support Vector. Since these vectors support the hyperplane, hence called a Support vector. How does SVM works?

2.4.1. Linear SVM:
The working of the SVM algorithm can be understood by using an example. Suppose we have a dataset that has two tags (green and blue), and the dataset has two features x1 and x2. We want a classifier that can classify the pair(x1, x2) of coordinates in either green or blue. Consider the below image:



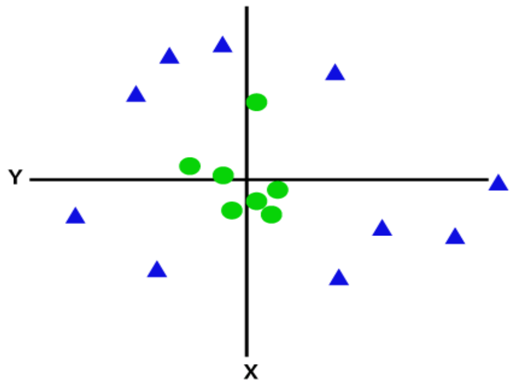
So as it is 2-d space so by just using a straight line, we can easily separate these two classes. But there can be multiple lines that can separate these classes. Consider the below image:



Hence, the SVM algorithm helps to find the best line or decision boundary; this best boundary or region is called as a **hyperplane**. SVM algorithm finds the closest point of the lines from both the classes. These points are called support vectors. The distance between the vectors and the hyperplane is called as **margin**. And the goal of SVM is to maximize this margin. The **hyperplane** with maximum margin is called the **optimal hyperplane**.

2.4.2. Non-Linear SVM:

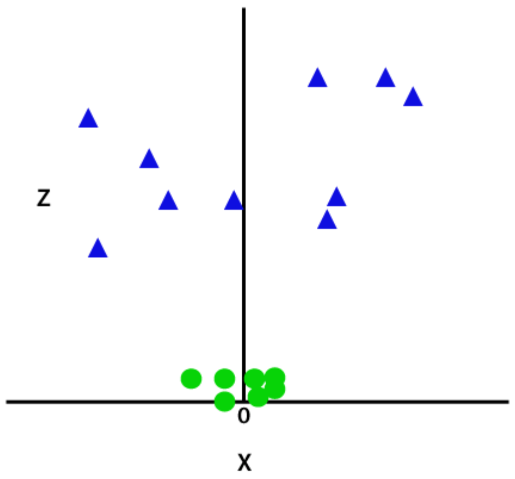
If data is linearly arranged, then we can separate it by using a straight line, but for non-linear data, we cannot draw a single straight line. Consider the below image:



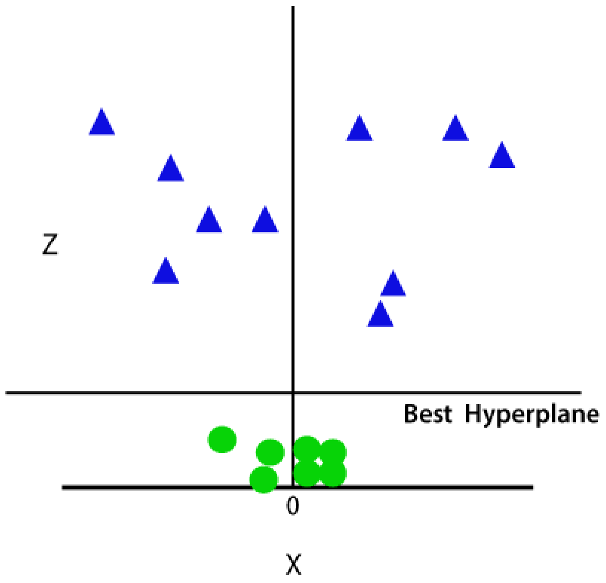
So to separate these data points, we need to add one more dimension. For linear data, we have used two dimensions x and y, so for non-linear data, we will add a third dimension z. It can be calculated as:

$z=x^2 +y^2$

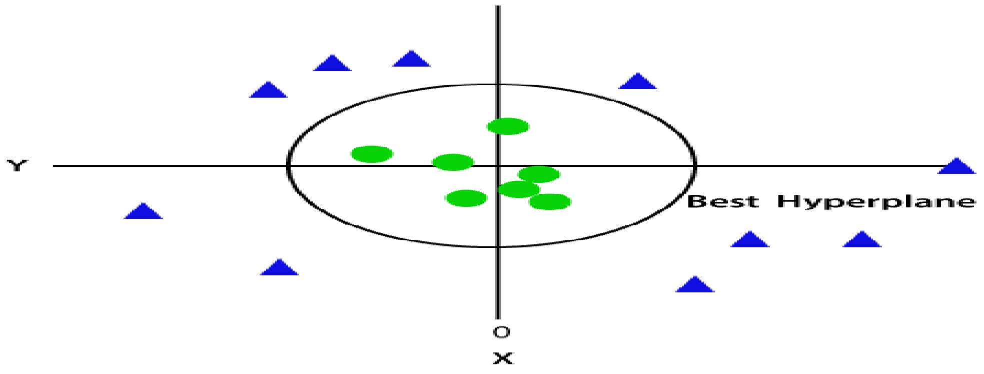
By adding the third dimension, the sample space will become as below image:



So now, SVM will divide the datasets into classes in the following way. Consider the below image:



Since we are in 3-d Space, hence it is looking like a plane parallel to the x-axis. If we convert it in 2d space with z=1, then it will become as:



Hence we get a circumference of radius 1 in case of non-linear data.

2.4.3. SVM Kernels

In practice, SVM algorithm is implemented with kernel that transforms an input data space into the required form. SVM uses a technique called the kernel trick in which kernel takes a low dimensional input space and transforms it into a higher dimensional space. In simple words, kernel converts non-separable problems into separable problems by adding more dimensions to it. It makes SVM more powerful, flexible and accurate. The following are some of the types of kernels used by SVM.

Linear Kernel

It can be used as a dot product between any two observations. The formula of linear kernel is as below

$$K(x,x_i)=\text{sum}(x*x_i)$$

From the above formula, we can see that the product between two vectors say x & xi is the sum of the multiplication of each pair of input values.

2.5. Unsupervised Machine Learning:

2.5.1. Introduction to clustering

As the name suggests, unsupervised learning is a machine learning technique in which models are not supervised using training dataset. Instead, models itself find the hidden patterns and insights from the given data. It can be compared to learning which takes place in the human brain while learning new things. It can be defined as:

"*Unsupervised learning is a type of machine learning in which models are trained using unlabeled dataset and are allowed to act on that data without any supervision.*"

Unsupervised learning cannot be directly applied to a regression or classification problem because unlike supervised learning, we have the input data but no corresponding output data. The goal of unsupervised learning is to **find the underlying structure of dataset, group that data according to similarities, and represent that dataset in a compressed format**

Example: Suppose the unsupervised learning algorithm is given an input dataset containing images of different types of cats and dogs. The algorithm is never trained upon the given dataset, which means it does not have any idea about the features of the dataset. The task of the unsupervised learning algorithm is to identify the image features on their own. Unsupervised learning algorithm will perform this task by clustering the image dataset into the groups according to similarities between images.

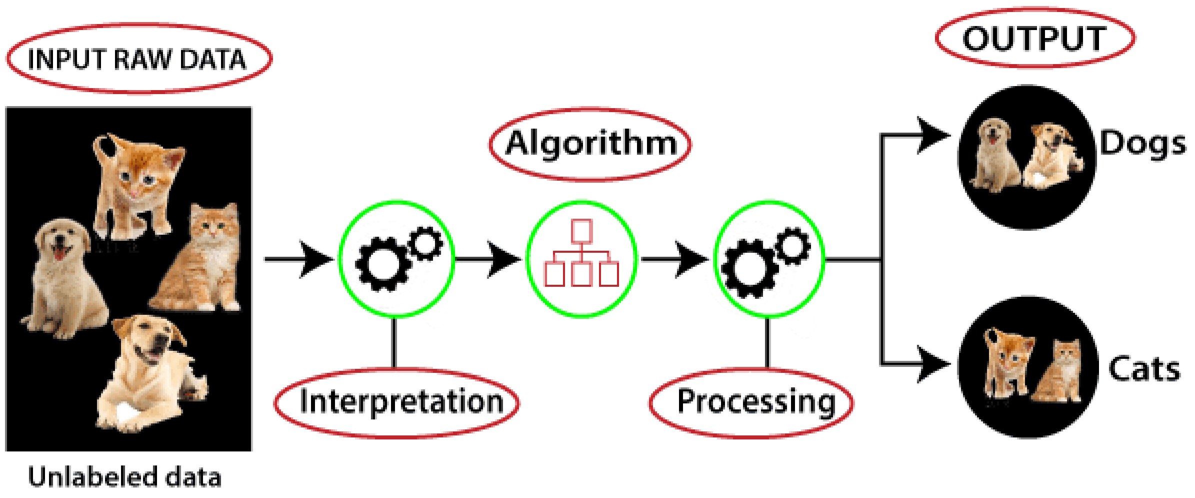
Why use Unsupervised Learning?

Below are some main reasons which describe the importance of Unsupervised Learning:

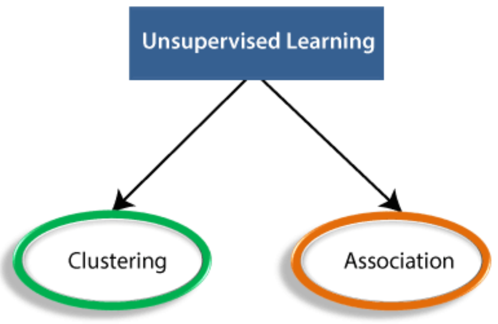
- Unsupervised learning is helpful for finding useful insights from the data.
- Unsupervised learning is much similar as a human learns to think by their own experiences, which makes it closer to the real AI.
- Unsupervised learning works on unlabeled and uncategorized data which make unsupervised learning more important.
- In real-world, we do not always have input data with the corresponding output so to solve such cases, we need unsupervised learning.

Working of Unsupervised Learning

Working of unsupervised learning can be understood by the below diagram:



Here, we have taken an unlabeled input data, which means it is not categorized and corresponding outputs are also not given. Now, this unlabeled input data is fed to the machine learning model in order to train it. Firstly, it will interpret the raw data to find the hidden patterns from the data and then will apply suitable algorithms such as k-means clustering, Decision tree, etc.
Once it applies the suitable algorithm, the algorithm divides the data objects into groups according to the similarities and difference between the objects.
Types of Unsupervised Learning Algorithm:
The unsupervised learning algorithm can be further categorized into two types of problems:



- **Clustering**: Clustering is a method of grouping the objects into clusters such that objects with most similarities remains into a group and has less or no similarities with the objects of another group. Cluster analysis finds the commonalities between the data objects and categorizes them as per the presence and absence of those commonalities.
- **Association**: An association rule is an unsupervised learning method which is used for finding the relationships between variables in the large database. It determines the set of items that occurs together in the dataset. Association rule makes marketing strategy more effective. Such as people who buy X item (suppose a bread) are also tend to purchase Y (Butter/Jam) item. A typical example of Association rule is Market Basket Analysis.

Unsupervised Learning algorithms:

Below is the list of some popular unsupervised learning algorithms:

- K-means clustering
- KNN (k-nearest neighbors)
- Hierarchal clustering
- Anomaly detection
- Neural Networks
- Principle Component Analysis
- Independent Component Analysis
- Apriori algorithm
- Singular value decomposition

Advantages of Unsupervised Learning

- Unsupervised learning is used for more complex tasks as compared to supervised learning because, in unsupervised learning, we don't have labeled input data.
- Unsupervised learning is preferable as it is easy to get unlabeled data in comparison to labeled data.

Disadvantages of Unsupervised Learning

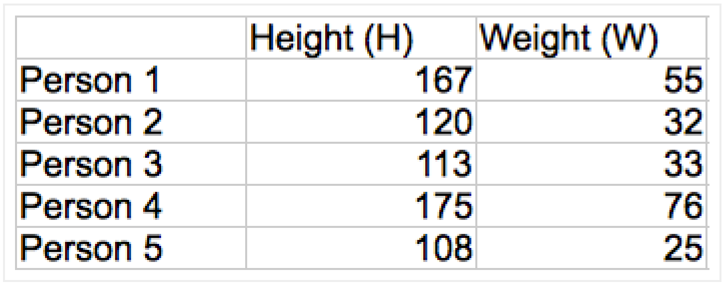
- Unsupervised learning is intrinsically more difficult than supervised learning as it does not have corresponding output.
- The result of the unsupervised learning algorithm might be less accurate as input data is not labeled, and algorithms do not know the exact output in advance.

Supervised Learning	Unsupervised Learning
Supervised learning algorithms are trained using labeled data.	Unsupervised learning algorithms are trained using unlabeled data.
Supervised learning model takes direct feedback to check if it is predicting correct output or not.	Unsupervised learning model does not take any feedback.
Supervised learning model predicts the output.	Unsupervised learning model finds the hidden patterns in data.
In supervised learning, input data is provided to the model along with the output.	In unsupervised learning, only input data is provided to the model.
The goal of supervised learning is to train the model so that it can predict the output when it is given new data.	The goal of unsupervised learning is to find the hidden patterns and useful insights from the unknown dataset.
Supervised learning needs supervision to train the model.	Unsupervised learning does not need any supervision to train the model.
Supervised learning can be categorized in **Classification** and **Regression** problems.	Unsupervised Learning can be classified in **Clustering** and **Associations** problems.
Supervised learning can be used for those cases where we know the input as well as corresponding outputs.	Unsupervised learning can be used for those cases where we have only input data and no corresponding output data.
Supervised learning model produces an accurate result.	Unsupervised learning model may give less accurate result as compared to supervised learning.
Supervised learning is not close to true Artificial intelligence as in this, we first train the model for each data, and then only it can predict the correct output.	Unsupervised learning is more close to the true Artificial Intelligence as it learns similarly as a child learns daily routine things by his experiences.
It includes various algorithms such as Linear Regression, Logistic Regression, Support Vector Machine, Multi-class Classification, Decision tree, Bayesian Logic, etc.	It includes various algorithms such as Clustering, KNN, and Apriori algorithm.

2.5.2. K-Mean Clustering

k-means clustering algorithm
One of the most used clustering algorithm is *k-means*. It allows to group the data according to the existing similarities among them in *k* clusters, given as input to the algorithm. I'll start with a simple example.
Let's imagine we have 5 objects (say 5 people) and for each of them we know two features (height and weight). We want to group them into *k=2* clusters.

Our dataset will look like this:



	Height (H)	Weight (W)
Person 1	167	55
Person 2	120	32
Person 3	113	33
Person 4	175	76
Person 5	108	25

First of all, we have to initialize the value of the centroids for our clusters. For instance, let's choose Person 2 and Person 3 as the two centroids *c1* and *c2*, so that *c1=(120,32)* and *c2=(113,33)*.
Now we compute the euclidian distance between each of the two centroids and each point in the data. If you did all the calculations, you should have come up with the following numbers:

	Distance of object from *c1*	**Distance of object from** *c2*
Person 1	52.3	58.3
Person 2	0	7.1
Person 3	7.1	0
Person 4	70.4	75.4
Person 5	13.9	9.4

At this point, we will assign each object to the cluster it is closer to (that is taking the minimum between the two computed distances for each object).

We can then arrange the points as follows:

Person 1 → cluster 1
Person 2 → cluster 1
Person 3 → cluster 2
Person 4 → cluster 1
Person 5→ cluster 2

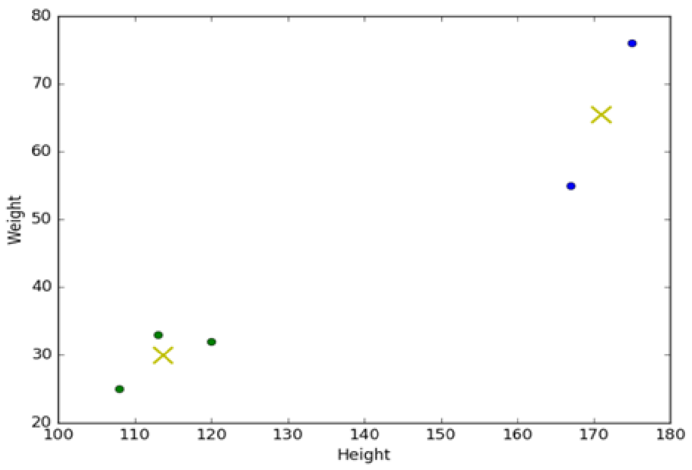
Let's iterate, which means to redefine the centroids by calculating the mean of the members of each of the two clusters.

So *c'1* = ((167+120+175)/3, (55+32+76)/3) = (154, 54.3) and *c'2* = ((113+108)/2, (33+25)/2) = (110.5, 29)

Then, we calculate the distances again and re-assign the points to the new centroids.

We repeat this process until the centroids don't move anymore (or the difference between them is under a certain small threshold).

In our case, the result we get is given in the figure below. You can see the two different clusters labelled with two different colours and the position of the centroids, given by the crosses.



How to apply k-means?
As you probably already know, I'm using Python libraries to analyze my data. The *k-means* algorithm is implemented in the *scikit-learn* package. To use it, you will just need the following line in your script:

What if our data is… non-numerical?

At this point, you will maybe have noticed something. The basic concept of *k-means* stands on mathematical calculations (means, euclidian distances). But what if our data is non-numerical or, in other words, *categorical*? Imagine, for instance, to have the ID code and date of birth of the five people of the previous example, instead of their heights and weights.
We could think of transforming our categorical values in numerical values and eventually apply *k-means*. But beware: *k-means* uses numerical distances, so it could consider close two really distant objects that merely have been assigned two close numbers.

k-modes is an extension of *k-means*. Instead of distances it uses *dissimilarities* (that is, quantification of the total mismatches between two objects: the smaller this number, the more similar the two objects). And instead of means, it uses *modes.* A mode is a vector of elements that minimizes the dissimilarities between the vector itself and each object of the data. We will have as many modes as the number of clusters we required, since they act as centroids.

Unit III
Ensemble and Probabilistic Learning

Ensemble Learning: Model Combination Schemes, Voting, Error-Correcting Output Codes, Bagging: Random Forest Trees, Boosting: Adaboost, Stacking.
Probabilistic Learning: Gaussian mixture models - The Expectation-Maximization (EM) Algorithm, Information Criteria, Nearest neighbour methods - Nearest Neighbour Smoothing, Efficient Distance Computations: the KD-Tree, Distance Measures.

3. Introduction:

Ensemble Learning

Ensemble learning usually produces more accurate solutions than a single model would.