EE2703 : Applied Programming Lab
Assignment 7
The Laplace Transform

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Introduction

In this assignment, we use a Signals tool box from SciPy to solve problems of continuous time domain using the Laplace Transform.

Question 1

Given system equation,

$$\frac{d^2x(t)}{dt^2} + 2.25x(t) = f(t) \tag{1}$$

whose initial conditions are $x(0) = 0$ and $\dot{x}(0) = 0$.

The input $f(t)$ and it's Laplace Transform F(s) are given by

$$f(t) = cos(1.5t)e^{-0.5t}u_0(t) \tag{2}$$

$$F(s) = \frac{s + 0.5}{(s + 0.5)^2 + 2.25} \tag{3}$$

The solution of $x(t)$ i.e displacement of spring for t=0 to t=50 seconds is shown in Figure 1. The amplitude of the signal increases for a while (for 10s) and then settles to a constant amplitude of 0.639.

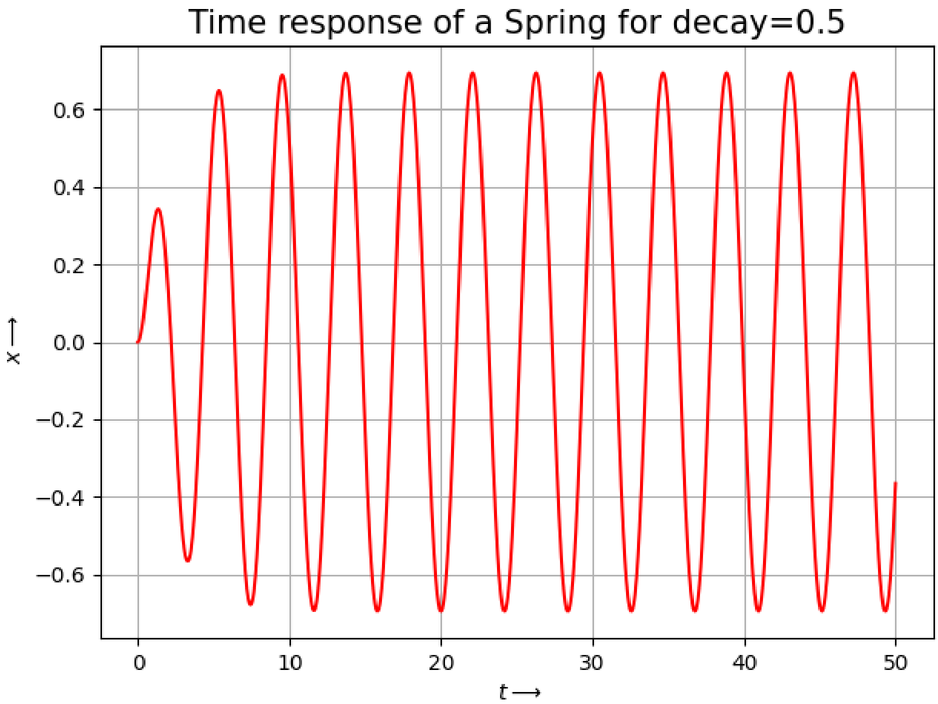

Figure 1: Displacement of spring for decay = 0.5

Question 2

We change the value of decay i.e coefficient of t in the exponential part of $f(t)$ to 0.05. The solution of $x(t)$ is shown in Figure 2. For decay=0.05 we can see that the amplitude of the signal grows exponentially between t=0 and t=50s

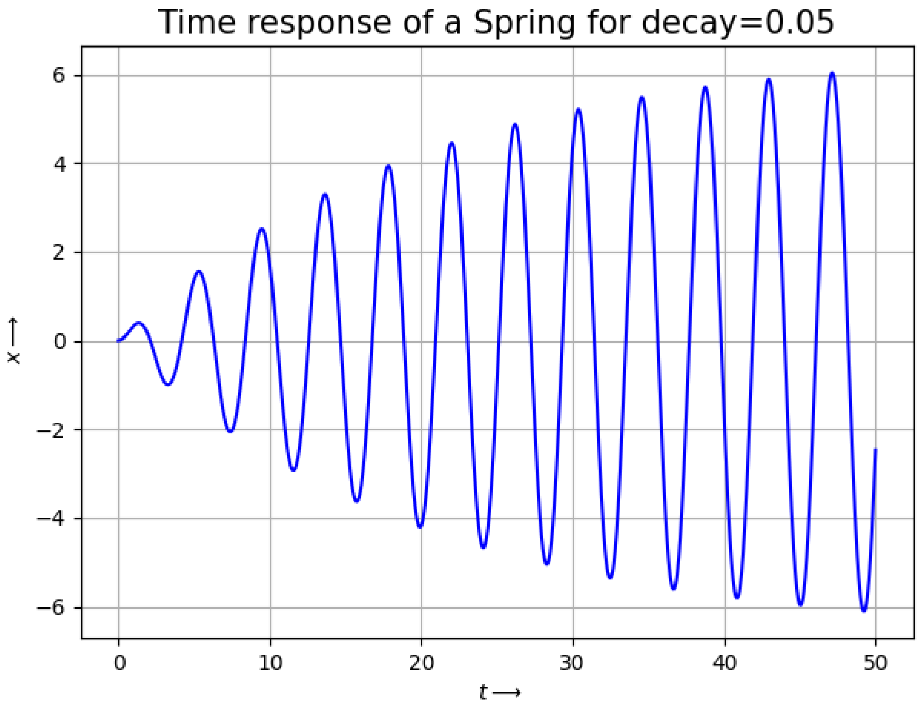

Figure 2: Displacement of spring for decay = 0.05

Question 3

The given spring system can be modelled as an LTI system with $f(t)$ as input and $x(t)$ as the output to the system. We vary the input by varying the frequency of the cosine term of the signal, and the corresponding output signals are plotted in Figure 3. This is done by finding the tranfer function of the LTI system and signal.lsim() function is used to find the output for a given input.

```
H = sp.lti(1,[1,0,2.25])    #Defining transfer function of system
t = np.linspace(0,50,1001)
w = np.arange(1.4,1.6,0.05)
for w0 in w:
    f = np.cos(w0*t)*np.exp(-0.05*t)
    t,y,temp =sp.lsim(H,f,t)     #Finding the response for input signals with various frequencies
    ax.plot(t,y,label='Freq. = %1.2f'%w0)
```

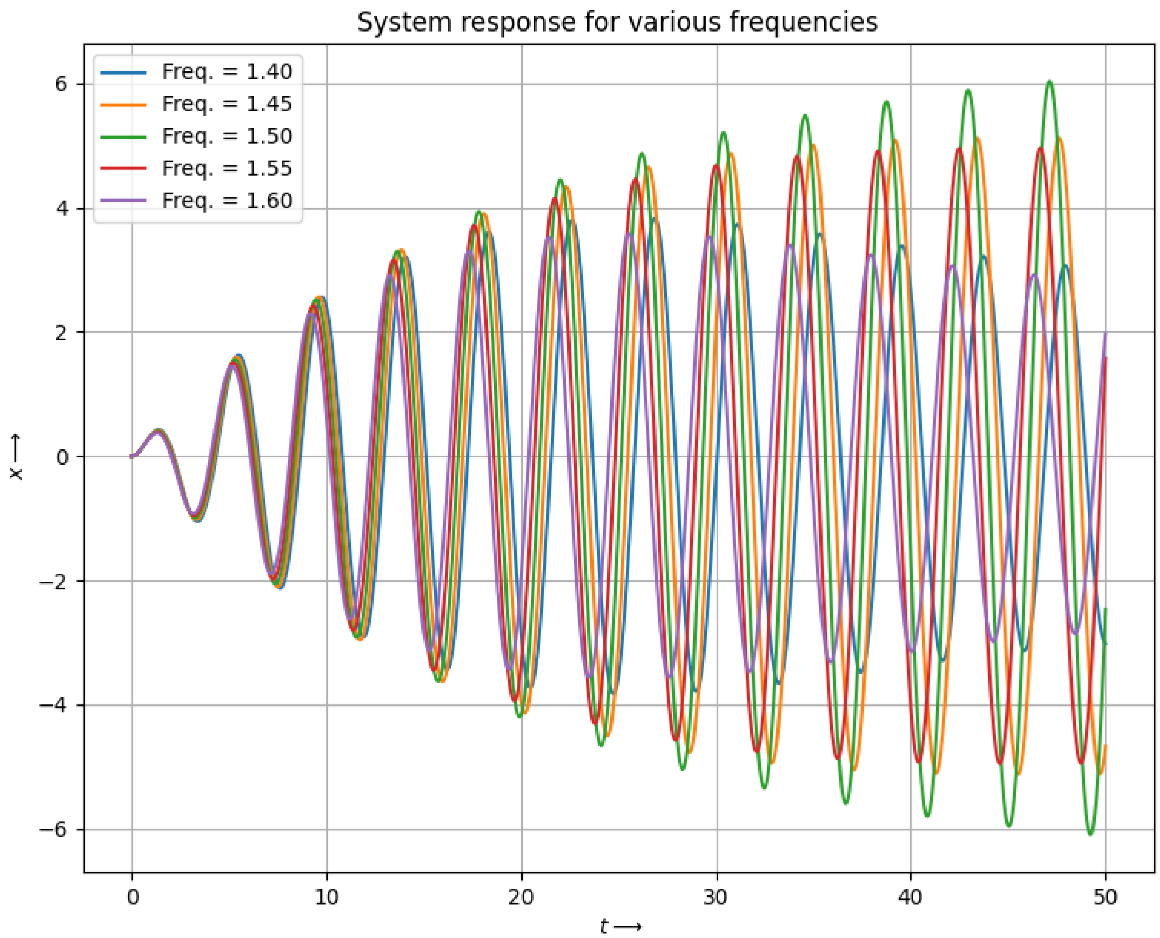

Figure 3: Displacement of spring for input signals with various frequencies

We can observe that the output signal attains maximum amplitude at a frequency of 1.50. This can be explained from the bode plots of the transfer function, as shown in Figure 4 and 5. The amplitude of the output signal is dependent on magnitude response which attains maximum at frequency of 1.50.

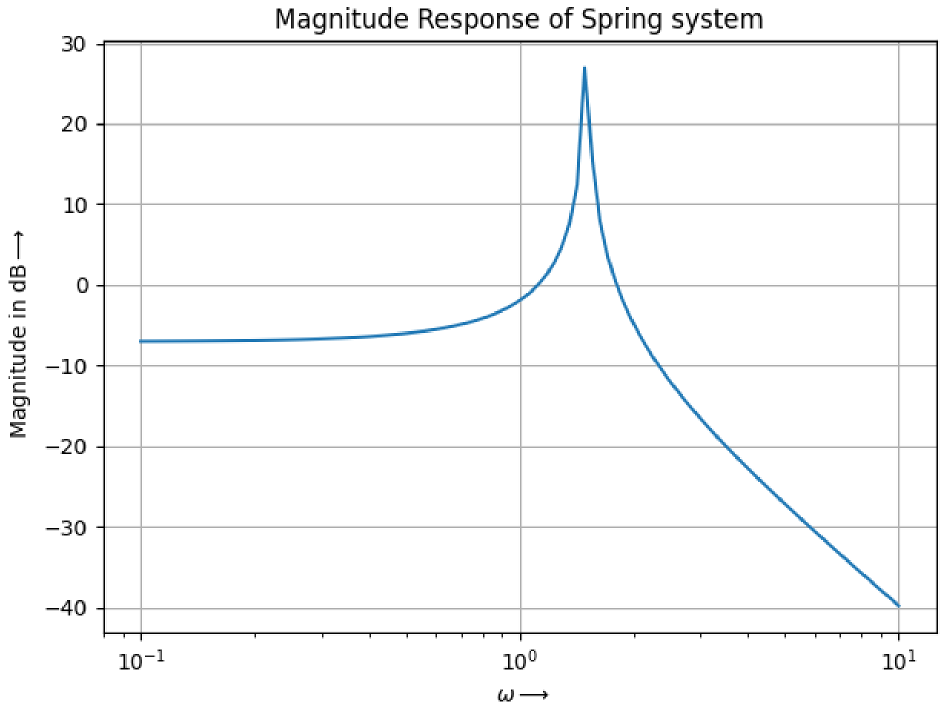

Figure 4: Magnitude response of LTI system

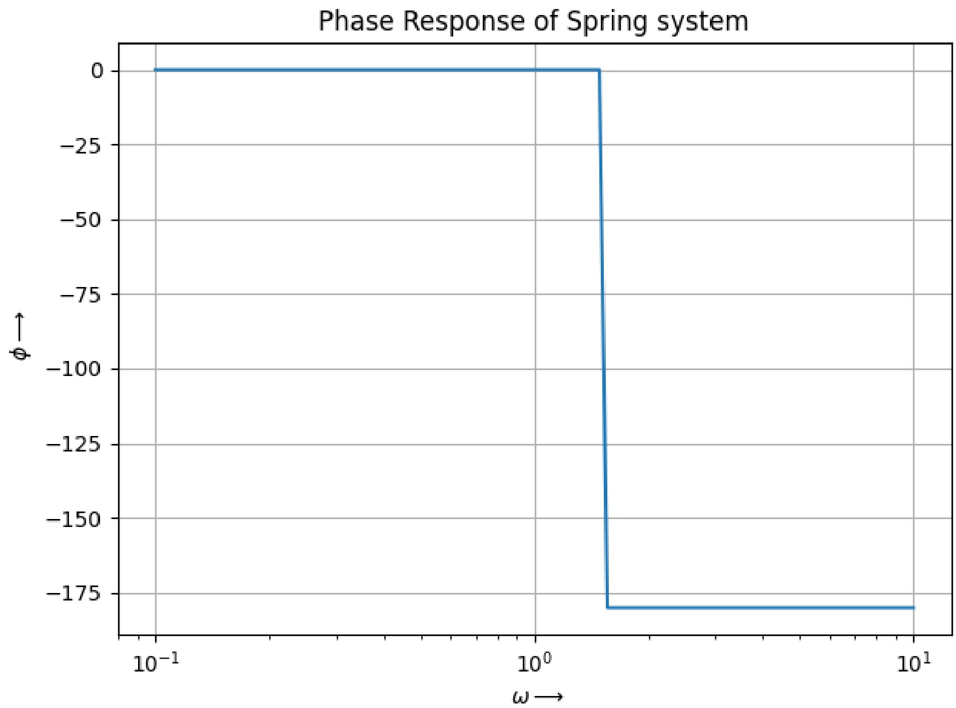

Figure 5: Phase response of LTI system

Question 4

Given a coupled spring system whose equations of motion are given by,

$$\ddot{x} + (x - y) = 0 \tag{4}$$

$$\ddot{y} + 2(y - x) = 0 \tag{5}$$

and the initial conditions of the system are $x(0) = 1$, $\dot{x}(0) = 0$, $y(0) = 0$ and $\dot{y}(0) = 0$. By solving and applying results from Laplace transform, we get

$$X(s) = \frac{s^2 + 2}{s^3 + 3s} \tag{6}$$

$$Y(s) = \frac{2}{s^3 + 3s} \tag{7}$$

From the Laplace transforms $X(s)$ and $Y(s)$, we solve for $x(t)$ and $y(t)$ using the function signal.impulse(). Figure 6 shows the time evolution from t=0 to t=20s.

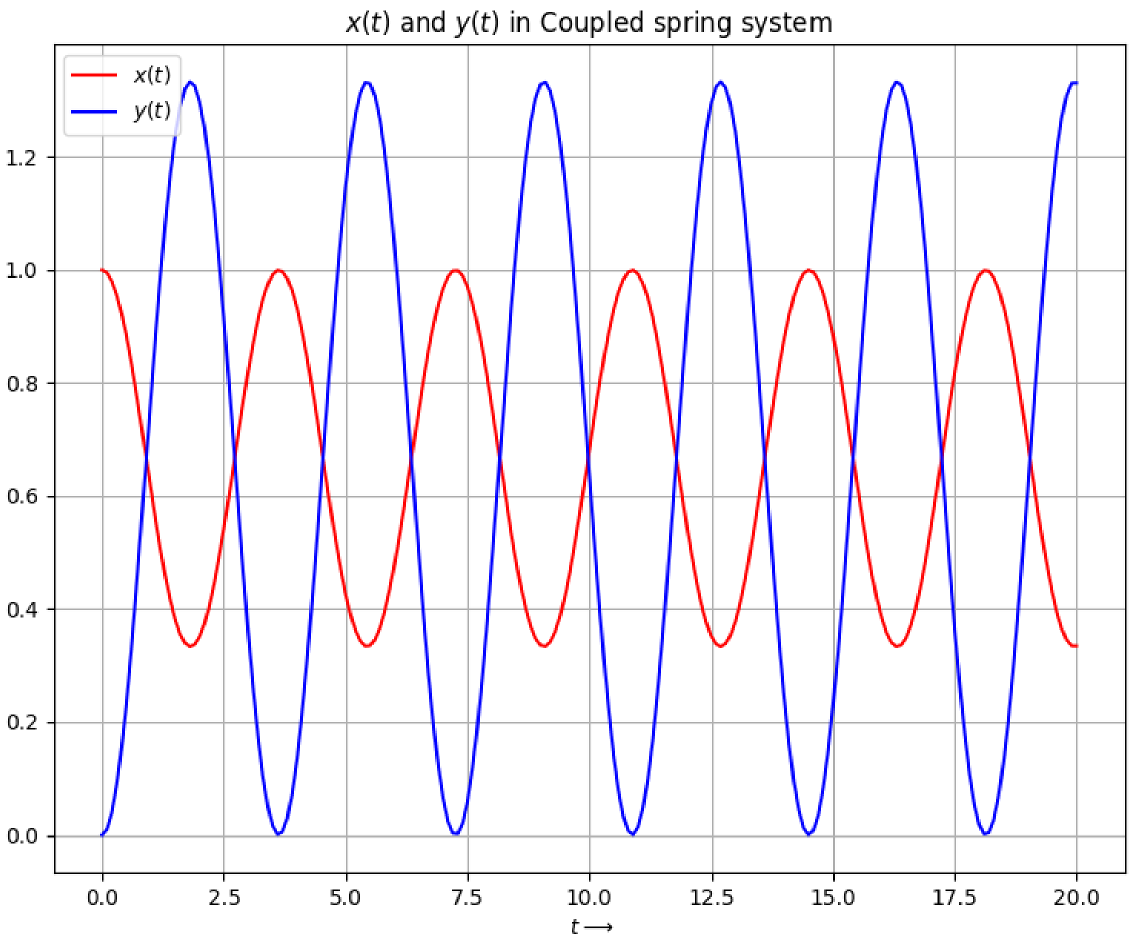

Figure 6: Plot of $x(t)$ and $y(t)$ from the coupled spring system

Question 5

Given a two-port RLC network with $R = 100\Omega$, L $= 1\mu H$ and $C = 1\mu F$. The system's transfer function is given by

$$H(s) = \frac{1}{LCs^2 + sRC + 1} \tag{8}$$

Figure 7 and 8 contains the Bode plots of magnitude and phase response of the transfer function.

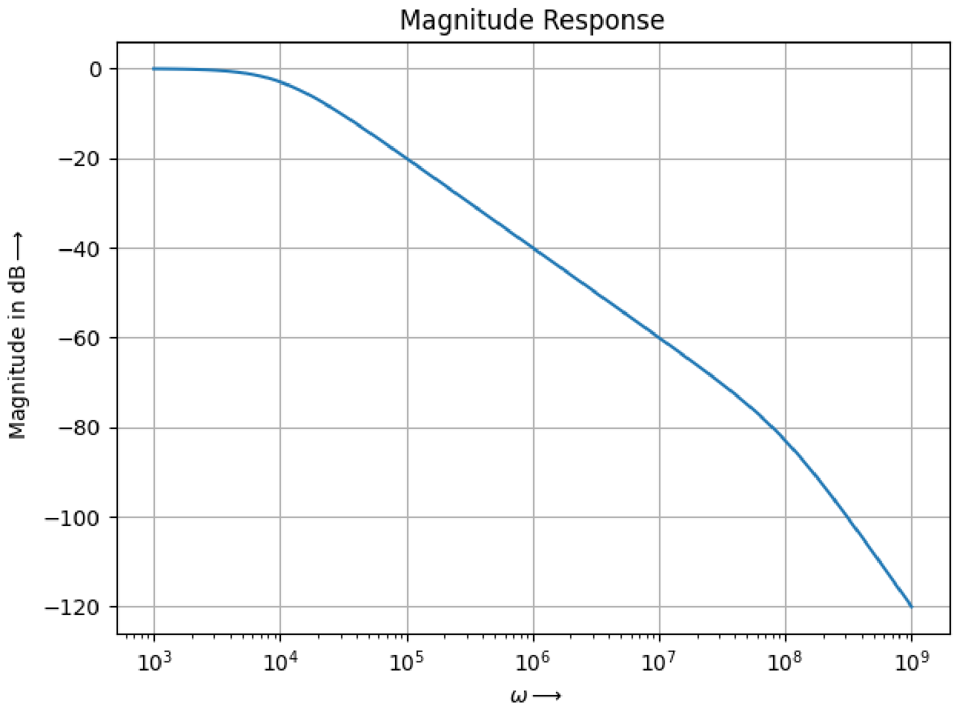

Figure 7: Magnitude Response of RLC system transfer function

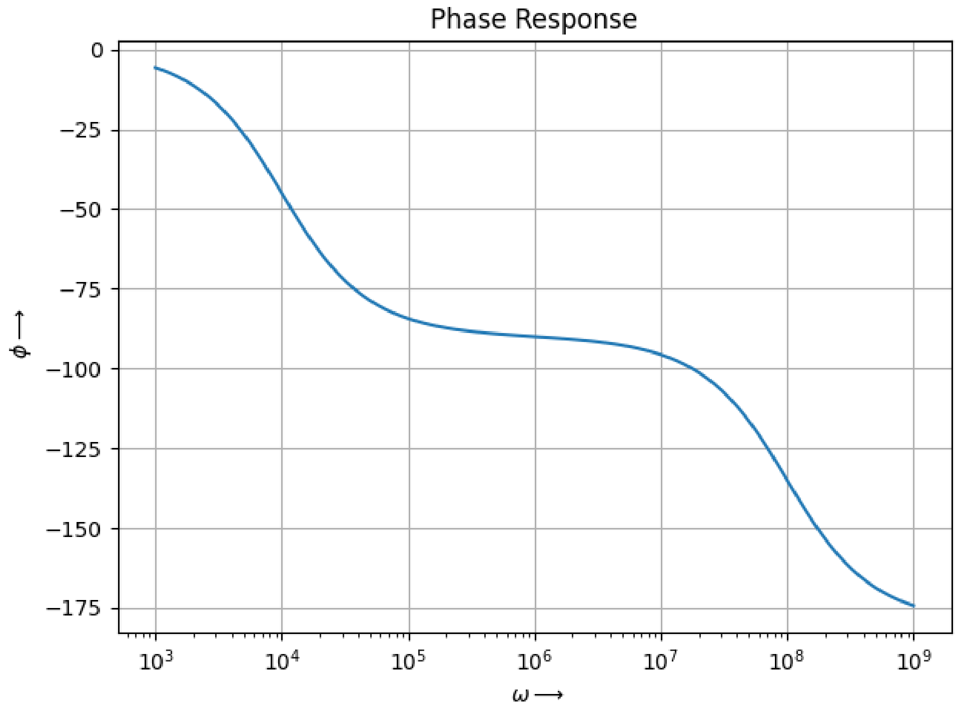

Figure 8: Phase Response of RLC system transfer function

Question 6

Given,

$$v_i(t) = cos(10^3 t)u(t) - cos(10^6 t)u(t) \tag{9}$$

The output voltage $v_o(t)$ is found from the system transfer function and $v_i(t)$ using signal.lsim() function. Figure 9 shows the output voltage signal from t=0 to t=30μs

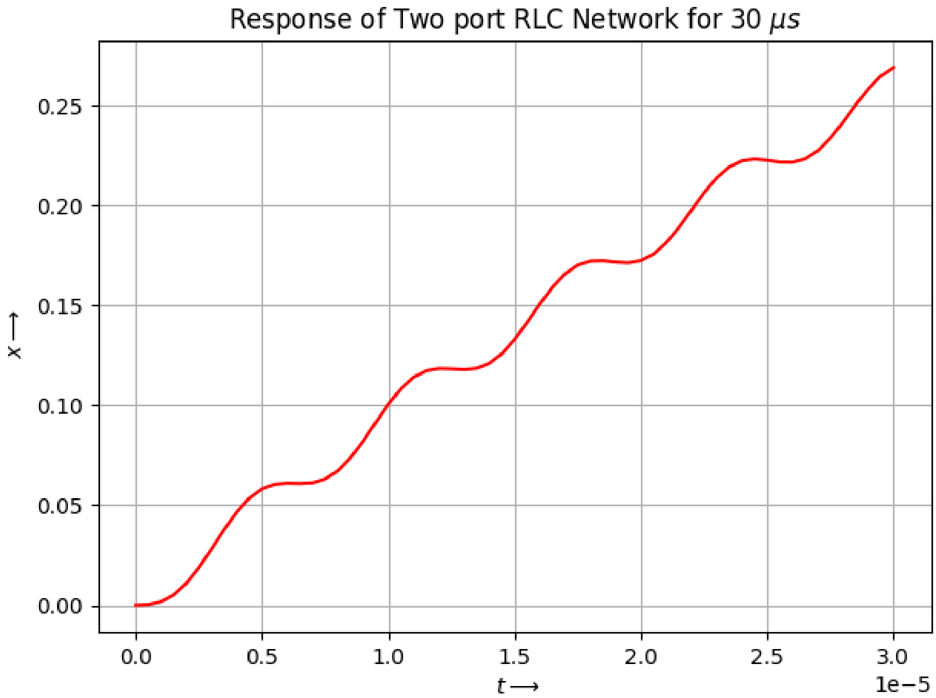

Figure 9: Output voltage signal

Figure 10 shows the output voltage signal from t=0 to t=10ms

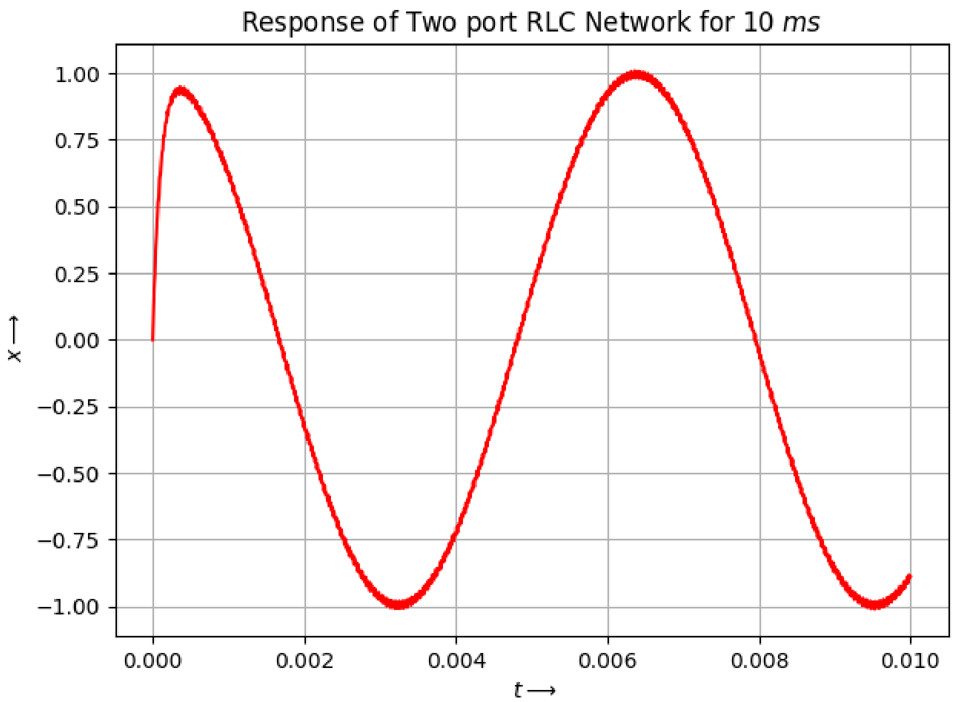

Figure 10: Output voltage signal

- The transient response of the system is rapidly increasing which can be seen in Figure 9. This is because the system has to charge up to match the input amplitude. This results in a phase difference between the input and the output. This can also be interpreted as a delay between the input and the output signals.
- From magnitude response of the system in Figure 7, we can observe that the attenuation at higher frequency (10^6) is quite high (-40dB), where as the system has roughly unity gain (0dB) at lower frequencies (10^3). Hence the system acts as a Low Pass Filter.
- Hence only the low frequency component i.e., $cos(10^3 t)u(t)$ of input voltage signal is passed to output with almost unity gain and a very low phase delay. This results the output to be of roughly same amplitude as input and same frequency, with a small delay to due to low phase difference.
- The high frequency component is attenuated by almost 100 times and has a phase delay of approximately 90 degrees. This signal is almost unseen compared to the response of the low frequency input, causing a very low noise which can be seen in Figure 10.