⇒ Relations:-

⇒ **ordered Pair**:- (a,b).

Pair of elements in specific order.

1) $(a,b) \neq (b,a)$.

⇒ Let A and B be two sets.

$$A \times B = \{(a,b) \mid a \in A \text{ and } b \in B\}$$

$A = \{1,2\}$; $B = \{3,4,5\}$

$A \times B = \{(1,3), (1,4), (1,5), (2,3), (2,4), (2,5)\}$

$|A \times B| = 6$.

⊛ → $|A| = n$; $|B| = m$

$|A \times B| = m \cdot n$

⊛ → $|P(A \times B)| = 2^{mn}$.

Ex:- $A = \{1,2,3\}$; $B = \{2,3,4\}$

$A \times B = \{(1,2), (1,3), (1,4), (2,2), (2,3), (2,4), (3,2), (3,3), (3,4)\}$.

"$=$" $= \{(2,2), (3,3)\}$

"$<$" $= \{(1,2), (1,3), (1,4), (2,3), (2,4), (3,4)\}$

"$x+y > 15$" $= \{\ \}$.

"$x+y \leq 7$" $= A \times B$

⇒ Relation from A to B is defined as

$$R \subseteq A \times B.$$

⇒ **operations**:- Let R, R_1 and R_2 be defined from A t

1.) – $R^c = \{(a,b) \mid (a,b) \notin R \text{ and } (a,b) \in A \times B\}$.

2.) – $R_1 \cap R_2 = \{(a,b) \mid (a,b) \in R_1 \text{ and } (a,b) \in R_2\}$

3.) – $R_1 \cup R_2 = \{(a,b) \mid (a,b) \in R_1 \text{ or } (a,b) \in R_2 \text{ or both}\}$

4.) $R \subseteq A \times B$.

Domain R = Set of all first coordinates in $R \subseteq A$

Range R = Set of all second coordinates in $R \subseteq B$.

eg:- $R = \{(1,4)\}$ $A = \{1,2,3\}$

$B = \{2,3,4\}$

Domain $R = \{1\}$

Range $R = \{4\}$.

5:- Inverse:-

$R \subseteq A \times B$

$R^{-1} \subseteq B \times A$ and is defined as

$R^{-1} = \{(b,a) \mid (a,b) \in R\}$.

⇒ $|A| = m$, $|B| = n$

① → The no. of ~~elements~~ relations from A to B.

= No. of subsets of $A \times B$.

$= |P(A \times B)|$

$= \boxed{2^{mn}}$

② → No. of relations from A to A : $|A| = n$ Given

$= \boxed{2^{n^2}}$

⇒ Composite Relations

If $R \subseteq A \times B$

$S \subseteq B \times C$

Then $RS \subseteq A \times C$ defined as

$RS = \{(a,c) \mid \exists\, b \in B$ such that $(a,b) \in R$ and $(b,c) \in S\}$.

eg:

Given $\begin{cases} A = \{1,2,3\}; \ B = \{2,3,4\}; \ C = \{4,5,6\} \\ R = \{(1,2), (1,4), (2,3), (2,4), (3,3)\} \subseteq A \times B \\ S = \{(2,4), (2,5), (3,6), (4,5)\} \subseteq B \times C. \end{cases}$

$RS = \{(1,4), (1,5), (2,6), (2,5), (3,6)\}$.

A R C RS

$1 - 2 \begin{cases} 4 & (1,4) \\ 5 & (1,5). \end{cases}$

$1 - 4 - 5$ (1,5)

$2 - 3 - 6 \equiv (2,6).$

$2 - 4 - 5 \equiv (2,5).$

$3 - 3 - 6 \equiv 3,6)$

Ques $R = \{(1,3), (2,4), (3,2)\}$

$S = \{(2,5), (2,6), (3,4), (4,5), (4,6)\}.$

$1 - 3 - 4 \equiv (1,4).$

$2 - 4 \begin{cases} 5 & \equiv (2,5). \\ 6 & \equiv (2,6). \end{cases}$

$3 - 2 \begin{cases} 5 & \equiv (3,5) \\ 6 & \equiv (3,6) \end{cases}$

$RS = \{(1,4), (2,5), (2,6), (3,5), (3,6)\}.$

⇒ Relations ~~on~~ A ε

$R \subseteq A \times A.$

$A = \{1, 2, 3, 4\}$

→ $A \times A = \{(1,1), (1,2), (1,3), (1,4), (2,1), (2,2), (2,3), (2,4), (3,1),$
$(3,2), (3,3), (3,4), (4,1), (4,2), (4,3), (4,4)$

≡ Universal Relation on A.

→ $\phi = \{\ \}$ → empty relation on A.

"Δ" $= \{(1,1), (2,2), (3,3), (4,4)\}$ ≡ equality

"$<$" $= \{(1,2), (1,3), (1,4), (2,3), (2,4), (3,4)\}$, less than

"$>$" $= \{(2,1), (3,1), (3,2), (4,1), (4,2), (4,3)\}$ greater than (inverse of <

"$\leq$" $= \{(1,1), (1,2), (1,3), (1,4), (2,2), (2,3), (2,4), (3,3), (3,4), (4$
less than equal to

"$\geq$" $= \{(1,1), (2,1), (2,2), (3,1), (3,2), (3,3), (4,1), (4,2), (4,3), (4,$
greater than equal to (inverse

"$|$" $= \{(1,1), (1,2), (1,3), (1,4), (2,2), (2,4), (3,3), (4,4)\}$

a devides b ≡ a|b (a,

"multiple of" $= \{(1,1), (2,1), (2,2), (3,1), (3,3), (4,1), (4,2), 4,4)\}$

"a multiple of b" (a,b

→ inverse of "|"

⇒ a relation R on A is

(1)→ Reflexive – if $(a,a) \in R \; \forall a \in A$.

yes → eg :- Reflexive – $A \times A$, Δ, $\leq$, $\geq$, "|", multiple of

NO → NOT Reflexive – ϕ, $<$, $>$,

eg :- $R_{15} = \{(1,1), (2,2), (3,3), (4,3)\}$ on $A = \{1,2,3,4\}$.
NOT reflexive.

(2) → Irreflexive – if $(a,a) \notin R \; \forall a \in A$.

yes → ϕ, $<$, $>$.

NO → $A \times A$, Δ, $\leq$, $\geq$, |, multiple of

$R_{15} = \{(1,1), (2,2), (3,3), (4,3)\}$ on $A = \{1,2,3,4\}$.
Not irreflexive

(3) → Symmetric – if $(a,b) \in R$
then $(b,a) \in R$, where $a,b \in A$.

yes → $A \times A$, ϕ, Δ,

NO → $<$, $>$, $\leq$, $\geq$, |, "multiple of"

eg: $R_{101} = \{(1,2), (2,1), (2,3)\}$ – Not symmetric

4) → Assymetric – if $(a,b) \in R$
then $(b,a) \notin R$, where $a,b \in A$.

yes → ϕ, $<$, $>$,

NO → $A \times A$, Δ, $\leq$, $\geq$, |, "multiple of

$R_{196} = \{(1,2)\}$ Asymmetric

5) → Antisymmetric – if $(a,b) \in R$
~~then~~ and $(b,a) \in R$.
then $a = b$, where $a,b \in A$

yes → ϕ, Δ, $<$, $>$, $\leq$, $\geq$, |. multiple of.

NO → $A \times A$,

$R_{201} = \{(1,2), (1,1), (2,2)\}$. Antisymmetry

⇒ Antisymmetric = Assymetric along with reflexive elements also allowed.

Every asymmetry is antisymmetric
But Not vice-versa.

6) → Transitive
if $(a,b) \in R$, and $(b,c) \in R$
then $(a,c) \in R$. where $a,b,c \in R$

Yes → $A \times A$, ϕ, Δ, $<$, $>$, $\leq$, $\geq$, $|$, "multiple of"

No →
$R_{210} = \{(1,2), (2,1)\}$ NOT transitive
$R_{420} = \{(1,2), (2,1) (1,1)\}$ NOT transitive as (2,2) missing

⇒ Let R, R_1 and R_2 be relations on A.

Let R, R_1 & R_2 be	R^{-1}	$R_1 \cap R_2$	$R_1 \cup R_2$
(1) Reflexive	✓	✓	✓
(2) Irreflexive	✓	✓	✓
(3) Symmetric	✓	✓	✓
(4) Asymmetric	✓	✓	✗
(5) Antisymmetric	✓	✓	✗ Need NOT be
(6) Transitive	✓	✓	✗

(✗) → Need not be

eg:- $A = \{1,2,3\}$
→ $R = \{(1,1), (2,2), (3,3), (1,3)\}$ – Reflexive.
$R^{-1} = \{(1,1), (2,2), (3,3), (3,1)\}$ – Reflexive

→ $R = \{(1,3), (2,3)\}$ – irreflexive
$R^{-1} = \{(3,1), (3,2)\}$ – irreflexive

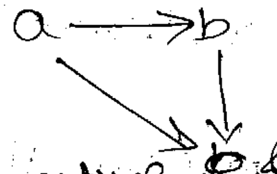
→ $R = \{(1,2), (2,1)\}$ – symmetric
$R^{-1} = \{(2,1), (1,2)\}$ – symmetric

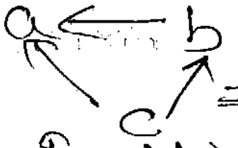
→ $R = \{(1,2)\}$ - Asymmetry

$R^{-1} = \{(2,1)\}$ — Asymmetric

→ $R = \{(1,2), (2,2)\}$ — Anti symmetric

$R = \{(2,1), (2,2)\}$ — Anti symmetric

→ $R =$ 

⇒ $R^{-1} =$ 

Transitive (a,b) & (b,c) ↓ (a,c)

$= (c,b)$ & (b,a) Transitive (c,a).

→ $R_1 = \{(1,2)\}$ — Asymmetric $\quad R_1 \cup R_2 = \{(1,2), (2,1)\}$

$R_2 = \{(2,1)\}$ — Asymmetric $\quad$ So, Not asymmetric

$R_1 \cap R = \{\ \}$ = Asymmetric

→ $\{(a,b)\,(b,a)\}$ Sym $\cup$ $\{(c,d),(d,c)\}$ Sym. $= \{(a,b)\,(b,a), (c,d), (d,c)\}$ Sym

→ $R_1 = \{1,2\}$ ⇒ Antisymmetric

$R_2 = \{2,1\}$ ⇒ Antisymmetric

$R_1 \cup R_2 = \{(1,2), (2,1)\}$ → Not antisymmetric

→ $R_1 = \{(1,2)\}$ Transitive

$R_2 = \{(2,1)\}$ Transitive

$R_1 \cup R_2 = \{(1,2), (2,1)\}$ Need Not be Transitive

$(n)\ (n^2-n$

$\frac{n^2-n}{n^2}$

⇒ Howo many? $|A| = n$.

(1) How many relations are there on A $= \underline{2^{n^2}}$

(2) ” ” reflexive ” ” ” ” ” $= \underline{2^{n(n-1)}}$.

(3) ” ” irreflexive ” ” ” ” ” $= \underline{2^{n(n-1)}}$.

(4) ” ” Symmetric ” ” ” ” ” $= \underline{2^{n(n+1)/2}}$.

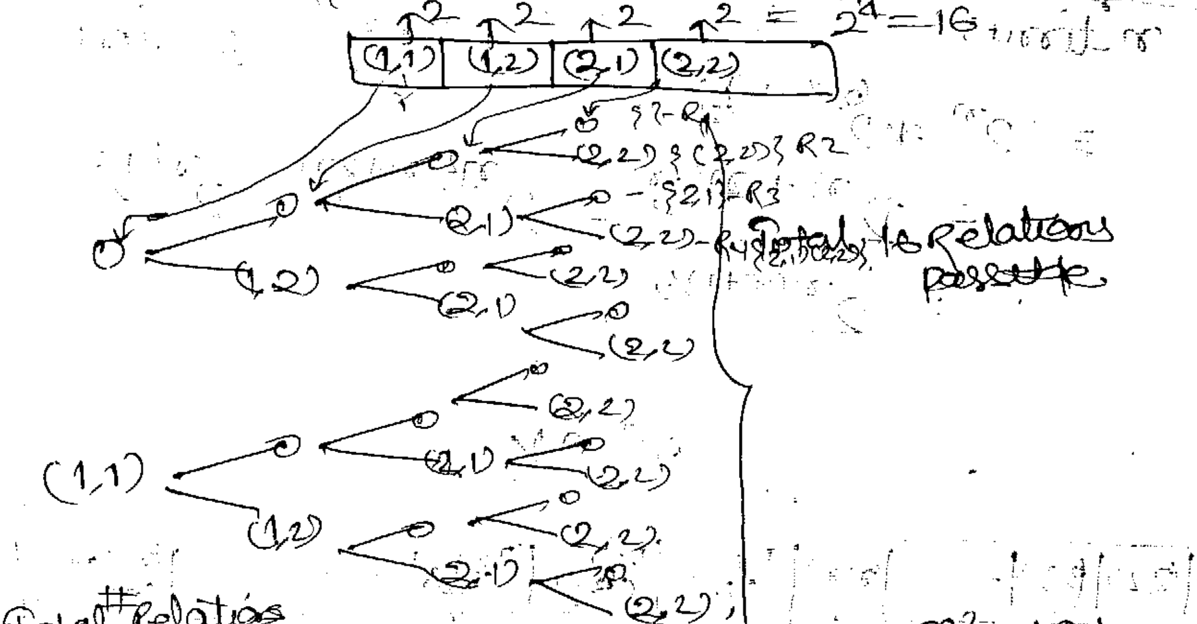
(5) ” ” Asymmetric ” ” ” ” ” $= \underline{3^{n(n-1)/2}}$.

(6) ” ” Antisymmetric ” ” ” ” ” $= \underline{2^n \times 3^{n(n-1)/2}}$.

(7) ” ” Reflexive & symmetric ” ” ” ” ” $= \underline{2^{n(n-1)/2}}$.

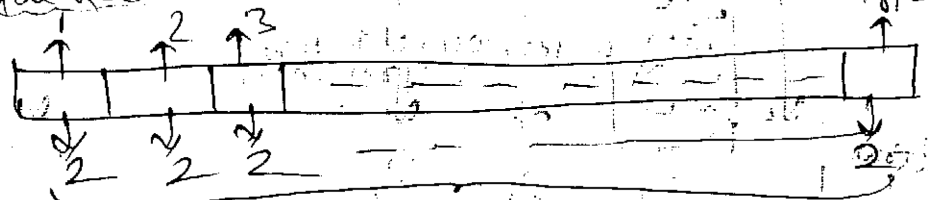
eg: $A = \{(1,2)\}$

$\therefore A \times A = \{(1,1), (1,2) (2,1) (2,2)\}$



Total # Relations

(1)→



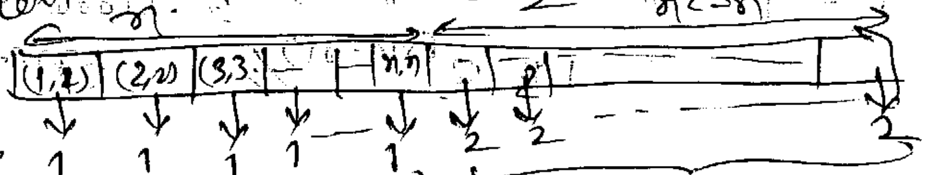
$|A| = n$

$|A \times A| = n^2$

n^2 times

Reflexive $= 2^{n^2}$

(2)→



There must be present in relation

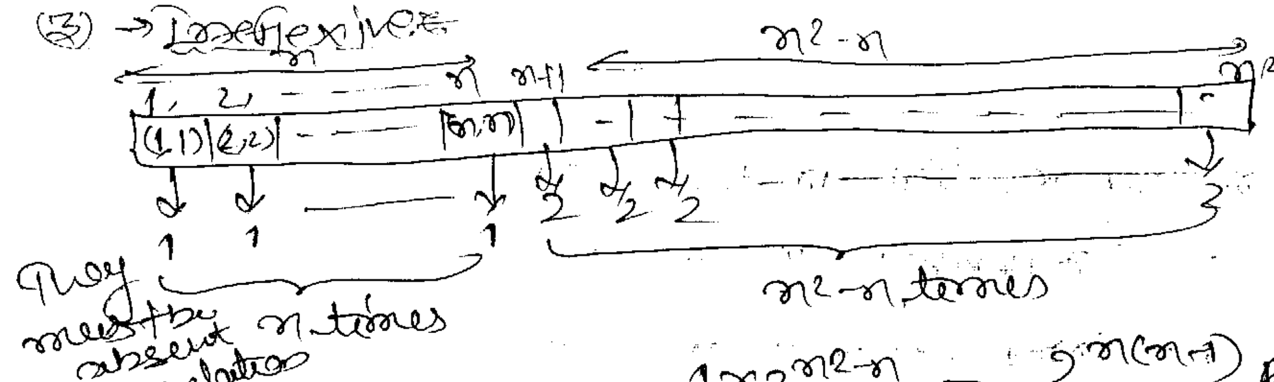
n times

n^2-n times

$1^n * 2^{n^2-n} = 1 * 2^{n^2-n} = 2^{n^2-n}$

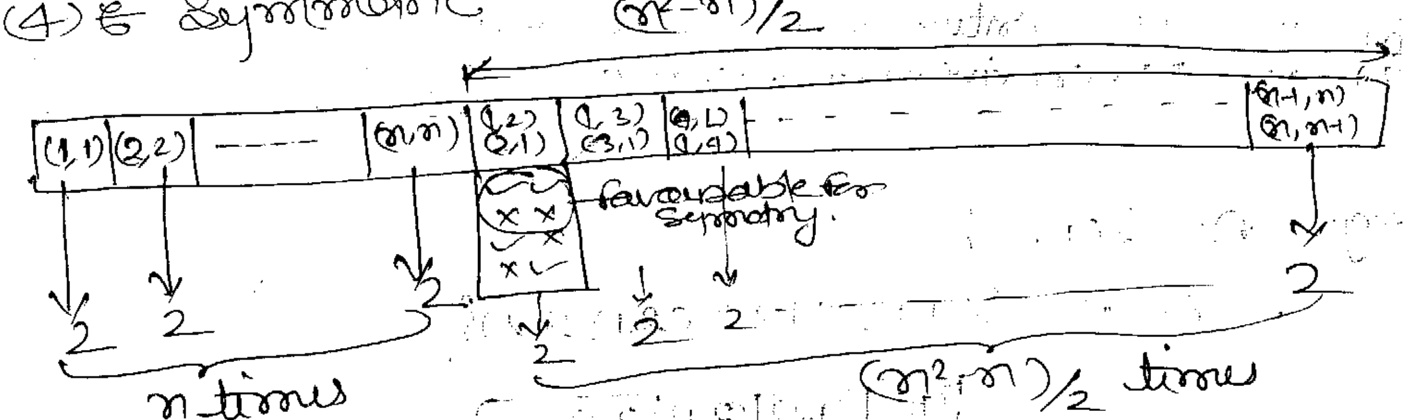
$= 2^{n(n-}$

(3) → ~~I~~ Irreflexive



$1 \times 2^{n^2-n} = 2^{n(n-1)}$ possible relations

(4) ~~E~~ Symmetric

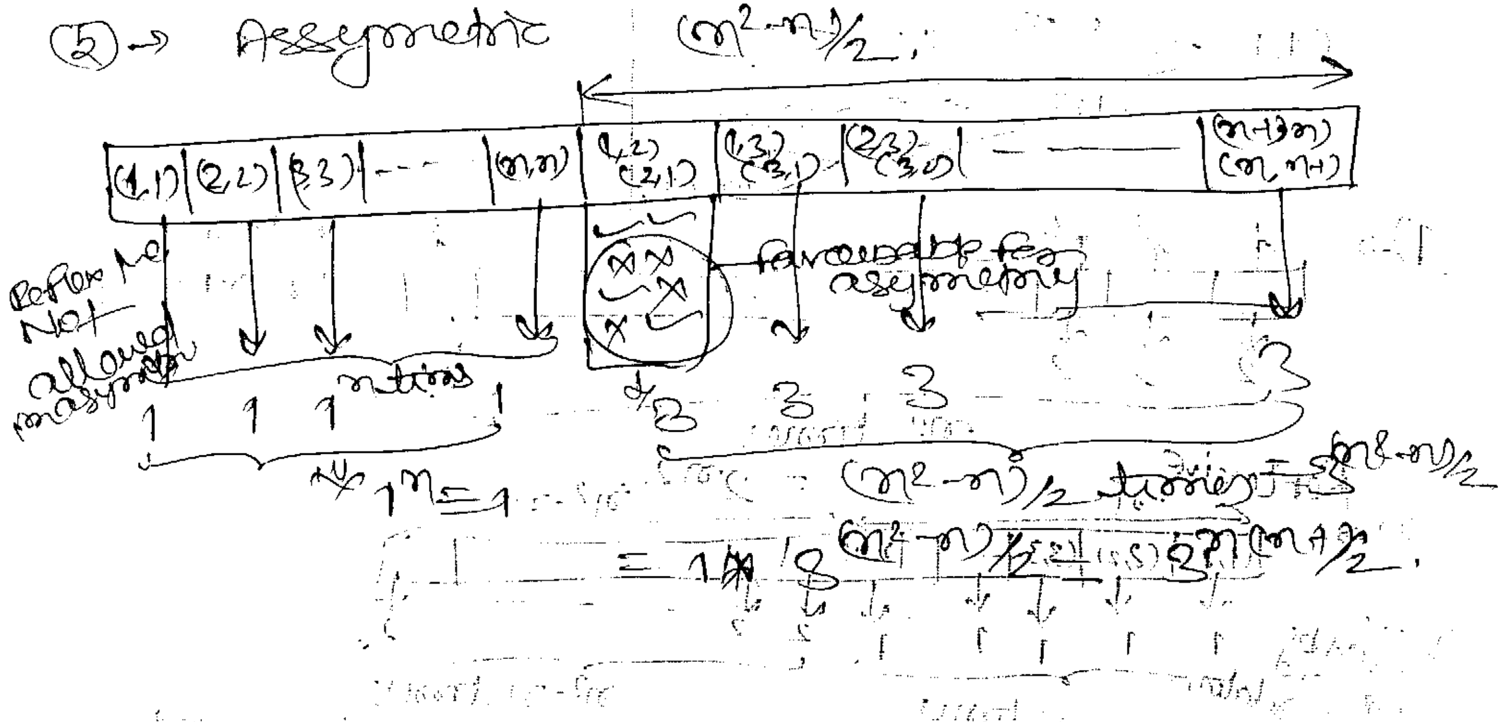


$= 2^n * 2^{(n^2-n)/2}$

$= 2^{n+\frac{n^2-n}{2}} = 2^{\frac{n^2-n+2n}{2}} = 2^{\frac{n^2+n}{2}}$

$= 2^{n(n+1)/2}$

(5) → Assymetric



$1^n = 1$

$= 1 \times 3^{(n^2-n)/2} = 3^{n(n-1)/2}$.

(6) ->

Ques: A = {1, 2, 3, 4, 5}. |A| = 5.

No. of relation = 2^{n^2}. = 2^{25}

No. of reflexive " = $2^{n(n-1)}$ = $2^{5\times4} = 2^{20}$

No. of irreflexive " = $2^{n(n-1)}$ = 2^{20}

" " Symmetric " = $2^{n(n+1)/2}$ = $2^{5\times6/2} = 2^{15}$

" " Asymmetric " = $3^{n(n-1)/2}$ = 3^{10}.

" " Antisymmetric " = $2^n \times 3^{n(n-1)/2}$ = $2^5 * 3^{10}$

" " Reflexive & symmetric " = $2^{n(n-1)/2}$. = 2^{10}.

⇒ Closures:

→ Reflexive Closure of R (Rr): Smallest reflexive relation containing R.

eg: A = {1, 2, 3, 4}.

R = {(1, 1), (2,2), (3,3), (2,4)}

~~r = {(4,4)}~~.

Rr = {(1, 1), (2,2), (3,3), (4,4), (2,4)}

~~Rr = R∆U~~

$$\boxed{R_r = R \cup \Delta}$$

→ Rr = R iff R is already reflexive.

→ % Symmetric Closure of R (Rs) ∈
Smallest symmetric Relation containing R.

Ex:- $A = \{1,2,3,4\}$

$R = \{(1,2), (2,1), (3,4)\}$ & $R^{-1} = \{(2,1), (1,2), (4,3)\}$

$Rs = \{\underbrace{(1,2), (2,1), (3,4)}_{R}, \underbrace{(4,3)}_{R^{-1}}\}$

$$\boxed{R_S = R \cup R^{-1}}$$

[R^+ is used for rel^n on finite sets for infinite sets R^∞ is used]

→ % Transitive closure of R (R^+) ∈
Smallest Transitive Relation containing R.

R^+ $|A| = n$

$R^+ = R \cup R^2 \cup R^3 \cup ---- \cup R^n$.

⇒ Representation of relation ∈

$R \subseteq A \times A$

→ matrix Relation

$M_R = [a[ij]]_{n\times n}$ Boolean matrix

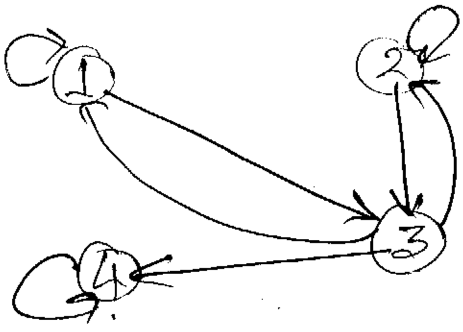
$$a_{ij} = \begin{cases} 0 & (i,j) \notin R \\ 1 & (i,j) \in R. \end{cases}$$

→ % $A = \{1,2,3,4\}$.

$R = \{(1,1), (1,3), (2,2), (2,3), (3,1), (3,2), (3,4), (4,4)\}$

$$M_R = \begin{array}{c|cccc} & 1 & 2 & 3 & 4 \\ \hline 1 & 1 & 0 & 1 & 0 \\ 2 & 0 & 1 & 1 & 0 \\ 3 & 1 & 1 & 0 & 1 \\ 4 & 0 & 0 & 0 & 1 \end{array}$$

Digraphs



28/11/2012

⇒ Warshall's algorithm (To find Transitive Closure)

A = {1, 2, 3, 4}.

R = {(1,2), (2,1), (3,4), (4,3)}.

$$M_R = \begin{array}{c|cccc} & 1 & 2 & 3 & 4 \\ \hline 1 & 0 & 1 & 0 & 0 \\ 2 & 1 & \cancel{0}1 & 0 & 0 \\ 3 & 0 & 0 & 0 & 1 \\ 4 & 0 & 0 & 1 & 0 \end{array}$$

Take first row & column. & Pick zeros that are either in row or column but NOT both. Now, cut rows, columns corresp to ~~zer~~ those zero's. & make rest zero's 1 if its in both

$$M_R^1 = \begin{array}{c|cccc} & 1 & 2 & 3 & 4 \\ \hline 1 & \overset{1}{\cancel{0}} & 1 & 0 & 0 \\ 2 & 1 & 1 & 0 & 0 \\ 3 & 0 & 0 & 0 & 1 \\ 4 & 0 & 0 & 1 & 0 \end{array}$$

Repeat for Row 2 & Column

$$m_R^2 = \begin{array}{c|cccc} & 1 & 2 & 3 & 4 \\ \hline 1 & 1 & 1 & 0 & 0 \\ 2 & 1 & 1 & 0 & 0 \\ 3 & 0 & 0 & 0 & 1 \\ 4 & 0 & 0 & 1 & \cancel{0}1 \end{array}$$

Repeat for " 3 & " 3.

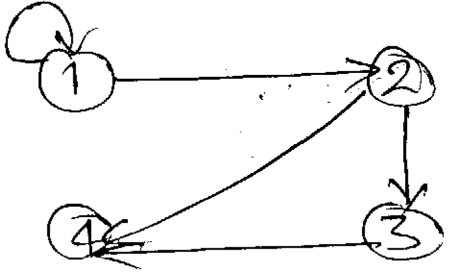
$$m_R^3 = \begin{array}{c|cccc} & 1 & 2 & 3 & 4 \\ \hline 1 & 1 & 1 & 0 & 0 \\ 2 & 1 & 1 & 0 & 0 \\ 3 & 0 & 0 & \overset{1}{\cancel{0}} & 1 \\ 4 & 0 & 0 & 1 & 1 \end{array}$$

" " " 4 & " 4

$$m_R^4 = \begin{array}{c|cccc} & 1 & 2 & 3 & 4 \\ \hline 1 & 1 & 1 & 0 & 0 \\ 2 & 1 & 1 & 0 & 0 \\ 3 & 0 & 0 & 1 & 1 \\ 4 & 0 & 0 & 1 & 1 \end{array} = M_{R^+}$$ ie matrix Relation for Transitive clos

$R^+ = \{(1,1)(1,2)(2,1)(2,2)(3,3)(3,4)(4,3)(4,4)\}$.

Ques find Transitive closure of R defined by



$$M_R = \begin{array}{c|cccc} & 1 & 2 & 3 & 4 \\ \hline 1 & 1 & 1 & 0 & 0 \\ 2 & 0 & 0 & 1 & 1 \\ 3 & 0 & 0 & 0 & 1 \\ 4 & 0 & 0 & 0 & 0 \end{array}$$

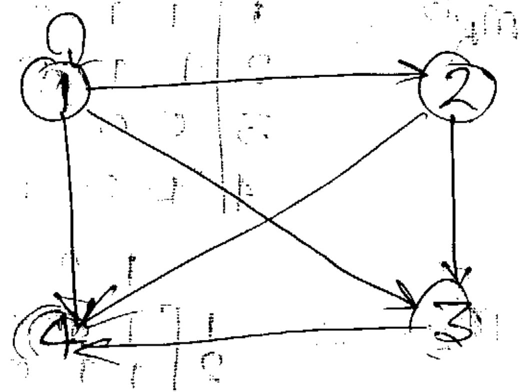
$$M_R^1 = \begin{array}{c|cccc} & 1 & 2 & 3 & 4 \\ \hline 1 & 1 & 1 & \cancel{0}1 & \cancel{0}1 \\ 2 & 0 & 0 & 1 & 1 \\ 3 & 0 & 0 & 0 & 1 \\ 4 & 0 & 0 & 0 & 0 \end{array}$$

$$M_R^2 = \begin{array}{c|cccc} & 1 & 2 & 3 & 4 \\ \hline 1 & 1 & 1 & 1 & 1 \\ 2 & 0 & 0 & 1 & 1 \\ 3 & 0 & 0 & 0 & 1 \\ 4 & 0 & 0 & 0 & 0 \end{array}$$

$$M_R^3 = \begin{array}{c|cccc} & 1 & 2 & 3 & 4 \\ \hline 1 & 1 & 1 & 1 & 1 \\ 2 & 0 & 0 & 1 & 1 \\ 3 & 0 & 0 & 0 & 1 \\ 4 & 0 & 0 & 0 & 0 \end{array} = M_R^4 = \begin{array}{c|cccc} & 1 & 2 & 3 & 4 \\ \hline 1 & 1 & 1 & 1 & 1 \\ 2 & 0 & 0 & 1 & 1 \\ 3 & 0 & 0 & 0 & 1 \\ 4 & 0 & 0 & 0 & 0 \end{array} = M_{R^{\dagger}}$$

Transitive closure of R.

$R^{\dagger} =$



→° Equivalence Relation:– A relation R on A is equivalence relation if it satisfies the following property.

(i) → R is reflexive,
& (ii) → R is symmetric.
& (iii) → R is transitive.

eg:– $A = \{1, 2, 3, 4\}$
$R = \{(1,1), (2,2), (3,3), (4,4)\}$.
reflexive, symmetric, transitive.
So, – An equivalence relation

eg:– $A = \{1, 2, 3, 4\}$
$R = \{(1,1), (1,2), (2,1), (2,2), (3,3), (3,4), (4,3), (4,4)\}$.
reflexive, symmetric, transitive.
So, an equivalence relation.

eg:– Set of integers Z
xRy iff (x+y) is even.

1) → x+x is even ∀x ∈ Z } Reflexive
xRx

2) → (x+y) is even. } symmetric
→ (y+x) is even

3) → (x+y) is even
& (y+z) is even } Transitive.
So (x+z) is even

→° Equivalence class:– Let R be an equivalen
relation on A and a∈A.
→ The equivalence class of a∈A, denoted as
[a] or $\bar{a}$, defined as –

$$[a] = \{ b \in A \mid (a,b) \in R \}.$$

Ex:– 1):– $A = \{1, 2, 3, 4\}$.
$R = \{(1,1), (2,2), (3,3), (4,4)\}$ is E·R.
$[1] = \{1\}$; $[2] = \{2\}$; $[3] = \{3\}$; $[4] = \{4\}$.
~~Distinct~~ equivalence classes = $\{\{1\}, \{2\}, \{3\}, \{4\}\}$

ex:-2 $A = \{1,2,3,4\}$

$R = \{(1,1), (1,2), (2,1), (2,2), (3,3) (3,4), (4,3), (4,4)\}$ E.R on A

$[1] = \{1,2\}$

$[2] = \{1,2\}$

$[3] = \{3,4\}$

$[4] = \{3,4\}$.

Distict equivalence classes $P = \{\{1,2\}, \{3,4\}\}$

→ Partition.

→ properties about equivalence class —

1) $a \in [a]$

2) $b \in [a] \rightarrow a \in [b]$.

3) $b \in [a]$ then $[a] = [b]$.

4) for any two classes $[a]$ & $[b]$: either $[a]=[b]$ or $[a] \cap [b] = \phi$.

eg:- A = Set of integers Z

xRy iff $x+y$ is even is an E.R on A.

How many distinct equivalence classes are there?

a) →1

b) →2

c) →3

d) →None

Sol:- $[1] = \{$Set of all odd Numbers$\}$

$[2] = \{$Set of all even Numbers$\}$.

∴ 2 Equivalence classes.

→ Partition of a Set A:-

A non-empty collection of non-empty set

$P = \{A_1, A_2, --- A_n\}$

Such that

(i) → $A_1 \cup A_2 \cup A_3 \cup \cdots \cup A_n = A$.

(ii) → $A_i \cap A_j = \phi \quad (i \neq j)$

is called partition of A

ex:- $A = \{1,2,3,4,5,6\}$

1) $P_1 = \{\{1\},\{2\},\{3\},\{4\},\{5\},\{6\}\}$ = 6-part partition of A

2) $P_2 = \{\{1,2\},\{3\},\{5\},\{4\},\{6\}\}$ = 5 " " "

3) $P_3 = \{\{1,2\},\{3,4\},\{5,6\}\}$ = 4 " " "

4) $P_4 = \{\{1,2,3\},\{4,5,6\}\}$ = 2 " " "

$P_5 = \{\{1,2,3,4,5,6\}\}$ — 1 part partition of set A.

$P_6 = \{\{1,2,3,4\}, \{4,5,6\}\}$ — Not a partition as $A_1 \cap A_2 \neq \phi$

<u>ex</u> $A = \{1,2,3,4\}$

$P = \{\{1,2\} \{3,4\}\}$ Derive equivalence relation for given partition

Assuming each part as an equivalence class

$\{1,2\} \rightarrow \{(1,2), (1,1), (2,1), (2,2)\}$,

$\cup$

$\{3,4\} \rightarrow \{(3,3), (3,4), (4,3), (4,4)\}$

$= \{(1,1), (1,2), (2,1) (2,2) (3,3), (3,4), (4,3), (4,4)\}$

→ So, Partition can be derived from E.R. as well as E.R. c
also be derived from partition

ex $A = \{1,2,3,4\}$

$P = \{\{1\}, \{2,3\}, \{4\}\}$. Find Equivalence Relation corresponding to P.

$\{1\} \rightarrow \{(1,1)\}$.

$\{2,3\} \rightarrow \{(2,2) (2,3), (3,2), (3,3)\}$;

$\{4\} \rightarrow \{(4,4)\}$;

$ER = \{(1,1), \underline{(2,2), (2,3), (3,2)} (3,3), (4,4)\}$.

So, Given an E.R. on A, we can find a partitio
of A and vice-versa.

∴ No. of E.R. on A are in 1-1 correspondes
with No. of partitions of A

$$\boxed{\begin{aligned} \therefore \text{No. of E.R. on A} &= \text{No. of Partitions of A } (\text{where } |A| = n). \\ &= B_n \\ &\text{[Bell No.]} \end{aligned}}$$

⇒ Formula for B_n.

$$** \boxed{\begin{gathered} B_n = \sum_{k=0}^{n-1} C(n-1, k) \cdot B_k. \\ B_0 = 1 \end{gathered}}$$

$\rightarrow B_1 = {}^0C_0 B_0$

$\boxed{B_1 = 1}$

$\rightarrow \quad B_2 = {}^1C_0 B_0 + {}^1C_1 B_1$

$= 1 \cdot 1 + 1 \cdot 1 = 2.$

$\boxed{B_2 = 2}$

$\rightarrow B_3 = {}^2C_0 B_0 + {}^2C_1 B_1 + {}^2C_2 B_2$

$= 1 \cdot 1 + 2 \cdot 1 + 1 \cdot 2$

$\boxed{B_3 = 5}$

$\longrightarrow B_4 = {}^3C_0 B_0 + {}^3C_1 B_1 + {}^3C_2 B_2 + {}^3C_3 B_3$

$\boxed{B_4 = 15}$; $B_5 = \underline{\underline{52}}$

(*) → A = {1, 2, 3, 4}

No. of E.R. on A = ________

a) → 2, b) → 5, [✓ c) → 15], d) → 24.

(*) R_1 is E.R. on A

R_2 is E.R on A

→ I_1: $R_1 \cap R_2$ is E.R. on A.

→ I_2: $R_1 \cup R_2$ is E.R. on A.

[✓ a) → only I true]

b) → " II "

c) → both true

d) → both false.

eg: A = {1, 2, 3, 4}

$R_1 = \{(1,1), (1,2), (2,1), (2,2), (3,3), (4,4)\}.$

$R_2 = \{(2,2), (2,3), (3,2), (3,3), (1,1), (4,4)\}.$

$R_1 \cap R_2 = \{(1,1), (2,2), (3,3), (4,4)\}.$

$R \cup R_2 = \{(1,1), (1,2), (2,1), (2,2)(2,3)(3,3), (3,2), (4,4)\}.$

$$\begin{array}{c} (1,2) \\ \underline{(2,3)} \\ (1,3) - \times \ldots \end{array}$$

⇛ Partially ordered Relations

A reflexive, antisymmetric and transitive Relation R on A is said to be - Partially ordered Relation (P.O.R.)

eg:- $Z, \leq$ is a Partially ordered Relation on Z.

a)→ $(a \leq a)$ $\forall a \in Z$ - Reflexive

b)→ $(a \leq b)$ & $(b \leq a)$ Then $(a = b)$ — Anti symmetric

c)→ $(a \leq b)$ & $(b \leq c)$ Then $(a \leq c)$ - Transitive.

So $\leq$ is P.O.R. on Z.

eg:- "|" on Z is

a)→ Reflexive
b)→ Antisymmetric
[✓ c)→ Transitive.]
d)→ all the above

(a)→ $0|0$ — Not reflexive as $0|0$ Not possible so $(0,0) \notin R$ for $0 \in Z$

(b)→ $-1|1$ & $1|-1$ but $1 \neq -1$ } so Not antisymmetric

c)→ $a|b$ & $b|c$ so $a|c$ so transitive

eg:- "|" on Z^+ is $Z^+ \equiv$ positive integers

a)→ reflexive
b)→ Antisymmetric
c)→ transitive
[✓ d)→ P.O.R.]

eg:- $\subseteq$ on $P(S)$.

a)→ reflexive : $a \subseteq a$ $\forall a \in P(S)$ every set is its subset
b)→ antisymmetric : $A \subseteq B$ & $B \subseteq A \rightarrow A = B$
c)→ transitive : $A \subseteq B$ & $B \subseteq C \rightarrow A \subseteq C$.
[✓ d)→ P.O.R.] So, P.O.R

⇒ **Definitions** POSet ∈ Let R be a P.O.R. on A.
Then <A, R> is said to be partially ordered set (PoSet).

ex:- $\langle Z, \le \rangle$ Poset.
$\langle Z^{+}, | \rangle$ Poset.
$\langle P(S), \subseteq \rangle$ Poset.

eg:- Arrange the elements (3, 5, 7, 2, 9, 8) means w.r.to some relation

→ $2 \le 3 \le 5 \le 7 \le 8 \le 9$

∵ all elements can be arranged i.e. all elements are related to each other
∴ it is Total order

→ But w.r.to "|"
2 | 3 | 5 - - - it can't be done as all element are NOT related to each other e.g. 2, 3.
∴ "|" is P.O.set NOT totally ordered.

⇒

R	R^{-1}
Reflexive	reflexive
Antisymm.	Antisym.
Transitive	Transitive
P.O.R.	P.O.R.

→ If <A, R> is poset, then <A, R^{-1}> is also poset.

→ <A, R> and <A, R^{-1}> are called dual of each other.

Notation of P.O.R

$$\boxed{\preccurlyeq}$$

⇒ **Comparable:** Let $\langle P, \preceq \rangle$ be a Poset.
Two elements $\{a, b\} \in P$ are said to be comparable if
either $a \preceq b$ or $b \preceq a$.

ex: $A = \{1, 2, 3, 4\}$.
$\langle A, \leq \rangle$ is poset

→ 1 and 3 comparable.
as $1 \leq 3$

→ 4 and 2 comparable
as $2 \leq 4$

→ as every pair is comparable
So Toset

$A = \{1, 2, 3, 4\}$
$\langle A, | \rangle$ is Poset.

→ 2 and 4 are comparabl
as $2 | 4$

→ 2 and 3 are NOT comparabl
as $2 \nmid 3$ & $3 \nmid 2$

→ as 2&3 NOT comparable. So NOT Toset; only POSET.

→ **Def: Toset:** A Poset $\langle P, \preceq \rangle$ in which every pair of elements are comparable, is called Totally ordered set (or) linearly ordered (or) chain

⇒ **Associated Relationship:** Let $\langle P, \preceq \rangle$ be a Po

$x \prec y$ iff $x \preceq y \wedge x \neq y$
$\underset{\text{associated relationship}}{\downarrow}$

ie x is associatively related to y iff x is partially related to y & $x \neq y$

→ **Covering:**

y covers x if $[x \prec y \wedge [x \preceq z \preceq y \Rightarrow x = z \text{ or } y = z]]$

ie. x and y are associated related and Nothing is in b/w x and

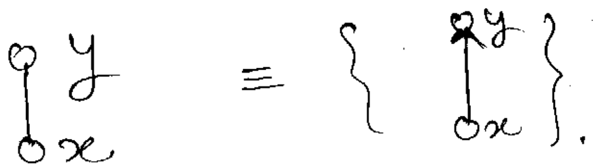
⇒ ~~Hasse Dia~~

⇒ Hasse Diagrams

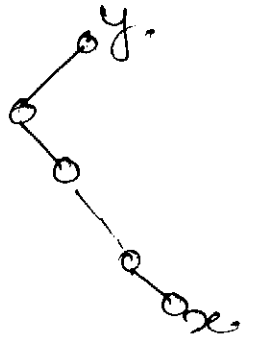
* Let $\langle P, \leq \rangle$ be a poset.

*> Every element of P is denoted by 'o'.

*> If y cover x Then



*> If $x \leq y$ and y doesn't cover x (ie. There is some element in b/w x and y).



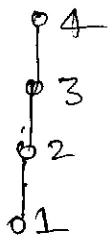
ex: Ex-1>:

$A = \{1, 2, 3, 4\}$.

$\langle A, \leq \rangle$ Poset

Hasse Diagram of Poset is always chain.

Chain



1

$\boxed{1, 2}$ $1 \leq 2$

$\boxed{\cancel{1}, 2, 3}$ $1 \leq 2 \leq 3$

$\boxed{\cancel{1}, \cancel{2}, 3, 4}$ $1 \leq 2 \leq 3 \leq 4$

ex: $A = \{1, 2, 3, 4\}$

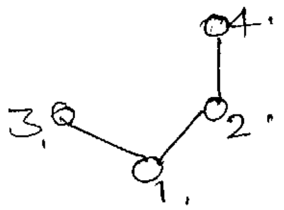
$\langle A, | \rangle$.

1

$\boxed{1, 2}$ $1|2$

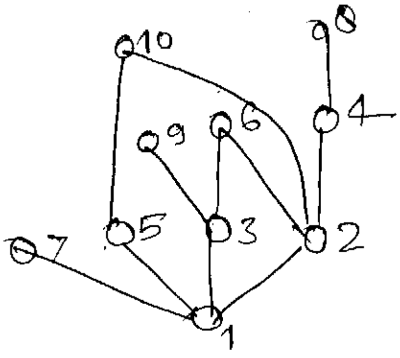
$\boxed{1, 3}$ ~~1|2|3~~ $1|3$.

$\boxed{\cancel{1}, 2, 4}$ $1|2|4$.

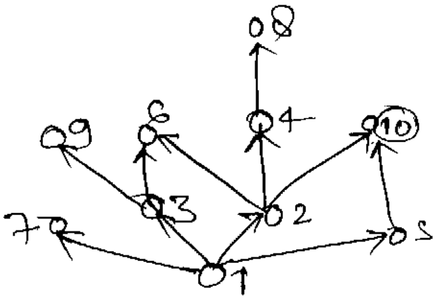


exe $A = \{1,2,3,4,5,6,7,8,9,10\}$, |

$\langle A, | \rangle$.



⇒



1

1 | 2

1 | 3

~~1~~ 2 | 4

1 | 5

~~1~~, 2, 3, | 6

1, | 7

~~1~~, ~~2~~, 4 | 8

~~1~~, 3 | 9

~~1~~, 2, 5 | 10

Using hasse diagrams, we can obtain many results.

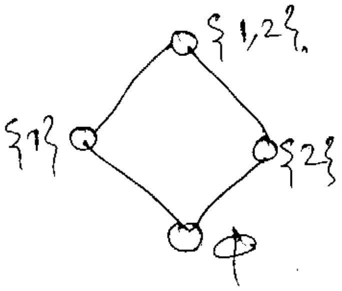
→ as if a path exists from a to b or b to a then they are comparable. otherwise NOT comparable.

→ we can also conclude relation from hasse diagram

exe $A = \{1,2\}$

$< P(A), \subseteq >$ Poset

$P(A) = \{\phi, \{1\}, \{2\}, \{1,2\}\}$.



φ

φ | {1}

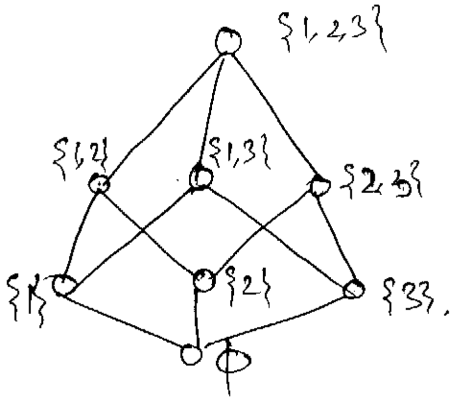
φ | {2}

φ | {1} | {2} | {1,2}

→eg $A = \{1,2,3\}$

$\langle P(A), \subseteq \rangle$.

$P(A) = \{ \phi, \{1\}, \{2\}, \{3\}, \{1,2\}, \{1,3\}, \{2,3\}, \{1,2,3\} \}$.



ϕ

$\boxed{\phi \mid \{1\}}$

$\boxed{\phi \mid \{2\}}$

$\boxed{\phi . \mid \{3\}}$

$\boxed{\phi \{1\},\{3\} \mid \{1,3\}}$

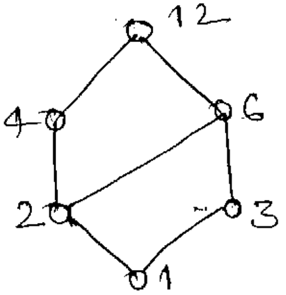
⇒ eg D_n = Set of positive divisors of n.

$D_8 = \{1, 2, 4, 8\}$

$D_{12} = \{1,2,3,4,6,12\}$.

. $\langle D_n, | \rangle$ is a Poset

eg: $\langle D_{12}, | \rangle$ $\{1,2,3,4,6,12\}$.



⇒ Special elements (of Posets)

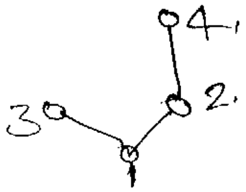
(I) maximal and minimal elements

Let $\langle P, \preceq \rangle$ be a poset

→ An element $M \in P$ is maximal if for no $x \in P$, $M \preceq x$ i.e. m is related to No element

→ An element $m \in P$ is minimal if for no $y \in P$ $y \preceq m$. i.e. No element is related to m

exe

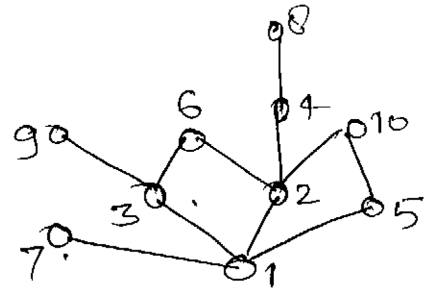


Poset

maximal → 3, 4

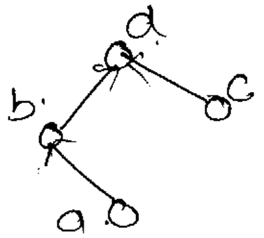
minimal → 1.

eg:-



maximal → 6,7,8,9,10
minimal → 1.

eg:-



maximal → d
minimal → a,c.

→ Every finite Poset <P,≤> has a maximal and minimal elements.

<Z+,|> max-?
min → {1}

eg: <Z,≤> Poset.

max → } NO.
min → }

Becz this is infinite Poset so maximal & minimal doesn't exist.

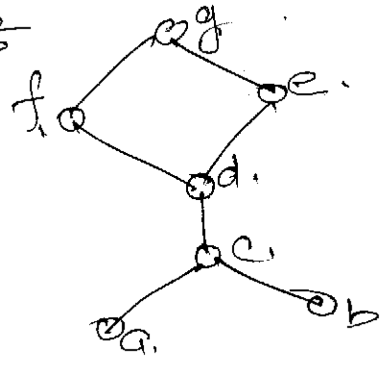
(II) Greatest and least elements:

Let <P,≤> be a Poset.

→ An element g∈P is greatest element if x≤g ∀x∈P (ie. if every element is related to g)

→ An element l∈P is least element if l≤y ∀y∈P (ie if l is related to every element

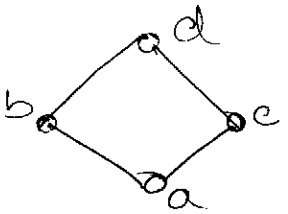
eg:



max → g
min → a, b

Greatest → g.
least → Not exist.

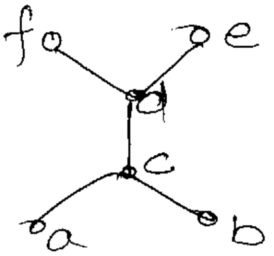
ex:-



Max → d. | Greatest → d.
min → a. | least → a.

→ If maximal is unique, then it is greatest.
→ If minimal is unique then it is least.

ex:-



max → f, e | Greatest → } No, exist.
min → a, b | least → }

1. ** → Greatest and least elements, if exist, are unique

(III): Upper bound and lower bounds

Let $<P, \preccurlyeq>$ be a poset & $A \subseteq P$.

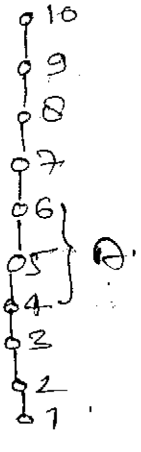
An element $u \in P$ is an upper bound of A.
If $x \preccurlyeq u \quad \forall x \in A$. (If every element of A is related to u).

A element $l \in P$ is an lower bound of A.
if $l \preccurlyeq y \quad \forall y \in A$ (If l is related to every element of A)

ex:- $P = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$,
$< P, \leq >$ Poset, let $A = \{4, 5, 6\}$

Upper bound A → ⑥, 7, 8, 9, 10. → least upper bound lub.

lower bound A → 4, 3, 2, 1.
1, 2, 3, ④ → greatest lower bound (g.l.b).



→ l.u.b. $\preccurlyeq$ u.b.'s

→ ~~glb~~ l.b's $\preccurlyeq$ g.l.b.
related to.

eg:- $<P, \preccurlyeq>$

$\{a,b\} \subseteq P.$

l.u.b.$\{a,b\} = a \vee b$ = a "join" b.

g.l.b$\{a,b\} = a \wedge b$ = a "meet" b.

	"$\leq$"	"|"	"$\subseteq$"
l.u.b$\{A,B\}$	max$\{A,B\}$	Lcm$\{A,B\}$	$A \cup B$.
G.L.b$\{A,B\}$	min$\{A,B\}$	GCD$\{A,B\}$	$A \cap B$.

⇒ **Lattice:-** A Poset $<P, \preccurlyeq>$ in which every two element subset has l.u.b. & g.l.b. is called a lattice.

Q1) → WB:- $a \equiv F$

$b \Leftrightarrow c$?

↳ $b \equiv c$.

→ $a \vee (b \wedge c)$

→ $F \vee (b \wedge c)$

→ $b \wedge c)$

→ b

29/11/2012

⇒ **Properties of lattice:-** Let $<L, \preccurlyeq>$ be a lattice.

1) → $a \preccurlyeq (a \vee b)$ l.u.p.

→ $b \preccurlyeq (a \vee b)$

2) → $(a \wedge b) \preccurlyeq a$

→ $(a \wedge b) \preccurlyeq b$

g.l.b.

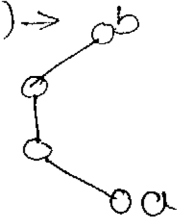
Case 1) →

$a \preccurlyeq b$

$a \vee b = b.$ } b covers a

$a \wedge b = a.$

Case (ii) →



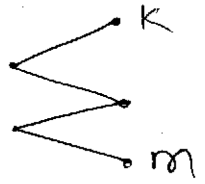
$a \preceq b$; $a \prec b$ (associated relationship).

$\left.\begin{matrix} a \preceq b \\ b \preceq b \end{matrix}\right\} \Rightarrow a \vee b = b$ - l.u.b.

$\left.\begin{matrix} a \preceq a \\ a \preceq b \end{matrix}\right\} \Rightarrow a \wedge b = a$ - g.l.b.

3) → Consistency ~~Property~~

$a \preceq b$ iff $a \vee b = b$ iff $a \wedge b = a$.

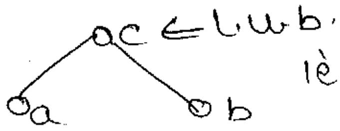


$K \vee m = K$.

$K \wedge m = m$.

Case (iii) →

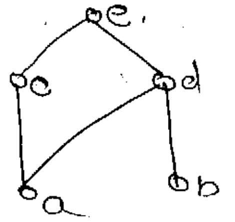
a) →



$a \vee b = c$

i.e. Go ~~to~~ upward and see where they join

b) →



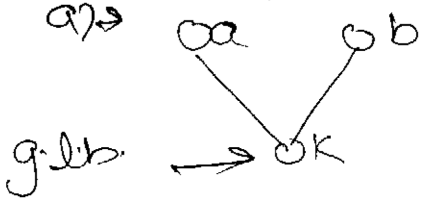
→ d, e ≡ u.p.

$a \vee b = d$.

Here they join at two places. So, two. u.p.s. So one of them is l.u.b. i.e. d. from lub defn

Case (iv) →

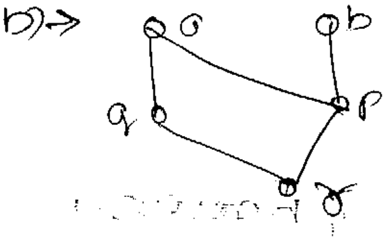
a) →



$a \wedge b = K$.

Go downward & see where they meet i.e.

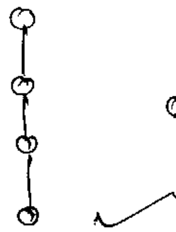
b) →

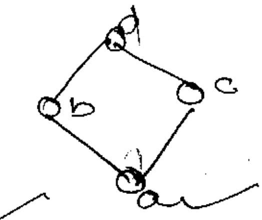


→ p, r ≡ lb's

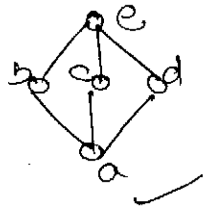
$a \wedge b = p$.

⇒ which of the following are not lattice

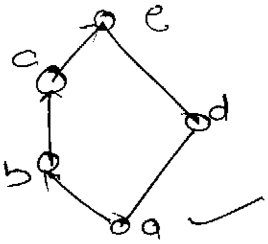

(1)



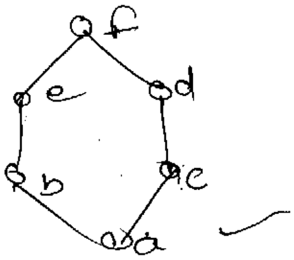
(2)



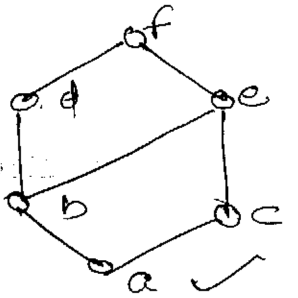
(3)



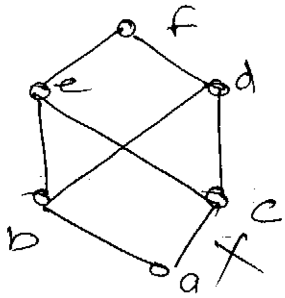
(4).



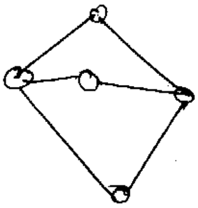
(5).



(6)



(7),


(8),

(1)→ chain — l.u.b & g.l.b {all exist (consistency).
∴ lattice

Result: Every chain is lattice

(2)→ All comparable pairs are related. So by consistency property. They have l.u.b & g.l.b.
So we need to check only non-comparable pairs.
Here only incomparable pairs b, c.
$b \vee c = d$
$b \wedge c = a$. } So lattice

(3)→ $b \vee c = e$ | $b \vee d = e$ | $c \vee d = e$
$b \wedge c = a$ | $b \wedge d = a$ | $c \wedge d = a$. } So lattice

(4)→ $b \vee d = e$. | $c \vee d = e$.
$b \wedge d = a$. | $c \wedge d = a$.

(5)→ $b \vee c = f$ | $b \vee d = f$ | $e \vee c = f$ | $e \vee d = f$
$b \wedge c = a$ | $b \wedge d = a$ | $e \wedge c = a$ | $e \wedge d = a$.

6)→ $b \vee c = e$ | $c \vee d = f$ | $d \vee e = f$

$b \wedge c = a$ | $c \wedge d = a$ | $d \wedge e = b$.

7)→ $b \vee c$ = No. l.u.b $\quad$ ($\to$ d, e, f = u.bs.)

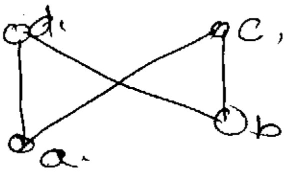
$b \wedge c = a$.

$b \vee c$ = d, e, f as $d \leq f$ & $e \leq f$
d & e are NOT related
So No L.U.P.

$e \vee d = f$

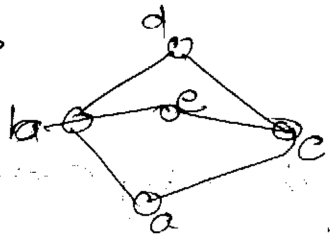
$e \wedge d$ = No glb $\longrightarrow$ c, b, a as $a \leq b$, $a \leq c$
& b & c are NOT comparable

Case(VI)→ whenever hasse diagram consists such a path a below—

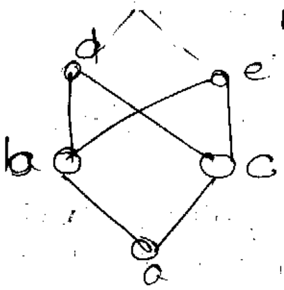


$a \vee b$ → doesn't exist
$c \wedge d$ → " " "

(8)→



⇒



No. l.u.b.

⇒ NOT lattice

(4)→ Let $\langle L, \leq \rangle$ be a lattice. Then the following properties holds.

i)→ Idempotent:–
$a \vee a = a$
$a \wedge a = a$.

ii)→ Commutative:–
$a \vee b = b \vee a$.
$a \wedge b = b \wedge a$.

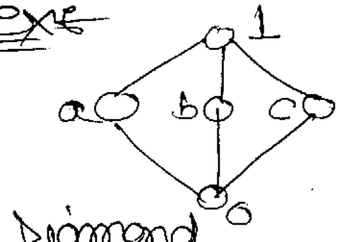
iii)→ Associative:–
$a \vee (b \vee c) = (a \vee b) \vee c$
$a \wedge (b \wedge c) = (a \wedge b) \wedge c$.

iv)→ Absorption:–
$a \wedge (a \vee b) = a$.
$a \vee (a \wedge b) = a$.

ex:- Which of the following properties need NOT be be satisfied in a lattice.

(a)→ Absorption.
✓(b)→ Distributive.
(c)→ Associative.
(d)→ Commutative

ex:-



Diamond lattice.

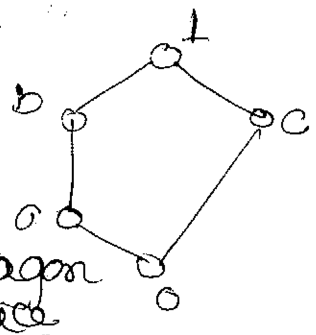
$a\vee(b\wedge c) = a\vee 0 = a$

$(a\vee b)\wedge(a\vee c)$

$1\wedge 1 = 1$

So, Distributive properties Need NOT be satisfied in a lattice.

ex:-



Pentagon lattice.

$a\vee(b\wedge c) = a\vee 0 = a.$

$(a\vee b)\wedge(a\vee c) = b\wedge 1 = b.$

So Distributive properties Need Not be satisfied.

②→ In any lattice $\langle L, \leq\rangle$, the following distributive inequalities are satisfied.

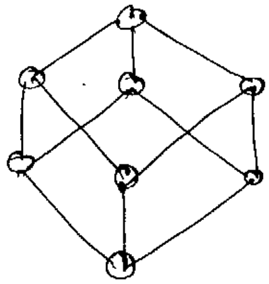
(i)→ $a\vee(b\wedge c) \preceq (a\vee b)\wedge(a\vee c)$.

(ii)→ $(a\wedge b)\vee(a\wedge c) \preceq a\wedge(b\vee c)$.

⇒ Definition:- A lattice in which distributive properties are satisfied, is called Distributive lattice.

ex:- $A = \{1,2,3\}$

$\langle P(A), \subseteq\rangle$ Distributive.



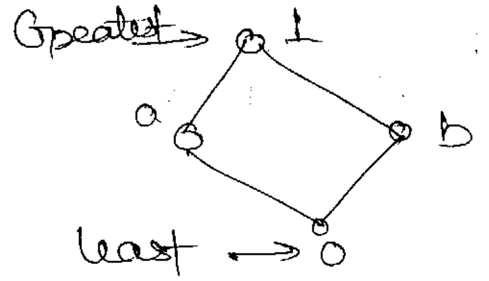
→ **Sublattice:** Let $\langle L, \preceq \rangle$ be lattice and $S \subseteq L$. If $\langle S, \preceq \rangle$ is lattice, then it is called sublattice.

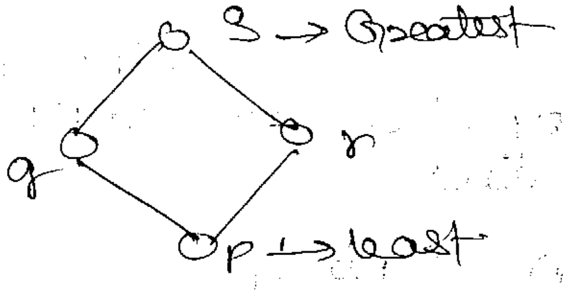
Result
*1) – A lattice is distributive iff it doesn't contains any sublattice isomorphic to diamond (or) polygon lattice.

→ **Bounded lattice:**
Greatest → 1.
Least → 0
A lattice in which the greatest and least elements exist is called bounded lattice.

Result
*1) – Every finite lattice is bounded. Infinite lattices need not be bounded.
$\langle Z, \leq \rangle$ is lattice.
but NOT bounded as its infinite Lattice

→ **Complement of an ~~lattice~~ element:**
Let $\langle L, \preceq \rangle$ be a bounded lattice. An element $b \in L$ is complement of $a \in L$
if $a \vee b = 1$ (greatest)
$a \wedge b = 0$ (least).



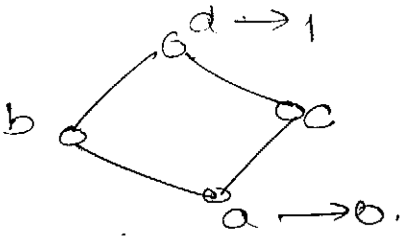


So in these Notations 1 & 0 are Nothing but Greatest & least elements respectively.
Spcl Notation of

1) → if b is complement of a.
Then a is complement of b.

2) → $1 \vee 0 = 1$
$0 \wedge 0 = 0$.

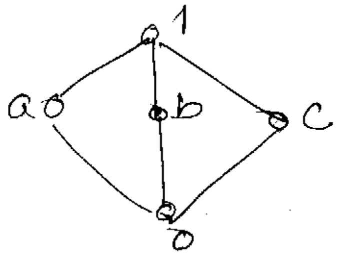
ex:



element	complement
a	d
b	c
c	b
d	a

as $b \wedge c = a$
$b \vee c = d$.

ex:

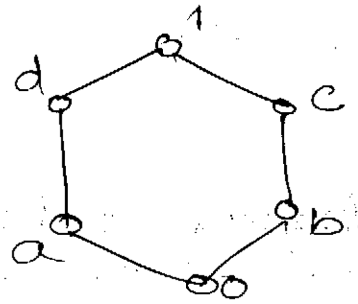


element	complement
0	1
a	b, c
b	a, c
c	a, b.
1	0

$a \wedge b = 0$
$a \vee b = 1$

$a \vee c = 1$
$a \wedge c = 0$

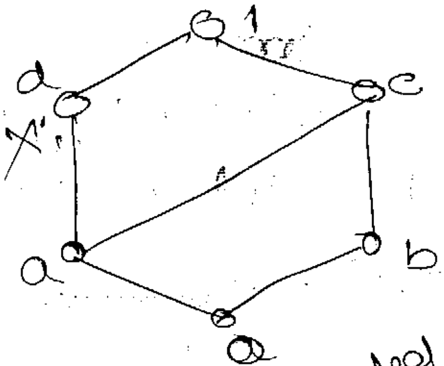
ex:



complemented lattice

element	complement
0	1
a	b, c
b	a, d
c	a, d
d	b, c.
1	0

ex:



NOT complemented lattice

element	Compleme
0	1
a	
b	d
c	
d	b
1	0

No comp men

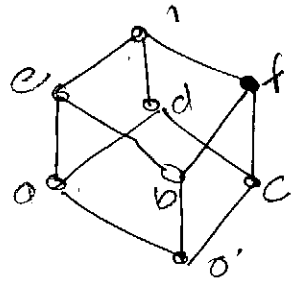
Definition: A bounded lattice in which complement of every element exist is called complemented lattice.

Result *) In a distributive bounded lattice, complement if exist are unique.

Def:- A bounded, distributive and complemented lattice is called Boolean algebra.

Ex:- $A = \{1, 2, 3\}$

$< P(A), \subseteq >$



Result:-

*1) Let $n = p_1 \cdot p_2 \cdot \cdots p_k$ where $p_1 \cdot p_2, \cdots p_k$ are distinct prime factors, $n \in Z^+$. Then D_n is Boolean algebra.

ex:- D_6 is boolean algebra. $6=2\times3$

D_{30} " " " $30=2\times3\times5$.

Result:- $n \in Z^+$, $p^2 | n$: p is prime no, then D_n is NOT boolean algebra.

*) $<P(A), \subseteq>$ is always Boolean algebra.

⇛ **Topological Sort:-** The linear order corresponding to a given partial order is the topological sort.

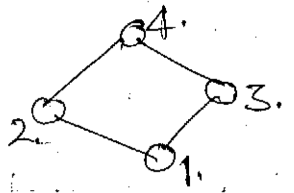
→ $<A, \leq>$ Partial order set

1) → Let 'a' be a minimal element.

2) → Put 'a' in the SORT & replace A by $A-\{a\}$.

3) → Continue steps 1 and 2 till $A=\{\}$

eg:-



$A=\{1,2,3,4\}$: ~~minimal~~

⇓

1 is minimal

SORT

$A = A-\{1\} = \{2,3,4\}$

1			

⇓

2 is minimal

$A = A-\{2\} = \{3,4\}$

1	2	

// 3 is minimal

SORT

1	2	3	

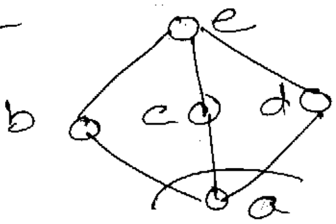
$A = A - \{3\} = \{4\}$ o 4

// 4 is minimal

$A = A - \{4\} = \{ \}$

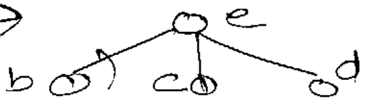
1	2	3	4

ex:



a

→



ab

→

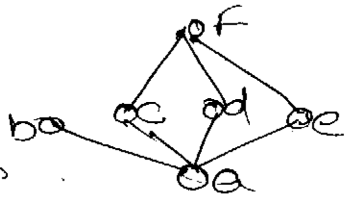
abc

→

abed

→

abcde.



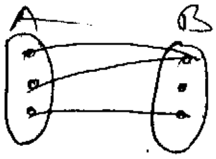
Ques: which of the following aren't possible topological sort for the above poset

a) → a c d e b f
b) → a b c d e f
c) → a e d c b f
d) → a e d c f b
e) → a f b c d e.

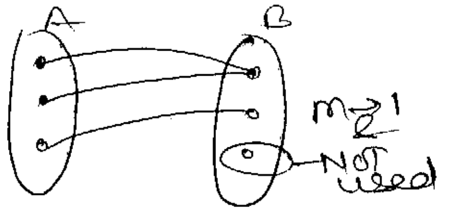
⇛ FUNCTIONS

A function from set A to set B is a rule which assign every element of A a unique element of set B.

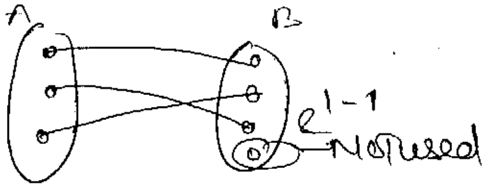
We write $f : A \to B$



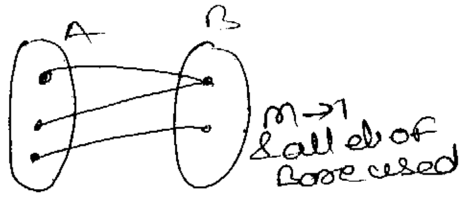
I



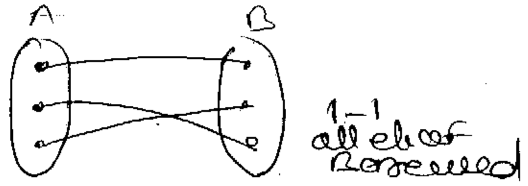
II



III



IV



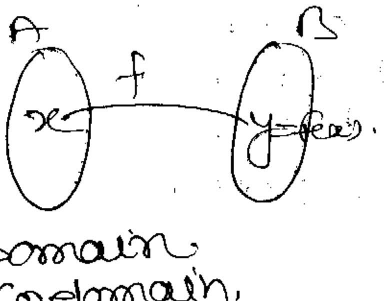
I, II → INTO fns (All els of set B are NOT used)

III, IV → ONTO fn (" " " " , B are used).

II, IV → 1-1 fn

IV → 1-1 and ONTO fn.

Terminologies

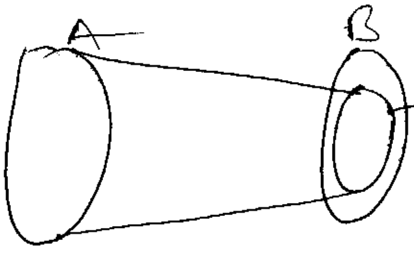


→ A → Domain

→ B → Codomain

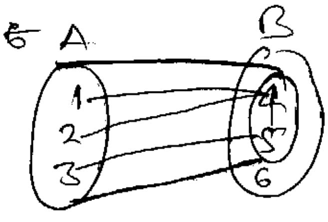
→ $f(x) = y$ ⇒ image of x under f

x = pre-image of y under f

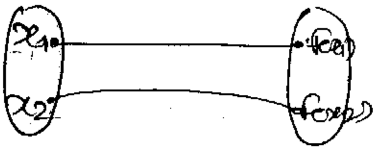


→ f(A)
= Dom (f)
= Range of f.

ex



→ **One-one fn (injection)**: If $x_1 \neq x_2$ Then $f(x_1) \neq f(x_2)$.



1-1

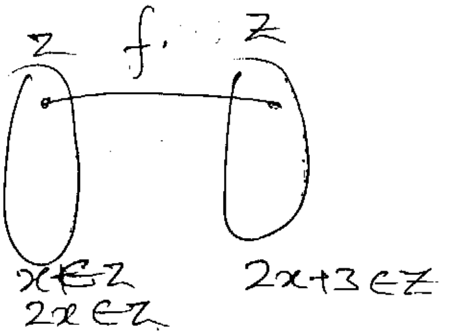
NOT 1-1

→ If $x_1 \neq x_2$ Then $f(x_1) \neq f(x_2)$
(or)
≡: If $f(x_1) = f(x_2)$ Then $x_1 = x_2$

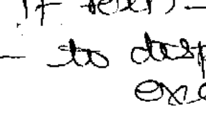
Exe 1) $f: Z \to Z$.
$f(x) = 2x+3$.
1-1 or NOT.?

→ $f(x_1) = f(x_2)$.
→ $2x_1+3 = 2x_2+3$.
→ $2x_1 = 2x_2$
→ $x_1 = x_2$.

So it is 1-1



2) $f: R \to R$
$f(x) = x^2$.



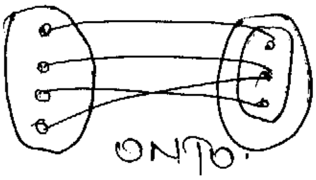
NOT 1-1.

To prove, we need to prove
If $f(x_1) = f(x_2)$ then $x_1 = x_2$
But to disprove one counter example is enough

3) $f: R^+ \to R^+$
$f(x) = x^2$
$f(x_1) = f(x_2)$
$x_1^2 = x_2^2$
$x_1 = \pm\sqrt{x_2^2}$
$x_1 = \pm x_2$
$\Rightarrow \boxed{x_1 = x_2}$

as Negative Nos NOT all
So unique

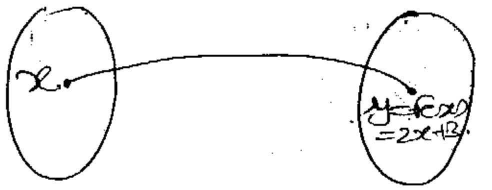
→ **ONTO fn (Surjection)**:
$f: A \to B$ is ONTO if
$\forall y \in B$; $\exists x \in A$ such that $\boxed{f(x) = y}$.
So, here range of fn = B.



ONTO.

ex: $f: Z \to Z$

$f(x) = 2x+3$ is ONTO or NOT?



$y = 2x+3$ (y in terms of x)

write x in terms of y

$2x = y-3$

$\boxed{x = \frac{y-3}{2}} \notin Z$

Let $y = 6$; $x = \frac{6-3}{2} = 3/2 \notin Z$.

So, it is NOT ONTO

ex: $f: R \to R$

$f(x) = 2x+3$.

$y = 2x+3$. (y in terms of x)

$x = \frac{y-3}{2} \in R$ (Domain)

So ONTO fn.

⇒ **Bijection:** A 1-1 and ONTO function is called "bijection".

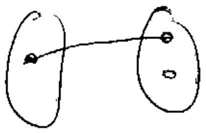
ex: Let $|A| = n$

$|B| = m$

$F: A \to B$

(i) → which of following is always true If F is ONTO?

a) $n \le m$ b) → $n \ge m$ c) → $m = n$ (d) None



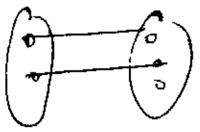
NOT ONTO

So $n \nleq m$

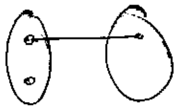
i.e. $\boxed{|A| \geq |B|}$

(2)→ If f is one one fn

a)→ $n \le m$ b)→ $n \ge m$ c)→ $m = n$ (d)→ None



$\boxed{|A| \le |B|}$



So. $|A| \le |B|$ for $f: A \to B$ to be one-one

It can't be 1-1.

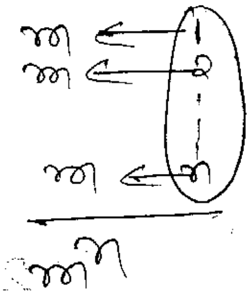
(3)→ If f is bijection ie (1-1 and ONTO)

a)→ $n \le m$ b)→ $n \ge m$ c)→ $n = m$ d)→ None

$\boxed{|A| = |B|}$

(4)→ The no. of distinct fns possible from A to B

(a)→ n^m (b)→ m^n. (c)→ ^mP_n (d)→ ^nP_m.



$\Rightarrow \boxed{|B|^{|A|}}$

Ex:- There are 97 distinct functions from x to y. Then which of the following is true

a)- $|X| = 1$; $|Y| = 97$

b)- $|X| = 97$ $|Y| = 1$.

c)- $|X| = 97$ $|Y| = 97$

d)- None

$|Y|^{|X|} = 97$

So → (b) $1^{97} = 1$ ⇒ Not possible

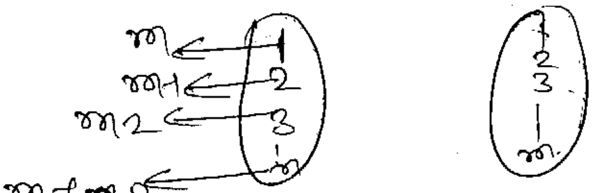
(a) $97^1 = 97$ - Correct

(5)→ The No of 1-1 fn possible from A to B.

~~So ([illegible])~~

a) → m^n b) → n^m c) → ^mP_n d) - ^nP_m

So For 1-1 fn $n \leq m$.



→ n-permutations of m-objects of B.

$$= \frac{m*(m-1)*\overset{= {}^mP_n}{\cdots}(m-(n-1))*(m-n)!}{(m-n)!} = \text{So } {}^mP_n.$$

(6)→ $|A| = n$; $|B| = m$.

$f: A \to B$.

No. of onto fns from A to B.

$$= \sum_{i=0}^{m} {}^mC_i (-1)^i (m-i)^n$$

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<u>Ques</u> No. of ONTO fns

$|A| = n$; $|B| = 2$

$$= {}^2C_0 (2-0)^n + {}^2C_1 (2-1)^n + \cancel{{}^2C_2 (2-2)^n} \to 0$$

$$= {}^2C_0 (2)^n - {}^2C_1 (1)^n$$

$$= \boxed{2^n - 2}$$

<u>Ques</u> → $|A| = 4$; $|B| = 3$.

find Nos. of ONTO fns.

$$^3C_0(3-0)^4 - {}^3C_1(3-1)^4 + {}^3C_2(3-2)^4$$

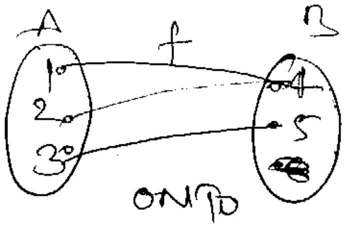
$$1*3^4 - 3*2^4 + 3(1)^4 = 81 - \overset{48}{\cancel{64}} + 3$$

= ~~84 - 64 = 20~~.

= 84 - 48

= <u>36</u>.

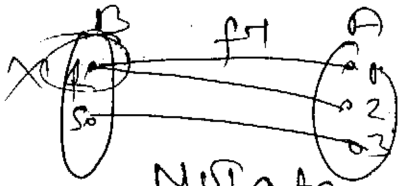
⊛→ f^{-1} (Inverse of a fn).



as $f = \{(1,4), (2,4), (3,5)\}$

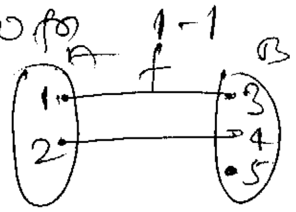
$f^{-1} = \{(4,1), (4,2), (5,3)\}$

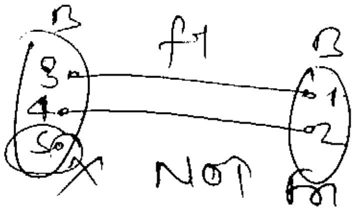
Not a fn



NOT a fn
element 4∈B don't e
any ele

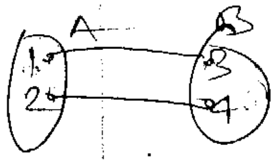
for INTO fn 1-1



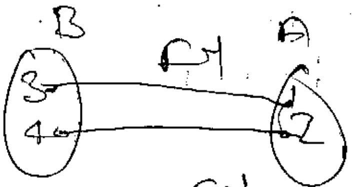


NOT fn, as all ele of B
aren't mapped

one-one & ONTO



$\Rightarrow$



f^{-1} exists.

⊛→ Results

f^{-1} exist iff f is bijection
(1-1 and ONTO).

If $f: A \to B$ is 1-1 & ONTO
Then $f^{-1}: B \to A$ is also (1-1 & ONTO) fn.

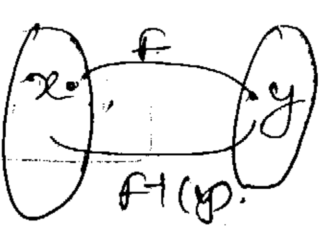
→⊛ finding inverse of a fn

$f(x) = 2x+3$.

$y = 2x+3$ (y in terms of x).

write x in terms of y.

$$\boxed{x = \frac{y-3}{2}}$$ formula of inverse.



$$\boxed{f^{-1}(y) = \frac{y-3}{2}}$$

$$\boxed{f^{-1}(x) = \frac{x-3}{2}}$$

ex:- (2) → $f(x) = \frac{x+3}{x+5}$ $f^{-1}(x) = ?$

$$y = \frac{x+3}{x+5}$$

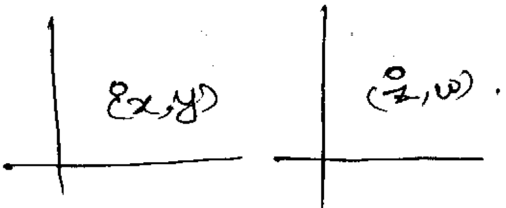
$$yx + y5 = x+3.$$

$$5y - 3 = x - yx = x(1-y)$$

$$x = \frac{5y-3}{1-y} = \frac{3-5y}{y-1}$$

** $$\boxed{f^{-1}(x) = \frac{3-5x}{x-1}}$$

ex: $f(x,y) = (x+y, x-y)$.
find f^{-1}



$(z,w) = (x+y, x-y)$ [z & w in terms of x & y].

write x & y in terms of z & w

on adding
$$\begin{aligned} z &= x+\cancel{y} \\ w &= x-\cancel{y} \end{aligned}$$
$(z+w) = 2x$; $\frac{(z+w)}{2} = x$

on subtracting
$$\begin{aligned} z &= \cancel{x}+y \\ -w &= \cancel{x}-y \\ \hline (z-w) &= 2y \end{aligned}$$
$(z-w)/2 = y$

$$\boxed{(x,y) = \left(\frac{z+w}{2}, \frac{z-w}{2}\right)}$$ inverse formula,

$$f^{-1}(z,w) = \left(\frac{z+w}{2}, \frac{z-w}{2}\right)$$

$$\boxed{f^{-1}(x,y) = \left(\frac{x+y}{2}, \frac{x-y}{2}\right)}$$

exe $f(x,y) = (x+2y,\ 2x-y)$ find f^{-1}

$(z,w) = (x+2y,\ 2x-y)$

$$\begin{array}{l|l} z = x+2y & 2(z = \cancel{x}+2y) \\ 2(w = 2x-\cancel{y}) & w = \cancel{2x}-y \\ \hline z+2w = 5x & 2z-w = 5y \\ \left(\frac{z+2w}{5}\right) = x & \left(\frac{2z-w}{5}\right) = y \end{array}$$

$f^{-1}(z,w) = \left(\frac{z+2w}{5}, \frac{2z-w}{5}\right)$

$f^{-1}(x,y) = \left(\frac{x+2y}{5}, \frac{2x-y}{5}\right)$

⇛ Composite functions

$f: A \to B$

$g: B \to C$

$gof: A \to C$ defined as

$$\boxed{gof(x) = g[f(x)]}$$

→ If gof is defined, fog need not be defined.

→ when gof and fog defined. It is NOT necessary that both are equal ie $gof \neq fog$

exe $f(x) = 2x^2$

$g(x) = x+3$

$f: R \to R$; & $g: R \to R$, } Bcoz f & g both are defined on same sets. So fog & gof both exists

$gof(x) = g[f(x)] = g(2x^2) = 2x^2+3$.

$fog(x) = f[g(x)] = f(x+3) = 2(x+3)^2$.

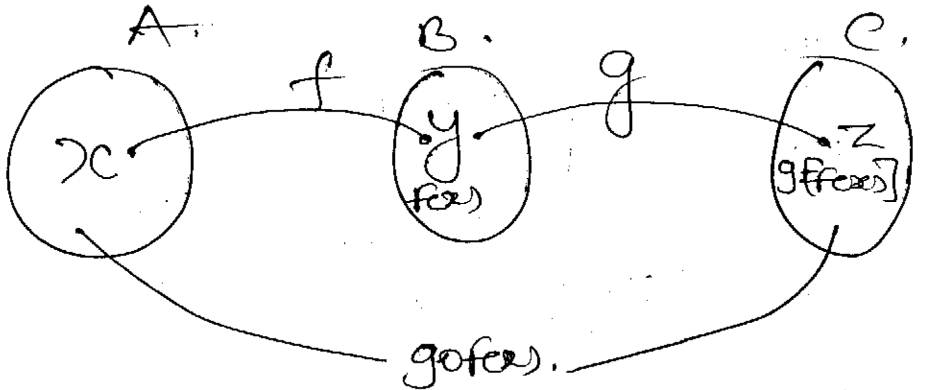
$fog \neq gof$

So fog & gof Need need to equal.

$f: A \to B$
$g: B \to C$ [illegible]

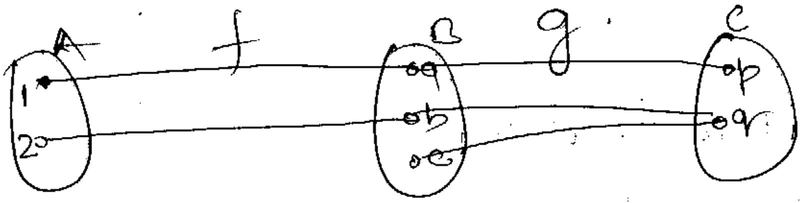
*→

I - (1)→ If f and g are one-to-one, then gof is 1-
(2)→ If f and g are ONTO, then gof is ONTO.
(3)→ If f and g are 1-1 & ONTO, then gof is 1-1 & ONT

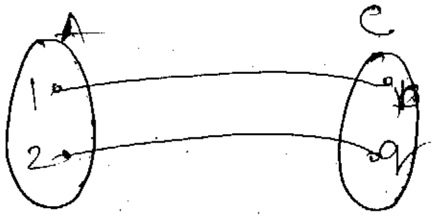


NOTE:- NOT conversely.

ex:



~~fo~~
$gof(1) = g[f(1)] = g(a)=p$
$gof(2) = g[f(2)] = g(b)=q$



gof is 1-1
and ONTO.
But g & f
are NOT 1-1 & ONTO
So, converse is NOT
necessarily true.

Result
*→
II - (1) If gof is 1-1, then f is 1-1
(2) If gof is ONTO, then g is ONTO.
(3) If gof is 1-1 and ONTO, then f is 1-1 and
g is ONTO

Problems

→ hog is 1-1 → g is 1-1.

→ fog is ONTO → f is ONTO.

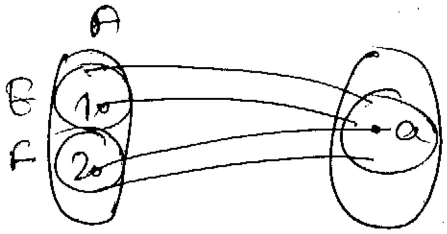
Ques→ CB/ 18 [illegible]/28

$f : A \to B$,

$S_1 : f(E \cup F) = f(E) \cup f(F)$ ✓

$S_2 : f(E \cap F) = f(E) \cap f(F)$ ✗

$E \subseteq A$
$F \subseteq A$.

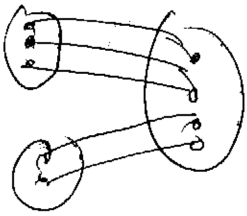


$E = \{1\}$
$F = \{2\}$

$E \cap F = \{\}$
$f(E \cap F) = \{\}$

$f(E) = \{a\}$
$f(F) = \{a\}$
$f(E) \cap f(F) = \{a\}$.

$\boxed{\text{So } f(E \cap F) \neq f(E) \cap f(F)}$



≡ So $f(E \cup F) = f(E) \cup f(F)$ ✓

So I→ $f(E \cup F) = f(E) \cup f(F)$

II→ $f(E \cap F) \subseteq f(E) \cap f(F)$.

III→ $f(E \cap F) = f(E) \cap f(F)$.
iff f is 1-1

Ques Let X, Y, Z be set of size x, y, z respectively

Let W = ~~XYZ~~ X×Y

E = set of all subsets of W.

No. of fns from Z to E is

$|W| = xy$

$|E| = 2^{xy}$

$f: Z \rightarrow E$ -

$|E|^{|Z|}$

$= (2^{xy})^z = 2^{xyz}$.

⇒ 13) – Paper 16 | WB.

$A \cap A \neq \phi$ = NOT ~~Re~~flexive

$A \cap B = \phi$. Then $B \cap A = \phi$ so symm.

let $A = \{1\}$, $B = \{2\}$, $C = \{1,3\}$.

$A \cap B = \phi$; $B \cap C = \phi$.

$A \cap C = \{1\} \neq \phi$ so Not Transitive.

19) → $x \preceq a_i$

$a_i \preceq y$