NUMERICAL Methods :-

→ Soln of Linear Systems
- LU Decomposition method.
- Gauss Elimination ,,
- Gauss Seidel ,,

→ Roots of Eqns $f(x)=0$
- Bisection method.
- Regula-falsi ,,
- Secant ,,
- Newton-Raphson ,,

→ Numerical Integration (~~Quadrature~~) (Quadrature) : $I = \int f(x)dx$
- Trapezoidal Rule.
- Simpsons Rule.

⇒ methods of solving
- Analytical
- Numerical.
  - Exact → LU Dec, → Gauss Elimin
    - → Round off error
  - Approximation (Trial & Error) (Iteration) methods.
    - → Round off error + Truncation Error

-ve : Over Estimate

+ve : under Estimate

Error = Exact − Approximate (Ex − Ap)

$$\text{Absolute Error} = |Ex - Ap|$$

$$\text{Relative Error} = \left|\frac{Ex - Ap}{Ex}\right|$$

$$\%\ \text{Relative Error} = \left|\frac{Ex - Ap}{Ex}\right| \times 100.$$

1) → LU Decomposition :

eg: $3x + 4y - 2z = 4$.
$x - y + 3z = -6$.
$4x - 2y - 7z = 5$.

$$\begin{bmatrix} 3 & 4 & -2 \\ 1 & -1 & 3 \\ 4 & -2 & -7 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 4 \\ -6 \\ 5 \end{bmatrix}.$$

$$A\hat{x} = B$$
$$\boxed{L}\boxed{U\hat{x}} = B.$$
$$\text{let } U\hat{x} = \hat{Z} \quad \text{--- (I)}$$
$$L\hat{Z} = B. \quad \text{--- (II)}.$$

A is LU decomposable iff all principle minors are non-zero ie $\neq 0$.

Principle minors whose diagonal is matching with principle diagonal of matrix, ie. No principle minor should be singular

Principle minors

$$\begin{bmatrix} \boxed{3} & 4 & -2 \\ 1 & -1 & 3 \\ 4 & -2 & -7 \end{bmatrix}$$

$|3| = 3 \neq 0$

$\begin{vmatrix} 3 & 4 \\ 1 & -1 \end{vmatrix} = -3 - 4 = -7 \neq 0$

$\begin{vmatrix} 3 & 4 & -2 \\ 1 & -1 & 3 \\ 4 & -2 & 7 \end{vmatrix} = 91$. So A is LU decomposable.

$A = LU$

$$\begin{bmatrix} 3 & 4 & -2 \\ 1 & -1 & 3 \\ 4 & -2 & -7 \end{bmatrix} = \begin{bmatrix} l_{11} & 0 & 0 \\ l_{21} & l_{22} & 0 \\ l_{31} & l_{32} & l_{33} \end{bmatrix} \begin{bmatrix} u_{11} & u_{12} & u_{13} \\ 0 & u_{22} & u_{23} \\ 0 & 0 & u_{33} \end{bmatrix}$$

$V = 6 + 6 = 12$ unknowns.
& we have $3 \times 3 = 9$ eqns.
So, we dont have unique soln.
So, we will apply either Do-little's method
or CROUT's method.

Dolittle's method $\Rightarrow$ make $l_{11} = l_{22} = l_{33} = 1$.
Crout's method $\Rightarrow$ $u_{11} = u_{22} = u_{33} = 1$.

So, Now we have 9 unknowns (v) & 9 eqns.
So we can easily solve,

$\Rightarrow$ Do little's method,

$$\begin{bmatrix} \boxed{3} & \boxed{4} & -\boxed{2} \\ \boxed{1} & -\boxed{1} & \boxed{3} \\ \boxed{4} & -\boxed{2} & \boxed{7} \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ l_{21} & 1 & 0 \\ l_{31} & l_{32} & 1 \end{bmatrix} \begin{bmatrix} u_{11} & u_{12} & u_{13} \\ 0 & u_{22} & u_{23} \\ 0 & 0 & u_{33} \end{bmatrix}.$$

$u_{11} = 3$
$u_{12} = 4$
$u_{13} = -2$

$1 = l_{21} * u_{11}$; $l_{21} = 1/3$.

$-1 = l_{21} \cdot u_{12} + u_{22} \Rightarrow u_{22} = -1 - \frac{1}{3} * 4 = -1 - \frac{4}{3}$
$= -\frac{7}{3}$.

$3 = \cancel{l_{31} \cdot u_{13} + l_{32} \cdot u_{23} +} u_{33}$,
$l_{21} \cdot u_{13} + u_{23}$ $\therefore u_{23} = 3 - \frac{1}{3} \cdot (-2)$
$= 3 + \frac{2}{3} = \frac{11}{3}$.

Similarly $\Rightarrow l_{31} \cdot u_{11} = 4$
$l_{31} = 4/3$
$\Rightarrow l_{31} \cdot u_{12} + l_{32} \cdot u_{22} = -2 \Rightarrow l_{32} = 22/7$.

$$A = \begin{bmatrix} 1 & 0 & 0 \\ 1/3 & 1 & 0 \\ 4/3 & 22/7 & 1 \end{bmatrix} \begin{bmatrix} 3 & 4 & -2 \\ 0 & -7/3 & 11/3 \\ 0 & 0 & - \end{bmatrix}.$$

U.

$\therefore$ Solve $L\hat{Z} = B$ to get $\begin{bmatrix} z_1 \\ z_2 \\ z_3 \end{bmatrix}$.
Then to solve $UX = \hat{Z}$ to get x, y, z.

If a matrix is NOT LU decomposable.
∴ exchange rows (to make non-zero Principal minor).

If all minors are zero, then we can't adjust
∴ ie. No solution exists.

⇒ Gauss Elimination

$$\begin{bmatrix} 3 & 4 & -2 \\ +1 & -1 & 3 \\ 4 & -2 & -7 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 9 \\ -6 \\ 5 \end{bmatrix}$$

$A \quad \hat{X} \quad B.$

$$A\hat{X} = B \underset{\text{forward Elimination}}{\Longrightarrow} U\hat{X} = B' \underset{\text{Back substitution}}{\Longrightarrow} \text{Solve to get } \hat{X}$$

Gauss Elimination with partial pivot.

⇒ Augmented Matrix

$$= \left[\begin{array}{ccc|c} \boxed{3} & 4 & -2 & 9 \\ 1 & -1 & 3 & -6 \\ 4 & -2 & -7 & 5 \end{array}\right]$$

Largest in this magnitude. column. So choose it as first pivot & exchange $R_1 \leftrightarrow R_3$.

$\xrightarrow{R(1,3)}$ $\left[\begin{array}{ccc|c} 4 & -2 & -7 & 5 \\ 1 & -1 & 3 & -6 \\ 3 & 4 & -2 & 4 \end{array}\right] \xrightarrow[R_3 - \frac{3}{4}R_1]{R_2 - \frac{1}{4}R_1} \left[\begin{array}{ccc|c} 4 & -2 & -7 & 5 \\ 0 & -1/2 & 19/4 & 29/4 \\ 0 & 11/2 & 15/4 & 1/4 \end{array}\right]$

→ Pivot. (If it was -4, then also pivot = (-4).)

It is bigger than $|1/2|$ so take it as pivot & $R_2 \leftrightarrow R_3$.

$\xrightarrow{R(2,3)}$

→ Pivot #2.

$$\left[\begin{array}{ccc|c} 4 & -2 & -7 & 5 \\ 0 & 11/2 & 15/4 & 1/4 \\ 0 & -1/2 & 19/4 & 29/4 \end{array}\right]$$

$$\Big\downarrow R_3 + \tfrac{1}{11}R_2$$

$$\left[\begin{array}{ccc|c} 4 & -2 & -7 & 5 \\ 0 & 11/2 & 15/4 & 1/4 \\ 0 & 0 & \_ & \_ \end{array}\right]$$

Pivot1 = 4
Pivot2 = 11/2.

a matrix of order n has (n-1) pivots.

Pivoting is done ∵ pivot always comes in denominator & if the number is very small ∴ error increases.

eg: if we chose small element as pivot.

$R_3 - \frac{1000}{(R_2)}$ if it is 1000 times less than R_3, so multiply R_2 by 1000 so little error may get increase.

So ~~we use~~ To avoid this problem. we use larg ~~small~~ element as pivot. so if we devide it with something. Then error will even decrease but NOT increase

So we use bigger element as pivot to reduce round-off errors.

Bcoz it always remains in denominator. So it should also NOT be 0. Bcoz it may make ∞. So we always

⇒ Gauss-Seidel Method: $\boxed{\text{Cramer's Rule} = O(n^4)}$

Gauss-Seidal method is faster than Gauss elimination method when coefficient matrix is SPARSE.
Such as in tri-diagonal matrix.

> Gauss Seidal has 2 method
→ Manual method ≡ for solving till 2 or 3 iterations. It is preferable.
→ Matrix method ≡ To check whether
→ convergence ?
or Divergence ?
→ Rate of convergence?

$\boxed{\begin{array}{l} +ve \equiv \text{convergence} \\ -ve \equiv \text{Divergence} \end{array}}$

eg:

$$3x + 4y - 2z = 4 \quad \therefore x = (4 - 4y + 2z)/3$$
$$x - y + 3z = -6 \quad \therefore y = 6 + x + 3z$$
$$4x - 2y - 7z = 5 \quad \therefore z = (-5 + 4x - 2y)/7$$

Iteration NO.	x	y	z
0	0	0	0
1	4/3	22/3	49/3
2	___	___	___

→ if not given use 0, 0, 0 by default.

(X − Y) ≡ Relative error
i.e. at each step
Relative error = latest − before

$$x = (4 - 0 + 0)/3 = 4/3$$
$$y = 6 + 4/3 = 22/3$$
$$z = (-5 + 4 \times 4/3 - 2 \times 22/3)/7 = -43/4$$

⇒ To identify convergence/ divergence.

$$3x + y = 4$$
$$-x + 4y = 5$$

$$\boxed{[L+D]\,X^{i+1} = -UX^{i} + B}$$

$$\boxed{X^{i+1} = [L+D]^{-1}(-U))X^{i} + (L+D)^{-1}B'}$$

$$A = L + D + U, \quad A = \begin{bmatrix} 3 & 4 & -2 \\ 1 & -1 & 3 \\ 4 & -2 & -7 \end{bmatrix}$$

$$L+D = \begin{bmatrix} 3 & 0 & 0 \\ 1 & -1 & 0 \\ 4 & -2 & -7 \end{bmatrix}$$

$$U = \begin{bmatrix} 0 & 4 & -2 \\ 0 & 0 & 3 \\ 0 & 0 & 0 \end{bmatrix}$$

$$X^{i+1} = \underbrace{\boxed{[L+D]^{-1}(-U)}}_{H}\,X^{i} + \underbrace{\boxed{(L+D)^{-1}B}}_{B'}$$

$$\underline{X^{i+1} = HX^{i} + B'}$$

Spectral Radial is the ~~max. of diagonal~~ mod of largest eigen value of H

$$Hx = \lambda x$$

$$0 < \rho(H) < 1$$

or $\rho(H) \geq 1$

$$ex = \begin{bmatrix} 3 & 1 \\ -1 & 4 \end{bmatrix}$$

$$H = \begin{bmatrix} 3 & 0 \\ -1 & 4 \end{bmatrix}^{-1} \begin{bmatrix} 0 & -1 \\ 0 & 0 \end{bmatrix}$$

$$= \frac{1}{12}\begin{bmatrix} 4 & 0 \\ 1 & 3 \end{bmatrix} \begin{bmatrix} 0 & -1 \\ 0 & 0 \end{bmatrix} = \frac{1}{12}\begin{bmatrix} 0 & -4 \\ 0 & -1 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & -\frac{1}{3} \\ 0 & -\frac{1}{12} \end{bmatrix}.$$

also may be 0.

$Eig(H) = \{0, -\frac{1}{2}\}$

$\rho(H) = \frac{1}{2} < 1 \Rightarrow$ So, it is convergent.

Rate of convergent $= \quad -\log_e \rho(H)$
$\nu(H).$ $\quad = -\ln \rho(H)$
$\quad = -\log_e(\frac{1}{2})$

$\Rightarrow$ Any Diagonally Dominant System is always convergence

$\Downarrow$

Every Diagonal element is greater in magnitude than every off-diagonal element

eg: $\begin{bmatrix} 0.1 & 3 \\ -1 & 0.1 \end{bmatrix} \Rightarrow$ Divergence,

eg:- $\begin{bmatrix} 3 & 5 \\ 1 & 4 \end{bmatrix} \Rightarrow$ Divergence

$= \begin{bmatrix} 4 & 3 \\ 1 & 2 \end{bmatrix} \Rightarrow$ can't say as one diagonal $4 > 3$ & other is $2 < 3$
So can't say

$$\boxed{\begin{array}{ll} \rho(H) < 1 & \text{Then } \nu(H) > 0 \\ \rho(H) \geq 1 & \text{Then } \nu(H) \leq 0 \end{array}}$$

⇒ Gauss Seidal & Jacobi ≡

Gauss Seidal → To derive new values, (new roots) we use latest update value.

Jacobi → To derive new root we fully use old root.

⇒ So Gauss seidal is faster than Jacobi ≡

$$\boxed{\gamma(\text{Gauss-Seidal}) = 2\gamma(\text{Jacobi})}$$

i.e. Rate of converge in G.S. is 2 times faster than Jacobi ⇒

Bcoz

$$\boxed{\rho(\text{Gauss-seidal}) = \rho^2(\text{Jacobi})}$$

So spectral Radius in G.S. is square of spectral Radius for Jacobi ≡

2) ⇒ Roots of Eqn ≡ [$f(x)=0$].

	Guess Value Required	Order of convergence	Guarantee	Iteration formula	
BISECTION	2	1	✓	$x_2 = \frac{x_0+x_1}{2}$	If $f_0 f_2 < 0$ ⇒ $x_1 \Leftarrow x_2$ else $x_0 \Leftarrow x_2$
REGULA-FALSI	2	1	✓	$x_2 = \frac{f_1 x_0 - f_0 x_1}{f_1 - f_0}$	,,
SECANT	2	1.62	✗	,,	$x_0 \Leftarrow x_1$, $x_1 \Leftarrow x_2$,
NEWTON-RAMPHSON.	1.	2.	✗	$\boxed{x_1 = x_0 - \left(\frac{f(x_0)}{f'(x_0)}\right)}$	$x_0 \Leftarrow x_1$

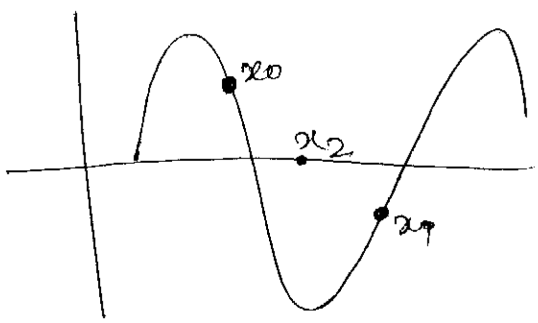
Intermediate value Theorem ≡

If $f(a) \cdot f(b) < 0$

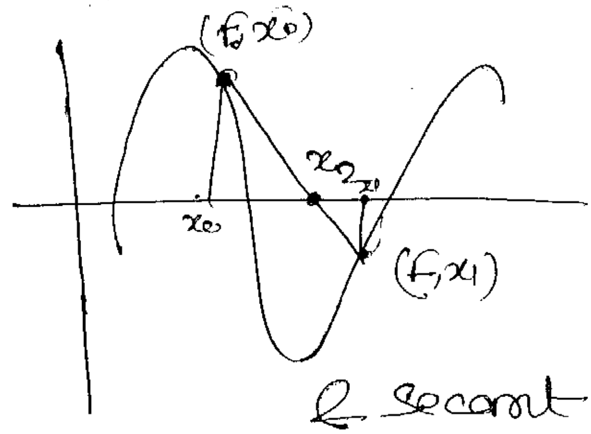
$\exists$ a root $x \in \{a, b\}$.

Newton Raphson doesn't work when first derivative vanishes i.e. $f'(x)=0$

Bisection

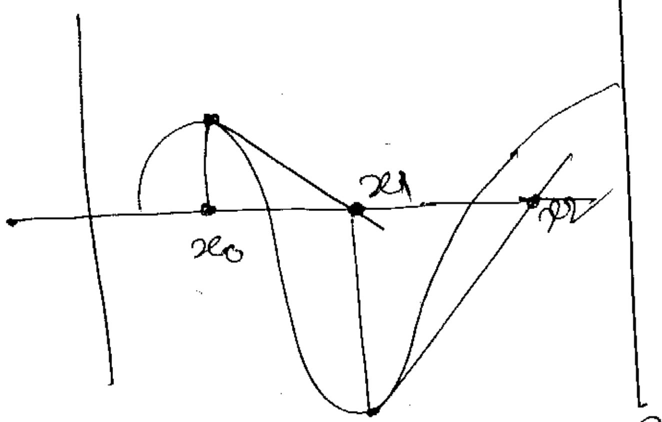


Regula Falsi



& Secant

Newton Raphson's



~~Applications of Newton Raphson~~

⇒ inverse of b, is the root of the eqn $f(x)=\frac{1}{x}-b=0$

Iteration eqn → $\boxed{X_{n+1}=X_n(2-bX_n)}$

⇒ inverse square root b, is the root of the eqn $f(x)=\frac{1}{x^2}-b$

Iteration eqn. $X_{n+1}=\frac{1}{2}X_n(3-bX_n^2)$

⇒ pth root of a given Number N, is root of the eqn

$f(x)=x^p-N=0$

Iteration eqn :- $X_{n+1}=\frac{(p-1)X_n^p+N}{pX_n^{p-1}}$

BISECTION

eg ≡ $x^2 - 2 = 0$.

$x_0 = 1$, $x_1 = 2$.

Iteration	x_0	x_1	x_2	f_0	f_1	f_2
1	1	2	1.5	-1	+2	0.25
2	1	1.5	1.25	-1	0.25	
	1.25	1.5	1.375	-0.1	0.25	

Guess value of Root is ~~done~~ chosen by seeing behaviour of function i.e. when fn changes its value from positive to Negative.

⇒ Regula Falsi

$$x_2 = \frac{f_1 x_0 - f_0 x_1}{f_1 - f_0}$$

Iteration	x_0	x_1	x_2	f_0	f_1	f_2
1	1	2	1.33	-1	+2	
2						
3						

Secant

1	1	2	1.33	-1	+2	
2						
3						

⇒ Newton Raphsone

$x \in (1,2)$

Start $x = 1.5$ (if Not Given).

Let $x = 2$ is Given

$x_1 = x_0 - \frac{f(x_0)}{f'(x_0)}$

Iteration	x_0	x_1
1	2	1.5
2	1.5	1.418

Setting up formula for Newton Ramphson.

$$\Rightarrow x_1 = x_0 - \left(\frac{x_0^2 - 2}{2x_0}\right)$$

$$x_1 = \left(\frac{x_0^2 + 2}{2x_0}\right)$$

$$x_{i+1} = \left(\frac{x_i^2 + 2}{2x_i}\right)$$

⇒ Newton Ramphson - after every iteration accuracy of digits is doubled.

i.e. after 1 iteration 1 digit
2 iteration 2 digit
3 iteration 4 digit
⋮
n iteration $2^{(n-1)}$ digit.

If $\boxed{\mathcal{E}_{i+1} \propto \mathcal{E}_i^{\text{order}}}$

So, for Bisection $\rightarrow \mathcal{E}_{i+1} = K\mathcal{E}_i$

for Regula-Falsi / Secant $\rightarrow \mathcal{E}_{i+1} = K\mathcal{E}_i^{1.62}$

for Newton-Ramphson $\rightarrow \mathcal{E}_{i+1} = K\mathcal{E}_i^2$

⇒ Error formula for Bisection method.

If $x_0 = a$; $x_1 = b$

$$E_{n\,max} = \frac{b-a}{2^n} \quad \left(\text{max. error in } n\text{th iteration}\right).$$

$$\therefore E_0 = b-a$$
$$E_1 = \frac{b-a}{2}$$
$$E_2 = \frac{b-a}{2^2}$$

Ques – $x \in (1,3)$

$\therefore$ max error after 10 iterations

$$\frac{b-a}{2^n} = \frac{3-1}{2^{10}} = \frac{2}{2^{10}} = \frac{1}{2^9} = \frac{1}{512}.$$

Ques – How many ~~Homy~~ iterations for Bisection $x \in (1,3)$ must be performed ~~to~~ error be within $\pm 10^{-5}$

$$10^{-5} = \frac{3-1}{2^n}$$
$$= 2^n = 2 * 10^5$$
$$n = \lceil \lg_2 2 * 10^5 \rceil$$
$$\approx \lceil 27.628 \rceil$$
$$\approx 28$$

Ques – How may iterations required to perform Bisection (a,b) to be error within $\pm E$

$$\Rightarrow E = \frac{b-a}{2^n}$$
$$\Rightarrow 2^n = \frac{b-a}{E}. \Rightarrow \boxed{n = \left\lceil \lg_2 \left(\frac{b-a}{E}\right) \right\rceil}$$

⇒ Numerical Integration   19/11/2012

Simple Trapezoidal Rule = 2 points ; Complex T.R = Any # of points
Simple Simpson's Rule = 3 points; Complex S.R = ~~Add~~ # of points
(to set even # of interval)

$(n_i = n_p - 1)$
(# of intervals = # of points - 1).

⇒ Integeral value

S.T.R ≡ $I = \frac{h}{2}(f_0 + f_1)$

S.S.R. ≡ $I = \frac{h}{3}(f_0 + 4f_1 + f_2)$

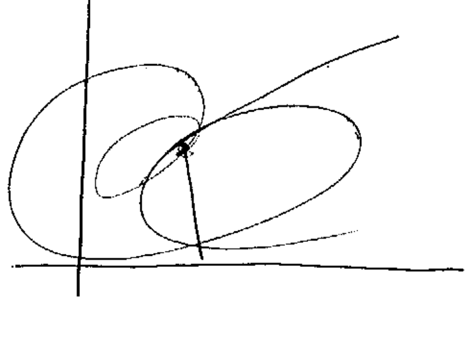
C.T.R ≡ $I = \frac{h}{2}(f_0 + 2f_1 + 2f_2 + \dots + f_5) = \frac{h}{2}(f_0+f_1) + \frac{h}{2}(f_1+f_2) + h/2(f_2+f_3) \dots$

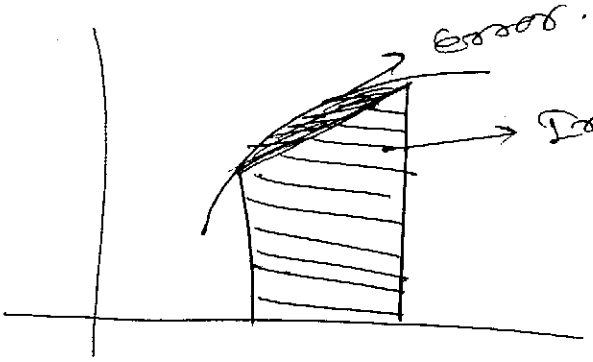
C.S.R ≡ $I = \frac{h}{3}(f_0 + 4f_1 + 2f_2 + 4f_3 + 2f_4 + 4f_5 + f_6)$

$= \frac{h}{3}(f_0 + 4f_1 + f_2) + \frac{h}{3}(f_2 + 4f_3 + f_4) \cdot$

Step Size (h) $= \frac{b-a}{n_i}$; n_i = no. of intervals,
where $(n_i = n_p - 1)$
∴ n_p = no. of points.

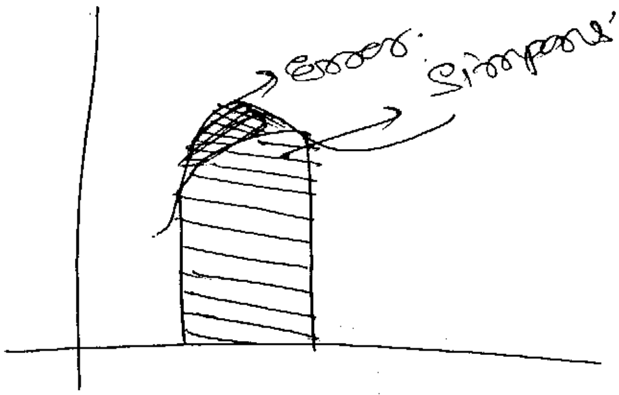
⇒ Error in answer (Truncation Error)





in Trapezoidal Rule, integration is performed by assuming straight line

while in simpson, integration is performed by assuming quadratic area.

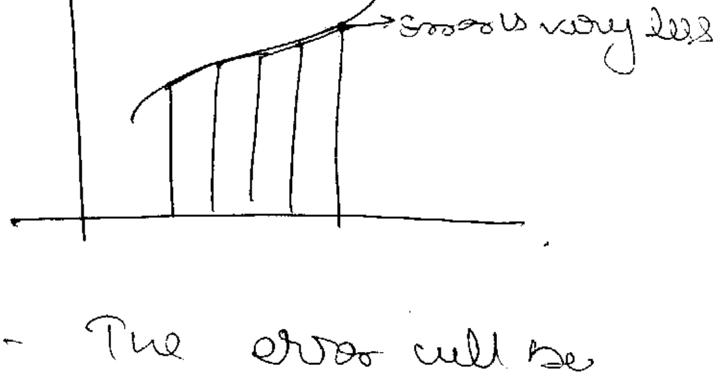


$C.T.R = O(h^2)$

$C.S.R = O(h^4)$ - stepsize reduced dynamically

→ it gives min error.

while in compound integration, error is very much reduced as any curve is divided in small-small intervals is very much close to linear system



~~C.T.R~~ ⇒ If h is 1/2 - The error will be reduced by 1/4 in C.T.R

1/16 in C.S.R

Truncation Error (Bound) (max. error) $= -\frac{h^3}{12} n_i f''(\xi_i)$ T.R.

$= -\frac{h^{2}}{12}\, (b-a)\, f''(\xi_i)$

$= -\frac{h^5}{90}\left(\frac{n_i}{2}\right) f^{iv}(\xi_i)$. S.R.

$a \le \xi_i \le b$

$\Rightarrow$ $\boxed{\text{Error} = \text{Exact} - \text{Approx.}}$

Error $\rightarrow$ $-ve$ | – over estimate

$+ve$ | – under estimate

$$\boxed{\begin{aligned} TE(\text{Bound}) &= +\frac{h^3}{12} n_i \cdot \max\left|f''(\xi_i)\right|. \\ &= +\frac{h^5}{90}\left(\frac{n_i}{2}\right) \max\left|f^{iv}(\xi_i)\right|. \end{aligned}}$$

Abs Error $= |Ex - Ap|$.

relative Error $= \left|\frac{Ex - Ap}{Ex}\right|$;

% relative Error $= \left|\frac{Ex - Ap}{Ex}\right| * 100$.

$\Rightarrow$ 5 questions

$\rightarrow$ Area under curve $I = \int_a^b f(x)\,dx$

$\rightarrow$ Error, Abs Error, rel Error, % rel Error

$\rightarrow$ Error Bound.

$\rightarrow$ Theory

→ Trapezoidal Rule produces exact answers for fn upto polynomial of degree 0 or 1.

→ Simpson's Rule produces exact answers for fn upto polynomial of degree 3.

⇒ T.R upto linear fn because $f''(x)$ for x = linear $f''(x) = 0$

S.R. upto cubic fn because $f^{iv}(x)$ for x = cubic $f^{iv}(x) = 0$

⇒ Answer ①

$$\int_1^2 \frac{1}{1+x}\,dx$$ using simple Trapezoidal.

$n = 2$ points

$n_i = 1$.

$h = \frac{b-a}{1} = \frac{2-1}{1} = 1.$

S.T.R,

x	$f(x) = 1/1+x$
1	$1/2$ ×1
2	$1/3$ ×1

$$I = \frac{h}{2}(f_0 + f_1)$$

$$= \frac{1}{2}\left(\frac{1}{2} + \frac{1}{3}\right) = \frac{5}{12}.$$

S.S.R ≡

$n_{pt} = 3$
$n_i = 2.$
$h = \frac{b-a}{2} = \frac{2-1}{2} = 0.5$

x	$f(x)$	
1	1/2	× 1
1.5	1/2.5	× 4
2.0	1/3	× 1

$$I = \frac{h}{3}(f_0 + 4f_1 + f_2)$$

$$= \frac{0.5}{3}\left(\frac{1}{2} + \frac{40}{25} + \frac{1}{3}\right)$$

⇒ C.S.R. ⇒ compound 5 points $n_i = 4$

$h = \frac{2-1}{4} = 0.25$

x	1	1.25	1.5	1.75	2.0
f(x) C.T.R.	1/2	1/2.25	1/2.5	1/2.75	1/3.
f(x) C.S.R.	"	"	"	"	"

$$C.S.R = \frac{h}{3}\left(\frac{1}{2} + 4\left(\frac{1}{2.25}\right) + 2\left(\frac{1}{2.5}\right) + 4\left(\frac{1}{2.75}\right) + \left(\frac{1}{3}\right)\right)$$

$$C.T.R = \frac{h}{2}\left(\frac{1}{2} + 2\left(\frac{1}{2.25}\right) + 2\left(\frac{1}{2.5}\right) + 2\left(\frac{1}{2.75}\right) + \left(\frac{1}{3}\right)\right)$$

② Error ⇒ Exact = $\ln(1+x)$

$\ln 3 - \ln 2$

⇒ Error = Exact − Approx.

③ ⇒ T.E. bound

$$T.E_{bound} = \frac{h^2}{12}\left(\frac{b-a}{\cancel{h}}\right) \max |f''(x_p)|$$

$$= \frac{(0.25)^2}{12}(2-1) * 1/4.$$

$f(x) = 1/_{1+x}$

$f'(x) = -1/_{(1+x)^2}$

$$f''(x) = \frac{2}{(1+x)^3} = \frac{2}{(1+1)^3} = 2/8 = 1/4.$$

max will be at 1 bcoz $1/_{1+x}$ is a decreasing function

If T.E. bound is given, calculate $n_i = ?$

let T.E. is 10^{-5}

$$10^{-5} = \frac{h^2}{12}(2-1) * 1/4$$

$$\sqrt{h^2} = \sqrt{48 * 10^{-5}}$$

Now using h, n_i can be calculated.

also ... be ... pivot

$\Rightarrow$ Same for S.R.

$$10^{-5} = \frac{h^4}{90} \cdot \frac{b-a}{2h} \max |f^{iv}(x)|.$$

$$10^{-5} = \frac{h^4}{180} (2-1) * \left(\frac{3}{4}\right)$$

$\Rightarrow$ ques (4) -

for $\int_1^2 \frac{1}{1+x} dx$ - 5 points.

$f''(x) = \frac{2}{(1+x)^3} \Rightarrow$ U. Estimate

= sign positive. So U.

for $= \int_2^1 \log x \, dx$

$f''(x) = -\frac{1}{x^2}$

= sign negative

So, over estimate