# 15093 project - portfolio optimization formulation

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December 2022

## 1 Portfolio Optimization

### 1.1 Variables

#### 1.1.1 Decision Variables

1. $x_i$: weight of stock $i$ in the portfolio at time $t$
2. $z_i$: binary variable indicating whether stock $i$ is in the portfolio at time $t$
3. $b_i$: dollar value of stock $i$ bought at time $t$

#### 1.1.2 Other Variables

1. $r_t^i$: return of stock $i$ at time $t$
2. $inventory_t^i$: vector whose indices contain number of shares bought for stock $i$ at time $t$
3. $V_t^i$: trading volume of stock $i$ at time $t$
4. $sector_{ij}$: binary variable indicating stock $i$ is in sector $j$
5. $price_t^i$: price of stock $i$ at time $t$
6. $budget_t$: budget at time $t$

#### 1.1.3 Indices

Generally:

- $i$: stock index, $i = 1, 2, ..., N$
- $j$: sector index, $j = 1, 2, ..., M$
- $t$: time index, $t = 0, 1, 2, ..., T$

## 1.2 Formulation

$$inventory^i_{t-1} = \begin{cases} 0, & \text{if } t = 0 \\ inventory^i_t, & \text{otherwise} \end{cases} \tag{1}$$

### 1.2.1 At every time step $t$, perform the following optimization

$$\max \quad \sum_{i=1}^{N} r_t^i x_i$$

$$\text{s.t.} \quad \text{x}\Sigma x \leq 0.5$$

$$\text{b}_i \times price_t^i \leq budget_t$$

$$\sum_{i=1}^{N} x_i = 1$$

$$\sum_{i=1}^{N} (z_i \times sector_{ij}) \leq 4, \quad \forall j \in \{1, ..., M\}$$

$$\sum_{i=1}^{N} z_i \geq K$$

$$\text{z}_i + z_j \leq 1, \quad \forall i, j \in \{1, ..., N\} : |corr(r_i, r_j)| > 0.3, i \neq j$$

$$\text{b}_i - inventory^i_{t-1} \leq 0.05 \times V_t^i, \quad \forall i \in \{1, ..., N\}$$

$$\text{inventory}^i_{t-1} - b_i \leq 0.05 \times V_t^i, \quad \forall i \in \{1, ..., N\}$$

$$\text{x}_i \geq 0, \quad \forall i \in \{1, ..., N\}$$

$$\text{b}_i \geq 0, \quad \forall i \in \{1, ..., N\}$$

$$\text{z}_i \in \{0, 1\}, \quad \forall i \in \{1, ..., N\}$$

$$\vdots$$

linking constraints

After each optimization update the following variables and plug them results into next iteration

- $inventory^i_t$ based on purchases at the beginning of month and sales at end of month
- $budget_t$ based on stock performance at end of month minus transaction costs of updating inventory (assume \$8.9 brokerage fee per transaction)