

Modelling explosive percolation under local rules: replicating paper results

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Shannon Moran



Degree product rule tempers explosive percolation in the absence of global information

Alexander J. Trevelyan, Georgios Tsekenis, and Eric I. Corwin

Materials Science Institute and Department of Physics, University of Oregon, Eugene, Oregon 97403, USA



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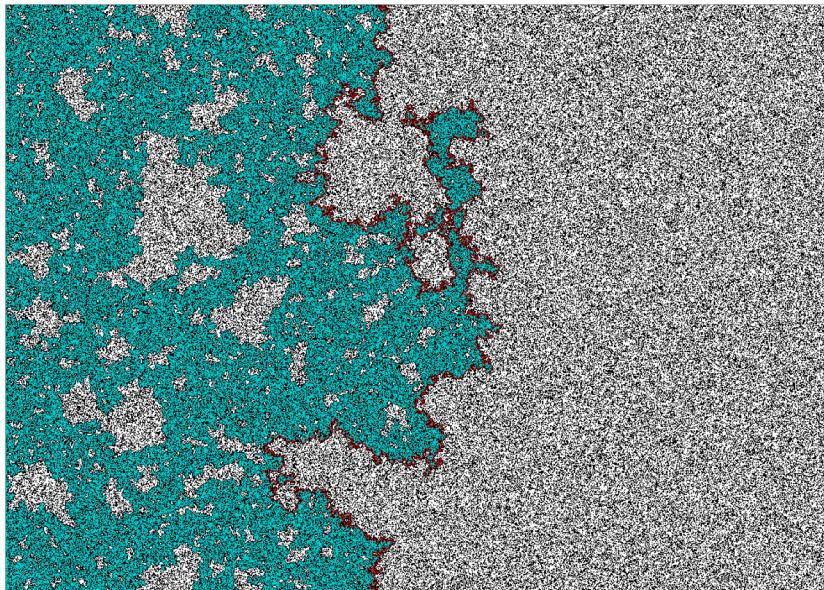
We introduce a guided network growth model, which we call the degree product rule process, that uses solely local information when adding new edges. For small numbers of candidate edges our process gives rise to a second-order phase transition, but becomes first order in the limit of global choice. We provide the set of critical exponents required to characterize the nature of this percolation transition. Such a process permits interventions which can delay the onset of percolation while tempering the explosiveness caused by cluster product rule processes.

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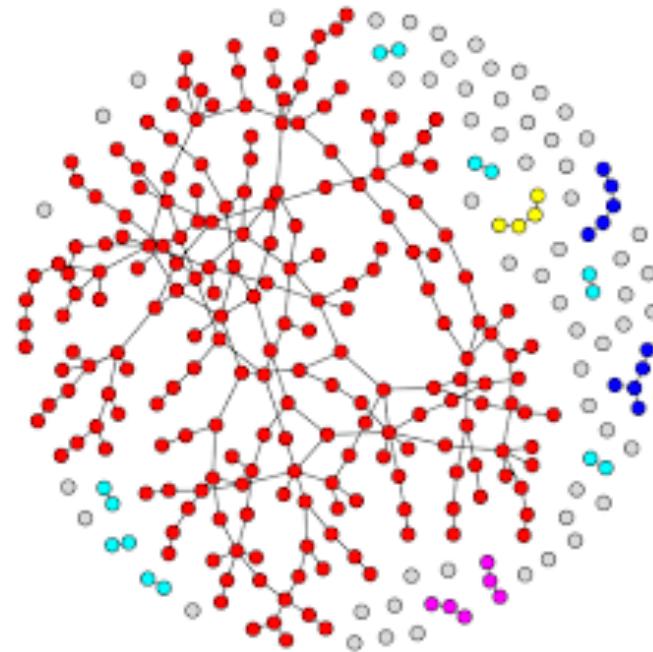
What are network growth models, how do they relate to percolation?

Site occupation under local rules



https://commons.wikimedia.org/wiki/File:Front_de_percolation.png

Edge occupation under local rules



<http://www.eng.ox.ac.uk/sen/files/course2016/lec1.pdf>

We can think of network percolation as occupied edges, rather than sites

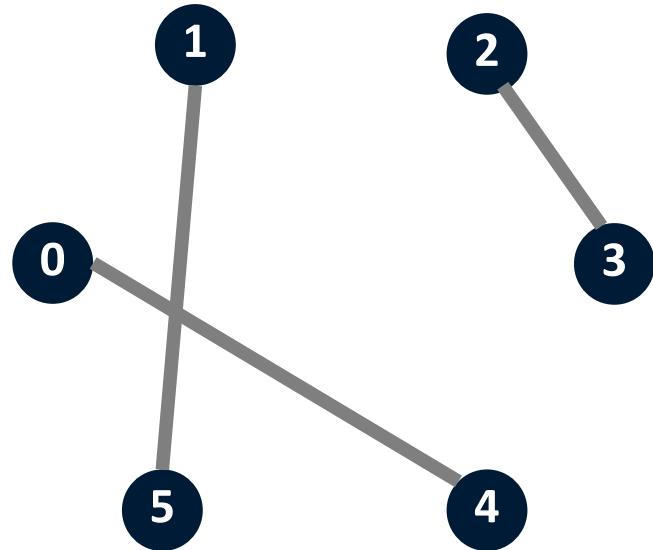
Building an Erdős-Rényi network

1. Initialize N nodes (unique order)
2. Pick two nodes at random: (tail, head) of edge
3. Add edge

Repeat 2&3 until t edges are added

$$\text{Edge density} = p = t/N$$

Proposed edge: (5,1)



3 clusters of 2 nodes each

Source: P. Erdős and A. Rényi, On Random Graphs, Publ. Math. 6, 290 (1959)



In this paper, the authors investigate three different network percolation methods

Erdős-Rényi	<i>Science</i> , 2009	Degree product rule (DPR)
<ol style="list-style-type: none">1. Initialize N nodes2. Pick two nodes at random3. Add edge <p>Repeat 2&3 until t edges are added to reach an edge density of p</p>	<ol style="list-style-type: none">1. Initialize N nodes2. Pick m sets of two nodes at random (“edge”)3. For each edge, calculate the total cluster size created by the edge4. Add edge that creates the smallest new cluster <p>Repeat 2&3 until t edges are added to reach an edge density of p</p>	<ol style="list-style-type: none">1. Initialize N nodes (unique order)2. Pick m sets of two nodes at random3. For each edge, calculate the “degree product” of the component nodes: $(d_1+1)(d_2+1)$4. Add edge that has the lowest “degree product” <p>Repeat 2&3 until t edges are added to reach an edge density of p</p>

Creates a “powder keg” for explosive percolation by suppressing cluster growth

Assert a second-order phase transition for small m and a first-order transition for global m



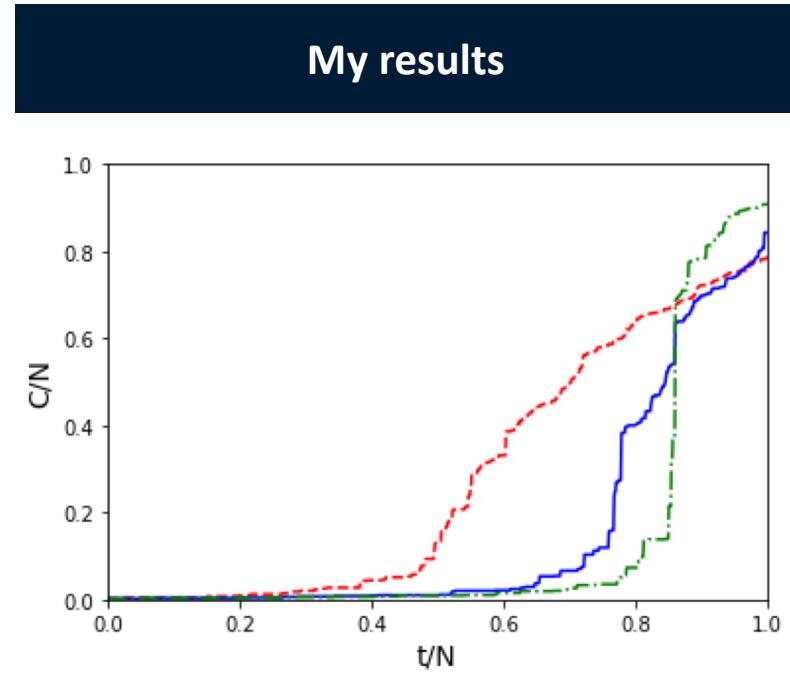
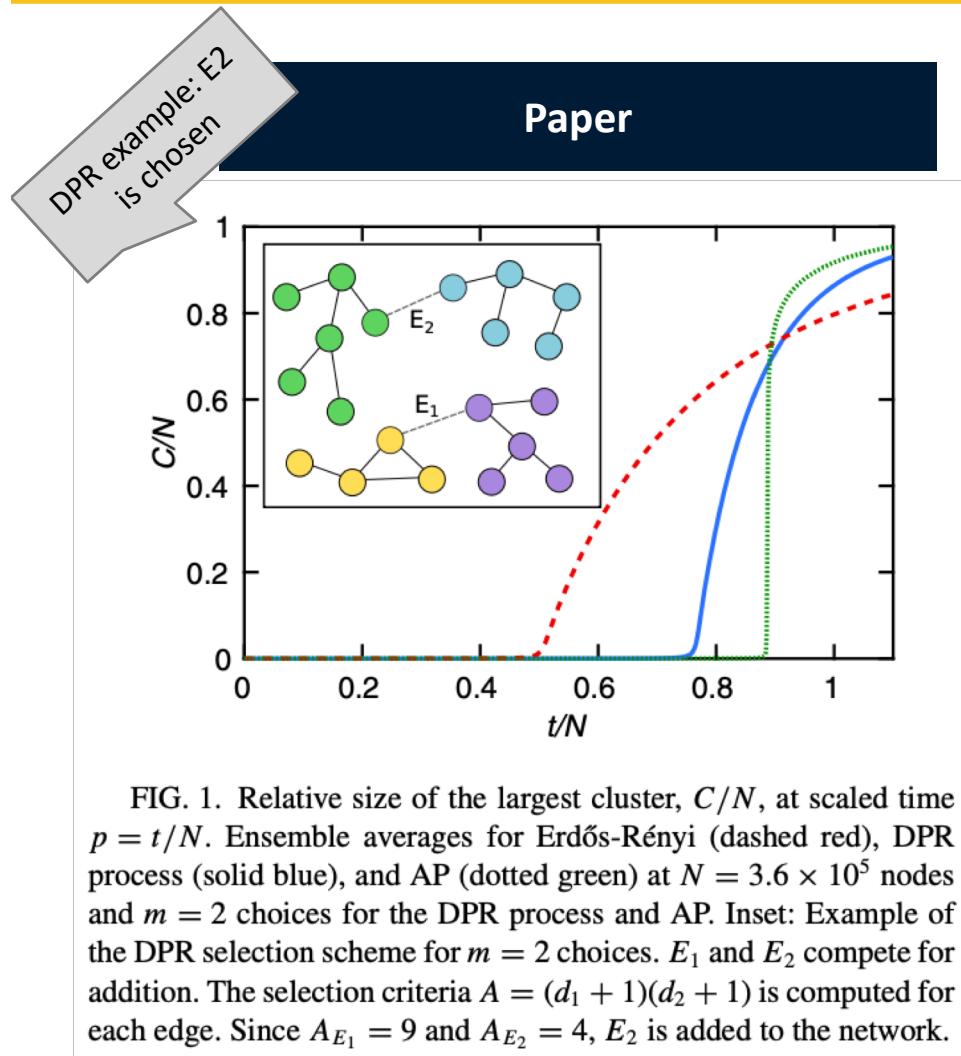
What is this paper looking to do? Why should we care?

- AP and DPR are two methods generally called “competitive edge addition” methods
- These methods can often lead to what is called explosive percolation “due to the abruptness with which the largest cluster grows from microscopic to system spanning”
- But why bother looking at DPR?
 - It relies only upon ***local node*** information— no need to calculate full cluster information for each edge addition

tion threshold. Second, the problem of classical percolation has long involved using a stable probability distribution to choose an edge at each timestep. Explosive percolation upended this notion by allowing the distribution to shift unpredictably depending on which edge is chosen, a characteristic potentially more in line with how certain types of real networks take shape



Figure 1: Validate that my model can replicate evolution of large cluster size over time



- Able to replicate the trends in cluster size evolution over edge density
- This is interesting: paper uses $N=3.6E5$ nodes, while I was only able to run $N=1E3$
- Critical point/nature of transition does not appear to be network-size-dependent

Figure 3: Validating that my model replicates the correct edge degree distribution

Paper

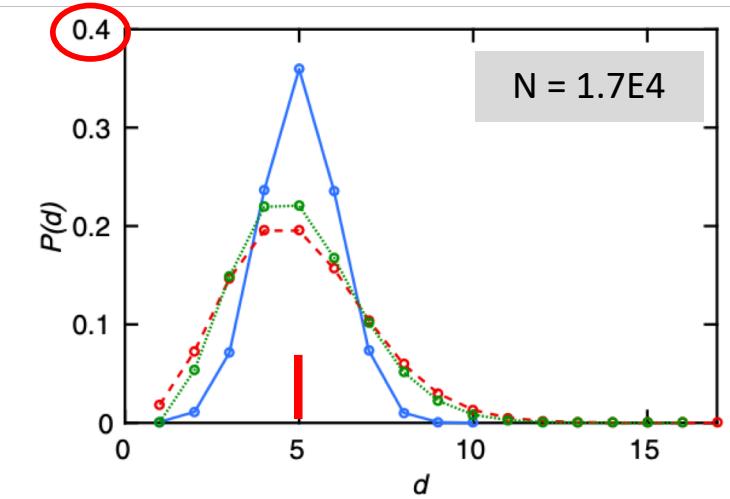
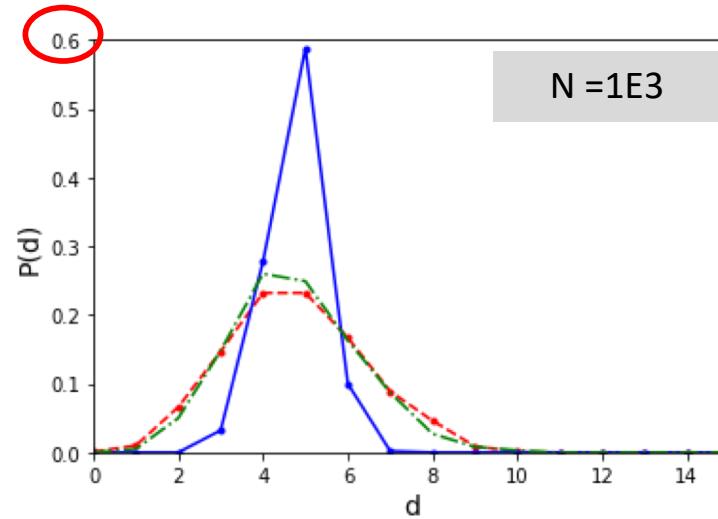


FIG. 3. The degree distributions at $p = 5$ for Erdős-Rényi (dashed red), degree product rule process (solid blue), and Achlioptas process (dotted green) at $N = 1.7 \times 10^4$ nodes and $m = 2$ choices for the DPR process and AP.

My results



- The micro-environment of my networks is the same as those in the paper-- good!
- However, my simulation sizes will start to present problems when I look to calculate critical exponents and study phase trans

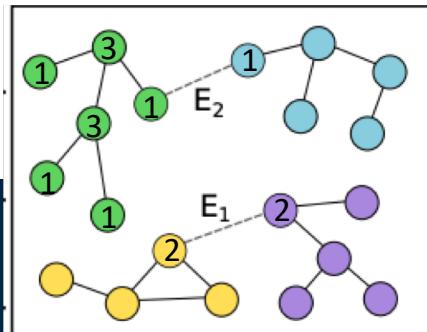
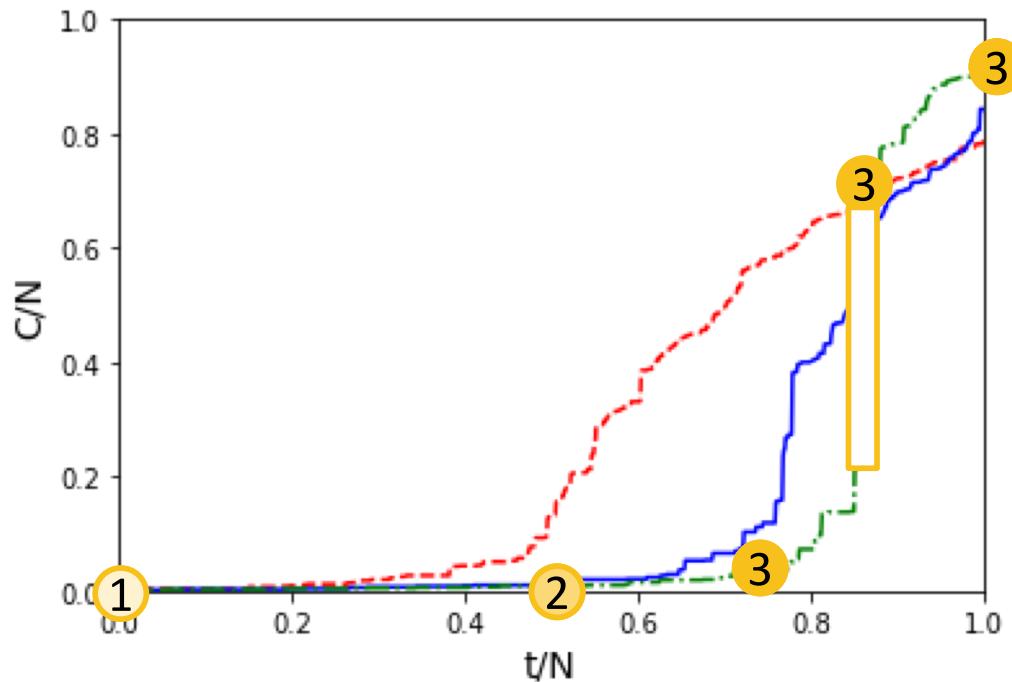


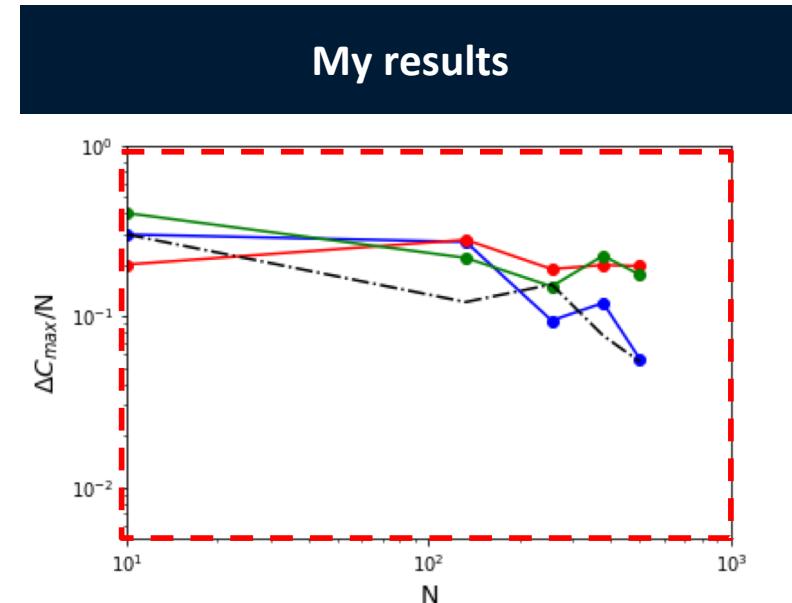
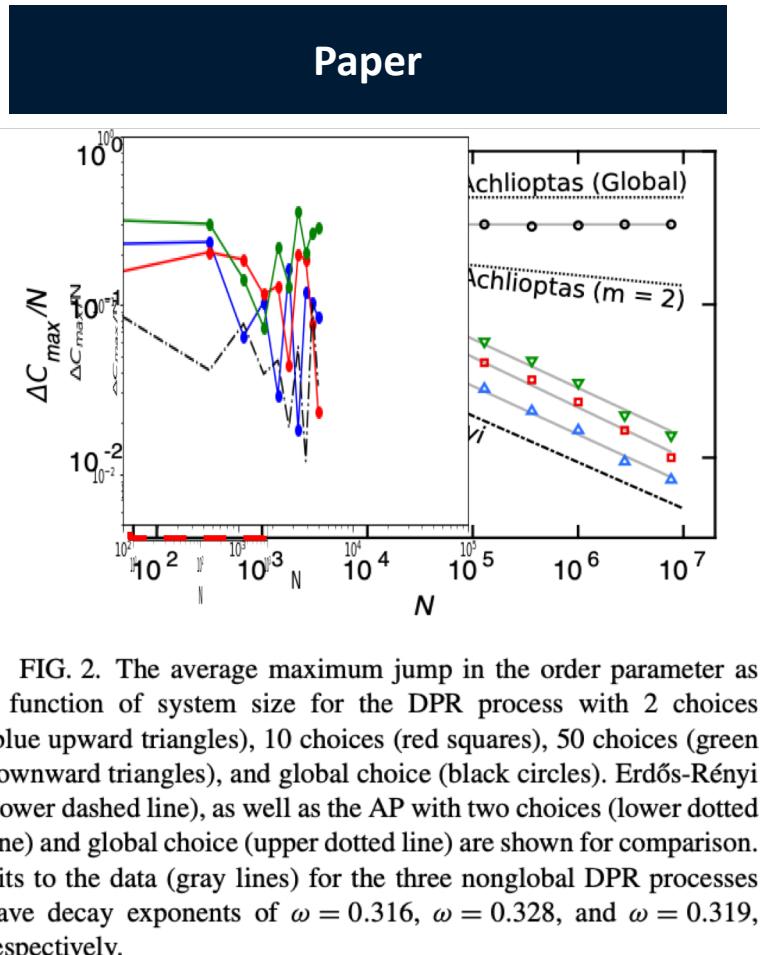
Figure 2: Troubles with system size begin when studying the “largest jump” order parameter (1/2)

Binary search to find “largest jump in C/N” order parameter for study



1. Divide x -range into two segments (“binary”)
2. Check in which range the jump in y is greatest
3. Re-do search in that region and repeat until x -range can no longer be divided

Figure 2: Troubles with system size begin when studying the “largest jump” order parameter (2/2)



- Characterizing the critical behavior will require accessing larger system sizes
- **Next steps:** Run systems longer on non-laptop systems
- **Also not yet added:** Global m search for network generation algorithms



DPR's critical behavior is independent of the number of choices at each edge addition step

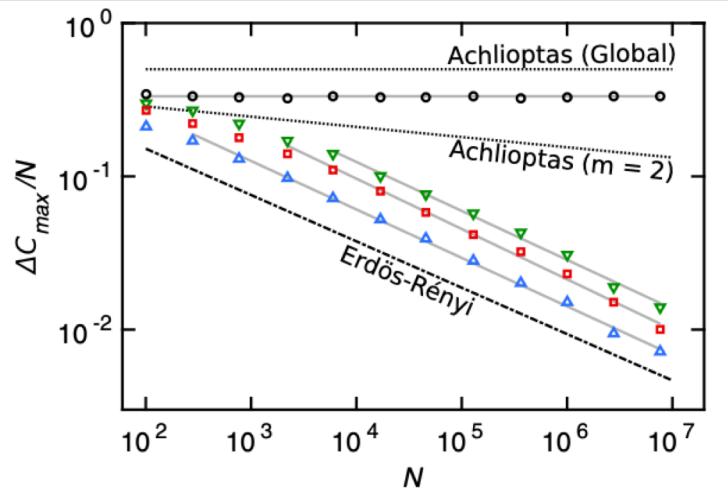


FIG. 2. The average maximum jump in the order parameter as a function of system size for the DPR process with 2 choices (blue upward triangles), 10 choices (red squares), 50 choices (green downward triangles), and global choice (black circles). Erdős-Rényi (lower dashed line), as well as the AP with two choices (lower dotted line) and global choice (upper dotted line) are shown for comparison. Fits to the data (gray lines) for the three nonglobal DPR processes have decay exponents of $\omega = 0.316$, $\omega = 0.328$, and $\omega = 0.319$, respectively.

- For the AP process, changing the number of edge choices (m) changes the decay exponent
- For DPR, the decay exponent is not a function of the number of choices
- **However**, global choice does lead to a phase transition (order parameter independent of N)



Critical behavior: Without larger systems, I cannot replicate the critical exponent calculations

Paper

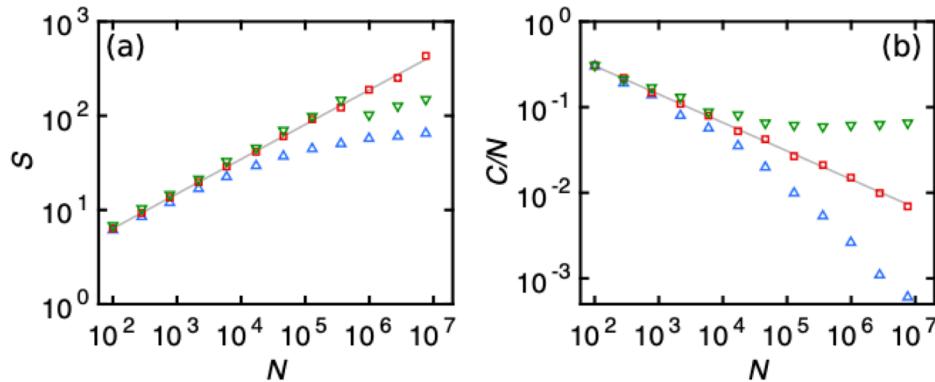
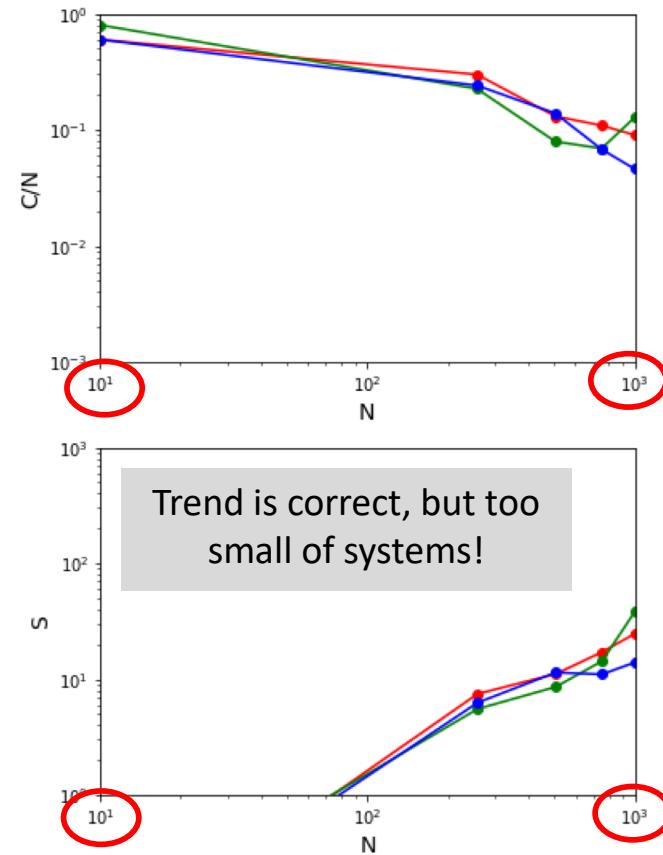


FIG. 4. Finite-size scaling for the critical exponents β/ν and γ/ν of the DPR process. (a) Mean cluster size S is plotted versus system size N . The fit at $p = p_c = 0.763$ (red squares) gives the value $\gamma/\nu = 0.37$. (b) Relative size of the largest cluster C/N is plotted versus system size N . The fit at $p = p_c = 0.763$ (red squares) gives the value $\beta/\nu = 0.33$. Breakdown of the power law scaling away from the critical point is shown in both (a) and (b) for $p = 0.75$ (blue upward triangles) and $p = 0.77$ (green downward triangles).

My results



Challenges and next steps: Larger systems!

- I was very limited in the sizes of systems I was able to analyze, which hindered my ability to replicate the analysis of the critical behavior
 - The paper went up to systems with 10^7 nodes; I had trouble even with 10^4 nodes for the AP network (the most computational intensive to run)
- I suspect I can get larger systems by running on not-my-laptop, and can parallelize network analysis to speed up overall performance
 - I'm currently also digging into the Networkx (python package) source code to see if I can find areas for optimization
- **Goal before report submission:** Run up to 10^5 systems

