

Protein folded states are kinetic hubs

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Understanding molecular kinetics, and particularly protein folding, is a classic grand challenge in molecular biophysics. Network models, such as Markov state models (MSMs), are one potential solution to this problem. MSMs have recently yielded quantitative agreement with experimentally derived structures and folding rates for specific systems, leaving them positioned to potentially provide a deeper understanding of molecular kinetics that can lead to experimentally testable hypotheses. Here we use existing MSMs for the villin headpiece and NTL9, which were constructed from atomistic simulations, to accomplish this goal. In addition, we provide simpler, humanly comprehensible networks that capture the essence of molecular kinetics and reproduce qualitative phenomena like the apparent two-state folding often seen in experiments. Together, these models show that protein dynamics are dominated by stochastic jumps between numerous metastable states and that proteins have heterogeneous unfolded states (many unfolded basins that interconvert more rapidly with the native state than with one another) yet often still appear two-state. Most importantly, we find that protein native states are hubs that can be reached quickly from any other state. However, metastability and a web of nonnative states slow the average folding rate. Experimental tests for these findings and their implications for other fields, like protein design, are also discussed.

Markov state model | network | protein folding

Molecular kinetics has fascinated biophysicists and biochemists for decades. From a biophysical point of view, it remains a mystery how systems with so many possible configurations can self-organize with such specificity and rapidity, carry out catalysis, and trigger signaling cascades. From a biomedical standpoint, protein misfolding causes many debilitating diseases, including Alzheimer's, Huntington, and Parkinson diseases (1). Understanding how proteins fold is a logical first step in understanding how they misfold and, more importantly, how to prevent or recover from misfolding; indeed, this approach is already proving valuable (2). Furthermore, a better understanding of protein folding mechanisms could lead to more efficient structure prediction (3, 4), for use in high throughput proteomics and studies of systems that defy experimental characterization, and better models for molecular kinetics could aid in computational drug and protein design.

What would the ultimate theory of molecular kinetics look like though? A natural way of answering this question is by analogy to well-established theories, such as Schrodinger's equation in the successful field of quantum mechanics. On the one hand, computational solutions to Schrodinger's equation have yielded quantitative agreement with and prediction of experimental observables. However, equally important is this theory's ability to yield insight into simple systems, such as the particle in a box, for the purposes of gaining an intuition for fundamental principles, like the quantization of energy and the role of molecular orbitals. Likewise, the ultimate theory of molecular kinetics should be capable of scaling from sophisticated models capable of quantitatively predicting experiments to simple models that yield mechanistic insight. At even the most fundamental levels of this hierarchy, such a theory ought to be at least qualitatively consistent with experimental observations and be capable of generating experimentally testable hypotheses. In particular, such a theory ought to provide insight

into protein folding as success in describing such drastic conformational changes would be evidence for the theory's ability to describe less extreme ones.

We propose that networks of metastable, or long-lived, states (5–8) could fulfill this role because they are implicit in even the most simple protein folding models; examples include $U \leftrightarrow N$ and $U \leftrightarrow I \leftrightarrow N$ where U is the unfolded state, I is an intermediate, and N is the native state. Networks called Markov state models (MSMs) make these implicitly considered properties explicit and have the potential to provide complete maps of a protein's free energy landscape, with nodes corresponding to metastable states (or free energy basins) and edges representing the probabilities of transitioning between pairs of these states (5–10).

A number of recent works have provided validation for these networks by showing that they can yield quantitative agreement with experimentally derived structures and folding rates (6, 11–13). In particular, the predicted native state from our villin model (based on calculated free energies) had an rmsd to the crystal structure of approximately 1.8 Å (6). The model also correctly predicted quantitative details of the kinetics, such as the absolute folding rate (to logarithmic accuracy). This degree of accuracy in predicted free energies, structures, and rates is crucial as all experimental measurements are functions of these properties. In all, the agreement between theory and experiment leads us to the conclusion that our models provide a sufficiently accurate reflection of reality.

To further flesh out this potential theory of molecular kinetics, we have delved into the nature of the free energy landscapes of the villin headpiece (HP-35 NleNle) (14) and a 39 residue fragment of NTL9 (15). Furthermore, because complex networks for real systems are difficult to comprehend, we construct simple, generic models that capture qualitative phenomena like apparent two-state folding and provide an intuition for molecular kinetics. Together, these models allow us to assess existing theories, which describe folding as a two-state process characterized by cooperative transitions across a dominant free energy barrier separating a rapidly mixing unfolded ensemble from the native state (16, 17).

The remainder of this paper will be organized around three key results. First, protein free energy landscapes can yield apparent two-state behavior even in the absence of a single dominant barrier. Second, protein unfolded states are heterogeneous, having multiple basins that interconvert more rapidly with the native state than one another. Third, protein native states are kinetic hubs: it is possible to reach them relatively quickly from anywhere in a network but it is also possible to get stuck in a web of nonnative states.

Results and Discussion

Apparent Two-State Behavior Can Occur In the Absence of a Kinetically Relevant Two-State Decomposition. Many proteins appear to fold via a single cooperative transition from a rapidly mixing

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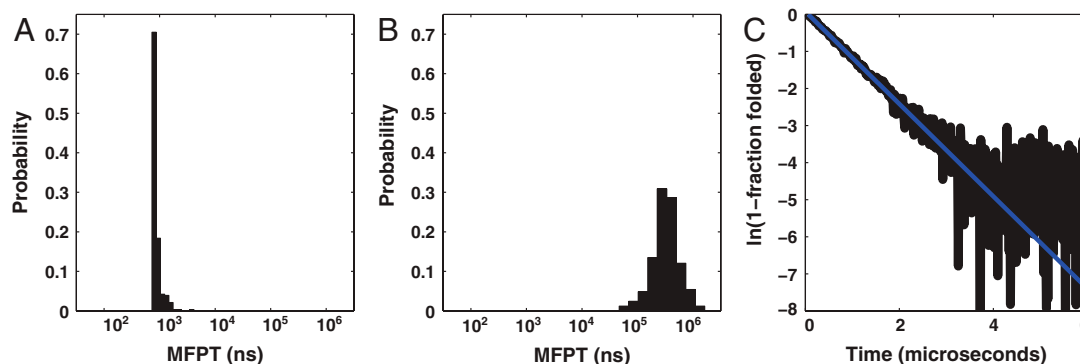


Fig. 3. Relaxation of villin from 500 state model. Distributions of the MFPTs from (A) unfolded states to the native state and (B) between unfolded states. (C) Relaxation kinetics with a 10:1 signal-to-noise ratio (black curve with Gaussian noise) and a single exponential fit (blue curve with $\tau = 810$ ns).

were used to choose the number of probes employed and their placement.

Proteins Have Heterogeneous Unfolded States with Multiple Basins that Interconvert More Rapidly with the Native State than Each Other.

We now investigate which of the simple network topologies is most representative of real protein free energy landscapes. As a first step, we have calculated that every state can reach the native basin of our villin model in one or two steps. This eliminates the possibility of a single pathway because states with that topology could require up to 499 steps to reach the native basin.

Determining whether the parallel pathway model (17, 33, 34) or the heterogeneous unfolded state model is more representative of villin requires a definition of the unfolded state(s). Because every nonnative state can reach the native basin in one or two steps it is natural to label every state that is not directly connected to the native state (332 in all) as unfolded and all other nonnative states (167 in all) as intermediates.

Taking this definition, we can now examine the distribution of MFPTs from each unfolded state to the native state as well as the distribution of MFPTs between all pairs of unfolded states. Doing so reveals that the average MFPT to the native state is 880 (+/− 270) ns, in reasonable agreement with the experimentally predicted folding time of 720 ns (14). Moreover, this value is much lower than the average MFPT between pairs of unfolded states (approximately 370 ns), as shown in Fig. 3 A and B. Considering every nonnative state as part of the unfolded ensemble also gives similar distributions (Fig. S2), implying that these results are robust to the exact definition of the unfolded state. Similar results are found for NTL9 as well (Fig. S3). Thus, we can conclude that the heterogeneous unfolded state model is most representative of our villin and NTL9 models and possibly proteins in general. This result is in contrast to existing theories of protein folding, which assume rapid equilibration within the unfolded ensemble (17, 35, 36).

Examination of representative structures suggests that nonnative interactions (often in the context of relatively compact conformations) and the enormity of conformational space are responsible for slow transitions between unfolded basins (Fig. S4). Nonnative contacts can easily have free energies on the order of native contacts, making nonnative states reasonably metastable. Once a set of nonnative contacts is broken, the probability of forming a particular set of other nonnative contacts is quite small due to the large number of other possibilities. This small probability is equivalent to a slow rate. In contrast, evolutionary pressure to fold makes transitioning to the native state reasonably probable, which equates to fast folding relative to slower transitions between unfolded basins.

The tight distribution of MFPTs to the native state is also consistent with our explanation of apparent two-state behavior. Due to experimental noise, it is difficult to justify using more than

one or two exponentials to fit the relaxation of our coarse-grained villin model with 500 states, as shown in Fig. 3C. Only with an extremely high signal to noise ratio can one accurately identify the deviations from single exponential relaxation shown in Fig. S5. We also note that more fine-grained models for villin can capture the burst phase in its relaxation (Fig. S6) but here we emphasize the ability of our coarse-grained model to capture the apparent two-state behavior that dominates this system's relaxation (14).

Our ability to reconcile our model with existing experimental data on the nature of the unfolded ensemble (specifically under native conditions, as opposed to the more rapidly mixing denatured state) indicates that more experiments will be required to definitively falsify or support our conclusions. For example, Nettels et al. have reported a 50 ns global relaxation time within the unfolded ensemble (37). Our model, however, would suggest that this may be due to relaxation within individual unfolded basins, not between them. This hypothesis is consistent with recent measurements of slow dynamics in the unfolded ensemble from the Lapidus lab (38, 39). Therefore, we suggest that this may be an interesting direction for future experimental work. In addition to existing methodologies for probing the unfolded ensemble, single molecule experiments monitoring multiple degrees of freedom could help to falsify or support our conclusions.

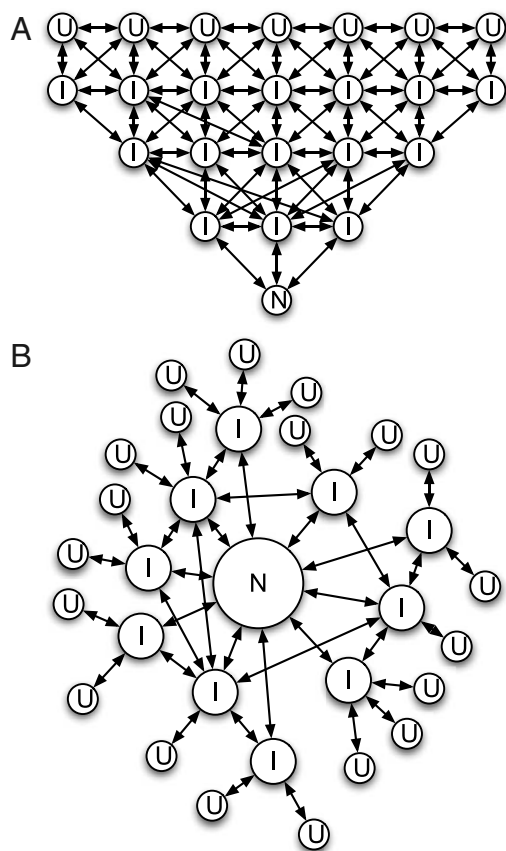
If our heterogeneous unfolded state model is indeed generally true then protein folding kinetics cannot be accurately described by two-states separated by a single barrier. Instead, folding must be understood in terms of multiple pathways starting from a number of distinct states. Mixing between pathways adds another layer of complexity to the folding process. Modeling the effects of mutations will thus require considering changes in the relative free energies of numerous states and barrier heights. Understanding the global effects of small changes on networks will likely also be important for protein design.

A Native Hub Allows Rapid Folding but Proteins Can Still Get Stuck In a Web of Nonnative States.

The accessibility of villin's native state implies the hub-like connectivity characteristic of small-world and scale-free networks (40, 41). We can test this hypothesis by counting the number of connections observed between states because only those transitions with probabilities above some threshold are observed with our finite sampling (all transitions would be observed with infinite sampling). Examining subsets of the states independently, one finds that the average degree (or number of connections) increases as one moves from the unfolded states to the native basin. The unfolded states have an average degree of 12 whereas the intermediate states have an average degree of 25. The native state acts as a hub, connected to 167 other states. Similar results are found for a small β -sheet peptide (42) and NTL9.

Due to the kinetic proximity of the native state with a 15 ns lagtime, we see that villin can fold in just 30 ns; however, such trajectories are rare because the metastability and connectivity of nonnative states makes taking a direct route to the native state improbable. Instead, villin will often spend considerable time in a web of nonnative states before finally folding, resulting in an average folding time on the microsecond timescale. In the future, it will be interesting to test whether slower folding proteins have unfolded states further from the native one or just more strongly metastable states, which equates to higher barriers and slower transitions between states. Preliminary analysis of NTL9 suggests

Another simple but more efficient strategy would be to start simulations from multiple conformations dispersed throughout phase space and run them long enough to ensure mixing between them and coverage of the entire space. In fact, Fig. 5 and Fig. S7 show that such a scheme is actually more valuable than a few long trajectories, using a relative entropy metric for MSMs from ref. 47



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to measure the information content of different datasets relative to our validated villin model. However, this trend can be seen to break down for simulations that are insufficiently long or too few as they are unlikely to reach every state or traverse every possible pathway between pairs of states. The simulation length at which this breakdown occurs decreases as the number of simulations increases though. Even better performance can be obtained using adaptive sampling algorithms (47, 48), which direct sampling to where it is needed most to improve a model.

Comparison to Previous Theories for Protein Folding. There is a long history of theoretical models for protein folding (28) so it is important to put our work in the context of these previous theoretical approaches. In particular, folding funnel models (17, 36, 43) have dominated much of how the field currently conceptualizes protein folding and hence it is natural to compare our model to such theories. One of the most similar funnel categories is type0B, which is characterized by overall downhill folding interrupted by a glass transition along the reaction coordinate (17). Whereas this regime does include slow dynamics between compact states, it also results in a small number of folding pathways relative to higher connectivity in the unfolded ensemble. In addition, this and other previous funnel-based models have explicitly described rapidly interconverting unfolded states, as reflected in the “bottleneck” discussed in previous works (34, 35), as well as the choice of structurally based reaction coordinates like the number of native contacts (Q) (17, 35), which directly requires that dynamics along orthogonal degrees of freedom, such as interconversion between unfolded conformations, is rapid compared to folding. In contrast, we find a large number of folding pathways, slow dynamics between unfolded states relative to folding, and no glass transition. Our folding rates are also quite similar, rather than the different rates characteristic of the folding pathways in type0B folding.

Other funnel models have recognized the possibility of a large number of folding pathways (17, 33, 43), but still in the context of fast dynamics within the unfolded basin relative to slower transitions to the folded state. Some have even gone so far as to assume global connectivity (49, 50); however, even these emphasize that local connectivity would dominate in the full dimensional conformational space and global connectivity only arises when projecting onto a few order parameters. Furthermore, they argue global connectivity will not give an activation barrier and, therefore, these models are primarily intended for studies of downhill folding or the early activationless stages of folding. Our model, on the other hand, has a native hub and slow dynamics in the unfolded state relative to faster folding regardless of the degree of coarse-graining one employs. We also demonstrate that this can result in apparent two-state folding (i.e., activated kinetics) and that this occurs in nondownhill folding proteins, such as the millisecond folding NTL9.

Conclusions

Many biological systems, ranging from signaling pathways to social networks, can be most naturally described as networks. As a field, we have now established an additional level to this hierarchy: a network theory for molecular kinetics that is able to map out the free energy landscapes of proteins and other macromolecules in their entirety.

Previous work has demonstrated that this network theory is capable of quantitative agreement with experiments (6, 11–13) and we have now shown that it can also scale down to simple, generic models. Using this theory at both the quantitative and qualitative levels, we have provided an intuition for conformational changes as drastic as protein folding and this intuition has led to experimentally testable insights into the nature of protein free energy landscapes.

We have focused on three insights from these network models, which appear to hold regardless of the degree of coarse-graining

one employs and can be reconciled with current experiments. First, even models that defy a kinetic decomposition into two states often give rise to apparent two-state behavior. Second, proteins have heterogeneous unfolded states (multiple basins that each interconvert more rapidly with the native state than with one another, preventing a kinetic decomposition into two states). Third, proteins have a native hub. Thus, it is possible to fold quickly from anywhere in the landscape but proteins often get stuck in a web of nonnative states before finally folding, greatly increasing the average folding time.

These properties are a natural result of reasonably strong nonnative interactions and the enormous number of nonnative conformations a protein can adopt, in combination with evolutionary pressure to fold quickly (for example, to avoid aggregation). Therefore, we suggest that these conclusions are likely true of proteins in general. Our approach also unifies other models for protein folding by recognizing that each of them builds upon elements, whether they are called folding nuclei (25) or foldons (27), which correspond to different types of metastable states.

We look forward to a fruitful future of drawing on network theory to better understand molecular kinetics and guide experiments probing both general properties and system specific details. In particular, can one reinterpret the many experiments that have been analyzed under a two-state assumption? If so, that could shed light on the chemistry of the underlying structures that leads to the network topology and dynamics described here. Moreover, can further experiments be designed to directly probe the unfolded state under native conditions (rather than with denaturant or high temperature, where mixing is more rapid) to directly test the predictions made here? We also hope to explore how the methodologies developed for building and understanding biomolecular networks may be applicable to other types of networks, especially as network theorists attempt to develop a general framework for understanding network dynamics.

Materials and Methods

Atomic Resolution Protein Folding Simulations and Networks. Ref. 6 describes the use of the MSMBuilder package (<https://simtk.org/home/msmbuilder/>) (23) to construct an MSM with 10,000 microstates for the villin headpiece (HP-35 NleNle). This model was based on approximately 450 all-atom, explicit solvent simulations, each up to 2 μ s in length, for a total simulation time of 354 μ s (51). Whereas the longest timescale transitions in the model from ref. 6 were found to be Markovian, implying memoryless transitions between metastable states, not every state was metastable. We used MSMBuilder to lump kinetically related microstates into 500 metastable macrostates to ensure a direct correlation between states in the MSM and free energy basins, as described in *SI Text*. This is equivalent to common experimental analyses in which the potential is smoothed and the friction is rescaled. We note, however, that the free energy landscape for this system is actually a hierarchy of basins so it is possible to build many valid MSMs with different numbers of states. As a result, one would not expect there to be exactly 500 experimentally detectable states. Regardless of the resolution at which one examines this hierarchy, however, requiring that each state is metastable ensures that they are directly related to a free energy basin. Thus, our networks of metastable states are an important step beyond previously described networks, which often used simpler approximations to define state boundaries and the transition rates between states (17, 34, 42, 52, 53). An additional 40,000 simulations, each up to 400 ns in length (for a total simulation time of 14 milliseconds), were also assigned to this MSM to explore the effect of using more simulations.

Preliminary results for a 39-residue fragment of NTL9 are based on an MSM built from approximately 1.5 milliseconds of simulation in implicit solvent with a different force field (13). Similarities between these two systems thus suggest our results are not a force field artifact.

Simple Models. We have designed three simple networks, depicted in Fig. 1, that capture the essence of various protein folding mechanisms. Each of these models has six metastable states with approximately the same equilibrium and transition probabilities so that differences between their behaviors may be attributed to differences in their topologies (*SI Text*).

The first model (S) has a single folding pathway. This model is a natural extension of the common $U \leftrightarrow I \leftrightarrow N$ model (19, 54) and is often used to

justify the expense of running long simulations as shorter ones could fail to reach every state.

The second model (P) has parallel folding pathways. Parallel folding pathways have been proposed for a number of systems (20, 21, 33, 51). In addition, this model emphasizes the need to observe numerous folding and unfolding transitions to obtain sufficient statistics on the entire process. The increased connectivity relative to S also results in faster timescales.

The third model (H) has a heterogeneous unfolded state—multiple unfolded basins that each interconvert more rapidly with the native state than with one another. Thus, there is no kinetic decomposition of this model

into two states, one folded and one unfolded. This model was inspired by a growing body of work on the presence of deep minima and gutters in unfolded regions of conformational space (38, 39, 55–57).

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