```
% Shannon Moran
% Math 671, HW 2, problem 4

function[]=Math671_HW2_p4b()

% Problem givens
q = linspace(2,7,6);
N = 2.^q;
sigma = 2;
h = (1./N)';

% Defing matrices for more efficient calculations
[err_FD, err_PS, err_G] = deal([], [] ,[]);

% Run over q for f1 = x
figure('Name','Function f=x','NumberTitle','off')
for i=1:length(q)
    % Set up grid and driving function
    xj = linspace(0,N(i)-1,N(i))/N(i);
    f1 = xj;
    % Set up numerical methods
    F_N = findF(N(i));
    D_FD = numericalFiniteDiff(N(i),sigma);
    D_PS = numericalPseudospectral(N(i),sigma);
    G = numericalG(N(i),sigma,xj);
    % Calculate approximation from numerical methods
    u_FD = real((inv(F_N)*inv(D_FD)*F_N)*f1')';
    u_PS = real((inv(F_N)*inv(D_PS)*F_N)*f1')';
    u_G = real(G*f1')';
    % Calculate analytical solution
    Phi1 = -((2*exp(2)).*xj-2.*xj+exp(2-2.*xj)-exp(2.*xj))./(8-8.*exp(2));
    [xj,u_FD,u_PS,u_G, Phi1]=deal([xj, 1],[u_FD u_FD(1)],[u_PS u_PS(1)],[u_G u_G(1)],[Phi1 Phi1(1)]);
    % Calculate 'error' for each method
    [err_FD(i), err_PS(i), err_G(i)] = deal(max(Phi1-u_FD),max(Phi1-u_PS),max(Phi1-u_G));
    % Plot
    subplot(3,2,i);
    plot(xj,u_FD,xj,u_PS,xj,u_G, xj, Phi1)
    title(['N = ' num2str(N(i))])
    if i==length(q)
        legend('Finite Difference','Pseudo-spectral','Greens FUnction','Analytical Solution','Location','eastoutside');
    end
end

sprintf('Results of numerical methods for f=x')
sprintf('Finite-difference')
table(h,err_FD',err_FD'./h,err_FD'./h.^2)
sprintf('Pseudospectral')
table(h,err_PS',err_PS'./h,err_PS'./h.^2)
sprintf('Greens function')
table(h,err_G',err_G'./h,err_G'./h.^2)

% Reset variables
[err_FD, err_PS, err_G] = deal([], [] ,[]);

% Run over q for f2 = sin(pi*x)
figure('Name','Function f=sin(pi*x)','NumberTitle','off')
for i=1:length(q)
    % Set up grid and driving function
    xj = linspace(0,N(i)-1,N(i))/N(i);
    f2 = sin(pi*xj);
    % Set up numerical methods
    F_N = findF(N(i));
    D_FD = numericalFiniteDiff(N(i),sigma);
    D_PS = numericalPseudospectral(N(i),sigma);
    G = numericalG(N(i),sigma,xj);
    % Calculate approximation from numerical methods
    u_FD = real((inv(F_N)*inv(D_FD)*F_N)*f2')';
    u_PS = real((inv(F_N)*inv(D_PS)*F_N)*f2')';
    u_G = real(G*f2')';
    % Calculate analytical solution at grid points
    Phi2 = (exp(-2.*xj).*(pi.*(exp(4.*xj)+exp(2))+2.*(exp(2)-1).*(exp(2.*xj).*sin(pi.*xj))))/(2*(exp(2)-1)*(4+pi^2));
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        [xj,u_FD,u_PS,u_G, Phi2]=deal([xj, 1],[u_FD u_FD(1)],[u_PS u_PS(1)],[u_G u_G(1)],[Phi2 Phi2(1)]);
        % Calculate 'error' for each method
        [err_FD(i), err_PS(i), err_G(i)] = deal(max(Phi2-u_FD),max(Phi2-u_PS),max(Phi2-u_G));
        % Plot
        subplot(3,2,i);
        plot(xj,u_FD,xj,u_PS,xj,u_G,xj,Phi2)
        title(['N = ' num2str(N(i))])
        if i==length(q)
            legend('Finite Difference','Pseudo-spectral','Greens FUnction','Analytical Solution','Location','eastoutside');
        end
    end

    sprintf('Results of numerical methods for f=sin(pi*x)')
    sprintf('Finite-difference')
    table(h,err_FD',err_FD'./h,err_FD'./h.^2)
    sprintf('Pseudospectral')
    table(h,err_PS',err_PS'./h,err_PS'./h.^2)
    sprintf('Greens function')
    table(h,err_G',err_G'./h,err_G'./h.^2)

    % Helper fxn: Find F_N matrix manually
    function[F_N] = findF(N)
    omega = exp(-2*pi*1i/N);
    F_N = zeros(N,N);
    for n=1:N
        for j=1:N
            F_N(n,j) = omega^((n-1)*(j-1));
        end
    end
    F_N = F_N/sqrt(N);
    end

    end

    % Numerical method: Green's function method
    function[G] = numericalG(N,sigma,xj)
    G = zeros(N,N);
    for j=1:N
        for k=1:N
            G(j,k) = c(xj(mod(j-k,N)+1),sigma,N);
        end
    end

    % Helper fxn: Green's function method circulant matrix
    function[c_out] = c(x,sigma,N)
    y = 0;
    g = cosh(sigma*(abs((x-1)-y)-0.5))/(2*sigma*sinh(0.5*sigma));
    c_out = g/N;
    end

    end

    % Numerical method: Finite difference method
    function[D_FD] = numericalFiniteDiff(N, sigma)
    h = 1/N;
    lambda = zeros(N,1);
    for n=1:N
        lambda(n) = (4*sin(pi*(n-1)*h)^2)/h^2 + sigma^2;
    end
    D_FD = diag(lambda);
    end

    % Numerical method: Pseudospectral
    function[D_PS] = numericalPseudospectral(N,sigma)
    d_n = zeros(N,1);
    for n=1:N/2
        d_n(n) = 2*pi*1i*(n-1);
    end
    for n=N/2:N
        d_n(n) = 2*pi*1i*(n-1-N);
    end
    D_PS = -diag(d_n)^2 + sigma^2*eye(N);
    end
```
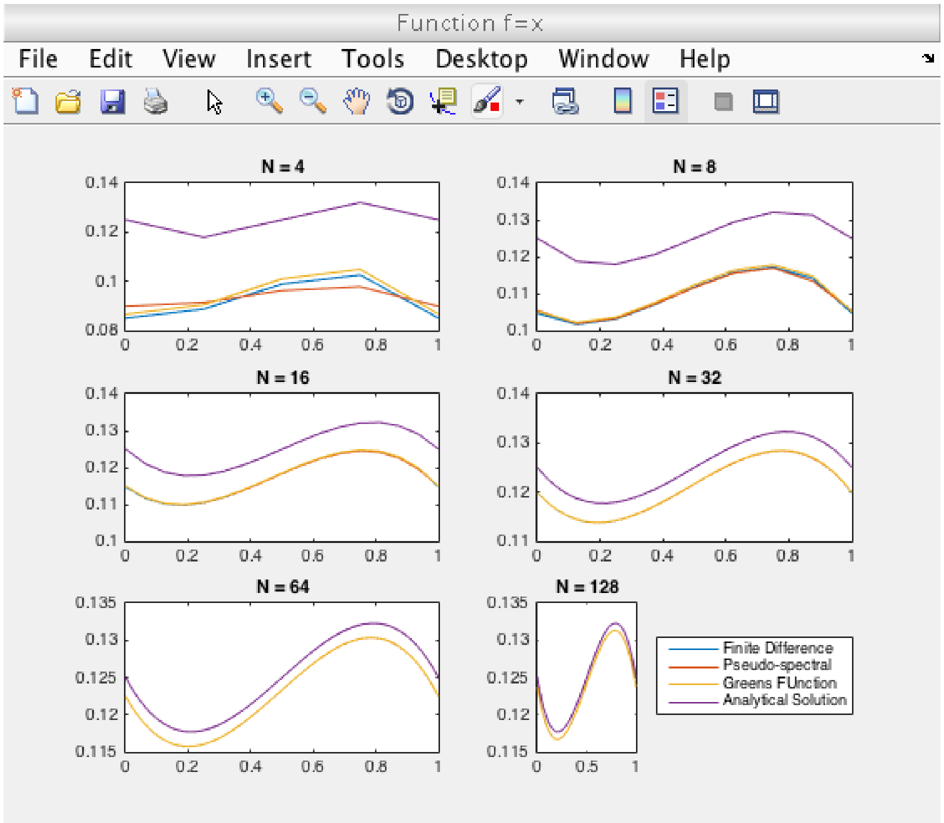
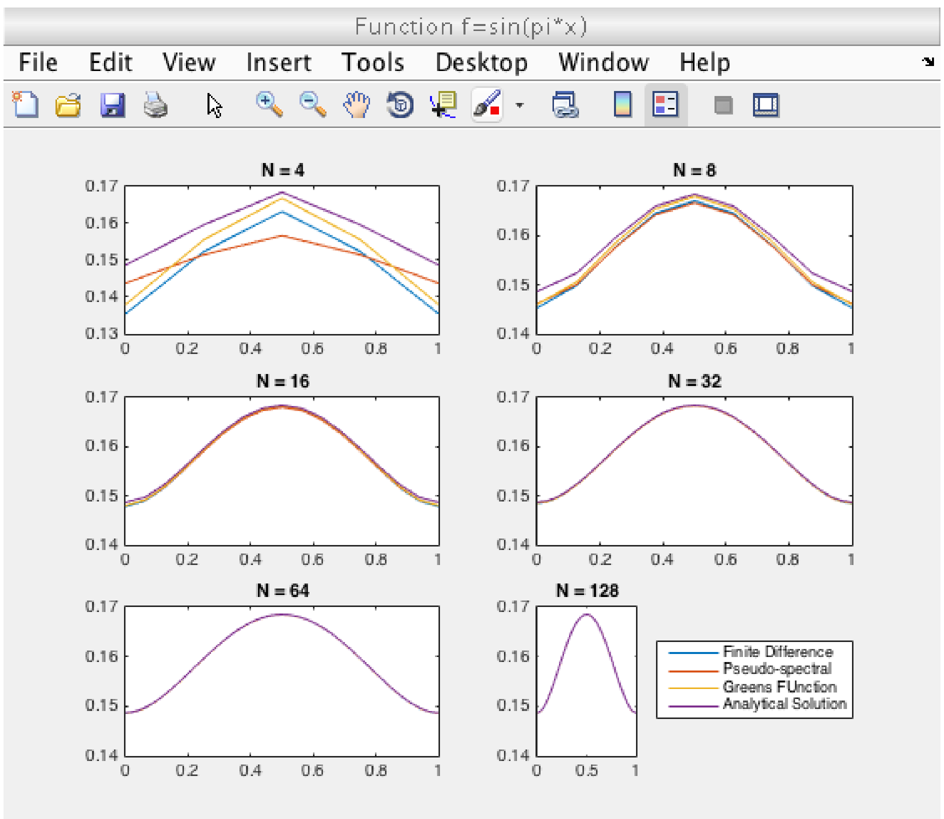

```
ans =
Results of numerical methods for f=x
ans =
Finite-difference
ans =
        h           Var2         Var3        Var4
    _________    _________    _______    _______

         0.25     0.040033    0.16013    0.64052
        0.125     0.020387     0.1631     1.3048
       0.0625     0.010242    0.16387     2.6219
      0.03125     0.005127    0.16406       5.25
     0.015625    0.0025643    0.16411     10.503
    0.0078125    0.0012822    0.16413     21.008
ans =
Pseudospectral
ans =
        h           Var2         Var3        Var4
    _________    _________    _______    _______

         0.25     0.035245    0.14098    0.56392
        0.125     0.019607    0.15686     1.2549
       0.0625     0.010053    0.16085     2.5737
      0.03125    0.0050792    0.16253     5.2011
     0.015625    0.0025521    0.16334     10.454
    0.0078125    0.0012792    0.16373     20.958
ans =
Greens function
ans =
        h           Var2         Var3        Var4
    _________    _________    _______    _______

         0.25     0.038439    0.15376    0.61502
        0.125     0.019866    0.15893     1.2714
       0.0625     0.010095    0.16153     2.5844
      0.03125    0.0050884    0.16283     5.2105
     0.015625    0.0025543    0.16348     10.463
    0.0078125    0.0012797     0.1638     20.967
ans =
Results of numerical methods for f=sin(pi*x)
ans =
Finite-difference
ans =
        h            Var2           Var3         Var4
    _________    __________    _________    _______

         0.25      0.013231     0.052923    0.21169
        0.125     0.0033839     0.027071    0.21657
       0.0625    0.00085093     0.013615    0.21784
      0.03125    0.00021305    0.0068175    0.21816
     0.015625    5.3281e-05      0.00341    0.21824
    0.0078125    1.3322e-05    0.0017052    0.21826
ans =
Pseudospectral
ans =
        h            Var2           Var3         Var4
    _________    __________    _________    _______

         0.25      0.011776     0.047104    0.18842
        0.125     0.0025436     0.020349    0.16279
       0.0625    0.00065917     0.010547    0.16875
      0.03125    0.00016646    0.0053266    0.17045
     0.015625    4.1792e-05    0.0026747    0.17118
    0.0078125    1.0469e-05    0.0013401    0.17153
ans =
Greens function
ans =
        h            Var2           Var3         Var4
    _________    __________    _________    _______

         0.25      0.010715     0.042859    0.17144
        0.125      0.002684     0.021472    0.17178
       0.0625     0.0006713     0.010741    0.17185
      0.03125    0.00016784    0.0053709    0.17187
     0.015625    4.1962e-05    0.0026855    0.17187
    0.0078125     1.049e-05    0.0013428    0.17188
```





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