

Problem 4a

Find the associated vorticity for the stream function given. Show that on the set of Reals in 2D, the integral of the vorticity is =1 for all values of delta.

```
Psi[x_, y_, delta_] := -1 / (2 * Pi) * Log[(x^2 + y^2 + delta^2)^(1/2)]  
omega[x_, y_, delta_] = -Laplacian[Psi[x, y, delta], {x, y}] // Simplify
```

Out[159]=
$$\frac{\delta^2}{\pi (x^2 + y^2 + \delta^2)^2}$$

```
In[163]:= Integrate[omega[x, y, delta], {x, -Infinity, Infinity}, {y, -Infinity, Infinity}] // Simplify
```

Out[163]= ConditionalExpression[1, Re[delta^2] >= 0 || delta^2 < Real]

```
In[161]:= Plot[{omega[x, 0, 0.2], omega[x, 0, 0.1], omega[x, 0, 0.05]}, {x, -4, 4},  
PlotRange -> Automatic, PlotLegends -> {"delta=0.2", "delta=0.1", "delta=0.05"}]
```

