Problem 4a

Find the associated vorticity for the stream function given. Show that on the set of Reals in 2D, the integral of the vorticity is =1 for all values of delta.

```
Psi[x_, y_, δ_] := -1 / (2 * Pi) * Log[(x^2 + y^2 + δ^2)^(1 / 2)]
ω[x_, y_, δ_] = -Laplacian[Psi[x, y, δ], {x, y}] // Simplify
```

Out[159]=
$$\frac{\delta^2}{\pi \left(x^2 + y^2 + \delta^2\right)^2}$$

In[163]:=
```
Integrate[ω[x, y, δ], {x, -∞, ∞}, {y, -∞, ∞}] // Simplify
```

Out[163]=
```
ConditionalExpression[1, Re[δ^2] ≥ 0 || δ^2 ∉ Reals]
```

In[161]:=
```
Plot[{ω[x, 0, 0.2], ω[x, 0, 0.1], ω[x, 0, 0.05]}, {x, -4, 4},
 PlotRange → Automatic, PlotLegends → {"δ=0.2", "δ=0.1", "δ=0.05"}]
```

Out[161]=