Name: ______________ Student ID: ______________ Web ID: ______________

Problem 1. (10 pts) For the following functions, rank them from the slowest (with the lowest complexity) to the fastest growing.

$(\lg n)^{\lg n}$, $\lg n!$, $(\lg n)!$, $n^{1/\lg n}$, 2^{2^n}

Problem 2. (10 pts) Is $4n^2 - 3n + 1 = O(n^2)$ correct? Disprove the statement, or prove it by determining the smallest **integer constant** c and then the smallest **integer constant** n_0 in the definition of O-notation.

Problem 3. (12 pts) For the following two recurrence relations, give their asymptotic growth rates using the Θ notation. Assume in each case that $T(n)$ is $\Theta(1)$ for $n \leq 4$. **Show your steps!**

(a) (6 pts) $T(n) = 2T(n/4) + n^2 \lg n$.

(b) (6 pts) $T(n) = T(n/3) + T(2n/3) + n$.

Problem 4. (8 pts) Given n items, with i-th item worth v_i dollars and weighing w_i kg, a porter develops an $O(nW)$ algorithm to pick as valuable a load as possible, where W is the maximum weight (in kg) he can carry in one load. Does this algorithm run in polynomial time? Why?