Name: Student ID: Algorithm ID:

Problem 1. (12 pts total) Taiwan Telecom wants to link n stations spread over Taiwan using communication channels. Each pair of stations has a different bandwidth available. Taiwan Telecom wants to select $n-1$ channels (the minimum possible) in such a way that all the stations are linked by the channels and the total bandwidth is **maximum**. Give an efficient algorithm for this problem and determine its time complexity.

Problem 2. (15 pts total) Give the following linear-programming system of difference constraints:

$$\begin{aligned}
x_1 - x_4 &\leq -2 \\
x_1 - x_5 &\leq -3 \\
x_2 - x_1 &\leq -4 \\
x_2 - x_3 &= -8 \\
x_3 - x_5 &\leq 2 \\
x_4 - x_3 &\leq -4 \\
x_5 - x_1 &\leq 3 \\
x_5 - x_4 &\leq 4
\end{aligned}$$

Show the constraint graph for these constraints, and solve for the unknowns $x_i, i = 1, \ldots, 5$, or explain why no solution exists.

Problem 3. (12 pts total) Let $G = (V, E)$ be a directed graph with edge costs modelled by the corresponding weights. The *bottleneck* of a path is defined as the **minimum** edge cost among all the edges on the path. Suppose that we want to find a **maximum** bottleneck path between each pair of vertices. Show how to modify Line 7 of Floyd-Warshall's all-pair shortest-path algorithm to solve this problem in $O(V^3)$ time.

Floyd-Warshall(W)

7. $d_{ij}^{(k)} = \min(d_{ij}^{(k-1)}, d_{ik}^{(k-1)} + d_{kj}^{(k-1)})$

Problem 4. (6 pts total) Give two methods that can increase the attendance of our class.