National Taiwan University
Department of Electrical Engineering
Algorithms, Fall 2019

Handout #26
December 27, 2019
TA: Chien-Hao Tsou

Sample Solutions to Quiz #3

1. (10) The optimal Huffman code is shown in Figure 1. The optimal cost is 67.

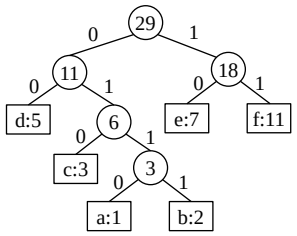

Figure 1: The optimal Huffman code.

2. (12)

(a) (6) The greedy choice is that the thief always takes as valuable a load as possible. For any optimal solution which is not the result of greedy choices, if we replace as many items per pound as possible with the most valuable items being left, the new total value must be larger than or equal to the original total value. Thus, this problem has the greedy-choice property.

(b) (6) **Step 1.** Calculate per pound for each item $u_i = v_i/w_i$, needs O(n).
Step 2. Sort u_i in non-increasing order by using merge sort, needs O($nlgn$).
Step 3. Pick items as much as possible until the knapsack is full or all items are taken away, needs O(n). So this greedy algorithm takes O($nlgn$) time complexity.

3. (10) Since there is a cycle $c \to f \to g \to c$, there is no topological ordering in the given graph.

4. (12) Construct an undirected graph G, where each vertex represents the corresponding unshaded square. The edges are constructed between each pair of neighboring unshaded squares. Then, the shortest tour can be found by applying BFS algorithm on G. Since the number of neighbors of each square is at most 4, it implies that $E = O(V)$ in G, where V and E denote the numbers of vertices and edges, respectively. So the time complexity is $O(V + E) = O(V)$.