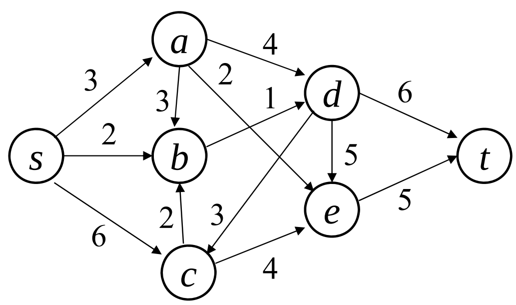
Homework #5 (due 6pm, January 3, 2020 in BL 406)

INSTRUCTIONS: Because we will make the sample solutions available on-line by 6pm January 3, no late submission will be accepted for this homework. Thank you very much for your cooperation! Collaboration policy: You can discuss the problems with other students, but you must write the final answers by yourself. Please specify all of your collaborators (names and student id's) for each problem. If you solve some problems by yourself, please also specify "no collaborators". Homework without collaborator specification will not be graded.

1. In the flow network shown below, the number beside an edge denotes its corresponding capacity. Apply the **Edmonds-Karp** algorithm to find a maximum flow from s to t in the network. Show **every augmentation path** (but you **do NOT need to show the whole network** to save time) and **explain why the flow you found is maximum.**



2. Problem 26-1 (pages 760–761).

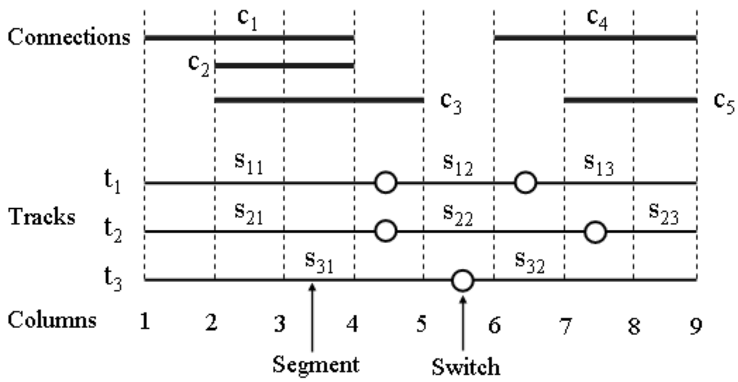
3. Problem 26-3 (pages 761–762).

4. A realtor would like to maximize the number of apartments sold. She has p apartments to sell and q potential customers for these apartments. She has m salesmen working for her. Each salesman is assigned a list of apartments and clients interested in these apartments. A salesman can sell an apartment to any of his customers. Salesman i can sell at most b_i apartments. Also, any apartment cannot be owned by more than one person. For $m = 2, p = 4, q = 5, b_1 = 3, b_2 = 1$, and the following assignments of customers and apartments to the salesmen, construct the flow network for the underlying problem. **How to find** the maximum number of apartments that can be sold? **(Hint: How can you constrain that salesman i can sell at most b_i apartments in this flow network?)**

Salesman	Customers	Apartments
1	1, 2, 3, 4	1, 2, 3
2	3, 4, 5	3, 4

5. The figure below shows a segmented routing structure in a row-based field-programmable gate array (FPGA). There are five connections, $c_1, c_2, \ldots, c_5$, to be routed on three segmented tracks, t_1, t_2, and t_3, with eight segments $s_{11}, s_{12}, \ldots, s_{32}$ in the row-based FPGA. A track can be partitioned into a set of segments by using switches. If a switch incident on two adjacent segments is "ON", then the two segments are electrically connected; otherwise, the two segments can be used independently. You are asked to route (place) the five connections on the three segmented tracks. Suppose each connection can use at most one segment for routing, i.e., 1-segment routing. In other words, a connection c_k of the

column span $[l_k, r_k]$ is said to be routed on a segment s_{ij} of track t_i if $c_k = [l_k, r_k]$ is placed within the column span of s_{ij}. For example, $c_3 = [2, 5]$ can be routed on segment s_{31} of track t_3 (which consumes only one segment) while it cannot route on track t_1 or t_2 (which would have consumed two segments, thus violating the constraint of 1-segment routing). Give an efficient algorithm to solve the 1-segment routing problem. What is the time complexity of your algorithm?



6. Concepts on polynomial-time complexity. (a) Exercise 34.1-4 (page 1060). (b) Professor Right finds a fast algorithm for the maximum flow problem on the network $G = (V, E)$ with the capacity $c(u, v)$ for the edge (u, v), which runs in $O(VE(\lg C)^2)$ time, where $C = \max_{(u,v)\in E} c(u, v)$. Is it a polynomial-time algorithm? Justify your claim.

7. Exercise 34.4-7 (page 1086).

8. Problem 34-1 (pages 1101–1102).

9. Problem 34-3 (pages 1103–1104).

10. (a) Exercise 17.1-3 (page 456). (b) Exercise 17.2-2 (page 459). (c) Exercise 17.3-2 (page 462).

11. Exercise 17.4-3 (page 471).

12. Problem 17-3 (pages 473–474).

13. (DIY Problem) For this problem, you are asked to design a problem *set* related to Chapter(s) 17, 26, and/or 34, and give a sample solution to your problem set. Grading on this problem will be based upon the *quality* of the designed problem as well as the *correctness* of your sample solution.