Opening up the court (surface) in tennis grand slams

September 15, 2018

Abstract

IDK writing is hard

1 Introduction

Rafael Nadal is known as the "King of Clay" in tennis, having won 11 out of his current 17 grand slams titles at the French Open (FO), which is played on a clay surface (Jurejko 2018). In contrast, his rival Roger Federer has won his most grand slam titles (8 out of 20) at Wimbledon (Wim.), which is played on grass. On the women's side, Serena Williams, winner of 24 grand slam titles, has been dominant both on hard court (7 titles at the Australian Open (AO) and 6 at US Open (USO)) and grass (7 at Wim.). These three players have different playstyles and origins, and they are legends in the sport of tennis. However, their success at grand slams seems to differ among the tournaments, which leads to question why that may be the case.

The four grand slams (AO, FO, Wim., USO) are played in different cities (Melbourne, Paris, London, New York City) at different starting dates (January, May, June, August) over the course of two weeks with 7 total rounds and 128 players. As a feature unique to tennis, all four slams are played on different surfaces (Plexicushion hardcourt, clay, grass, DecoTurf hardcourt). It is often thought that top players achieve better results on some surfaces than others. For example, there is the "home court advantage:" do players from the country of the grand slam tournament perform better than at other grand slams? Recently, Spanish players, led by Nadal, seem to have dominated the clay court scene (Lewit 2018) making it seem like players of a certain country may perform better on some surfaces than others.

In this paper, we analyze the results of grand slam players from 2013-2017, and we analyze the surface effect on players at the four grand slams, first broadly at a high level and then narrowing down to player effects. More specifically,

1. Determine if results in terms of wins/losses and point distribution differ across the four surfaces
2. Examine more fine grained attributes such as winners, aces, and unforced errors across the four surfaces
3. Assess how individuals differ across the four surfaces, with special focus on Nadal, Federer, and Williams.

As to issue (1) quantifying the high-level effect of court surface on players, there has not been much written about with regards to tennis. There are materials available in the literature for forecasting the outcome of tennis matches (Klaassen and Magnus 2003; Newton and Keller 2005; McHale and Morton 2011; Kovalchik 2016)] or for assessing whether points within a match are independent and identically distributed (Klaassen and Magnus 2001). (Knottenbelt, Spanias, and Madurska 2012) do take into account surface in their model but do not compare the results of one surface to another and (O'Donoghue and Ingram 2001) concludes that players have more injuries on grass comparead to other courts. Other sports analyses do take into account surface type such as grass vs. turf in soccer and football. Results from these studies show that surface type does have an effect on the game, either directly or indirectly [Andersson, Ekblom, and Krustrup (2008); Gains et al. (2010);].

For issue (2) we examine the match attributes more unique to tennis such as winners (when a player hits a shot that the opponent is unable to return), aces (a serve an opponent is unable to return), and unforced errors (UE) (when a player makes a simple mistake resulting in a loss of the point). Specifically, serving is thought to be of greater importance at Wimbledon since the ball travels faster on grass. In general, players hope to maximize the winner to UE ratio (W/UE), which may lead to playing more aggressively on faster courts and more conservatively on slower courts such as clay (Paxinos 2007).

Finally, for issue (3) the player analyses, we take two approaches. On one hand, we use a hierarchical model with fixed and random effects taking into account player and tournament attributes using the complete data set and to fairly compare players to one another. This allows us to leverage tournament information to players without many matches played and to help account for noise. On the other hand, since Nadal, Federer, and Williams are extremely popular and have had much success in the 2013-2017 time range, we have enough data points with feature attributes to examine the effects individually. In both cases, we examine whether our models pass "common sense" tests like how the models in (Thomas et al. 2013) show that commonly well known hockey players also have high status in the model. We also examine whether these players do have surface apparent effects.

We use models that take into account both individual and group effects such as in the Gaussian-process player production basketball model or predicting individual soccer performance (Page, Barney, and McGuire 2013; Egidi and Gabry 2018). For our hierarchical model across all players, we model whether a player or lost a match based on player attributes along with opponent rank. For the individual models, we instead model percent of points won which can better show how dominant a player is in a match. For model selection, we use AIC to compare models from one another (Wasserman 2004). For the individual models, we do not split the data into training and test sets due to the relatively low number of observations for the individuals. However, when the data becomes available, we would like to test these models on the 2018 grand slam results.

Few academic papers have been written about Nadal, Federer, or Williams. One paper studies Federer's odds of winning when Nadal suddenly withdrew from Wimbledon and shows that Federer was too heavily favored by bookmakers (Leitner, Zeileis, and Hornik 2009). One analysis of Williams shows how she has gotten better with age, even past the point when other greats began to decline, but the study does not look at surface type (Morris 2015).

Readers may object that we are looking at differences between grand slams, which each have their own time period, weather conditions, play time conditions, and "home court effects" instead of differences in surfaces alone. However, (1) grand slam data is the most readily available and most complete which makes it the best choice at the moment for modelling, (2) we adjust for these confounders where we can, and (3) analyzing the difference in the grand slams is still useful as they are considered to be the most prestigious events in tennis.

The rest of this paper is organized as follows. In Section Data we describe our grand slam tennis data. In Section Early Data Analysis we examine the data at a high level and use clustering whether to determine how the courts differ from one another. In Section Methods we describe our models with (1) complete data with few features and (2) partial data with more features. In Section Results we describe the results of our modelling and also examine the play of Nadal, Federer, and Williams. Finally in Section Discussion, we discuss future work and extensions or our models.

2 Data and EDA

2.1 Data

The primary data consists of 5080 matches split evenly over the four grand slams and the two leagues: ATP (men's) and WTA (women's). There are 5080 observations in this data set, which corresponds to a complete data set for the period of 2013-2017 for four grand slams, two, leagues, and 7 rounds. Each match has 80 attributes, many of which are redundant. We focus on the following attributes for both the winner and loser of the match: games won, points won, retirement, break points faced, break points saved, aces, country of origin, and player attributes. Additionally, we take into account the number of sets in a match, the surface type, and round of the tournament. A subset of the data is shown in Table 1.

The secondary data is partial point by point data for grand slam matches. In this data set, each row is a point in a match with details on who won the point, whether a player had a forced or unforced error, winner, ace, or net point win, and serve speed. There is also tournament info such as surface, year, and time start. Additional attributes include rally length, winner and final score of the match, retirement, and minutes played. However, these additional attributes are only available for about 1/8 of the data.

Table 1: Example of the complete grand slam data. It includes winner and loser attributes, match attributes, and tournament attributes. Not all attributes are shown here.

Winner	Tournament	Year	W. IOC	W. Points	W. Rank	L. Points	L. Rank
Serena Williams	Australian Open	2013	USA	52	3	18	110
Serena Williams	Australian Open	2013	USA	70	3	41	112
Roger Federer	Australian Open	2013	SUI	95	2	63	46
Roger Federer	Australian Open	2013	SUI	111	2	86	40
Rafael Nadal	Roland Garros	2013	ESP	140	4	115	59
Rafael Nadal	Roland Garros	2013	ESP	113	4	90	35

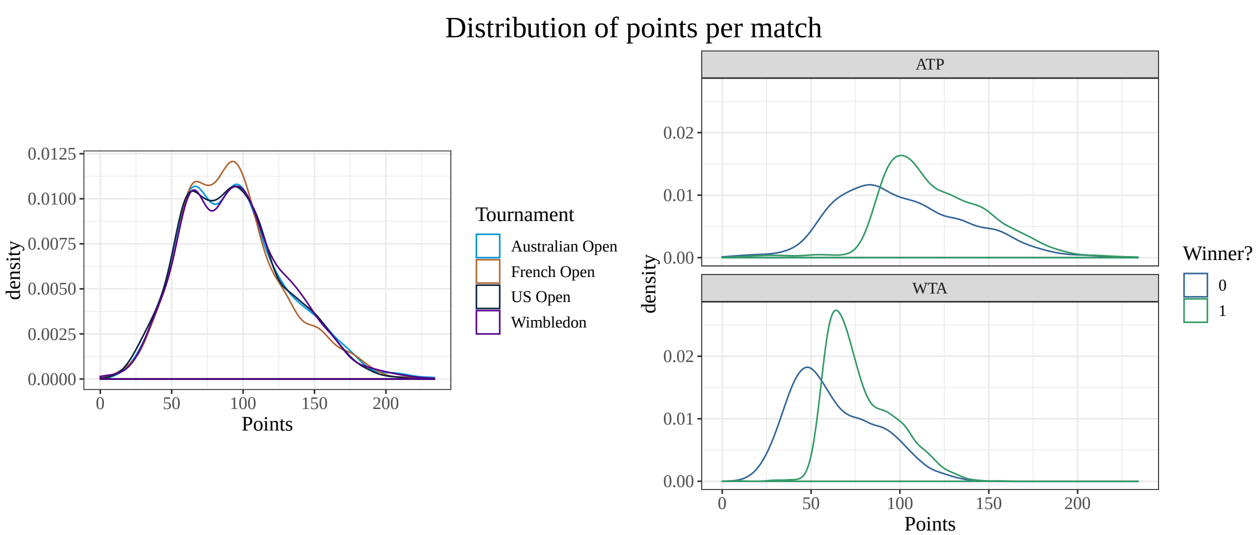
We aggregate the partial point by point data into partial match data, summing the total number of errors, winners, aces, net points, and taking the average service speed for each player in the match. We then join the partial match data with the complete data to find the score and outcome of the match. We do this by joining the features of tournament, year, player one, and player two. This final data set consists of 3977 observations (compared to the 5080 in the original grand slams). The median rank of the winners for the original data is 28, and the median rank for the winners of the partial data is 21. The same trend is true for the loser rank. This means that it is more likely for better players to have court by court data, which may effects our analysis for models when using this more detailed yet partial data. Additionally, we found that no women's point by point data was recorded for 2015.

All data is obtained from Jeff Sackmann's open website via the `R` package `deuce` (Sackmann 2018; Kovalchik 2017). All steps of our analysis from collection to dissemination are freely available online.

2.2 Early Data Analysis

2.2.1 Examining the distribution of points earned

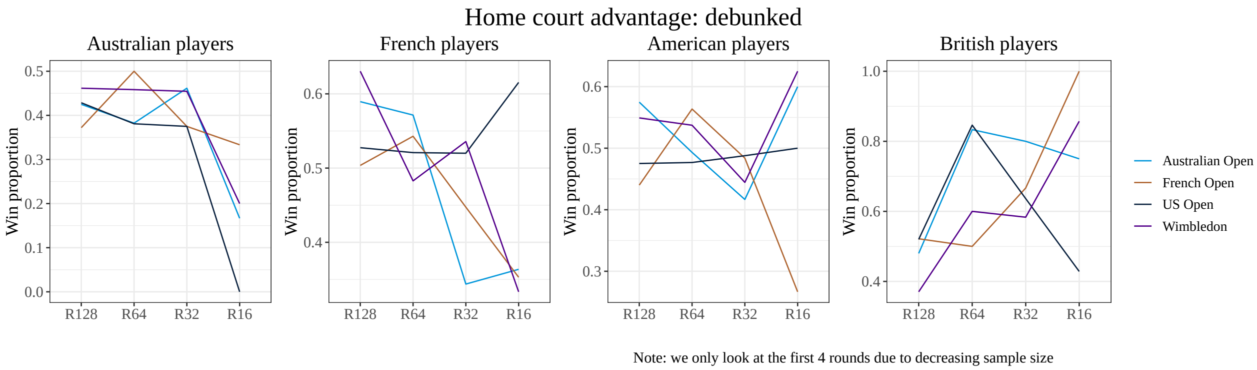
We first examine the distribution of points earned per match to assess normality. The distribution of points earned is approximately normal. This distribution is similar across tournament, with Wimbledon differing slightly from the other grand slams. As expected, there are more points earned in the WTA than the ATP due to the differing numbers of games played. Also unsurprisingly, the winners of the match tended to earn more points than the losers.



2.2.2 Home court advantage

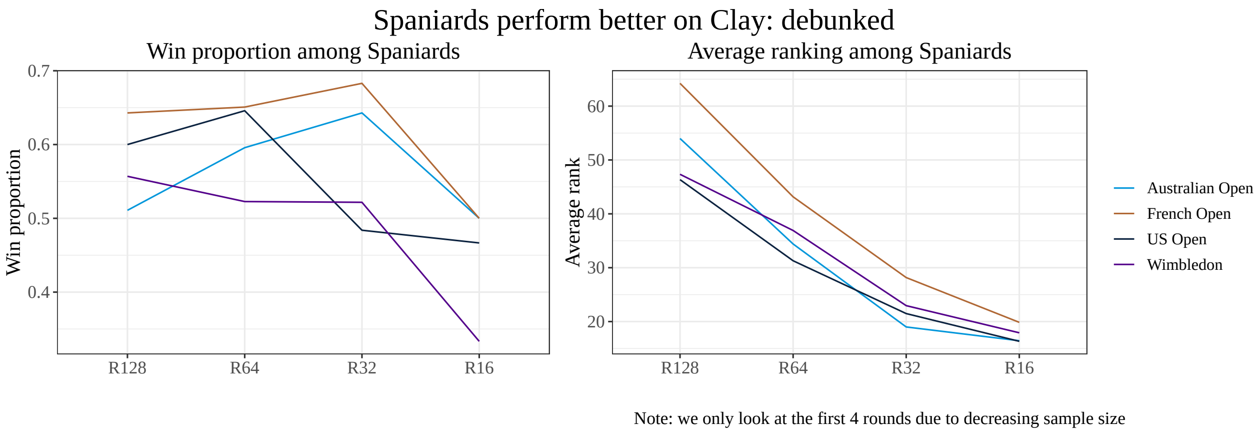
It is commonly thought that there is a home court advantage in grand slam games (SOURCE). In our data we find this to be true (i.e. French players win the French open more than French players win other slams). But, we also know that the home team is given preference for wild card bids (SOURCE) so potentially citizens of a particular country play in "their" tournament more often than they play in other tournaments. We also find this to be true in our data for France, The United States, and Australia (i.e. the proportion of French players in the French Open is greater than the proportion of French players in other slams).

Therefore, we want to see how the proportion of wins for the home country changes across the different tournaments. If there was really a home court advantage, the proportion of French wins each round would be higher at the French Open than the Australian Open, the US Open, and Wimbledon. The same would be true for Australia and the US. But, we see that this isn't the case. After accounting for the number of players from each country, we don't find a home court advantage in the grand slam.



2.3 Spaniards on clay

It is also commonly thought that Spaniards play better on clay. We are interested in whether Spaniards win the French Open more than they win in other tournaments. It does appear that Spaniards are winning the French Open more than they are winning other tournaments. But, this result is not significant. In addition, the ranking of Spaniards in the French Open is, on average, higher than other tournaments, which may help explain this common misconception, although this result is not significant either.



2.4 Tall players and aces

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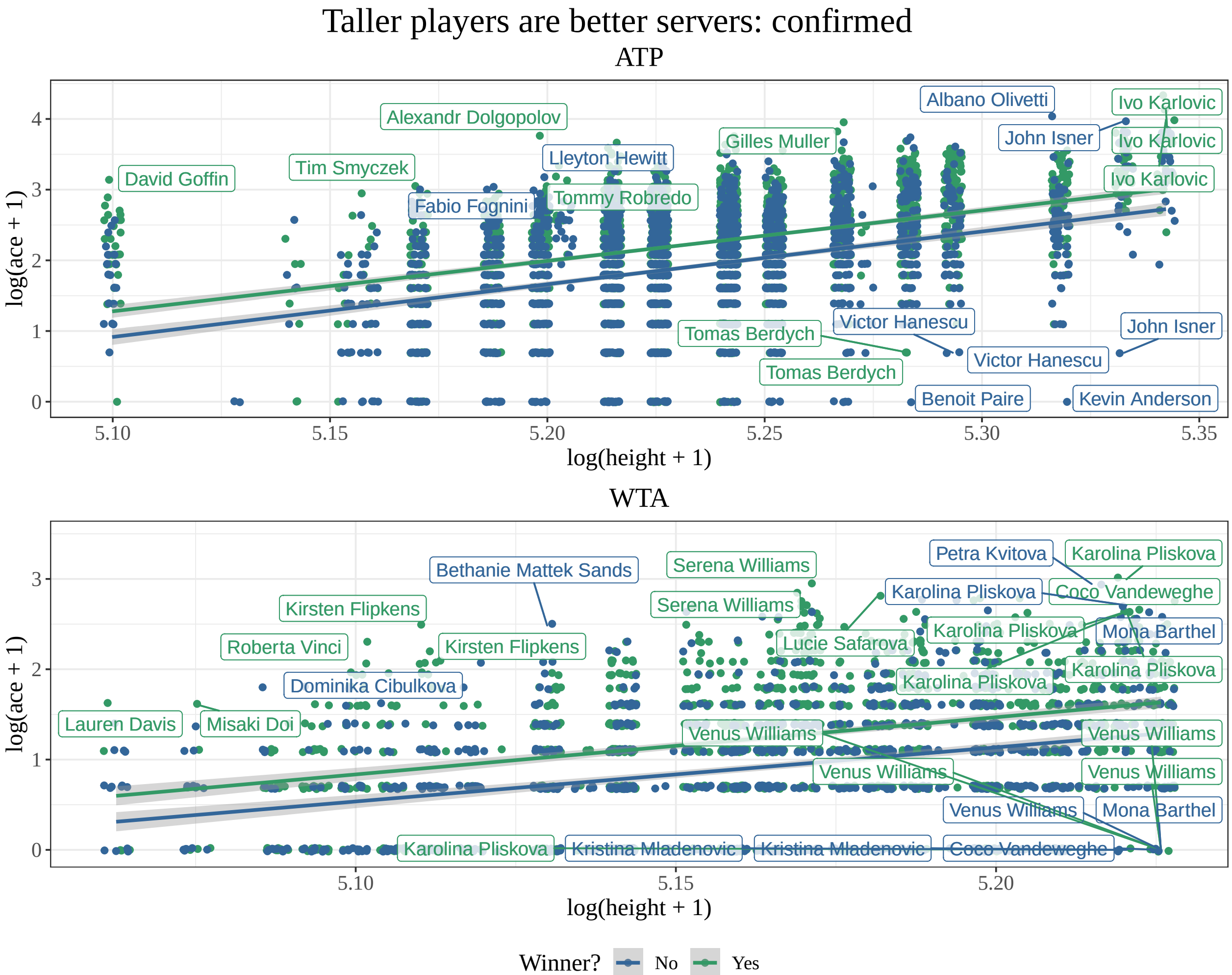


3 Methods

3.1 Mixed-effects models for all grand slams

We use a mixed-effects modeling approach to examine the grand slam results from all players from 2013-2017. This allows us to include fixed effects, which remain the same for each player, as well as include player-level effects. Since players appear multiple times in the data, the observations are not independent and including a player-level effect allows us to account for this dependence. It also provides a way to examine individual player effects.

3.1.1 Logistic regression for wins

We initially planned to use points won by an individual as our outcome variable, but found that it may not be an ideal measure of player performance. For instance, players may score few points in a match due to a poor performance, but they may also score few (relative) points if they win a short match. Modeling the number of points won is also complicated by the difference between length of matches for men (best of 5 sets)

Table 2: AIC summary of the seven logistic regression models fitted. Including individual effects for each grand slam, without including any country effects, leads to the best model fit according to AIC.

Model	AIC	EDF
nocourt_logistic	11398.51	69
base_logistic	11404.48	72
country_logistic	11374.86	19
ind_logistic	11395.69	79
ind_logistic_noioc	11354.20	19
ind_int_logistic	11365.07	13
ind_year_logistic	11366.68	23

and women (best of 3 sets). For these reasons, we chose to model whether a player won the match using a mixed-effects logistic regression approach, with the following predictors:

- `ioc_fac`: Country that the player represents
- `tournament`: Which Grand Slam the match was played at
- `late_round`: Indicator for whether the match occured in Round of 16 or later
- `rank`: Rank of player at the time of the match
    - Included on log scale
- `opponent_rank`: Rank of opponent at the time of the match
    - Included on log scale
- `year`: Factor variable with a level for each year included in the dataset (2013-2017)
- `atp`: Indicator that the match was played in the ATP league instead of the WTF league.

We considered the following models:

- No mixed effects
    - `nocourt_logistic`: $\text{logit}(\pi_i) = \beta X$, X does not include `tournament`
        - If this model fits well, court surface does not have an effect on winning probability
    - `base_logistic`: $\text{logit}(\pi_i) = \beta X$
        - If this model fits better than the mixed-effects models, performance does not depend on individual-level or IOC-level
- IOC-level effect
    - `country_logistic`: $\text{logit}(\pi_i) = \beta X + \beta_{T[i]} T$, $\beta_{T[i]} = \alpha_i C_i$
- Player-level effect
    - `ind_logistic`: $\text{logit}(\pi_i) = \beta X + \beta_{T[i]} T$, $\beta_{T[i]} = \alpha_i P_i$
        - Includes both a fixed-effect term for IOC and a player-level effect for each grand slam
    - `ind_logistic_noioc`: $\text{logit}(\pi_i) = \beta X + \beta_{T[i]} T$, where X does not include IOC and T represents indicators for tournament, $\beta_{T[i]} = \alpha_i P_i$
        - Simpler than the `ind_logistic` model, since we exclude IOC effects.
    - `ind_int_noioc`: $\text{logit}(\pi_i) = \beta X + \beta_{0[i]}$, where X does not include IOC, $\beta_{0[i]} = \alpha_i$
        - Similar to `ind_logistic_noioc`, but includes a single player-level effect rather than a player-level effect for each grand slam.
    - `ind_year_logistic`: $\text{logit}(\pi_i) = \beta X + \beta_{D[i]}$, where X does not include IOC and D represents indicators for years, $\beta_{Y[i]} = \alpha_i Y_i$
        - Rather than including a player-level effect for grand slam, includes a player-level effect for each year in the dataset

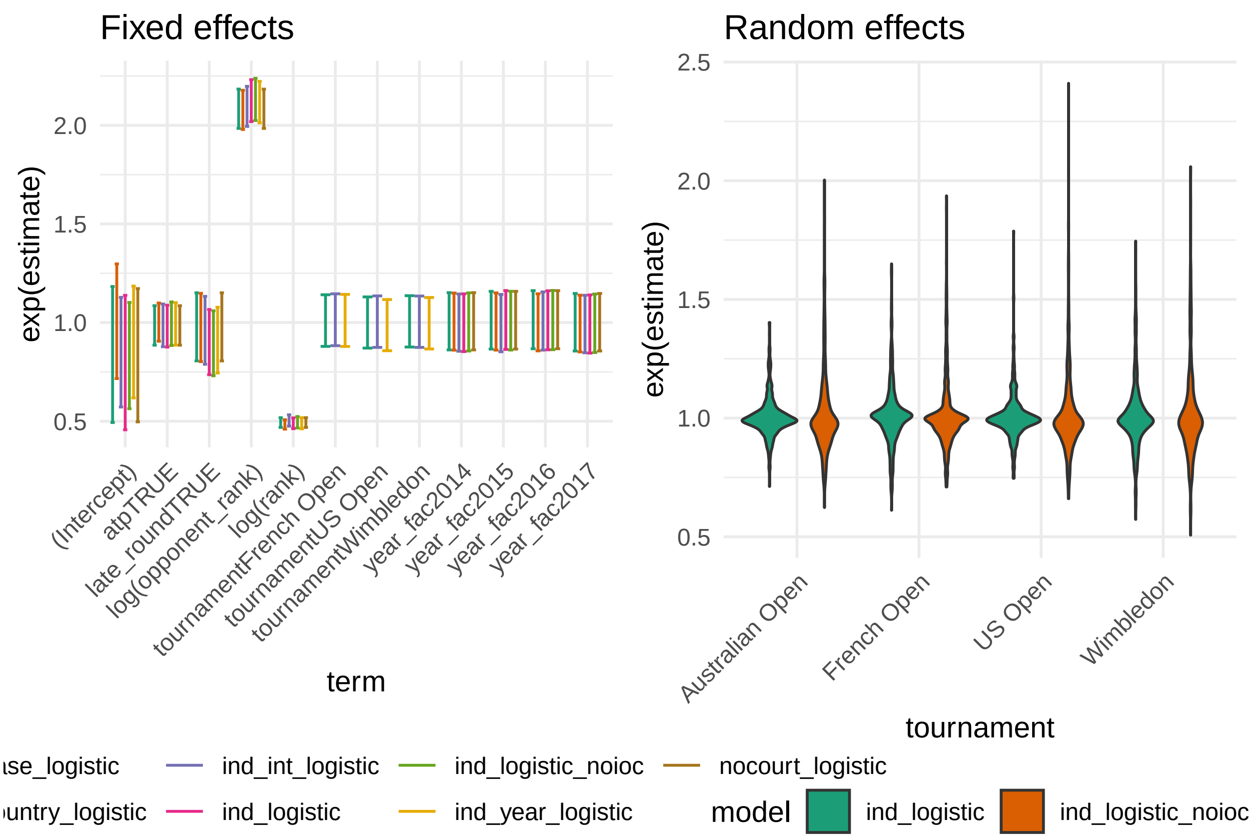
Based on AIC, the `ind_logistic_noioc` model performed best after accounting for the number of variables in the model.

If we examine the fixed effects for all seven models, and the random effects for the `ind_logistic` and `ind_logistic_noioc`, we see that the model structure chosen does not substantially change the parameter

Table 3: Correlation matrix for random effects

	Australian Open	French Open	US Open	Wimbledon
tournamentAustralian Open	1.00	0.00	0.94	0.93
tournamentFrench Open	0.00	1.00	-0.05	-0.36
tournamentUS Open	0.94	-0.05	1.00	0.88
tournamentWimbledon	0.93	-0.36	0.88	1.00

estimates. Across all models, `rank` and `opponent_rank` appear to best explain the probability of winning the match. Other fixed effects in the models, like whether it was a late round in the grand slam, the year, or ATP/WTF, were not found to be significantly different than zero.

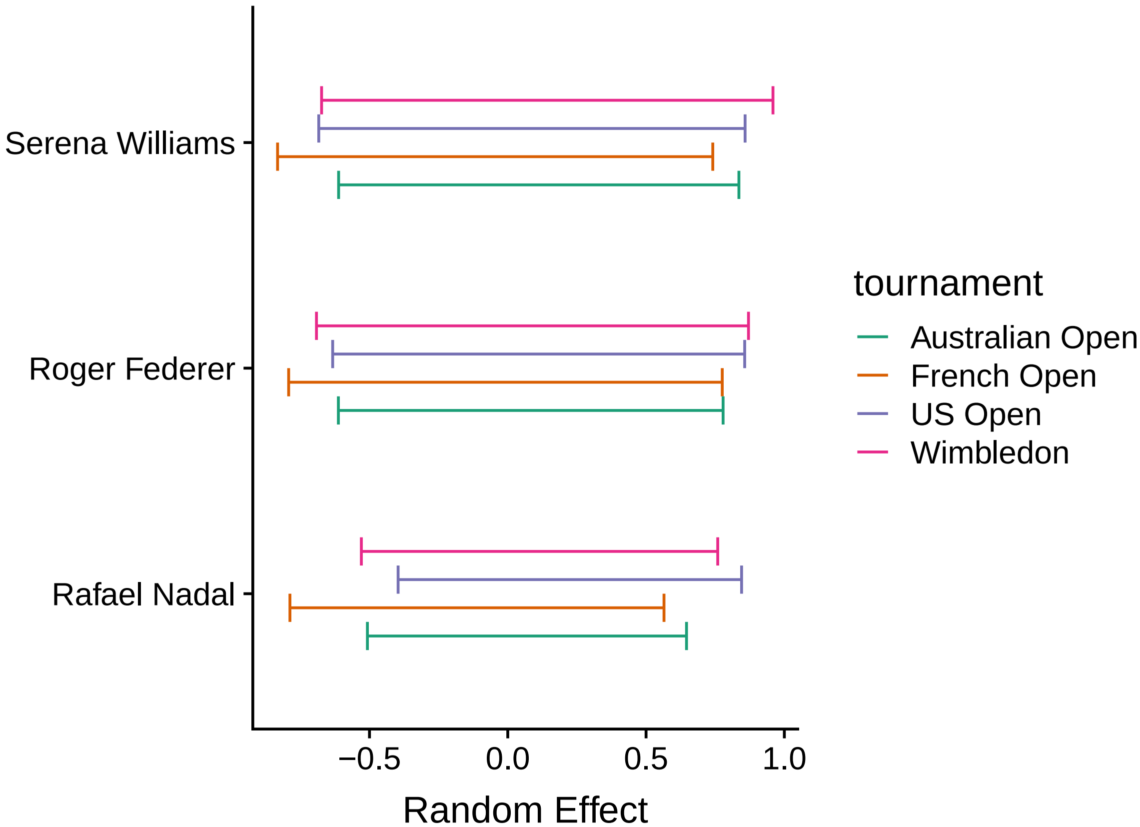


Looking at the `ind_logistic_noioc` random effects more in-depth, we see that the effects for the Austrialian Open, US Open, and Wimbledon are all quite correlated. This suggests that, after accounting for additional variables such as rank and opponent rank, differences in win probability are not detectable between the two types of hard court and grass.

If we look at our big three, their individual effects are not very informative (especially after taking uncertainty into consideration).

Table 4: Number of matches played for Nadal, Federer, and Williams from 2013-2017 at each of the grand slams.

Tournament	Nadal	Federer	Williams
Australian Open	20	28	30
French Open	29	14	23
US Open	21	22	26
Wimbledon	11	29	21
Total	**81**	**93**	**100**



3.1.2 Linear regression for other stats

Rank and opponent's rank clearly have the largest effect on predicting the winner. Looking at the individual effects for Serena, Nadal, and Federer; they actually fall near the average for all four grand slams. We now look at a similar model, but instead of using 'win' as the outcome variable, we'll look at game statistics that may be more sensitive to court surface – aces, winners, net points won, and unforced errors – and see if individual differences are detectable.

(some plots that show a) Serena/Federer are stronger servers, b) Federer makes fewer unforced errors at Wimbledon, c) something about Nadal)

3.2 Modelling players individually

In Table 4, we display the number of matches Nadal, Federer, and Williams have played from 2013-2017. Over that time span, Nadal won 6 grand slams, Federer won 1, Williams won 8. Despite this, Federer played more total matches in Nadal. All three players were absent for exactly three slams during this time period due to external factors (Nadal: (1 AO, 0 FO, 1 Wim, 1 USO), Federer: (0 AO, 2 FO, 0 Wim, 1 USO), Serena: (0 AO, 1 FO, 1 Wim, 1 USO)). Not unexpectedly, Nadal has the most wins on clay (29), Federer has the most wins at Wimbledon (29) and Williams the most on hardcourt (30 at AO and 21 at USO). Despite having played more matches than Nadal, Federer only has 1 grand slam to show compared to Nadal's six, which indicates that Federer made it deeper into tournaments on average but had difficulties winning the championships. For the WTA, Williams had her second most successful five-year span at the grand slams over this time period, winning 8.

We fit individual linear models for these three individuals (subsetting the partial data to their matches only), estimating the percent of points won in a match using forward-backwards stepwise regression. The lower model is the percent of points regressed on the opponent rank and indicator variables for surface with one court as the reference variable (in the below equation, the reference surface as FO),

$$E[Y_{\%pts.won}|X] = \beta_0+\beta_1 X_{opp.rank}+\beta_{2,AO}\mathcal{I}(X_{sur.} = AO)+\beta_{2,Wim.}\mathcal{I}(X_{sur.} = Wim.)+\beta_{2,USO}\mathcal{I}(X_{sur.} = USO).$$

The upper model is the lower model with the additional variables of winner to unforced error ratio (W/UE), average serve speed, percent aces, percent break points, percent net points won, and their interaction effects with court type. We do not display the full best fit models here, but they are available online. We do report the effects ($\pm 2\cdot$ Std. Err.) of significant variables and their interaction terms, adjusting for the other covariates. Since we postulated that Nadal is best on clay, Federer on grass, and Williams on hard court, specifically, AO, we use the FO, Wim., and AO, respectively, as the reference variable in the model. For Nadal and Federer we use forward-backwards stepwise regression, using AIC as the criterion, beginning with the full model. For Williams, we also use forwards-backwards stepwise regression but begin at the low model since she has more missing data.

3.3 Nadal: King of Clay: confirmed

Looking at Nadal individually, the model with the lowest AIC has positive coefficients for opponent rank, percent of aces, and percent of break points won. It has negative coefficients for AO, USO, and Wim. compared to FO and for percent of net points won. There are no interaction effects in this model.

The standardized residuals plot shown in Figure 1 (along with the other linear regression diagnostics plots not shown here) shows that the model is a good fit for predicting the percent of points won. For the significant variables ($\alpha = 95\%$), while adjusting for the other variables, we expect percent of points won to be 0.034 ± 0.028 less at the AO and 0.046 ± 0.033 less at Wim compared to the FO; to increase by $1.5 \times 10^{-4} \pm 1.3 \times 10^{-4}$ for a one unit increase in opponent rank; a $0.0012 \pm 5.6 \times 10^{-4}$ increase for a .01 increase in percent of break points won; and to decrease by $8.8 \times 10^{-4} \pm 5.5 \times 10^{-4}$ for a .01 increase in percent of net points won. As such, we see clear evidence Nadal performs better at the FO compared to the other grand slams.

3.4 Federer: Wimbledon extraordinaire: confirmed

For Federer, the model with the lowest AIC has negative coefficients for AO, FO, and USO. vs Wim., and W/UE, It has positive coefficients for opponent rank, percent of break points won, and all the interaction terms: court and W/UE.

The standardized residuals plot shown in Figure 1 shows that the model is a fairly good fit for predicting the percent of points won, but almost seem to see two clusters of residuals splitting with predicted point

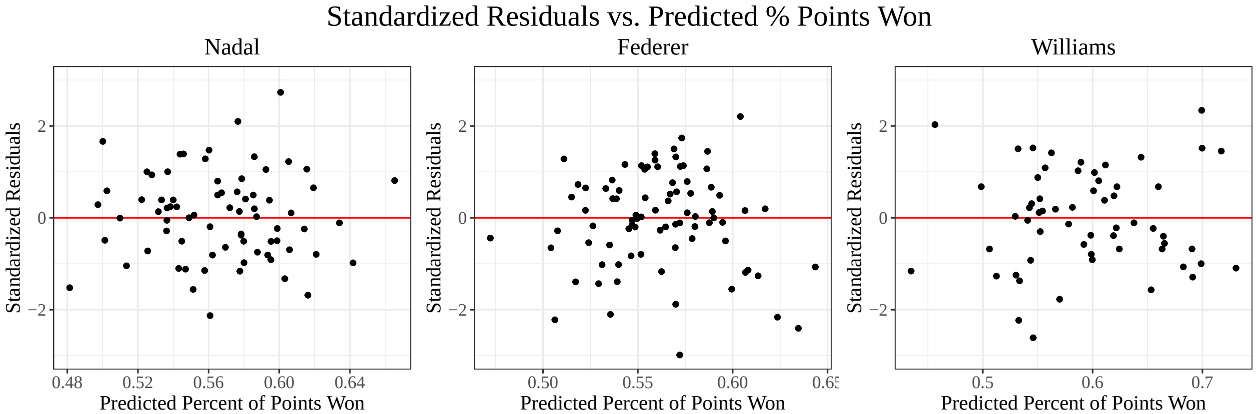


Figure 1: Standardized residuals vs percent of points won best fit model from forward-backwards stepwise regression for Nadal's, Federer's, and William's, respectively.}

percentage of about 0.63. For the significant variables ($\alpha = 95\%$), while adjusting for the other variables, we expect percent of points won to be 0.04 ± 0.034 less at AO compared to the Wim.; 0.058 ± 0.046 less at FO compared to the Wim.; 0.007 ± 0.093 less at USO compared to the Wim.; to decrease by 0.022 ± 0.019 for a one unit increase in W/UE at AO, 0.073 ± 0.066 for a one unit increase in W/UE at FO, and 0.041 ± 0.033 for a one unit increase in W/UE at USO. This is strong evidence that Federer performs best at Wimbledon but possibly does better at the USO, depending on his W/UE ratio.

3.5 Williams: hard court magician only: debunked

For Williams, the model with the lowest AIC selected 22 coefficients, which makes it the largest model of the three. Since Williams only has 59 observations, we believe this model is overfitting and so caution the reader to be wary of any inference from this model. It should also be noted that no women's point by point data was recorded for 2015 (in which she won three grand slams) so that year is excluded from this model. The coefficients are surface, opponent rank, W/UE, average serve speed, percent aces, percent break points won, percent net points won, and interaction effects with court and W/UE, court and average service speed, court and percent aces, and court and break points won.

The standardized residuals plot shown in Figure 1 shows that the model is a fair fit for predicting the percent of points won, but we tend to overestimate the percent of points won in comparison with Nadal's model. For the significant variables ($\alpha = 95\%$), while adjusting for the other variables, we expect percent of points won to increase by $4.8 \times 10^{-4} \pm 3.2 \times 10^{-4}$ for a 1 unit increase in oponent rank; to increase by 0.034 ± 0.02 for a one unit increase in W/UE at FO, to increase by 0.092 ± 0.045 for a one unit increase in W/UE at USO, and to increase by 0.0038 ± 0.066 for a one unit increase in W/UE at Wim. compared to AO; to increase by $2 \times 10^{-4} \pm 1.6 \times 10^{-4}$ for a .01 increase in percent of aces at the French Open; and to increase by 0.0024 ± 0.0011 for a .01 increase in percent of break points won at the French Open.

For this, we do not see that Williams is dominant on the hard courts compared to the other courts. Rather, she has great results on all the courts and has the titles to prove it. Her significant interaction effects show that if she focuses on acing her opponent and capitalizing on breakpoints, then she has a better chance at winning more points at the French Open than at the Australian Open.

4 Discussion

5 References

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