ELEC 4700 Assignment 2: Finite Difference Method

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1 Part 1: Rectangular Region with Isolating Conducting Sides

1. Code to solve for the electrostatic potential of a 1-D case with the following boundary conditions, V=V_0 at x=0 and V=0 at x=L, was developed. It was selected that V_0 = 5 volts, the length was 30 units. A plot of the solution is show below;

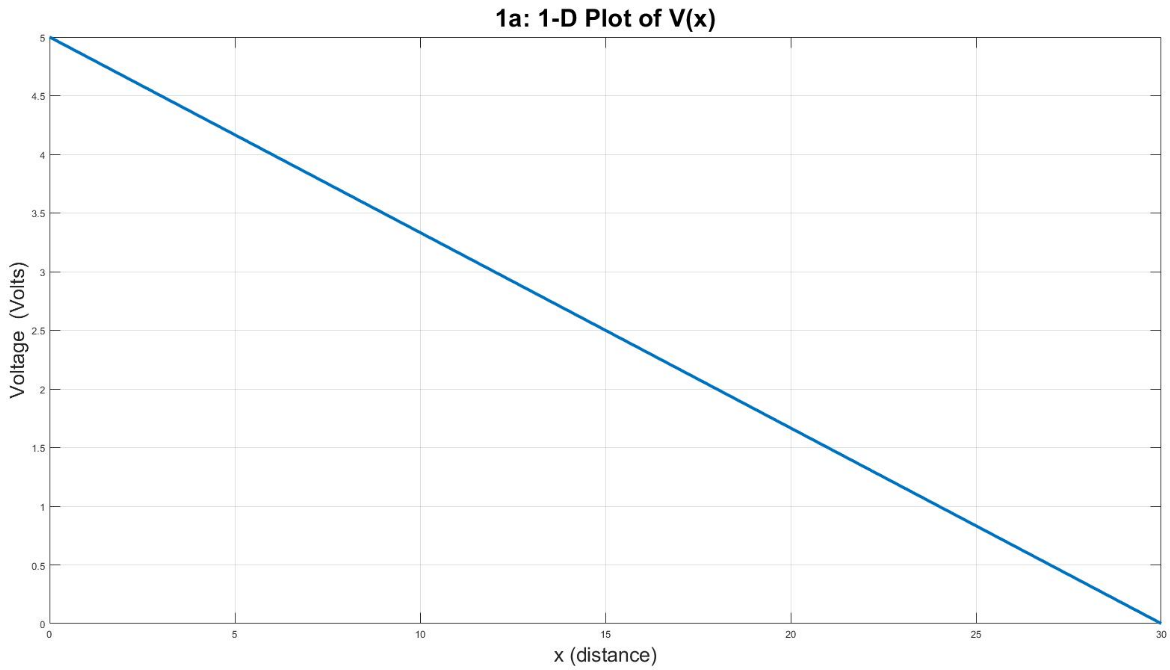

Figure 1: Plot of V(x)

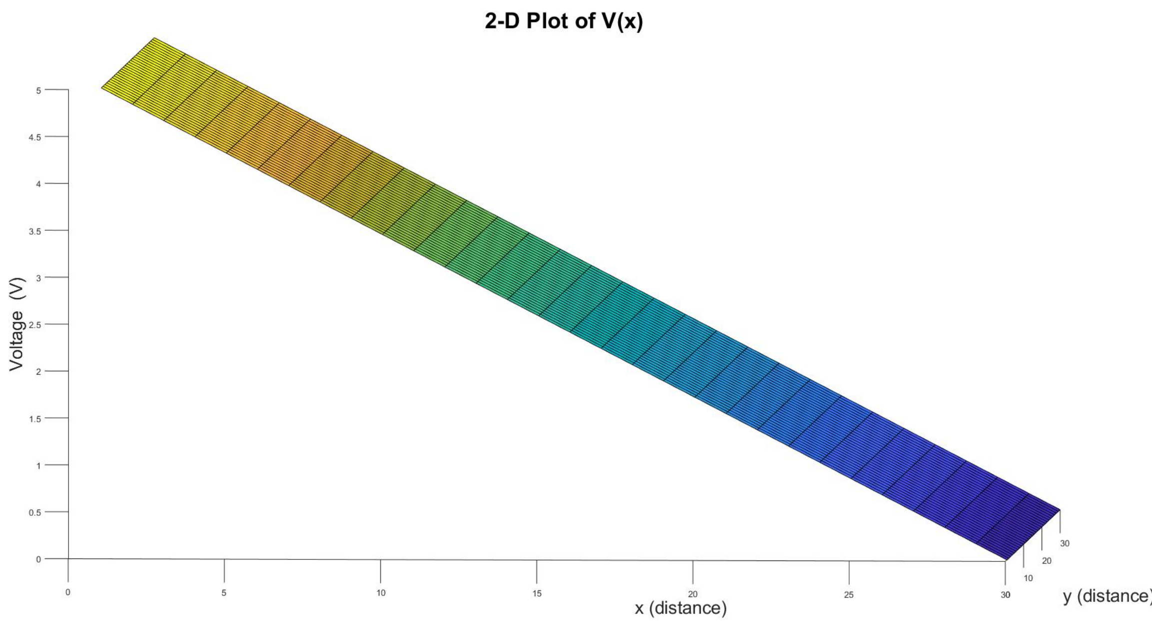

Figure 2: Surface plot of V(x)

2. The two-dimensional electrostatic potential was calculated according to the specified boundary conditions using two methods; finite difference and an analytical solution. For both methods, the length of the region was 30 units, the width was 20 units and V_0 was 5 volts. A plot of the simulated solution is shown;

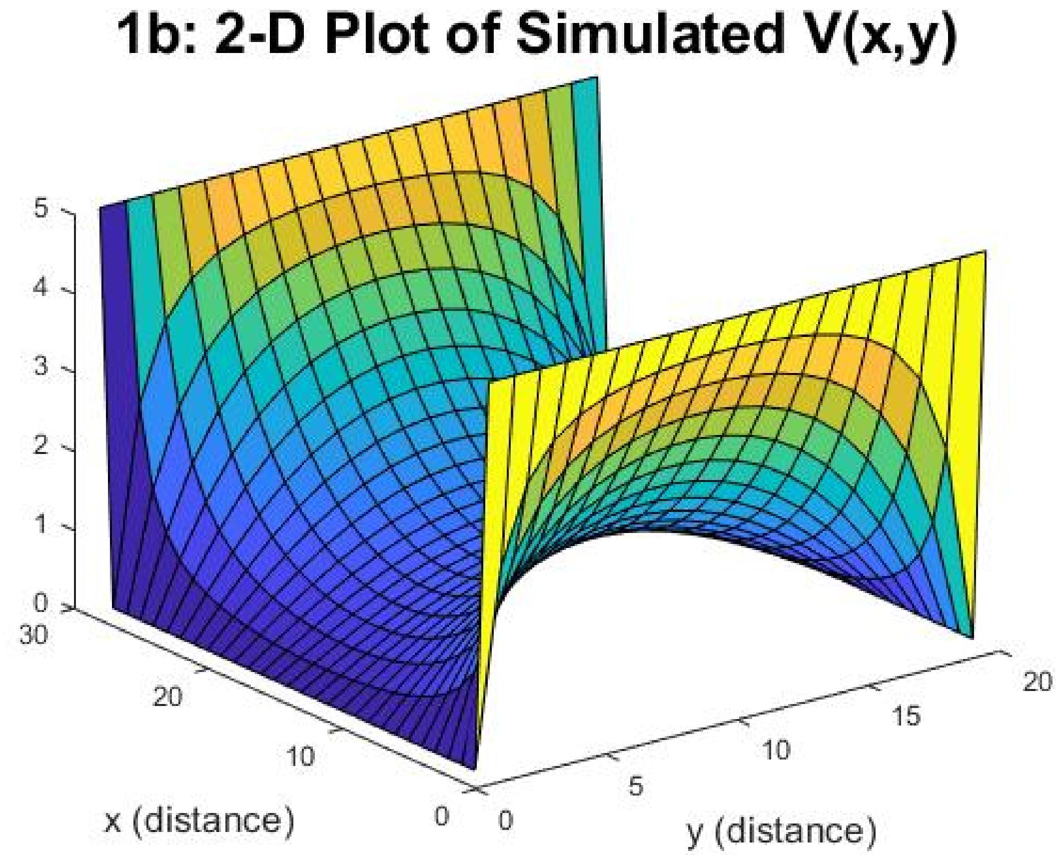

Figure 3: Simulated surface plot of V(x,y)

The following equation was used to calculate the analytical solution;

$$V(x,y) = \frac{4V_0}{\pi} \sum_{n=1,3,5...}^{\infty} \frac{cosh(\frac{n\pi x}{a})}{ncosh(\frac{n\pi b}{a})} sin\Big(\frac{n\pi y}{a}\Big) \tag{1}$$

where a is the width of the region and b is half of the length of the region. The solution was calculated up to n= 280 (140 iterations). The number of iterations is limited by the cosh term in the analytical solution. Matlab is able to evaluate coshx until the argument is less than or equal to $\pm$ 710. This is sensible as;

$$\lim_{x\to\pm\infty} cosh(x) = \infty$$

Thus, for the dimensions used in this problem, the series was stopped at n=280. Consequently, an infinite summation cannot be preformed to solve the analytical case. A plot of the analytical solution is shown;

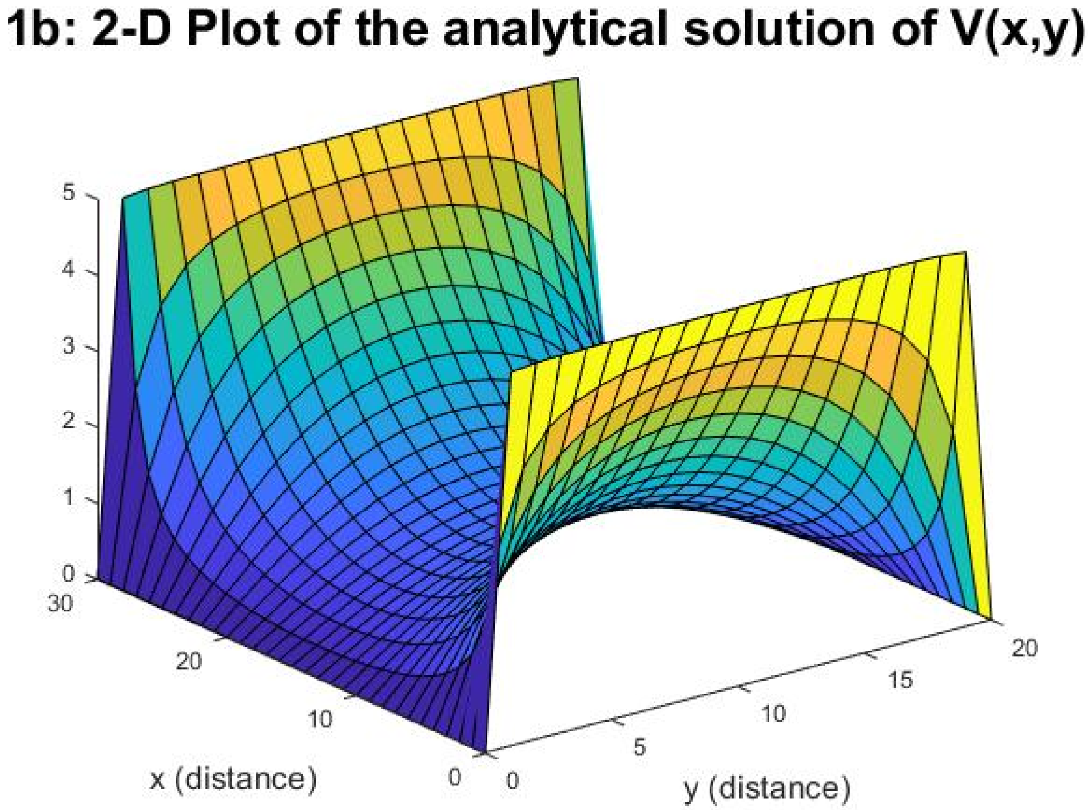

Figure 4: Analytical surface plot of V(x,y)

Decreasing the mesh size results in a longer cputime. For a mesh size of 1, the simulated solution requires 0.1719 seconds whereas the analytical solution requires 0.2031 seconds. Whereas for a mesh size of 0.05, the simulated solution requires 956.9063 seconds whereas the analytical solution requires 7.8125 seconds. The simulated solution converges to the result much slower than the analytical solution. Both methods converge to similar results.

The simulation is an acceptable solution in this case (a simple model). However, in general it is hard to implement for complex problems with intricate geometries.

2 Part 2: Addition of the bottleneck region

1. Highly resistive bottleneck regions were added to the simulation developed in part 1. The length of the region was 30 units, the width was 20 units and V_0 was 5 volts. The width and height of the bottleneck was chosen to be a third of the entire rectangular

region's width of height. A plot of the conductivity is shown;

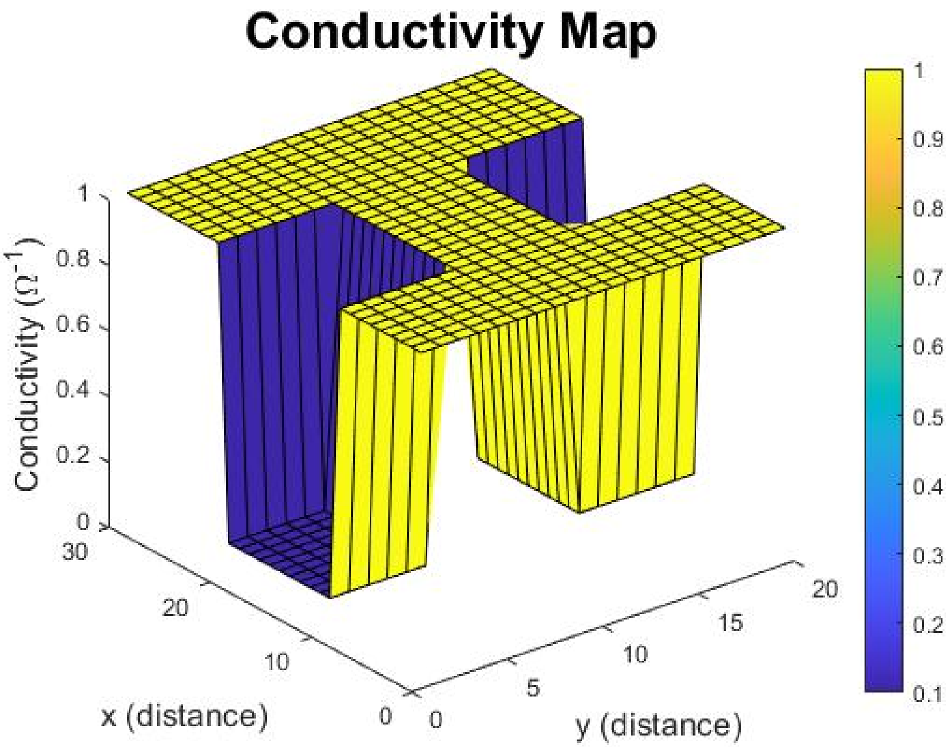

Figure 5: Surface plot of the conductivity

As expected, the conductivity within the bottleneck is 10^{-2} Ω^{-1} and is 1 Ω^{-1} outside. A surface plot of the voltage is shown;

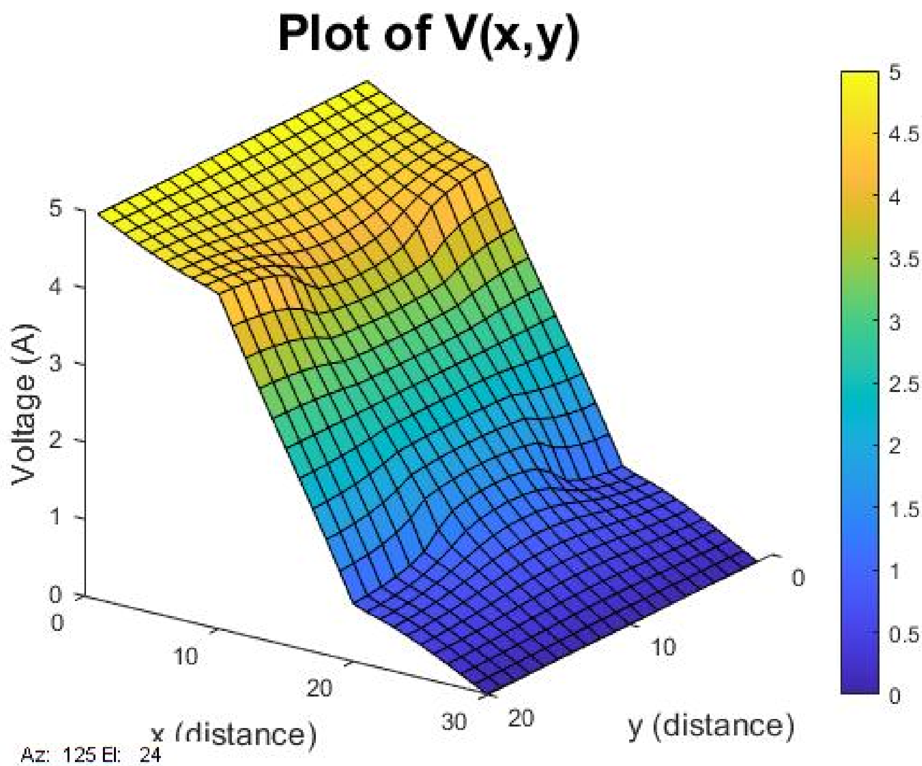

Figure 6: Surface plot of voltage, V(x,y)

The voltage is a maximum (V_0) at x=0 and is 0 V at x=L. The voltage distribution isn't uniform due to the bottleneck. A plot of the electric field is below;

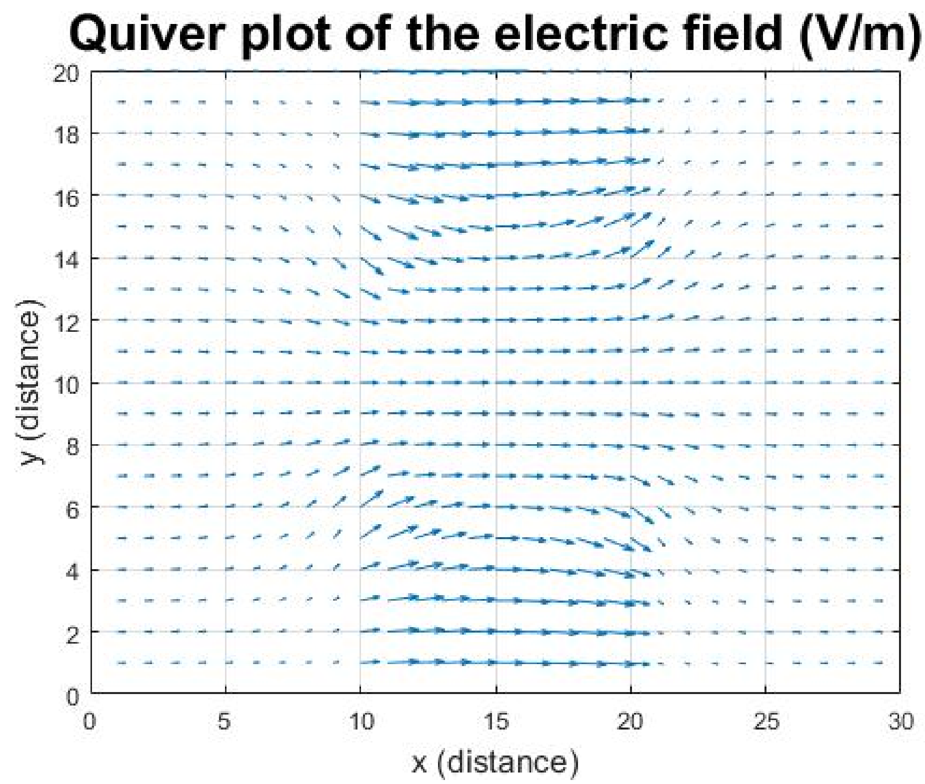

Figure 7: Quiver plot of electric filed, E(x,y)

The electric filed is a maximum inside and between the bottleneck. A plot of the current density is shown below;

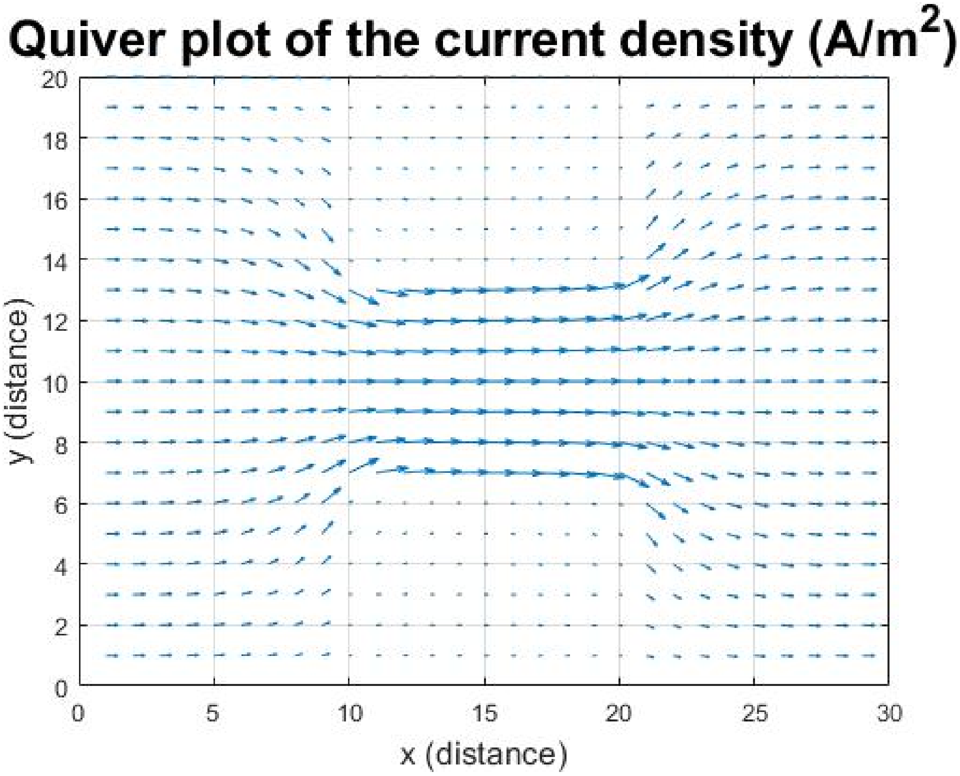

Figure 8: Surface plot of current density, J(x,y)

The current density is maximized between the bottleneck regions and is minimized within the bottleneck. This can be explained due to the large conductivity within the bottleneck. The current was calculated as $\boxed{2.0812A}$.

2. The mesh size was adjusted to investigate the relationship between mesh size and current. For the simulation, the length of the region was 30 units, the width was 20 units and V_0 was set to 1 volt. A plot of the calculated current at various mesh sizes is shown below;

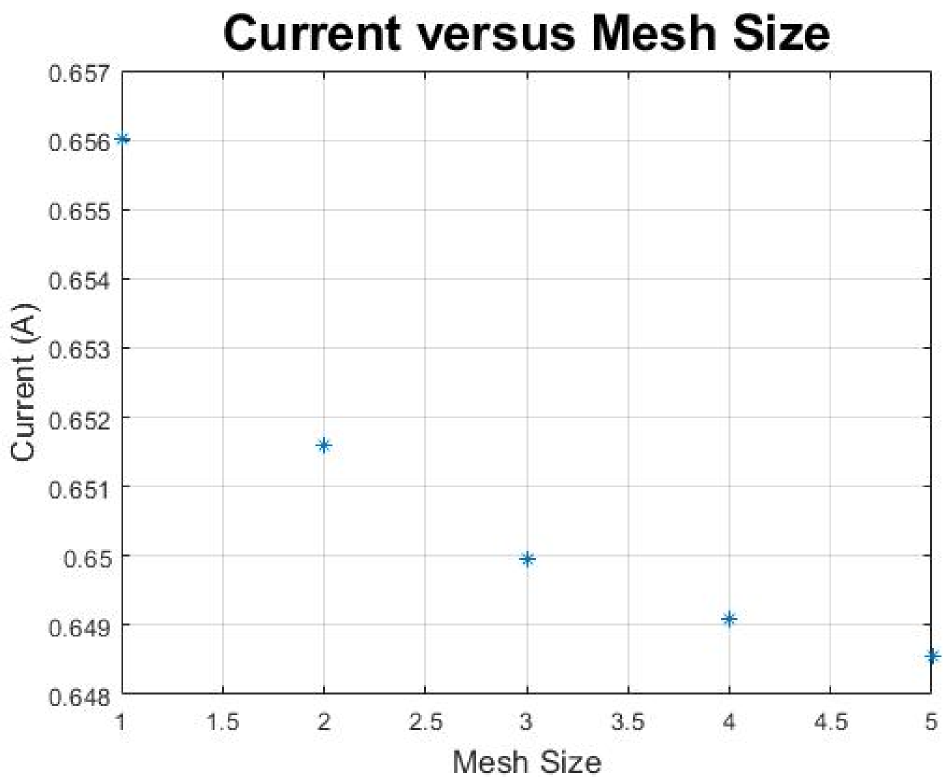

Figure 9: Plot of current for several mesh sizes (mesh density = 1/mesh size)

As the mesh size increased, the current decreased. Thus as the mesh density increases, the current increases. This is expected as a higher mesh density produces a more accurate result.

3. The bottleneck size was adjusted to investigate the relationship between bottleneck size and current. For the simulation, the length of the region was 30 units, the width was 20 units and V_0 was set to 1 volt. A plot of the calculated current at various mesh sizes is shown below;

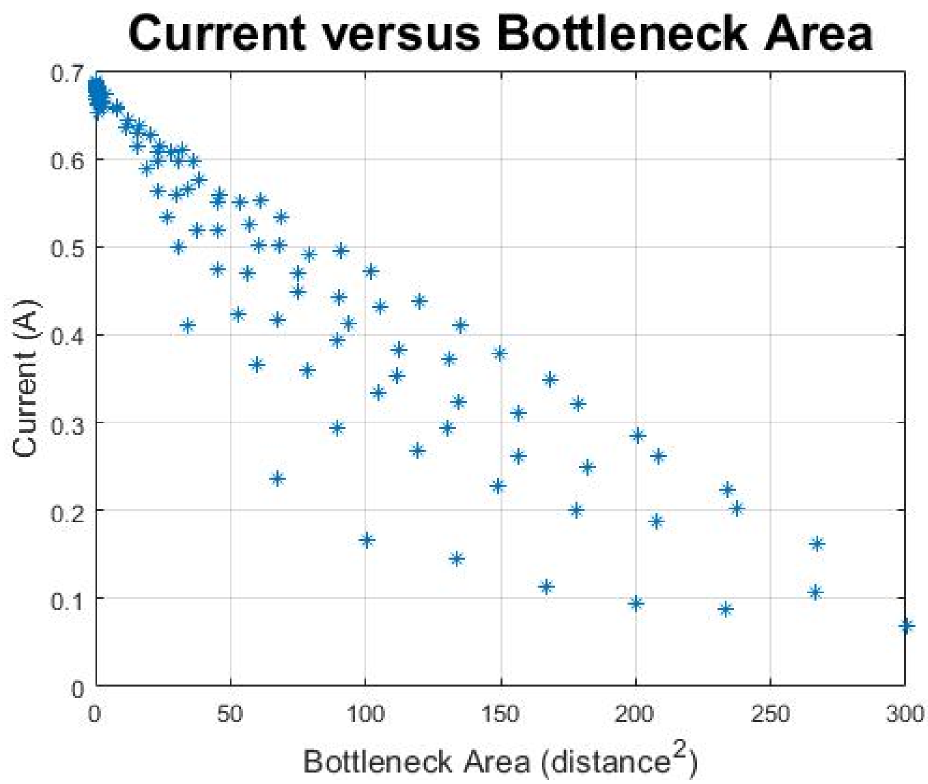

Figure 10: Plot of calculated current for various bottleneck areas

As the bottleneck size increases, the current decreases. The bottleneck controls how much current can flow between the region. Thus, as the bottleneck area increases, the current is restricted and is decreased. The above plot was obtained by varying both the bottleneck width and height, resulting in a wide spread of current values for various bottleneck area. Since the distance between the bottlenecks has the largest impact on the current, the plot below shows the calculated current when varying the bottleneck width (keeping the length constant).

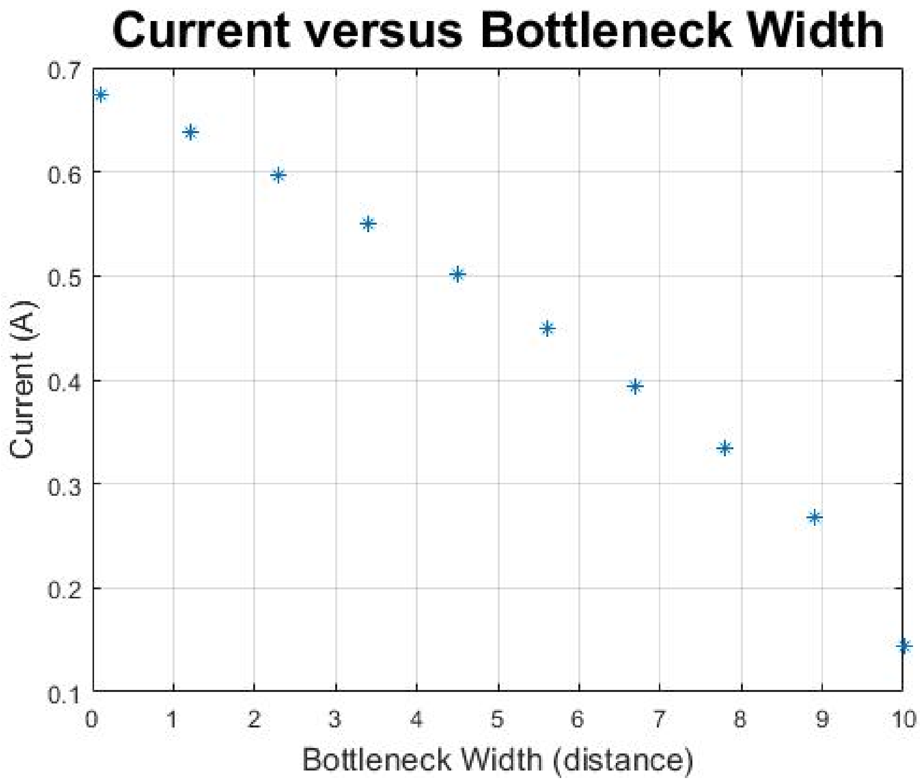

Figure 11: Plot of calculated current for various bottleneck widths

Thus, the current is inversely proportional to the bottleneck width.

4. The bottleneck conductivity was adjusted to investigate the relationship between conductivity and current. For the simulation, the length of the region was 30 units, the width was 20 units and V_0 was set to 1 volt. A plot of the calculated current at various mesh sizes is shown below;

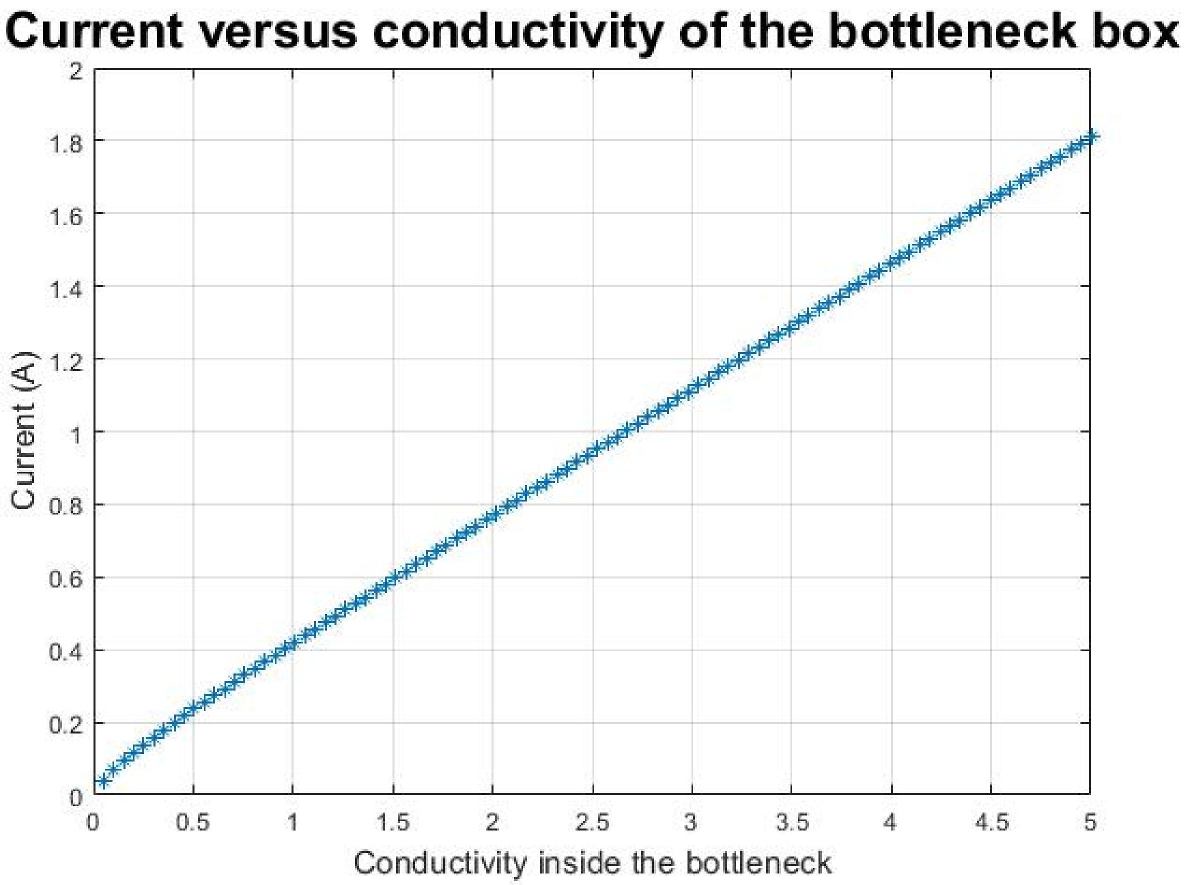

Figure 12: Plot of calculated current for various bottleneck conductivity

As the conductivity inside the bottleneck increased, the current increased.