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Modified Cour-Palais/Christiansen damage equations for double-wall structures

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Abstract

Ballistic limit equations (BLEs) are used for the damage prediction of spacecraft in a meteoroid and space debris environment. For double-wall configurations, the Cour-Palais/Christiansen equations have been modified to yield a general approach including the influence of the shield thickness. The ratio of shield thickness to particle diameter is considered as additional parameter in the equations. These equations result in the single-wall equation when the shield thickness approaches zero. The modifications can also be applied to other BLEs. Impact tests have been performed in order to validate the modified equations. In this paper, the test results are compared to the modified BLEs. Especially in the hypervelocity region, the new equations are more suitable for configurations with very thin shields than the original ones.

Keywords: Ballistic limit equation; Double-wall structure; Bumper shield thickness

1. Introduction

The design of protection schemes for spacecraft in a meteoroid and space debris environment is based on ballistic limit equations (BLEs). In case of double-wall structures where a bumper shield protects the main wall, BLEs published by Christiansen [1] are most common. They allow to compute either the needed back-up wall thickness in order to defeat a certain impacting particle or they are used to compute the critical particle diameter depending on the impact velocity for a given configuration. These equations yield reliable results for the design as well as for the damage prediction. Doing so they assume that the bumper shield is well designed which means its thickness is sufficient to destroy the impacting particle. On the other hand, these equations do allow to describe the influence of the shield thickness on the ballistic limit only at low velocities properly.

The aim of this work is to modify the existing equations allowing to consider the bumper shield thickness also for higher velocities. By this, the equations become more versatile and they are suitable to be used in combination with mathematical optimization procedures to find mass effective protection concepts [2]. The

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Nomenclature

Notations

d	diameter [cm]
F_2^*	factor (double wall)
k	parameter for failure mode
K_∞	material parameter
m	mass [g]
$r_{S/D}$	ratio of back wall thickness
S	spacing [cm]
t	thickness [cm]
v	velocity [km/s]
α	impact angle
ρ	density [g/cm$_3$]
ϱ	areal density [kg/mm^2]
σ	yield stress [ksi]

Subscripts

lim	limit
n	normal
p	particle
s	shield
tot	total
w	back wall

modifications presented are an improvement of an approach, published in [3]. Experiments were performed, in order to verify the suggested modifications.

2. Modifications of the ballistic limit equations

The BLEs, given by Christiansen are [1] as follows:

$$d_\mathrm{p} = \left(\left(t_\mathrm{w}\left(\frac{\sigma_\mathrm{w}}{40}\right)^{0.5} + t_\mathrm{s}\right)\Big/(0.6(\cos\alpha)^{5/6}\rho_\mathrm{p}^{0.5}\upsilon^{2/3})\right)^{18/19}$$
$$\text{for } \upsilon_\mathrm{n} < 3\,\mathrm{km/s}, \tag{1}$$

$$d_\mathrm{p} = 3.918\, t_\mathrm{w}^{2/3}\rho_\mathrm{p}^{-1/3}\rho_\mathrm{p}^{-1/9}\upsilon_\mathrm{n}^{-2/3}S^{1/3}\left(\frac{\sigma_\mathrm{w}}{70}\right)^{1/3}$$
$$\text{for } \upsilon_\mathrm{n} > 7\,\mathrm{km/s}. \tag{2}$$

For velocities between 3 and 7 km/s a linear interpolation is used (see Fig. 1). The yield stress, σ_w has to be given in [ksi].

If rearranged, Eq. (2) gives the sizing equation allowing to compute the back-up wall thickness:

$$t_\mathrm{w} = 0.178 m_\mathrm{p}^{1/2}\rho_\mathrm{s}^{1/6}\upsilon_\mathrm{n}S^{-1/2}\left(\frac{70}{\sigma_\mathrm{w}}\right)^{1/2}$$
$$\text{for } \upsilon_\mathrm{n} > 7\,\mathrm{km/s}. \tag{3}$$

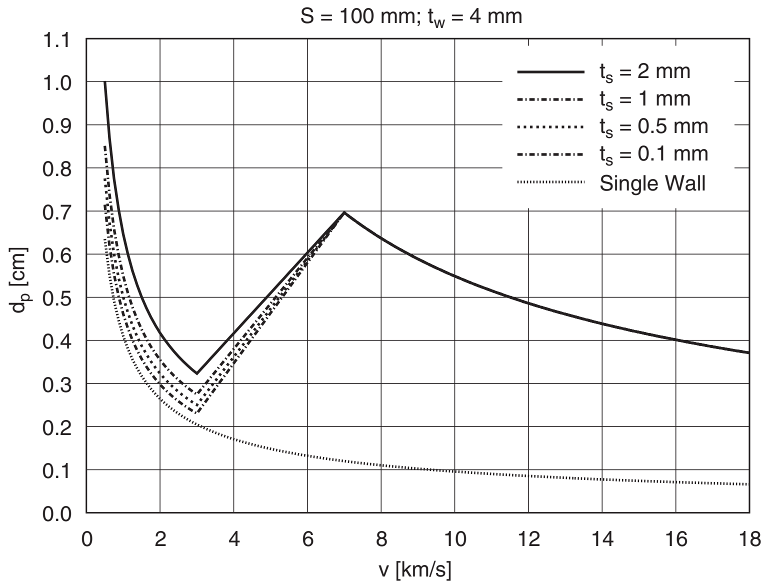

Fig. 1. Critical particle diameter as a function of impact velocity for Christiansen equations and various shield thicknesses.

In addition, for the shield thickness the following sizing equations are given:

$$t_s = 0.25 d_p \left(\frac{\rho_p}{\rho_s}\right) \quad \text{when} \quad S/d_p < 30, \tag{4}$$

$$t_s = 0.20 d_p \left(\frac{\rho_p}{\rho_s}\right) \quad \text{when} \quad S/d_p \geqslant 30. \tag{5}$$

For a double-wall structure with a 4 mm thick back-up wall, 100 mm spacing and different shield thicknesses the result of these equations is presented in Fig. 1. The typical three velocity regions (low velocity, shatter velocity and hypervelocity) are shown in the diagram. It can be seen in Fig. 1 and from Eq. (2) that the shield thickness does not affect the result in the hypervelocity region. Christiansen states that the equation is only valid for shields thicker than (0.20–0.25) d_p.

In order to describe the influence of the shield thickness three modifications are introduced:

- a different BLE for the low-velocity region is derived;
- the limit velocity between the low-velocity and the shatter region is dependent on the shield thickness/particle diameter ratio (t_s/d_p);
- in the hypervelocity region the back-up wall thickness has to be increased if $(t_s/d_p) < (0.20-0.25)$.

The suggested modifications are based on results published in literature.

2.1. Low-velocity region

Based on results published by Gehring et al. [4] the crater depth in a protected semi-infinite target plus the protection shield thickness is equal to the crater depth in a semi-infinite target alone (Fig. 2):

$$p_{\infty 1} = p_{\infty 2} + t_s. \tag{6}$$

For single thin sheets very often BLEs are given, based on the crater depth in a semi-infinite target:

$$t_w = k p_\infty, \tag{7}$$

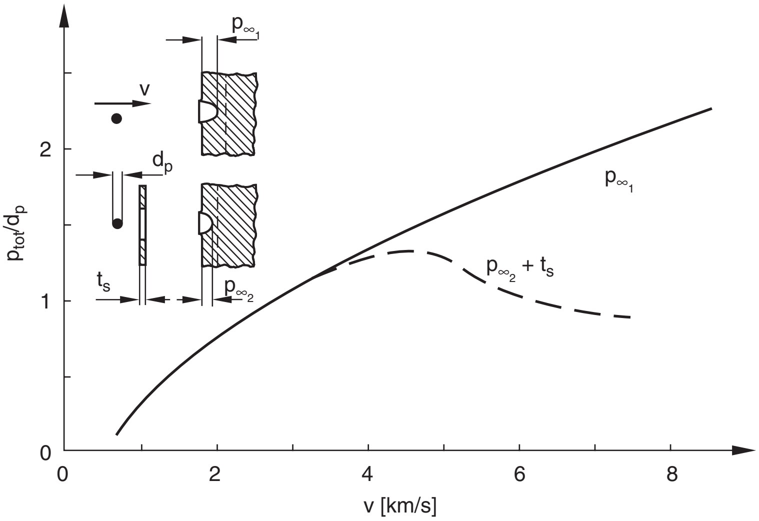

Fig. 2. Crater depths in semi-infinite targets [4].

Table 1
Failure modes [5]

k	Failure mode
>3.0	No spallation on the rear side
>2.2	No detached spall on the rear side
>1.8	No perforation

where k is dependent on the failure mode as presented in Table 1 [5]. For a protected wall in the low-velocity region, the following equation can thus be derived:

$$t_w = kp_{\infty 2} = k(p_{\infty 1} - t_s). \tag{8}$$

For p_∞ several equations are published. The following equation from [6] is used here:

$$p_\infty = K_\infty m_p^{0.352} \rho^{1/6} \upsilon_n^{2/3}, \tag{9}$$

where K_∞ is dependent on the target material. For aluminum alloys it is equal to 0.42 [6]. If a spherical particle is assumed, the following BLE results for the low-velocity region ($\upsilon_n < \upsilon_{lim}$):

$$d_p = \left(\frac{t_w/k + t_s}{0.796 K_\infty \rho_p^{0.518} \upsilon_n^{2/3}} \right)^{18/19} \quad \text{for} \ \ \upsilon_n < \upsilon_{lim}. \tag{10}$$

2.2. Limit velocity: low velocity—shatter velocity

In the original BLEs, the shield thickness does not affect the limit velocity between low velocity and shatter region (see Fig. 1). In 1965, Maiden et al. [7] published results presented in Fig. 3. For a bumper shield-protected semi-infinite target the total penetration depths p_{tot} (shield thickness plus crater depth in a semi-infinite target) versus velocity are shown for different (t_s/d_p)-ratios. Above a certain velocity (limit velocity), the crater depth decreases as a result from the fragmentation of the particle. It can be seen that the limit velocity increases with decreasing (t_s/d_p)-ratio. This was confirmed recently by Pikutowski [8]. In Fig. 4 from [8] the velocity leading to first fragmentation of spherical particles is shown versus the (t_s/d_p)-ratio. The crosses in Fig. 4 are the maximum values of the crater depth for the three different (t_s/d_p)-ratios in Fig. 3. A good correlation can be seen between both results achieved independently. The limit velocity should be above

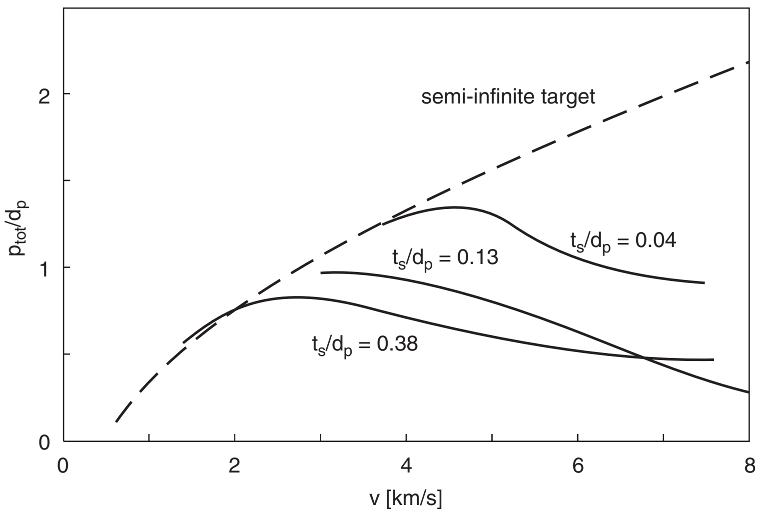

Fig. 3. Total penetration depth versus velocity [7].

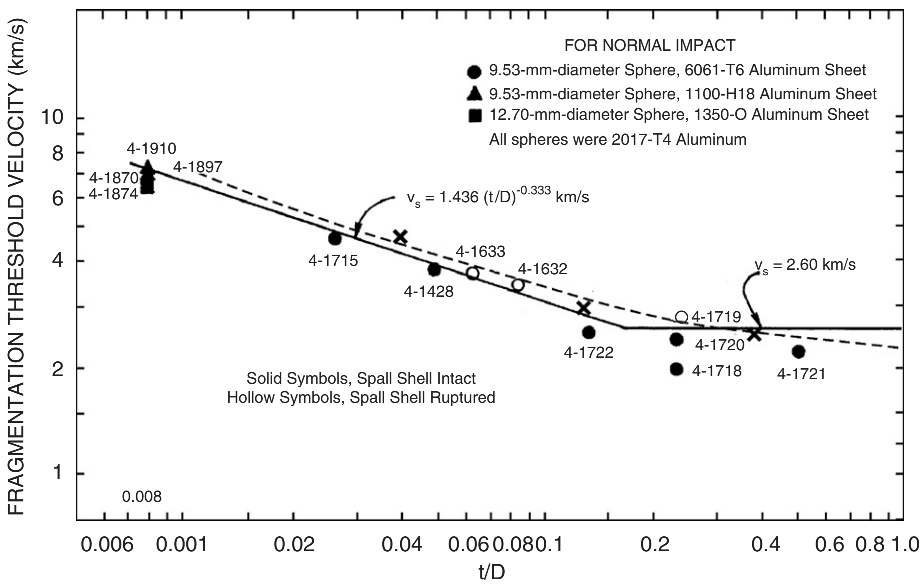

Fig. 4. Limit velocity depending on the (t_s/d_p)-ratio [8].

Pikutowski's limit-curve as it describes the onset of particle fragmentation. A regression results in the following equation for the limit velocity depending on the (t_s/d_p)-ratio (dashed line in Fig. 4):

$$\upsilon_{\lim} = 1.853 + 0.397\left(\frac{t_s}{d_p}\right)^{-0.565}. \tag{11}$$

The presented results were achieved for aluminum projectiles and aluminum targets and are not valid for other materials.

2.3. Hypervelocity region

It is expected that in the hypervelocity region the total wall thickness (shield thickness and back-up wall thickness) will have a minimum at a certain shield thickness as shown in Fig. 5 [9]. From Fig. 5 it can be concluded that the back-up wall thickness is nearly constant above a (t_s/d_p)-ratio of 0.25. This is where the Christiansen equation is valid. Thinner shields lead to a sharp increase in the back-up wall thickness. If the shield thickness approaches zero the back-up wall acts as a single wall. In [3] a factor F_2^* depending on (t_s/d_p) was introduced to take this into account. The factor F_2^* models the behavior below $(t_s/d_p)_{crit}$ with the following boundary conditions:

(1) at $(t_s/d_p)_{crit}$ it converges into the double-wall equation) $\Rightarrow F_2^* = 1.0$
(2) the convergence should be steady $\Rightarrow dF_2^*/d(t_s/d_p) = 0.0$ at $(t_s/d_p)_{crit}$
(3) at $t_s = 0$, the calculated particle diameter should be the same as for the single wall equation of the back-up wall

This leads to a general formulation of F_2^*:

$$\begin{aligned} &F_2^* = 1; && \left(\frac{t_s}{d_p}\right) \geqslant (t_s/d_p)_{crit}; \\ &F_2^* = r_{S/D} - 10\frac{t_s}{d_p}(r_{S/D} - 1) + 25\left(\frac{t_s}{d_p}\right)^2 (r_{S/D} - 1); && \left(\frac{t_s}{d_p}\right) < (t_s/d_p)_{crit}. \end{aligned} \tag{12}$$

The coefficient $r_{S/D}$ is the ratio between the required back-up wall thickness for a shield thickness, $t_s = 0$ and the back-up wall thickness for $t_s/d_p = (t_s/d_p)_{crit}$ at a velocity of 7 km/s:

$$r_{S/D} = \frac{t_w,\ \text{required}\ (t_s = 0)}{t_w, \text{reqiured}\left(t_s/d_p = (t_s/d_p)_{crit}\right)}, \quad \text{at}\ \upsilon = 7\,\text{km/s}. \tag{13}$$

The factor F_2^* is presented in Fig. 6 dependent on the (t_s/d_p)-ratio.

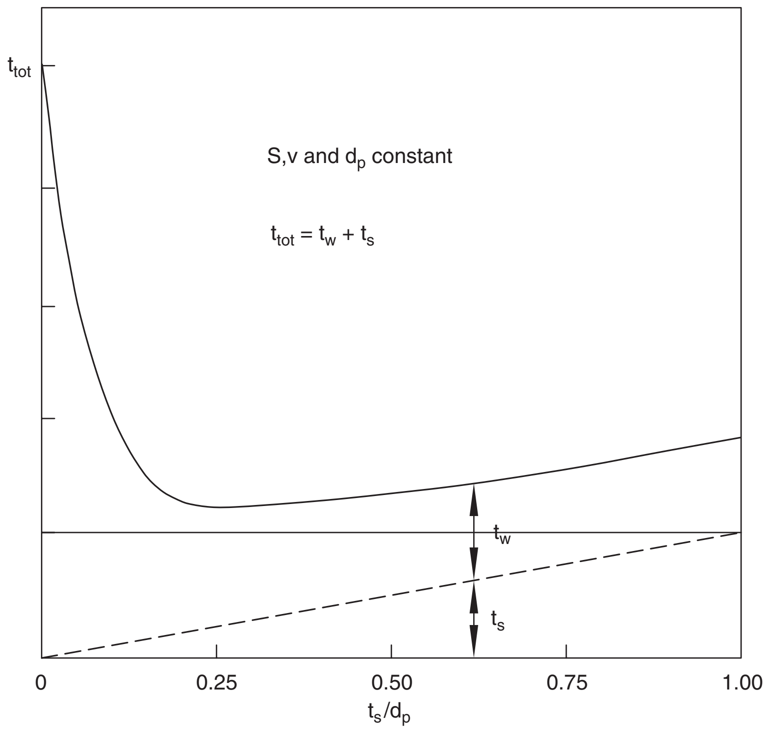

Fig. 5. Required total thickness versus shield thickness to particle diameter ratio [9].

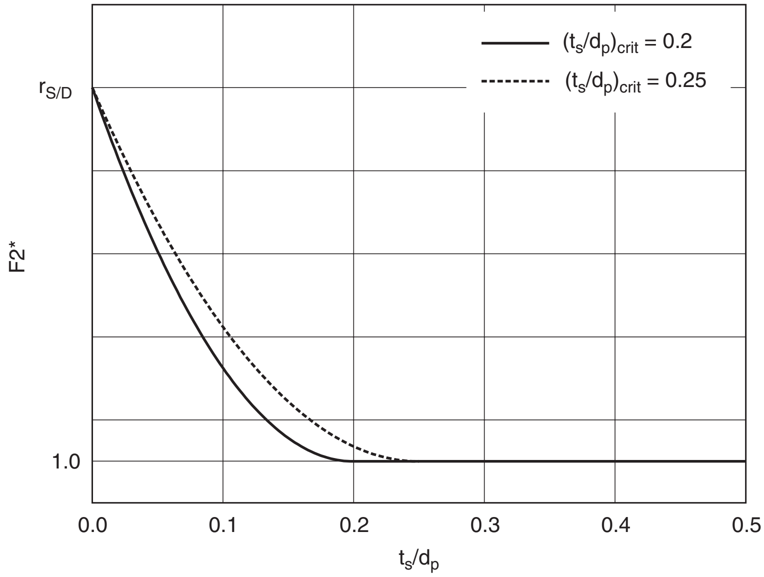

Fig. 6. F_2^* depending on (t_s/d_p) for $(t_s/d_p)_{crit} = 0.200; 0.250$.

2.4. Modified ballistic limit equations

With the developed modifications the influence of the shield thickness can be described more consistently. The BLEs change as follows: Eq. (1) is replaced by Eq. (10) which is applicable if $0 < \upsilon_n < \upsilon_{lim}$ as defined in Eq. (11). The change of Eq. (3) for the back-up wall thickness is straight forward. The original equation has to be multiplied by F_2^*:

$$t_w = 0.178 F_2^* m_p^{1/2} \rho_s^{1/6} \upsilon_n S^{-1/2} \left(\frac{70}{\sigma_w}\right)^{1/2} \quad \text{for } \upsilon_n > 7\,\text{km/s}. \tag{14}$$

Finally, Eq. (2) has to be multiplied by $F_2^{*-2/3}$:

$$d_p = 3.918 F_2^{*-2/3} t_w^{2/3} \rho_p^{-1/3} \rho_s^{-1/9} \upsilon_n^{-2/3} S^{1/3} \left(\frac{\sigma_w}{70}\right)^{1/3} \quad \text{for } \upsilon_n > 7\,\text{km/s}. \tag{15}$$

As F_2^* is dependent on d_p, this new equation becomes nonlinear and d_p has to be computed by an iterative procedure where the starting value for F_2^* is equal to 1.0.

Fig. 7 shows the computed critical particle diameter for the modified Christiansen BLEs discussed above. The modifications clearly show the expected convergence towards the single-wall equation.

In this work the modifications are applied to the Christiansen BLEs. As they are general, it should be no problem to use them in combination with other BLEs as well.

3. Experimental verification

In order to verify the modified BLEs some tests were performed at Ernst–Mach-Institut (EMI). Two configurations with different bumper shield thicknesses were tested. The tests are summarized in Table 2. The bumper shields are made of Al 2024T3 and the back-up walls of Al 7075 with 75 ksi yield strength in tension. The projectiles consist of Al 99.9%.

In Fig. 8, the original and the modified BLEs are plotted together with the test results for the configuration with 1 mm shield thickness. The failure of the test with a 5 mm sphere at 7.07 km/s was not predicted.

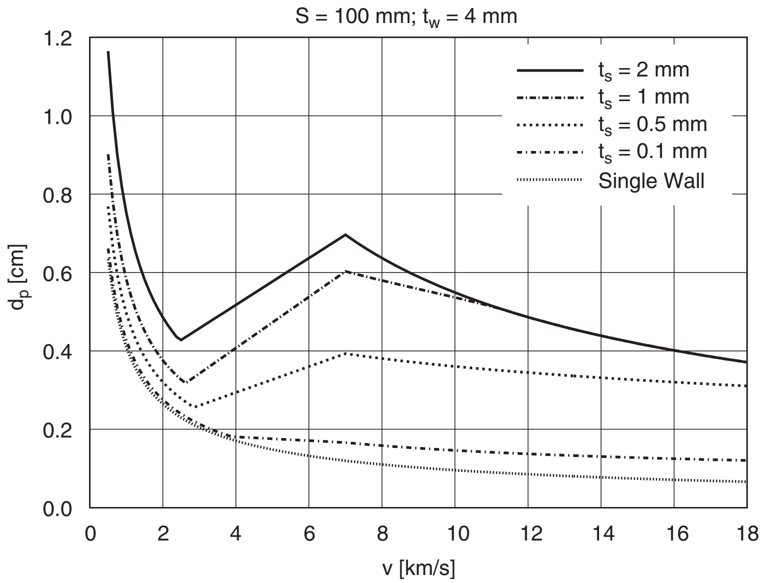

Fig. 7. Critical particle diameter as a function of impact velocity and various shield thicknesses—modified equations.

Table 2
Test configurations

Test no. (EMI no.)	t_s [mm]	t_w [mm]	S [mm]	d_p [mm]	v [mm]	Result [km/s]
4436	0.46	2.00	100	2.5	3.80	Above BL
4438	0.46	2.00	100	4.0	7.02	Above BL
4444	0.46	2.00	100	3.0	7.00	Below BL
4445	0.46	2.00	100	3.5	6.89	Above BL
4435	1.00	2.00	100	2.5	2.57	Closely above BL
4441	1.00	2.00	100	5.0	7.07	Far above BL
4442	1.00	2.00	100	4.0	7.17	Far below BL
4443	1.00	2.00	100	4.5	7.11	Below BL

Although the (t_s/d_p)-ratio is equal to 0.2 Christiansen's equation is not valid. The reason is the ratio of spacing to particle diameter which is equal to 20 here. According to Eq. (4) the (t_s/d_p)-ratio should be equal to 0.25 in this case. If the factor F_2^* is computed, applying $(t_s/d_p)_{crit} = 0.25$ the dashed line in Fig. 8 is achieved giving a safe prediction of the ballistic limit.

For the shield with 0.46 mm thickness, Fig. 9 presents the original and the modified BLEs as well as the test results. It is shown that the critical particle diameter is now reduced to 3.3 mm at about 7 km/s. This behavior is well predicted with the modified equations applying F_2^* based on $(t_s/d_p)_{crit} = 0.20$ for $S/d_p = 30$ (Eq. (5)).

The presented results show that the introduced modifications to the Christiansen double-wall BLEs lead to an improvement at velocities of about 7 km/s, allowing to predict the ballistic limit depending on the shield thickness. More tests are needed to investigate the predictions at velocities of about 2.5–3.5 km/s.

4. Conclusions

In this paper a modification of Cour-Palais/Christiansen ballistic limit equation (BLEs) for double-wall structures was introduced allowing to describe the effect of the shield thickness. The suggested modifications are based on phenomena published in the literature and combine single-wall and double-wall equations such

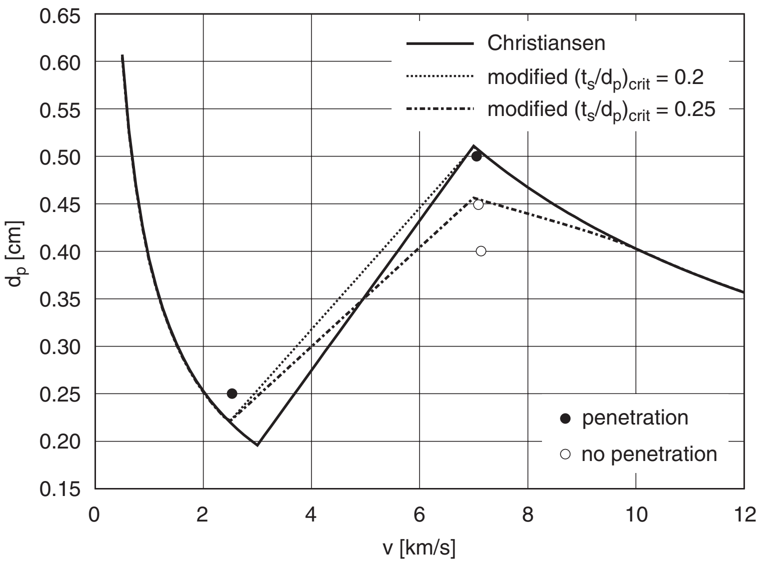

Fig. 8. Experimental verification—shield thickness: 1.00 mm.

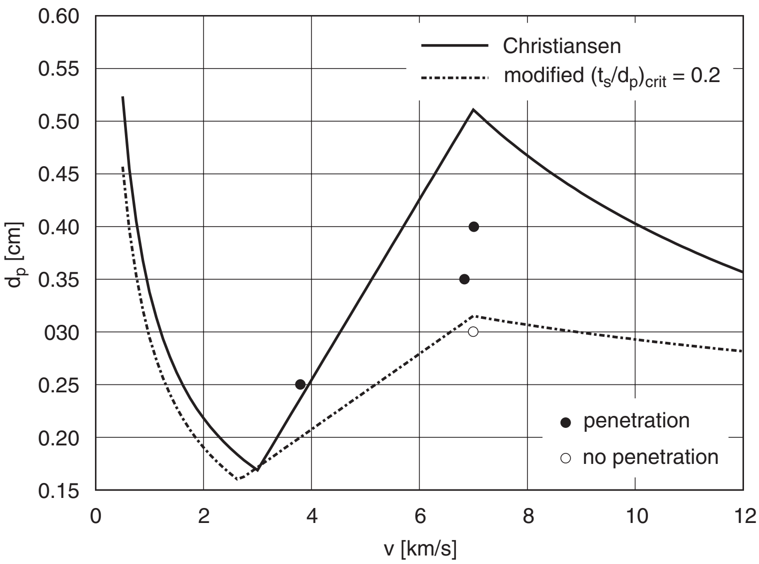

Fig. 9. Experimental verification—shield thickness: 0.46 mm.

that for a shield thickness equal to zero results for the single-wall are achieved. The use of these modifications is not limited to the Christiansen equations alone.

An experimental investigation confirmed the suggested approach and showed a substantial improvement of the predictions for relatively thin shields. Further experiments are needed to verify the predictions at velocities of about 2.5–3.5 km/s. In addition, it should be investigated how the limit velocity between shatter and hypervelocity region (here 7 km/s) is dependent on the shield thickness. Due to experimental limitations no results are available allowing to derive equations.

The aim of this work is to establish BLEs to be used for mathematical optimization. An investigation of existing BLEs for optimization purposes showed a large demand for further research in this field in order to derive more general equations not only for double-wall structures.

Acknowledgments

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References

[1] Christiansen EL. Design and performance equations for advanced meteroid and debris shields. Int J Impact Eng 1993;14:145–56.
[2] Wohlers W, Reimerdes HG. Analytical optimization of protection systems. Int J Impact Eng 2003;29:803–19.
[3] Reimerdes HG, Stecher BG, Lambert M. Ballistic limit equations for the Columbus-double bumper shield concept. In: Proceedings of the first European conference on space debris, Darmstadt, Germany, 1993. p. 433–40.
[4] Gehring Jr JW, et al. Experimental studies concerning the meteoroid hazard to aerospace materials and structures. J Spacecraft 1965;2:731–7.
[5] Cour-Palais BG. Hypervelocity impact investigations and meteroid shielding experience related to Apollo and Skylab. Orbital debris, NASA CP-2360, 1985. p. 247–75.
[6] Forst VC. Meteroid damage assessment. NASA SP-8042, 1970.
[7] Maiden CJ, et al. Thin sheet impact. NASA CR-295, 1965.
[8] Piekutowski AJ. Fragmentation-initiation threshold for spheres impacting at hypervelocity. Int J Impact Eng 2003;29:563–74.
[9] Gehring Jr JW. Theory of impact on thin targets and shields and correlation with experiments. In: Kinslow R, editor. High-velocity impact phenomena. New York, NY: Academic Press; 1970. p. 105–57.