The Data One sample z-test Two sample z-test Chi-squared test Summary

Inference for Categorical Data

Shannon Pileggi

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Overview of Statistical Methods

Quantitative response (means)

- one sample t-test
- paired t-test
- two sample t-test
- anova

Categorical response (proportions)

- one sample z-test
- ► N/A for STAT 217
- two sample z-test
- chi-squared test

Gardasil vaccinations

One sample z-test

- ▶ HPV is a sexually transmitted virus with links to certain types of cancer
- ▶ the FDA approved the Gardasil vaccination in 2006 to protect against HPV
- ▶ Gardasil is a three shot sequence recommended for women age 9-26
- ▶ the study subjects are 1413 females aged 11-26 who
 - 1. made their first "Gardasil visit" to a Johns Hopkins Medical Institution clinic between 2006 and 2008, and
 - 2. had 12 months to complete the regimen

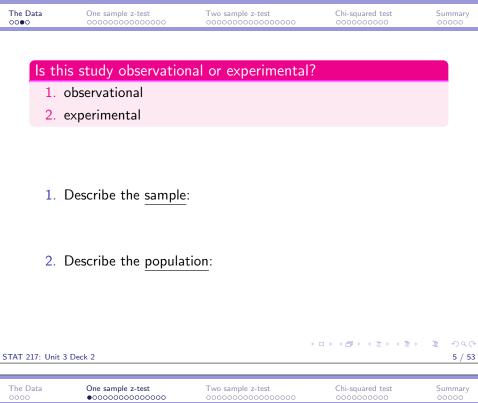
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The first 6 observations in the data set

>	> head(gardasil)										
	AgeGroup	Race	Completed	${\tt InsuranceType}$							
1	18-26	white	yes	military							
2	18-26	white	yes	military							
3	18-26	white	no	private payer							
4	11-17	white	yes	military							
5	11-17	other/unknown	no	military							
6	11-17	black	no	medical assistance							

11-17; 18-26 AgeGroup white, black, Hispanic, other/unknown Race completion of three-shot sequence 12 months (yes, no) Completed medical assistance, private payer, hospital based, military InsuranceType

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One sample z-test

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The research question: Suppose the CDC claims that the completion rate for the Gardasil vaccination is 35%. Do we have evidence for or against this claim?

- 1. What types of variables do we have?
- > head(gardasil) Completed yes 2 yes 3 no yes 5 no
- 2. How many groups are we studying?
- 3. How can we approach the problem?

> addmargins(table(gardasil\$Completed))

no yes Sum 944 469 1413

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Parameter of interest

What is the parameter of interest?

- 1. whether or not a female completes the shot sequence
- 2. the observed sample proportion of females that complete the shot sequence
- 3. the population proportion of females that complete the shot sequence
- 4. the number of females that complete the shot sequence
- 5. the population mean number of shots that a female gets
- 6. if the observed sample proportion of females that complete the shot sequence is different than 0.35

Answering the research question

We can answer the research question of interest with:

- ► A confidence interval for *p*
- A hypothesis test of H_0 : p = 0.35 vs H_a : $p \neq 0.35$ (this is the one sample z-test)

The value of p in our hypothesis statement is called p_0 . Here, $p_0 = 0.35$. This is what we assume to be true about p under the null hypothesis.

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The possibilities

469 out of 1413 (33.2%) completed the vaccination sequence

 $H_0: p = 0.35 \text{ vs } H_a: p \neq 0.35$

- 1. It could be that the population proportion that completes the vaccination sequence really is 0.35, and we observed 33.2% by random chance in our sample. This idea corresponds to the null hypothesis.
- 2. It could be that the population proportion that completes the vaccination sequence is not 0.35. This idea corresponds to the alternative hypothesis.

Conditions required for the one sample z-test

1. independent observations

One sample z-test

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2. at least 10 observed 'successes' and 10 observed 'failures'

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One sample z-test

The test statistic looks like a z-score $\left(z = \frac{x - \mu}{\sigma}\right)$:

 ${\sf test\ statistic} = \frac{{\sf sample\ proportion} - {\sf null\ hypothesis\ proportion}}{{\sf test\ statistic}}$ standard error under H_0

$$z = rac{\hat{p} - p_0}{se_{p_0}} = rac{\hat{p} - p_0}{\sqrt{rac{p_0(1 - p_0)}{n}}} =$$

This is a **z** test statistic because the test statistic follows a standard normal distribution (a normal distribution with a mean of 0 and a standard deviation of 1).

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Results in R

> prop.test(469,1413,p=0.35,correct=F)

One sample z-test

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1-sample proportions test without continuity correction

data: 469 out of 1413, null probability 0.35 X-squared = 2.0308, df = 1, p-value = 0.1541 alternative hypothesis: true p is not equal to 0.35 95 percent confidence interval: 0.3078495 0.3568977 sample estimates: 0.3319179

Even though we performed the one sample z-test, R reports a chi-squared test statistic. This has equivalent results to the z test statistic because $z = \sqrt{\chi_1^2}$.

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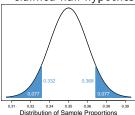
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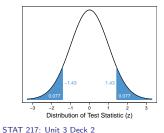
Summarv

Interpret the p-value

A sample proportion of $\hat{p} = 0.332$ is 1.43 standard errors below the claimed null hypothesis proportion of 0.35.



▶ If the proportion of females that complete the Gardasil vaccination sequence really is 0.35, the probability that we would see a sample proportion less than 0.332 or greater than 0.368 is 0.154.



▶ If the proportion of females that complete the Gardasil vaccination sequence really is 0.35, the probability that we would see a

test statistic less than -1.43 or greater than 1.43 is 0.154.

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Conclusion in context

1. Decision about H_0 :

One sample z-test

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2. Statement about the parameter tested in context of the research question:

3. Provide a deeper connection of how this relates to the research question:

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This decision could have been the result of... (mark all that apply)

- 1. a Type I error
- 2. a Type II error
- 3. a correct decision

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A 95% confidence interval for the population proportion would...

- 1. include 0
- 2. not include 0
- 3. include 0.95
- 4. not include 0.95
- 5. include 0.35
- 6. not include 0.35
- 7. include 0.15
- 8. not include 0.15

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Review: Conditions required for the confidence interval

1. independent observations

whether or not one female completes the shot sequence is independent from the other females' completion of the shot sequence

2. at least 10 'successes' and 10 'failures'

we have 469 females that completed the sequence (>10) and 944 females that did not complete the sequence (>10) so this condition is satisfied

Review: 95% confidence interval for p

$$\hat{
ho}\pm z^* imes\sqrt{rac{\hat{
ho}(1-\hat{
ho})}{n}}$$

based on a 95% conf. level $z^* = 1.96$

$$\textit{se} = \sqrt{\frac{0.332 \times (1 - 0.332)}{1413}} = 0.0125$$

One sample z-test

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 $0.332 \pm 1.96 \times 0.0125$

 0.332 ± 0.025

95% CI for p: (0.307, 0.357)

What is the margin of error for this confidence interval?

- 1. 0.0125
- 2. 0.025
- 3. 0.05
- 4. 0.95
- **5**. 1.96

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Which of the following is a correct interpretation of the (0.31, 0.36) interval?

- 1. We are 95% confident that the population proportion of females that complete the Gardasil vaccination sequence is in the interval (0.31, 0.36).
- 2. We are 95% confident that among the 1413 females in this study, the proportion that complete the Gardasil vaccination sequence is in the interval (0.31, 0.36).
- 3. Between 31 to 36% of the time, we are 95% confident that we will reject the null hypothesis.
- 4. Between 31 to 36% of the time, we are 95% confident that we will find a statistically significant result.

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The research question: Does completion rate differ by age group?

First six observations:

>	head(gardas	il)
	AgeGroup	Completed
1	18-26	yes
2	18-26	yes
3	18-26	no
4	11-17	yes
5	11-17	no
6	11-17	no

- 1. What types of variables do we have?
- 2. How many groups are we studying?
- 3. How can we approach the problem?

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no yes Sum

11-17 454 247 701

18-26 490 222 712

Sum 944 469 1413

Which two proportions should you compare to determine if completion rate differs by age group?

1. 701/1413 vs 712/1413

- _ | |
- 2. 469/1413 vs 944/1413
- 3. 701/1413 vs 469/1413
- 4. 247/701 vs 222/712
- 5. 247/469 vs 222/469
- 6. 490/944 vs 222/469
- 7. 454/701 vs 247/701

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Parameters of interest:

 p_1 = population proportion of 11-17 year old females who complete the Gardasil vaccination sequence

 p_2 = population proportion of 18-26 year old females who complete the Gardasil vaccination sequence

We can answer the research question of interest with:

- ▶ A hypothesis test of H_0 : $p_1 = p_2$ vs H_a : $p_1 \neq p_2$ (this is the two sample z-test)
- ▶ A confidence interval for $p_1 p_2$. Three possible scenarios:

No difference
$$p_1 - p_2 = 0 \rightarrow p_1 = p_2$$

Difference
$$p_1 - p_2 > 0 \rightarrow p_1 > p_2$$

Difference
$$p_1 - p_2 < 0 \rightarrow p_1 < p_2$$



Which of these scenarios represent a two sample comparison?

- 1. We take a random sample of 100 Cal Poly students and test proportion that participate in Greek life differs from 0.2.
- 2. We take a random sample of 100 Cal Poly students and test if the proportion that participate in Greek life differs among males and females.
- 3. We take a random sample of 100 Cal Poly students and ask them in their Sophomore and Senior year if they participate in Greek life, and we want to know if the proportion that participates in Greek life changes over year in school.
- 4. We take a random sample of 100 Cal Poly students and test if the proportion that participate in Greek life differs by where they grew up (ie, west, south, northeast, midwest).

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Summarv

Conditions required for the CI for $p_1 - p_2$

- 1. independent observations
- 2. check to see that you have at least 10 observed 'successes' and 10 observed 'failures' in each group

Confidence interval for $p_1 - p_2$

$$(\hat{p}_1 - \hat{p}_2) \pm z^* \times se$$

11-17 females: $\hat{p}_1 = 247/701 = 0.352$ 18-26 females: $\hat{p}_1 = 222/712 = 0.312$

$$se = \sqrt{\frac{\hat{p}_1(1-\hat{p}_1)}{n_1} + \frac{\hat{p}_2(1-\hat{p}_2)}{n_2}}$$

$$= \sqrt{\frac{0.352(1-0.352)}{701} + \frac{0.312(1-0.312)}{712}}$$

$$= 0.025$$

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Interpret the CI

- ▶ We are 95% confident that the difference in the population proportion of 11-17 year old females and 18-26 year old females that complete the Gardasil vaccination sequence is in the interval (-0.009, 0.089).
- ▶ Because this interval includes 0, it is plausible that the population proportion of 11-17 year old females who complete the Gardasil vaccination sequence is equal to the population proportion of 18-26 year old females who complete the Gardasil vaccination sequence

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Researchers want to know if wearing glasses differs by males and females. We calculated a 95% CI for the true proportion difference of men and women $(p_{men} - p_{women})$ who wear glasses who wear glasses to be (0.01,0.03). Which of the following statements is $\underline{\text{true}}$? Mark $\underline{\text{all}}$ that apply.

- 1. In general, the proportion of men and women who wear glasses is small.
- 2. There is evidence that a higher proportion of males wear glasses compared to females.
- 3. There is evidence that a higher proportion of females wear glasses compared to males.
- 4. It is plausible that the proportion of females who wear glasses is equal to the proportion of males who wear glasses.

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We calculated a 95% CI for the true proportion difference of men who wear glasses and women who wear glasses to be (0.01,0.03).

If I used the same data to calculate a CI for $p_{women} - p_{men}$ instead of $p_{men} - p_{women}$, what would the CI be?

- 1. the same
- 2. (-0.03, 0.01)
- 3. (-0.03, -0.01)
- 4. (-0.01, 0.03)
- 5. not enough information to determine

Consider doing the two sample z-test for H_0 : $p_1=p_2$ vs H_a : $p_1\neq p_2$. Based on the 95% CI for p_1-p_2 of (-0.009, 0.089), you would expect to

- 1. reject H_0
- 2. fail to reject H_0
- 3. accept H_0
- 4. find a statistically significant result
- 5. not enough information to determine

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Conditions required for the two sample z-test

1. independent observations

2. check to see that you have at least 10 observed 'successes' and 10 observed 'failures' in each group

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Results in R

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> prop.test(c(247,222),c(701,712),correct=F)

2-sample test for equality of proportions without continuity c

data: c(247, 222) out of c(701, 712)X-squared = 2.62, df = 1, p-value = 0.1055 alternative hypothesis: two.sided 95 percent confidence interval: -0.008517967 0.089630022 sample estimates: prop 1 prop 2 0.3523538 0.3117978

Even though we performed the one sample z-test, R reports a chi-squared test statistic. This has equivalent results to the z test statistic because $z = \sqrt{\chi_1^2}$.

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The test statistic

$$z=\frac{\left(\hat{p}_1-\hat{p}_2\right)-0}{se_0}$$

where se_0 is the standard error calculated assuming H_0 true.

1.
$$\hat{p} = \frac{x_1 + x_2}{n_1 + n_2} = \frac{247 + 222}{701 + 712} = \frac{469}{1413} = 0.332$$

$$se_0 = \sqrt{\frac{\hat{p}(1-\hat{p})}{n_1} + \frac{\hat{p}(1-\hat{p})}{n_2}}$$

$$= \sqrt{\hat{p}(1-\hat{p})\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}$$

$$= \sqrt{0.332(1-0.332)\left(\frac{1}{701} + \frac{1}{712}\right)}$$

$$= 0.025$$

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 H_0 : $p_1 = p_2$ vs H_a : $p_1 \neq p_2$, p-value = 0.1055

Which is a *correct* interpretation of this *p*-value?

1. Fail to reject H_0 .

One sample z-test

- 2. The probability that the two proportions are equal is 0.1055.
- 3. The probability that we committed a type I error is 0.1055.
- 4. The probability that we observe a test statistic as extreme or more extreme than 1.60 if the two proportions really are equal is 0.1055.
- 5. The probability that we observe a test statistic as extreme or more extreme than 1.60 if the two proportions really are unequal is 0.1055.

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Conclusion in context

1. Decision about H_0 :

2. Statement about the parameter tested in context of the research question:

3. Provide a deeper connection of how this relates to the research question:

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Do people who drink caffeinated beverages have a higher occurrence of heart disease than people who do not drink caffeinated beverages? 200 caffeinated beverage drinkers and 150 non-caffeinated beverage drinkers report whether or not they have heart disease.

To answer this question would you use proportions or means AND paired or not paired measurements?

- 1. Two proportions from not paired measurements
- 2. Two proportions from paired measurements
- 3. Two means from not paired measurements
- 4. Two means from paired measurements

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 $\frac{\mbox{The research question:}}{\mbox{type?}} \mbox{ Does completion rate differ by insurance}$

First six observations:

>	head(garda	sil)
	Completed	${\tt InsuranceType}$
1	yes	military
2	yes	military
3	no	private payer
4	yes	military
5	no	military
6	no	medical assistance

- 1. What types of variables do we have?
- 2. How many groups are we studying?
- 3. How can we approach the problem?

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 no
 yes
 Sum

 hospital based
 45
 39
 84

 medical assistance
 220
 55
 275

 military
 209
 122
 331

 private payer
 470
 253
 723

 Sum
 944
 469
 1413

no yes hospital based 0.5357143 0.4642857 medical assistance 0.800000 0.2000000 military 0.6314199 0.3685801 private payer 0.6500692 0.3499308

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The chi-squared test

Equivalent ways of stating the hypotheses:

H₀: the two variables are independent

*H*_a: the two variables are dependent

*H*₀: the two variables are not associated

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*H*_a: the two variables are associated

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Gardasil data

	hospital	medical		private	
	based	assistance	military	payer	Total
yes					469
no					944
Total	84	275	331	723	1413

If there was no association between Completed and InsuranceType (ie, H_0 true), how many females would you expect to see complete the shot sequence on the hospital based insurance?

- 1. What is the overall proportion of completion?
- 2. Apply that to the total number on hospital based insurance.

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Expected cell counts

If H_0 true,

 $\frac{\mathsf{Expected cell count}}{\mathsf{Total sample size}} = \frac{\mathsf{Row total} \times \mathsf{Column total}}{\mathsf{Total sample size}}$

	hospital	medical		private	
	based	assistance	military	payer	Total
yes	39 (27.9)	55 (91.2)	122 (109.9)	253 (240.0)	469
no	45 (56.1)	220 (183.7)	209 (221.1)	470 (483.0)	944
Total	84	275	331	723	1413

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Conditions for the χ^2 test

1. observations are independent within each of the groups

2. expected cell count ≥ 5 in all cells

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> chisq.test(gardasil\$Completed,gardasil\$InsuranceType,correct=F)

Pearson's Chi-squared test

data: gardasil\$Completed and gardasil\$InsuranceType X-squared = 31.283, df = 3, p-value = 7.411e-07

What can we conclude? we (do/do not) have any evidence of an association between completion and insurance type.

- 1. Reject H_0 ; do
- 2. Reject H_0 ; do not
- 3. Fail to reject H_0 ; do

One sample z-test

4. Fail to reject H_0 ; do not

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The test statistic

	hospital	medical		private	
	based	assistance	military	payer	Total
yes	39 (27.9)	55 (91.2)	122 (109.9)	253 (240.0)	469
no	45 (56.1)	220 (183.7)	209 (221.1)	470 (483.0)	944
Total	84	275	331	723	1413

The **chi-squared test statistic** summarizes the difference between the observed and expected counts.

$$\chi^2 = \sum \frac{\text{(observed count} - expected count)}^2}{\text{expected count}}$$

with $df = (\text{number of rows } -1) \times (\text{number of columns } -1)$

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Chi-squared test Summary

If your data can go in a 2x2 contingency table, the two proportion z-test and the χ^2 test will give you equivalent results.

Two sample z-test

> addmargins(table(gardasil\$AgeGroup,gardasil\$Completed))

no yes Sum 11-17 454 247 701 18-26 490 222 712 Sum 944 469 1413

One sample z-test

> prop.test(c(247,222),c(701,712),correct=F)

> chisq.test(gardasil\$AgeGroup,gardasil\$Completed,correct=F)

Both test results yield a p-value of 0.1055.

Two sample z-test: Fail to reject H_0 ; we do not have evidence that the

proportion of 11-17 year olds that complete the vaccination sequence differs from the proportion of 18-26 year olds that

complete the vaccination sequence

Chi-squared test: Fail to reject H_0 ; there is no evidence of an association

between age group and completion of the vaccination sequence

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Interpreting the p-value

One sample z-test

The *p*-value represents the **strength** of the **evidence**:

- ▶ small p-values mean you have strong evidence of an association between two variables
- ▶ small p-values do not mean you have evidence of a strong association between two variables
- large p-values mean there is no evidence of an association

Other measures represent the **strength** of the **association**:

- ightharpoonup difference of means: $(\bar{x}_1 \bar{x}_2)$
- ightharpoonup difference of proportions: $(\hat{p}_1 \hat{p}_2)$

The **strength** of the **association** can help you assess if an observed difference is meaningful.

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Attending religious services weekly by gender

	Case A				Case E	3	Case C		С
	Yes	No	n	Yes	No	n	Yes	No	n
Female	52	48	100	104	96	200	5200	4800	10,000
Male	50	50	100	100	100	200	5000	5000	10,000
	$\hat{p}_F = 0.52$			$\hat{p}_F = 0.52$			$\hat{p}_F = 0.52$		
	$\hat{p}_{M} = 0.50$			$\hat{p}_M = 0.50$		$\hat{p}_M = 0.50$			
	$\chi^2 = 0.08$		$\chi^2 = 0.16$		$\chi^2 = 8.0$				
	<i>p</i> -value= 0.78			<i>p</i> -value= 0.69		<i>p</i> -value= 0.005			

Case (A/B/C) shows the strongest association between gender and attendance; case (A/B/C) shows the strongest evidence of an association between gender and attendance.

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Two sample z-test

Chi-squared test

Summary

Group Exercise

A researcher investigated if the proportion of non-athletes (group 1) and athletes (group 2) that are on the Dean's List differs.

```
data: c(1000, 1100) out of c(10000, 10000)
X-squared = 5.3206, df = 1, p-value = 0.02108
alternative hypothesis: two.sided
95 percent confidence interval:
```

-0.018495941 -0.001504059

One sample z-test

sample estimates: prop 1 prop 2 0.10 0.11

What is the best conclusion from this analysis? We (do/do not) have strong statistically significant evidence that the proportion of students on the Dean's list differs by athlete status, and the effect of athlete status is (strong/weak)

1. do; strong

3. do; weak

2. do not; strong

4. do not; weak

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Different methods

Method	Use	Variables	Estimation	Testing
Single mean	quantitative response	one quantitative variable	CI for μ	$H_0: \mu = \mu_0$
(one-sample t-test)	in single group			
*Two means	quantitative response	one quantitative variable and	CI for $\mu_1 - \mu_2$	H_0 : $\mu_1 = \mu_2$
(two-sample t-test)	in two groups	one categorical variable		
Dependent means	quantitative response	two paired	CI for μ_d	$H_0: \mu_d = 0$
(paired t-test)	measured on same observation	quantitative variables		
*ANOVA	quantitative response	one quantitative variable and	Tukey pairwise	$H_0: \mu_1 = \mu_2 = \cdots = \mu_g$
	in > 2 groups	one categorical variable	intervals	
Single proportion	categorical response	one categorical variable	CI for p	$H_0: p = p_0$
(one-sample z-test)	in single group			
*Two proportions	categorical response	two categorical variables	CI for $p_1 - p_2$	$H_0: p_1 = p_2$
(two-sample z-test)	in two groups			
*X2 test	categorical response	two categorical variables	N/A	H ₀ : no association/
	in ≥ 2 groups			vars independent

^{*}The starred methods can answer the question "Is there an association?" If we reject H_0 , then we conclude that some sort of association is present in the two variables.



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