Response functions as a new concept to study local dynamics in traffic networks

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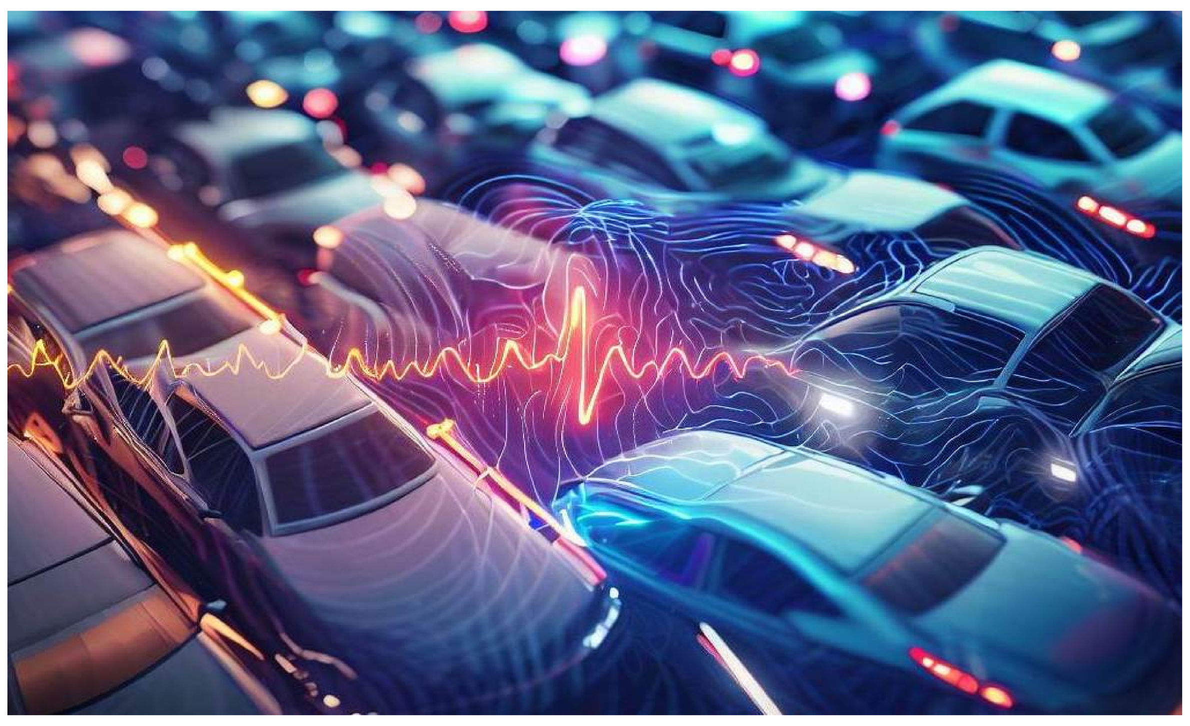
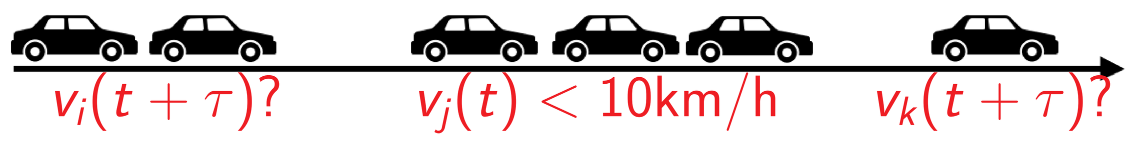
Open-Minded

Motivation





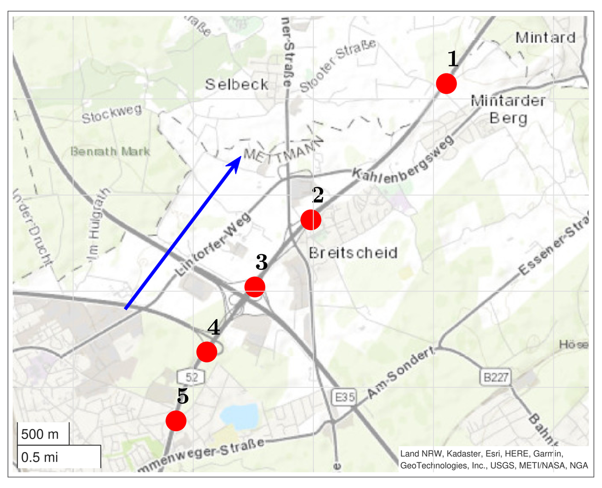
Several questions to be addressed:

- How does the velocity respond to congestion?
- What causes the velocity response to congestion?
- What features are present in the velocity response?

We aim to reveal the basic characteristics of velocity responses among neighbouring sections at first hand [1].

Description of traffic data

- collected by **inductive loop detectors** and provided by Straßen.NRW.
- for five motorway sections near Breitscheid of NRW, Germany



- from 00:00 to 23:59 of 64 workdays in 2017 with **one-minute resolution**
- Velocity $v_i(t)$ is averaged by

$$v_i(t) = \frac{\sum_l q_{il}(t)}{\sum_l \rho_{il}(t)} , \quad i = 1, \cdots, 5 ,$$

with traffic flows $q_{il}(t)$ and flow densities $\rho_{il}(t)$ on lanes l of section i at time t .

Responses and congestion correlator

Indicator function of heavy congestion

$$\varepsilon_j(t) = \begin{cases} 1, \text{ if } v_j(t) < 10 \text{ km/h} \\ 0, \text{ otherwise} . \end{cases}$$

Correlator of congestion indicators of motorway sections i and j

$$\Theta_{ij}(\tau) = \frac{1}{T-\tau} \sum_{t=1}^{T-\tau} \tilde{\varepsilon}_i(t+\tau)\tilde{\varepsilon}_j(t) .$$

$\tilde{\varepsilon}_j(t)$ represents $\varepsilon_j(t)$ normalized to zero mean and unit standard deviation.

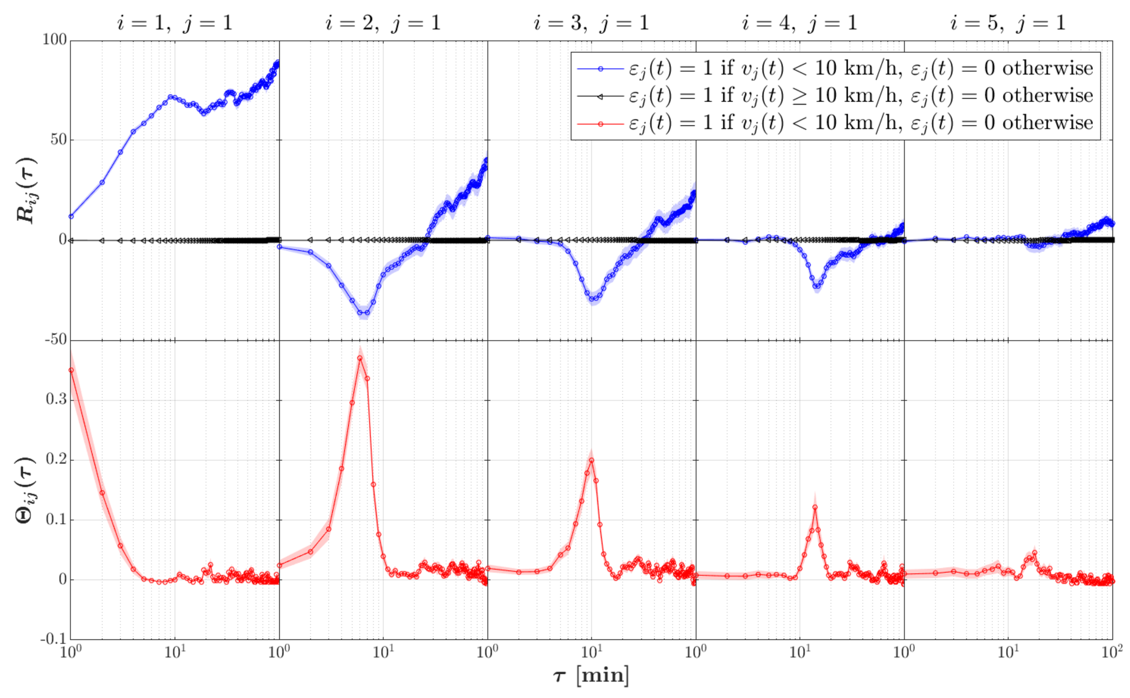
Response of velocities to congestion

$$R_{ij}(\tau) = \frac{\sum_{t=1}^{T-\tau} \big(v_i(t+\tau) - v_i(t)\big)\varepsilon_j(t)}{\sum_{t=1}^{T-\tau} \varepsilon_j(t)}$$

describes, on average, the velocity change on motorway section i at a time $t+\tau$, conditioned on **heavy congestion** on motorway section j at an earlier time t.

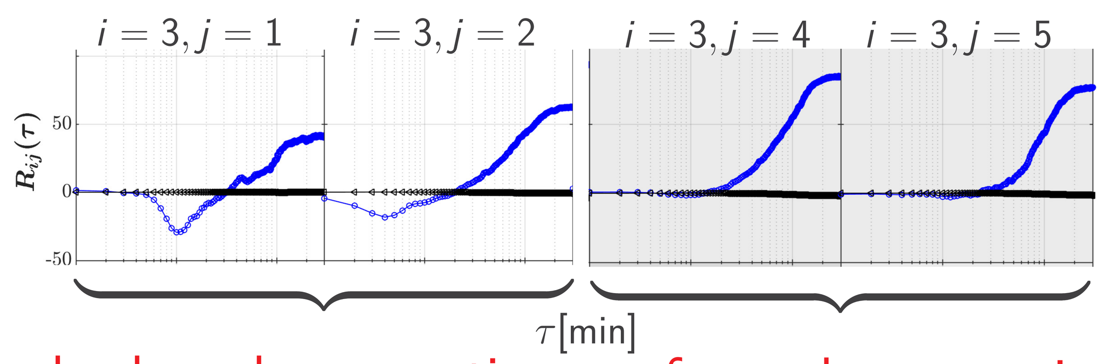
Characteristics of velocity response

- **Velocity response and congestion correlation**



i: impacted section, j: heavily congested section

- **Two kinds of response behavior**

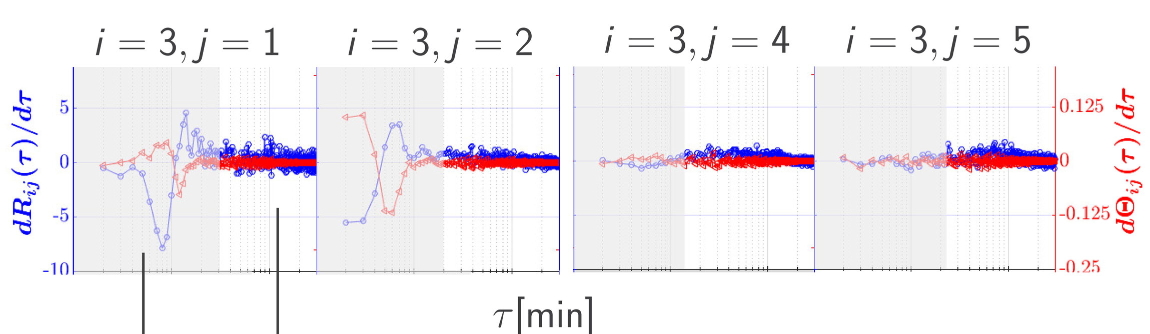


backward propagation of heavy congestion

forward propagation of heavy congestion

Causes of velocity response

- **Phase separation**



Non-steady phase
Transient response
(Phase 1)

$\frac{d\Theta_{ij}}{d\tau} \to 0$ Steady phase
Long-term response
(Phase 2)

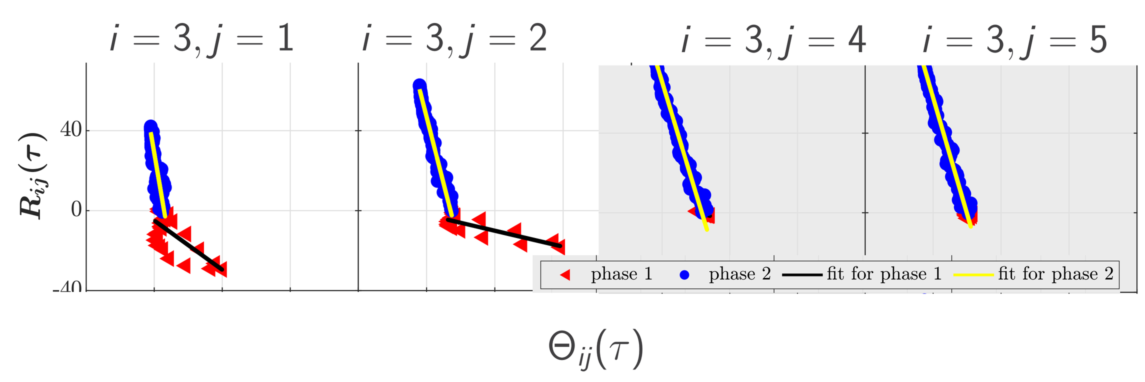
Define a function analogous to traffic resilience

$$\zeta_{ij}(\tau') = \int_0^{\tau'} R_{ij}(\tau) d\tau = \sum_{\tau=0}^{\tau'} R_{ij}(\tau)\Delta\tau$$

The critical time to separate two response phases

$$\tau_0 = \underset{\tau' \in [1,\ \tau_{\max}]}{\operatorname{argmin}} \ \zeta_{ij}(\tau')$$

- **Relations between velocity response and congestion correlation**



For each response phase (PR), a linear relation can be found

$$R_{ij}(\tau)|_{\text{RP }k} \sim -\Theta_{ij}(\tau)|_{\text{RP }k} .$$

Congestion correlation might be viewed as a cause of the velocity response, regardless of other possible reasons

SDW model for transient responses

Susceptible-decelerated-withdrawing (SDW) model, inspired by SIR epidemic model

$$\begin{array}{c} \text{susceptible velocity} \\ S_{ij}(\tau) \end{array} \xrightarrow{\frac{\beta_{ij} D_{ij}(\tau) S_{ij}(\tau)}{V_{ij}(0)}} \begin{array}{c} \text{decelerated velocity} \\ D_{ij}(\tau) \end{array} \xrightarrow{\gamma_{ij} D_{ij}(\tau)} \begin{array}{c} \text{withdrawing velocity} \\ W_{ij}(\tau) \end{array}$$

β_{ij}: deceleration rates, γ_{ij}: recovery rates.

$$\frac{dS_{ij}(\tau)}{d\tau} = -\frac{\beta_{ij} D_{ij}(\tau) S_{ij}(\tau)}{V_{ij}(0)} ,$$

$$\frac{dD_{ij}(\tau)}{d\tau} = \frac{\beta_{ij} D_{ij}(\tau) S_{ij}(\tau)}{V_{ij}(0)} - \gamma_{ij} D_{ij}(\tau) ,$$

$$\frac{dW_{ij}(\tau)}{d\tau} = \gamma_{ij} D_{ij}(\tau) .$$

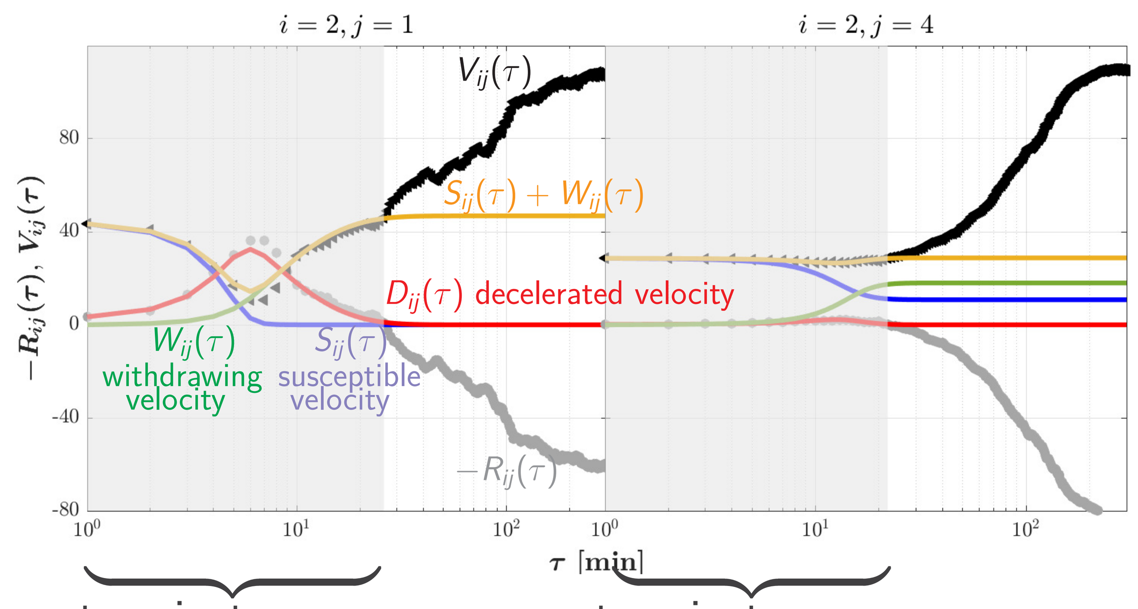
where $\quad V_{ij}(0) = D_{ij}(\tau) + S_{ij}(\tau) + W_{ij}(\tau)$.

Model simulations

- **An example of simulation results**

backward propagation of heavy congestion

forward propagation of heavy congestion



transient response

transient response

- **Explanations by fitted β_{ij} and γ_{ij}**

acceleration is impossible | heavy congestion | acceleration is possible

back section | middle section | front section

back section		front section
$v_i(t)\downarrow$	$\Longleftarrow \beta_{ij} \Longrightarrow$	$v_i(t)\downarrow$
recovered slowly	$\Longleftarrow \gamma_{ij} \Longrightarrow$	recovered quickly
$\lvert\Delta v_i(t)\rvert\uparrow$	$\Longleftarrow \beta_{ij}/\gamma_{ij} \Longrightarrow$	$\lvert\Delta v_i(t)\rvert\uparrow$
$\Downarrow$		$\Downarrow$
$R_{ij}(\tau)\vert_{RP\ 1}$ strong		$R_{ij}(\tau)\vert_{RP\ 1}$ weak

Conclusions

- The **backward and forward propagation** of heavy congestion leads to different behaviors in velocity response.
- The **correlation of heavy congestion** might be as one of causes for velocity responses.
- Two response phases: a phase for the **transient response** and a phase for the **long-term response**.
- The transient response can be explained by the **SDW model**.

References

[1] Wang, Schreckenberg, and Guhr, Physica A 626, 129116 (2023), preprint: arXiv:2211.08232.

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