Quasi-stationary states in temporal correlations and their transitions in traffic systems: Cologne orbital motorway as an example

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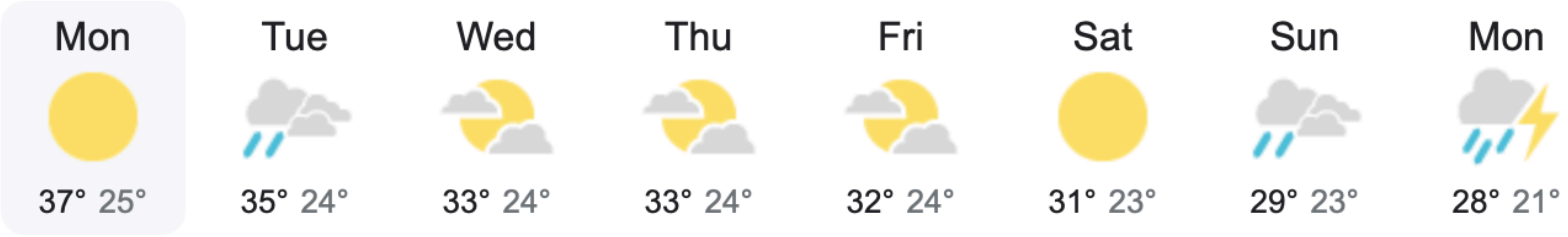
Background—patterns

Weather in Naples from Aug. 21-28

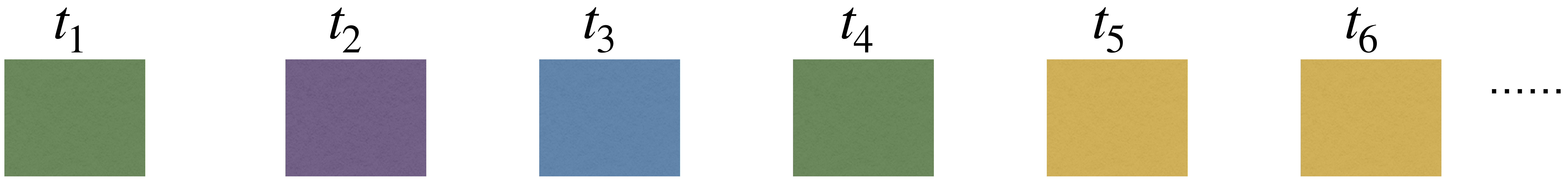
From https://weather.com



Six basic emotions of human beings

Ref. https://online.uwa.edu/infographics/basic-emotions/

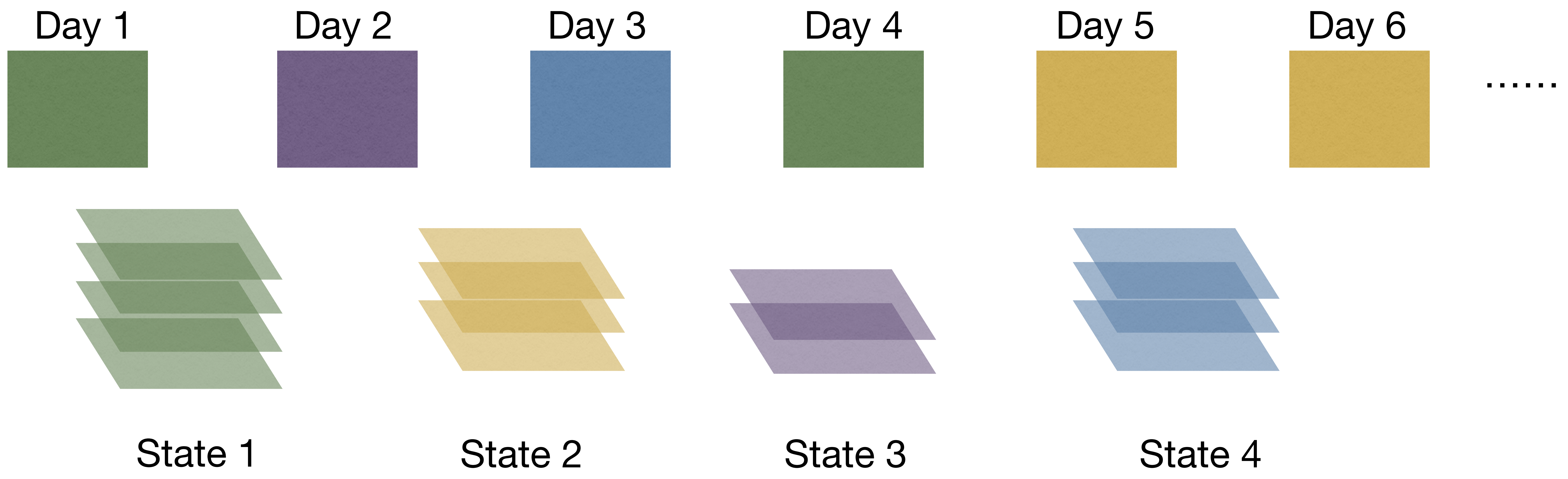
Patterns in matrices



Background—quasi-stationary states

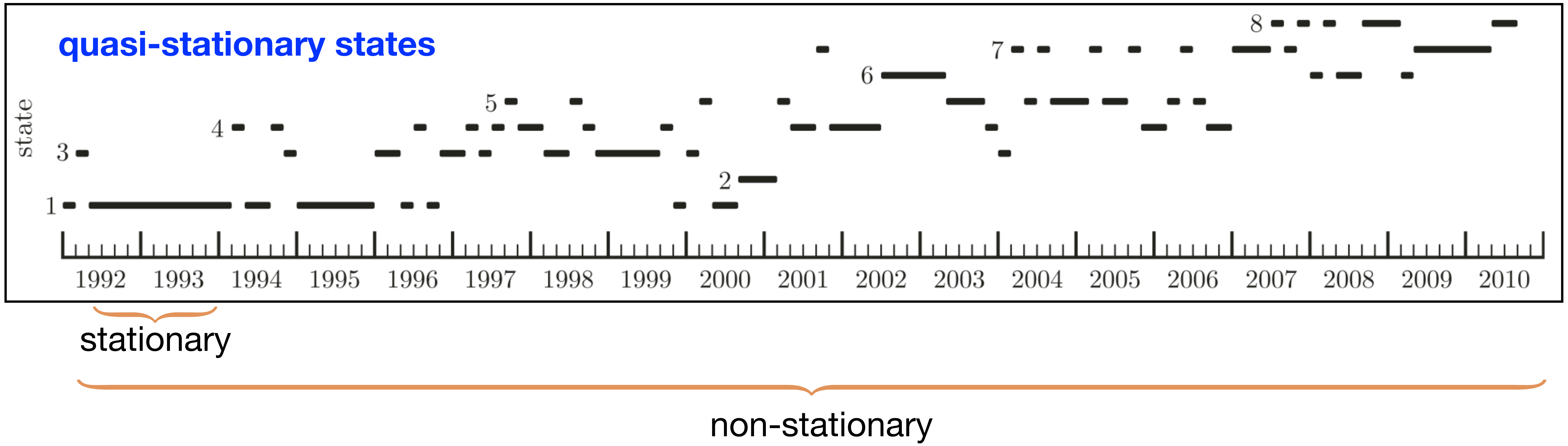
Patterns in correlation matrices

Classify matrices



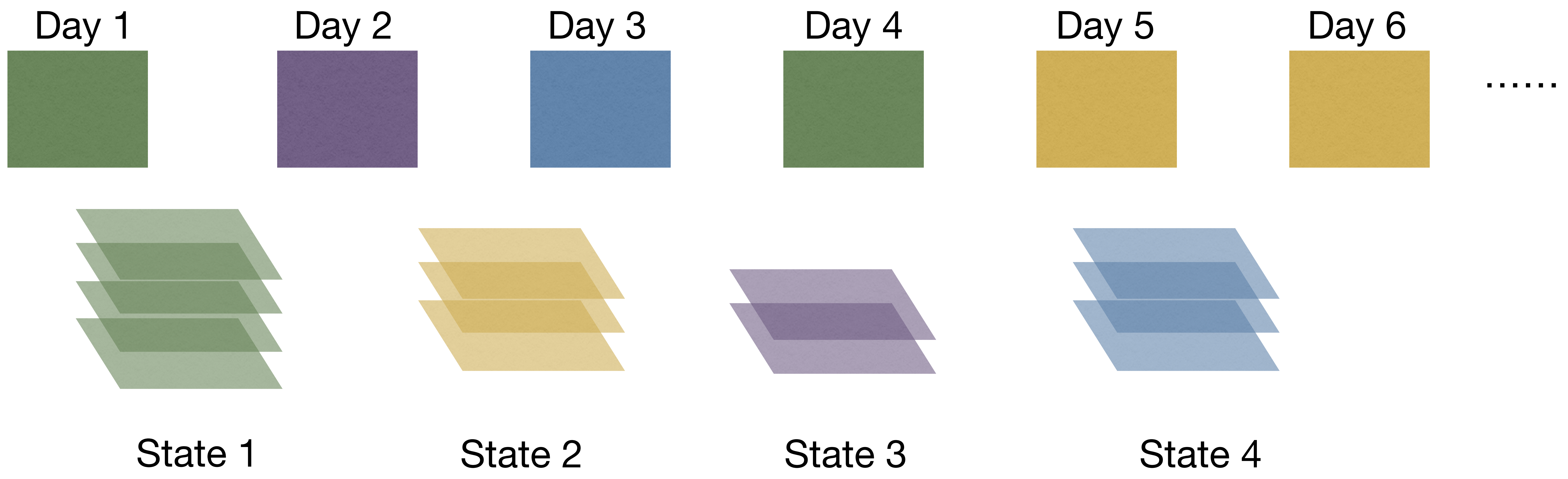
For financial markets

Ref. Münnix, Shimada, Schäfer, Leyvraz, Seligman, Guhr, and Stanley. Scientific Reports, 2:644, 2012



Patterns in correlation matrices

Classify matrices



For traffic systems

If we can find such quasi-stationary states for each day, they can be

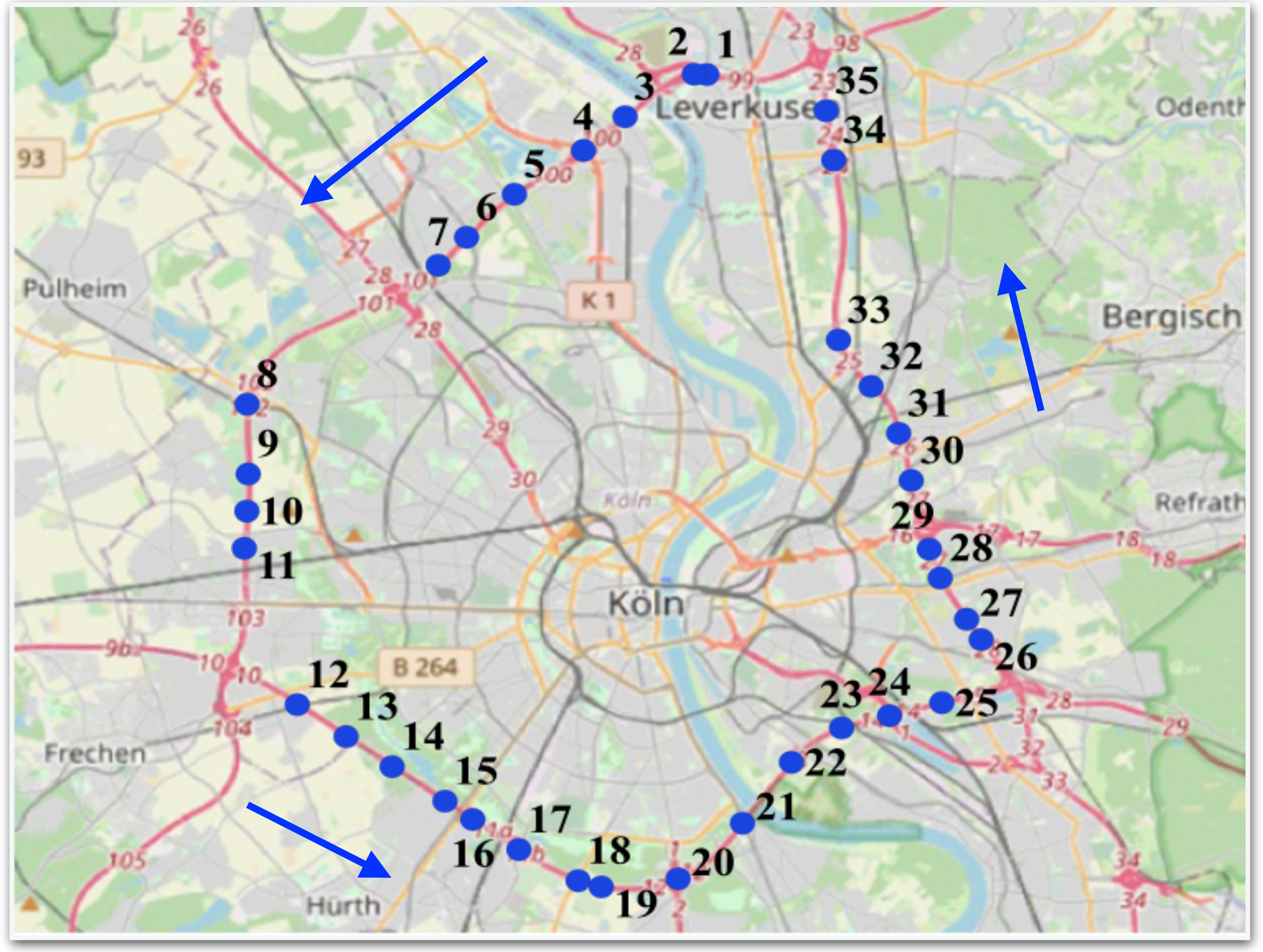
- as a **precursor** in traffic states
- for **forecasting** traffic patterns
- as a guidance of traffic control, traveling strategy, etc.

Picture from Google Maps

Background—features in traffic systems

For traffic systems

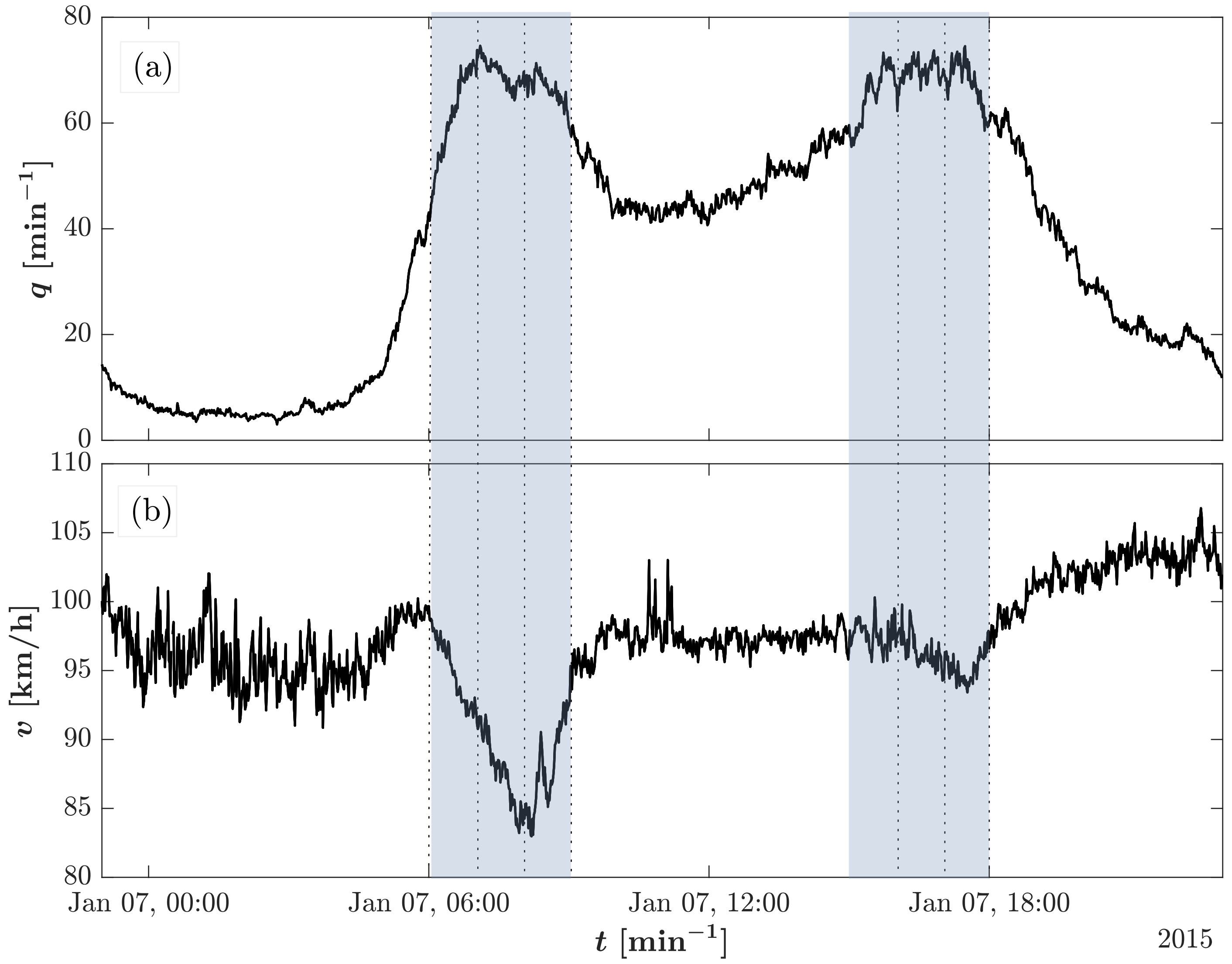
Information of space



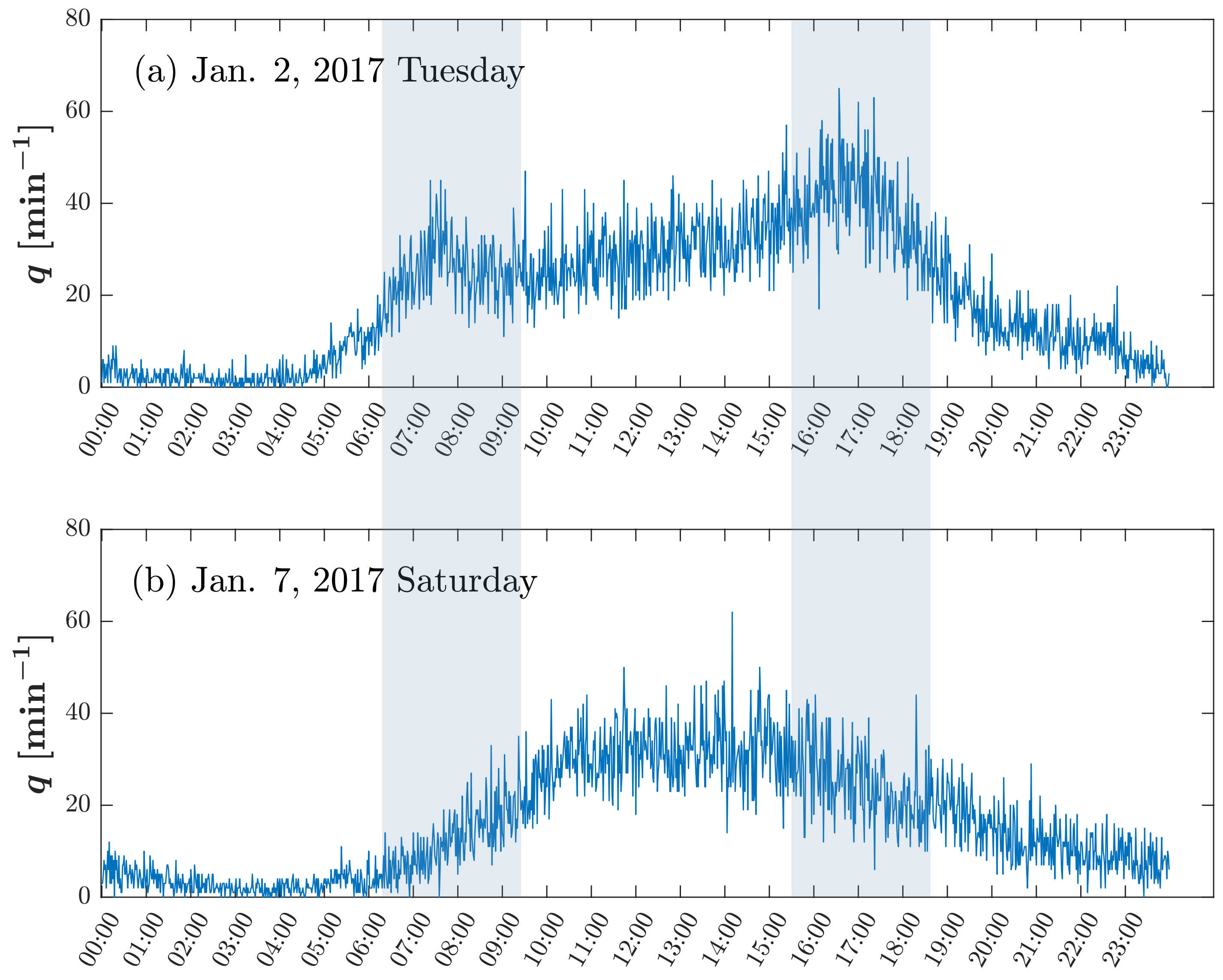
35 sections on Cologne orbital motorway, Germany

Information of time

different traffic behaviors in traffic flow and velocity in each work day



different traffic behaviors on workdays and weekends



Background—motivations

For traffic systems

Day 1 Day 2 Day 3 Day 4 Day 5 Day 6 ……

How to identify quasi-stationary states in traffic systems regarding such kind of matrices?

Information of time

Information of space

Traffic states

Congested traffic states

Free traffic states

Picture from The New Yorker Times

Picture from BBC

Identification of quasi-stationary states—correlation matrices

$T \times T$ temporal correlation matrix

$$\boxed{D = \frac{1}{K} M^\dagger M}$$

M is a $K \times T$ data matrix normalized zero mean and unit variance. Here, $K = 39$ and $T = 96$

$T \times T$ temporal covariance matrix

$$\Sigma = \frac{1}{K} A^\dagger A$$

A is a $K \times T$ data matrix normalized zero mean.

spectrum decomposition

$$\Sigma = \sum_{t=1}^{T} \Theta_t V(t) V^\dagger(t)$$

reduced-rank covariance matrix

$$\tilde{\Sigma} = \sum_{t=a}^{b} \Theta_t V(t) V^\dagger(t)$$

diagonal matrix of the square roots of the diagonal elements in $\tilde{\Sigma}$

$$\tilde{\sigma} = \operatorname{diag}\left(\tilde{\sigma}_1, \ldots, \tilde{\sigma}_T\right)$$

reduced-rank correlation matrix

$$\boxed{\tilde{D} = \tilde{\sigma}^{-1} \tilde{\Sigma} \tilde{\sigma}^{-1}}$$

Identification of quasi-stationary states—decomposition of matrices

$$\boxed{D = \frac{1}{K} M^{\dagger} M}$$

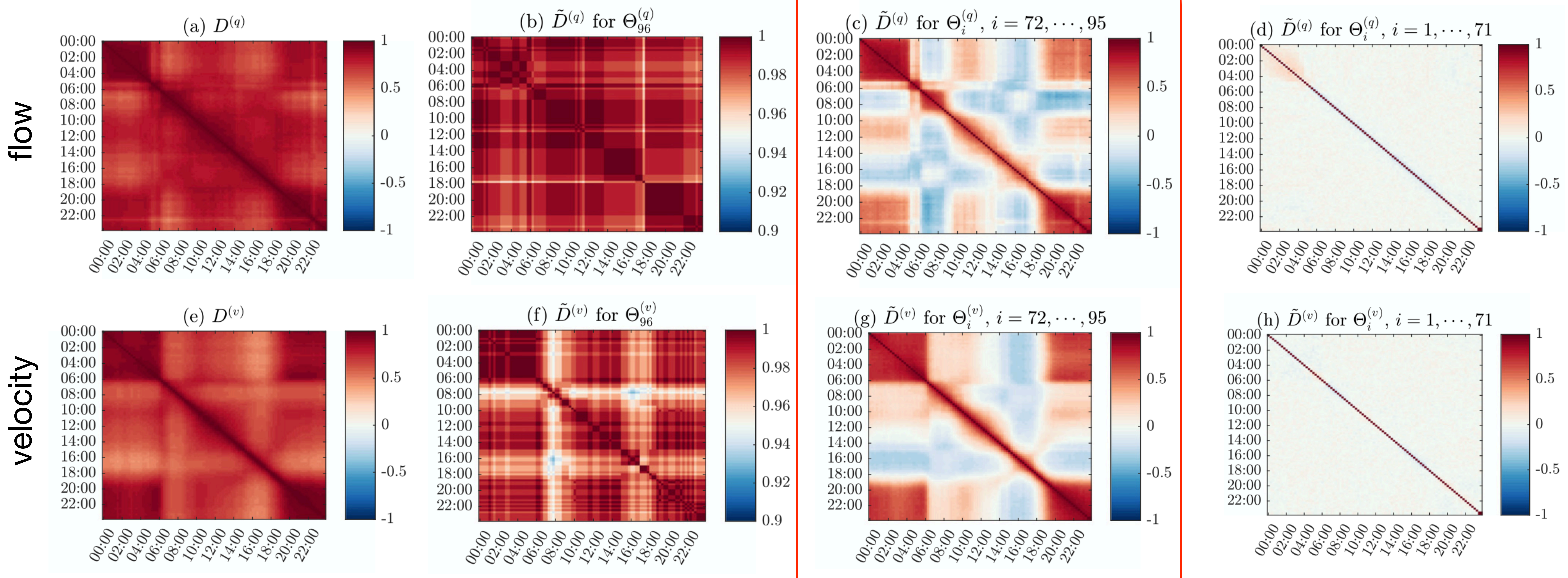
$$\boxed{\tilde{D} = \tilde{\sigma}^{-1} \tilde{\Sigma} \tilde{\sigma}^{-1}}$$

all eigenvalues

largest eigenvalue

middle eigenvalues

small eigenvalues



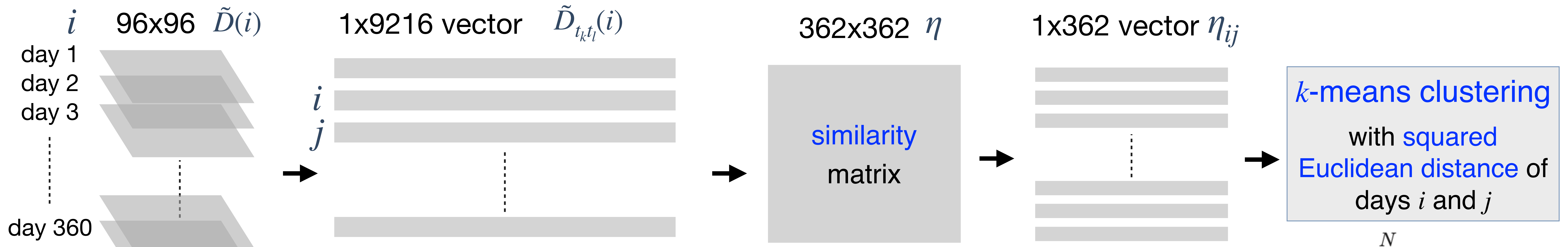
Total information

Information of positions

Information of time

Noise

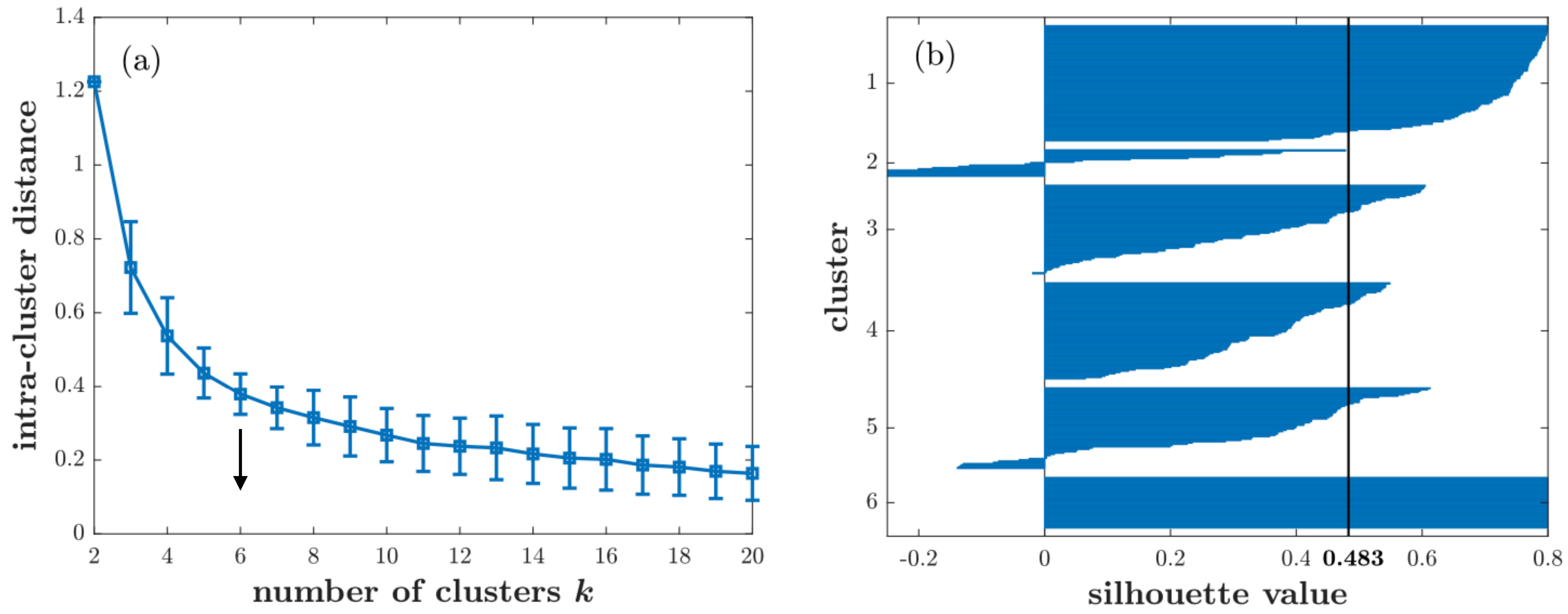
Identification of quasi-stationary states—clustering



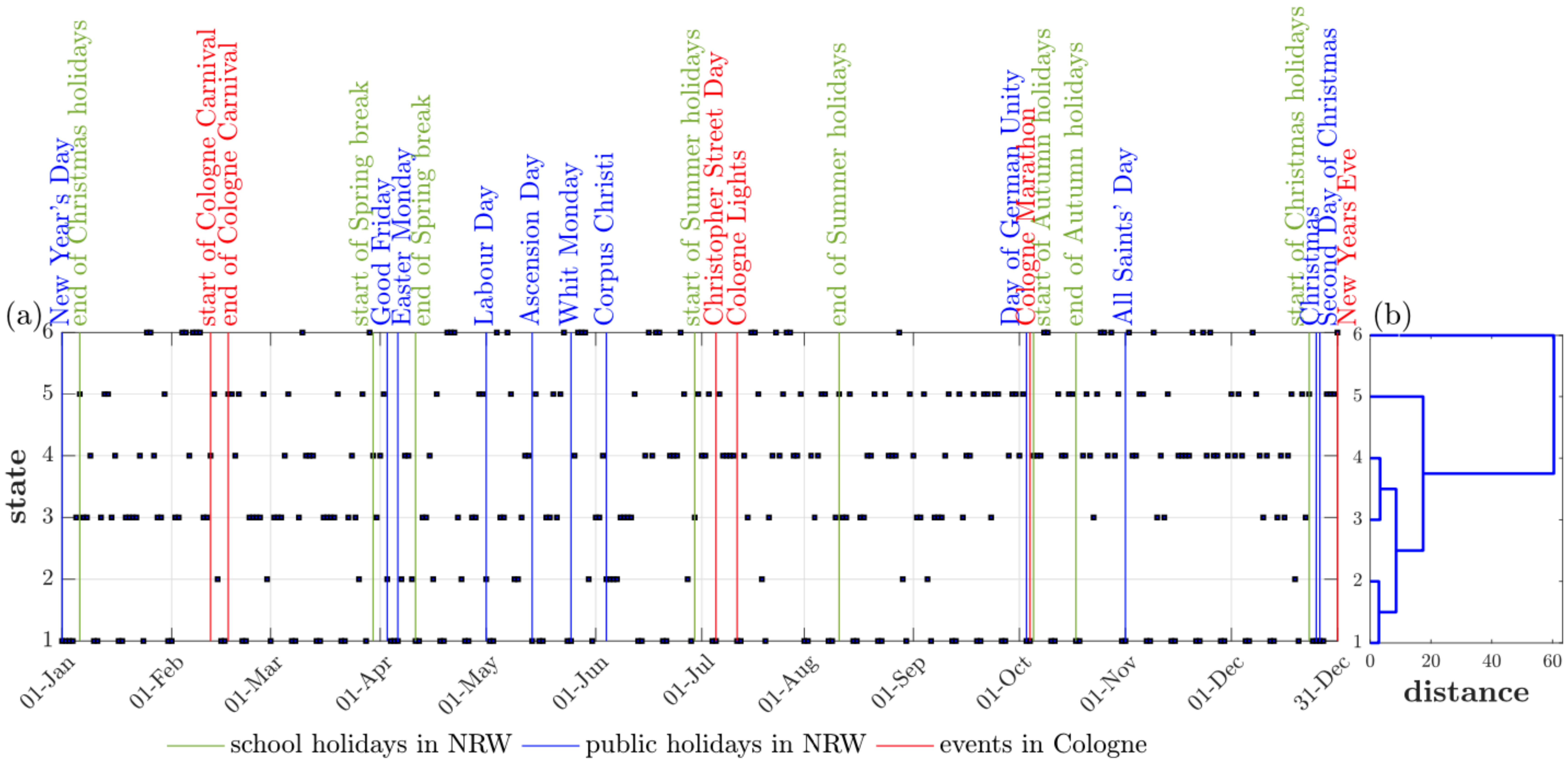
$$\eta_{ij} = \left\langle \left| \tilde{D}_{t_k t_l}(i) - \tilde{D}_{t_k t_l}(j) \right| \right\rangle_{t_k t_l}$$

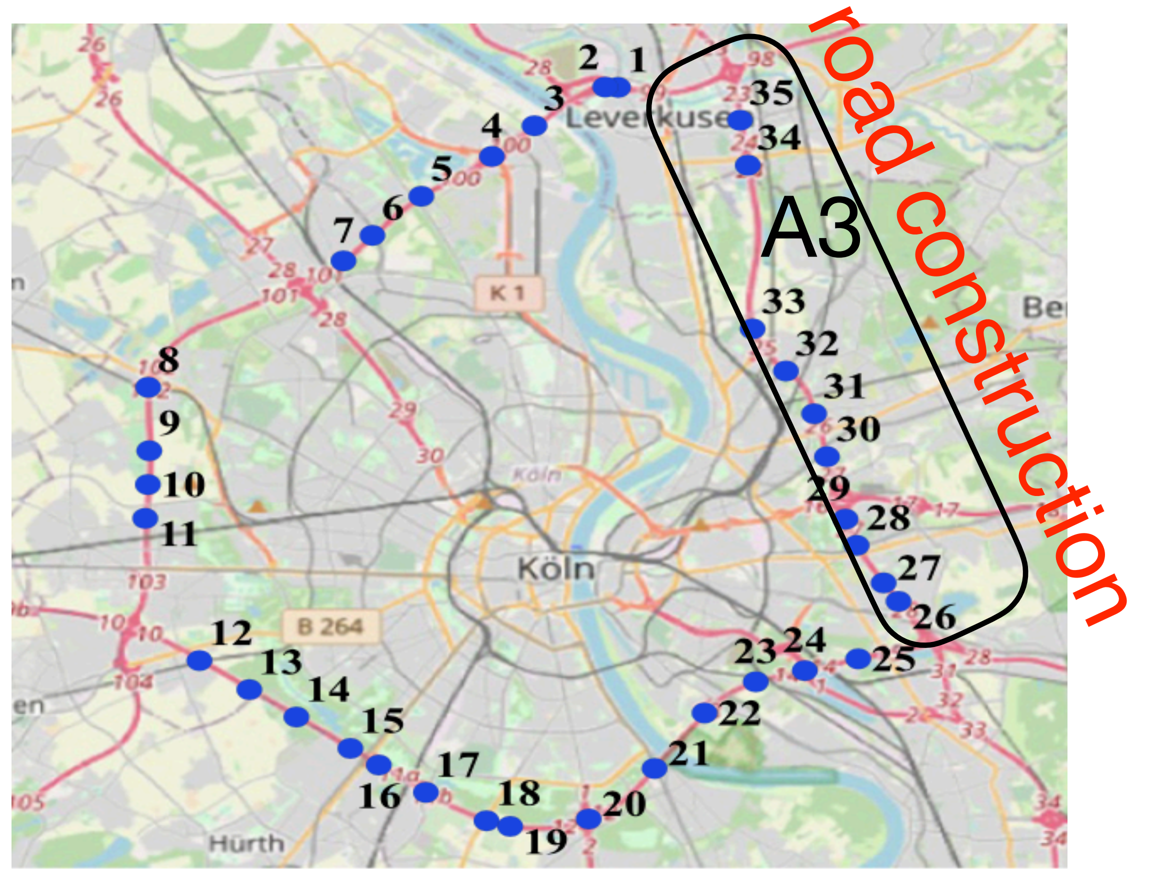
$$d_{ij} = \sum_{n=1}^{N} (\eta_{in} - \eta_{jn})^2$$

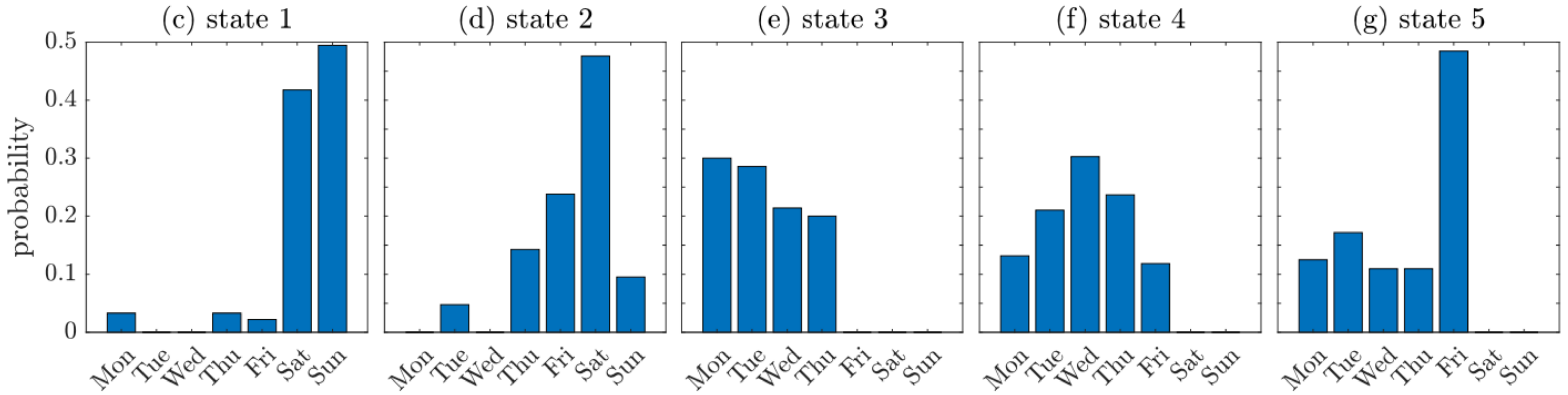
- determine number of clusters:
  - given a k, run 500 times of k-means clustering
  - obtain 500 averaged intra-cluster distances
  - obtain a mean value and a standard deviation
  - find the k corresponding to the minimal standard deviation
- with the best k , perform k-means clustering
- validate the consistency within clusters by silhouette values



Identification of quasi-stationary states—time evolution of quasi-stationary states







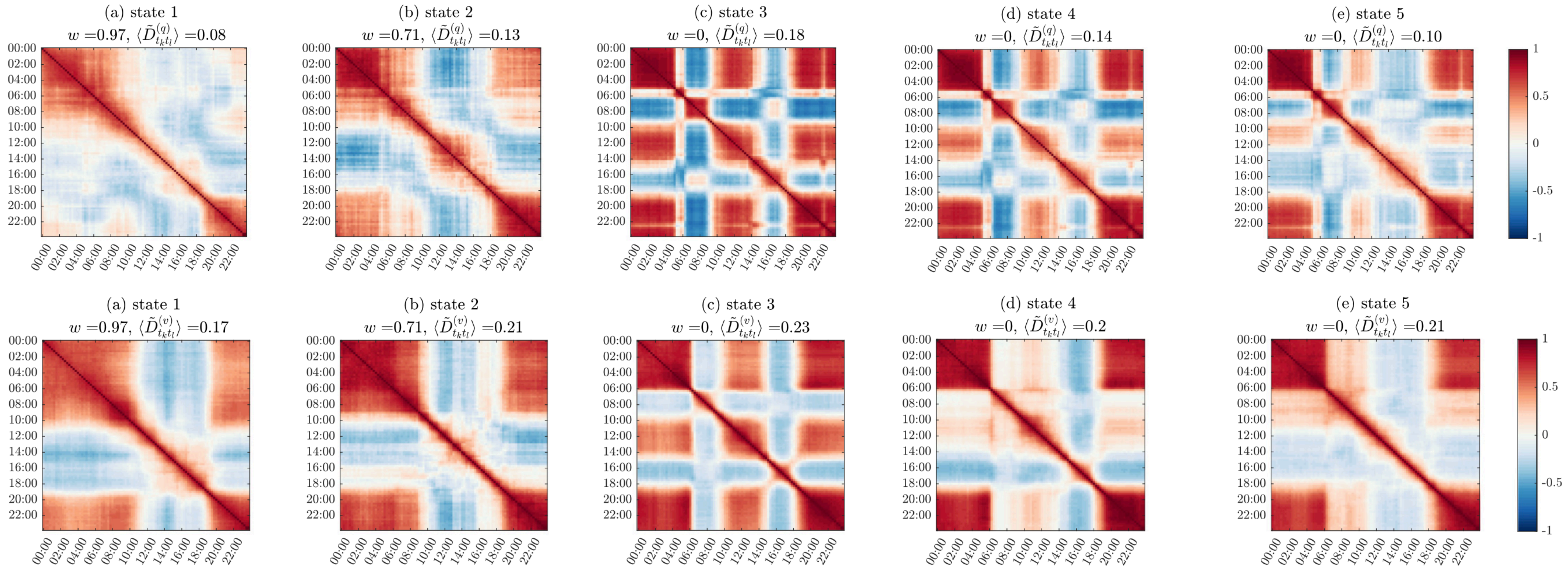
- state 1: almost for holidays
- state 2: both workdays and holidays
- states 3,4,5: only for workdays

Identification of quasi-stationary states—correlation structures

holiday state	mixed state	workday state Mon.-Thur.	workday state Mon.-Fri.	workday state mainly Fri.

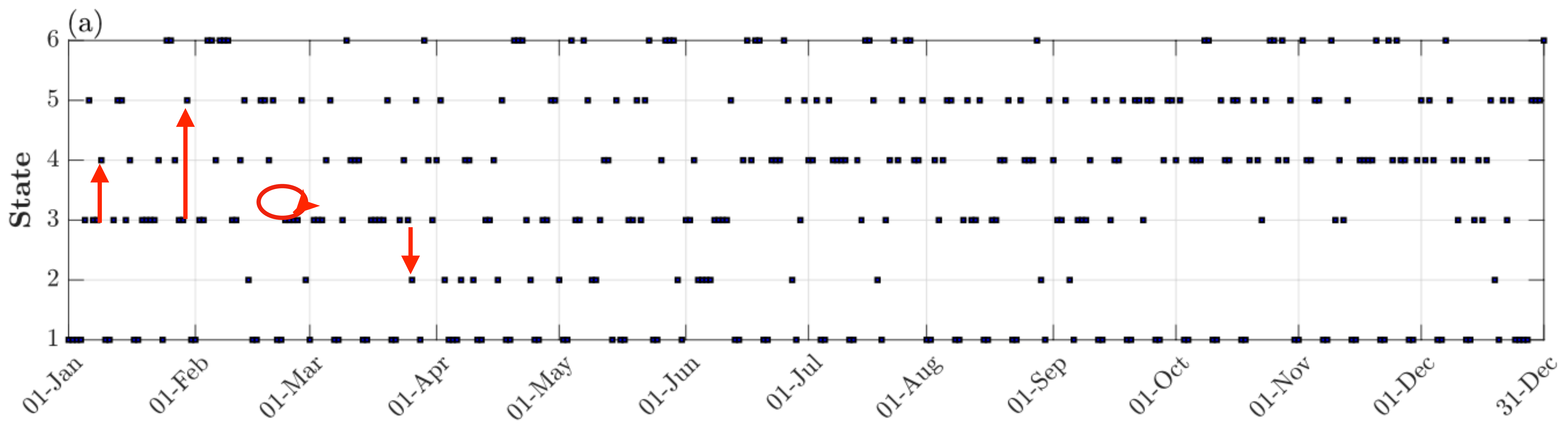
flow

velocity



How does each quasi-stationary state change to others?

Transitions between quasi-stationary states—Transition probabilities



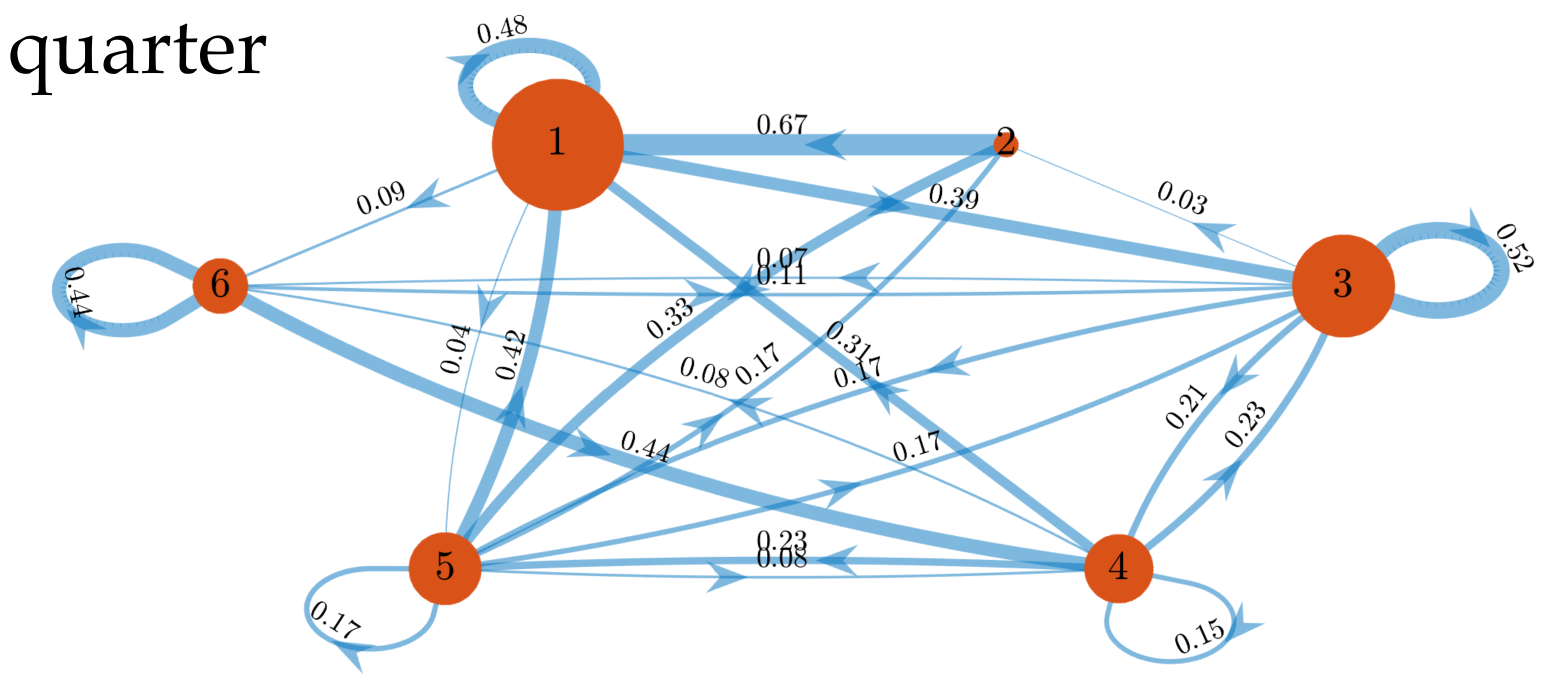
transition probability

$$p_{ij}^{(\tau)} = \Pr\Big(X(t+\tau) = j \,|\, X(t) = i\Big)$$
$$= \frac{n_{ij}^{(\tau)}}{\sum_{q=1}^{\mathcal{K}} n_{iq}^{(\tau)}}$$

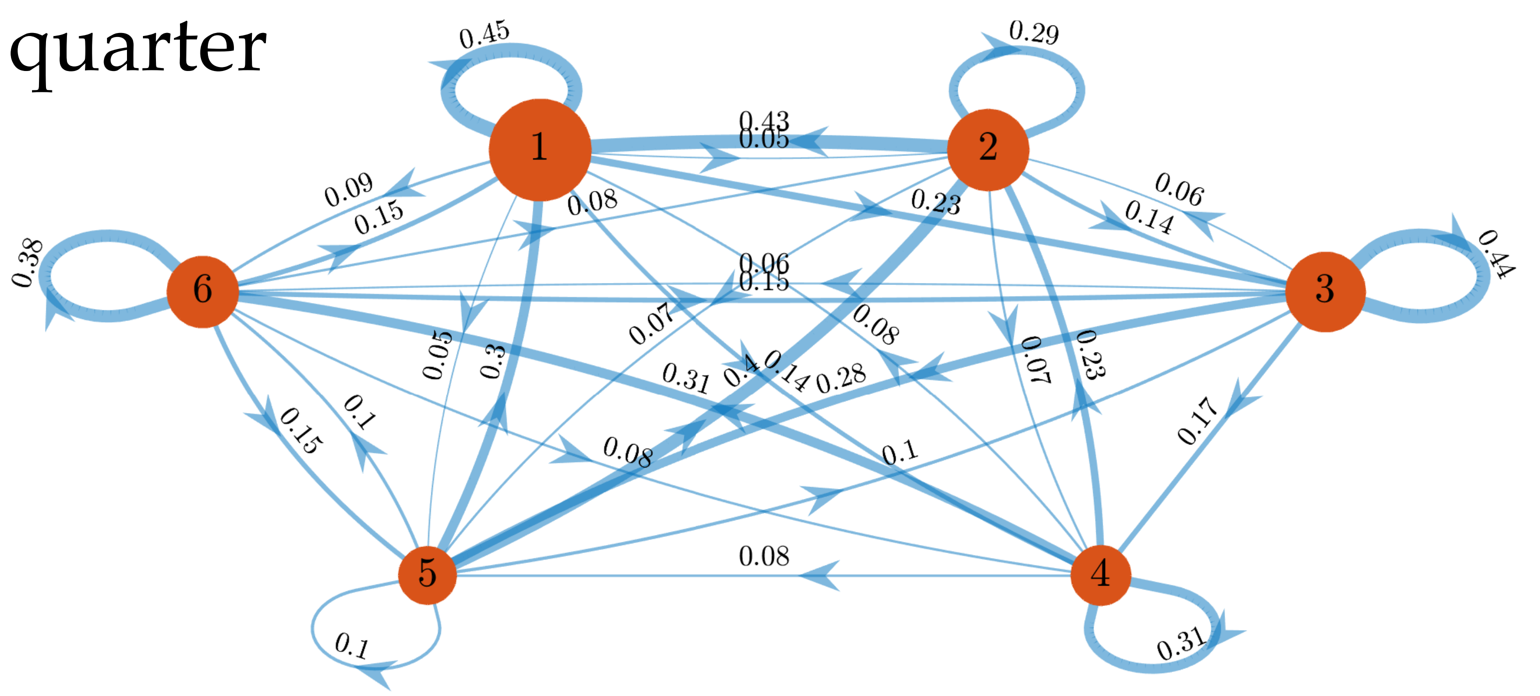
transition matrix

$$P^{(\tau)} = \begin{bmatrix} p_{11}^{(\tau)} & \cdots & p_{1n}^{(\tau)} \\ \vdots & \ddots & \vdots \\ p_{N1}^{(\tau)} & \cdots & p_{NN}^{(\tau)} \end{bmatrix}$$

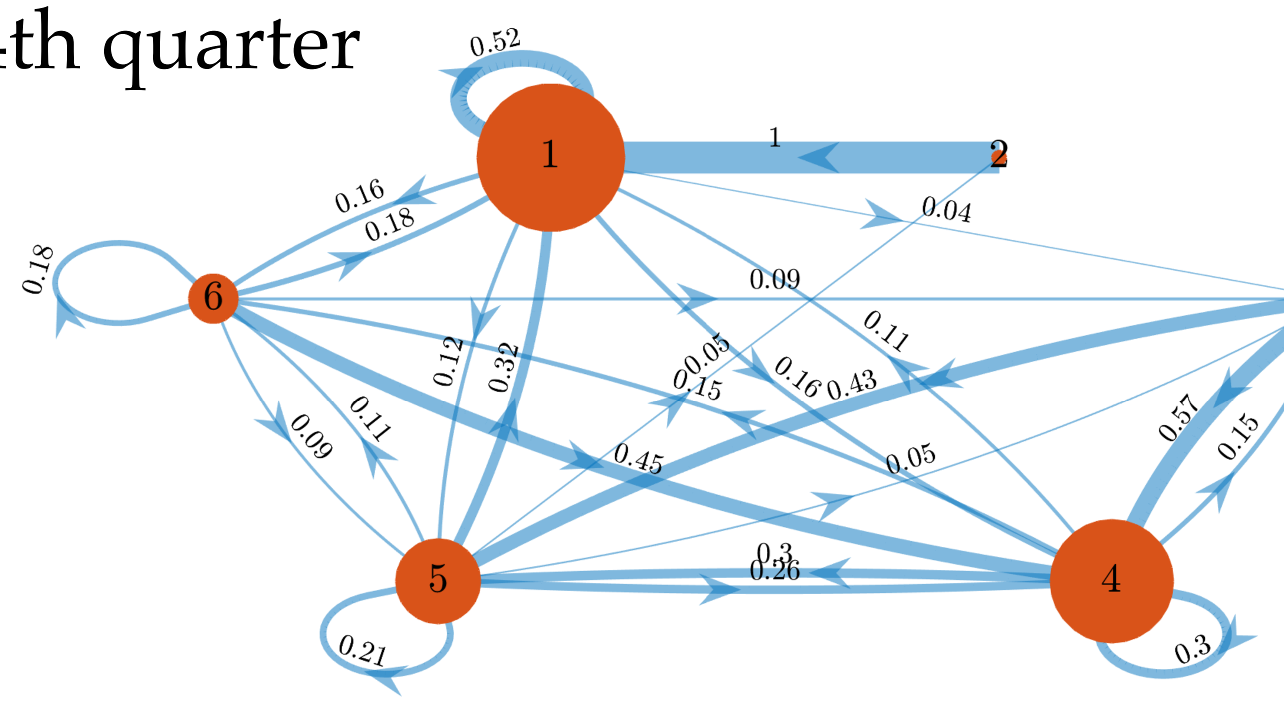
1st quarter



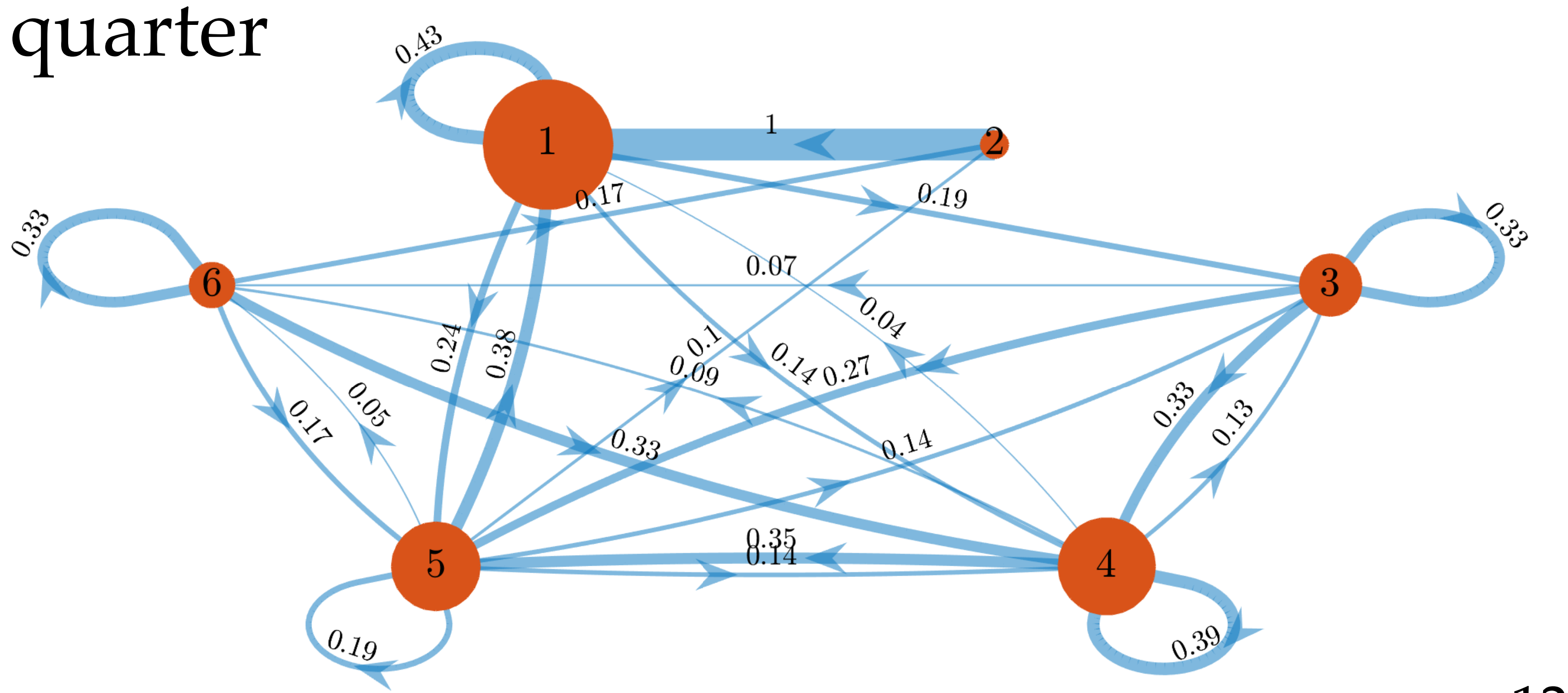
2nd quarter



4th quarter



3rd quarter

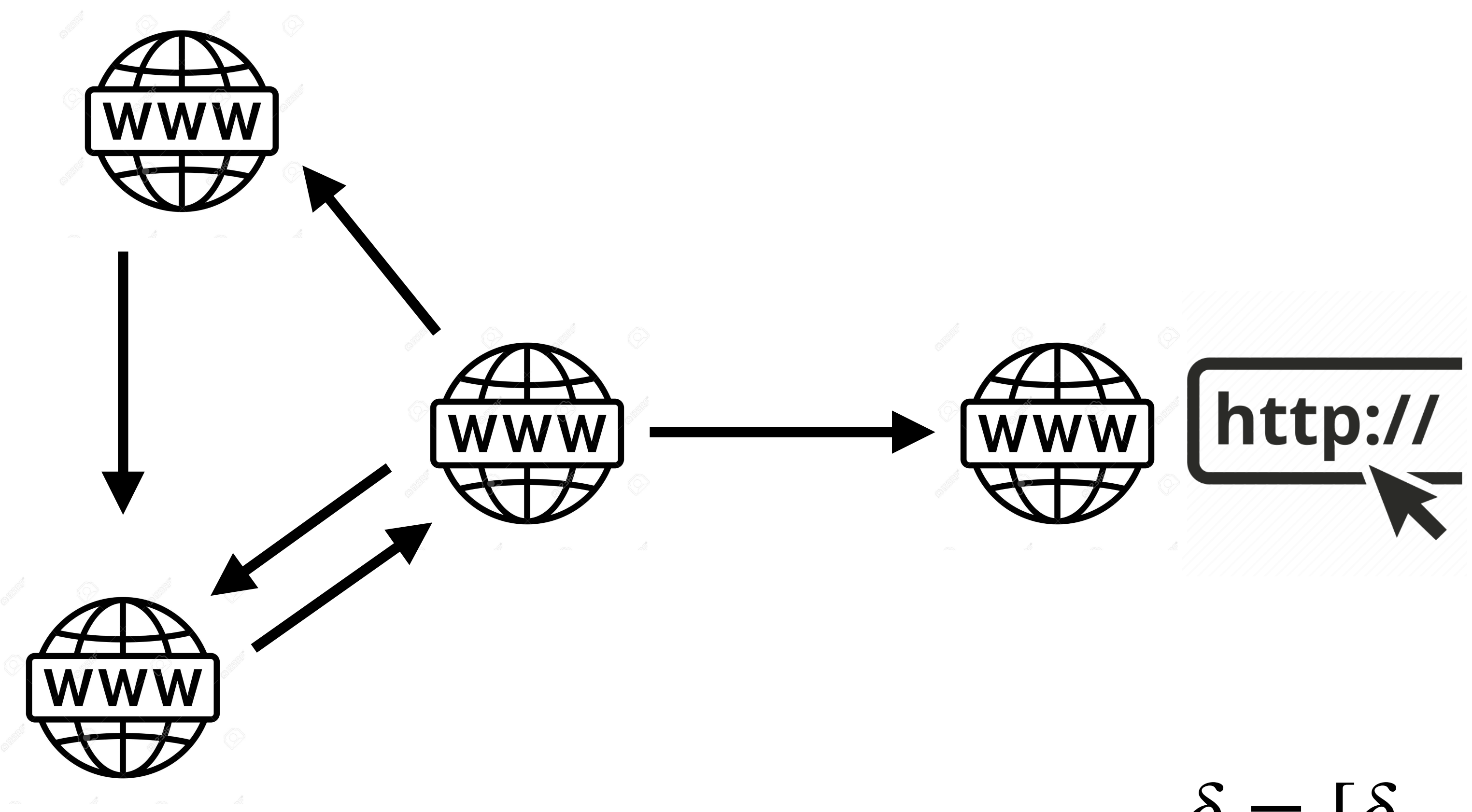


Which state dominates the system and how this dominant state evolve with time?

Transitions between quasi-stationary states—PageRank algorithm

- PageRank, is the first algorithm used by Google Search to rank web pages.
- It assumes that more important websites are likely to receive more links from other websites.

α (weights hyperlink structure)



$$H = \begin{bmatrix} H_{11} & \cdots & H_{1n} \\ \vdots & \ddots & \vdots \\ H_{n1} & \cdots & H_{nn} \end{bmatrix}$$

Hyperlink matrix

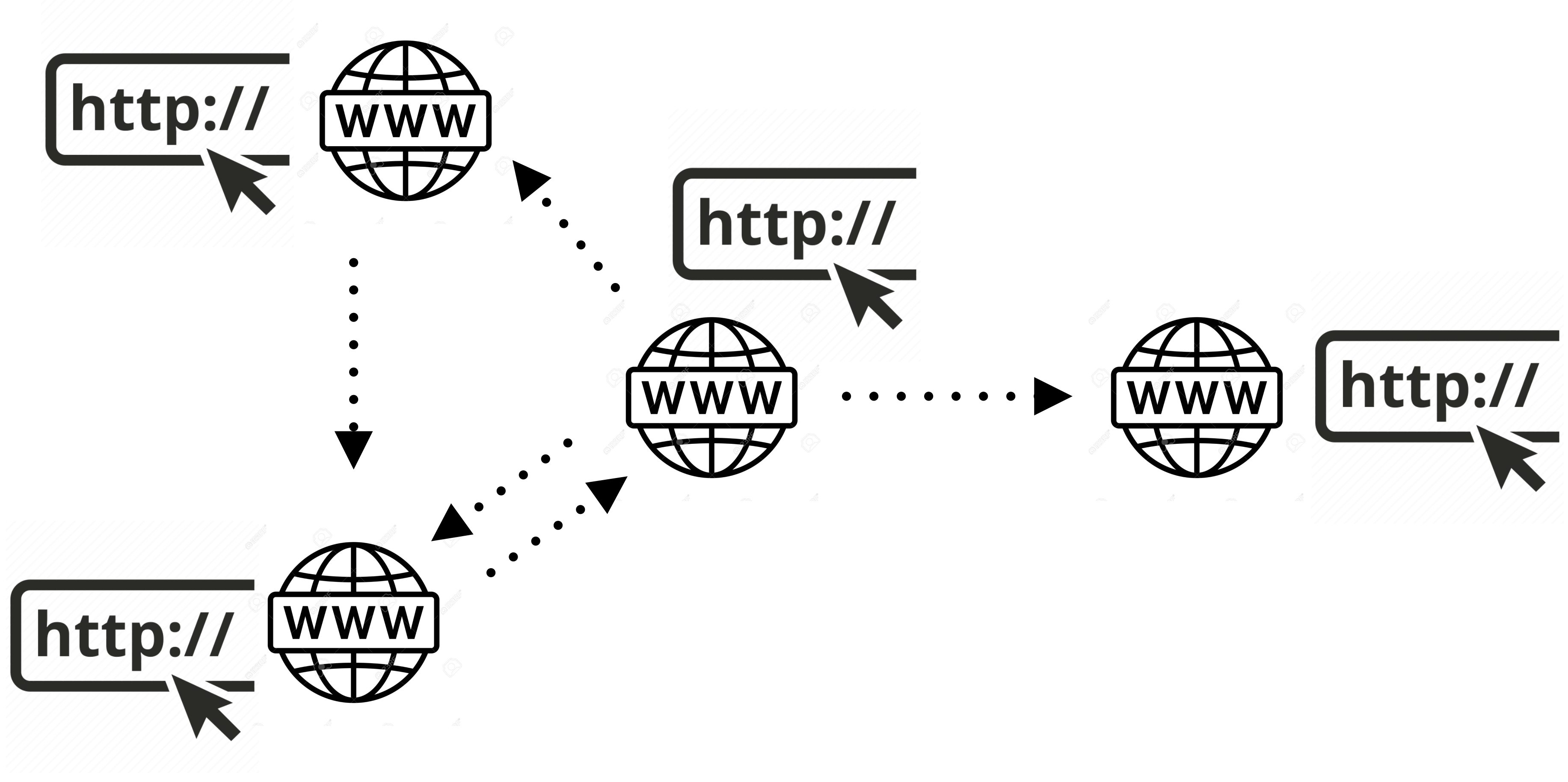
$$\delta = [\delta_1, \ \cdots, \ \delta_n]$$

$$\delta_i = \begin{cases} 0, & i \text{ has no link to others} \\ 1, & \text{otherwise} \end{cases}$$

$$w = [1/n \ \cdots \ 1/n]$$

$$S = H + \delta w$$

$1 - \alpha$ (weights random events)



$$\mathbf{1} = [1, \ \cdots, \ 1]$$

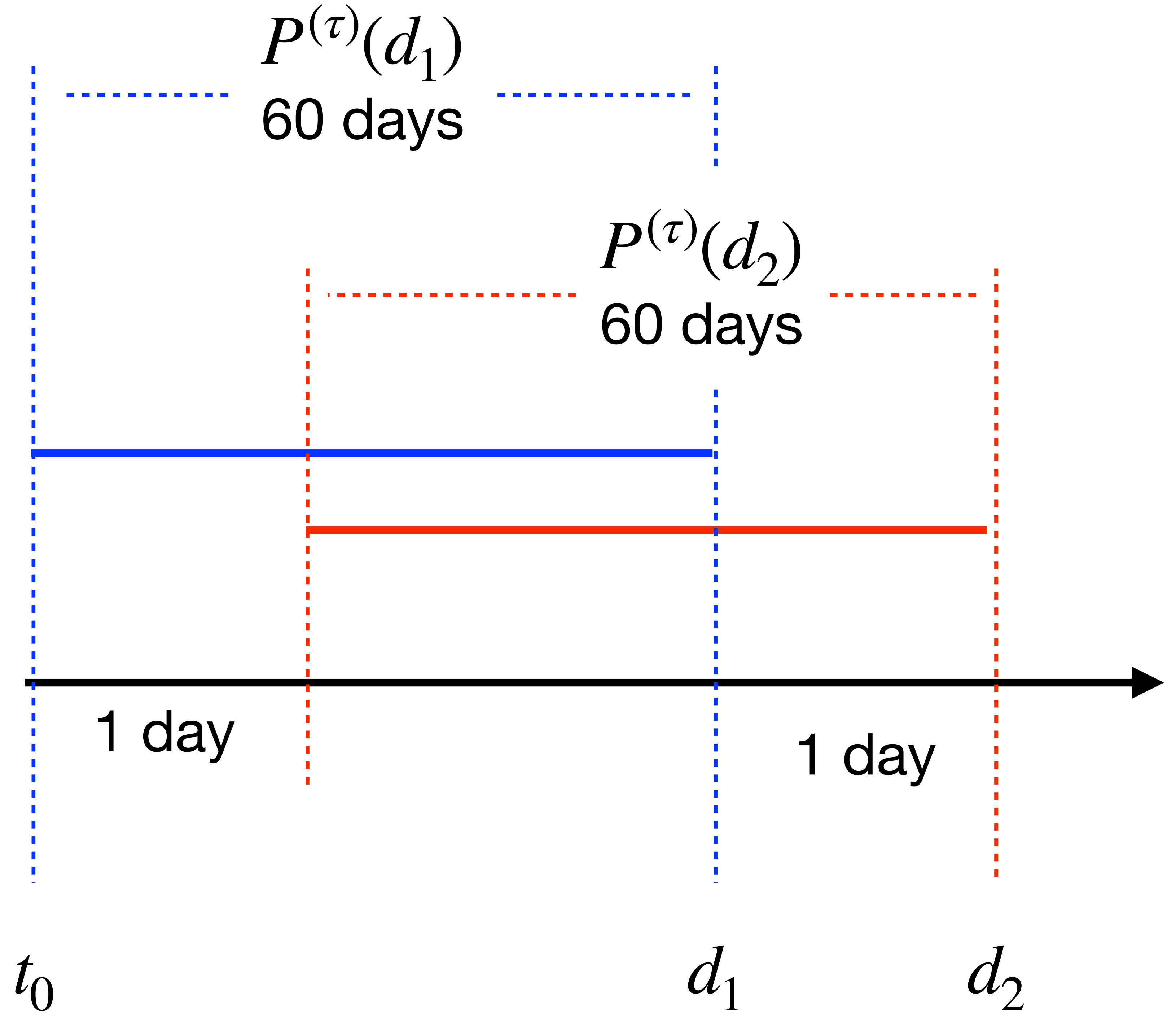
$$\zeta = [1/n \ \cdots \ 1/n]$$

Google matrix $G = \alpha S + (1 - \alpha)\mathbf{1}\zeta$

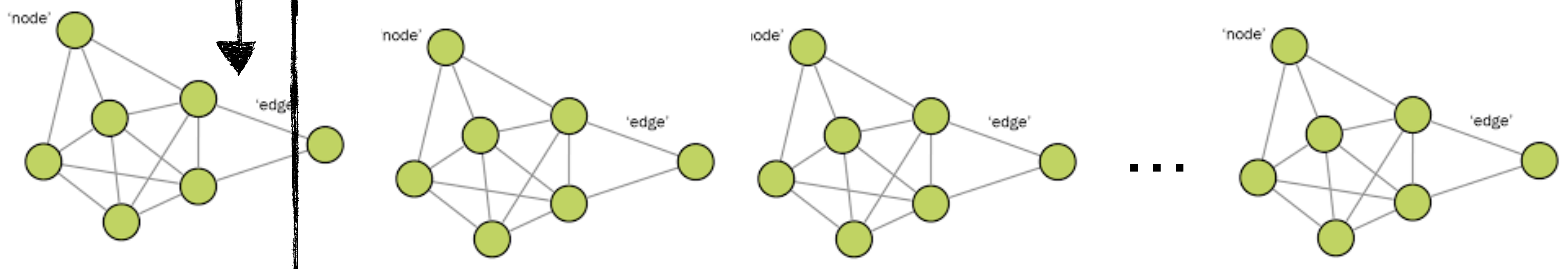
Refs.

- Brin and Page. Computer Networks and ISDN Systems, 30(107-117), 1998.
- Page, Brin, Motwani and Winograd. Technical report, Stanford InfoLab, 1999.
- Langville and Meyer. Princeton University Press, 2011.

Transitions between quasi-stationary states—PageRank score



transition matrices $P^{(\tau)}(d_1),\ P^{(\tau)}(d_2), P^{(\tau)}(d_3), \dots, P^{(\tau)}(d_n)$



Google matrices $G(d) = \alpha(P^{(\tau)}(d) + \delta w) + (1-\alpha)\mathbf{1}\zeta$

Stationary distributions
by power method

$$\pi(d) = [\pi_1(d) \ \cdots \ \pi_n(d)]$$
$$\pi(d)G(d) = \pi(d)$$

PageRank scores
measures the importance of states

$$\pi_i(d)$$

Dominant state for day d

$$\underset{i}{\text{argmax}}\ \pi_i(d)$$

Transitions between quasi-stationary states—Dominant states in quasi-stationary systems

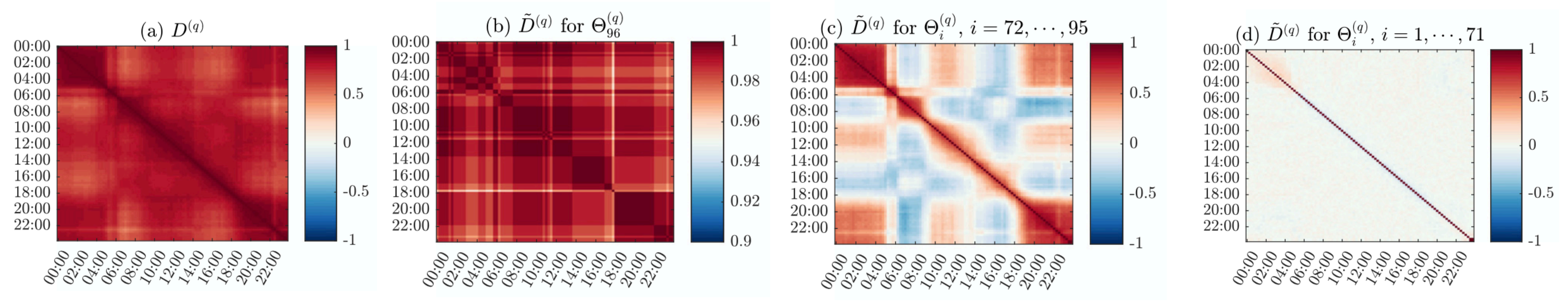
$$G(d) = \alpha(P^{(\tau)}(d) + \delta w) + (1-\alpha)\mathbf{1}\zeta$$

$$\pi(d)G(d) = \pi(d)$$

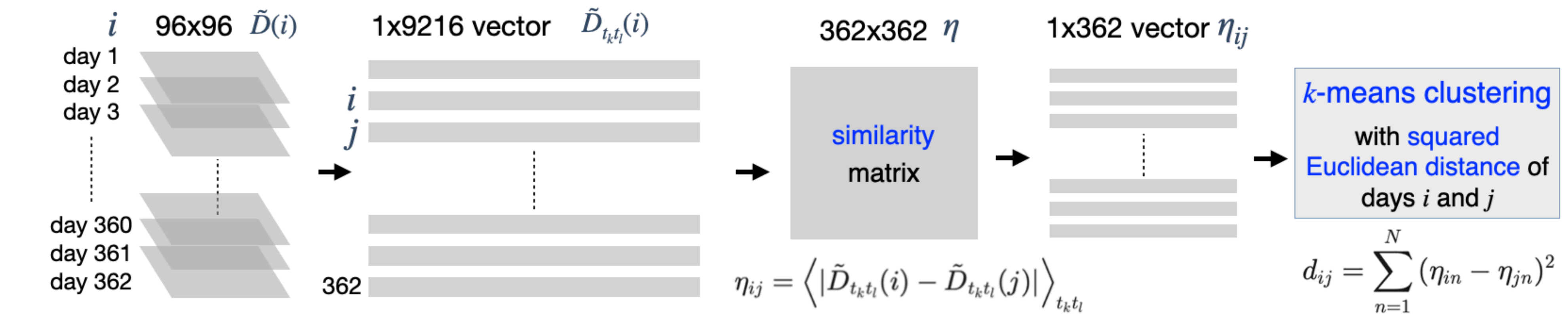
- For one-step transition, $\tau = 1$ day
- $1-\alpha$: occurring probability of random events
- PageRank score $\pi_i(d)$: reflects the importance of a state
- for $\alpha = 1$, no random events, dominant states depend on the seasons
- reducing α prolongs the time period dominated by the holiday state (state 1)
- Random events, e.g., coronavirus breakout, violent typhoon

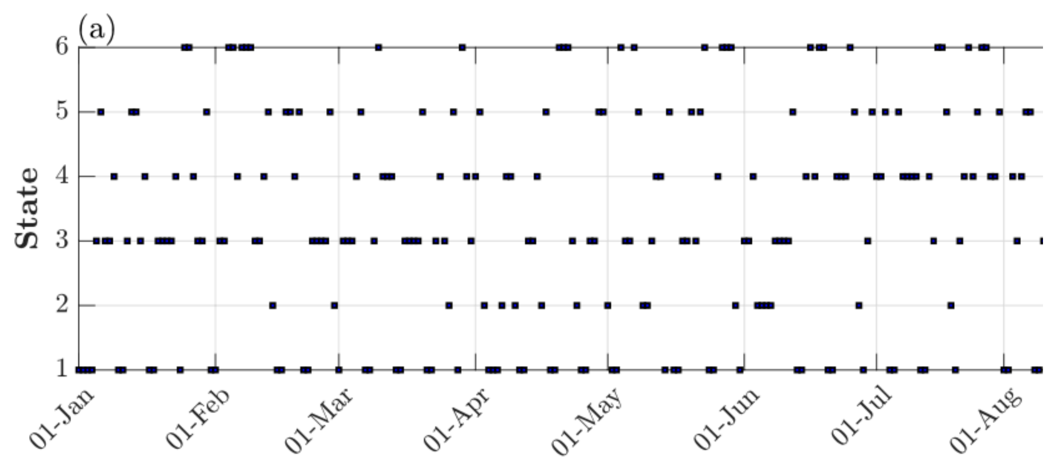
Summary

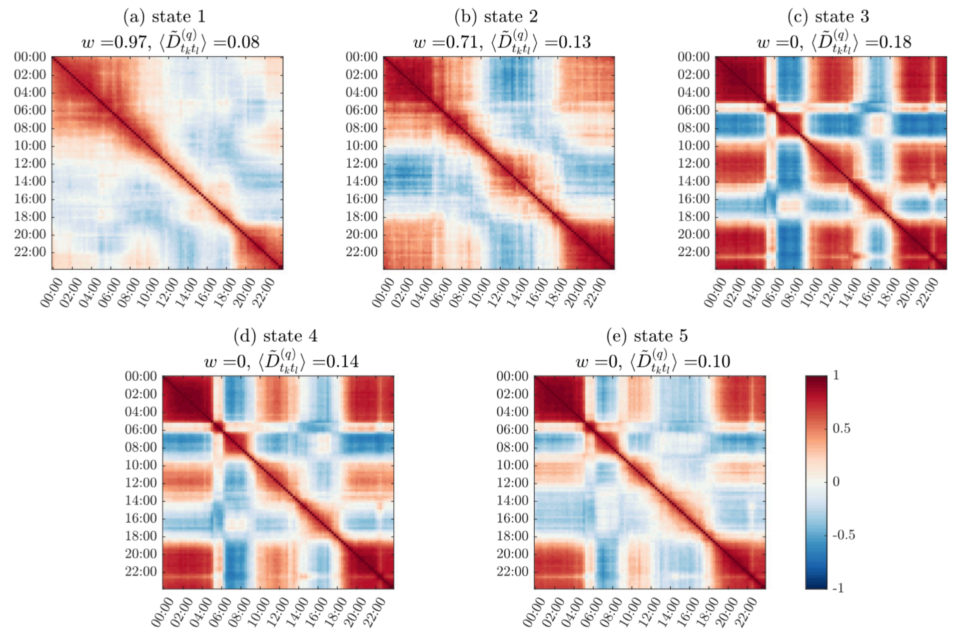
- Found distinct structural features depending on time, i.e., the rich non-Markovian features of traffic, in the reduced-rank correlation matrix of traffic flows



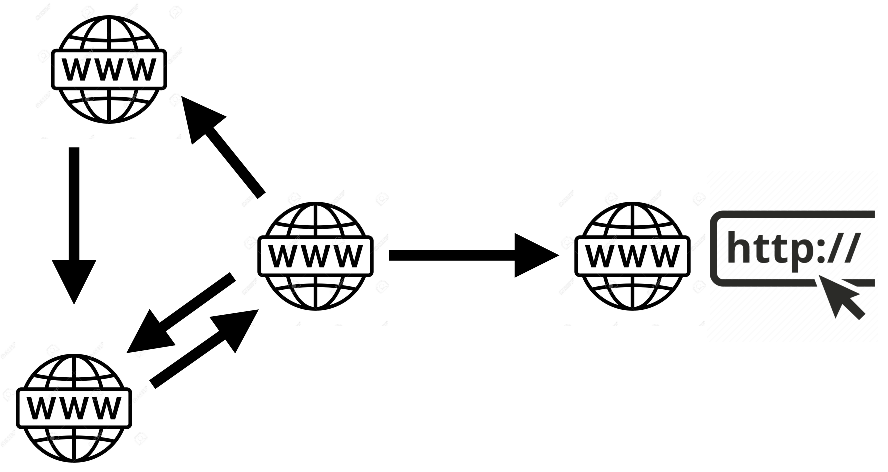
- Identified six quasi-stationary states by k-means clustering with different correlation structures

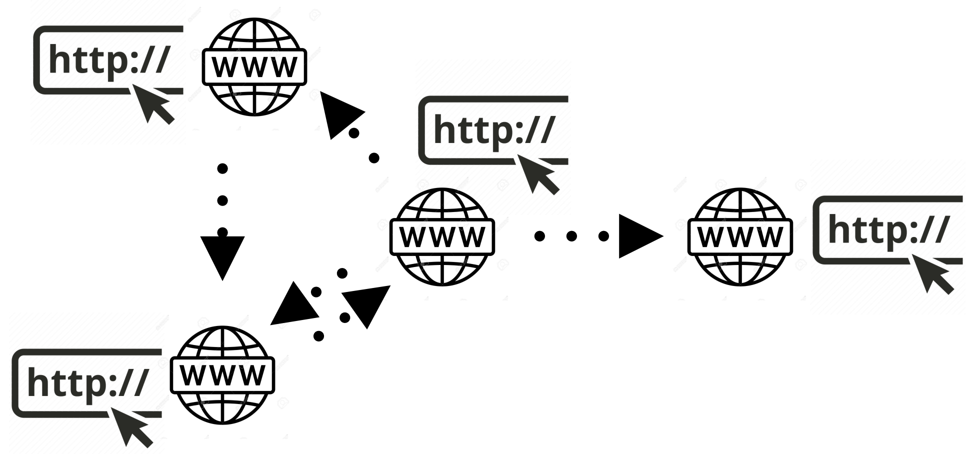


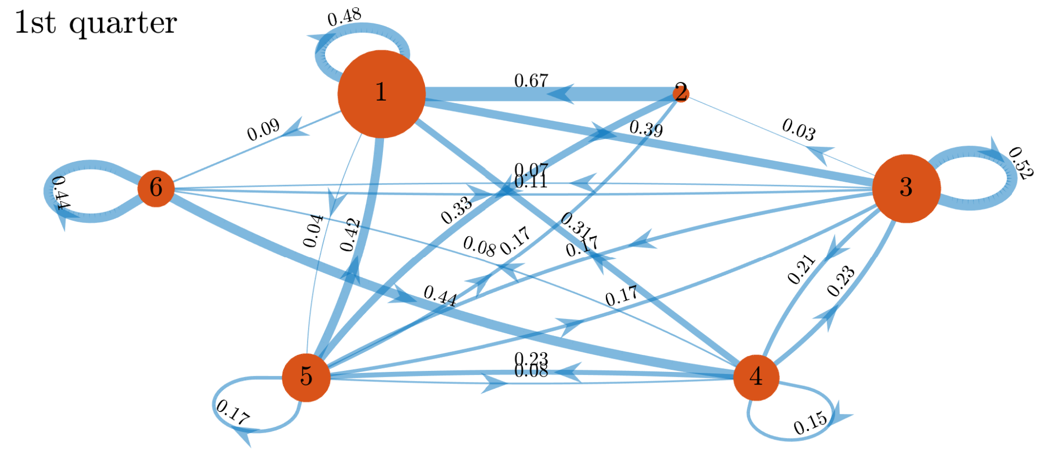


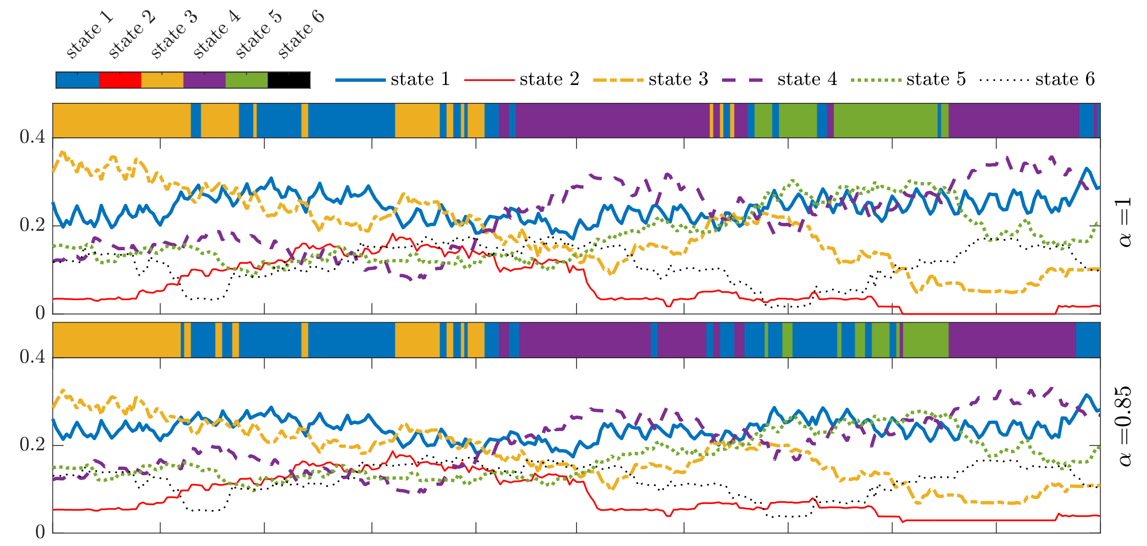


- Identified dominant states with PageRank algorithm, and found the dominant state depends on the seasons.
- Increasing the influence of random events prolongs the time period dominated by the holiday state.









Thank you for your attention!

Thanks to my collaborators

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Prof. Dr. Michael Schreckenberg

Prof. Dr. Thomas Guhr

Wang, Gartzke, Schreckenberg, and Guhr, J. Stat. Mech. **2020** 103404 (2020)

Wang, Schreckenberg, and Guhr, accepted by J. Stat. Mech., arXiv:2302.14596 (2023)

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