Faculty of Physics

UNIVERSITÄT DUISBURG ESSEN

Open-Minded

Microscopic understanding of price cross-responses between stocks

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DPG conference, Dresden, March 2017

Contents

Introduction —order book

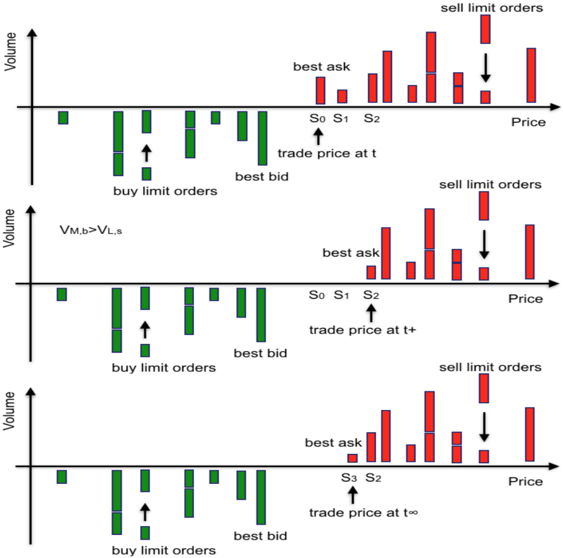
BÖRSE
FRANKFURT

Bid/Ask-Übersicht

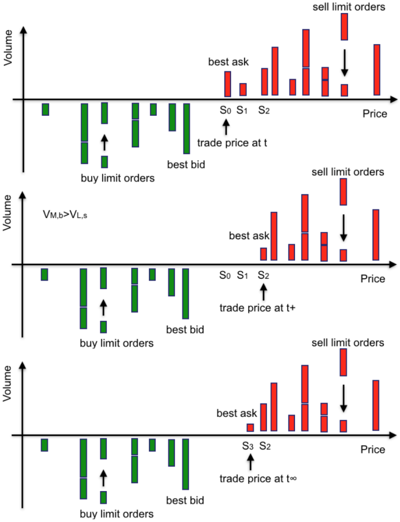
Bid	Bid Vol	*limitierte Kaufaufträge*	*limitierte Verkaufaufträge*	Ask Vol	Ask
69,140	98			416	69,180
69,130	346			527	69,190
69,120	410			954	69,200
69,110	1.760			1.608	69,210
69,100	2.479			1.112	69,220
69,090	1.242			707	69,230
69,080	1.131			2.023	69,240
69,070	2.744			548	69,250
69,060	1.073			366	69,260
69,050	910			427	69,270

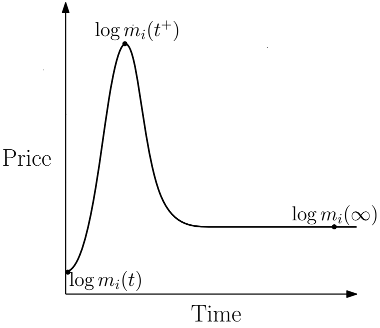
ref. https://www.traders-only.com/orderbuch/

Introduction —price formation

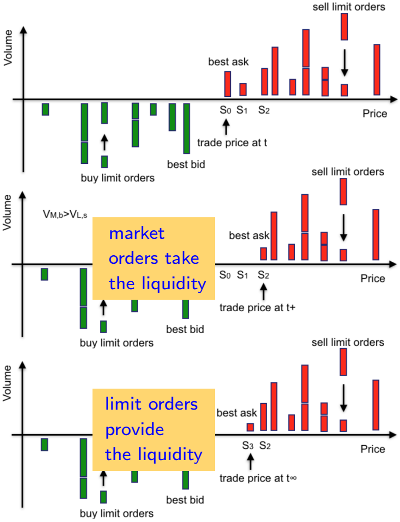


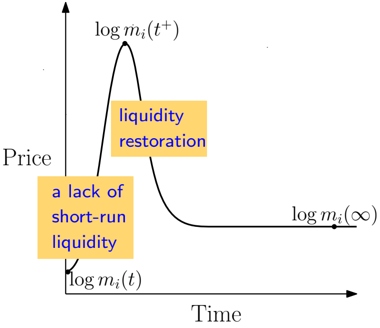
Introduction —price formation





Introduction —price formation

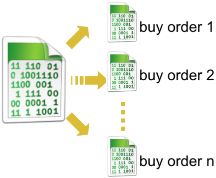




- in a liquidity market, shares can be rapidly bought or sold with little impact on stock price.
- market liquidity measured by spread between best ask and best bid.

Empirical results —correlations of trade signs

Order splitting



Trade sign

$$\varepsilon = \begin{cases} +1 & , \quad \text{for a buy market order} \\ -1 & , \quad \text{for a sell market order} \end{cases}$$

Correlation of trade signs in single stocks

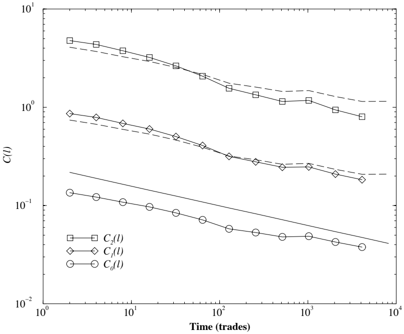
$$\begin{aligned} \mathcal{C}_0(l) &= \langle \varepsilon_{n+l}\varepsilon_n \rangle - \langle \varepsilon_n \rangle^2 \\ \mathcal{C}_1(l) &= \langle \varepsilon_{n+l}\varepsilon_n \ln V_n \rangle \\ \mathcal{C}_2(l) &= \langle \varepsilon_{n+l} \ln V_{n+l}\varepsilon_n \ln V_n \rangle \end{aligned}$$

fitted by

$$\mathcal{C}_0(l) \simeq \frac{C_0}{l^\gamma} \ , \quad (l > 1) \ ,$$

where $\gamma = 1/5$ for France-Telecom.
(long memory for the sign correlation)

Bouchaud, Gefen, Potters, Wyart,
Quantitative Finance, 4, 176 (2004).



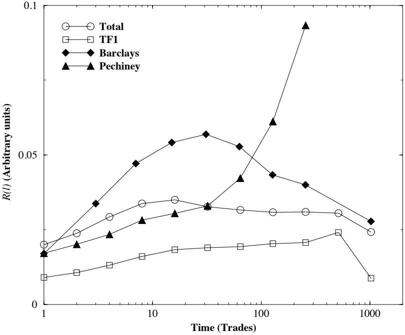
Empirical results —price self-responses

Price self–response

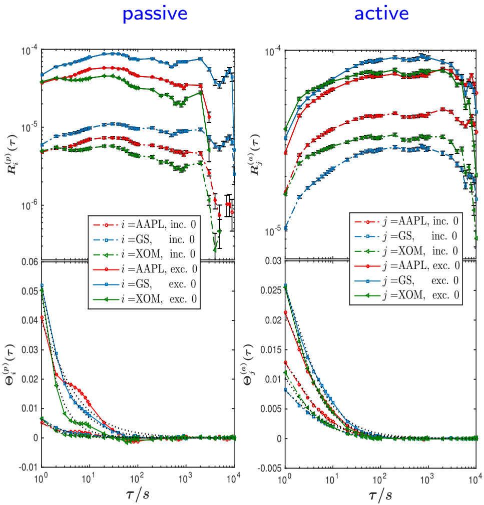
measures how much price changes after time τ, on average, conditioned on an initial buy or sell market order.

$$R_{ii}(\tau) = \left\langle \Big(S_i(t+\tau) - S_i(t) \Big) \varepsilon_i(t) \right\rangle_t$$

Bouchaud, Gefen, Potters, Wyart, Quantitative Finance, 4, 176 (2004).



Empirical results — average cross–responses and average sign cross–correlators



Wang, Schäfer, and Guhr, Eur. Phys. J. B 89, 207 (2016)

Price cross-response of stock i to stock j

$$R_{ij}(\tau) = \left\langle \Big(\log m_i(t+\tau) - \log m_i(t) \Big) \varepsilon_j(t) \right\rangle_t$$

cross–correlator of trade signs

$$\Theta_{ij}(\tau) = \left\langle \varepsilon_i(t+\tau)\varepsilon_j(t) \right\rangle_t$$

passive and active cross-response

$$R_i^{(p)}(\tau) = \langle R_{ij}(\tau) \rangle_j \quad , \quad R_j^{(a)}(\tau) = \langle R_{ij}(\tau) \rangle_i$$

passive and active sign correlators

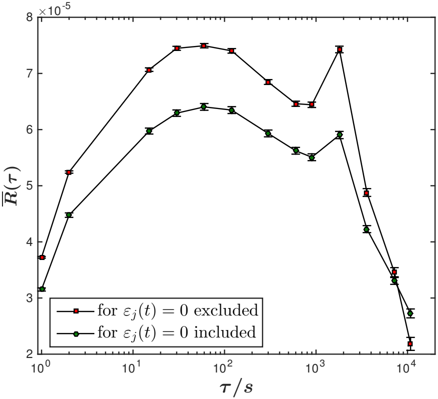
$$\Theta_i^{(p)}(\tau) = \langle \Theta_{ij}(\tau) \rangle_j \quad , \quad \Theta_j^{(a)}(\tau) = \langle \Theta_{ij}(\tau) \rangle_i$$

(long memory for sign cross-correlations)

sign correlators are fitted by

$$\Theta_i(\tau) = \frac{\theta_i}{\left(1 + (\tau/\tau_i^{(0)})^2\right)^{\gamma_i/2}}$$

Empirical results — market responses



- doubly averaged response for the market

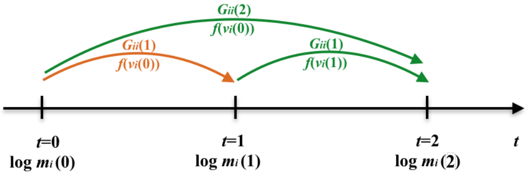
$$\overline{R}(\tau) = \langle\langle R_{ij}(\tau)\rangle_j\rangle_i$$

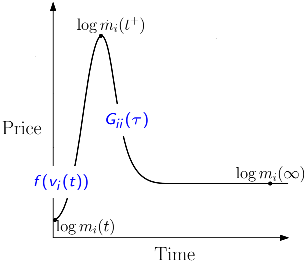
  excluding $i = j$.
- 99 stocks from 10 economic sectors in 2008
- for each sector, first 9 or 10 stocks with largest average market capitalization

Market efficiency is violated on short time scales, but restored on longer time scales.

Wang, Schäfer and Guhr, Eur. Phys. J. B 89, 105 (2016)

Price impact model —single stocks





$$\log m_i(1) = \log m_i(0) + G_{ii}(1) f\left(v_i(0)\right)\varepsilon_i(0) + \eta_{ii}(0)$$

$$\begin{aligned} \log m_i(2) &= G_{ii}(1) f\left(v_i(1)\right)\varepsilon_i(1) + \eta_{ii}(1) \\ &+ G_{ii}(2) f\left(v_i(0)\right)\varepsilon_i(0) + \eta_{ii}(0) \\ &+ \log m_i(0) \end{aligned}$$

$$\begin{aligned} \log m_i(t) &= \sum_{t'<t} G_{ii}(t-t') f\left(v_i(t')\right)\varepsilon_i(t') + \sum_{t'<t} \eta_{ii}(t') \\ &+ \log m_i(-\infty) \end{aligned}$$

$v_i(t)$: traded volume

$\eta_{ii}(t)$: random variable

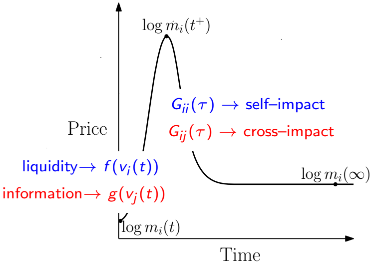
$f(v_i(t))$: impact function of traded volumes

$G_{ii}(\tau)$: 'bare' impact function of time lags for a single trade

Bouchaud, Gefen, Potters, Wyart, Quantitative Finance, 4, 176 (2004).

Price impact model —across stocks

$$\begin{aligned}\log m_i(t) &= \sum_{t'<t}\left[G_{ii}(t-t')f\left(v_i(t')\right)\varepsilon_i(t')+\eta_{ii}(t')\right]\\ &+ \sum_{t'<t}\left[G_{ij}(t-t')g\left(v_j(t')\right)\varepsilon_j(t')+\eta_{ij}(t')\right]\\ &+ \log m_i(-\infty)\end{aligned}$$



Price impact model —across stocks

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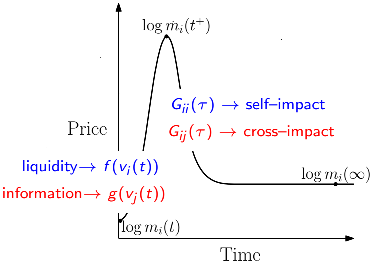


cross–response function of stock i to stock j is

$$R_{ij}(\tau) = \Big\langle \Big(\log m_i(t+\tau) - \log m_i(t) \Big) \varepsilon_j(t) \Big\rangle_t$$

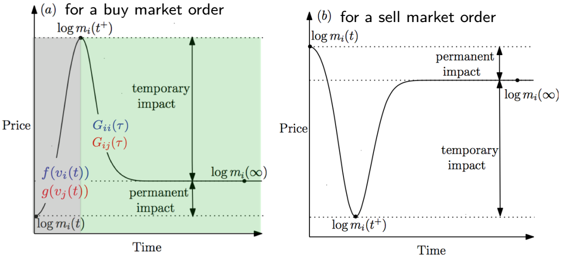
passive and active cross–response functions

$$R_i^{(p)}(\tau) = \langle R_{ij}(\tau) \rangle_j \quad \text{and} \quad R_i^{(a)}(\tau) = \langle R_{ji}(\tau) \rangle_j$$

Wang and Guhr, arXiv:1609.04890
see also: Benzaquen, Mastromatteo, Eisler and Bouchaud, arXiv:1609.02395

Price impact model —across stocks

- impact function of time lag (sketches)



Assume impact function

$$G(\tau) = \frac{\Gamma_0}{\left[1 + \left(\frac{\tau}{\tau_0}\right)^2\right]^{\beta/2}} + \Gamma$$

temporary or permanent impact is determined by data fits.

- average impacts of traded volumes (based on empirical analysis)

$$\left\langle f_i^{(p)}(v_i)\right\rangle_t \sim v_i^{\delta_{ip}}, \qquad \left\langle g_i^{(p)}(v_j)\right\rangle_{t,j} \sim v_j^{\delta_{jp}}$$

$$\left\langle f_i^{(a)}(v_j)\right\rangle_{t,j} \sim v_j^{\delta_{ja}}, \qquad \left\langle g_i^{(a)}(v_i)\right\rangle_t \sim v_i^{\delta_{ia}}$$

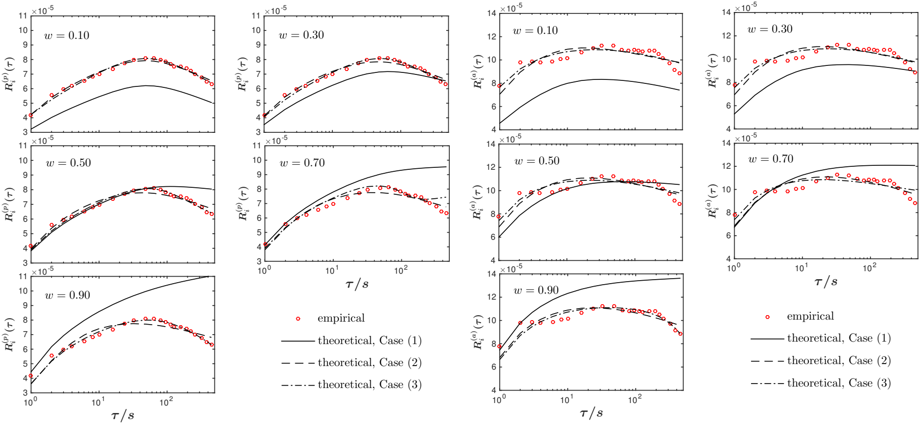
$\delta_{ip}, \delta_{jp}, \delta_{ja}, \delta_{ia} \sim 0.5 \pm 0.2$ for small volumes of most stocks

$\langle f_i^{(p)}(v_i)\rangle_t$, $\langle g_i^{(p)}(v_j)\rangle_{t,j}$, $\langle f_i^{(a)}(v_j)\rangle_{t,j}$, $\langle g_i^{(p)}(v_i)\rangle_t \rightarrow$ independent of time lag

Price impact model —simulations and data fits

passive cross–responses

active cross–responses

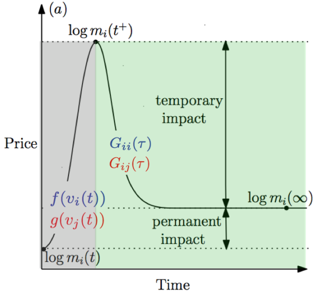


stock i is MSFT in 2008, and the pairwise stocks j are other 30 stocks with the largest average number of daily trades in S&P 500 index of 2008.

Wang and Guhr, arXiv:1609.04890

Price impact model —impact functions

sketch of price impacts



after averaging,

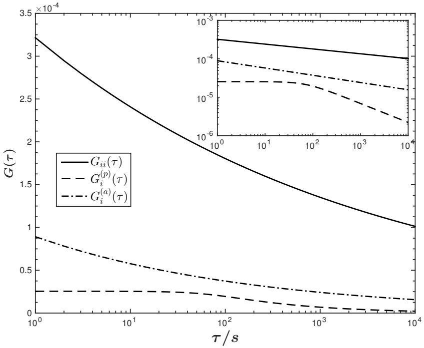
$$G_{ij}(\tau) \to G_i^{(p)}(\tau),\, G_i^{(a)}(\tau)$$

simulated impact function

$$G(\tau) = \frac{\Gamma_0}{\left[1+\left(\frac{\tau}{\tau_0}\right)^2\right]^{\beta/2}} + \Gamma$$

Wang and Guhr, arXiv:1609.04890

simulations of impact functions

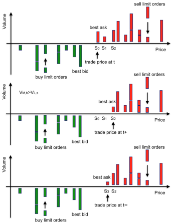


MSFT in 2008

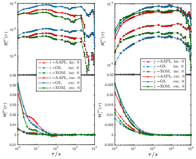
impact functions	Γ $(\times 10^{-10})$	Γ_0 $(\times 10^{-4})$	τ_0 [s]	β
$G_{ii}(\tau)$	0.5	5.12	0.025	0.13
$G_i^{(p)}(\tau)$	0	0.25	70.873	0.49
$G_i^{(a)}(\tau)$	0	2.57	0.004	0.19

Summary

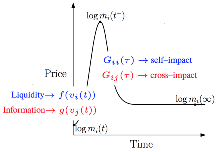
- price formation:
  - interaction of market orders and limit orders
  - liquidity plays important role



- empirical results:
  - average cross–responses and sign correlators
  - market responses



- price impact model:
  - a self– and a cross–impact function
  - comparison of empirical and simulated results
  - self–, active and passive impact functions



Our papers

[1] Shanshan Wang, Rudi Schäfer, and Thomas Guhr. Cross-response in correlated financial markets: individual stocks, *The European Physical Journal B* **89**, 105 (2016)

[2] Shanshan Wang, Rudi Schäfer, and Thomas Guhr. Average cross-responses in correlated financial market, *The European Physical Journal B* **89**, 207 (2016)

[3] Shanshan Wang and Thomas Guhr. Microscopic understanding of cross-responses between stocks: a two-component price impact model, arXiv:1609.04890

[4] Shanshan Wang. Trading strategies for stock pairs regarding to the cross-impact cost, https://ssrn.com/abstract=2897711

Thank you for your attentions!