Response functions as a new concept to study local dynamics in traffic networks

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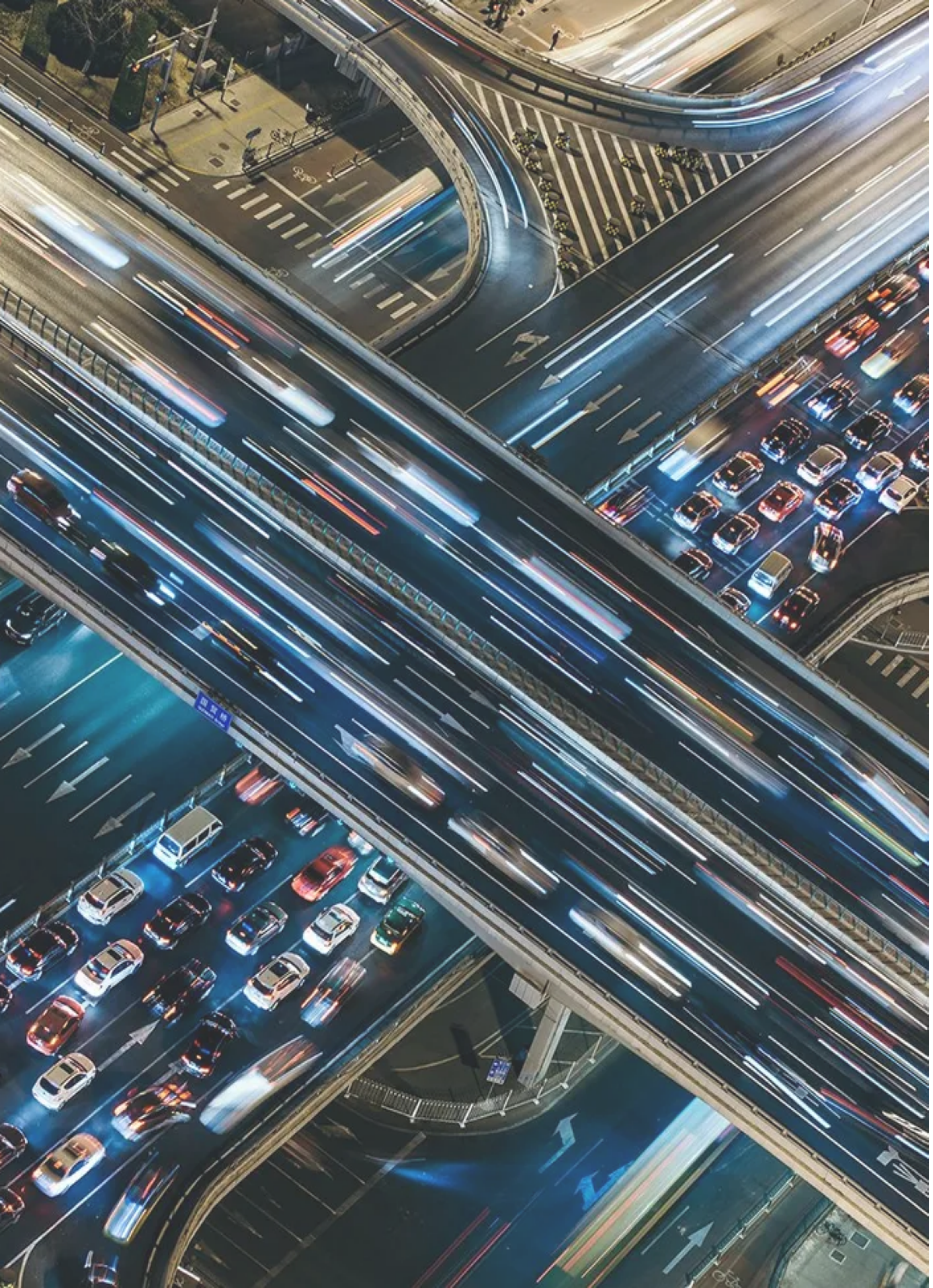
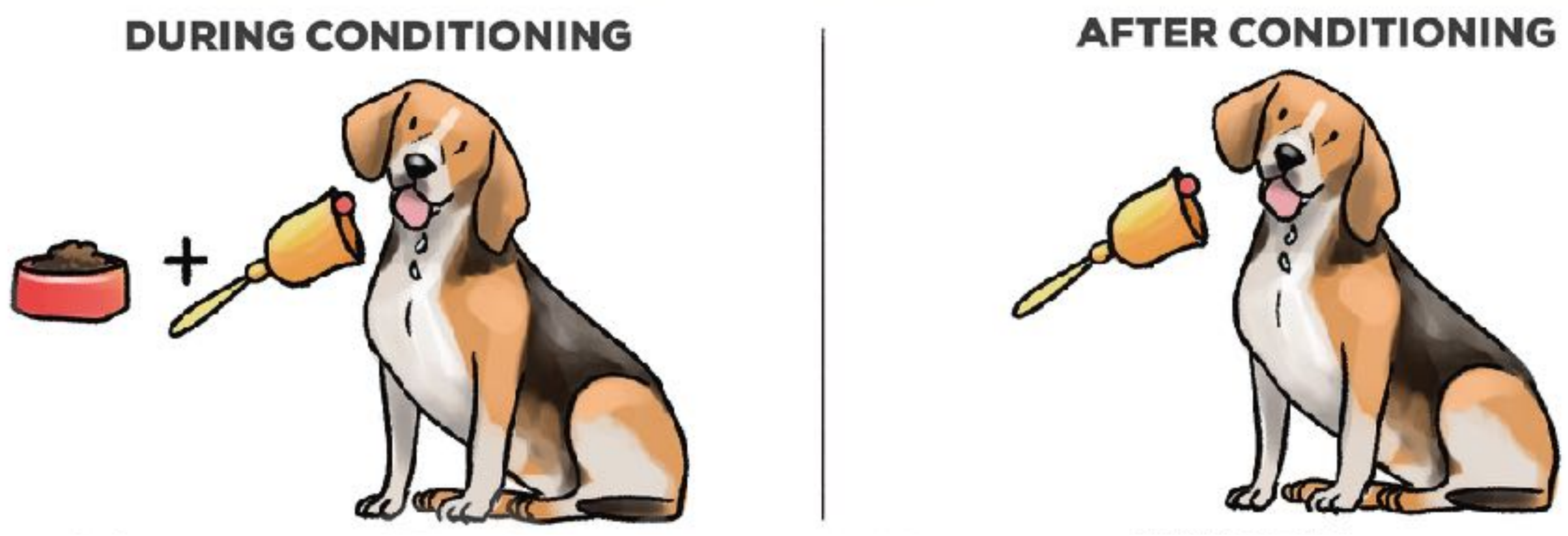


Contents

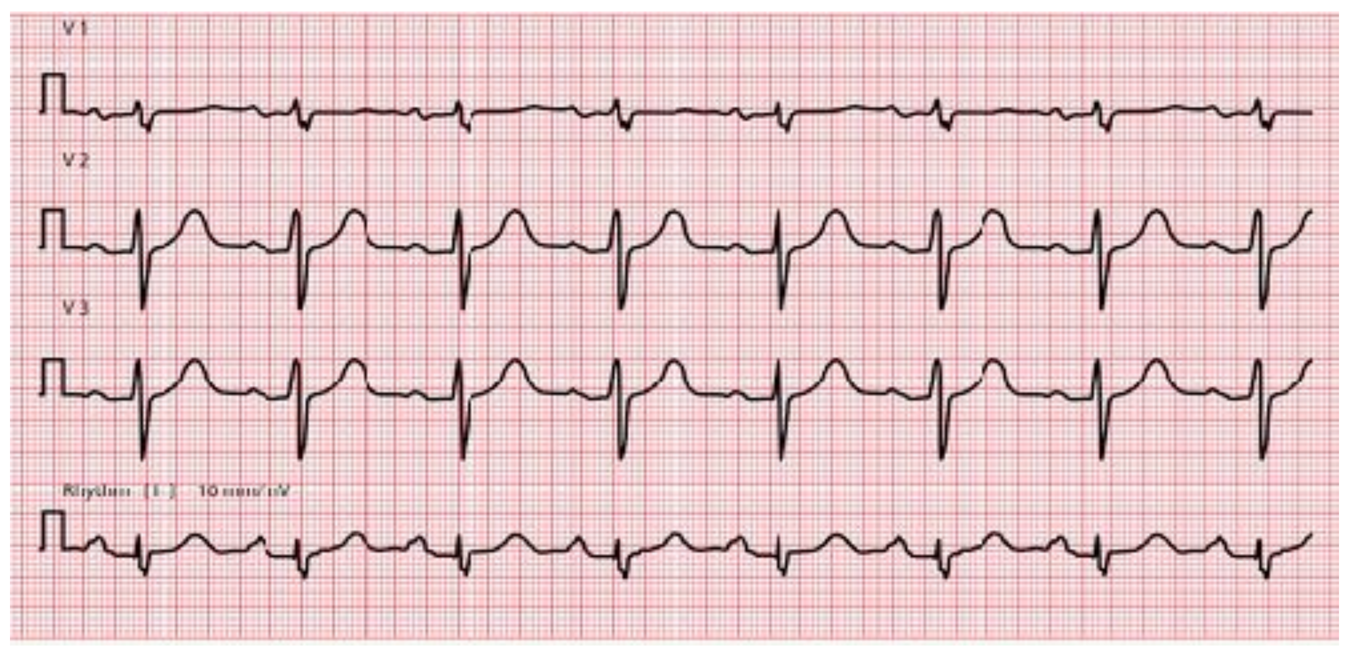
Background—what is the response?

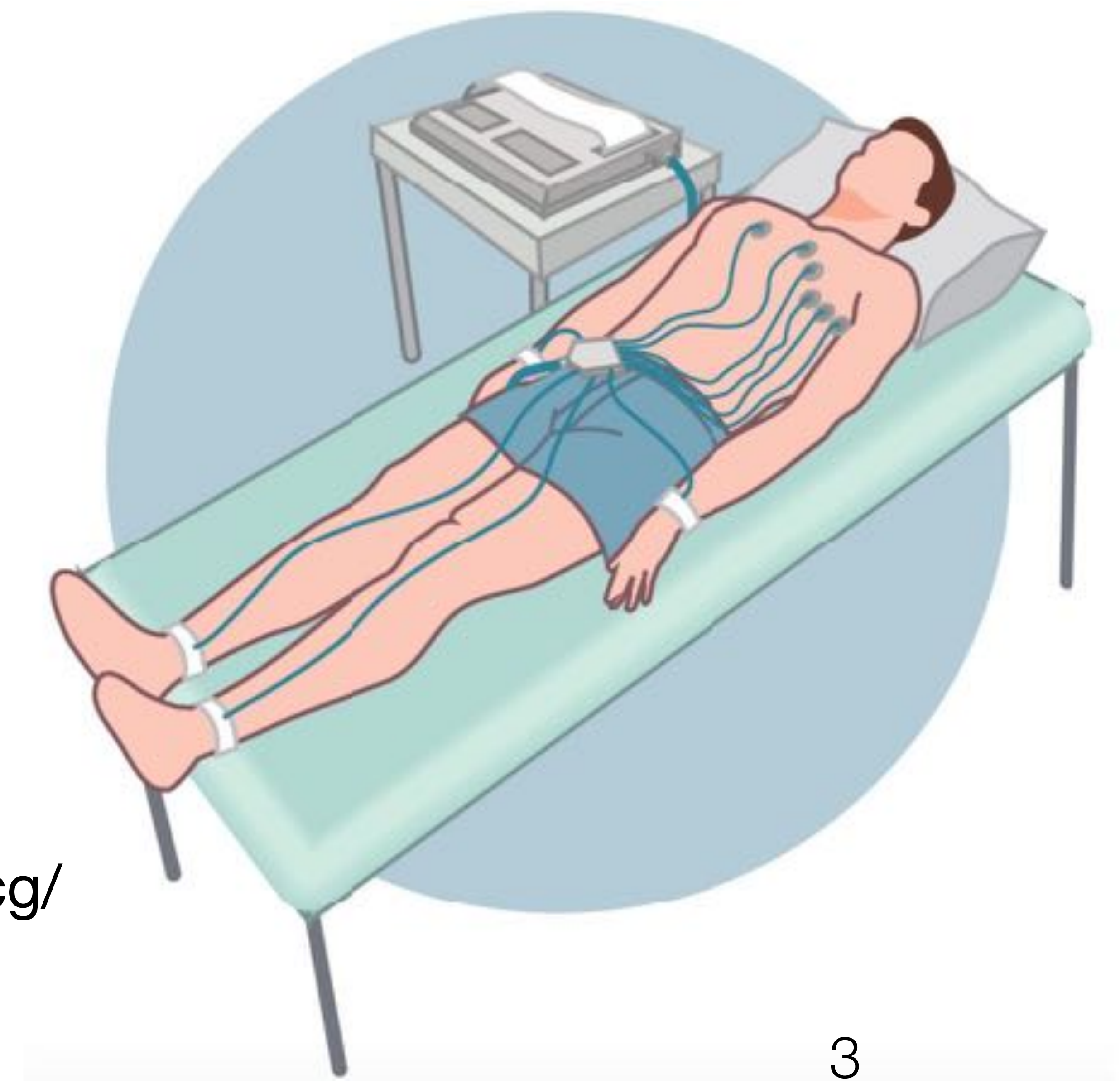
Pavlov's dogs



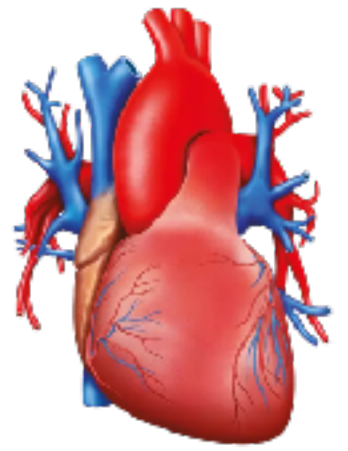
Ref. https://practicalpie.com/stimulus-response-theory/

Electrocardiogram (ECG) test





Ref. https://www.ihealthcentre.ca/heart-tests/ecg/

Stimulus	Responder
	
	

Response process:

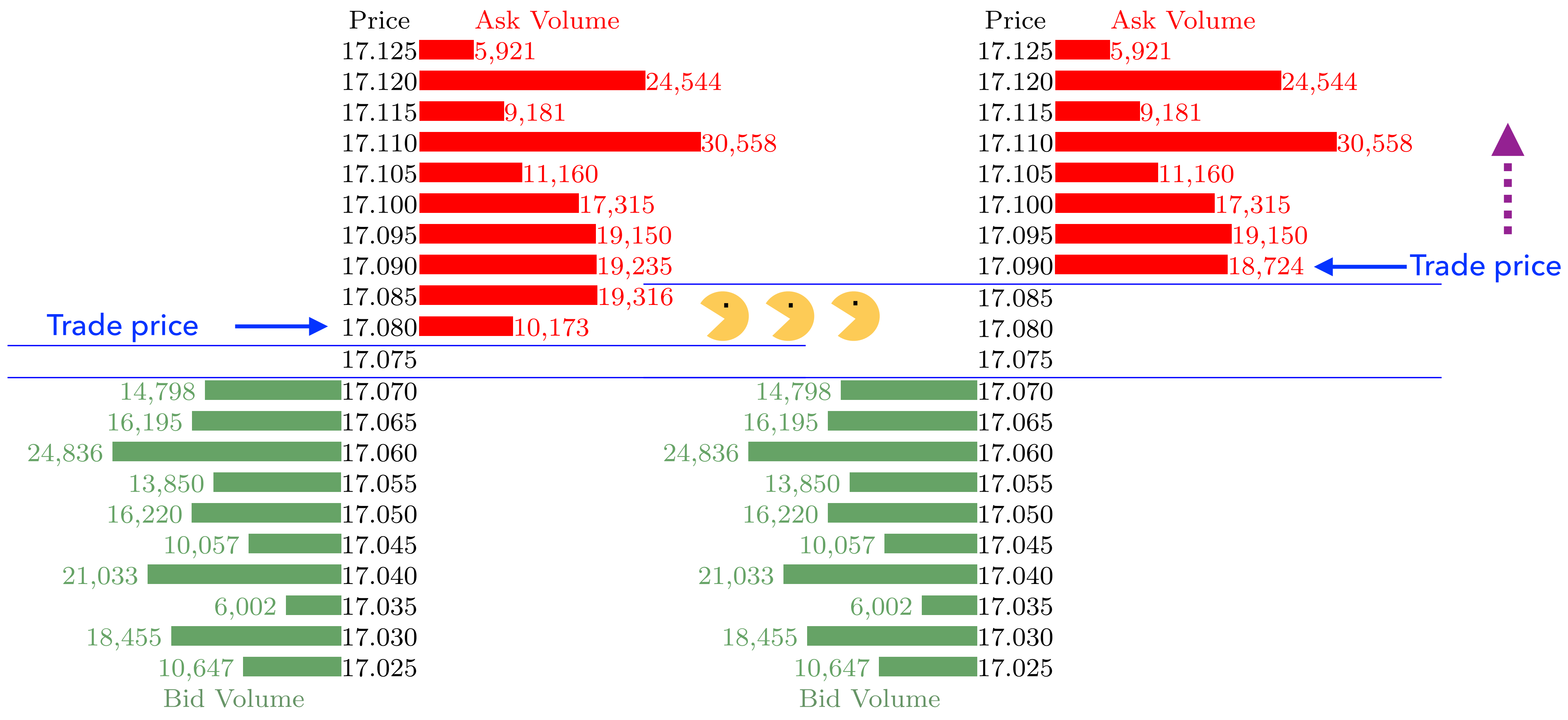
- Stimulus send signals
- Responder receives and reacts to signals
- Time delay between signal generating and reactions

Background—response functions for financial markets

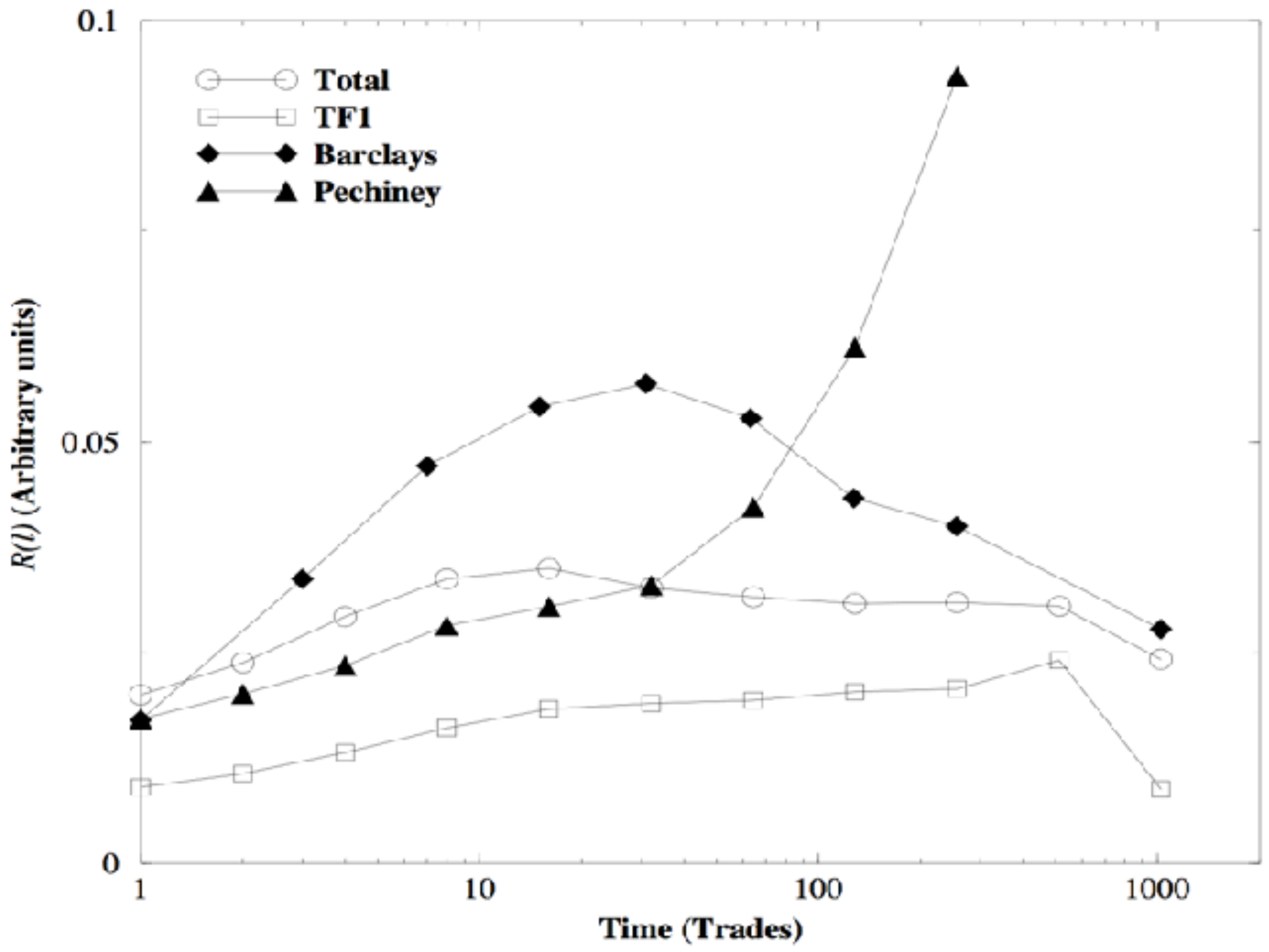
Financial markets

Stimulus: trades $\varepsilon_j(t)$

Responder: stock prices $S_i(t)$



Response function $R_{ij}(\tau) = \Big\langle \big(S_i(t+\tau) - S_i(t)\big)\varepsilon_j(t) \Big\rangle_t$



Ref. J.-P. Bouchaud, Y. Gefen, M. Potters, M. Wyart. Quantitative Finance, 4(2), 176 (2004).

If $i = j$, self-response

If $i \neq j$, cross-response

Refs.

- Wang, Schäfer and Guhr, Eur. Phys. J. B 89,105 (2016);
- Wang, Schäfer and Guhr, Eur. Phys. J. B 89, 207 (2016);
- Henao-Londono, Krause and Guhr, Eur. Phys. J. B 94, 78 (2021);
- Henao-Londono and Guhr, Physica A 589,126587 (2022)
- Grimm and Guhr, Eur. Phys. J. B 92, 133 (2019)

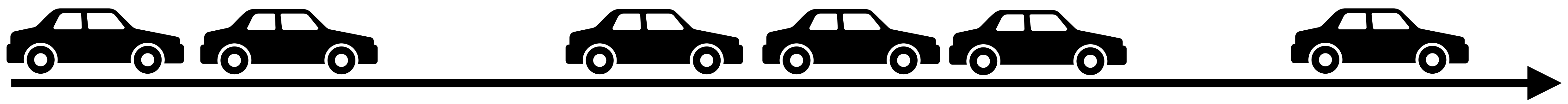
Background—several questions to be addressed

Traffic networks

Velocity?
at $t + \tau$ on
section i_2

Heavy congestion
at t on section j

Velocity?
at $t + \tau$ on
section i_1



	Stimulus	**Responder**
Pavlov's dogs		
Traffic networks		

Several questions to be addressed

- How does the velocity respond to congestion?
- What causes the velocity response to congestion?
- What features are present in the velocity response?

Response functions

- **Indicator function of heavy congestion**

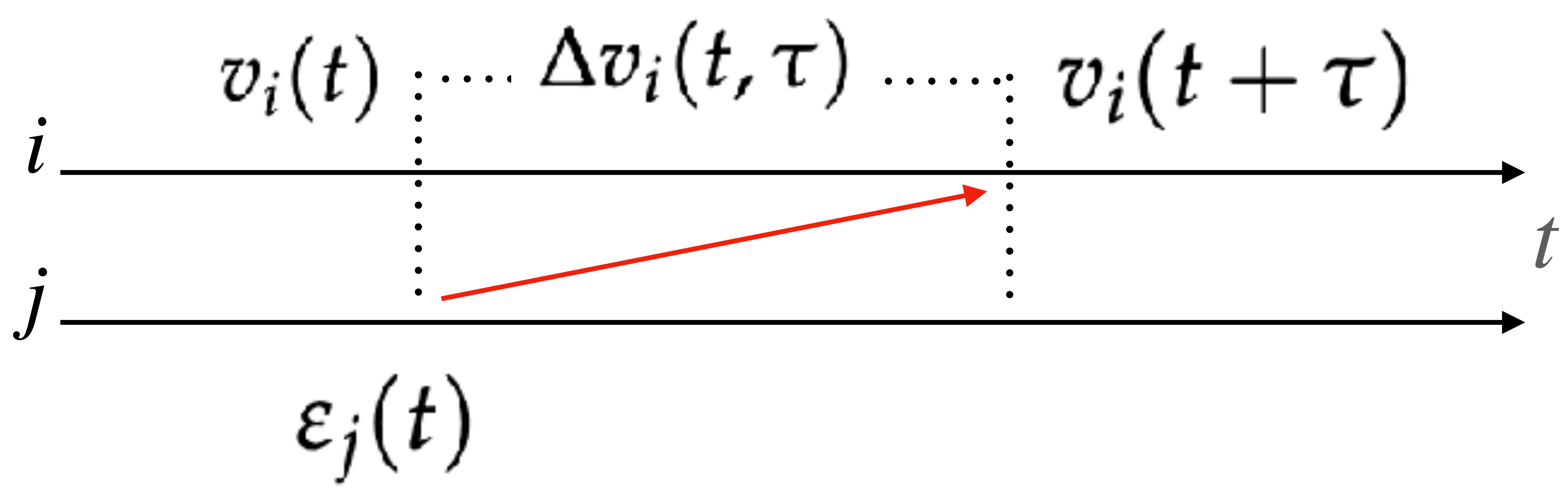
$$\varepsilon_j(t) = \begin{cases} 1, & \text{if } v_j(t) < 10 \text{ km/h} \\ 0, & \text{otherwise} \end{cases}$$

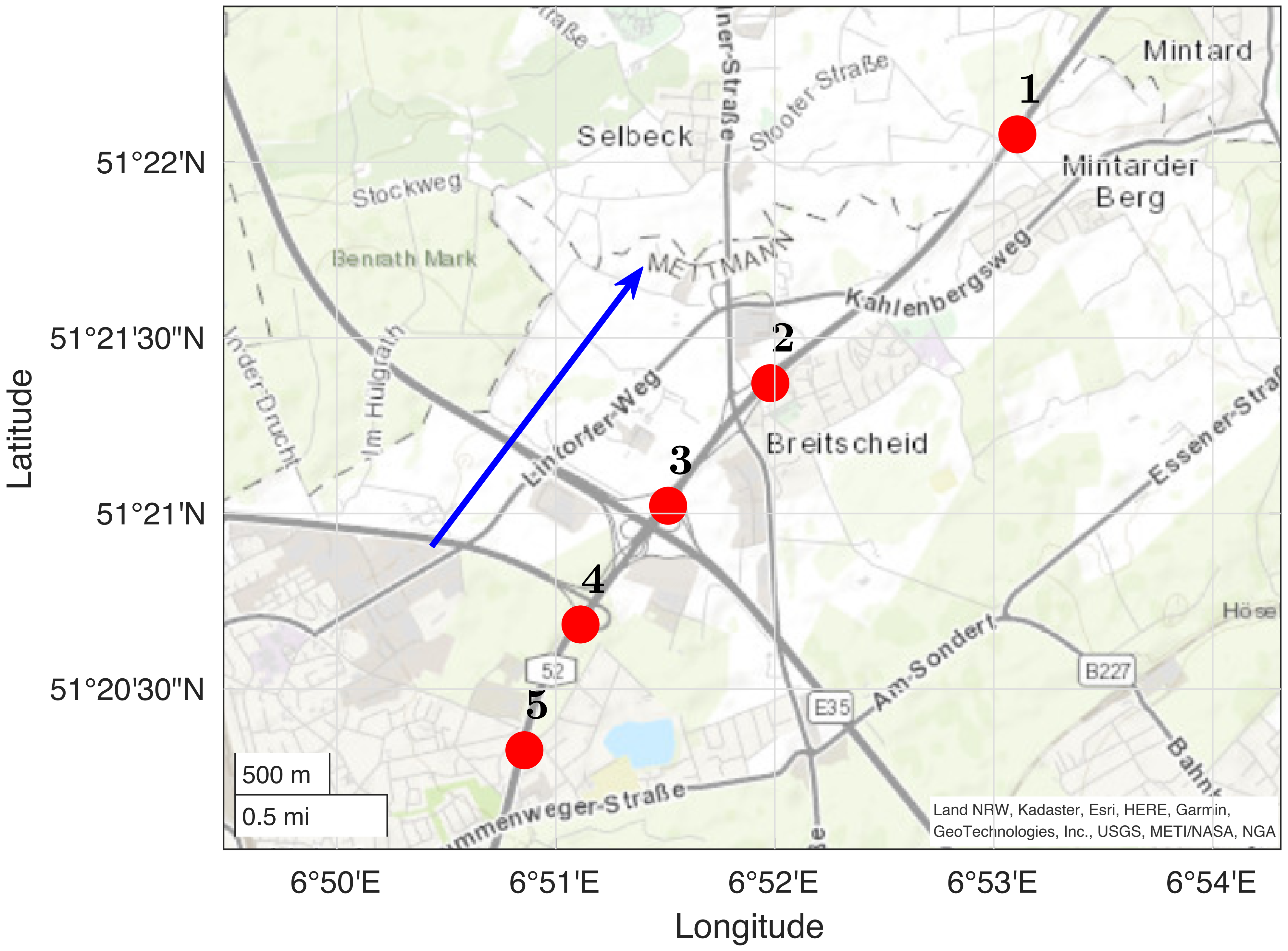
- **Correlator of congestion indicators**

$$\Theta_{ij}(\tau) = \frac{1}{T-\tau} \sum_{t=1}^{T-\tau} \tilde{\varepsilon}_i(t+\tau)\tilde{\varepsilon}_j(t)$$

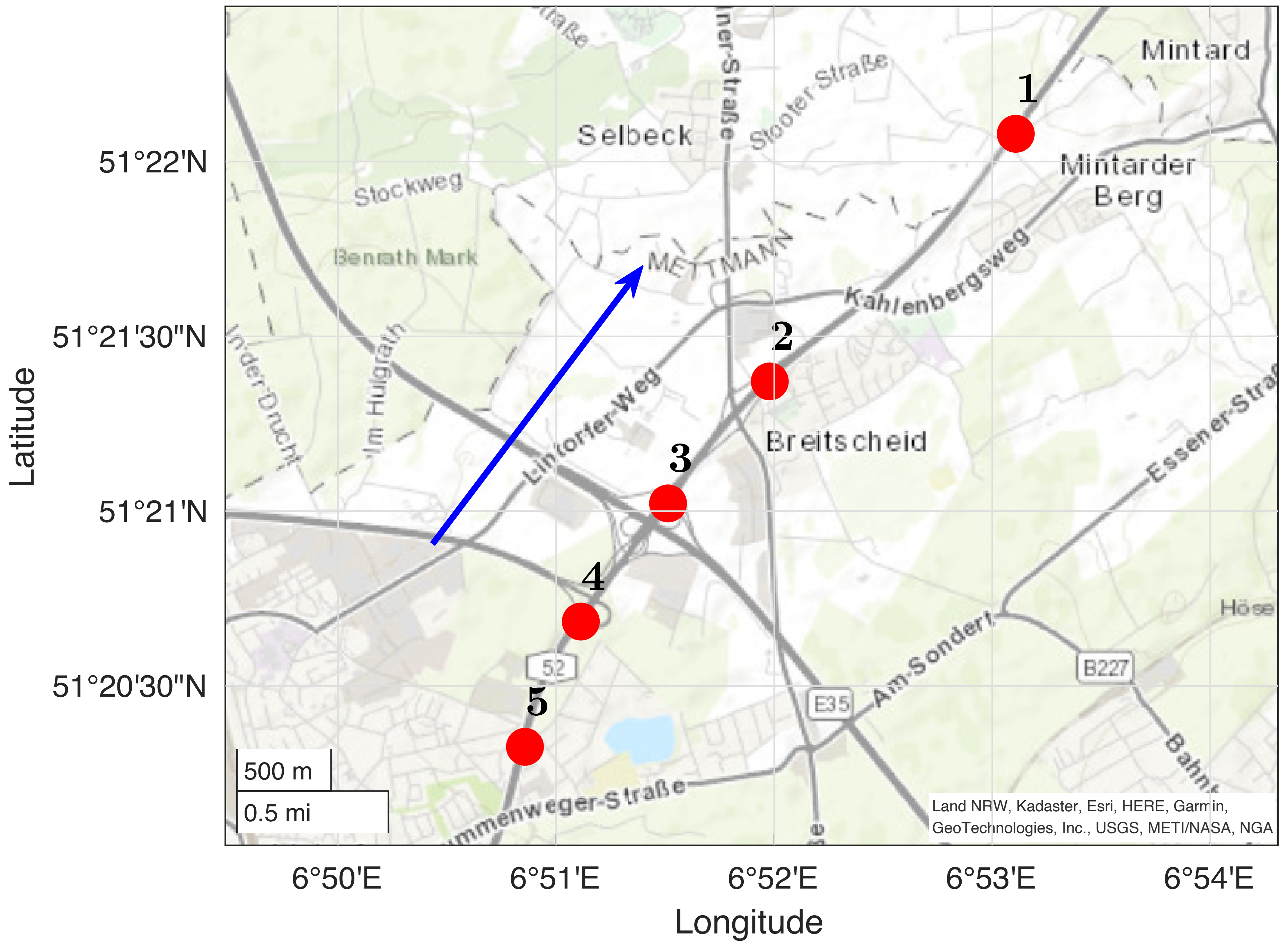
- **Response of velocities to congestion**

$$R_{ij}(\tau) = \frac{\sum\limits_{t=1}^{T-\tau} \big(v_i(t+\tau) - v_i(t)\big)\varepsilon_j(t)}{\sum\limits_{t=1}^{T-\tau} \varepsilon_j(t)}$$

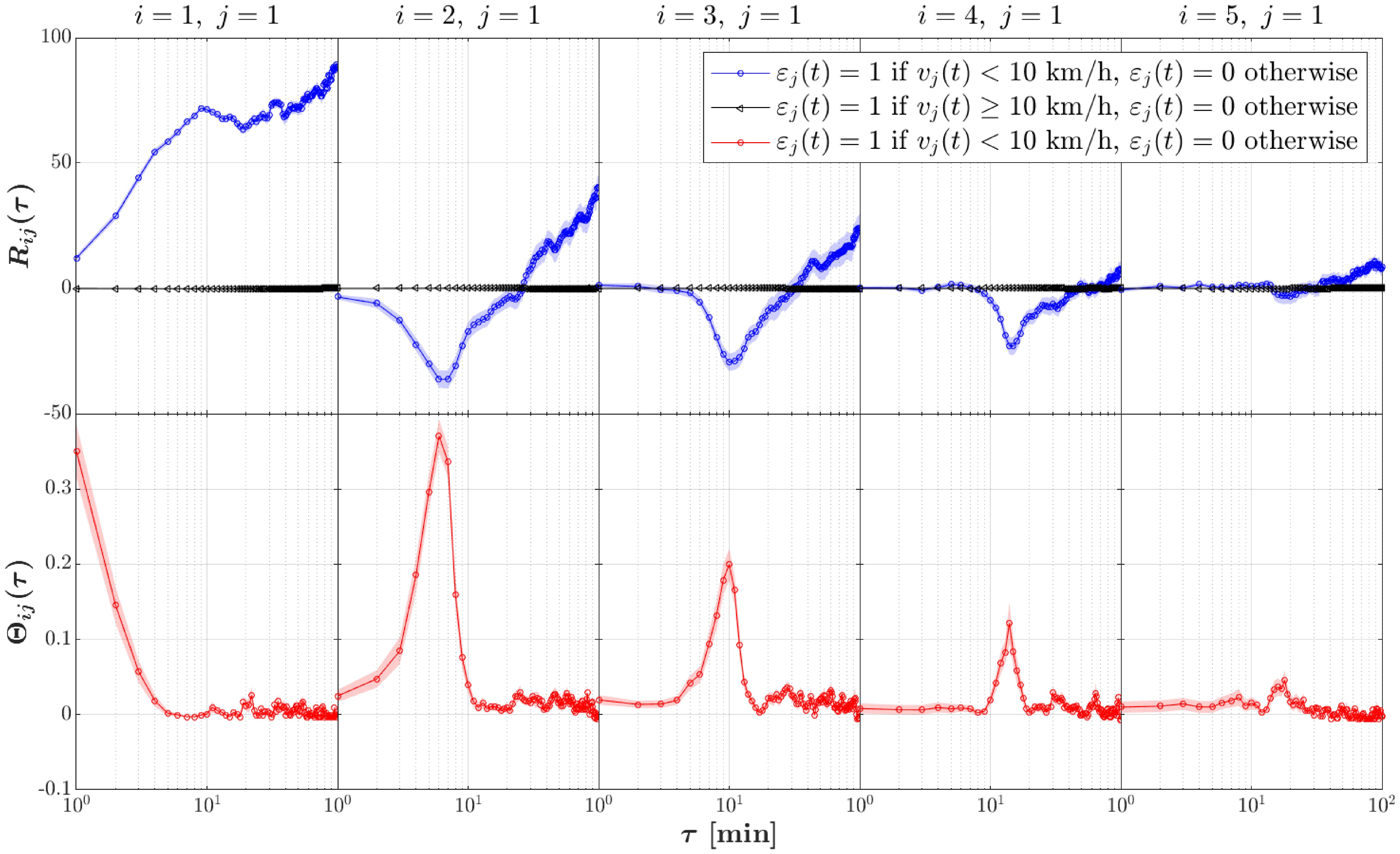




Empirical results—velocity response and indicator correlators



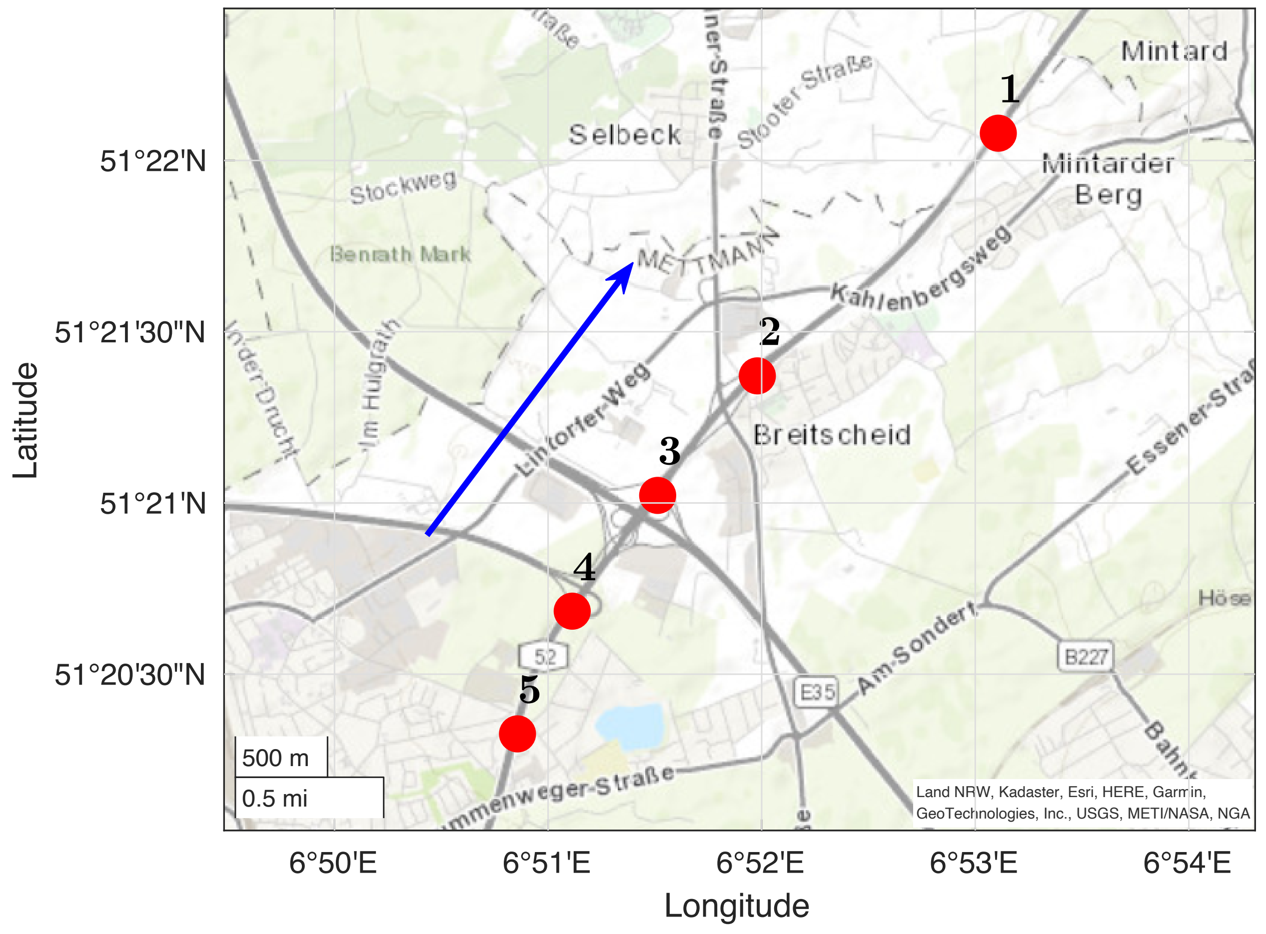
i: heavily congested section, j: impacted section

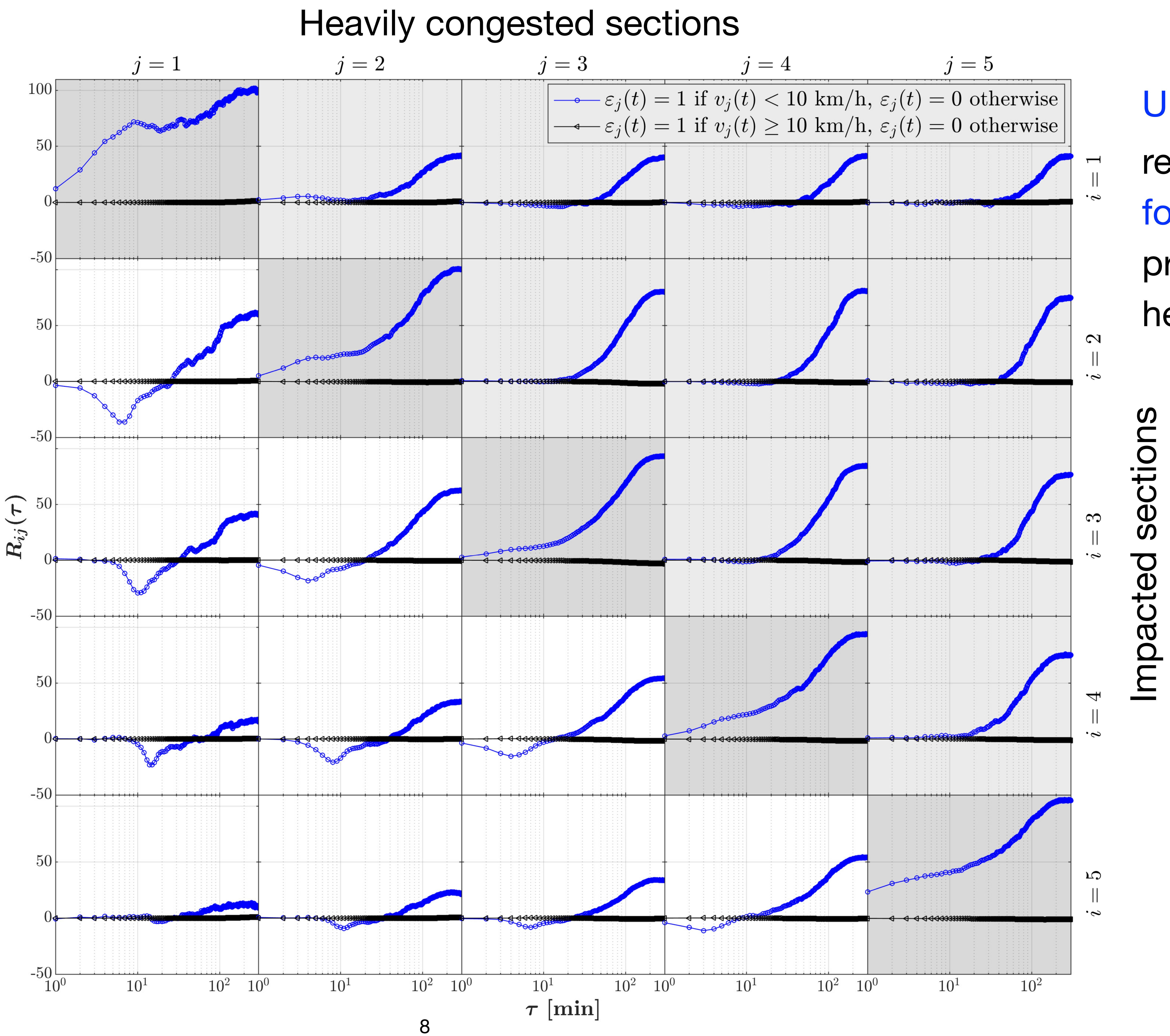


used traffic data:

- collected by inductive loop detectors
- velocities of 5 motorway sections near Breitscheid of NRW
- between 00:00 and 23:59 from 64 workdays in 2017

Empirical results—Two kinds of response behavior





Up triangle: response due to forward propagation of heavy congestion

Bottom triangle: response due to backward propagation of heavy congestion

Empirical results—phase separation

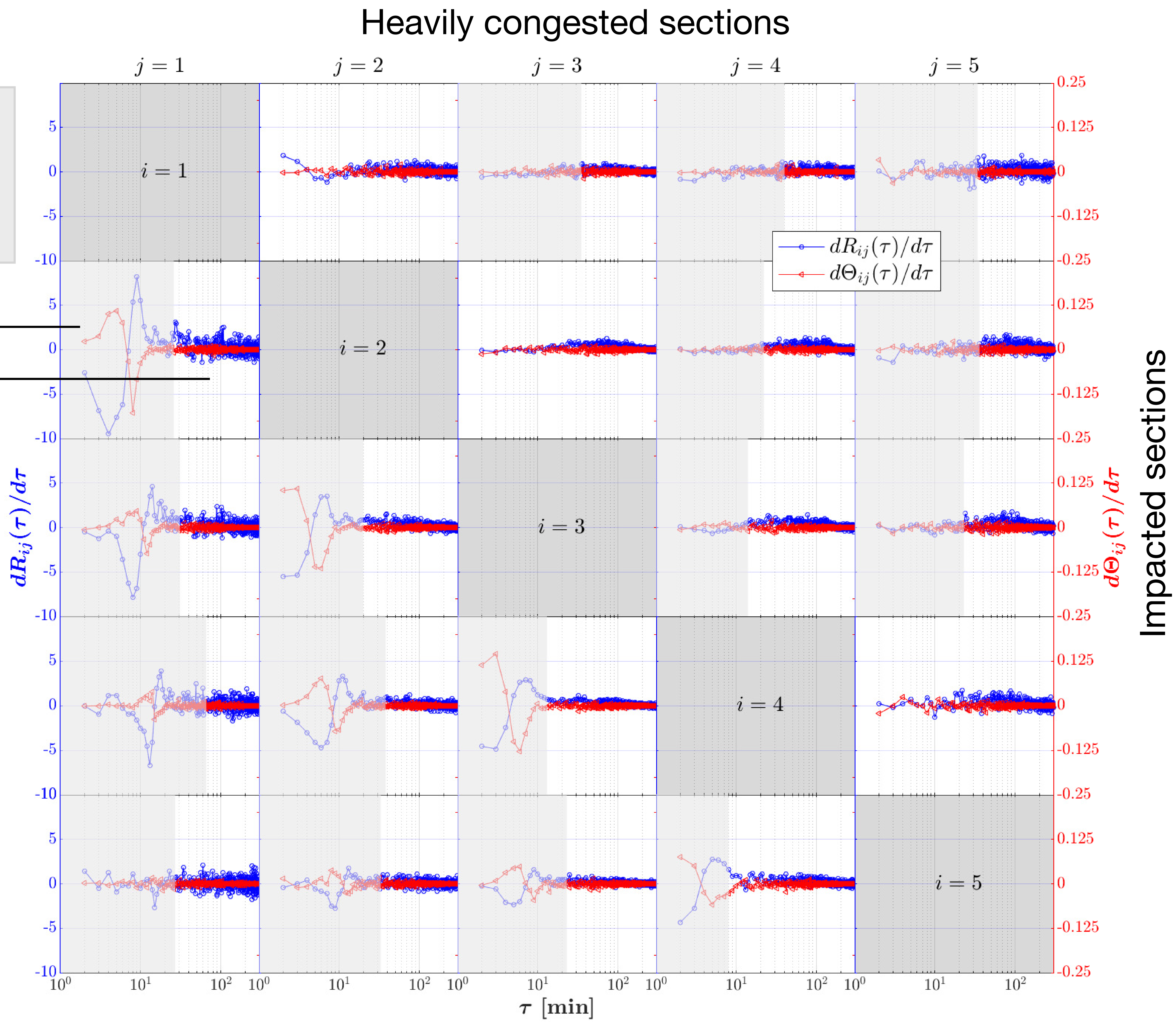
$$\frac{dR_{ij}(\tau)}{d\tau} = \frac{R_{ij}(\tau+\Delta\tau)-R_{ij}(\tau)}{\Delta\tau}$$

$$\frac{d\Theta_{ij}(\tau)}{d\tau} = \frac{\Theta_{ij}(\tau+\Delta\tau)-\Theta_{ij}(\tau)}{\Delta\tau}$$

$$\frac{d\Theta_{ij}}{d\tau} \to 0$$ steady state

Non-steady phase
Transient response

Steady phase
Long-term response



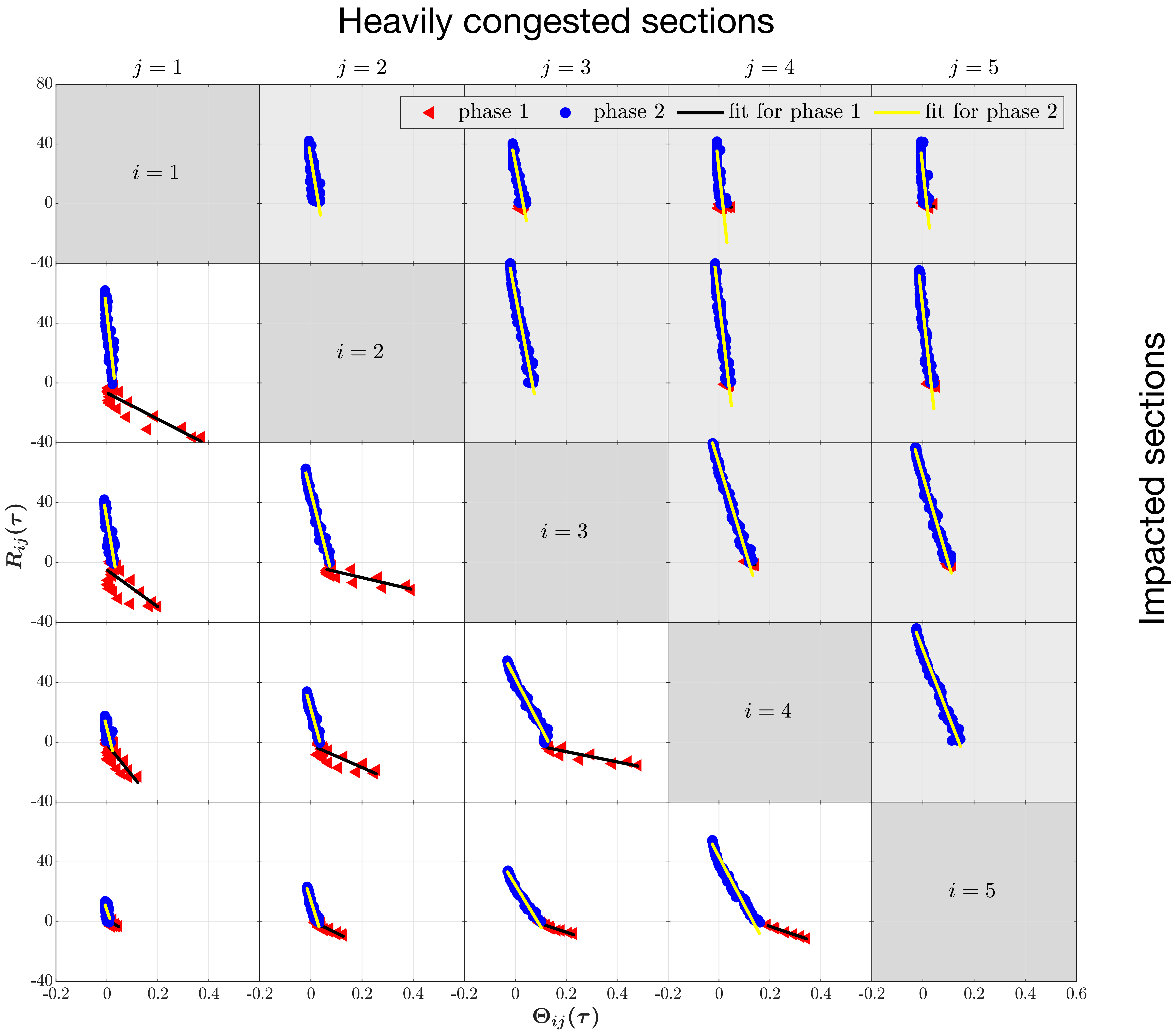
Large changes of congestion correlation corresponds to the large changes of velocity response to congestion

Empirical results—relations between velocity responses and indicator correlators

For each response phase (PR), a linear relation between velocity responses and correlations of congestion indicators can be found.

$$R_{ij}(\tau)|_{\mathrm{RP}\ k} \sim -\Theta_{ij}(\tau)|_{\mathrm{RP}\ k}$$

Congestion correlation might be viewed as a cause of the velocity response, regardless of other possible reasons



SDW model for transient responses

Susceptible velocity $S_{ij}(\tau)$ $\xrightarrow{\frac{\beta_{ij} D_{ij}(\tau) S_{ij}(\tau)}{V_{ij}(0)}}$ Decelerated velocity $D_{ij}(\tau)$ $\xrightarrow{\gamma_{ij} D_{ij}(\tau)}$ Withdrawing velocity $W_{ij}(\tau)$

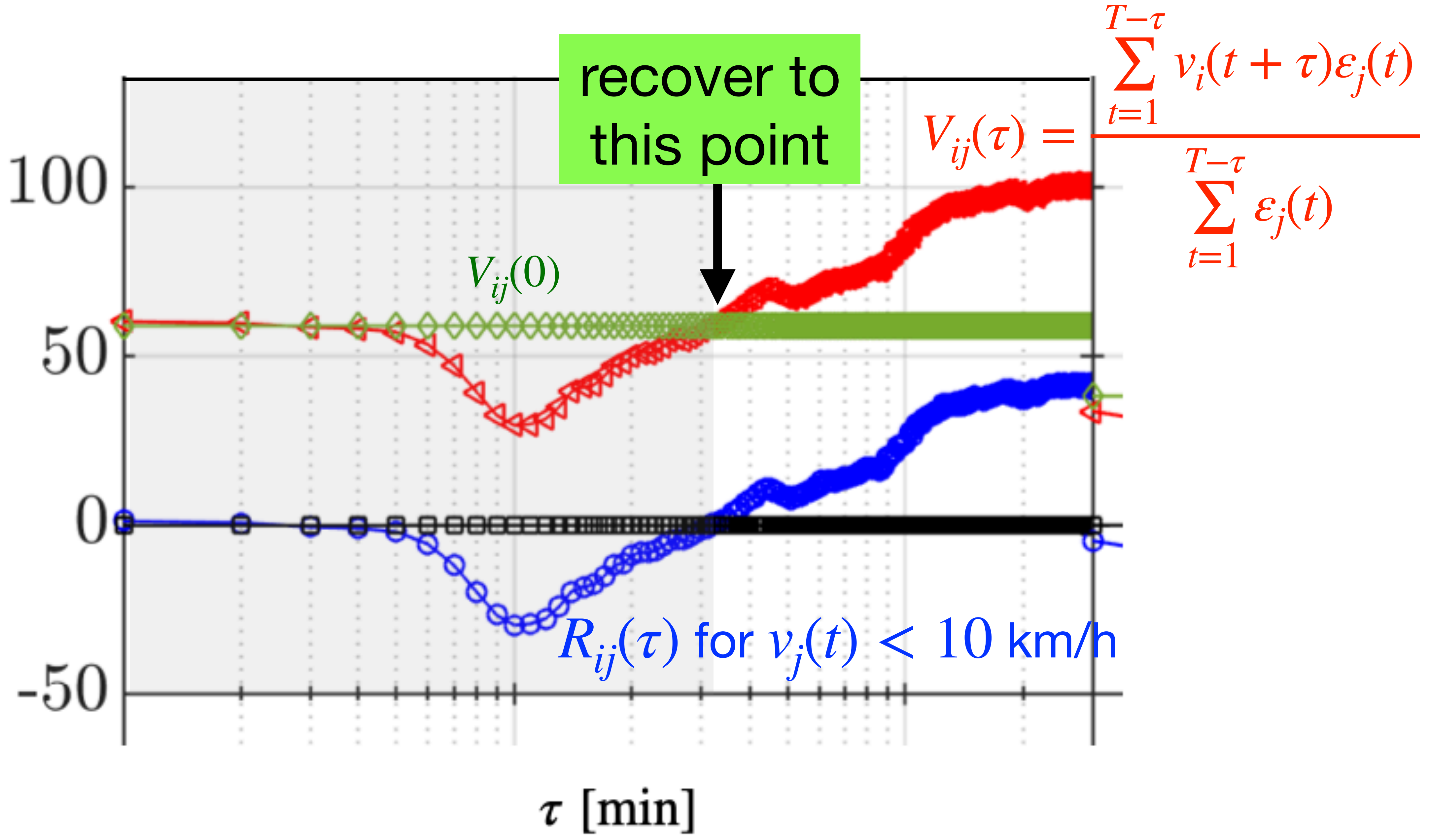
β_{ij} : deceleration rate

γ_{ij} : recovery rate

Susceptible-decelerated-withdrawing (SDW) model, inspired by SIR epidemic model

$$
\begin{cases}
\dfrac{dS_{ij}(\tau)}{d\tau} = \dfrac{S_{ij}(\tau+\Delta\tau) - S_{ij}(\tau)}{\Delta\tau} = -\dfrac{\beta_{ij} D_{ij}(\tau) S_{ij}(\tau)}{V_{ij}(0)} , \\[2ex]
\dfrac{dD_{ij}(\tau)}{d\tau} = \dfrac{D_{ij}(\tau+\Delta\tau) - D_{ij}(\tau)}{\Delta\tau} = \dfrac{\beta_{ij} D_{ij}(\tau) S_{ij}(\tau)}{V_{ij}(0)} - \gamma_{ij} D_{ij}(\tau) , \\[2ex]
\dfrac{dW_{ij}(\tau)}{d\tau} = \dfrac{W_{ij}(\tau+\Delta\tau) - W_{ij}(\tau)}{\Delta\tau} = \gamma_{ij} D_{ij}(\tau) .
\end{cases}
$$

$$V_{ij}(0) = D_{ij}(\tau) + S_{ij}(\tau) + W_{ij}(\tau)$$



SDW model for transient responses

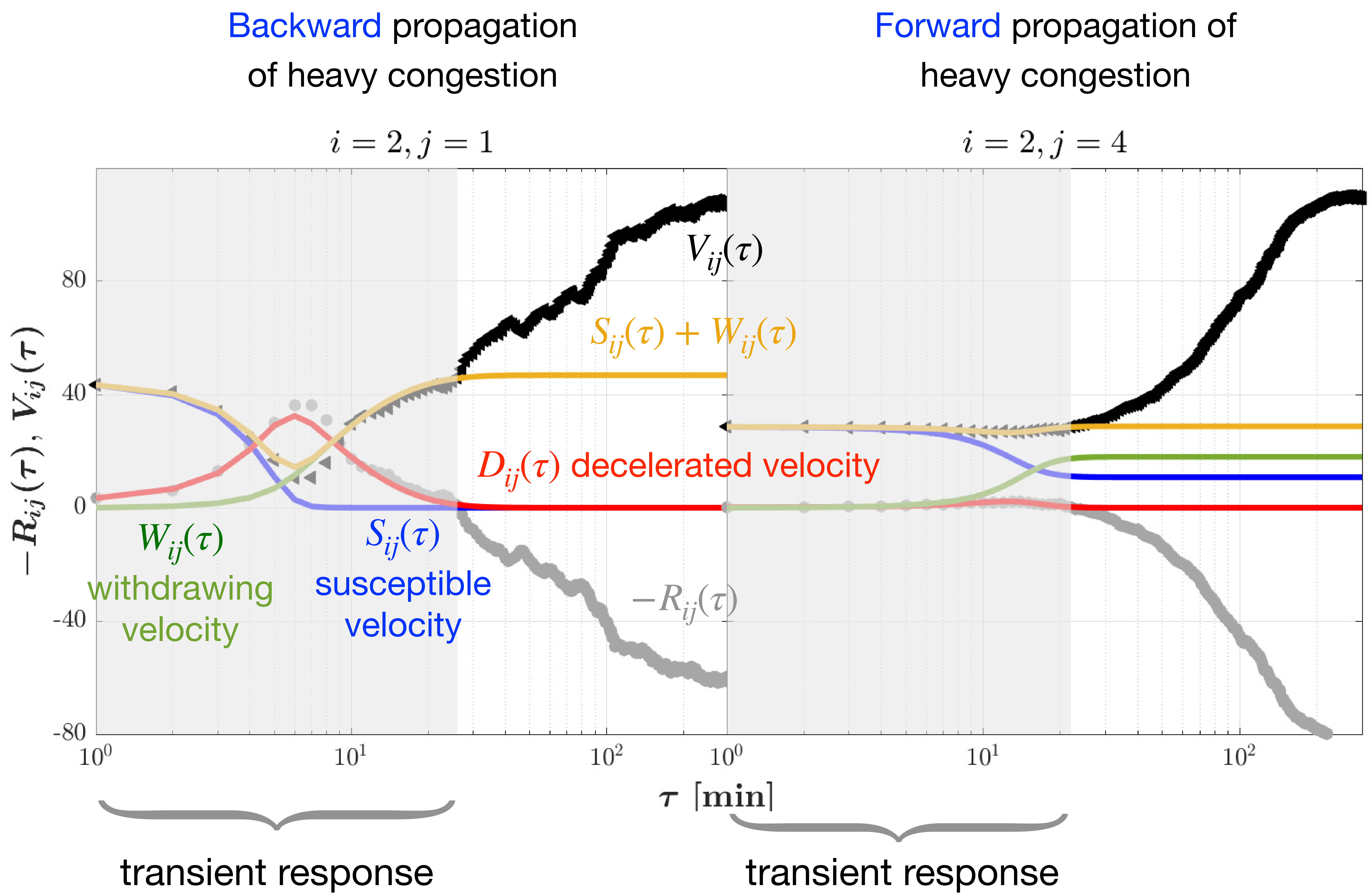
Procedure of simulations:

1. Give the initial values
   $W_{ij}(0),\ D_{ij}(0),$
   $S_{ij}(0) = V_i(0)|_j - D_{ij}(0) - W_{ij}(0)$

2. Simulate model in parameter space
   deceleration rate $\beta_{ij} \in [0,2]$
   and recovery rate $\gamma_{ij} \in [0,2]$
   with the increment 0.01,
   to get $S_{ij}(\tau)$, $D_{ij}(\tau)$ and $W_{ij}(\tau)$

3. Find best β_{ij} and γ_{ij} by minimizing

$$\Xi(\beta_{ij}, \gamma_{ij}) = \sum_{\tau=1}^{\tau_{\max}} (-R_{ij}(\tau) - D_{ij}(\tau|\beta_{ij}, \gamma_{ij}))^2$$

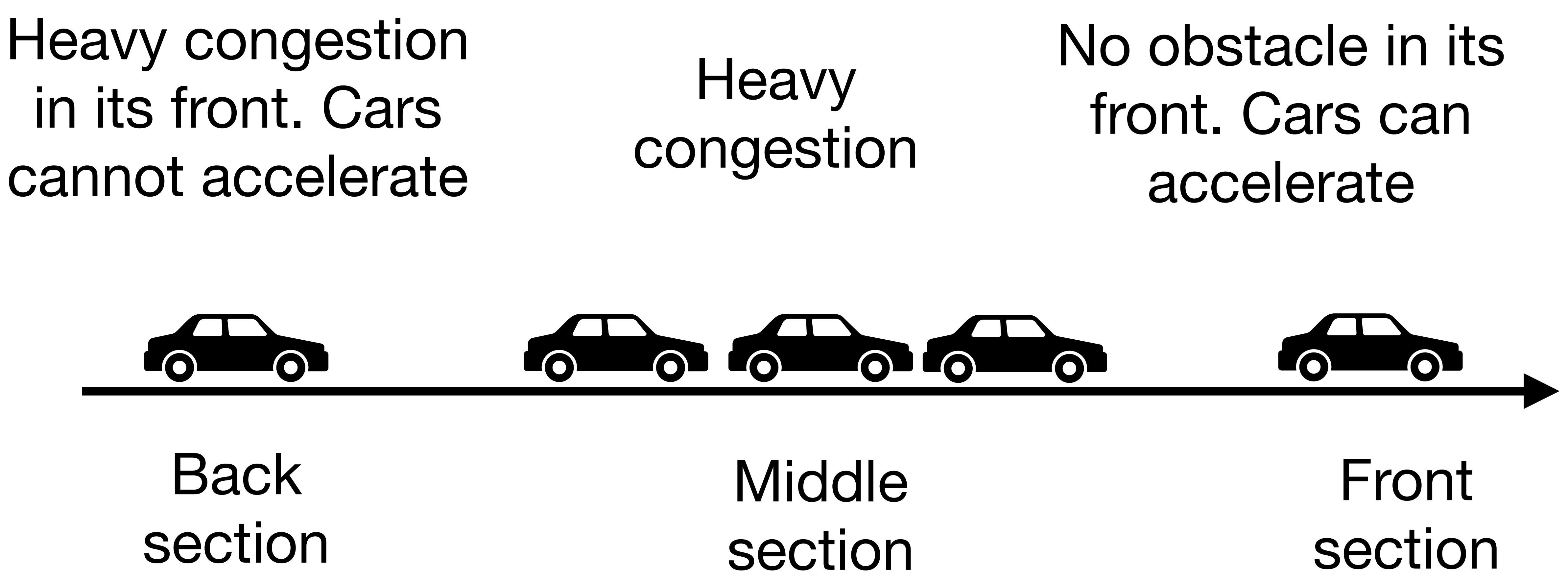


SDW model for transient responses

The fitted deceleration rates $\beta_{ij,\mathrm{fit}}$

i \ j	1	2	3	4	5
1	-	0	1.25	1.38	1.97
2	1.19	-	0	1.09	0.84
3	0.97	1.15	-	1.29	0.79
4	0.60	1.24	1.39	-	0
5	0.13	0.88	1.76	1.80	-

Both front and back motorway sections are influenced to a similar extent



The grey (white) cells: the forward (backward) propagation of heavy congestion of section j

The fitted recovery rates $\gamma_{ij,\mathrm{fit}}$

i \ j	1	2	3	4	5
1	-	0	0.87	1.00	1.95
2	0.16	-	2.00	0.72	0.60
3	0.19	0.30	-	0.91	0.54
4	0.26	0.42	0.53	-	0
5	1.92	0.56	1.06	0.99	-

Velocity on front section is more quick to recover from impact of congestion

The ratio $\beta_{ij,\mathrm{fit}}/\gamma_{ij,\mathrm{fit}}$

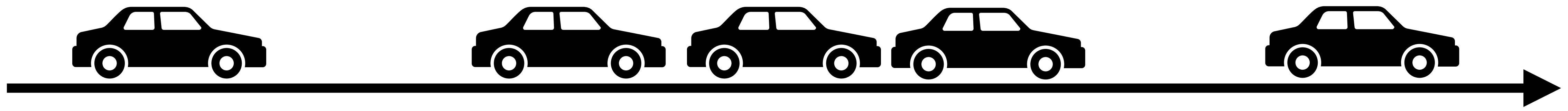
i \ j	1	2	3	4	5
1	-	-	1.44	1.38	1.01
2	7.44	-	0	1.51	1.40
3	5.11	3.83	-	1.42	1.46
4	2.31	2.95	2.62	-	-
5	0.07	1.57	1.66	1.82	-

β/γ determines the dynamics of velocity deceleration.

Velocity of back sections is more likely to decrease, leading to significant responses

Summary

- Introduced the response function of velocities to heavy congestion
- The backward and forward propagation of heavy congestion leads to different behaviors in velocity response
- Velocity response has a linear relation with the correlation of heavy congestion
- Two response phases: a phase for the transient response and a phase for the long-term response
- The transient response can be explained by our SDW model
- Helpful for traffic management or navigation systems with congestion pre-warning.



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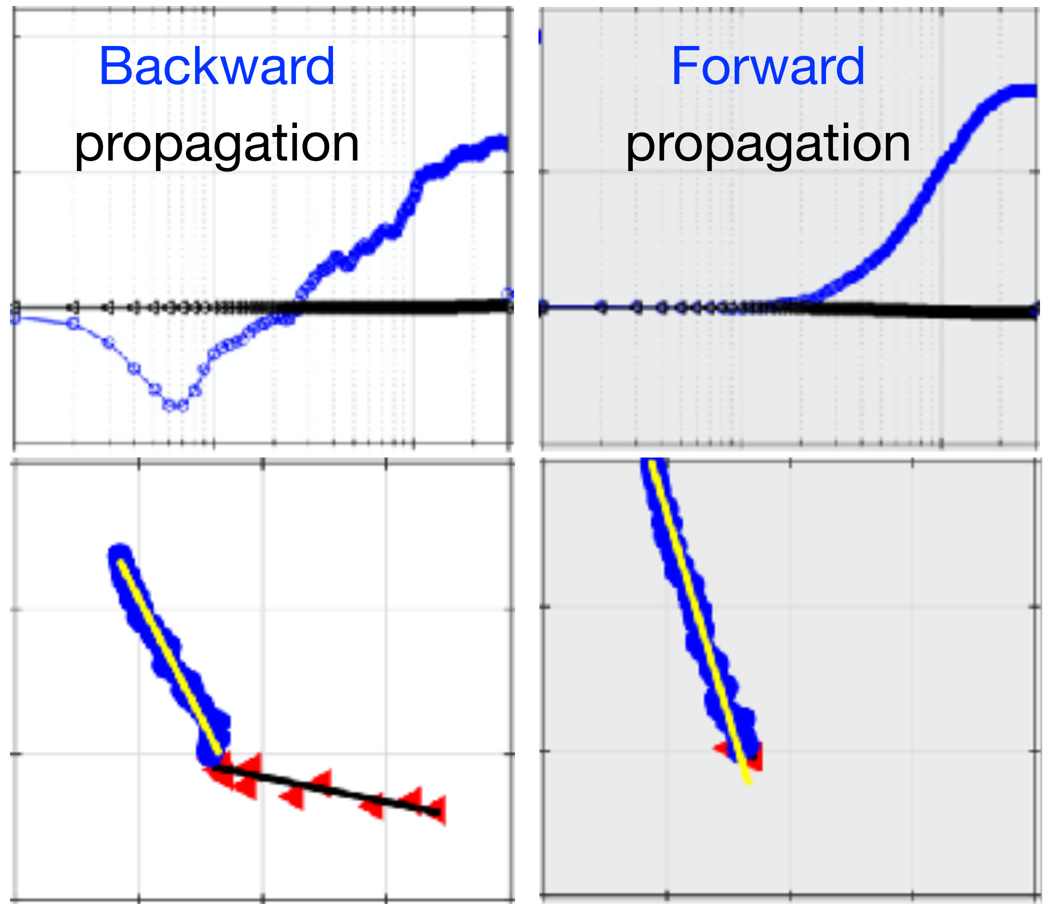

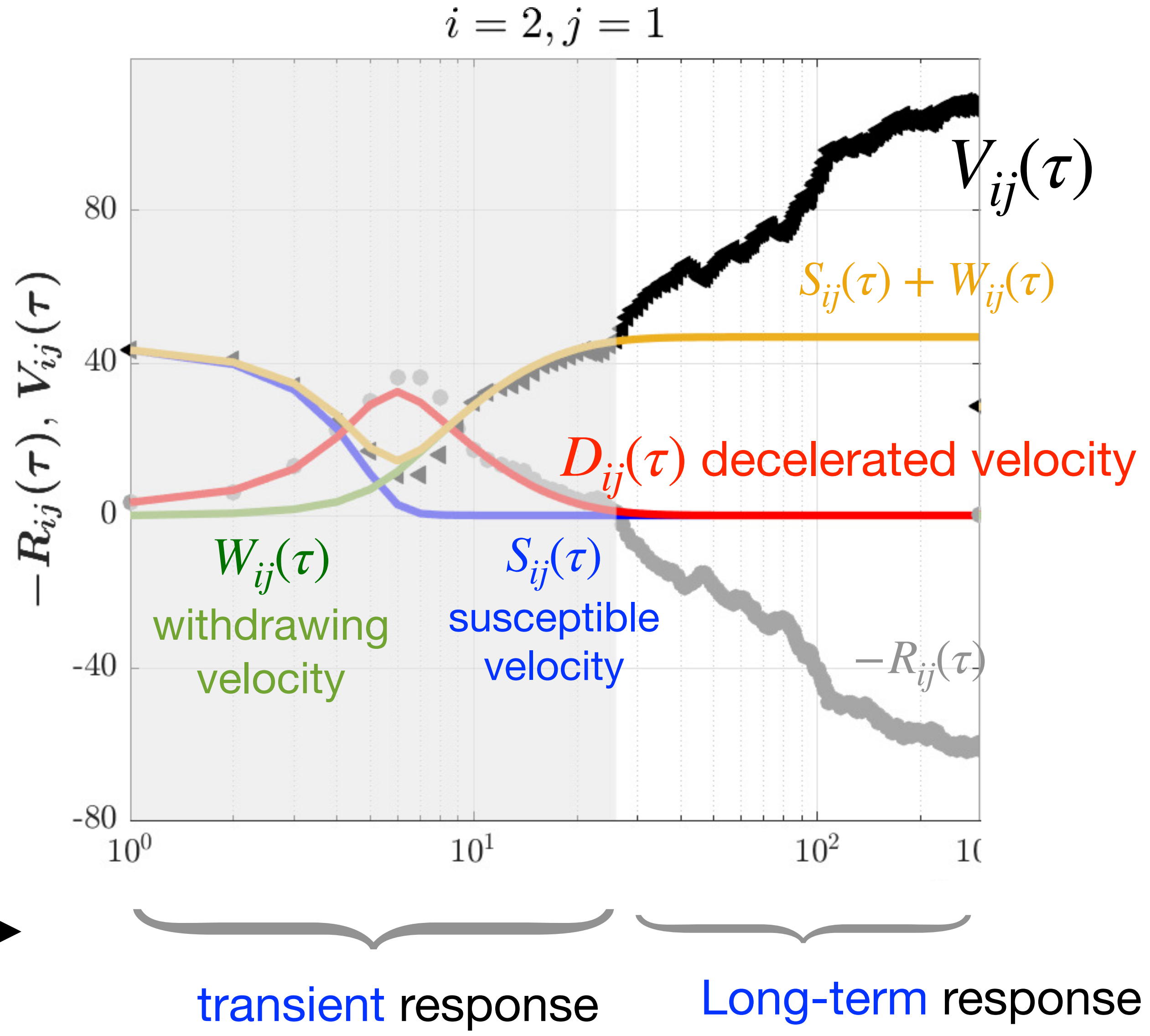

Thank you for your attention!

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"Between stimulus and response there is a space. In that space is our power to choose our response. In our response lies our growth and our freedom." — Viktor E. Frankl