Identifying Subdominant Collective Effects in a Large Motorway Network

Shanshan Wang, Michael Schreckenberg and Thomas Guhr

Faculty of Physics, University of Duisburg-Essen

UNIVERSITÄT DUISBURG ESSEN

Open-Minded

Motivation

- **Collective motion** has been revealed in a large traffic network as a whole [1].
- **A certain hierarchy of correlations** in traffic networks due to the presence and to the extent of collectivity is to be revealed.

- We aim to **identify subdominant collectivities** affecting different, large parts of a traffic network [2].

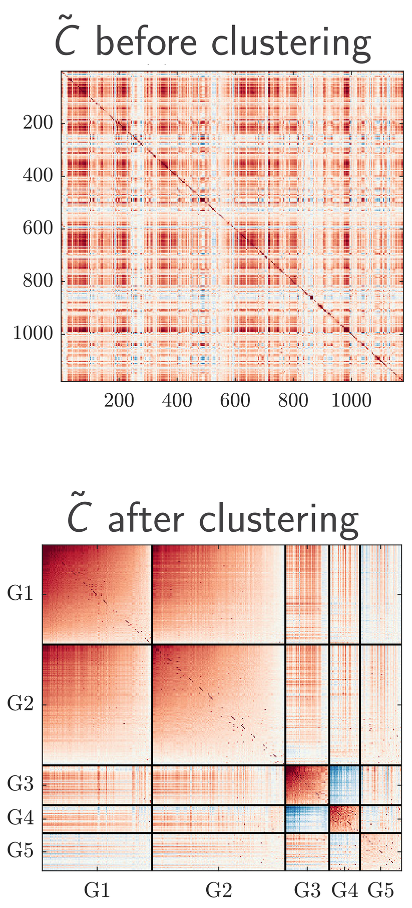
Description of traffic data

- Empirical traffic data in **one-minute resolution** for $N = 1179$ motorway sections in NRW, Germany were collected by **inductive loop detectors** and provided by Straßen.NRW.
- Velocity $v_n(t)$ entering a data matrix is averaged by

$$v_n(t) = \frac{\sum_l q_{nl}(t)}{\sum_l \rho_{nl}(t)} , \quad n = 1, \cdots , N ,$$

with traffic flows $q_{nl}(t)$ and flow densities $\rho_{nl}(t)$ on lanes l of section n at time t .

Reduced-rank correlation matrices



Given a $N \times T$ data matrix A normalized to zero mean, removing the largest singular value from it yields [3]

$$\tilde{A} = \sum_{n=1}^{L-1} S_n U_n V_n^\dagger$$

$N \times N$ reduced-rank covariance matrix

$$\tilde{\Sigma} = \frac{1}{T} \tilde{A} \tilde{A}^\dagger$$

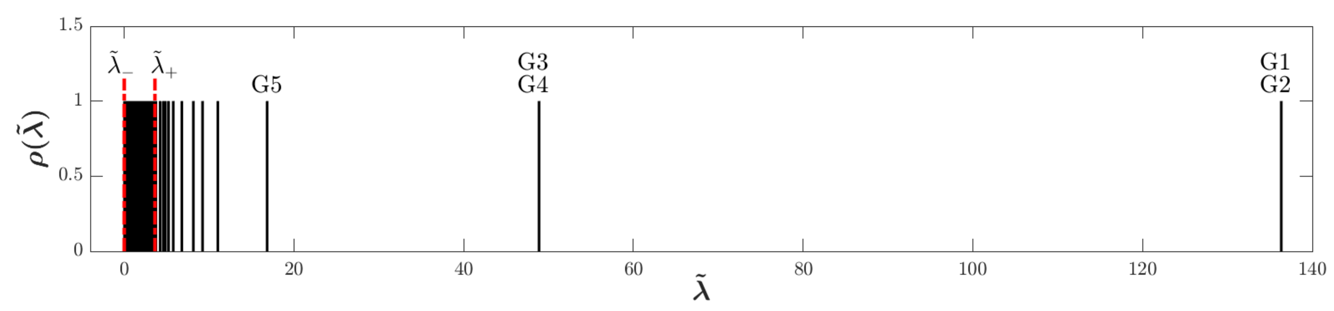
$N \times N$ reduced-rank correlation matrix

$$\tilde{C} = \tilde{\sigma}^{-1} \tilde{\Sigma} \tilde{\sigma}^{-1}$$

where $L = \min(N, T)$ and $\tilde{\sigma} = \mathrm{diag}\,(\tilde{\sigma}_1, \cdots , \tilde{\sigma}_N)$ with $\tilde{\sigma}_n = \sqrt{\tilde{\Sigma}_{nn}}$

Spectral information

spectral density of $\tilde{C}$: $\rho(\tilde{\lambda}) = \sum_{n=1}^{N} \delta(\tilde{\lambda} - \lambda_n)$



$N \times (k-1)$ matrix of eigenvectors for the largest $k-1$ eigenvalues from $\tilde{C}$

$$\tilde{u}^{(k-1)} = \left[\tilde{u}_{N-(k-1)+1} \cdots \tilde{u}_N \right]$$

Each row of $\tilde{u}^{(k-1)}$ is as an observation for clustering

References

[1] S. Wang *et al.*, *J. Stat. Mech.* (2021) 123401

[2] S. Wang *et al.*, *J. Stat. Mech.* (2022) 113402

[3] A.J. Heckens *et al.*, *J. Stat. Mech.* (2020) 103402

Procedure of optimized k-means clustering

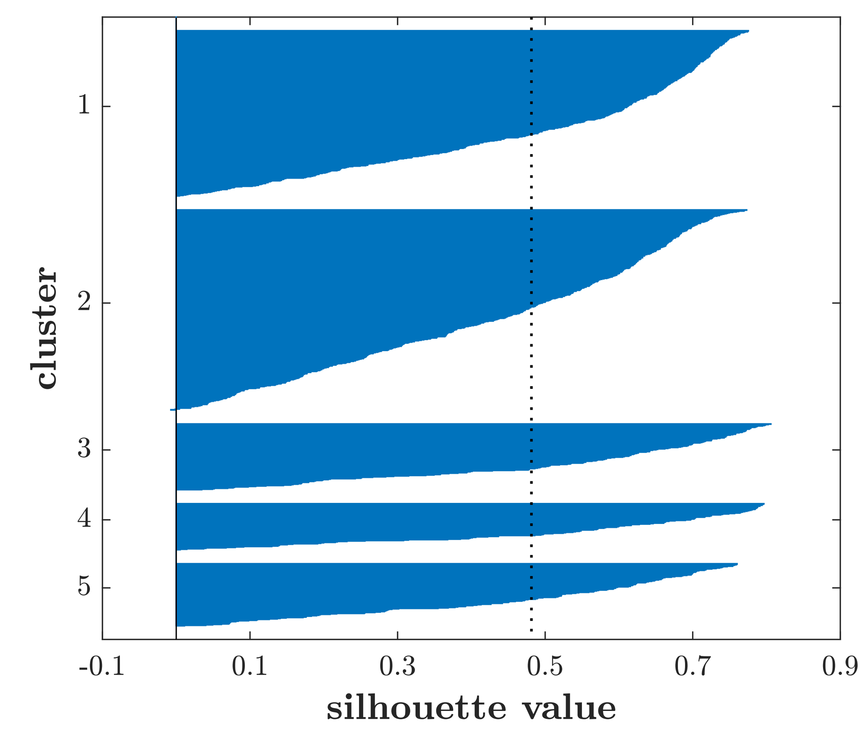
a. Perform k-means clustering with squared Euclidean distance

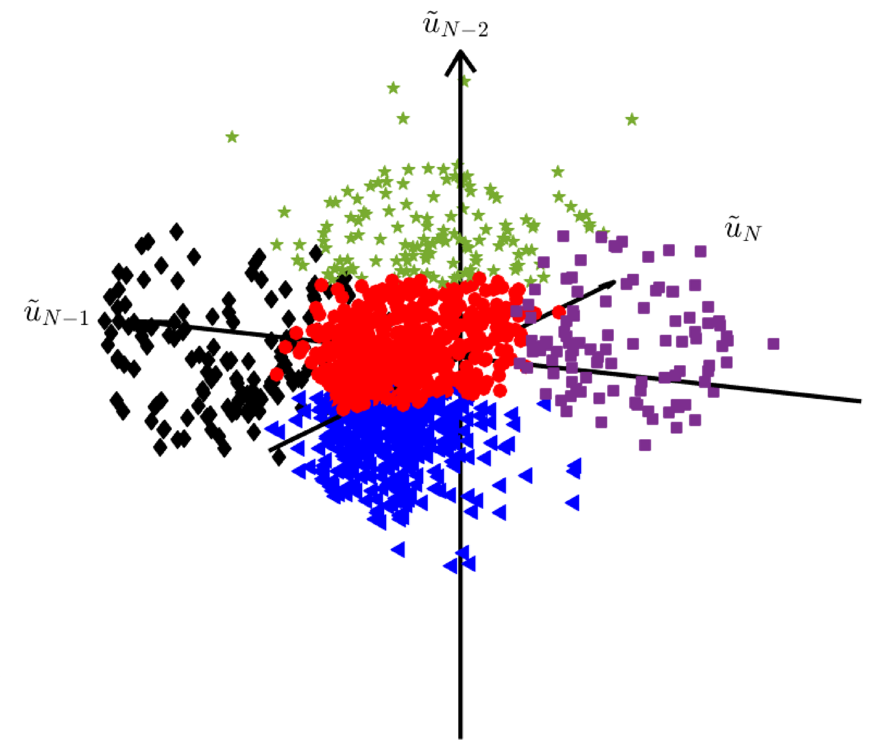
$$d_{ij} = \sum_{m=N-k+2}^{N} (\tilde{u}_{im} - \tilde{u}_{jm})^2 .$$

b. Refine the clustering as follows:
   1. Validate consistency within clusters by silhouette values and reassign observations with negative silhouette values to $(k+1)$-th cluster.
   2. Reassign each observation in $(k+1)$-th cluster separately to all $k+1$ clusters and calculate the silhouette value of that observation for every assignment.
   3. Reassign that observation to the cluster with the maximal silhouette value.
   4. Repeat step 1 for once.
   5. Repeat steps 2–4 until assignments remain the same or iterations reach the maximum.

c. Validate consistency within clusters again.

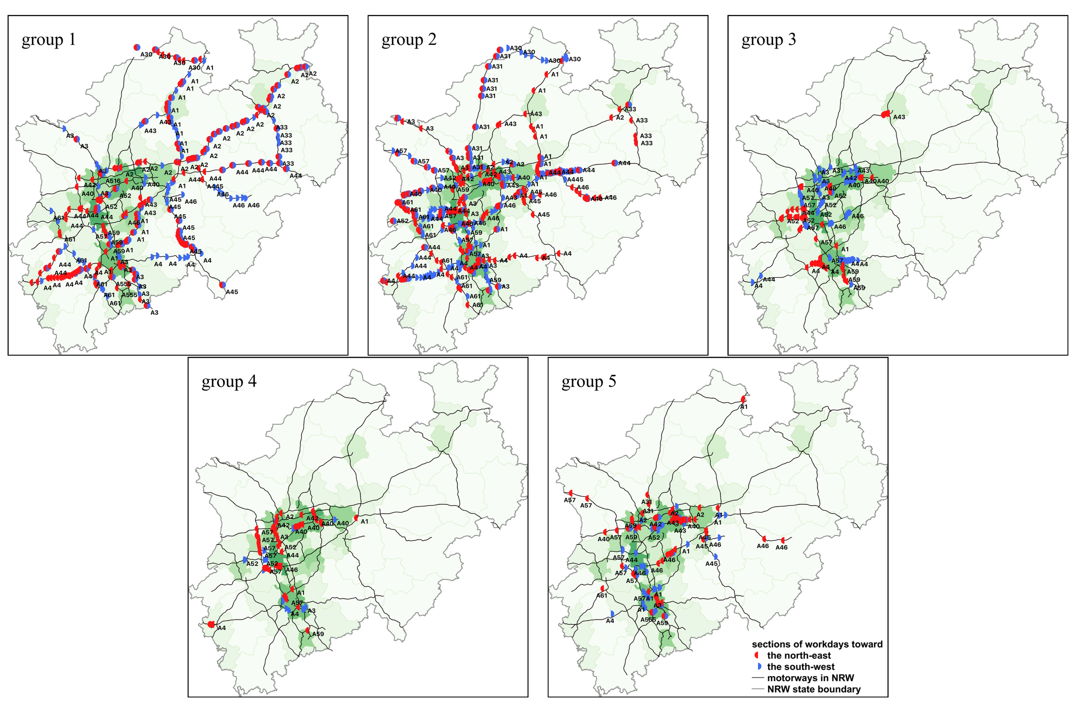
d. Reorder indices of $k+1$ clusters regarding eigenvectors.



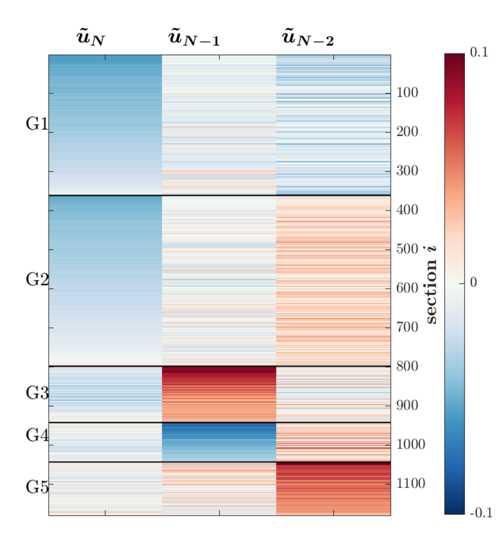


Features of strongly correlated groups

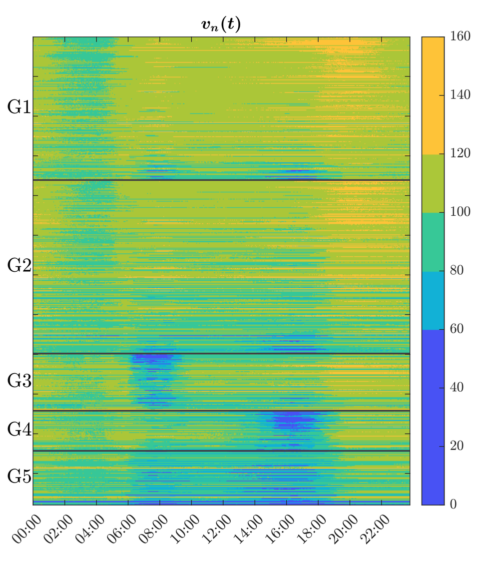
Geographic features



Spectral features



Traffic behavior



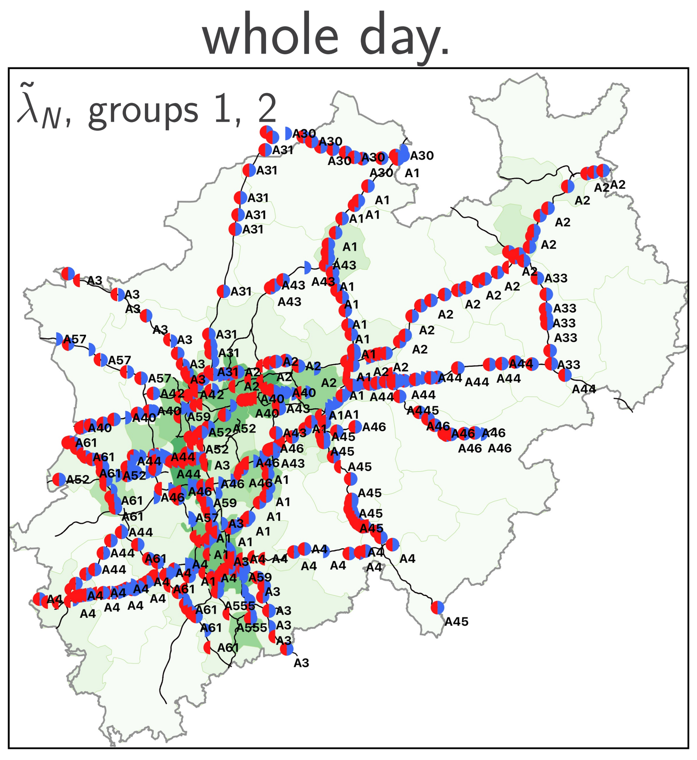
For $k = 4$, clustering results in 5 groups

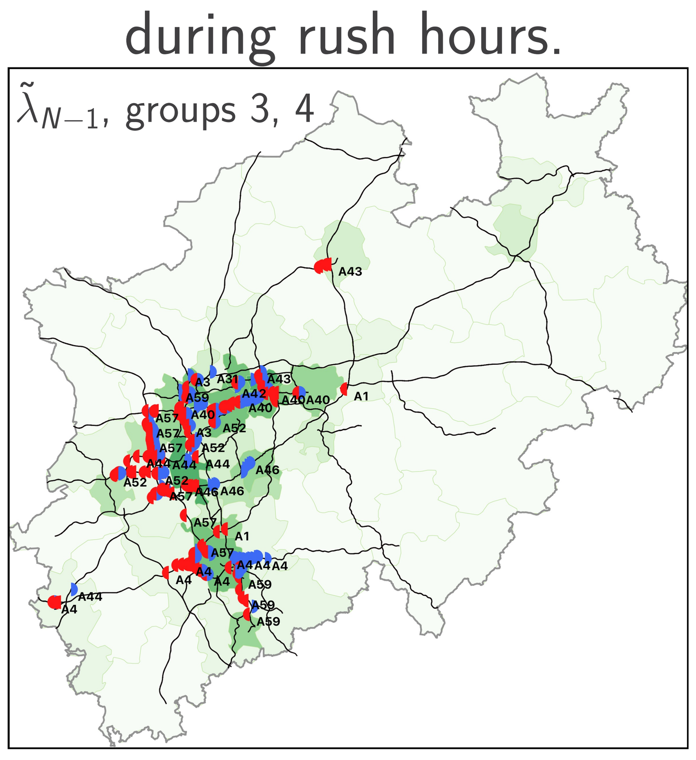
Eigenvalues related to geographic distributions

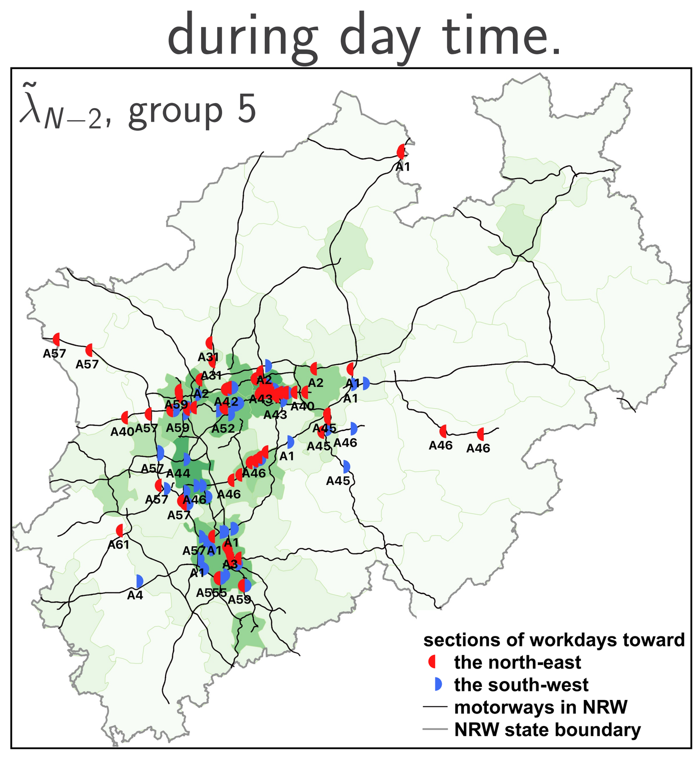
$\tilde{\lambda}_N$ is related to free traffic phases during a whole day.

$\tilde{\lambda}_{N-1}$ is related to congested traffic phases during rush hours.

$\tilde{\lambda}_{N-2}$ is related to slightly congested traffic phases during day time.







Conclusions

- We extracted information from the **virtual network** induced by correlations and mapped it on the **real motorway network**.
- The **developed approach** clearly identified strongly correlated groups.
- Uncovered **subdominant collectivities** provided a new insight into traffic networks.

PAPER

CODE

shanshan.wang@uni-due.de, https://github.com/shannwang