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No 1(a)

(i) Deterministic signal.

It is a signal whose behaviour can be precisely predicted at any point of time or described mathematically with a function.

for example; a sinusoidal signal expressed as;

$$y(t) = A\cos(2\pi ft + \phi)$$ where A - Amplitude, f - frequency, ϕ - phase shift.

(ii) Random process.

It is also referred to as stochastic process that refers to a collection of random variables indexed by time or space. It cannot be predicted with certainty but characterised by probabilistic descriptions.

(iii) Strict sense stationary of a random process.

A random process is said to be strictly stationary if it's statistical properties do not change over time.

That is to say a random process $\{X(t), t \in J\}$ is strict-sense stationary if, for all $t_1, t_2, ..., t_r \in \mathbb{R}$ and all $\Delta \in \mathbb{R}$, the joint CDF of;

$X(t_1), X(t_2), ..., X(t_r)$ is the same as the joint CDF of;

$X(t_1+\Delta), X(t_2+\Delta), ..., X(t_r+\Delta)$.

That is for all real numbers $x_1, x_2 ... x_r$ we have:

$$F_{X(t_1)X(t_2)...X(t_r)}(x_1, x_2, ..., x_r) = F_{X(t_1+\Delta)X(t_2+\Delta)...X(t_r+\Delta)}(x_1, x_2, ..., x_r).$$

(iv) Wide sense stationary of a random process.

A random process is said to be wide sense stationary if it's mean function and it's correlation function do not change by shifts in time.

More precisely: $X(t)$ is wide sense stationary if for all $t_1, t_2 \in \mathbb{R}$ and all $\Delta \in \mathbb{R}$

1. $E[X(t_1)] = E[X(t_2)]$
2. $E[X(t_1)X(t_2)] = E[X_1(t_1+\Delta)X(t_2+\Delta)]$

In the first equation (condition) states that the mean function $\mu X(t)$ is not a function of time t thus can be written as $\mu X(t) = \mu X$

In the second condition, states that the correlation function $R_X(t_1, t_2)$ is only a function of $\tau = t_1 - t_2$, and not t_1 and t_2 individually, thus can be written as;

$R_X(t_1, t_2) = R_X(t_1 - t_2) = R_X(\tau)$.

NO 1(b)

Autocorrelation is a statistical measure used to evaluate the similarity between a signal and a delayed version of itself over various time lags.

The autocorrelation function of a process $X(t)$ is defined as the expectation of the product of two random variables, $X(t_1)$ and $X(t_2)$ obtained by observing the process $X(t)$ at times t_1 and t_2 respectively.

$$R_x(t_1, t_2) = E[X(t_1)X(t_2)]$$

Properties

① The mean-square value of the process may be obtained from $R_x(\tau)$ simply by putting $\tau = 0$ in the equation; $R_x(\tau) = E[X(t+\tau)X(t)]$ for all t

$$\therefore \quad R_x(0) = E[X(t)X(t)]$$

$$\underline{R_x(0) = E(X^2(t))}$$

② The autocorrelation function $R_x(\tau)$ is an even function of τ that is $R_x(\tau) = R_x(-\tau)$

③ The autocorrelation function $R_x(\tau)$ has its maximum magnitude at $\tau = 0$, that is

$$|R_x(\tau)| \leq R_x(0)$$

To prove this consider the non-negative quantity

$$E[(X(t+\tau) \pm X(t))^2] \geq 0 \quad \text{and by expansion}$$

$$E(X^2(t+\tau)) \pm 2E[X(t+\tau)X(t)] + E[X^2(t)] \geq 0$$

with property ① $R_x(0) = E[X^2(t)]$ and $R_x(\tau) = E[X(t+\tau)X(t)]$

$$\therefore \quad 2R_x(0) \pm 2R_x(\tau) \geq 0$$

This implies that $\quad -R_x(0) \leq R_x(\tau) \leq R_x(0)$ #

NO 1 C

$$Y(t) = \int_0^t X(\tau)\,dt \qquad X(t) = A\cos(2\pi f_c t)$$

$$X(\tau) = A\cos(2\pi f_c \tau)$$

$$Y(t) = \int_0^t A\cos 2\pi f_c \tau$$

$$= \frac{A}{2\pi f_c} \sin 2\pi f_c \tau \Big|_0^t$$

$$= \frac{A}{2\pi f_c}(\sin 2\pi f_c t - \sin 0)$$

$$Y(t) = \frac{A \sin 2\pi f_c t}{2\pi f_c}$$

$$E[Y(t)] = E\left[\frac{A \sin 2\pi f_c t}{2\pi f_c}\right]$$

$$E[Y(t)] = \frac{\sin 2\pi f_c t}{2\pi f_c} E[A]$$

but $E[A] = 0 = \mu_A$

$\therefore E[Y(t)] = 0$ which is a constant

for $Var(y(t)) = Var\left(\frac{A \sin 2\pi f_c t}{2\pi f_c}\right)$

$$\cancel{= Var\left(\frac{\sin 2\pi f_c t}{2\pi f_c}\right) Var(A)}$$

$$\cancel{= E\left[\frac{\sin 2\pi f_c t}{2\pi f_c}\right.}$$

$$Var(y(t)) = \frac{\sin^2(2\pi f_c t)}{(2\pi f_c)^2} \cdot Var(A) \qquad \text{from } f(x) = \frac{e^{-\frac{x-\mu}{2\delta^2}}}{\sqrt{2\pi\delta^2}}$$

$$= \frac{\sin^2 2\pi f_c t}{4\pi^2 f_c^2} \delta_A^2$$

$$\cancel{f_Y} \quad f_{Y(t)} = \frac{1}{\sqrt{\frac{2\pi \cdot \sin^2 2\pi f_c t\, \delta_A^2}{4\pi^2 f_c^2}}} \cdot e^{-\left(\frac{y-0}{\frac{2\sin^2 2\pi f_c t\, \delta_A^2}{4\pi^2 f_c^2}}\right)^2}$$

$$= \frac{\sqrt{2\pi} f_c}{\sin 2\pi f_c t \cdot \delta_A} e^{\frac{-2\pi^2 f_c^2 y}{\sin^2 2\pi f_c t}}$$

$$\therefore y(t) \sim N\left\{0, \frac{\delta_A^2 \sin^2 2\pi f_c t}{4\pi^2 f_c}\right\}$$

(ii) Determine if $y(t)$ is stationary or not.

for ~~$\mu_{y(t)} =$~~

$E[y(t)] = \mu_y = 0$ constant

$Var[y(t)] = \frac{\delta_A^2 \sin^2 2\pi f_c t}{4\pi^2 f_c^2}$ which depends on time t.

This shows that the process is not stationary.

(iii)

for a random process to be ergodic, it has to be stationary. from the above (ii) $y(t)$ is not stationary, hence also not ergodic.

NO 2 (a)

A base band signal is the orignal signal, unmodulated data directly from the source with low frequencies that can be transmitted over short distances.

while

A carrier signal is a high frequency signal or periodic waveform that is modulated to carry information over long distances.

NO 2(b)(i) modulation index

$e_c = E_c \sin \omega_c t$ for carrier signal and modulating signal $e_m = E_m \sin \omega_m t$

$e_c = 50 \sin 2\pi \times 10^5 t$ $\quad$ $e_m = 10 \sin 2\pi \times 500 t$

Modulation Index $\mu = \frac{E_m}{E_c} = \frac{10}{50}$

$\mu = \frac{1}{5}$

(ii) Side band frequencies.

$f_{USB} = f_c + f_m$ $\quad$ $f_c = 10^5 Hz = 100 KHz$

$f_{LSB} = f_c - f_m$ $\quad$ $f_m = 500 Hz = 0.5 KHz$

f_{USB} (upper side band) $= f_c + f_m = 100 + 0.5$

$f_{USB} = 100.5 KHz$

f_{LSB} (lower side band) $= f_c - f_m = 100 - 0.5$

$f_{LSB} = 99.5 KHz$

(iii) Amplitude of each side band frequencies.

for upper side band $\Rightarrow A_{USB} = \frac{\mu E_c}{2}$

$A = \frac{\frac{1}{5} \times 50}{2} = 5V$

for lower side band $\Rightarrow A = \frac{\mu E_c}{2} = \frac{\frac{1}{5} \times 50}{2} = 5V$

(iv) Band width required

$BW = 2 f_m$ and $f_m = 500 Hz$

$= 2 \times 500 Hz$

$= 1000 Hz$

(v) Total power delivered by the load of 600 ohms

$$P_{total} = P_c\left(1 + \frac{\mu^2}{2}\right) \quad \text{and} \quad P_c = \frac{E_c^2}{2R}$$

$$P_{total} = \frac{E_c^2}{2R}\left(1 + \frac{\mu^2}{2}\right)$$

$$= \frac{(50)^2}{600}\left(1 + \frac{\left(\frac{1}{5}\right)^2}{2}\right)$$

$$\underline{P_{total} = 2.125\,W}$$

NO 2 (c)

Internal noise is the disturbance that originates within the communication system it's self while External noise originates from outside the communication system and interfere with the transimission of signals.

Internal noise can be of Thermal noise, Shot noise, partition noise, low frequency or flinke noise and high frequency or transmit time noise while external noise can be of Atmospheric noise, Extraterreshrial noise and industrial noise.

Internal noise can affect the clarity and processing of messages while External noise can dirupt transmission and reception of signals

NO 2 (d)(i)

$P_n = 8 \times 10^{-17}\,W$ at $T = 20^\circ c$

$T = 273 + 20 = 293K$

from eqn:

$P_n = KTB$ where K - Boltzmann's constant $[1.38 \times 10^{-23}]$
T - Temperature
B - Bandwidth

finding B of speedn

for $8 \times 10^{-17} = 1.38 \times 10^{-23} \times 293 \times B$

$$\underline{B = 1.9785 \times 10^{4}\,Hz}$$

for $T = 60^\circ c$

$T = 60 + 273 = 333K$

$P_n = 1.38 \times 10^{-23} \times 333 \times 1.9785 \times 10^{4}$

$\underline{P_n = 9.092 \times 10^{-17} \, W}$

No 2 d (ii)

$T = 90^{\circ} K$

$P_n = 1.38 \times 10^{-23} \times 90 \times 1.9785 \times 10^{4}$

$\underline{P_n = 2.4573 \times 10^{-17} \, W}$

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